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ONE CLASS OF OPTIMUM LINEAR EQUALLY SPACED ANTENNA ARRAYS WITH --ETC(U)

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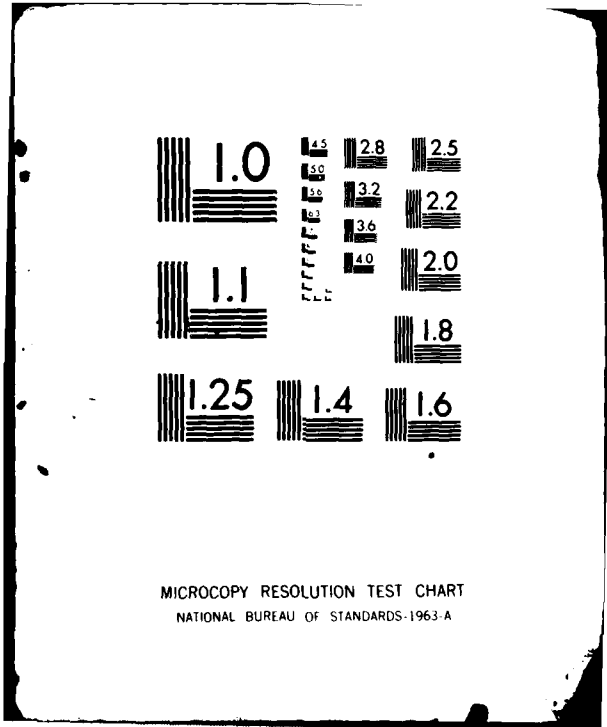
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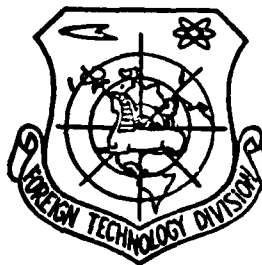
FOREIGN TECHNOLOGY DIVISION



ONE CLASS OF OPTIMUM LINEAR EQUALLY SPACED ANTENNA ARRAYS
WITH SUMMARY OR DIFFERENCE RADIATION PATTERNS

by

B. M. Minkovich



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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<i>А а</i>	A, a	Р р	<i>Р р</i>	R, r
Б б	<i>Б б</i>	B, b	С с	<i>С с</i>	S, s
В в	<i>В в</i>	V, v	Т т	<i>Т т</i>	T, t
Г г	<i>Г г</i>	G, g	У у	<i>У у</i>	U, u
Д д	<i>Д д</i>	D, d	Ф ф	<i>Ф ф</i>	F, f
Е е	<i>Е е</i>	Ye, ye; E, e*	Х х	<i>Х х</i>	Kh, kh
Ж ж	<i>Ж ж</i>	Zh, zh	Ц ц	<i>Ц ц</i>	Ts, ts
З э	<i>З э</i>	Z, z	Ч ч	<i>Ч ч</i>	Ch, ch
И и	<i>И и</i>	I, i	Ш ш	<i>Ш ш</i>	Sh, sh
Й й	<i>Й й</i>	Y, y	Щ щ	<i>Щ щ</i>	Shch, shch
К к	<i>К к</i>	K, k	Ъ ъ	<i>Ъ ъ</i>	"
Л л	<i>Л л</i>	L, l	Ы ы	<i>Ы ы</i>	Y, y
М м	<i>М м</i>	M, m	Ь ь	<i>Ь ь</i>	'
Н н	<i>Н н</i>	N, n	Э э	<i>Э э</i>	E, e
О о	<i>О о</i>	O, o	Ю ю	<i>Ю ю</i>	Yu, yu
П п	<i>П п</i>	P, p	Я я	<i>Я я</i>	Ya, ya

*ye initially, after vowels, and after ъ, ы; e elsewhere.
When written as ë in Russian, transliterate as y \ddot{e} or \ddot{e} .

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh ⁻¹
cos	cos	ch	cosh	arc ch	cosh ⁻¹
tg	tan	th	tanh	arc th	tanh ⁻¹
ctg	cot	cth	coth	arc cth	coth ⁻¹
sec	sec	sch	sech	arc sch	sech ⁻¹
cosec	csc	csch	csch	arc csch	csch ⁻¹

Russian English

rot curl
lg log

EDITED TRANSLATION

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ONE CLASS OF OPTIMUM LINEAR EQUALLY SPACED
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RADIATION PATTERNS

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ONE CLASS OF OPTIMUM LINEAR EQUALLY SPACED ANTENNA ARRAYS WITH
SUMMARY OR DIFFERENCE RADIATION PATTERNS.

B. M. Minkovich.

The optimum pattern $F^0(x)$ minimizes the integral $\int_{-1}^1 |F(x)|^2 (1-x^2)^{-1/2} w(x) dx$.
With a suitable selection of $w(x) \geq 0$ many of the known problems of
optimization coincide with the examined problem. The function $w(x)$
produces a system of orthogonal polynomials $p_n(x)$ through which $F^0(x)$
is expressed and the corresponding excitation currents, through their
coefficients.

INTRODUCTION.

The structure of radiation patterns which are optimum in the
Dol'f-Chebyshev sense often turns out to be unsatisfactory in view of
the constancy, or finally, of the slow drop of the level of side

lobes. This shortcoming led to the appearance of antenna arrays for which the concept of optimality has a different sense. In particular, arrays were constructed which are optimum with respect to the front-to-rear factor [knd] [1, 2], with respect to the minimum noise temperature [3], with respect to the efficiency of the main lobe [4], with respect to an assigned knd with a minimum of radiation power in the side lobes [5, 6], with respect to the maximum reduced curvature of the difference pattern [7], and a number of others.

In the majority of the mentioned works during solution of the stated problems matrix methods are used, or, what is the same thing, methods of systems of linear algebraic equations. These methods, which possess a number of advantages, especially with respect to universality, turn out to be marginally satisfactory when the number of elements of the matrix becomes extremely large.

Recently, for calculation of optimum arrays, L. G. Sodin [6] drew upon polynomials which are orthogonal on a unit circle. Below, it will be shown that in the case of equally spaced arrays with a summary or difference radiation pattern the apparatus of polynomials, which are orthogonal in the integral $[-1, 1]$ with a non-negative weight $w(x)$, may be widely used for controlling the radiation power distribution in space. With an increase in the number of radiators, when the matrix methods become awkward, the use of orthogonal

polynomials, thanks to the presence of asymptotic formulas, provides an advantage both during calculation of excitation currents and of the resulting pattern.

SUMMARY ANTENNA ARRAYS.

A summary radiation pattern with a maximum on the normal to a symmetrically excited in-phase array may be written in the form of a polynomial - expansion in Chebyshev polynomials:

$$(1) \quad F_{MS}(x) = I_0(1 - \delta) + 2 \sum_{n=1}^N I_n T_{2n-1}(x).$$

where $I_n = I_{-n}$ - excitation currents,

$$\begin{aligned} x &= \cos Du = \cos \theta, \\ D &= nd/\lambda, \\ n &= \cos \theta, \end{aligned}$$

θ - angle read from the axis of the array,

$T_m(x)$ - Chebyshev polynomial of the first type,

d - distance between elements of the array,

λ - wavelength in free space,

δ - here, and subsequently, a number assuming two values: 0 and 1 - respectively for arrays of an odd and even number of elements, $M+1=2N-\delta+1$ - number of elements of the array.

Let us examine only the case when $D \leq \pi/2$ ($d \leq 0.5\lambda$) and variable x , depending on D , runs through the interval

$$x \in [-1, -a] + [a, 1], \text{ where } a = \cos D.$$

Let us introduce the following quadratic functional into the examination

$$(2) \quad P_s = \int_{-1}^1 |F_{Ms}(x)|^2 \frac{w(x)}{\sqrt{1-x^2}} dx,$$

in which case it is demanded of the weight function $w(x)$ that it be non-negative and even.

With the selected D and M we shall consider as optimum the summary array in which, with a given value of the functional $P_s=1$, the pattern $F_{Ms}(x)$ acquires the maximum of possible values in the direction $x_0=1$ ($\theta_0=\pi/2$).

Arrays, optimum for the direction $x_0 \neq 1$, may be obtained from the optimum array for the direction $x_0=1$ by introduction into the current distribution of a linear phase shift $\phi = Du_0$ [2, 6] and the weight

$w(x)$, in the general case, may be taken in the following form:

$$(3) \quad w(x) = \begin{cases} W(x) & x \in [-b, -a] + [a, b], \\ 0 & x \in [-1, -b] + (-a, a) + (b, 1) \end{cases} \quad 0 \leq a \leq b < 1.$$

An adequate selection of $W(x)$ and b makes it possible to obtain, as particular cases, the problems of optimization mentioned in the introduction. However, by selecting the weight $W(x)$ so that it changes along the radiated interval it is possible to accomplish more complex regulation, ascribing a different value to the power radiated in different directions. In directions where $W(x)$ has an increased value, the level of radiation decreases. Let us note that when the weight $w(x)$ is natural, i.e., $b=1$ and $W(x)=1$, then we obtain an array with the maximum knd.

Let us solve the problem of optimization: let us find the optimum (in the sense indicated above) radiation pattern $F_{MS}^0(x)$ and the corresponding excitation currents I_n^0 .

In principle the solution of the problem is not complex. In fact, let a system of polynomials $\{p(x)\}$ be known which are orthogonal in the interval $[-1, 1]$ with the weight

$$(4) \quad w_0(x) = w(x) (1-x^2)^{-1/2},$$

in which case, thanks to the parity of $w_0(x)$, the parity of the polynomial $p_n(x)$ coincides with the parity of its main term x^n . Then the random pattern $F_{MS}(x)$ may be represented in the form of a sum of polynomials of parity identical with the parity $M=2N-6$:

$$(5) \quad F_{MS}(x) = \sum_{\substack{n=0 \\ n=2 \\ n=M}}^M 'a_n p_n(x),$$

where the prime indicates that summation proceeds on indices of parity M . Functional (2) in light of the orthogonality of polynomials $p_n(x)$ is equal to

$$P_S = \sum_{\substack{n=0 \\ n=2 \\ n=M}}^M |a_n|^2 h_n,$$

where

$$(6) \quad h_n = \int_{-1}^1 |p_n(x)|^2 \frac{w(x)}{\sqrt{1-x^2}} dx.$$

It is easy to establish that $F_{MS}(1) = \max$ with $P_S = 1$, when a_n with an accuracy up to a constant is equal to

$$(7) \quad a_n^* = p_n(1)/h_n.$$

Consequently, the optimum pattern

$$F_{MS}(x) = \sum_{n=0}^M h_n^{-1} \rho_n(1) \rho_n(x) = 1/2 \left\{ \sum_{n=0}^M h_n^{-1} \rho_n(1) \rho_n(x) + \right. \\ \left. + (-1)^n \sum_{n=0}^M h_n^{-1} \rho_n(-1) \rho_n(x) \right\}.$$

Using the Kristoffel'-Darbu formula [8, 9] which is valid for orthogonal polynomials, we obtain a final expression for the optimum pattern

$$(8) \quad F_{MS}(x) = \frac{\kappa_M}{h_M \kappa_{M+1}} \frac{x \rho_M(1) \rho_{M+1}(x) - \rho_{M+1}(1) \rho_M(x)}{x^2 - 1},$$

where k_n - coefficient $p_n(x)$ with the main term x^n .

Using Horner's diagram we may write the polynomial $F_{MS}^0(x)$ in the explicit form:

$$(9) \quad F_{MS}^0(x) = \sum_{m=0}^N b_{2m-1}^M x^{2m-1}.$$

Then the excitation currents I_n^0 as coefficients of expansion into a series on Chebyshev polynomials, are equal to:

$$(10) \quad I_n^0 = \frac{1}{\pi} \sum_{m=0}^N b_{2m-1}^M \int_{-1}^1 x^{2m-1} \frac{T_{2n-1}(x)}{\sqrt{1-x^2}} dx = \\ = \sum_{m=0}^N b_{2m-1}^M C_{2m-1}^{2n-1} 2^{-2m+1}.$$

The structure (10) is similar to the structure of the analogous

formula for calculating currents of Dol'f-Chebyshev arrays [10], however, the absence of factors representing numbers close to one in a high power, significantly simplifies calculations.

In principle the problem of calculation of an optimum array is solved: the pattern is equal to (8), currents - (10). Actually, however, for calculation of an optimum array we need to construct polynomials which are orthogonal in $[-1, 1]$ with a weight $w_0(x)$.

INFORMATION ON ORTHOGONAL POLYNOMIALS.

The theory of orthogonal polynomials is presented in detail in [8, 9]. Here only some of the general properties of orthogonal polynomials are presented.

The weight negative function $w(x)$ in section $[a_1, b_1]$ uniquely defines a system of orthogonal polynomials $\{p_n(x)\}$ with an accuracy up to a constant factor for each polynomial. Moments of the weight function

$$C_n = \int_{a_1}^{b_1} w(x) x^n dx$$

make it possible to write the orthogonal polynomial in the form of a determinant

$$p_n(x) = \frac{\kappa_n}{G_{n-1}} \begin{vmatrix} c_0 & c_1 & \dots & c_n \\ c_1 & c_2 & \dots & c_{n+1} \\ \dots & \dots & \dots & \dots \\ c_{n-1} & c_n & \dots & c_{2n-1} \\ 1 & x & \dots & x^n \end{vmatrix}; \quad G_n = \begin{vmatrix} c_0 c_1 & \dots & c_n \\ c_1 c_2 & \dots & c_{n+1} \\ \dots & \dots & \dots \\ c_n c_{n+1} & \dots & c_{2n} \end{vmatrix}.$$

Three sequential polynomials are connected by the linear recurrent relationship

$$p_{n+1}(x) = (A_n x + B_n) p_n(x) - C_n p_{n-1}(x) \quad n = 1, 2, 3 \dots$$

Thus, when an analytical expression for the polynomials $p_n(x)$ is absent the optimum pattern (8) is represented in the form of a sum of determinants. However, even in this case the calculation of currents (10) may turn out to be relatively simpler than the calculation of the inverse matrix. In the same case when the analytical form of $p_n(x)$ is known not only is the calculation of currents considerably simplified, but the type of optimum diagram becomes known ahead of time, before the calculation of currents, in a closed form.

Below it is shown how to use polynomials in which analytical expressions are known.

CLASSICAL POLYNOMIALS.

Let us set $a=0$; $b=1$ and weight

$$(11) \quad w(x) = (1-x^2)^\lambda, \quad x \in [-1, 1] \quad \lambda \geq 0.$$

Such a selection corresponds to an antenna array with a distance between elements of $d=0.5\lambda$, in which, by selection of the Jacobian weight (11) it is possible to regulate the level of side lobes, inasmuch as with $\lambda > 0$ the structure of the weight is such that the value of side lobes is increased. Weight $w_0(x) = (1-x^2)^{\lambda-1/2}$ and interval $[-1, 1]$ make it possible to establish directly [8, 9] that the corresponding orthogonal polynomials are Gegenbauer polynomials or ultraspherical polynomials

$$(12) \quad P_{M+1}(x) = C_{M+1}^{\lambda}(x) = \sum_{m=0}^M a_{2m+1}^{M+1} x^{2m+1},$$

where

$$(13) \quad a_{2m+1}^{M+1} = (-1)^{N-m} \frac{\Gamma(N-m+\lambda+1-\delta) 2^{2m+1-\delta}}{(N-m)! (2m+1-\delta)! \Gamma(\lambda)}.$$

With $\lambda=0$, 1 , and $1/2$ the Gegenbauer polynomials coincide

respectively with Chebyshev polynomials (of the 1-st and 2-nd types) and Legendre polynomials. Using the formula of differentiation of Gegenbauer polynomials

$$(1-x)^2 \frac{d}{dx} C_n^\lambda(x) = -nx C_n^\lambda(x) + (n+2\lambda-1) C_{n-1}^\lambda(x)$$

and the condition of standardization

$$C_n^\lambda(1) = \frac{\Gamma(n+2\lambda)}{n! \Gamma(2\lambda)},$$

we find that the optimum pattern $F_{MS}^0(x)$ is equal to the derivative of $C_n^\lambda(x)$:

$$(14) \quad F_{MS}^0(x) = A_{MS} \frac{d}{dx} \{C_{M+1}^\lambda(x)\},$$

in which case

$$A_{MS} = \frac{\Gamma(\lambda) \Gamma(M+2\lambda)}{2 \Gamma(\lambda+1/2) \Gamma(2\lambda) (M+1)!}, \quad F_{MS}^0(1) = A_{MS} \times \frac{\Gamma(M+2\lambda+2)}{2(\lambda+1/2) \Gamma(2\lambda) M!}.$$

With $\lambda=0$ and optimization of kind we obtain

$$(15) \quad F_{MS}^0(x) = \frac{1}{2\pi(M+1)} \frac{d}{dx} \{\cos(M+1) \arccos x\} = \frac{1}{2\pi} \frac{\sin(M+1)\theta}{\sin \theta},$$

where

$$x = \cos \theta.$$

Let us note that the case $\lambda=0$ is special and in (13) and (14) with $\lambda=0$ $\Gamma(\lambda)$ must be replaced by a constant.

It is easy to interpret relationship (14). Extremal orthogonal polynomials, in particular Gegenbauer polynomials, having all zeros in the examined interval, increase abruptly on the edges of the interval, and the area of abrupt growth goes to the formation of a narrow main lobe.

Thanks to this it is possible, irrespective of optimization, to obtain "good" radiation patterns, formally applying relationship (14) and also (8) to non-orthogonal (with the given weight) but extremal polynomials, for example, of the type of Chebyshev-Akhiezer polynomials [11].

From (13) and (14) it is not difficult to find b_{2m-1}^{2N-1} and the excitation currents (10) of the optimum summary array. Practically, however, the currents are more simply calculated using a trigonometric representation of Gegenbauer polynomials:

$$C_n^\lambda(\cos \theta) = \sum_{m=0}^n \frac{(\lambda)_m (\lambda)_{n-m}}{m! (n-m)!} \cos[(n-2m)\theta],$$

where

$$(\lambda)_m = \Gamma(m+\lambda)/\Gamma(\lambda).$$

Then the optimum pattern

$$(16) \quad F_{MS}^o(\cos \theta) = A_{MS} \sum_{m=0}^{M+1} \frac{(\lambda)_m (\lambda)_{M+1-m}}{m! (M+1-m)!} (M+1-2m) \frac{\sin[(M+1-2m)\theta]}{\sin \theta}$$

Here each of the terms corresponds to a uniform distribution along $(M+1-2m)$ central elements of the array. Thanks to this the excitation currents satisfy the simple recurrent relationship

$$(17) \quad I_{N-n}^o = I_{N-n+1}^o + A_{MS} \frac{(\lambda)_n (\lambda)_{M+1-n}}{n! (M+1-n)!} (M+1-2n) \quad n=0, 1, \dots, N-\delta,$$

in which case I_N^o is the current of the final element, $I_{N+1}^o=0$. Calculation is elementary and is actually reduced to the writing out of expression (16).

Explicit expressions for the optimum pattern (14) and (16) and for excitation currents (17) are obtained. However, this is not the only merit of the examined method of calculation of antenna arrays. It was noted above that it has certain advantages when the number of elements of the array is large and the use of matrix methods encounters certain difficulties. Then it is possible to use asymptotic relationships for polynomials. Since we are interested in

the asymptotics, including the point $\theta=0$ ($x=1$), let us use Khil's formula for Gegenbauer polynomials [8, p. 205].

$$(18) \quad C_n^{\lambda}(\cos \theta) \simeq \frac{\Gamma(\lambda + 1/2) x^{-1/2} \Gamma(n + 2\lambda)}{(n + \lambda)^{1/2} n! \Gamma(2\lambda)} \left(\frac{\theta}{\sin \theta}\right)^{1/2} \frac{J_{\lambda-1/2}[(n + \lambda)\theta]}{(\sin \theta)^{\lambda-1/2}},$$

which makes it possible for large M to write the optimum pattern through known functions:

$$(19) \quad F_{MS}^{\lambda}(\cos \theta) \simeq \frac{2\lambda + 1}{M + 1 + 2\lambda} F_{MS}^{\lambda}(1) \left(\frac{\theta}{\sin \theta}\right)^{1/2} f_{MS}^{\lambda}(\theta),$$

where

$$(20) \quad f_{MS}^{\lambda}(\theta) = \frac{\Lambda_{\lambda-1/2}[(M + \lambda)\theta] - \cos \theta \Lambda_{\lambda-1/2}[(M + 1 + \lambda)\theta]}{\sin^2 \theta};$$

$$f_{MS}^{\lambda}(0) = \frac{M + 2\lambda + 1}{2\lambda + 1};$$

$\Lambda_{\nu}(x)$ - lambda function [12].

With small $\theta \ll 1$

$$(21) \quad f_{MS}^{\lambda}(\theta) \simeq \frac{1}{2} \left(\frac{\theta}{\sin \theta}\right)^2 \left\{ \frac{[2(M + \lambda + 1/2) + 1/2] \cos^2 \frac{\theta}{2} - 1}{2\lambda + 1} \Lambda_{\lambda+1/2}(x_1) + \right.$$

$$\left. + \left(\frac{\sin \theta}{\theta}\right)^2 \Lambda_{\lambda-1/2}(x_1) + \frac{x_1^2 \sin^2 \frac{\theta}{2}}{(2\lambda + 1)(2\lambda + 3)} \Lambda_{\lambda+3/2}(x_1) \right\},$$

where

(22) $x_1 = (M + \lambda + 1/2)\theta.$

When $\theta \ll 1$ and $M \gg 1$, then

(23)
$$I_{ms}(\theta) \approx \frac{M + \lambda}{2\lambda + 1} \left(\frac{\theta}{\sin \theta}\right)^2 \Lambda_{\lambda+1/2}(x_1) \approx \frac{M + \lambda}{2\lambda + 1} \Lambda_{\lambda+1/2}(x_1).$$

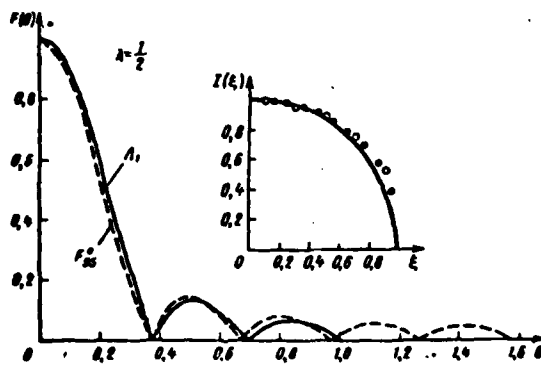


Fig. 1.

Consequently, in the first approximation the envelope of the current distribution is described by the law

(24)
$$I_{\epsilon} \approx (1 - \epsilon^2)^{\lambda}.$$

where normalization of the coordinate of the radiator position

relative to the center of the array

$$(25) \quad \xi_n = \frac{2n-\delta}{2N-\delta+1} \quad n = \delta, 1, 2, \dots, N.$$

It is obvious [8, p. 205, 251] that (23) and (24) not only satisfactorily describe the radiation pattern and the current distribution of an optimum array with a large number of radiators $M+1 \gg 1$, but also make it possible to obtain a concept of the character of the pattern and of the current distribution, when the number of radiators is small. This is confirmed by Fig. 1 where an example is given of calculation of patterns $(16) \Lambda_1(x_1)$ and of corresponding currents with $\lambda=1/2$ and $M+1=10$. Fig. 1 also shows the current distribution for $M+1=20$. The coincidence is very good; $(\theta/\sin\theta)^2$ in (23) may only improve it.

Let us note in conclusion that there is also a possibility of using a generalization of classical polynomials, Pollachek's polynomials [8, 9], for calculation of arrays.

CLASS OF POLYNOMIALS EXAMINED BY S. N. BERNSHTEYN AND G. SEGYE.

Let us examine the application of another class of polynomials for which expressions may be given in an explicit form.

First, the polynomials themselves. Orthogonal polynomials corresponding to weight functions

$$(26) \quad w_\alpha(x) = (1-x^2)^\alpha [\rho(x)]^{-1} \quad \alpha = \mp 1/2,$$

where $\rho(x)$ - positive in the section $[-1, 1]$ polynomial of the precise power l , may be represented in the following manner:

$$(27) \quad p_n(\cos \theta) = \sqrt{\frac{2}{\pi}} [C(\theta) \cos n\theta + S(\theta) \sin n(\theta)], \quad l < 2n; \alpha = -\frac{1}{2};$$

$$(28) \quad p_n(\cos \theta) = \sqrt{\frac{2}{\pi}} \left\{ C(\theta) \frac{\sin(n+1)\theta}{\sin \theta} - S(\theta) \frac{\cos(n+1)\theta}{\sin \theta} \right\},$$

$$l < 2(n+1); \alpha = 1/2.$$

Real functions $C(\theta)$ and $S(\theta)$ are expressed through so-called normalized representation [8, 13] of the trigonometric polynomial

$$(29) \quad \rho(\cos \theta) = |h(z)|^2 \quad z = e^{i\theta}.$$

For the normalized representation the polynomial $h(z)$ satisfies two additional conditions: $h(z)$ does not approach zero in the circle $|z| < 1$ and $h(0)$ - real and positive, thanks to which the representation of the form (29) is unique.

With $h(z)$ are connected the functions $C(\theta)$ and $S(\theta)$:

$$(30) \quad h(z) = C(\theta) + iS(\theta) \quad z = e^{i\theta}.$$

When

$$\rho(\cos \theta) = a_0 + 2 \sum_{\kappa=0}^l a_{\kappa} \cos \kappa \theta,$$

then

$$h(z) = \sum_{\kappa=0}^l d_{\kappa} z^{\kappa},$$

in which case coefficients d_{κ} are real and may be found from the system of equations

$$(31) \quad a_{\kappa} = d_{\kappa} d_{\kappa} + d_1 d_{\kappa+1} + \dots + d_{l-\kappa} d_l \quad \kappa = 0, 1, \dots, l.$$

Polynomial $\rho(x)$ may be used as a supplementary means of regulating the power distribution with respect to angles, for example for suppression of the level of fringe radiation in a given sector.

Let us pause briefly on the application of polynomials of the form (27). Inasmuch as in the examined case the polynomial $\rho(x)$ is even, of the power $l=2L$, we introduce functions $C(\theta)$ and $S(\theta)$ in the following form:

$$(32) \quad C(\theta) = \sum_{n=0}^L d_{2n} \cos 2n\theta;$$

$$(33) \quad S(\theta) = \sum_{n=1}^L d_{2n} \sin 2n\theta;$$

where d_{2n} - real numbers.

Then it is not difficult to find that polynomial (27)

$$(34) \quad P_n(\cos \theta) = \sqrt{\frac{2}{\pi}} \sum_{k=0}^L d_{2k} \cos(n - 2k)\theta$$

and according to (8) with an accuracy to a constant

$$(35) \quad F_{MS}^*(\cos \theta) = \sum_{k=0}^L d_{2k} \frac{[\sin(M+1-2k)\theta]}{\sin \theta}.$$

To each term in (35) there corresponds a constant amplitude distribution along $(M+1-2k)$ central elements of the array. Such a form of excitation of the array [14], when the larger part of it remains excited uniformly, with the exception of $2L$ end elements from each side, turns out to be quite convenient from the standpoint of constructing a feeder system which makes it possible to reduce the fringe radiation without a noticeable reduction of the gain. In [14] it was necessary to apply certain connections to the excitation currents in order to obtain the optimum, mathematically convenient,

procedure for regulating the fringe radiation of the antenna array.

The use of orthogonal polynomials of the form (27) provides the new possibility of optimum suppression or more complex regulation of fringe radiation of the array with $d=0.5\lambda$ by means of selection of the polynomial $\rho(x)$, and moreover with no limitations on currents. The number of terms in $C(\theta)$, not equal to zero ($d_{2k}=0; k=0, 1, \dots$), is equal to the number of discrete terms in the amplitude distribution. The representation of $C(\theta)$ and $S(\theta)$ in the form (32) and (33) corresponds to the random even weight polynomial

$$(36) \quad \rho(x) = a_0 + 2 \sum_{k=1}^L a_{2k} \cos 2k\theta = a_0 + 2 \sum_{k=1}^L a_{2k} T_{2k}(x),$$

where d_{2k} and a_{2k} are connected by relationship (31).

To polynomials (28) there corresponds a more complex structure of excitation currents, the envelope of which is equal to $1-\xi^2$. However, when $C(\theta)$ and $S(\theta)$ have the form (32) and (33), only currents of $2L$ end elements of the array are also subject to changes.

DIFFERENCE ANTENNA ARRAYS.

Let us show that the majority of results obtained for summary arrays carry over directly to difference arrays.

A difference radiation pattern with the "main" zero along the normal to an antisymmetrically excited array ($I_n = -I_{-n}$; $I_0 = 0$) may be represented in the form of a sum

$$(1') \quad F_{MD}(x) = 2 \sum I_n U_{2n-1}(x) = \sqrt{1-x^2} \Pi_{M-1}(x),$$

where

the polynomial of the degree $M-1$ $\Pi_{M-1}(x) = 2 \sum_{n=1}^M I_n Q_{2n-1}(x)$, $U_m(x) = \sin m \arccos x$, is a Chebyshev polynomial of the 2nd type

$$Q_{m-1}(x) = \frac{U_m(x)}{\sqrt{1-x^2}}.$$

For a difference array the basic parameter is curvature of the pattern in the "main" zero, located between two opposite-phase main lobes:

$$(37) \quad s(x_0) = \left. \frac{dF_{MD}(x)}{du} \right|_{u=x_0} = -D \sqrt{1-x^2} \left. \frac{dF_{MD}(x)}{dx} \right|_{x=x_0}.$$

As in the case of summary arrays, we introduce the quadratic functional

$$(2') \quad P_D = \int_{-1}^1 |F_{MD}(x)|^2 \frac{w(x)}{\sqrt{1-x^2}} dx = \int_{-1}^1 |\Pi_{M-1}(x)|^2 w(x) \sqrt{1-x^2} dx.$$

With selected D and M we shall consider as optimum the difference array in which with an assigned value of the functional $P_n=1$ the curvature $s(x_0)$ acquires the maximum of the possible values in the direction $x_0=1$ ($\vartheta_0 = \pi/2$). The linear phase shift makes it possible to obtain arrays which are optimum with $x_0 \neq 1$.

Let us represent the difference pattern $F_{MD}(x)$ in the form of a sum of polynomials, orthogonal on $[-1, 1]$ with respect to weight:

$$(4') \quad w_1(x) = \sqrt{1-x^2} w(x),$$

$$(5') \quad F_{MD}(x) = \sqrt{1-x^2} \sum_{n=0}^{M-1} a_n p_n(x).$$

According to (34) the curvature of the diagram $F_{MD}(x)$ in point $x_0=1$

$$s(1) = D \sum_{\substack{n=0 \\ \Delta n=2}}^{M-1} a_n p_n(1).$$

Thanks to the selection of the variable in the form $x = \cos Du$, the problem of optimization of the difference pattern is completely analogous to the problem of optimization of a summary pattern. Here also $a_n^0 = p_n(1)/h_n$ and the optimum difference pattern

$$(8') \quad F_{MD}^0(x) = \sqrt{1-x^2} \frac{\kappa_{M-1}}{\kappa_M h_{M-1}} \frac{x P_{M-1}(x) P_M(x) - P_M(1) P_{M-1}(x)}{x^2 - 1}$$

The difference between $F_{MS}^0(x)$ and $F_{MD}^0(x)$ with an identical number of array elements is in the factor $\sqrt{1-x^2}$ and in the decrease of the subscripts by one for $F_{MD}^0(x)$ in comparison with $F_{MS}^0(x)$. For a difference pattern polynomials are used which are orthogonal with respect to weight $w_1(x) = \sqrt{1-x^2}\omega(x)$, for a summary pattern - with respect to weight $w_0(x) = (1-x^2)^{-1/2}\omega(x)$. However, when the weight $w(x)$ contains a Jacobian factor of the form $(1-x^2)^\lambda$, then the difference between weights $w_1(x)$ and $w_0(x)$ is not fundamental and leads only to an increase in the parameter λ by one in the difference case in comparison with the summary case. In particular, the natural weight of the difference pattern corresponds to $\lambda=1$, and not $\lambda=0$, as for the summary pattern. Inasmuch as structurally formulas (8) and (8') differ only by a simple factor $\sqrt{1-x^2} = \sin\theta$, then it is not difficult to write directly many of the results for difference arrays, with which it is easy to find the analogy in the summary case.

When $F_{MD}^0(x)$ is written in the polynomial form

$$(9') \quad F_{MD}^0(x) = \sqrt{1-x^2} \sum_{m=1}^N b_{2m-1}^{M-1} x^{2m-1}$$

then the excitation currents

$$I_n^0 = \frac{1}{\pi} \sum_{m=1}^N b_{2m-1}^{M-1} \int_{-1}^1 x^{2m-1} U_{2m-1}(x) dx.$$

The integral on the right is expressed through table [15]:

$$\int_0^{\pi} \cos^p x \cos qx dx = [1 + (-1)^{p+q}] \frac{\pi}{2^{p+1}} \frac{\Gamma(p+1)}{\Gamma\left(\frac{p+q}{2}+1\right) \Gamma\left(\frac{p-q}{2}+1\right)},$$

and finally we obtain

$$(10^*) \quad I_n^0 = (2n - \delta) \sum_{m=n}^N b_{2m-1}^{M-1} \frac{C_{2m-1}^{n-n}}{(2m - \delta) 2^{2m-1}}.$$

Let us turn our attention to the factor in front of the sum $(2n - \delta)$, which shows that the optimum current distributions for difference arrays are to a certain degree a deformation of the linear amplitude distribution - optimum with $w(x) = 1$; $a = 0$; $b = 1$.

Let us cite, without derivation, only some of the relationships for calculation of difference arrays using classical polynomials

$$(14^*) \quad F_{MD}^0(x) = A_{MD} \sqrt{1-x^2} \frac{d}{dx} [C_M^{\lambda}(x)],$$

where

$$A_{MD} = \frac{M+1}{M+2\lambda+1} A_{MS}.$$

When $\lambda=1$ and we subject to optimization the curvature in zero with an assigned radiation power, we obtain the known result:

$$(15^*) \quad F_{MD}^2(x) = \sqrt{1-x^2} \frac{d}{dx} \left\{ \frac{\sin(M+1) \arccos x^2}{\sqrt{1-x^2}} \right\} = \\ = \frac{(M+1) \sin \theta \cos(M+1) \theta - \cos \theta \sin(M+1) \theta}{\sin^3 \theta}$$

$$(16^*) \quad F_{MD}^2(\cos \theta) = A_{MD} \sum_{m=0}^M \frac{(\lambda)_m (\lambda)_{M-m}}{m! (M-m)!} (M-2m) \sin[(M-2m)\theta],$$

$$(17^*) \quad I_{N-m}^0 = \frac{A_{MD}}{2} \frac{(\lambda)_m (\lambda)_{M-m}}{m! (M-m)!} (M-2m) \quad m = 0, 1, \dots, N-1.$$

Let us note that currents (17^{*}) with an accuracy to a constant are equal to coefficients of an optimum pattern (16^{*}):

$$(23^*) \quad I_{MD}^0(x_2) \approx (2\lambda+1)^{-1} x_2 \Lambda_{\lambda+1/2}(x),$$

$$(22^*) \quad x_2 = (M+\lambda-1/2)\theta,$$

$$(24^*) \quad I_0^0 \approx I_n (1 - \xi_n^2)^{-1}.$$

Formulas, the numbers of which contain the prime "0" are analogs of formulas under the same numbers for summary arrays. As we see, the analogy between summary and difference patterns is quite extensive.

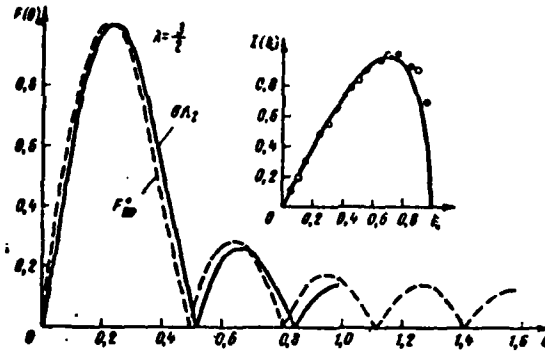


Fig. 2.

Fig. 2 shows an example of calculation of patterns (16') and (23') and of corresponding currents with $\lambda=3/2$ and $M+1=10$. The current distribution is also given for $M+1=20$. Coincidence of precise and asymptotic curves is not bad for the difference case.

CONCLUSION.

The article describes a method for calculating equally spaced antenna arrays ($d < 0.5\lambda$) with continuous summary $F_S(x)$ or difference $F_D(x)$ radiation patterns. With an assigned value $F_S(x)$ in the direction of the main maximum or curvature $F_D(x)$ in the direction of

the "main" zero the optimum patterns ensure the minimum of integral $\int_{-1}^1 |F(x)|^2 (1-x^2)^{-1/2} w(x) dx$. The weight function $w(x) \geq 0$ not only determines the type of problem of optimization (for example, optimization of kind or the power of side lobes, or regulation of fringe radiation of another, more complex character) but also produces a system of polynomials $\{p_n(x)\}$ which are orthogonal with the weight $w(x) (1-x^2)^{-1/2}$ in the section $[-1, 1]$. Through these polynomials $p_n(x)$ we write the optimum patterns and through their coefficients, the current distribution. Inasmuch as the theory of orthogonal polynomials is most completely developed for $w(x)$, which do not identically approach zero in the entire section of the interval $[-1, 1]$, the most complete results are obtained through calculation of arrays with $d=0.5\lambda$, for which the generalized angular coordinate x runs through the entire interval $[-1, 1]$. In this case the polynomials, for which explicit expressions are known, can be used extensively. In particular for the Jacobian weight $w(x) = (1-x^2)^{\lambda}$ the optimum patterns are proportional to the derivative of Gegenbauer polynomials. And here not only are explicit expressions obtained for optimum diagrams and current distribution, but also their asymptotic representations with an increase in the number of elements of the array, in which case these results, to an identical degree, apply both to summary and to difference radiation patterns. The theory of orthogonal polynomials provides a new approach to arrays with partially decaying amplitude distributions for which the current

distribution in the center is uniform. Thus at the present time the theory of orthogonal polynomials may be effectively applied to arrays with a distance between elements equal to the half-wave.

The asymptotic relationships for optimum solutions, when $d < 0.5 \lambda$ and in general when $w(x)$ approaches zero identically on the entire section or section of the interval $[-1, 1]$ require asymptotic relationships for corresponding, orthogonal polynomials, the theory of which, unfortunately, has been little developed. Here we may point to the works of N. I. Akhizer (see, for example, [16]).

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