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DETERMINING CRACK TIP FIELD PARAMETERS  
FOR ELASTIC-PLASTIC MATERIALS VIA AN  
ESTIMATION SCHEME

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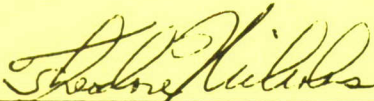
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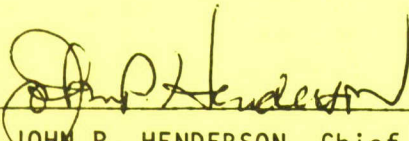
This technical report has been reviewed and is approved for publication.



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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  This report reviews the theoretical basis of an estimation scheme which appeared in the literature. A computer program was written for estimating the elastic-plastic fracture mechanics parameters for five different crack geometries. The parameters evaluated are J-integral, load-line displacement, and crack mouth opening displacement. Included in the report are the program listing and an example of an input and output.		

## FOREWORD

This technical report was prepared by the Aerospace Mechanics Division of the University of Dayton Research Institute. This study was conducted by the authors from November 1979 to November 1980 as a part of the USAF Contract F33615-78-C-5184 with the Air Force Wright Aeronautical Laboratory/Materials Laboratory. The contract, which was initiated under Project No. 2418, Task 24180306, was administered under the direction of the Air Force Materials Laboratory. Dr. Theodore Nicholas of the Metals and Ceramics Division of the Materials Laboratory was the Project Monitor for this study of the application of the Nonlinear Fracture Mechanics (NLFM) parameters to the study of Fatigue Crack Growth.

The authors wish to express their appreciation to Ms. Carol Bruner and Mr. D. Roalef for preparation of the computer program and modules which appear in this report. The methods which appear in this report were used by the authors in computation of analytical NLFM parameters for their fatigue crack growth work.

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SECTION 1  
INTRODUCTION

The estimating scheme described in this report is the direct result of prior work by Hutchinson, and co-workers<sup>1-2</sup> and by Shih and co-workers<sup>3-6</sup>. This estimating scheme is used to determine a fracture mechanics parameter (the J Integral) which describes the intensity of the Hutchinson<sup>7</sup>, Rice and Rosengren<sup>8</sup> (HRR) stress-strain field at the crack tip in an elastic-plastic material. The estimating scheme can also be used to determine load-load line displacement and crack mouth opening displacement. The J-Integral parameter determined by the estimating scheme can be used to obtain the crack tip opening displacement when the HRR field singularity describes the crack tip singular behavior.

The two-fold purpose of the report is (a) to review the theoretical basis for the estimation scheme and (b) to describe a computer program which utilizes the estimating scheme to yield values for the J-Integral (J), the load-line displacement ( $\Delta$ ), and the crack mouth opening displacement (CMOD or  $\delta$ ) for five different structural crack geometries.

## SECTION 2

### THEORETICAL BACKGROUND

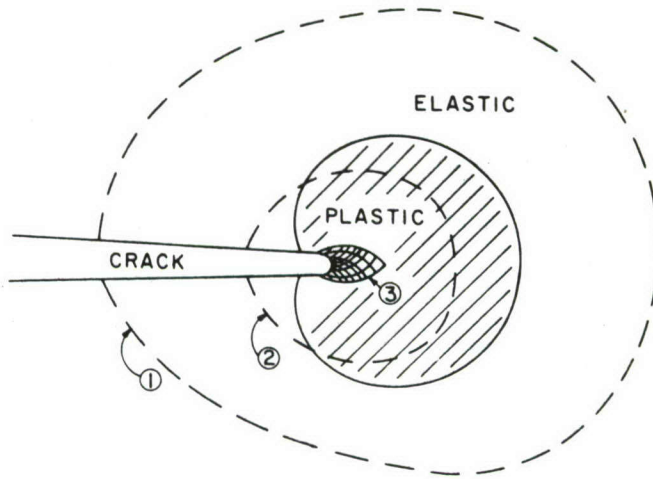
This section of the report has been prepared to provide background information relative to the elastic-plastic fracture mechanics parameters which are derived by the computer program described in Section 3.

#### 2.1 ELASTIC-PLASTIC FIELD PARAMETERS

The solutions for the in-plane tensile opening mode (Mode I) type of deformation problems are the primary interest in the present work as well as fracture mechanics in general. However, the mathematical difficulties have prevented detailed treatment of elastic-plastic problems. Except in some out-of-plane, tearing mode (Mode III) type of problems<sup>9,10</sup>, rigorous mathematical solutions of elastic-plastic problems are not available in general; the available limited cases of elastic-plastic crack tip stress analyses will be reviewed in this subsection.

Figure 1 shows the crack tip and area ahead of the crack tip<sup>11</sup>. The region ahead of the crack-tip is divided into three distinct zones: (1) elastic, (2) elastic-plastic, and (3) intensity non-linear (large strains and rotations, and ductile cavities) zone. The elastic zone (1) controls the behavior when the plastic zone size is small compared to the elastic zone and the geometry. In this case, referred to as small scale yielding, linear elastic fracture mechanics (LEFM) is applicable. If the plastic zone size is large, compared to the case of small scale yielding, LEFM is not applicable.

The intense elastic-plastic stress-strain field contained within zone 2 of Figure 1 is further expanded in Figure 2. When the intensely deformed process zone is small compared to the size of the elastic-plastic zone under consideration, the deformation theory of plasticity for a power hardening material can be used to obtain stress-strain solutions ahead of the crack tip outside the intensely deformed process zone as suggested by Hutchinson<sup>7</sup> and Rice and Rosengren<sup>8</sup>. These authors expressed power hardening using a stress ( $\sigma$ )-strain ( $\epsilon$ ) relationship given by:



- Zone 1 = An Elastic Field Surrounding the Crack Tip
- Zone 2 = An Elastic-Plastic Field Surrounding the Crack Tip
- Zone 3 = An Intense Zone of Deformation

Figure 1. Crack-Tip Stress and Strain Fields Surrounding the Crack (Reference 11).

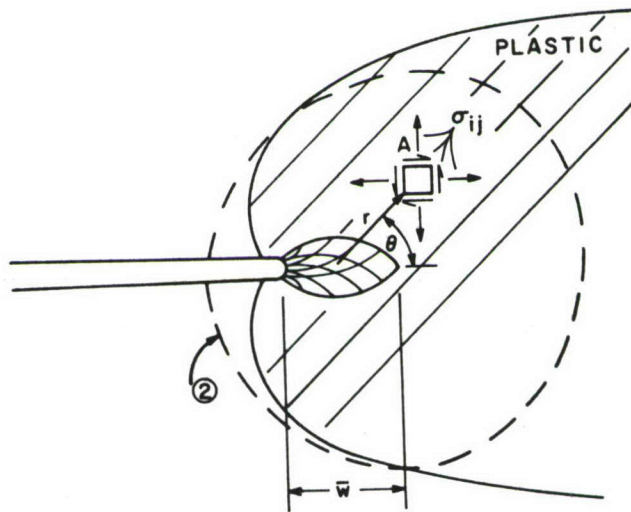


Figure 2. Expanded View of the Elastic-Plastic Stress-Strain Field (Reference 11).

$$\sigma = \sigma_0 \left( \frac{\epsilon}{\epsilon_0} \right)^N \quad (1)$$

where  $\sigma_0$  and  $\epsilon_0$  are reference stresses and strains, respectively, and  $N$  is the strain hardening exponent.

When Equation 1 is used to model the behavior of the material in the plastic range, the stress, strain, and displacement functions for the crack tip region are given by Equation 2, regardless of the amount of plastic deformation:

$$\sigma_{ij} = K_\sigma \tilde{\sigma}_{ij}(\theta, N) \cdot r^{-\frac{N}{N+1}} \quad (2a)$$

$$\epsilon_{ij} = K_\epsilon \tilde{\epsilon}_{ij}(\theta, N) \cdot r^{-\frac{1}{N+1}} \quad (2b)$$

$$u_i = K_\epsilon \tilde{u}_i(\theta, N) \cdot r^{\frac{1}{N+1}} \quad (2c)$$

where  $K_\sigma$  and  $K_\epsilon$  are the magnitudes of singularities of appropriate quantities with  $K_\epsilon$  being a function of  $K_\sigma$ . The functions  $\tilde{\sigma}_{ij}$ ,  $\tilde{\epsilon}_{ij}$  and  $\tilde{u}_i$  depend on angle and exponent  $N$  in Equation 1. Equations 2a, 2b and 2c have been referred to as the set of "HRR" field equations after the initial investigators (References 7 and 8).

In the derivation of the HRR field equations, the  $J_2$  deformation theory of plasticity was used to describe the material behavior. If the loading is proportional, the field solutions obtained using both the  $J_2$ -deformation theory and the more realistic  $J_2$ -incremented theory of plasticity are the same. However, when unloading occurs during deformation, the loading path may be different from the assumed proportional loading and the validity of the above solutions is not guaranteed.

The magnitude of the singularity in Equation 2 can be written in terms of  $J$  as when the process zone is small (Reference 11):

$$\sigma_{ij} = \sigma_0 \left( \frac{J}{r \sigma_0 \epsilon_0} \right)^{\frac{N}{N+1}} \tilde{\sigma}_{ij}(\theta, N) \quad (3a)$$

$$\epsilon_{ij} = \epsilon_0 \left( \frac{J}{r \sigma_0 \epsilon_0} \right)^{\frac{1}{N+1}} \tilde{\epsilon}_{ij}(\theta, N) \quad (3b)$$

$$u_i = \epsilon_0 \left( \frac{J}{\sigma_0 \epsilon_0} \right)^{\frac{1}{N+1}} r^{\frac{N}{N+1}} \tilde{u}_i(\theta, N) \quad (3c)$$

$$J = \int_{\Gamma} [w n_1 - n_j \sigma_{ij} u_{j,i}] ds \quad (4)$$

$$W = \int_0^{\epsilon_{ij}} \sigma_{ij} d\epsilon_{ij}$$

In equation 4,  $\Gamma$  is any contour that surrounds the crack tip,  $ds$  and  $n_j$  are the element length and outward normal to  $\Gamma$ ,  $u_{j,i}$  are the displacement gradients and  $\sigma_{ij}$  and  $\epsilon_{ij}$  are the stress and strain tensors. The  $J$  has been determined to be independent of the location of contour curve  $\Gamma$ .

Hence, for a given cracked material, with the assumption of power law hardening behavior, deformation theory of plasticity, and proportional loading, there exists a unique elastic-plastic stress and strain field which is characterized by its intensity  $J$ . In a manner similar to the LEFM approach where the stress intensity factor ( $K$ ) measures the intensity of stress and strain within the elastic crack tip field, the parameter  $J$  defines the intensity of the elastic-plastic crack tip field, the parameter  $J$  defines the intensity of the elastic-plastic stress and strain in the crack tip field and thus provides a basis for a nonlinear fracture mechanics approach. The use of  $J$  to define the level of elastic-plastic stresses and strains around the crack tip requires that the intensely deformed process zone is small.

For ideally plastic materials, the relationship between the  $J$ -integral and the crack tip opening displacement ( $\delta_t$ ) defined by the opening distance between the intercepts of two  $45^\circ$  lines drawn back from the crack tip, with the deformed profile of the stationary crack as shown in Figure 3 is given by References 12 and 13:

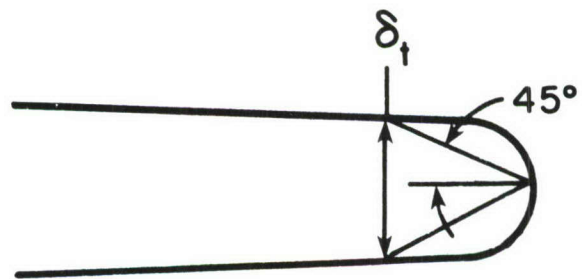


Figure 3. Definition of Crack Tip Opening Displacement  $\delta$ .

$$\delta_t = d_n \frac{J}{\sigma_0} \quad (5)$$

where  $d_n$  is a constant. For strain hardening materials, Shih and co-workers (References 4 and 14) have shown that Equation 5 can be used to relate  $J$  and  $\delta_t$  when the constant  $d_n$  is replaced with a function which is strongly dependent on the strain hardening exponent and mildly dependent on the ratio  $\sigma_0/E$  ( $E$  = Young's modulus). The functional relation for a strain-hardening material is described in Figure 4; Figure 4a applies for plane strain conditions while Figure 4b applies for plane stress conditions. Figure 4 has been derived assuming that Equation 1 describes the material.\* On the basis of Equations 3 and 5,  $\delta_t$ , the crack tip opening displacement, is also a parameter which characterizes the intensity of the stress-strain HRR field. The above HRR field equations are applicable only for the case of stationary cracks.

## 2.2 PARAMETER DETERMINATION

For elastic-plastic materials, the parameters: the J-integral ( $J$ ), the crack mouth opening displacement ( $\text{CMOD}=\delta$ ), and the load-line displacement due to the presence of a crack ( $\Delta_c$ ) can be approximated by the contributions due to their linearly elastic and plastic parts (References 3, 4, and 16):

$$\begin{aligned} J &= J^e + J^p \\ \delta &= \delta^e + \delta^p \\ \Delta_c &= \Delta_c^e + \Delta_c^p \end{aligned} \quad (6)$$

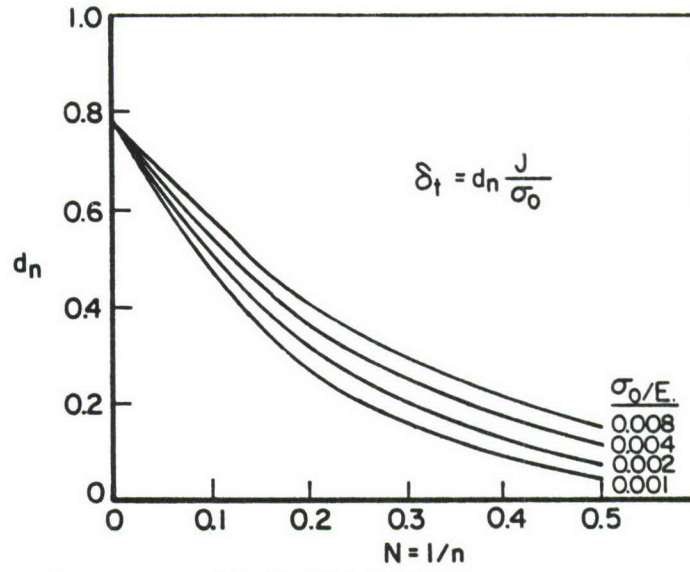
where superscripts e and p denotes elastic and plastic, respectively.

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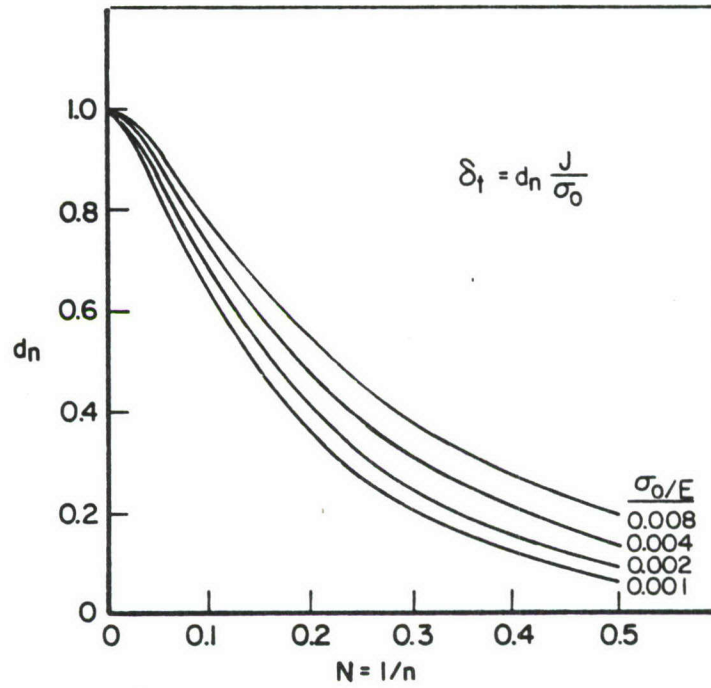
\* If the material is described by the relationship:

$$\frac{\epsilon}{\epsilon_0} = \alpha \left( \frac{\sigma}{\sigma_0} \right)^n \text{ and } \alpha = 1$$

then  $d_n$  is as presented in Figure 4. However, if  $\alpha \neq 1$  then  $d_n$  is equal to the product of the value of  $d_n$  given in Figure 4 and the quantity of  $\alpha^{1/n}$  (Reference 15).



(a) PLAIN STRAIN



(b)

Figure 4. Variation of  $d_n$  with  $N$  and  $\sigma_0/E$  (a) Plane Strain (b) Plane Stress (Assuming Equation 1 Applies).

Reasonable estimates of the total load-line displacement ( $\Delta$ ) for the structural geometry can be obtained by summing the contributions due to the presence of the crack, i.e.  $\Delta_c$  from Equation 6, and to that due to the structural geometry without a crack ( $\Delta_n$ ), (Reference 3):

$$\Delta = \Delta_c + \Delta_n \quad (7)$$

The error in using Equation 7 is small when the distance between the load points is much smaller than other structural dimensions.

The purpose of this subsection is to outline the analysis that must be accomplished in conjunction with Equation 6 to derive estimates of the elastic-plastic parameters. The following three paragraphs present: the elastic formulation, the plastic (strain-hardening) formulation and the elastic-plastic transition formulations, respectively.

#### 2.2.1 Linearly Elastic Contribution

For linearly elastic materials, the elastic crack parameters appearing in Equation 6 can be expressed in the form:

$$\begin{aligned} \frac{J^e}{\sigma_0 \epsilon_0 a} &= \left( \frac{\sigma^\infty}{\sigma_0} \right)^2 \hat{J}^e(a/b) \\ \frac{\delta^e}{\epsilon_0 a} &= \left( \frac{\sigma^\infty}{\sigma_0} \right) \hat{\delta}^e(a/b) \end{aligned} \quad (8)$$

$$\frac{\Delta_c^e}{\epsilon_0 a} = \left( \frac{\sigma^\infty}{\sigma_0} \right) \hat{\Delta}_c^e(a/b)$$

where  $\sigma^\infty$  is the remotely applied stress, and  $\sigma_0$  and  $\epsilon_0$  are reference stresses and strains related by the expression  $\sigma_0 = E\epsilon_0$ . Functions  $\hat{J}^e$ ,  $\hat{\delta}^e$ , and  $\hat{\Delta}_c^e$  are functions only of the ratio of crack length to width ( $a/b$ ). These functions can be found in Reference 17 for different finite width crack geometries.

### 2.2.2 Plastic (Strain Hardening) Contribution

To derive the plastic crack parameters given in Equation 6, certain assumptions are required. First, the material is assumed to behave according to a power hardening constitutive ( $\sigma$ - $\epsilon$ ) equation of the form

$$\frac{\epsilon}{\epsilon_0} = \alpha \left( \frac{\sigma}{\sigma_0} \right)^n \quad (9)$$

where  $\alpha$  is a dimensionless constant,  $\sigma_0$  and  $\epsilon_0$  are reference stresses and strains, and  $n$  is the hardening exponent. Note that the exponent in Equation 9 is in the inverse of the exponent in Equation 1. For  $n=1$ , the material behaves as a linearly elastic material; as  $n$  approaches infinity, the material behaves more and more like a perfectly plastic material. Generalizing Equation 9 to multi-axial states via the  $J_2$  deformation theory results in:

$$\frac{\epsilon_{ij}}{\epsilon_0} = \frac{3}{2} \alpha \left[ \frac{\sigma_e}{\sigma_0} \right]^{n-1} \frac{S_{ij}}{\sigma_0} \quad (10)$$

where  $\epsilon_{ij}$ ,  $S_{ij}$ , and  $\sigma_e$  are the strain deviator, the stress deviator, and the effective stress ( $= \sqrt{3/2 S_{ij} \cdot S_{ij}}$ ), respectively. Ilyushin (Reference 8) first noted that the boundary value problems which (1) have an externally applied, monotonically increasing load or displacement and (2) are based on Equation 10, have some special properties. He showed that the stress at each point in the body varies linearly with a single load parameter when tractions are prescribed on all boundaries and the directions of these tractions remain fixed while their magnitudes are everywhere linearly proportional to the load parameter. Since the stress components at each point are proportional, the solution based on  $J_2$  deformation theory also applies for incremental plasticity theory when the load parameter is monotonically increasing.

The functional dependence of the field parameters (stress, strain, and displacement) on the applied load (or displacement) also means that crack tip parameters can be uniquely related to the remotely applied load ( $\sigma^\infty$ ) via the following expressions (References 3 and 4):

$$\begin{aligned} \frac{J^P}{\sigma_0 \epsilon_0 a} &= \left(\frac{\sigma^\infty}{\sigma_0}\right)^{n+1} \cdot \hat{J}\left(\frac{a}{b}, n\right) \\ \frac{\delta^P}{\epsilon_0 a} &= \left(\frac{\sigma^\infty}{\sigma_0}\right)^n \cdot \hat{\Delta}^P\left(\frac{a}{b}, n\right) \\ \frac{\Delta_C^P}{\epsilon_0 a} &= \left(\frac{\sigma^\infty}{\sigma_0}\right)^n \cdot \hat{\Delta}_C^P\left(\frac{a}{b}, n\right) \end{aligned} \quad (11)$$

where  $\hat{J}^P$ ,  $\hat{\delta}^P$ , and  $\hat{\Delta}_C^P$  are functions only of  $\frac{a}{b}$  and  $n$ . The reader will note that the functional forms given by Equation 11 are similar to Equation 8 and, in fact, they reduce to Equation 8 when  $n=1$ .

An alternate form of Equation 11 that has been used previously (References 4 and 5) and which is used in the computer program discussed in Section 3 is:

$$\begin{aligned} J^P &= \alpha \sigma_0 \epsilon_0 a f_1(a/b) \cdot h_1\left(\frac{a}{b}, n\right) \cdot \left(\frac{P}{P_0}\right)^{n+1} \\ \delta^P &= \alpha \epsilon_0 a f_2(a/b) h_2\left(\frac{a}{b}, n\right) \cdot \left(\frac{P}{P_0}\right)^n \\ \Delta_C^P &= \alpha \epsilon_0 a f_3(a/b) h_3\left(\frac{a}{b}, n\right) \cdot \left(\frac{P}{P_0}\right)^n \end{aligned} \quad (12)$$

where  $P$  is the applied load (per unit thickness),  $P_0$  is the limit load (per unit thickness),  $f_1, f_2$  and  $f_3$  are functions only of geometry and crack length while  $h_1, h_2$  and  $h_3$  depend on geometry, crack length,

and strain hardening exponent. Shih and co-workers (References 4 and 5) have tabulated the functions  $h_1$ ,  $h_2$ ,  $h_3$  for a number of geometries for the conditions of plane stress and plane strain. From the reference tabulated data, these functions can be obtained by interpolation for any value within the  $\frac{a}{b}$  and  $n$  limits given; thus, the plastic (strain-hardening) component of Equation 6 can be computed for any given applied load  $P$  (or  $\sigma_\infty$ ) from Equation 12 (or 11).

### 2.2.3 Elastic to Plastic Transition

In the two preceding paragraphs, we reviewed the procedures for estimating the parameters of Equation 6 when the material was either linearly elastic or plastically strain hardening. Under the assumptions for small scale yielding, Bucci, et al. (Reference 16) observed that the transition region between fully linear elastic and fully large-scale plastic deformation could be more accurately modeled if the physical crack length term, i.e. "a" in the elastic components of Equation 6 was replaced with an effective crack length term ( $a_e$ ). Bucci, et al. suggested an effective crack length based on adding the Irwin plastic zone size ( $r_y$ ) to the physical crack length:

$$a_e = a + r_y \quad (13)$$

where

$$r_y = \frac{1}{\beta\pi} \left( \frac{K}{\sigma_0} \right)^2 \quad (14)$$

with  $\beta = 2$  for plane stress and  $\beta = 6$  for plane strain. The stress intensity factor is represented by  $K$ .

Subsequently, Shih, et al. (Reference 4) defined the effective crack length with a modification to account for strain hardening and for the relationship of the load to limit load. The Shih, et al. formulation for the effective crack length is:

$$a_e = a + \phi r_y \quad (15)$$

where

$$r_y = \frac{1}{\beta\pi} \left( \frac{n-1}{n+1} \right) \left( \frac{K}{\sigma_0} \right)^2 \quad (16)$$

and

$$\phi = \frac{1}{1 + (P/P_0)^2} \quad (17)$$

The factor  $\phi$  provides a correction to  $r_y$  in the case of small scale yielding, and also limits the contribution from the plastically adjusted crack length for large-scale yielding so that the values of  $\frac{J}{J^P}$ ,  $\frac{\delta}{\delta^P}$ , and  $\frac{\Delta_C}{\Delta_C^P}$  approach unity.

The version of Equation 6 embedded in the computer program discussed in Section 3 is given by:

$$\begin{aligned} J &= J^e(a_e) + J^P(a, n) \\ \delta &= \delta^e(a_e) + \delta^P(a, n) \\ \Delta_C &= \Delta_C^e(a_e) + \Delta_C^P(a, n) \end{aligned} \quad (18)$$

where  $a_e$  is the effective crack length defined in Equation 15. The elastic contribution given by Equation 8 is thus modified by the replacement of the physical crack length by the effective crack length. The plastic contribution is given by Equation 12.

### 2.3 FORMULATIONS FOR DIFFERENT GEOMETRIES

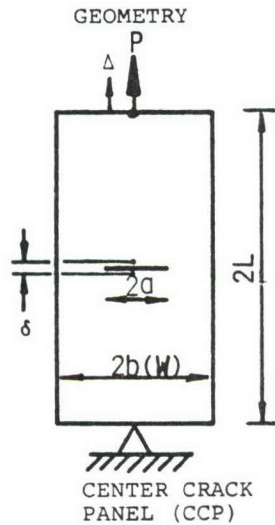
This subsection has been designed to present all the relevant information and equations that would allow one to use the estimating procedure outlined in subsection 2.2 for five different structural geometries. The equations and data presented herein have been previously presented by others (References 2 through 5 and 17) and are only repeated here for completeness. The information for each structural geometry is summarized using a figure and two supporting tables. The five structural geometries considered are listed in Table 1 along with the individual figure and table numbers which present the supporting information.

Each of the five supporting figures all are arranged in the same way so that each geometry's relevant equations and data appear in the same location in the figures. The arrangement of the figures is such that a schematic of the geometry appears in the upper left hand side and the individual versions of Equation 18 (the elastic-plastic parameters) can be found in the upper right hand side. The remainder of the figure is devoted to presenting equations that further define the elastic and plastic components of Equation 18 and to defining the tables that contain discrete value information on the plastic functions  $h_1$ ,  $h_2$ , and  $h_3$ .

In order to ensure that the information in Figures 5 through 9 is properly interpolated, a few additional notes are presented here. First,  $E'$  is Young's modulus ( $E$ ) for plane stress and is equal to  $E/(1 - \nu^2)$  for plane strain where  $\nu$  is the Poisson's ratio of the material. The loads,  $P$  and  $P_0^T$ , are the applied load and the theoretical limit load for a perfectly plastic material ( $n = \infty$ ) respectively; both are expressed per unit thickness.

TABLE 1  
 LIST OF THE CRACK GEOMETRIES CONSIDERED AND THE FIGURES AND  
 TABLES WHICH SUPPORT THEIR ANALYSIS

<u>STRUCTURAL CRACK GEOMETRY</u>	<u>SUPPORTING FIGURES AND TABLES</u>
CENTER CRACK PANEL	Figure 5, Tables 2a and 2b
COMPACT TENSION CRACK	Figure 6, Tables 3a and 3b
DOUBLE EDGE CRACKED PANEL	Figure 7, Tables 4a and 4b
SINGLE EDGE CRACKED PANEL LOADED IN THREE-POINT BENDING	Figure 8, Tables 5a and 5b
SINGLE EDGE CRACKED PANEL LOADED IN TENSION	Figure 9, Tables 6a and 6b



ELASTIC-PLASTIC PARAMETERS

$$J = \frac{\pi a_e}{4b^2} F^2(z) \frac{P^2}{E'} + \alpha \sigma_0 \epsilon_0 \frac{a(b-a)}{b} h_1(a/b, n) (P/P_0^T)^{n+1}$$

$$\delta = \frac{2a_e}{b} V_1(z) \frac{P}{E'} + \alpha \epsilon_0 a h_2(a/b, n) (P/P_0^T)^n$$

$$\Delta_c = \frac{2a_e}{b} V_2(z) \frac{P}{E'} + \alpha \epsilon_0 a h_3(a/b, n) (P/P_0^T)^n$$

$$\Delta_n = \frac{PL}{EB} + \sqrt{3} \alpha \epsilon_0 L \left( \frac{\sqrt{3P}}{4b\sigma_0} \right)^n$$

$$\Delta = \Delta_n + \Delta_c$$

SUPPORTING ELASTIC FUNCTIONS:  $F$ ,  $V_1$  AND  $V_2$  with  $z = a_e/b$

$$F(z) = \sqrt{\sec\left(\frac{\pi z}{2}\right)}$$

$$V_1(z) = -0.071 - 0.535z + 0.169z^2 + 0.02z^3 - 1.071(1/z) \ln(1-z)$$

$$V_2(z) = -1.071 + 0.250z - 0.357z^2 + 0.121z^3 - 0.047z^4 + 0.008z^5 - 1.071(1/z) \ln(1-z)$$

THEORETICAL PLASTIC LIMIT LOAD:  $P_0^T$

Plane Stress Condition

$$P_0^T = \frac{4}{\sqrt{3}} \sigma_0 (b-a)$$

Plane Strain Condition

$$P_0^T = 2\sigma_0 (b-a)$$

SUPPORTING PLASTIC FUNCTIONS:  $h_1$ ,  $h_2$ , and  $h_3$

Plane stress condition tabularized in Table 2a

Plane strain condition tabularized in Table 2b

Figure 5. Elastic-Plastic Parameters for the Center-Cracked Panel (References 4, 5 and 17).

TABLE 2a

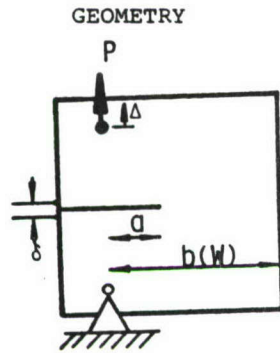
$h_1$ ,  $h_2$  and  $h_3$  FOR THE PLANE STRESS CCP IN TENSION (REFERENCE 5).

	$n = 1$	$n = 2$	$n = 3$	$n = 5$	$n = 7$	$n = 10$	$n = 13$	$n = 16$	$n = 20$
$a/b = 1/4$	$h_1$	2.544	2.972	3.140	3.195	3.106	2.896	2.467	2.196
	$h_2$	3.116	3.286	3.304	3.151	2.926	2.595	2.081	1.814
	$h_3$	0.611	1.010	1.352	1.830	2.083	2.191	2.122	1.792
$a/b = 3/8$	$h_1$	2.344	2.533	2.515	2.346	2.173	1.953	1.766	1.431
	$h_2$	2.710	2.621	2.414	2.032	1.753	1.473	1.279	0.988
	$h_3$	0.807	1.195	1.427	1.594	1.570	1.425	1.267	0.994
$a/b = 1/2$	$h_1$	2.206	2.195	2.057	1.809	1.632	1.433	1.300	1.000
	$h_2$	2.342	2.014	1.703	1.299	1.071	0.871	0.757	0.557
	$h_3$	0.927	1.186	1.256	1.178	1.040	0.867	0.758	0.560
$a/b = 5/8$	$h_1$	2.115	1.912	1.690	1.407	1.221	1.012	0.853	0.573
	$h_2$	1.968	1.458	1.126	0.785	0.617	0.474	0.383	0.256
	$h_3$	0.975	1.053	0.970	0.763	0.620	0.478	0.386	0.273
$a/b = 3/4$	$h_1$	2.073	1.708	1.458	1.208	1.082	0.956	0.745	0.532
	$h_2$	1.611	0.970	0.685	0.452	0.361	0.292	0.216	0.148
	$h_3$	0.933	0.802	0.642	0.450	0.361	0.292	0.216	0.149

TABLE 2b

 $h_1, h_2$  AND  $h_3$  FOR THE PLANE STRAIN CCP IN TENSION (REFERENCE 5).

	$n = 1$	$n = 2$	$n = 3$	$n = 5$	$n = 7$	$n = 10$	$n = 13$	$n = 16$	$n = 20$
$a/b = 1/4$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	2.535 2.680 0.536	3.009 2.989 0.911	3.212 3.014 1.217	3.289 2.847 1.639	3.181 2.610 1.844	2.915 2.618 1.554	2.625 1.971 1.802	2.340 1.712 1.637	2.028 1.450 1.426
$a/b = 3/8$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	2.344 2.347 0.699	2.616 2.391 1.059	2.648 2.230 1.275	2.507 1.876 1.440	2.281 1.580 1.396	1.969 1.276 1.227	1.709 1.065 1.050	1.457 0.890 0.888	1.193 0.715 0.719
$a/b = 1/2$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	2.206 2.028 0.803	2.291 1.856 1.067	2.204 1.600 1.155	1.968 1.230 1.101	1.759 1.002 0.968	1.522 0.799 0.796	1.323 0.664 0.665	1.155 0.564 0.565	0.978 0.466 0.469
$a/b = 5/8$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	2.115 1.705 0.844	1.960 1.322 0.937	1.763 1.035 0.879	1.616 0.696 0.691	1.169 0.524 0.522	0.863 0.358 0.361	0.628 0.250 0.251	0.458 0.178 0.178	0.300 0.114 0.115
$a/b = 3/4$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	2.072 1.345 0.805	1.732 0.857 0.700	1.471 0.596 0.555	1.108 0.361 0.359	0.895 0.254 0.254	0.642 0.167 0.168	0.461 0.114 0.114	0.337 0.081 0.081	0.216 0.051 0.052



COMPACT TENSION SPECIMEN (CT)

ELASTIC-PLASTIC PARAMETERS

$$J = \frac{a_e}{b^2} \cdot F^2(z) \frac{P^2}{E} + \alpha \sigma_0 \varepsilon_0 (b-a) h_1(a/b, n) (P/P_0^T)^{n+1}$$

$$\delta = V_1(z) \frac{P}{E} + \alpha \varepsilon_0 a h_2(a/b, n) (P/P_0^T)^n$$

$$\Delta_c = V_2(z) \frac{P}{E} + \alpha \varepsilon_0 a h_3(a/b, n) (P/P_0^T)^n$$

$$\Delta_n = 0$$

$$\Delta = \Delta_n + \Delta_c$$

SUPPORTING ELASTIC FUNCTIONS:  $F$ ,  $V_1$ , and  $V_2$  with  $z = \frac{a_e}{b}$

$$F(z) = \frac{(2+z)}{\sqrt{z}(1-z)^{1.5}} (0.886+4.64z-13.32z^2+14.72z^3-5.6z^4)$$

$$V_1(z) = (5.435+43.315z-83.166z^2+57.694z^3)/(1-z^2)$$

and

$$V_2(z) = (0.995+27.977z-27.209z^2+11.062z^3)/(1-z^2)$$

THEORETICAL PLASTIC LIMIT LOAD:  $P_0^T$

Plane Stress Condition

$$P_0^T = 1.072\eta (b-a)\sigma_0$$

Plane Strain Condition

$$P_0^T = \frac{2.52}{\sqrt{3}}\eta (b-a)\sigma_0$$

$$\text{with } \eta = \sqrt{\left(\frac{2a}{b-a}\right)^2 + 2\left(\frac{2a}{b-a}\right) + 2} - \left(\frac{2a}{b-a} + 1\right)$$

SUPPORTING PLASTIC FUNCTIONS:  $h_1$ ,  $h_2$ ,  $h_3$

Plane stress condition summarized in Table 3a

Plane strain condition summarized in Table 3b

Figure 6. Elastic-Plastic Parameters for the Compact Tension Specimen (References 4, 5 and 17).

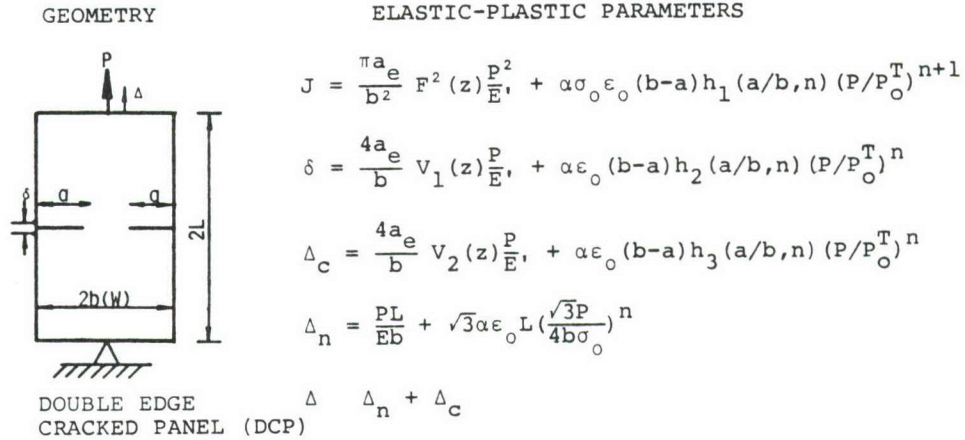
TABLE 2b  
 $h_1, h_2$  AND  $h_3$  FOR THE PLANE STRAIN CCP IN TENSION (REFERENCE 5).

	n = 1	n = 2	n = 3	n = 5	n = 7	n = 10	n = 13	n = 16	n = 20
$a/b = 1/4$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	1.609 17.552 9.670	1.464 12.042 7.996	1.284 10.706 7.205	1.060 8.736 5.944	0.903 7.316 5.000	0.729 5.744 3.945	0.601 4.629 3.191	0.511 3.746 2.591	0.395 2.916 2.023
$a/b = 3/8$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	1.552 12.410 7.800	1.249 8.203 5.734	1.047 6.538 4.615	0.801 4.563 3.253	0.647 3.447 2.475	0.484 2.442 1.765	0.377 1.830 1.330	0.284 1.360 0.990	0.220 1.019 0.746
$a/b = 1/2$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	1.398 9.155 6.288	1.084 5.673 4.149	0.901 4.212 3.107	0.686 2.801 2.087	0.558 2.123 1.590	0.436 1.571 1.181	0.356 1.245 0.938	0.298 1.026 0.774	0.238 0.814 0.614
$a/b = 5/8$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	1.274 7.471 5.419	1.031 4.483 3.375	0.875 3.347 2.536	0.695 2.367 1.804	0.593 1.923 1.468	0.494 1.539 1.176	0.423 1.292 0.988	0.370 1.116 0.853	0.310 0.928 0.710
$a/b = 3/4$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	1.234 6.252 4.767	0.977 3.780 2.922	0.833 2.893 2.242	0.683 2.135 1.657	0.598 1.775 1.379	0.506 1.437 1.116	0.431 1.204 0.936	0.373 1.030 0.800	0.314 0.857 0.666
$a/b \rightarrow 1$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	1.133 5.288 4.231	1.010 3.536 2.829	0.775 2.412 1.930	0.680 1.905 1.524	0.650 1.734 1.387	0.620 1.592 1.274	0.490 1.232 0.985	0.470 1.166 0.933	0.420 1.029 0.824

TABLE 3b

$h_1, h_2$  AND  $h_3$  FOR THE PLANE STRAIN COMPACT TENSION SPECIMEN (REFERENCE 5).

	$n = 1$	$n = 2$	$n = 3$	$n = 5$	$n = 7$	$n = 10$	$n = 13$	$n = 16$	$n = 20$
$a/b = 1/4$ $\begin{cases} h_1 \\ h_2 \\ h_3 \end{cases}$	2.227 17.883 9.852	2.048 12.481 8.506	1.783 11.675 8.170	1.475 10.788 7.774	1.334 10.538 7.706	1.248 10.745 7.942	1.258 11.460 8.517	1.325 12.570 9.371	1.566 14.563 10.887
$a/b = 3/8$ $\begin{cases} h_1 \\ h_2 \\ h_3 \end{cases}$	2.148 12.644 7.944	1.716 8.176 5.760	1.392 6.521 4.643	0.970 4.319 3.103	0.693 2.970 2.139	0.443 1.794 1.292	0.276 1.102 0.793	0.176 0.686 0.494	0.098 0.370 0.266
$a/b = 1/2$ $\begin{cases} h_1 \\ h_2 \\ h_3 \end{cases}$	1.935 9.327 6.406	1.509 5.846 4.268	1.242 4.304 3.157	0.919 2.747 2.024	0.685 1.912 1.413	0.461 1.199 0.888	0.314 0.788 0.585	0.216 0.530 0.393	0.132 0.317 0.236
$a/b = 5/8$ $\begin{cases} h_1 \\ h_2 \\ h_3 \end{cases}$	1.763 7.612 5.521	1.449 4.572 3.431	1.237 3.423 2.583	0.974 2.359 1.787	0.752 1.810 1.373	0.602 1.319 1.000	0.459 0.983 0.746	0.347 0.749 0.568	0.248 0.485 0.368
$a/b = 3/4$ $\begin{cases} h_1 \\ h_2 \\ h_3 \end{cases}$	1.709 6.370 4.857	1.424 3.948 3.048	1.263 3.179 2.456	1.033 2.337 1.807	0.864 1.876 1.450	0.717 1.441 1.114	0.575 1.124 0.869	0.448 0.887 0.686	0.345 0.665 0.514
$a/b \rightarrow 1$ $\begin{cases} h_1 \\ h_2 \\ h_3 \end{cases}$	1.568 5.388 4.310	1.450 3.738 2.990	1.350 3.093 2.474	1.180 2.433 1.946	1.080 2.121 1.697	0.950 1.795 1.436	0.850 1.573 1.258	0.730 1.333 1.066	0.630 1.136 0.909



SUPPORTING ELASTIC FUNCTIONS:  $F$ ,  $V_1$ , and  $V_2$  with  $z = \frac{ae}{b}$

$$F(z) = (1.122 - 0.561z - 0.205z^2 + 0.471z^3 - 0.190z^4) / \sqrt{1-z}$$

$$V_1(z) = \left(\frac{2}{\pi z}\right) \left[ 0.459 \left(\sin \frac{\pi z}{2}\right) - 0.065 \left(\sin \frac{\pi z}{2}\right)^3 - 0.007 \left(\sin \frac{\pi z}{2}\right)^5 \right. \\ \left. + \cosh^{-1} \left(\sec \frac{\pi z}{2}\right) \right]$$

$$V_2(z) = \left(\frac{2}{\pi z}\right) \left[ 0.0629 - 0.0610 \left(\cos \frac{\pi z}{2}\right)^4 - 0.0019 \left(\cos \frac{\pi z}{2}\right)^8 \right. \\ \left. + \ln \left(\sec \frac{\pi z}{2}\right) \right]$$

THEORETICAL PLASTIC LIMIT LOAD:  $P_0^T$

Plane Stress Condition

$$P_0^T = \frac{4}{\sqrt{3}} \sigma_0 (b-a)$$

Plane Strain Condition

$$P_0^T = 5.94 \sigma_0 (b-a)$$

SUPPORTING PLASTIC FUNCTIONS:  $h_1$ ,  $h_2$ , and  $h_3$

Plane stress condition tabulated in Table 4a

Plane strain condition tabulated in Table 4b

Figure 7. Elastic-Plastic Parameters for Double-Edge Cracked Panel (References 4, 5 and 17).

TABLE 4a

$h_1, h_2$  AND  $h_3$  FOR THE PLANE STRESS DECP IN TENSION (REFERENCE 5).

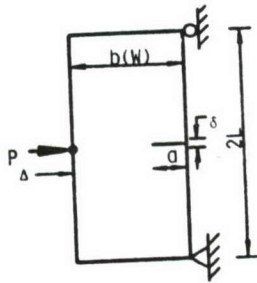
	n = 1	n = 2	n = 3	n = 5	n = 7	n = 10	n = 13	n = 16	n = 20
$a/b = 1/4$	$h_1$	1.01100	1.22620	1.35600	1.48280	1.54340	1.57770	1.59370	1.58790
	$h_2$	1.72580	1.81860	1.88590	1.91670	1.90480	1.85350	1.80190	1.69980
	$h_3$	0.29565	0.53672	0.76993	1.16890	1.48990	1.81520	2.02200	2.19810
$a/b = 3/8$	$h_1$	1.29310	1.41760	1.42740	1.34110	1.23740	1.09370	0.96971	0.67438
	$h_2$	2.59390	2.39340	2.22100	1.86440	1.58780	1.28340	1.06810	0.70934
	$h_3$	0.65822	1.03710	1.29460	1.52010	1.54690	1.41240	1.22720	1.06810
$a/b = 1/2$	$h_1$	1.47460	1.46570	1.37830	1.16790	1.01040	0.84503	0.73169	0.20790
	$h_2$	3.51420	2.82070	2.33660	1.66980	1.27680	0.94377	0.76195	0.23176
	$h_3$	1.18380	1.58120	1.69130	1.56290	1.31980	0.08000	0.80855	0.26553
$a/b = 5/8$	$h_1$	1.58600	1.45380	1.28450	1.03760	0.88209	0.73733	0.64944	0.02024
	$h_2$	4.55860	3.14460	2.31800	1.44890	1.06070	0.79048	0.65694	0.02766
	$h_3$	1.93220	2.13830	1.94990	1.44400	1.09380	0.80888	0.66455	0.03169
$a/b = 3/4$	$h_1$	1.65200	1.42590	1.11760	0.97914	0.83350	0.70092	0.63039	0.00006
	$h_2$	5.89560	3.37110	2.21450	1.29740	0.96595	0.74142	0.63559	0.00013
	$h_3$	3.06300	2.67070	2.06130	1.31380	0.97799	0.74708	0.63801	0.00016

TABLE 4b

 $h_1, h_2$  AND  $h_3$  FOR THE PLANE STRAIN DECP IN TENSION (REFERENCE 5).

	n = 1	n = 2	n = 3	n = 5	n = 7	n = 10	n = 13	n = 16	n = 20
$a/b = 1/4$	$h_1$	128.40	28.67	155.43	749.07	7530.80	75618.00	741530.00	15600000.00
	$h_2$	3.33	7.41	16.55	78.46	362.04	34044.00	326360.00	6719300.00
	$h_3$	0.57	2.20	6.82	49.33	297.75	43022.00	459780.00	10421000.00
$a/b = 3/8$	$h_1$	6.41	14.56	30.42	120.99	461.24	20374.00	168810.00	2278000.00
	$h_2$	5.00	9.61	18.64	67.83	244.20	11491.00	80300.00	1062000.00
	$h_3$	1.27	4.22	11.17	59.32	266.53	16698.00	123400.00	1693100.00
$a/b = 1/2$	$h_1$	7.31	14.59	27.07	87.53	275.57	8109.40	43102.00	343780.00
	$h_2$	6.78	11.22	18.94	53.76	155.16	4008.90	20719.00	173640.00
	$h_3$	2.28	6.39	14.29	55.49	187.17	5610.10	29420.00	244640.00
$a/b = 5/8$	$h_1$	7.87	13.40	21.62	53.51	128.26	1558.40	5376.80	28037.00
	$h_2$	8.79	11.91	16.73	34.22	73.40	783.60	2623.40	13141.00
	$h_3$	3.73	8.28	14.83	38.63	91.15	1030.70	3458.70	17316.00
$a/b = 3/4$	$h_1$	8.19	12.40	17.72	33.06	55.12	221.79	518.00	894.59
	$h_2$	11.37	12.21	14.07	20.72	31.92	117.99	262.33	473.82
	$h_3$	5.91	9.89	13.89	23.82	38.01	142.31	315.28	575.54

GEOMETRY



SINGLE EDGE  
CRACKED PANEL IN  
THREE-POINT BENDING  
(STB)

ELASTIC-PLASTIC PARAMETERS

$$J = \frac{9\pi a_e L^2}{b^4} F^2(z) \frac{P^2}{E} + \alpha \sigma \epsilon_0 (b-a) h_1 \left(\frac{a}{b}, n\right) \left(\frac{P}{P_O^T}\right)^{n+1}$$

$$\delta = \frac{12a_e L}{b^2} V_1(z) \frac{P}{E} + \alpha \epsilon_0 a h_2 \left(\frac{a}{b}, n\right) \left(\frac{P}{P_O^T}\right)^n$$

$$\Delta_c = \frac{6L^2}{b^2} V_2(z) \frac{P}{E} + \alpha \epsilon_0 a h_3 \left(\frac{a}{b}, n\right) \left(\frac{P}{P_O^T}\right)^n$$

$$\Delta_n = \frac{(1-\nu^2) PL^3}{6EI} + \frac{PL}{b} \left[ \frac{3(1+\nu)}{2E} - \frac{3(1-\nu^2)}{10E} - \frac{3\nu(1-\nu^2)}{4E} \right] - \frac{0.21(1-\nu^2)P}{E}$$

$$\Delta = \Delta_n + \Delta_c$$

SUPPORTING ELASTIC FUNCTIONS:  $F$ ,  $V_1$ , and  $V_2$  with  $z = \frac{a_e}{b}$

$$F(z) = 1.09 - 1.735z + 8.2z^2 - 14.18z^3 + 14.57z^4$$

$$V_1(z) = 0.76 - 2.28z + 3.87z^2 - 2.04z^3 + 0.66/(1-z)^2$$

$$V_2(z) = \left(\frac{z}{1-z}\right)^2 (5.58 - 19.57z + 36.82z^2 - 34.94z^3 + 12.77z^4)$$

where

$$z = a/b$$

THEORETICAL PLASTIC LIMIT LOAD:  $P_O^T$

PLANE STRESS CONDITION

$$P_O^T = 0.536 \sigma_0 (b-a)^2 / L$$

PLANE STRAIN CONDITION

$$P_O^T = 0.728 \sigma_0 (b-a)^2 / L$$

SUPPORTING PLASTIC FUNCTIONS:  $h_1$ ,  $h_2$ ,  $h_3$

Plane stress condition summarized in Table 5a

Plane strain condition summarized in Table 5b

Figure 8. Elastic-Plastic Parameters for the Single-Edge Cracked Panel in Three-Point Bending (References 4, 5 and 17).

TABLE 5a

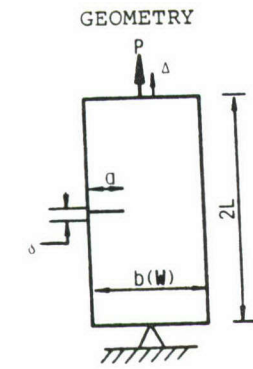
$h_1, h_2$  AND  $h_3$  FOR THE PLANE STRESS SECP UNDER THREE-POINT BENDING  
(REFERENCE 5).

	$n = 1$	$n = 2$	$n = 3$	$n = 5$	$n = 7$	$n = 10$	$n = 13$	$n = 16$	$n = 20$
$a/b = 1/4$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	0.869 5.690 4.007	0.731 4.503 8.812	0.629 3.680 7.189	0.479 2.614 4.731	0.370 1.947 3.388	0.246 1.290 2.204	0.174 0.897 1.517	0.117 0.603 1.012	0.059 0.307 0.508
$a/b = 3/8$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	0.963 5.085 4.420	0.797 3.732 5.533	0.680 2.929 4.482	0.527 2.071 3.172	0.418 1.580 2.409	0.307 1.134 1.726	0.232 0.841 1.277	0.174 0.626 0.948	0.105 0.381 0.575
$a/b = 1/2$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	1.019 4.768 4.604	0.767 3.120 4.085	0.621 2.320 3.092	0.453 1.547 2.081	0.324 1.077 1.442	0.202 0.655 0.874	0.128 0.410 0.545	0.081 0.259 0.344	0.030 0.097 0.129
$a/b = 5/8$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	1.051 4.551 4.617	0.786 2.830 3.434	0.649 2.118 2.599	0.494 1.455 1.794	0.357 1.023 1.258	0.235 0.656 0.803	0.173 0.472 0.577	0.105 0.286 0.349	0.047 0.130 0.158
$a/b = 3/4$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right\}$	1.067 4.385 4.394	0.786 2.656 3.012	0.643 1.967 2.235	0.474 1.329 1.510	0.343 0.928 1.052	0.230 0.601 0.680	0.167 0.427 0.483	0.110 0.280 0.316	0.044 0.114 0.129

TABLE 5b

$h_1, h_2$  AND  $h_3$  FOR THE PLANE STRAIN SECP UNDER THREE-POINT BENDING  
(REFERENCE 5).

	$n = 1$	$n = 2$	$n = 3$	$n = 5$	$n = 7$	$n = 10$	$n = 13$	$n = 16$	$n = 20$
$a/b = 1/4$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right.$	1.203 5.799 4.083	1.034 4.665 9.726	0.930 4.006 8.362	0.762 3.080 5.863	0.633 2.454 4.466	0.523 1.934 3.421	0.396 1.446 2.542	0.303 1.088 1.901	0.215 0.758 1.318
$a/b = 3/8$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right.$	1.334 5.182 4.505	1.149 3.931 6.008	1.018 3.199 5.031	0.840 2.384 3.737	0.695 1.927 3.016	0.556 1.471 2.302	0.442 1.153 1.803	0.360 0.928 1.452	0.265 0.684 1.070
$a/b = 1/2$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right.$	1.409 4.869 4.687	1.094 3.283 4.332	0.922 2.527 3.489	0.675 1.868 2.352	0.495 1.192 1.662	0.331 0.773 1.079	0.211 0.480 0.669	0.135 0.304 0.424	0.074 0.165 0.230
$a/b = 5/8$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right.$	1.456 4.638 4.705	1.070 2.861 3.490	0.896 2.156 2.700	0.631 1.369 1.722	0.436 0.907 1.142	0.255 0.518 0.652	0.142 0.287 0.361	0.084 0.166 0.209	0.041 0.081 0.102
$a/b = 3/4$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right.$	1.477 4.474 4.491	1.145 2.754 3.141	0.974 2.096 2.404	0.693 1.361 1.556	0.500 0.936 1.068	0.348 0.618 0.704	0.223 0.388 0.441	0.140 0.239 0.272	0.075 0.127 0.144



SINGLE EDGE CRACKED  
PANEL IN TENSION (SET)  $\Delta = \Delta_n + \Delta_c$

ELASTIC-PLASTIC PARAMETERS

$$J = \frac{\pi a_e F^2(z) P^2}{E' b^2} + \alpha \sigma_0 \epsilon_0 \frac{a(b-a)}{b} h_1(a/b, n) \left(\frac{P}{P_0^T}\right)^{n+1}$$

$$\delta = \frac{4a_e V_2(z) P}{b E'} + \alpha \epsilon_0 a h_3(a/b, n) (P/P_0^T)^n$$

$$\Delta_c = \frac{4a_e V_1(z) P}{b E'} + \alpha \epsilon_0 a h_3(a/b, n) (P/P_0^T)^n$$

$$\Delta_n = \frac{2PL}{Eb} + \sqrt{3} \alpha \epsilon_0 L \left(\frac{\sqrt{3}P}{2b\sigma_0}\right)^n$$

SUPPORTING ELASTIC FUNCTIONS:  $F$ ,  $V_1$  and  $V_2$  with  $z = \frac{a_e}{b}$

$$F(z) = \frac{\sqrt{(2/\pi z) \tan(\pi z/2)} [0.752 + 2.02z + 0.37\{1 - \sin(\pi z/2)\}^3]}{\sec(\pi z/2)}$$

$$V_1(z) = [1.46 + 3.42\{1 - \cos(\pi z/2)\}] (\sec \pi z/2)^2$$

$$V_2(z) = [z/(1-z)^2] [0.99 - z(1-z)(1.3 - 1.2z + 0.7z^2)]$$

THEORETICAL PLASTIC LIMIT LOAD:  $P_0^T$

PLANE STRESS CONDITION

$$P_0^T = 1.072 \eta (b-a) \sigma_0$$

PLANE STRAIN CONDITION

$$P_0^T = \frac{2.52}{\sqrt{3}} \eta (b-a) \sigma_0$$

$$\text{with } \eta = \sqrt{\left(\frac{a}{b-a}\right)^2 + 1} - \left(\frac{a}{b-a}\right)$$

SUPPORTING PLASTIC FUNCTIONS:  $h_1, h_2, h_3$

Plane stress condition summarized in Table 6a

Plane strain condition summarized in Table 6b

Figure 9. Elastic-Plastic Parameters for Single-Edge Cracked Panel in Tension (References 4, 5 and 17).

TABLE 6a

$h_1$ ,  $h_2$ , AND  $h_3$  FOR THE PLANE STRESS SECP IN TENSION  
(REFERENCE 5).

	$n = 1$	$n = 2$	$n = 3$	$n = 5$	$n = 7$	$n = 10$	$n = 13$	$n = 16$	$n = 20$
$a/b = 1/4$	$h_1$	3.261	2.919	2.115	1.531	0.960	0.615	0.400	0.230
	$h_2$	4.672	4.300	3.695	2.532	1.755	1.053	0.419	0.237
	$h_3$	10.090	6.488	4.362	2.185	1.239	0.630	0.224	0.123
$a/b = 3/8$	$h_1$	2.809	2.365	1.943	1.367	1.009	0.677	0.342	0.226
	$h_2$	4.465	3.426	2.632	1.685	1.181	0.762	0.372	0.244
	$h_3$	5.047	2.653	1.604	0.812	0.525	0.328	0.157	0.102
$a/b = 1/2$	$h_1$	2.459	1.665	1.254	0.776	0.510	0.286	0.096	0.047
	$h_2$	4.369	2.726	1.909	1.093	0.694	0.380	0.124	0.061
	$h_3$	3.095	1.429	0.871	0.461	0.286	0.155	0.051	0.025
$a/b = 3/8$	$h_1$	2.070	1.408	1.105	0.755	0.551	0.363	0.172	0.107
	$h_2$	4.297	2.552	1.837	1.160	0.816	0.523	0.242	0.150
	$h_3$	2.270	1.127	0.771	0.478	0.336	0.215	0.146	0.062
$a/b = 3/4$	$h_1$	1.696	1.142	0.910	0.624	0.447	0.280	0.118	0.067
	$h_2$	4.240	2.468	1.805	1.147	0.798	0.490	0.203	0.115
	$h_3$	1.983	1.087	0.784	0.494	0.344	0.211	0.136	0.050

TABLE 6b

$h_1$ ,  $h_2$ , AND  $h_3$  FOR THE PLANE STRAIN SECP IN TENSION  
(REFERENCE 5).

	$n = 1$	$n = 2$	$n = 3$	$n = 5$	$n = 7$	$n = 10$	$n = 13$	$n = 16$	$n = 20$
$a/b = 1/4$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right.$	4.338 4.756 10.271	4.767 4.559 7.635	4.639 4.281 5.873	3.815 3.391 3.695	3.056 2.639 2.483	2.170 1.808 1.496	1.548 1.253 0.970	1.105 0.875 0.654	0.712 0.552 0.405
$a/b = 3/8$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right.$	3.81 4.544 5.137	3.250 3.493 2.992	2.626 2.669 1.904	1.680 1.571 0.923	1.064 0.946 0.515	0.539 0.458 0.240	0.276 0.229 0.119	0.142 0.116 0.060	0.060 0.048 0.025
$a/b = 1/2$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right.$	3.398 4.447 3.151	2.302 2.765 1.537	1.694 1.888 0.912	0.928 0.954 0.417	0.514 0.507 0.215	0.213 0.204 0.085	0.090 0.085 0.036	0.039 0.036 0.015	0.012 0.011 0.004
$a/b = 5/8$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right.$	2.859 4.374 2.311	1.795 2.439 1.084	1.299 1.622 0.681	0.697 0.806 0.329	0.378 0.423 0.171	0.153 0.167 0.067	0.063 0.067 0.027	0.026 0.027 0.011	0.008 0.008 0.003
$a/b = 3/4$ $\left\{ \begin{array}{l} h_1 \\ h_2 \\ h_3 \end{array} \right.$	2.342 4.316 2.018	1.607 2.515 1.104	1.245 1.789 0.765	0.769 1.027 0.435	0.477 0.619 0.262	0.233 0.296 0.125	0.116 0.146 0.062	0.059 0.073 0.031	0.021 0.027 0.011

SECTION 3  
COMPUTER PROGRAM DESCRIPTION

The purpose of this section is to describe the computer program (EST) listed in Appendix A. The EST computer program was written in FORTRAN IV for interactive use on the CYBER 175. The first subsection describes its function and the major elements of the program using flow diagrams. The second subsection describes the program output and various methods for presenting the output data. The third subsection presents the input and how one should prepare the material properties data. The final subsection describes the format of Appendix A.

### 3.1 PROGRAM FUNCTION

The purpose of the EST computer program is to calculate the value of the J-Integral (J), the crack mouth opening displacement (CMOD =  $\delta$ ), and the load-line displacement ( $\Delta$ ), as well as their elastic-plastic components. The material is assumed to behave according to Ramberg-Osgood type of stress-strain relationship:

$$\frac{\epsilon}{\epsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0}\right)^n \quad (19)$$

where  $\epsilon_0$  and  $\sigma_0$  are reference strains and stresses related by  $\epsilon_0 = \sigma_0/E$ , E is the elastic modulus, and  $\alpha$  and n are power hardening constants. The program calculates the parameters according to the estimating procedures defined within subsection 2.2 and uses the equations and data presented in Figures 5 through 9 and Tables 2 through 6 in subsection 2.3. The values of functions  $h_1$ ,  $h_2$  and  $h_3$  are known for the particular exponent n and crack length a. In EST, the intermediate values of  $h_1$ ,  $h_2$  and  $h_3$  are evaluated using a two-dimensional linear interpolation algorithm from the tabulated values.

One modification of the theory was incorporated into the EST computer program to change the level of the theoretical limit load so that it more closely approximated the observed experimental results. The change involved utilizing a limit load ( $P_o$ ) calculated from the product of a constant  $k$  (dependent on material and geometry) and the theoretical limit load, i.e.

$$P_o = kP_o^T \quad (20)$$

in all those equations which utilized the limit load in the calculation of the elastic-plastic parameters. The value of  $k$  is an input to the computer program and would typically be set equal to 1 for the first set of runs. Subsequent runs made with this interactive program would then be made to bring the theoretical load-displacement behavior more in line with the observed behavior.

### 3.2 OVERALL FLOW DIAGRAM

Flow diagrams of the computer program are shown in Figures 10 and 11. The call to the subroutine RDDATA will initiate the entering of the test conditions interactively. Tabulated values of  $h_1$ ,  $h_2$  and  $h_3$  in Tables 2 through 6 are organized into separate subroutines in the program. Once the geometry and stress condition are interactively chosen, the program calls the corresponding subroutine with  $h_1$ ,  $h_2$  and  $h_3$ .

With the input material constants and crack length information, the program makes a call to subroutine TBL2. Subroutine TBL2 is a module which returns the linearly interpolated values of  $h_1$ ,  $h_2$  and  $h_3$  to the program for the given  $n$  and  $a/b$  condition.

After the interpolation of the corresponding  $h_1$ ,  $h_2$  and  $h_3$ , the program will make calls to the calculating subroutines JKCAL, LDSCAL and CODCAL. The subroutine JKCAL is written in such a way that for the selected geometry, it will compute  $K$ ,  $P_o^T$ ,  $\gamma_y$  and, elastic and plastic components of the

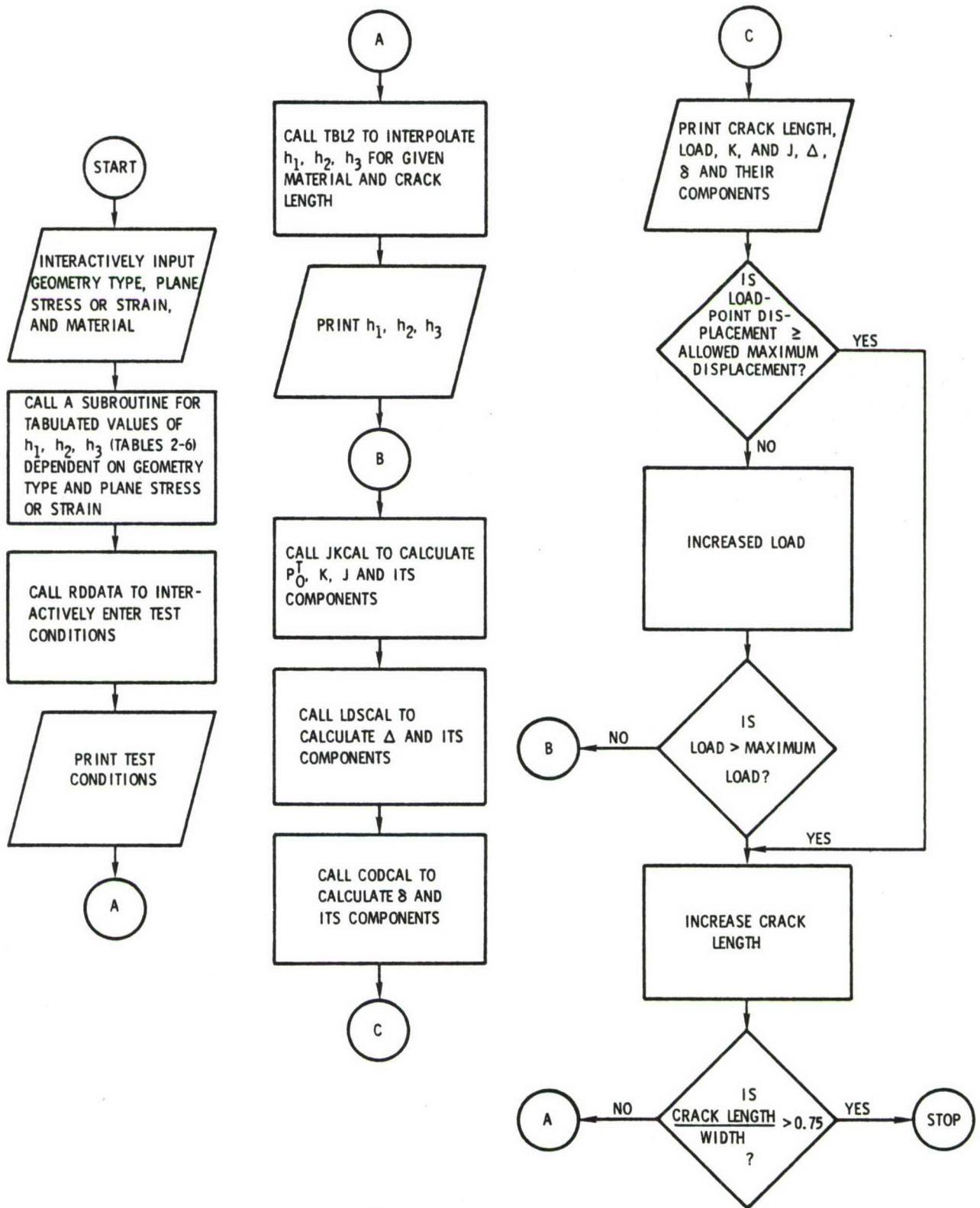


Figure 10. Overall Flow Diagram of the Computer Program.

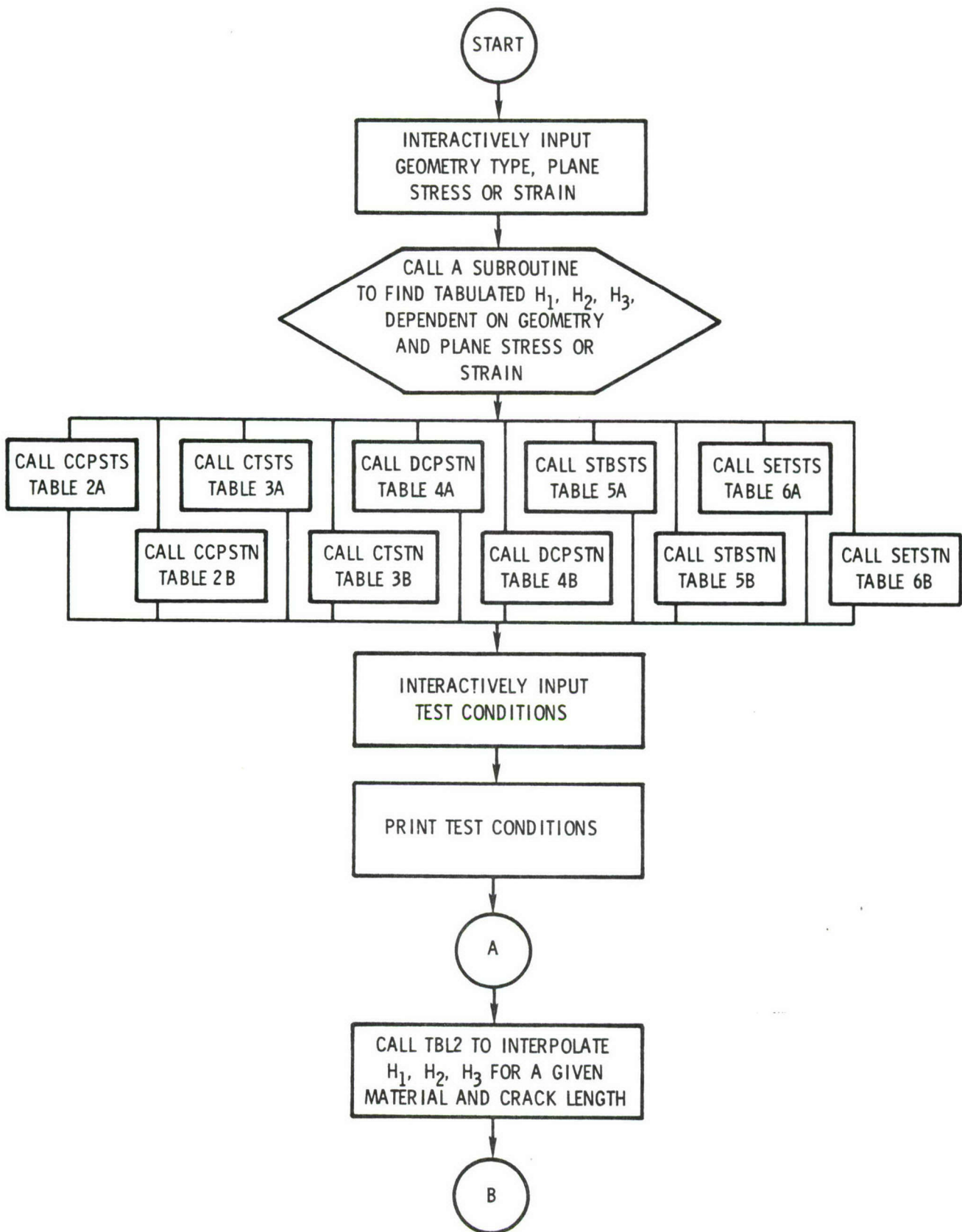


Figure 11a. Detailed Flow Diagram of the Computer Program.

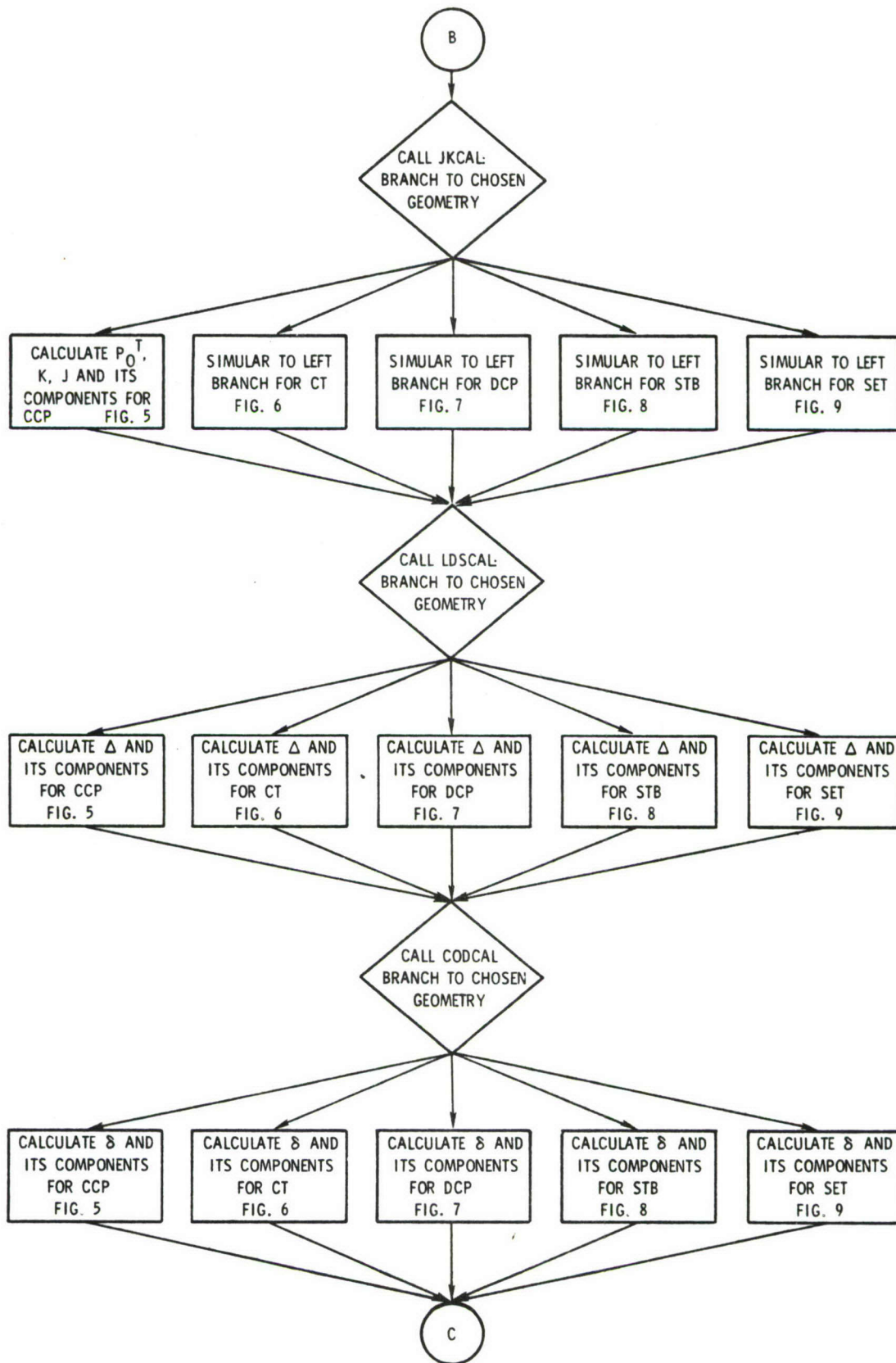


Figure 11b. Detailed Flow Diagram of the Computer Program.

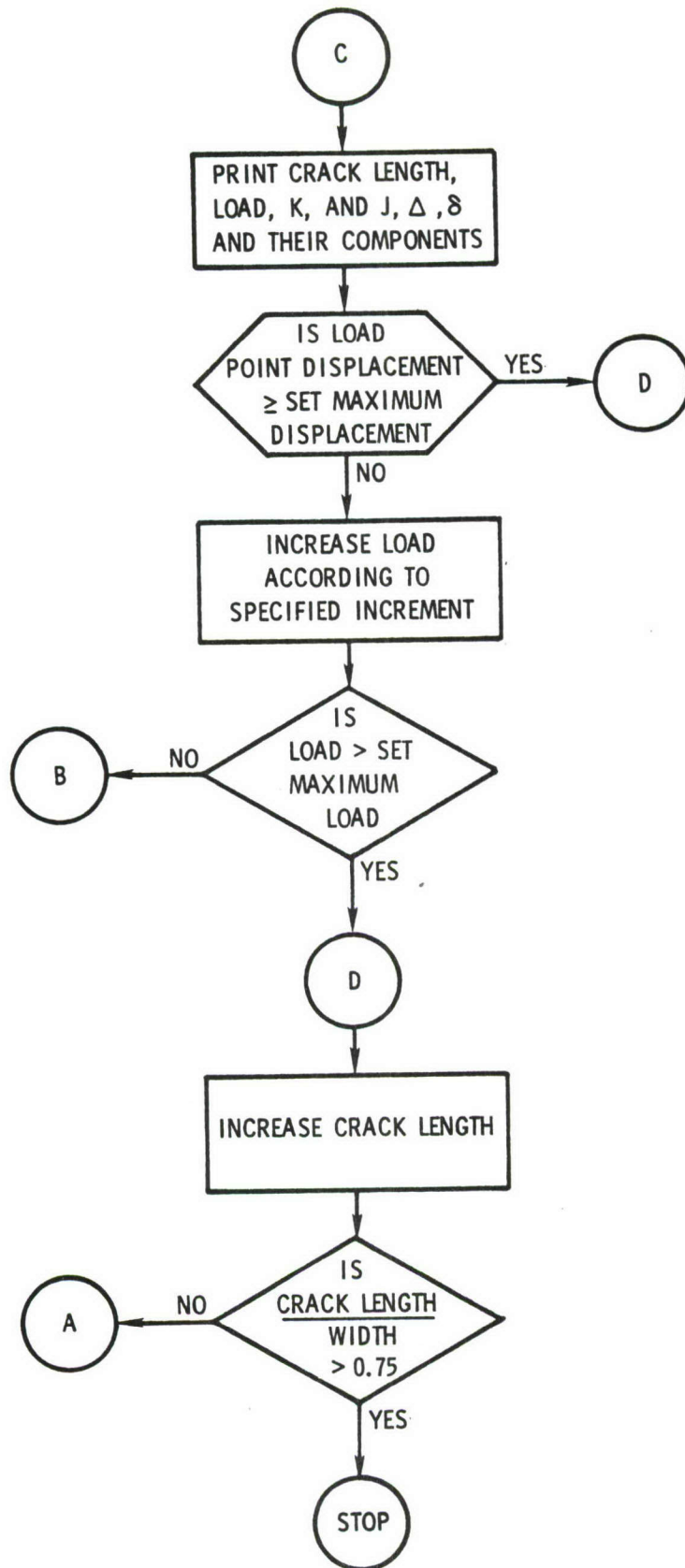


Figure 11c. Detailed Flow Diagram of the Computer Program.

J-integral and return these values to the main program. Sub-routines LDSCAL and CODCAL compute and return the values of  $\Delta_n^e$ ,  $\Delta_C^p$ ,  $\Delta_C^e$  and,  $\delta^e$  and  $\delta^p$  respectively, to the main program.

Once these calculations are completed for the given crack length and load, calculated values are printed. Then the load values and crack length values are increased and calculations are performed as described in the flow diagrams in Figures 10 and 11. This procedure continues until a/b exceeds 0.75.

### 3.3 PROGRAM OUTPUT

The program has been organized so that the output is currently restricted to yield information on only one geometry and one material for each run. The major variable controlling output is physical crack length; the other parameters are described as a function of increasing load for each crack length. Table 7 has been prepared to provide a listing of the output parameter symbols and their description. Appendix B provides the output for a typical run. Table 8 describes some of the cross-correlations which are possible based on the EST program output. Figure 12 was prepared to schematically describe how several of the possible cross-correlations would appear. Alternate presentation schemes are possible through a slight reconfiguration of the algorithm described in the flow diagrams.

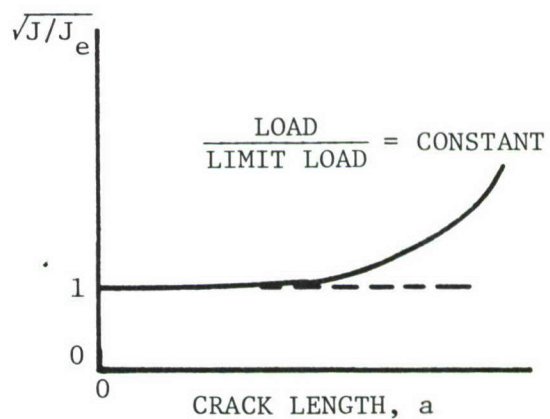
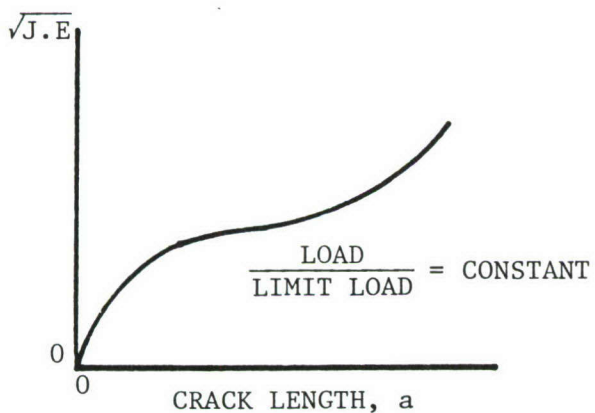
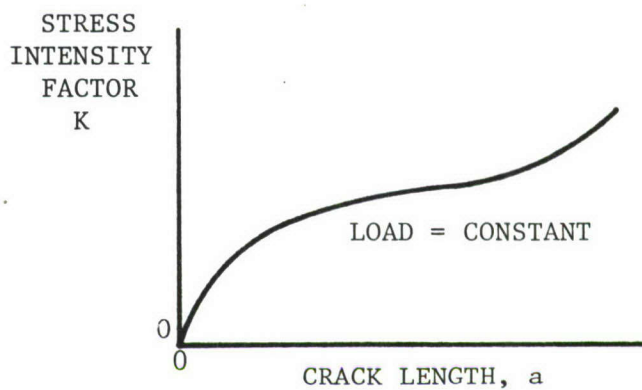
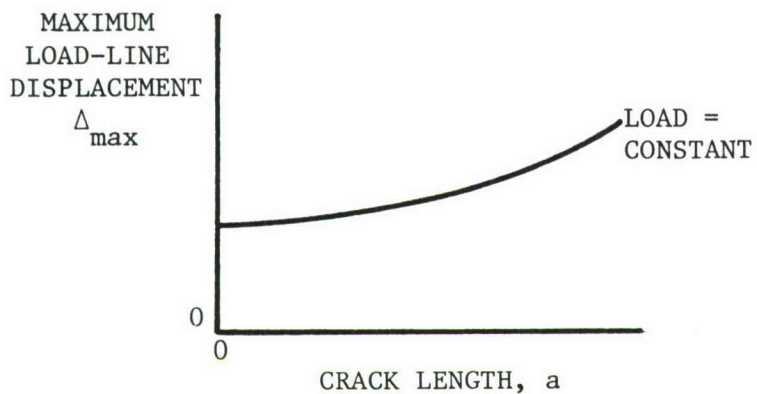
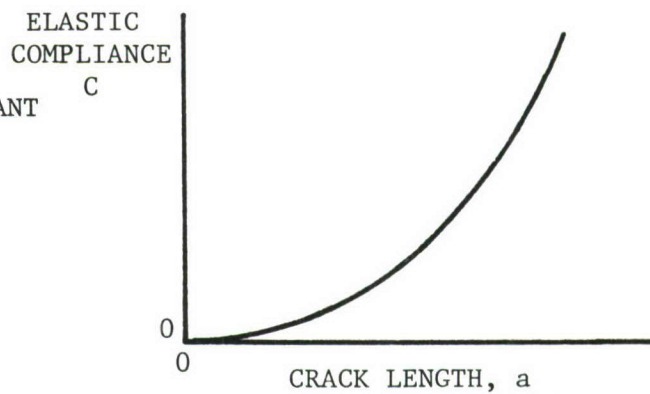
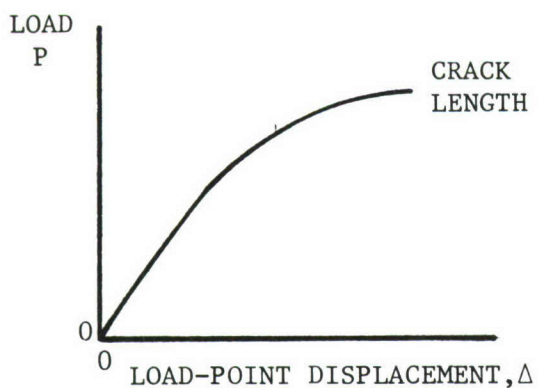


Figure 12. Sample of Program Output Presented in Schematic Diagrams.

TABLE 7  
PARAMETER OUTPUT AND THEIR SYMBOLS

<u>Computer Symbol</u>	<u>Text Symbol</u>	<u>Description</u>	<u>Units (English/Metric)</u>
A	a	Physical Crack Length	in/m
AE	$a_e$	Effective Crack Length	in/m
ALPHA	$\alpha$	Hardening Constant	--
CMOD	$\delta$	Crack Mouth Opening Displacement	in/m
E	E	Elastic (or Young's) Modulus	ksi/Pa
EC	$\Delta_c^e$	Elastic Crack (Load-line) Displacement	in/m
EN	$\Delta_n^e$	Elastic No-Crack (Load-line) Displacement Component	in/m
EPO	$\epsilon_0$	Reference Strain	--
H1	$h_1(a/b, n)$	J-Integral Plastic Function	--
H2	$h_2(a/b, n)$	CMOD Plastic Function	--
H3	$h_3(a/b, n)$	Plastic Load-line Displacement Function	--
J	J	J-Integral	kip-in/N-m
K	K or SIF	Stress Intensity Factor	ksi $\sqrt{\text{in}}$ /Pa $\sqrt{\text{m}}$
LC	k	Limit Load Correction Factor	--

TABLE 7 (Continued)  
PARAMETER OUTPUT AND THEIR SYMBOLS

<u>Computer Symbol</u>	<u>Text Symbol</u>	<u>Description</u>	<u>Units (English/Metric)</u>
LDISP	$\Delta$	Load Point Displacement	in/m
N	n	Ramberg-Osgood Exponent	--
PC	$\Delta_C^P$	Plastic Crack (Load Line) Displacement	in/m
PN	$\Delta_n^P$	Plastic No-Crack (Load Line)	in/m
P/PO	$P/P_0$	Ratio of Load to Limit Load	--
PZC	--	Plastic Zone Correction Factor (Monotonic=1; Cyclic=2)	--
SIGO	$\sigma_0$	Yield Strength	ksi/Pa
SPAN	2L	Gauge Length	in/m
SQRT(EJ)	$\sqrt{EJ}$	Square Root of (E.J)	ksi $\sqrt{\text{in}}$ /Pa $\sqrt{\text{m}}$
SQRT(J/J <sup>e</sup> )	$\sqrt{J/J^e}$	Square Root of (J/J <sup>e</sup> )	--
TH	--	Thickness of the Specimen	in/m

TABLE 8

## POTENTIAL CROSS-CORRELATIONS POSSIBLE WITH PROGRAM OUTPUT

Primary Independent Variable (Abcissa)	Dependent Variable (Ordinate)	Secondary Independent Variable (Multiple Curves)
--	-------------------------------	--

## BASIC EXPERIMENTAL PARAMETERS

Load-line Displacement	Load	Crack Length
<ul style="list-style-type: none"> <li>● Total</li> <li>● Elastic Component</li> <li>● Plastic Component</li> </ul>		
Crack Length	Elastic Compliance	---
	Maximum Load-line Displacement	Load
	Load to Limit Load Ratio	Load
	Maximum Crack Mouth Opening Displacement	Load

## FATIGUE AND FRACTURE RELATED PARAMETERS

Crack length	Plastic Zone Size	Load, Field Parameters
	Effective Crack Length	Load, Field Parameters
	Stress Intensity Factor	Load
	J-Integral (HRR Field)	Load, Displacement, Load to Limit Load Ratio
	<ul style="list-style-type: none"> <li>● <math>\sqrt{J \cdot E}</math></li> <li>● <math>\sqrt{J/J_e}</math></li> </ul>	
	Crack Tip Opening Displacement	Load
Load-Line Displacement	J-Integral (HRR Field)	Crack Length
	<ul style="list-style-type: none"> <li>● <math>\sqrt{J \cdot E}</math></li> <li>● <math>\sqrt{J/J_e}</math></li> </ul>	

### 3.4 PROGRAM INPUT REQUIREMENTS

Necessary inputs to the EST computer program are material, structural geometry, and loading properties. A summary listing of the input parameters and their symbols are presented in Table 9 and a sample input listing is provided in Appendix C. As can be noted from this listing, the material hardening parameter  $\alpha$  does not appear. The reason for this is that EST calculates  $\alpha$  (ALPHA) internally assuming (1) that the constitutive equation is the Ramberg-Osgood stress-strain relationship:

$$\frac{\epsilon}{\epsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0}\right)^n \quad (21)$$

(2) that the reference stresses and strains are related by  $\sigma_0 = E\epsilon_0$ , (3) that

$$\alpha\epsilon_0 = \alpha\left(\frac{\sigma_0}{E}\right) = 0.002 \quad (22)$$

and (4) that the hardening exponent  $n$  is obtained by least squares fitting procedures. Equation 22 was derived with the use of Figure 13 and an analysis of Equation 21 for the condition  $\sigma = \sigma_0$  which shows

$$\epsilon_Y = \epsilon \Big|_{\sigma=\sigma_0} = \epsilon_0(1+\alpha) \quad (23)$$

When the EST program is compiled and run, the program prompts the user for necessary information. Input of the geometry, as well as selection of plane stress or plane strain condition allow the program to select the appropriate table of values for  $h_1$ ,  $h_2$  and  $h_3$  functions presented in subsection 2.3. The EST program calculates the EPFM parameters for different loads, starting from the user selected minimum load and ending at the user selected maximum load with load increments selected by the user for a given initial  $a/b$  ratio. By a suitable selection of crack length increment, it is possible to calculate the EPFM parameters of different crack lengths starting from the initial  $(a/b)$  for different loads.

TABLE 9  
EST COMPUTER PROGRAM INPUTS

<u>EST SYMBOL</u>	<u>TEXT SYMBOL</u>	<u>PROPERTY</u>	<u>DEFAULT VALUE</u>	<u>ENGLISH/METRIC UNITS</u>
SIGO	$\sigma_0$	Material 0.2 Percent Yield Strength		ksi/Pa
N	n	Ramberg-Osgood Exponent		---
E	E	Elastic Modulus		ksi/Pa
LC	k	Correction for Limit Load	1.0	---
PZC		Plastic Zone Correction Factor	1.0	---
---		Plane Stress/Plane Strain Condition		---
---		Specimen Geometry (CCP, CT, DCP, STB, SET)		---
W	2b	Structural Width		in/m
---	L	Half Span		in/m
TH		Thickness		in/m
A/B	a/b	Initial Crack Length/Width Ratio	0.25	---
		Crack Length Increment		in/m
		<u>Load/Displacement</u>		
PMAX		Maximum Load		kips/N
PMIN		Minimum Load	0.0	kips/N
		Load Increment		kips/N
DISMAX		Maximum Load-Line Displacement		in/m

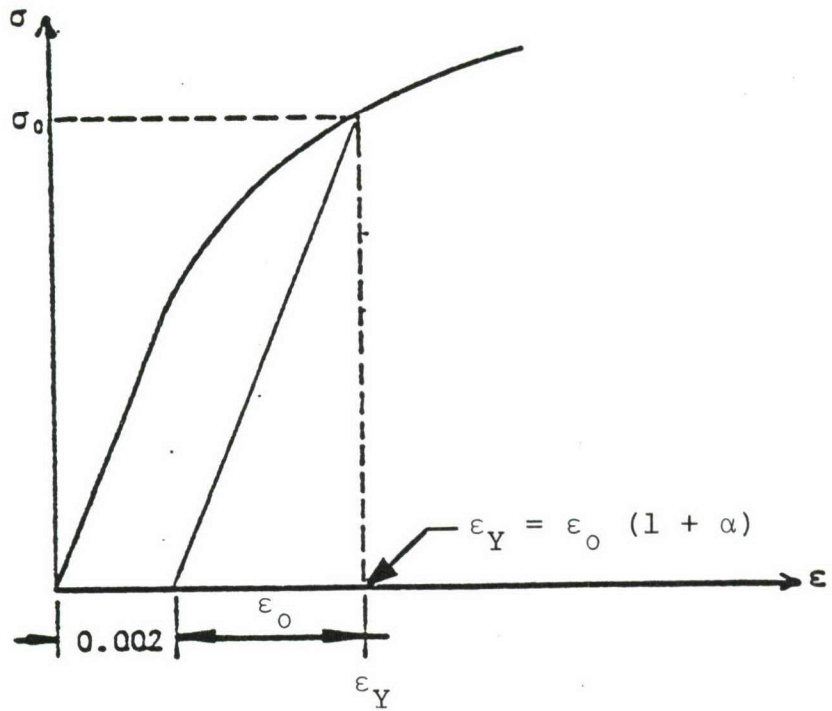


Figure 13. Material Stress-Strain Curve Describing the Values of  $\alpha$  and  $\epsilon_0$ .

In the input prompts, the plastic zone correction factor (PC) is taken as either 1 or 2. The value PC is used in Equation 16 as a multiplication factor on the yield strength  $\sigma_0$ ; when PC = 1 the loading is monotonically increasing, when PC = 2 the loading is cyclic.

There are several restrictions or bounds on the calculations that result from input or default conditions. The most obvious bounds are the limits on the load or load-line displacement values that are created by input. However, because the tabular values of the  $h_1$ ,  $h_2$ , and  $h_3$  functions are limited to the range of  $a/b$  between 0.25 and 0.75, the EST program frustrates any attempt to compute the value of these functions outside the allowable  $a/b$  range by resetting upper and lower limits. Another default condition occurs whenever the handling exponent (n) exceeds 20 in order to ensure that the  $h_1$ ,  $h_2$ ,  $h_3$  functions are not extrapolated beyond the tabulated values given in subsection 2.3.

## SECTION 4

### SUMMARY

A review of the theoretical basis for the estimation scheme proposed by Hutchinson and co-workers<sup>1,2</sup> and Shih and co-workers<sup>3,6</sup> was presented in previous sections. A computer program was written to implement the above estimation scheme. Certain modifications were incorporated into the program such that the limit load behavior computed by the program is compatible with the experimentally observed limit load behavior. The computer program can be used to calculate the elastic-plastic behavior of five different specimen geometries. Possible useful outputs from the program are given in Table 8.

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APPENDIX A

A LISTING OF THE FORTRAN IV  
PROGRAM EST  
DESIGNED TO RUN INTERACTIVELY  
ON THE CDC CYBER 175  
COMPUTER SYSTEM

```

PROGRAM EST(INPUT,OUTPUT,TAPE6,TAPE1=INPUT)
COMMON/A/E,EA,PLCOR,SIG0,XLL,B,FACTOR,XN,ALPHA,EP0,TH,PP1,IPL,
$ A,H1,H2,H3,PP,ZZ,ZZE,P0,AE,XKSQ,XJ,BETA,PI
COMMON/B/DELTE,DELTP,DELTA
COMMON/C/DELCE,DELCP,DELNE,DELNP,DELC,DELN,DELT,DELE,DELP
COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
COMMON/E/ PMIN,PMAX,PIN,ZZ0,AIN,DISMAX
COMMON/F/NUMAB,NUMN,ITYPE
COMMON/X/PPP0,XJJE

```

```

C
C PROGRAM EST CALCULATES J, DISPLACEMENT, AND DELTA
C FOR THE FOLLOWING SPECIMENS:
C

```

- C 1) CENTER CRACKED PANEL (CCP) --- PLANE STRAIN OR PLANE STRESS
- C 2) COMPACT TENSION SPECIMEN (CT) --- PLANE STRAIN
- C 3) DOUBLE-EDGE CRACK PANEL (DCP) --- PLANE STRAIN
- C 4) SINGLE-EDGE CRACK PANEL IN 3-POINT BENDING --- PLANE STRAIN
- C 5) SINGLE-EDGE CRACK PANEL IN TENSION (SET) --- PLANE STRAIN

```

C THE VALUES OF DISPLACEMENT VS. J, LOAD, AND DELTA ARE
C WRITTEN TO A FILE USED TO GENERATE PLOTS (UNIT 7).
C

```

```

11 REWIND 6
XNU=0.3
IFLAG=0
PRINT2100
2100 FORMAT(*0ENTER SPECIMEN GEOMETRY*,/,
$ * (CCP,CT,DCP,STB,SET)==| *)
READ(1,100)JTYPE
100 FORMAT(A3)
PRINT2200
2200 FORMAT(* PLANE STRESS (0) OR PLANE STRAIN (1) ? *)
READ*,IPL
PRINT2500
2500 FORMAT(* MATERIAL ==| *)
READ(1,2600)MAT
2600 FORMAT(A10)
WRITE(6,200)MAT
200 FORMAT(* MATERIAL: *,A10)
IF(JTYPE.EQ.3HCCP.AND.IPL.EQ.0)
$ CALL CCPSTS
IF(JTYPE.EQ.3HCCP.AND.IPL.EQ.1)
$ CALL CCPSTN
IF(JTYPE.EQ.3HCT .AND. IPL.EQ.0)
$ CALL CTSTS
IF(JTYPE.EQ.3HCT .AND. IPL.EQ.1)
$ CALL CTSTN
IF(JTYPE.EQ.3HDCP .AND. IPL.EQ.0)
$ CALL DCPSTS
IF(JTYPE.EQ.3HDCP .AND. IPL.EQ.1)
$ CALL DCPSTN
IF(JTYPE.EQ.3HSTB .AND. IPL.EQ.0)
$ CALL STBSTS
IF(JTYPE.EQ.3HSTB .AND. IPL.EQ.1)
$ CALL STBSTN

```

```

      IF(JTYPE.EQ.3HSET .AND. IPL.EQ.0)
$ CALL SETSTS
      IF(JTYPE.EQ.3HSET .AND. IPL.EQ.1)
$ CALL SETSTN
2300  IF(IFLAG.EQ.0) PRINT2300
      FORMAT(*0INVALID GEOMETRY*)
      IF(IFLAG.EQ.0) GO TO 11
C
C   READ VARIABLES INTERACTIVELY
C
      CALL RDDATA
      A0=ZZ0*B
      A=A0
      BETA=2.0
      IF(IPL .NE. 0) BETA=6.0
      WRITE(6,300)
300  FORMAT(6X,*TH*,9X,*SPAN*,11X,*N*,13X,*SIG0*,7X,*PZC*,14X,
$      *E*,8X,*WIDTH*,10X,*LC*,9X,*ALPHA*,10X,*EP0*,/)
      EA=E
      IF(IPL.NE.0) EA=E/(1.-XNU**2)
      EP0=SIG0/E
      ALPHA=0.002/EP0
      SPAN=2*XLL
      B2=B
      IF(ITYPE.EQ.1.OR.ITYPE.EQ.3) B2=B*2
      WRITE(6,400) TH, SPAN, XN, SIG0, PLCOR, E, B2, FACTOR, ALPHA, EP0
400  FORMAT(10G13.4)
12   ZZ=A/B
      IF(XN.GT.20.0) XN=20.0
      WRITE(6,1300) XN, ZZ
1300 FORMAT(*0 X = *,G14.4,5X,*A/B = *,G14.4)
C
C   INTERPOLATE FOR VALUES OF H1,H2,H3
C
      CALL TBL2(X,Y,Z1,NUMN,NUMAB,NUMN,XN,ZZ,H1,IER)
      CALL TBL2(X,Y,Z2,NUMN,NUMAB,NUMN,XN,ZZ,H2,IER)
      CALL TBL2(X,Y,Z3,NUMN,NUMAB,NUMN,XN,ZZ,H3,IER)
      WRITE(6,1400) H1,H2,H3
1400 FORMAT(* H1 = *,F12.4,3X,*H2 = *,F12.4,3X,*H3 = *,F12.4)
      IF(METENG .EQ. 1HM) WRITE(6,1500)
      IF(METENG .EQ. 1HE) WRITE(6,1600)
1500 FORMAT(*0ALL RESULTS IN METRIC UNITS*,/)
1600 FORMAT(*0ALL RESULTS IN ENGLISH UNITS*,/)
      IF(ITYPE.EQ.1) WRITE(6,500)
500  FORMAT(T4,*2A*,T8,*2A/W*,T15,*2AE*,T21,*P/P0*,T31,
$ *LOAD*,T42,*K*,T53,*J*,T59,*SQRT(EJ)*,T68,*SQRT(J/JE)*,
$ T81,*CMOD*,T91,*LDISP*,T102,*EN*,T111,*PN*,T119,*EC*,T128,*PC*)
      IF(ITYPE.NE.1) WRITE(6,600)
600  FORMAT(T5,*A*,T9,*A/W*,T15,*AE*,T21,*P/P0*,T31,
$ *LOAD*,T42,*K*,T53,*J*,T59,*SQRT(EJ)*,T68,*SQRT(J/JE)*,
$ T81,*CMOD*,T91,*LDISP*,T102,*EN*,T111,*PN*,T119,*EC*,T128,*PC*)
      P1=PMIN
13   CONTINUE

```

```

C
C      CALCULATE J AND K
CALL JKCAL

C
C      CALCULATE LOAD-POINT DISPLACEMENT
CALL LDSCAL

C
C      CALCULATE CRACK-OPENING DISPLACEMENT
CALL CODCAL

C
XK=SQRT(XKSQ)
XXJ=SQRT(XJ*EA)
XJJE=SQRT(XJJE)
AE2=AE
A2=A
IF(ITYPE .EQ. 1) A2=A*2
IF(ITYPE .EQ. 1) AE2=AE*2
WRITE(6,700)A2,ZZ,AE2,PPP0,PP1,XK,XJ,XXJ,XJJE,DELTA,DELT,DELNE,
$DELNP,DELCE,DELCP
700  FORMAT(1H ,2F5.3,F6.3,E10.3,F9.3,3E10.3,F10.7,E10.3,E9.3,E10.3,
$ 3E9.3)
IF(DELT .GE. DISMAX) GO TO 15
PP1=PP1+PIN
IF (PP1 .LT. PMAX) GO TO 13
15  CONTINUE
A=A+AIN
IF (A/B .LE. 3./4.) GO TO 12
STOP
END
C

```

SUBROUTINE RDDATA

C  
C  
C

\*\* INITIALIZES VARIABLES AND PROMPTS FOR USER INPUT \*\*

```

COMMON/A/E,EA,PLCOR,SIG0,XLL,B,FACTOR,XN,ALPHA,EPO,TH,PP1,IPL,
$A,H1,H2,H3,PP,ZZ,ZZE,P0,AE,XKSQ,XJ,BETA,PI
COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
COMMON/F/NUMAB,NUMN,ITYPE
COMMON/E/ PMIN,PMAX,PIN,ZZO,AIN,DISMAX
DATA PMIN/0.0/,PIN/.25/,PMAX/5.3/,ZZO/.25/,XN/36./,
$ SIG0/33E3/,PLCOR/1./,E/17E6/,XLL/2./,B/1.995/,
$ TH/0.498/,AIN/0.1/,FACTOR/1./,DISMAX/0.01/
PRINT1600
READ(1,2000)METENG
PRINT100,PMIN
READ(1,2000)KANS
IF(KANS.NE.1HY)READ*,PMIN
PRINT200,PIN
READ(1,2000)KANS
IF(KANS.NE.1HY)READ*,PIN
PRINT300,PMAX
READ(1,2000)KANS
IF(KANS.NE.1HY)READ*,PMAX
PRINT500,ZZO
READ(1,2000)KANS
IF(KANS.NE.1HY)READ*,ZZO
PRINT600,XN
READ(1,2000)KANS
IF(KANS.NE.1HY)READ*,XN
PRINT700,SIG0
READ(1,2000)KANS
IF(KANS.NE.1HY)READ*,SIG0
PRINT800,PLCOR
READ(1,2000)KANS
IF(KANS.NE.1HY)READ*,PLCOR
PRINT900,E
READ(1,2000)KANS
IF(KANS.NE.1HY)READ*,E
PRINT1000,XLL
READ(1,2000)KANS
IF(KANS.NE.1HY)READ*,XLL
PRINT1100,B
READ(1,2000)KANS
IF(KANS.NE.1HY)READ*,B
IF(ITYPE.EQ.1.OR.ITYPE.EQ.3)B=B/2.0
PRINT1200,TH
READ(1,2000)KANS
IF(KANS.NE.1HY)READ*,TH
PRINT1300,AIN
READ(1,2000)KANS
IF(KANS.NE.1HY)READ*,AIN
PRINT1400,FACTOR
READ(1,2000)KANS
IF(KANS.NE.1HY)READ*,FACTOR
PRINT1500,DISMAX
READ(1,2000)KANS
IF(KANS.NE.1HY)READ*,DISMAX

```

```

100  FORMAT(*0IF THE FOLLOWING VALUES ARE CORRECT,*,/,
      $* RESPOND WITH A "Y". IF THEY ARE INCORRECT,*,/,
      $* ANSWER "N", THEN ENTER THE CORRECT VALUE*,/,
      $*OMINIMUM LOAD (KIPS/N) = *,G10.5,* (Y/N)? *)
200  FORMAT(* LOAD INCREMENT (KIPS/N) = *,G11.5,* (Y/N)? *)
300  FORMAT(* MAXIMUM LOAD(KIPS/N) = *,G7.2,* (Y/N)? *)
500  FORMAT(* INITIAL A/B = *,G11.5,* (Y/N)? *)
600  FORMAT(* RAMBERG-OSGOOD EXPONENT - N = *,G11.5,* (Y/N)? *)
700  FORMAT(* YIELD STRENGTH - SIG0 (KSI/PA) = *,G14.5,* (Y/N)? *)
800  FORMAT(* PLASTIC ZONE CORRECTION FACTOR - PZC= *,G14.5,* (Y/N)? *)
900  FORMAT(* ELASTIC MODULUS - E (KSI/PA) = *,G14.5,* (Y/N)? *)
1000 FORMAT(* HALF SPAN (IN/M) = *,G11.5,* (Y/N)? *)
1100 FORMAT(* WIDTH - W (IN/M) = *,G11.5,* (Y/N)? *)
1200 FORMAT(* THICKNESS - TH (IN/M) = *,G12.5,* (Y/N)? *)
1300 FORMAT(* CRACK LENGTH INCREMENT (IN/M) = *,G10.5,* (Y/N)? *)
1400 FORMAT(* CORR. FACTOR FOR LIMIT LOAD - LC = *,G10.5,* (Y/N)? *)
1500 FORMAT(* MAXIMUM DISPLACEMENT (IN/M) = *,G10.5,* (Y/N)? *)
1600 FORMAT(* E(NGLISH) OR M(ETRIC) ? *)
2000 FORMAT(A1)
      RETURN
      END

```

SUBROUTINE TBL2 (X, Y, Z, NX, NY, NDX, XO, YO, ZO, IER)

TBL2 FROM THE COMPUTER CENTER LIBRARY OF 6600 ROUTINES

SUBROUTINE TBL2

PERFORMS DOUBLE LINEAR INTERPOLATION FROM A TABLE OF VALUES  
OF Z VERSES X AND Y.

USAGE

CALL TBL2 (X,Y,Z,NX,NDX,XO,YO,ZO,IER)

DESCRIPTION OF PARAMETERS

X VALUES OF THE FIRST INDEPENDENT VARIABLE IN INCREASING  
ORDER  
Y VALUES OF THE SECOND INDEPENDENT VARIABLE IN INCREASING  
ORDER  
Z VALUES OF THE DEPENDENT VARIABLE. Z(I,J) IS THE VALUE  
OF Z CORRESPONDING TO X(I), Y(J) AND IS REFERENCED AS  
Z((J-1)\*NDX+I)  
NX NUMBER OF ENTRIES IN THE X ARRAY  
NY NUMBER OF ENTRIES IN THE Y ARRAY  
NDX DIMENSION IN THE X DIRECTION OF THE Z ARRAY (NDX.GE.NX)  
XO X COORDINATE OF POINT AT WHICH INTERPOLATION IS DESIRED  
YO Y COORDINATE OF POINT AT WHICH INTERPOLATION IS DESIRED  
ZO THE COMPUTED INTERPOLATED VALUE  
IER = 0 INTERPOLATION SUCESSFULLY PERFORMED  
= 1 EXTRAPOLATION SUCESSFULLY PERFORMED  
= 2 ERROR CONDITION. EITHER TWO ADJACENT INDEPENDENT  
VARIABLES ARE NOT IN INCREASING ORDER OR NX IS GREATER  
THAN NDX

METHOD

XO AND YO ARE LOCATED RELATIVE TO THE X AND Y ARRAYS. IF  
EXTRAPOLATION IS NOT NECESSARY, REPEATED BISECTION OF THE  
INDICES OF THE LISTS OF X AND Y VALUES IS CARRIED OUT UNTIL  
BOTH ARGUMENTS ARE ISOLATED BETWEEN PAIRS OF CONSECUTIVE  
VALUES. AFTER LOCATION OF THE NEAREST PAIR OF ARGUMENTS FOR  
EACH OF THE TWO INDEPENDENT VARIABLES, DOUBLE LINEAR INTER-  
POLATION (EXTRAPOLATION) IS PERFORMED.

EXAMPLE - GIVEN THE FUNCTION  $Z = X+2Y$ ,  $X=1,2$  AND  $Y=0,1$  STORAGE  
AS A ONE DIMENSIONAL OR TWO DIMENSIONAL ARRAY COULD BE AS

FOLLOWS  $Z(1) = Z(1,1) = 1$        $Z(3) = Z(1,2) = 3$   
 $Z(2) = Z(2,1) = 2$        $Z(4) = Z(2,2) = 4$

WHERE THE DIMENSION OF Z IS Z(4) OR Z(2,2) AND NDX = 2

OR  $Z(1) = Z(1,1) = 1$        $Z(5) = Z(1,2) = 3$   
 $Z(2) = Z(2,1) = 2$        $Z(6) = Z(2,2) = 4$

WHERE THE DIMENSION OF Z IS Z(16) OR Z(4,4) AND NDX = 4.

DIMENSION X(1), Y(1), Z(1)

IF (NX.GT.NDX) GO TO 12

```

C      DETERMINE WHETHER EXTRAPOLATION IS NECESSARY IN THE X DIRECTION
      I1 = 1
      IF (XO.LT.X(1)) GO TO 4
      IF (XO.GT.X(NX)) GO TO 3
C
C      LOCATE XO WITHIN X(I) ARRAY FOR INTERPOLATION
      IER = 0
      I2 = NX
1  IF ((I2-I1).LT.2) GO TO 5
      I = (I1+I2)/2
      IF (XO.LT.X(I)) GO TO 2
      I1 = I
      GO TO 1
2  I2 = I
      GO TO 1
C
C      SET INDICES AND FLAG FOR EXTRAPOLATION
3  I1 = NX-1
4  IER = 1
C
C      DETERMINE WHETHER EXTRAPOLATION IS NECESSARY IN THE Y DIRECTION
5  J1 = 1
      IF (YO.LT.Y(1)) GO TO 9
      IF (YO.GT.Y(NY)) GO TO 8
C
C      LOCATE YO WITHIN Y(I) ARRAY FOR INTERPOLATION
      J2 = NY
6  IF ((J2-J1).LT.2) GO TO 10
      J = (J1+J2)/2
      IF (YO.LT.Y(J)) GO TO 7
      J1 = J
      GO TO 6
7  J2 = J
      GO TO 6
C
C      SET INDICES AND FLAG FOR EXTRAPOLATION
8  J1 = NY-1
9  IER = 1
C
C      COMPUTE ZO USING BIVARIATE INTERPOLATION
10 DIV = (X(I1+1) - X(I1)) * (Y(J1+1) - Y(J1))
      IF (DIV) 12,12,11
11 I11 = (J1-1)*NDX + I1
      I12 = I11 + NDX
      X20 = X(I1+1) - XO
      X01 = XO - X(I1)
C      ZO = (Y2-YO)((X2-XO)Z11 + (XO-X1)Z21) +
C          (YO-Y1)((X2-XO)Z12 + (XO-X1)Z22) / (X2-X1)(Y2-Y1)
      ZO = ((Y(J1+1)-YO)*(X20*Z(I11) + X01*Z(I11+1)) +
+          (YO-Y(J1)) * (X20*Z(I12) + X01*Z(I12+1))) / DIV
      RETURN
12 IER = 2
      RETURN
      END

```

## SUBROUTINE JKCAL

C  
C  
C \*\* CALCULATES ELASTIC AND PLASTIC COMPONENTS OF J \*\*

COMMON/A/E,EA,PLCOR,SIG0,XLL,B,FACTOR,XN,ALPHA,EP0,TH,PP1,IPL,  
\$ A,H1,H2,H3,PP,ZZ,ZZE,P0,AE,XKSQ,XJ,BETA,PI  
COMMON/X/PPP0,XJJE  
COMMON/F/NUMAB,NUMN,ITYPE  
FSQCCP(Z) = 1./((COS((3.14159/2.)\*Z))  
FSQCT(Z) = (((2+Z)\*(0.886+4.64\*Z-13.32\*Z\*\*2+14.72\*Z\*\*3  
\$ -5.6\*Z\*\*4)\*\*2)/(Z\*(1-Z)\*\*3)  
FSQSTB(Z) = (1.09-1.735\*Z+8.2\*Z\*\*2-14.18\*Z\*\*3+14.57\*Z\*\*4)\*\*2  
FSQDCP(Z) = (1.122-0.561\*Z-0.205\*Z\*\*2+0.471\*Z\*\*3-0.19\*Z\*\*4)\*\*2/(1-Z)  
FSQST(Z) = (2/(PI\*Z))\*TAN(PI\*Z/2.)\*((0.752+2.02\*Z+0.37  
\$ \*(1-SIN(PI\*Z/2.))\*\*3)/COS(PI\*Z/2.))\*\*2  
PI=3.14159  
PP=PP1/TH  
ZZ=A/B  
GO TO (1,2,3,4,5)ITYPE  
C  
1 CCP SPECIMEN  
P0=2.\*SIG0\*(B-A)\*FACTOR  
IF(IPL.NE.0) P0=2.31\*SIG0\*(B-A)\*FACTOR  
XKSQ=(PP\*\*2\*PI\*A\*FSQCCP(ZZ))/(4.\*B\*\*2)  
PHI=1./(1.+(PP/P0)\*\*2)  
RY=(XN-1.)\*XKSQ/(BETA\*PI\*(XN+1.)\*(SIG0\*PLCOR)\*\*2)  
AE=A+PHI\*RY  
ZZE=AE/B  
XJE=(PI\*AE\*FSQCCP(ZZE)\*(PP\*\*2))  
\$ /(EA\*4.\*B\*\*2)  
XJP=ALPHA\*SIG0\*EP0\*A\*((B-A)/B)\*H1\*(PP/P0)\*\*(XN+1.)  
GO TO 6  
C  
2 CT SPECIMEN  
XNETA=SQRT(((2.\*A)/(B-A))\*\*2+((4.\*A)/(B-A))  
\$ +2.)-(2.\*A)/(B-A)+1.)  
P0=1.071\*XNETA\*(B-A)\*SIG0\*FACTOR  
IF(IPL.NE.0) P0=1.455\*XNETA\*(B-A)\*SIG0\*FACTOR  
XKSQ=(PP\*\*2\*A\*FSQCT(ZZ))/(B\*\*2)  
PHI=1./(1.+(PP/P0)\*\*2)  
RY=(XN-1.)\*XKSQ/(BETA\*PI\*(XN+1.)\*(SIG0\*PLCOR)\*\*2)  
AE=A+PHI\*RY  
ZZE=AE/B  
XJE=(AE\*FSQCT(ZZE)\*(PP\*\*2))/(EA\*B\*\*2)  
XJP=ALPHA\*SIG0\*EP0\*(B-A)\*H1\*(PP/P0)\*\*(XN+1)  
GO TO 6  
C  
3 DCP SPECIMEN  
P0=2.31\*SIG0\*(B-A)\*FACTOR  
IF(IPL.NE.0) P0=5.94\*SIG0\*(B-A)\*FACTOR  
XKSQ=(PP\*\*2\*PI\*A\*FSQDCP(ZZ))/(2.\*B)\*\*2  
PHI=1./(1.+(PP/P0)\*\*2)  
RY=(XN-1.)\*XKSQ/(BETA\*PI\*(XN+1.)\*(SIG0\*PLCOR)\*\*2)  
AE=A+PHI\*RY  
ZZE=AE/B  
XJE=(PI\*AE\*FSQDCP(ZZE)\*(PP\*\*2))/(EA\*B\*\*2)  
XJP=ALPHA\*SIG0\*EP0\*(B-A)\*H1\*(PP/P0)\*\*(XN+1.)  
GO TO 6

```

C      STB SPECIMEN
4      P0=(0.536*SIG0*(B-A)**2/XLL)*FACTOR
      IF(IPL.NE.0) P0=(0.728*SIG0*(B-A)**2/XLL)*FACTOR
      XKSQ=9.*PP**2*XLL**2*PI*A*FSQSTB(ZZ)/((B)**4)
      PHI=1./(1.+(PP/P0)**2)
      RY=(XN-1.)*XKSQ/(BETA*PI*(XN+1.)*(SIG0*PLCOR)**2)
      AE=A+PHI*RY
      ZZE=AE/B
      XJE=(9.*PI*AE*XLL**2*FSQSTB(ZZE)*PP**2)/(EA*(B)**4)
      XJP=ALPHA*SIG0*EP0*(B-A)*H1*(PP/P0)**(XN+1.)
      GO TO 6
C      SET SPECIMEN
5      XNETA=SQRT(1+(A/(B-A))**2)-(A/(B-A))
      P0=1.455*XNETA*(B-A)*SIG0*FACTOR
      IF(IPL.EQ.0) P0=1.072*XNETA*(B-A)*SIG0*FACTOR
      XKSQ=(PP**2*PI*A*FSQST(ZZ))/(B**2)
      PHI=1.0/(1.0+(PP/P0)**2)
      RY=(XN-1.0)*XKSQ/(BETA*PI*(XN+1.0)*(SIG0*PLCOR)**2)
      AE=A+PHI*RY
      ZZE=AE/B
      XJE=(PI*AE*FSQST(ZZE)*(PP**2))/(EA*B**2)
      XJP=ALPHA*SIG0*EP0*A*((B-A)/B)*H1*(PP/P0)**(XN+1.)
6      XJ=XJE+XJP
      XJEE=(XKSQ/EA)
      IF(XJEE.EQ.0.0) XJJE=0.0
      IF(XJEE.NE.0.0) XJJE=(XJ/XJEE)
      PPP0=PP/P0
      RETURN
      END

```





## SUBROUTINE CCPSTS

COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG

COMMON/F/NUMAB,NUMN,ITYPE

DATA X/1.,1.5,2.,3.,5.,7.,10.,13.,16.,20./

DATA Y/0.125,0.2,0.25,0.375,0.5,0.625,0.75,3\*0.0/

DATA Z1/

\$ 2.80, 3.23, 3.54, 4.00, 4.52, 4.76, 4.86, 3.25, 3.95, 3.65,

\$ 2.63, 3.00, 3.15, 3.42, 3.62, 3.65, 3.43, 2.95, 2.89, 2.55,

\$ 2.54, 2.82, 2.97, 3.14, 3.19, 3.11, 2.90, 2.65, 2.47, 2.20,

\$ 2.34, 2.40, 2.53, 2.52, 2.35, 2.17, 1.95, 1.77, 1.61, 1.43,

\$ 2.21, 2.24, 2.20, 2.06, 1.81, 1.63, 1.43, 1.30, 1.17, 1.00,

\$ 2.11, 2.07, 1.91, 1.69, 1.41, 1.22, 1.01, 0.85, 0.71, 0.57,

\$ 2.07, 1.89, 1.71, 1.46, 1.21, 1.08, 0.96, 0.75, 0.65, 0.53,

\$ 30\*0.0/

DATA Z2/

\$ 3.53, 3.86, 4.11, 4.64, 4.83, 4.94, 4.89, 4.40, 3.37, 3.11,

\$ 3.28, 3.45, 3.60, 3.73, 3.61, 3.55, 3.25, 2.95, 2.57, 2.48,

\$ 3.12, 3.24, 3.29, 3.30, 3.15, 2.93, 2.59, 2.29, 2.08, 1.81,

\$ 2.71, 2.65, 2.62, 2.41, 2.03, 1.75, 1.47, 1.28, 1.13, 0.99,

\$ 2.34, 2.18, 2.01, 1.70, 1.30, 1.07, 1.87, 0.76, 0.67, 0.56,

\$ 1.97, 1.72, 1.46, 1.13, 0.79, 0.62, 0.47, 0.38, 0.31, 0.26,

\$ 1.61, 1.22, 0.97, 0.69, 0.45, 0.36, 0.29, 0.22, 0.18, 0.15,

\$ 30\*0.0/

DATA Z3/

\$ 0.35, 0.49, 0.64, 0.95, 1.54, 2.05, 2.63, 3.35, 3.38, 3.11,

\$ 0.52, 0.69, 0.90, 1.24, 1.77, 2.09, 2.37, 2.55, 2.57, 2.48,

\$ 0.61, 0.82, 1.01, 1.35, 1.83, 2.08, 2.19, 2.12, 2.01, 1.79,

\$ 0.81, 1.00, 1.19, 1.43, 1.59, 1.57, 1.43, 1.27, 1.13, 0.99,

\$ 0.93, 1.09, 1.19, 1.26, 1.18, 1.04, 0.87, 0.76, 0.67, 0.56,

\$ 0.98, 1.07, 1.05, 0.97, 0.76, 0.62, 0.48, 0.39, 0.32, 0.27,

\$ 0.93, 0.89, 0.80, 0.64, 0.45, 0.36, 0.29, 0.22, 0.18, 0.15,

\$ 30\*0.0/

WRITE(6,100)

100 FORMAT(\* CENTER CRACK PANEL --- PLANE STRESS\*,//)

ITYPE=1

NUMAB=7

NUMN=10

IFLAG=1

RETURN

END

```

SUBROUTINE CCPSTN
COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
COMMON/F/NUMAB,NUMN,ITYPE
DIMENSION XX(7),XY(7),XZ1(49),XZ2(49),XZ3(49)
DATA XX/ 1.,1.5,2.,3.,5.,7.,10./
DATA XY/ 0.25,0.3,0.4,0.5,0.6,0.7,0.75/
DATA XZ1/
$ 2.570, 2.900, 3.110, 3.350, 3.490, 3.430, 3.230,
$ 2.450, 2.720, 2.890, 3.100, 3.110, 2.950, 2.720,
$ 2.300, 2.450, 2.550, 2.600, 2.450, 2.230, 1.950,
$ 2.190, 2.250, 2.270, 2.180, 1.930, 1.710, 1.440,
$ 2.120, 2.060, 2.030, 1.830, 1.530, 1.350, 1.060,
$ 2.100, 1.900, 1.870, 1.630, 1.230, 1.110, 0.840,
$ 2.100, 1.850, 1.800, 1.570, 1.240, 1.040, 0.799/
DATA XZ2/
$ 2.790, 2.990, 3.090, 3.140, 3.000, 2.790, 2.470,
$ 2.630, 2.800, 2.820, 2.790, 2.560, 2.280, 1.970,
$ 2.350, 2.380, 2.320, 2.160, 1.800, 1.520, 1.270,
$ 2.090, 1.990, 1.870, 1.610, 1.230, 0.996, 0.776,
$ 1.810, 1.580, 1.420, 1.140, 1.780, 0.610, 0.450,
$ 1.540, 1.230, 1.050, 0.630, 0.490, 0.360, 0.250,
$ 1.400, 1.080, 0.899, 0.637, 0.401, 0.295, 0.204/
DATA XZ3/
$ 0.548, 0.752, 0.942, 1.270, 1.730, 1.970, 2.070,
$ 0.600, 0.830, 1.010, 1.290, 1.620, 1.770, 1.750,
$ 0.710, 0.920, 1.070, 1.260, 1.380, 1.350, 1.190,
$ 0.798, 0.949, 1.060, 1.150, 1.100, 0.959, 0.771,
$ 0.810, 0.900, 1.000, 0.970, 0.810, 0.600, 0.600,
$ 0.820, 0.820, 0.780, 0.720, 0.530, 0.360, 0.360,
$ 0.814, 0.763, 0.733, 0.593, 0.399, 0.294, 0.204/
WRITE(6,100)
100 FORMAT(* CENTER CRACK PANEL --- PLANE STRAIN*,//)
DO 10 I=1,7
X(I)=XX(I)
Y(I)=XY(I)
10 CONTINUE
DO 20 I=1,49
Z1(I)=XZ1(I)
Z2(I)=XZ2(I)
Z3(I)=XZ3(I)
20 CONTINUE
ITYPE=1
IFLAG=1
NUMAB=7
NUMN=7
RETURN
END

```

## SUBROUTINE CTSTS

COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG

COMMON/F/NUMAB,NUMN,ITYPE

DIMENSION XX(10),XY(10),XZ1(72),XZ2(72),XZ3(72)

DATA XX/1.,2.,3.,5.,7.,10.,13.,16.,20.,0.0/

DATA XY/0.25,0.30,0.375,0.45,0.50,0.625,0.75,1.00,0.0,0.0/

DATA XZ1/

\$ 1.61, 1.46, 1.28, 1.06, 0.90, 0.73, 0.60, 0.51, 0.39,

\$ 1.60, 1.35, 1.24, 0.94, 0.79, 0.59, 0.49, 0.39, 0.28,

\$ 1.55, 1.25, 1.05, 0.80, 0.65, 0.48, 0.38, 0.28, 0.22,

\$ 1.47, 1.11, 0.93, 0.78, 0.58, 0.46, 0.35, 0.24, 0.20,

\$ 1.40, 1.08, 0.90, 0.69, 0.56, 0.44, 0.36, 0.30, 0.24,

\$ 1.27, 1.03, 0.88, 0.69, 0.59, 0.49, 0.42, 0.37, 0.31,

\$ 1.23, 0.98, 0.83, 0.68, 0.60, 0.51, 0.43, 0.37, 0.31,

\$ 1.13, 1.01, 0.78, 0.68, 0.65, 0.62, 0.49, 0.47, 0.42/

DATA XZ2/

\$17.55,12.04,10.71, 8.74, 7.32, 5.74, 4.63, 3.75, 2.92,

\$15.10,10.27, 8.70, 6.80, 5.10, 3.90, 3.31, 2.20, 1.71,

\$12.41, 8.20, 6.54, 4.56, 3.45, 2.44, 1.83, 1.36, 1.02,

\$10.31, 6.50, 5.02, 3.32, 2.51, 1.73, 1.32, 1.09, 0.82,

\$ 9.16, 5.67, 4.21, 2.80, 2.12, 1.57, 1.25, 1.03, 0.81,

\$ 7.47, 4.48, 3.35, 2.37, 1.92, 1.54, 1.29, 1.12, 0.93,

\$ 6.25, 3.78, 2.89, 2.14, 1.78, 1.44, 1.20, 1.03, 0.86,

\$ 5.29, 3.54, 2.41, 1.91, 1.73, 1.59, 1.23, 1.17, 1.03/

DATA XZ3/

\$ 9.67, 8.00, 7.21, 5.94, 5.00, 3.95, 3.19, 2.59, 2.02,

\$ 8.80, 7.10, 6.10, 6.88, 3.60, 2.82, 2.22, 1.69, 1.30,

\$ 7.80, 5.73, 4.62, 3.25, 2.48, 1.77, 1.33, 0.99, 0.75,

\$ 6.68, 4.73, 3.68, 2.42, 1.81, 1.27, 1.00, 1.80, 0.60,

\$ 6.29, 4.15, 3.11, 2.09, 1.59, 1.18, 0.94, 0.77, 0.61,

\$ 5.42, 3.38, 2.54, 1.80, 1.47, 1.18, 0.99, 0.85, 0.71,

\$ 4.77, 2.92, 2.24, 1.66, 1.38, 1.12, 0.94, 0.80, 0.67,

\$ 4.23, 2.83, 1.93, 1.52, 1.39, 1.27, 0.99, 0.93, 0.82/

WRITE(6,100)

100 FORMAT(\* COMPACT TENSION SPECIMEN---PLANE STRESS\*,//)

ITYPE=2

NUMAB=8

NUMN=9

DO 1 I=1,10

X(I)=XX(I)

1 Y(I)=XY(I)

DO 2 I=1,72

Z1(I)=XZ1(I)

Z2(I)=XZ2(I)

2 Z3(I)=XZ3(I)

IFLAG=1

RETURN

END

SUBROUTINE CTSTN

COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG

COMMON/F/NUMAB,NUMN,ITYPE

DIMENSION XX(10),XY(10),XZ1(63),XZ2(63),XZ3(63)

DATA XX/1.,2.,3.,5.,7.,10.,13.,16.,20.,0.0/

DATA XY/0.25,0.3,0.375,0.45,0.5,0.625,0.75,3\*0.0/

DATA XZ1/

\$ 2.227, 2.048, 1.783, 1.475, 1.334, 1.248, 1.258, 1.325, 1.566,  
 \$ 2.200, 1.870, 1.540, 1.190, 0.960, 0.780, 0.700, 0.500, 0.520,  
 \$ 2.148, 1.716, 1.392, 0.970, 0.693, 0.443, 0.276, 0.176, 0.098,  
 \$ 2.040, 1.580, 1.280, 0.900, 0.540, 0.390, 0.280, 0.200, 0.060,  
 \$ 1.935, 1.509, 1.242, 0.919, 0.685, 0.461, 0.314, 0.216, 0.132,  
 \$ 1.763, 1.449, 1.237, 0.974, 0.752, 0.602, 0.459, 0.347, 0.248,  
 \$ 1.709, 1.424, 1.263, 1.033, 0.864, 0.717, 0.575, 0.448, 0.345/

DATA XZ2/

\$17.883,12.481,11.675,10.788,10.538,10.745,11.460,12.570,14.563,  
 \$15.850,10.430, 8.900, 6.020, 5.250, 3.840, 3.700, 7.750, 9.200,  
 \$12.644, 8.176, 6.521, 4.319, 2.970, 1.744, 1.102, 0.686, 0.370,  
 \$10.550, 6.650, 5.030, 3.180, 2.200, 0.820, 0.830, 0.520, 0.310,  
 \$ 9.327, 5.846, 4.304, 2.747, 1.912, 1.199, 0.788, 0.530, 0.317,  
 \$ 7.612, 4.572, 3.423, 2.359, 1.810, 1.319, 0.983, 0.749, 0.485,  
 \$ 6.370, 3.948, 3.179, 2.337, 1.876, 1.441, 1.124, 0.887, 0.685/

DATA XZ3/

\$ 9.852, 8.506, 8.170, 7.774, 7.706, 7.942, 8.517, 9.371,10.887,  
 \$ 9.000, 7.100, 4.170, 4.660, 3.540, 3.050, 2.600, 5.700, 6.700,  
 \$ 7.944, 5.760, 4.643, 3.103, 2.139, 1.292, 0.793, 0.494, 0.266,  
 \$ 7.020, 4.730, 3.560, 2.360, 1.630, 0.910, 0.580, 0.360, 0.200,  
 \$ 6.406, 4.268, 3.157, 2.024, 1.413, 0.888, 0.585, 0.393, 0.236,  
 \$ 5.521, 3.431, 3.179, 1.787, 1.373, 1.000, 0.746, 0.568, 0.368,  
 \$ 4.857, 3.048, 2.456, 1.807, 1.450, 1.114, 0.869, 0.686, 0.514/

IFLAG=1

ITYPE=2

DO 10 I=1,10

X(I)=XX(I)

10 Y(I)=XY(I)

DO 20 I=1,63

Z1(I)=XZ1(I)

Z2(I)=XZ2(I)

20 Z3(I)=XZ3(I)

NUMAB=7

NUMN=9

WRITE(6,100)

100 FORMAT(\* COMPACT TENSION --- PLANE STRAIN\*,//)

RETURN

END

```

SUBROUTINE DCPSTN
COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
COMMON/F/NUMAB,NUMN,ITYPE
DIMENSION XX(9),XY(9),XZ1(45),XZ2(45),XZ3(45)
DATA XX/1., 2., 3., 5., 7., 10., 13., 16., 20./
DATA XY/0.25, 0.375, 0.5, 0.625, 0.75, 4*0.0/
DATA XZ1/
$ 5.01,128.4,28.67,155.43,749.07,7530.8,75618.,741530.,156.E5,
$ 6.41,14.56,30.42,120.99,461.24,3354.2,20374.,168810.,2278.E3,
$ 7.31,14.59,27.07,87.53,275.57,1505.9,8109.4,43102.,343780.,
$ 7.87,13.40,21.62,53.51,128.26,455.26,1558.4,5376.8,28037.,
$ 8.19,12.4,17.72,33.06,55.12,105.32,221.79,518.,894.59/
DATA XZ2/
$ 3.33,7.41,16.55,78.46,362.04,3507.8,34044.,326360.,6719300.,
$ 5.00,9.61,18.64,67.83,244.2,1676.3,11491.,80300.,1062000.,
$ 6.78,11.22,18.94,53.76,155.16,784.89,4008.9,20719.,173640.,
$ 8.79,11.91,16.73,34.22,73.4,238.5,783.6,2623.4,13141.,
$ 11.37,12.21,14.07,20.72,31.92,58.76,117.99,262.33,473.82/
DATA XZ3/
$ 0.57,2.2,6.82,49.33,297.75,3720.9,43022.,459780.,10421.E3,
$ 1.27,4.22,11.17,59.32,266.63,2211.5,16698.,123400.,1693100.,
$ 2.28,6.39,14.29,55.49,187.17,1051.1,5610.1,29420.,244640.,
$ 3.73,8.28,14.83,38.63,91.15,310.61,1030.7,3458.7,17316.,
$ 5.91,9.89,13.89,23.82,38.01,70.98,142.31,315.28,575.54/
DO 10 I=1,9
X(I)=XX(I)
Y(I)=XY(I)
10 CONTINUE
DO 20 I=1,45
Z1(I)=XZ1(I)
Z2(I)=XZ2(I)
Z3(I)=XZ3(I)
20 CONTINUE
WRITE(6,100)
100 FORMAT(* DOUBLE-EDGE CRACK PANEL --- PLANE STRAIN*,//)
ITYPE=3
IFLAG=1
NUMAB=5
NUMN=9
RETURN
END

```

```

SUBROUTINE STBSTN
COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
COMMON/F/NUMAB,NUMN,ITYPE
DIMENSION XX(10),XY(10),XZ1(63),XZ2(63),XZ3(63)
DATA XX/1.,2.,3.,5.,7.,10.,13.,16.,20.,0.0/
DATA XY/0.25,0.3,0.35,0.4,0.5,0.6,0.75,3*0.0/
DATA XZ1/
$ 1.195, 1.034, 0.930, 0.765, 0.633, 0.523, 0.396, 0.303, 0.215,
$ 1.270, 1.040, 0.920, 0.740, 0.570, 0.450, 0.320, 0.240, 0.150,
$ 1.340, 1.050, 0.910, 0.720, 0.540, 0.410, 0.280, 0.200, 0.120,
$ 1.370, 1.070, 0.910, 0.700, 0.510, 0.370, 0.240, 0.160, 0.090,
$ 1.398, 1.094, 0.922, 0.675, 0.495, 0.331, 0.211, 0.135, 0.074,
$ 1.360, 1.100, 0.920, 0.670, 0.480, 0.320, 0.250, 0.110, 0.150,
$ 1.208, 1.145, 0.974, 0.693, 0.500, 0.348, 0.223, 0.140, 0.075/
DATA XZ2/
$ 5.799, 4.665, 4.006, 3.080, 2.454, 1.934, 1.446, 1.088, 0.758,
$ 5.520, 4.180, 3.450, 2.670, 2.060, 1.580, 1.100, 0.830, 0.550,
$ 5.300, 4.620, 3.120, 2.330, 1.760, 1.300, 0.870, 0.650, 0.410,
$ 5.140, 3.630, 2.860, 2.050, 1.520, 1.080, 0.700, 0.520, 0.300,
$ 4.869, 3.283, 2.527, 1.686, 1.192, 0.773, 0.480, 0.304, 0.165,
$ 4.650, 3.030, 2.300, 1.440, 0.980, 0.630, 0.380, 0.250, 0.110,
$ 4.474, 2.754, 2.096, 1.361, 0.936, 0.618, 0.388, 0.239, 0.128/
DATA XZ3/
$ 4.083,10.099, 8.413, 5.864, 4.466, 3.421, 2.542, 1.901, 1.318,
$ 4.280, 7.600, 6.350, 4.290, 3.340, 2.470, 1.590, 1.250, 0.900,
$ 4.440, 6.330, 5.140, 3.540, 2.740, 1.920, 1.230, 0.930, 0.650,
$ 4.560, 5.500, 4.420, 3.050, 2.290, 1.550, 0.980, 0.700, 0.470,
$ 4.687, 4.442, 3.509, 2.353, 1.663, 1.079, 0.669, 0.424, 0.230,
$ 4.670, 3.800, 2.300, 1.900, 1.320, 1.830, 0.510, 0.280, 0.140,
$ 1.491, 3.159, 2.407, 1.557, 1.068, 0.704, 0.441, 0.272, 0.144/
DO 10 I=1,10
X(I)=XX(I)
Y(I)=XY(I)
10 CONTINUE
DO 20 I=1,63
Z1(I)=XZ1(I)
Z2(I)=XZ2(I)
Z3(I)=XZ3(I)
20 CONTINUE
WRITE(6,100)
100 FORMAT(* SINGLE EDGE CRACK PANEL IN 3-POINT BENDING ---*
$ * PLANE STRAIN*,//)
ITYPE=4
IFLAG=1
NUMAB=7
NUMN=9
RETURN
END

```

```

SUBROUTINE SETSTN
COMMON/A/E,EA,PLCOR,SIG0,XLL,B,FACTOR,XN,ALPHA,EP0,TH,PP1,IPL,
$A,H1,H2,H3,PP,ZZ,ZZE,P0,AE,XKSQ,XJ,BETA,PI
COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
COMMON/E/PMIN,PMAX,PIN,ZZ0,AIN,DISMAX
COMMON/F/NUMAB,NUMN,ITYPE
DIMENSION XX(7),XY(7),XZ1(42),XZ2(42),XZ3(42)
DATA XX/1.,2.,3.,5.,7.,10.,0.0/
DATA XY/0.25,0.3,0.35,0.4,0.5,0.6,0.75/
DATA XZ1/
$ 4.338, 4.768, 4.639, 3.815, 3.056, 2.170,
$ 4.200, 3.880, 3.250, 2.520, 2.033, 1.490,
$ 4.020, 3.340, 2.650, 1.900, 1.450, 1.010,
$ 3.820, 3.030, 2.230, 1.460, 1.040, 0.660,
$ 3.398, 2.302, 1.694, 0.928, 0.514, 0.213,
$ 2.960, 1.910, 1.400, 1.690, 0.330, 0.110,
$ 2.342, 1.607, 1.245, 0.769, 0.477, 0.233/
DATA XZ2/
$ 4.756, 4.559, 4.281, 3.391, 2.639, 1.808,
$ 4.660, 3.670, 3.350, 2.310, 1.790, 1.260,
$ 4.590, 3.280, 2.790, 1.750, 1.270, 0.870,
$ 4.530, 3.030, 2.230, 1.870, 0.910, 0.580,
$ 4.447, 2.765, 1.888, 0.954, 0.507, 0.204,
$ 4.880, 2.520, 1.400, 0.810, 0.380, 0.150,
$ 4.316, 2.515, 1.789, 1.027, 0.619, 0.296/
DATA XZ3/
$10.270, 7.635, 5.874, 3.695, 2.483, 1.496,
$ 8.150, 4.120, 3.000, 1.930, 1.560, 0.900,
$ 6.050, 3.000, 1.880, 1.290, 1.030, 0.580,
$ 4.600, 2.370, 1.350, 0.880, 0.590, 0.350,
$ 3.151, 1.537, .9125, .4172, .2151, .0851,
$ 2.520, 1.150, 0.750, 0.500, 0.060, 0.040,
$ 2.018, 1.105, .7655, .4349, .2617, .1254/
WRITE(6,100)
100  FORMAT(* SINGLE EDGE CRACK PANEL IN TENSION --- *
$ *PLANE STRAIN*,//)
ITYPE=5
IFLAG=1
NUMAB=7
NUMN=6
DO 10 I=1,7
X(I)=XX(I)
Y(I)=XY(I)
10  CONTINUE
DO 20 I=1,42
Z1(I)=XZ1(I)
Z2(I)=XZ2(I)
Z3(I)=XZ3(I)
20  CONTINUE
RETURN
END

```

```

SUBROUTINE DCPSTS
COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
COMMON/F/NUMAB,NUMN,ITYPE
DIMENSION XX(9),XY(9),XZ1(45),XZ2(45),XZ3(45)
DATA XX/1.,2.,3.,5.,7.,10.,13.,16.,20./
DATA XY/0.25,0.37,0.5,0.62,0.75,0.0,0.0,0.0,0.0/
DATA XZ1/
$ 1.011, 1.226, 1.356, 1.483, 1.543, 1.578, 1.594, 1.591, 1.588,
$ 1.293, 1.418, 1.427, 1.341, 1.237, 1.094, 0.970, 0.873, 0.674,
$ 1.475, 1.466, 1.378, 1.168, 1.010, 0.845, 0.732, 0.625, 0.208,
$ 1.586, 1.454, 1.284, 1.038, 0.882, 0.737, 0.649, 0.466, 0.020,
$ 1.652, 1.425, 1.118, 0.979, 0.833, 0.701, 0.630, 0.297, 0.000/
DATA XZ2/
$ 1.726, 1.819, 1.886, 1.917, 1.905, 1.853, 1.802, 1.746, 1.700,
$ 2.594, 2.393, 2.221, 1.864, 1.588, 1.283, 1.068, 0.922, 0.709,
$ 3.514, 2.821, 2.337, 1.670, 1.277, 0.944, 0.762, 0.630, 0.232,
$ 4.559, 3.145, 2.318, 1.449, 1.061, 0.790, 0.657, 0.473, 0.028,
$ 5.896, 3.371, 2.214, 1.297, 0.966, 0.741, 0.634, 0.312, 0.000/
DATA XZ3/
$ 0.295, 0.537, 0.770, 1.169, 1.490, 1.815, 2.022, 2.124, 2.198,
$ 0.658, 1.037, 1.295, 1.520, 1.547, 1.412, 1.227, 1.068, 0.829,
$ 1.184, 1.581, 1.691, 1.563, 1.320, 0.080, 0.808, 0.662, 0.265,
$ 1.932, 2.138, 1.950, 1.444, 1.094, 0.809, 0.664, 0.487, 0.032,
$ 3.063, 2.670, 2.061, 1.314, 0.978, 0.747, 0.638, 0.318, 0.000/
WRITE(6,100)
100 FORMAT(* DOUBLE EDGE CRACK PANEL --- PLAIN STRESS*,//)
ITYPE=3
IFLAG=1
NUMAB=5
NUMN=9
DO 10 I=1,9
X(I)=XX(I)
Y(I)=XY(I)
10 CONTINUE
DO 20 I=1,45
Z1(I)=XZ1(I)
Z2(I)=XZ2(I)
Z3(I)=XZ3(I)
20 CONTINUE
RETURN
END

```

SUBROUTINE STBSTS

COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG

COMMON/F/NUMAB,NUMN,ITYPE

DIMENSION XX(9),XY(9),XZ1(45),XZ2(45),XZ3(45)

DATA XX/1.,2.,3.,5.,7.,10.,13.,16.,20./

DATA XY/0.25,0.37,0.5,0.62,0.75,0.0,0.0,0.0,0.0/

DATA XZ1/

\$ 0.869, 0.731, 0.629, 0.479, 0.370, 0.246, 0.174, 0.117, 0.059,

\$ 0.963, 0.797, 0.680, 0.527, 0.418, 0.307, 0.232, 0.174, 0.105,

\$ 1.019, 0.767, 0.621, 0.453, 0.324, 0.202, 0.128, 0.081, 0.030,

\$ 1.051, 0.786, 0.649, 0.494, 0.357, 0.235, 0.173, 0.105, 0.047,

\$ 1.067, 0.786, 0.643, 0.474, 0.343, 0.230, 0.167, 0.110, 0.044/

DATA XZ2/

\$ 5.690, 4.503, 3.680, 2.614, 1.947, 1.290, 0.897, 0.603, 0.307,

\$ 5.085, 3.732, 2.929, 2.071, 1.580, 1.134, 0.841, 0.626, 0.381,

\$ 4.768, 3.120, 2.320, 1.547, 1.077, 0.655, 0.410, 0.259, 0.097,

\$ 4.551, 2.830, 2.118, 1.455, 1.023, 0.656, 0.472, 0.286, 0.130,

\$ 4.385, 2.656, 1.967, 1.329, 0.928, 0.601, 0.427, 0.280, 0.114/

DATA XZ3/

\$ 4.007, 8.812, 7.189, 4.731, 3.388, 2.204, 1.517, 1.012, 0.508,

\$ 4.420, 5.533, 4.482, 3.172, 2.409, 1.726, 1.277, 0.948, 0.575,

\$ 4.604, 4.085, 3.092, 2.081, 1.442, 0.874, 0.545, 0.344, 0.129,

\$ 4.617, 3.434, 2.599, 1.794, 1.258, 0.803, 0.577, 0.349, 0.158,

\$ 4.394, 3.012, 2.235, 1.510, 1.052, 0.680, 0.483, 0.316, 0.129/

DO 10 I=1,9

X(I)=XX(I)

Y(I)=XY(I)

10 CONTINUE

DO 20 I=1,45

Z1(I)=XZ1(I)

Z2(I)=XZ2(I)

Z3(I)=XZ3(I)

20 CONTINUE

WRITE(6,100)

100 FORMAT(\* SINGE EDGE CRACK PANEL IN 3-POINT BENDING ---\*

\$ \*PLANE STRESS\*,//)

ITYPE=4

IFLAG=1

NUMAB=5

NUMN=9

RETURN

END

```

SUBROUTINE SETSTS
COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
COMMON/F/NUMAB,NUMN,ITYPE
DIMENSION XX(9),XY(9),XZ1(45),XZ2(45),XZ3(45)
DATA XX/1.,2.,3.,5.,7.,10.,13.,16.,20./
DATA XY/0.25,0.37,0.5,0.62,0.75,0.0,0.0,0.0,0.0/
DATA XZ1/
$ 3.140, 3.261, 2.919, 2.115, 1.531, 0.960, 0.615, 0.400, 0.230,
$ 2.809, 2.365, 1.943, 1.367, 1.009, 0.677, 0.474, 0.342, 0.226,
$ 2.459, 1.665, 1.254, 0.776, 0.510, 0.286, 0.164, 0.096, 0.047,
$ 2.070, 1.408, 1.105, 0.755, 0.551, 0.363, 0.248, 0.172, 0.107,
$ 1.696, 1.142, 0.910, 0.624, 0.447, 0.280, 0.181, 0.118, 0.067/
DATA XZ2/
$ 4.672, 4.300, 3.695, 2.532, 1.755, 1.053, 0.656, 0.419, 0.237,
$ 4.465, 3.426, 2.632, 1.685, 1.181, 0.762, 0.524, 0.372, 0.244,
$ 4.369, 2.726, 1.909, 1.093, 0.694, 0.380, 0.216, 0.124, 0.061,
$ 4.297, 2.552, 1.837, 1.160, 0.816, 0.523, 0.353, 0.242, 0.150,
$ 4.240, 2.468, 1.805, 1.147, 0.798, 0.490, 0.314, 0.203, 0.115/
DATA XZ3/
$10.090, 6.488, 4.362, 2.185, 1.239, 0.630, 0.362, 0.224, 0.123,
$ 5.047, 2.653, 1.604, 0.812, 0.525, 0.328, 0.223, 0.157, 0.102,
$ 3.095, 1.429, 0.871, 0.461, 0.286, 0.155, 0.088, 0.051, 0.025,
$ 2.270, 1.127, 0.771, 0.478, 0.336, 0.215, 0.146, 0.100, 0.062,
$ 1.983, 1.087, 0.784, 0.494, 0.344, 0.211, 0.136, 0.058, 0.050/
WRITE(6,100)
100  FORMAT(* SINGLE EDGE CRACK PANEL IN TENSION ---*
$ *PLANE STRESS*,//)
ITYPE=5
IFLAG=1
NUMAB=5
NUMN=9
DO 10 I=1,9
X(I)=XX(I)
X(I)=XY(I)
10  CONTINUE
DO 20 I=1,45
Z1(I)=XZ1(I)
Z2(I)=XZ2(I)
Z3(I)=XZ3(I)
20  CONTINUE
RETURN
END

```

APPENDIX B

A SAMPLE OUTPUT LISTING OF  
PROGRAM EST

(INPUT IS DEFINED IN APPENDIX C)

..REWIND,TAPE6  
 ..COPY,TAPE6  
 MATERIAL: COPPER  
 COMPACT TENSION --- PLANE STRAIN

TH .4980 SPAN 4.000 N 36.00 SIGO .3300E+05 PZC 1.000 E .1700E+08 WIDTH 1.995 LC 1.000 ALPHA 1.030 EPO .1941E-02

N = 20.00 A/B = .2500  
 H1 = 1.5660 H2 = 14.5630 H3 = 10.8870

ALL RESULTS IN ENGLISH UNITS

A	A/W	AE	P/P0	LOAD	K	J	SQRT(EJ)	SQRT(J/JE)	CMOD	LDISP	EN	PN	EC	PC
.499	.250	.499	0.	0.000	0.	0.	0.	0.000000	0.	0.	0.	0.	0.	0.
.499	.250	.499	.505E-04	.500	.350E+01	.656E-06	.350E+01	1.0000000	.114E-05	.617E-06	0.	0.	.617E-06	.124E-87
.499	.250	.499	.101E-03	1.000	.700E+01	.262E-05	.700E+01	1.0000000	.229E-05	.123E-05	0.	0.	.123E-05	.130E-81
.499	.250	.499	.151E-03	1.500	.105E+02	.590E-05	.105E+02	1.0000000	.343E-05	.185E-05	0.	0.	.185E-05	.433E-78
.499	.250	.499	.202E-03	2.000	.140E+02	.105E-04	.140E+02	1.0000000	.457E-05	.247E-05	0.	0.	.247E-05	.136E-75
.499	.250	.499	.252E-03	2.500	.175E+02	.164E-04	.175E+02	1.0000000	.572E-05	.309E-05	0.	0.	.309E-05	.118E-73
.499	.250	.499	.303E-03	3.000	.210E+02	.236E-04	.210E+02	1.0000000	.686E-05	.370E-05	0.	0.	.370E-05	.454E-72
.499	.250	.499	.353E-03	3.500	.245E+02	.321E-04	.245E+02	1.0000000	.800E-05	.432E-05	0.	0.	.432E-05	.991E-71
.499	.250	.499	.404E-03	4.000	.280E+02	.420E-04	.280E+02	1.0000000	.915E-05	.494E-05	0.	0.	.494E-05	.143E-69
.499	.250	.499	.454E-03	4.500	.315E+02	.531E-04	.315E+02	1.0000001	.103E-04	.556E-05	0.	0.	.556E-05	.151E-68
.499	.250	.499	.505E-03	5.000	.350E+02	.656E-04	.350E+02	1.0000001	.114E-04	.617E-05	0.	0.	.617E-05	.124E-67
.499	.250	.499	.555E-03	5.500	.385E+02	.794E-04	.385E+02	1.0000001	.126E-04	.679E-05	0.	0.	.679E-05	.835E-67
.499	.250	.499	.605E-03	6.000	.420E+02	.945E-04	.420E+02	1.0000001	.137E-04	.741E-05	0.	0.	.741E-05	.476E-66
.499	.250	.499	.656E-03	6.500	.455E+02	.111E-03	.455E+02	1.0000001	.149E-04	.803E-05	0.	0.	.803E-05	.236E-65
.499	.250	.499	.706E-03	7.000	.490E+02	.129E-03	.490E+02	1.0000001	.160E-04	.864E-05	0.	0.	.864E-05	.104E-64
.499	.250	.499	.757E-03	7.500	.525E+02	.148E-03	.525E+02	1.0000002	.172E-04	.926E-05	0.	0.	.926E-05	.413E-64
.499	.250	.499	.807E-03	8.000	.560E+02	.168E-03	.560E+02	1.0000002	.183E-04	.988E-05	0.	0.	.988E-05	.150E-63
.499	.250	.499	.858E-03	8.500	.595E+02	.190E-03	.595E+02	1.0000002	.194E-04	.105E-04	0.	0.	.105E-04	.505E-63
.499	.250	.499	.908E-03	9.000	.630E+02	.213E-03	.630E+02	1.0000002	.206E-04	.111E-04	0.	0.	.111E-04	.158E-62
.499	.250	.499	.959E-03	9.500	.665E+02	.237E-03	.665E+02	1.0000003	.217E-04	.117E-04	0.	0.	.117E-04	.467E-62

N = 20.00 A/B = .3001  
 H1 = .5193 H2 = 9.1852 H3 = 6.6892

ALL RESULTS IN ENGLISH UNITS

A	A/W	AE	P/P0	LOAD	K	J	SQRT(EJ)	SQRT(J/JE)	CMOD	LDISP	EN	PN	EC	PC
.599	.300	.599	0.	0.000	0.	0.	0.	0.000000	0.	0.	0.	0.	0.	0.
.599	.300	.599	.594E-04	.500	.400E+01	.855E-06	.400E+01	1.0000000	.137E-05	.794E-06	0.	0.	.794E-06	.241E-86
.599	.300	.599	.119E-03	1.000	.799E+01	.342E-05	.799E+01	1.0000000	.274E-05	.159E-05	0.	0.	.159E-05	.253E-80
.599	.300	.599	.178E-03	1.500	.120E+02	.770E-05	.120E+02	1.0000000	.412E-05	.238E-05	0.	0.	.238E-05	.840E-77
.599	.300	.599	.238E-03	2.000	.160E+02	.137E-04	.160E+02	1.0000000	.549E-05	.318E-05	0.	0.	.318E-05	.265E-74
.599	.300	.599	.297E-03	2.500	.200E+02	.214E-04	.200E+02	1.0000000	.686E-05	.397E-05	0.	0.	.397E-05	.230E-72
.599	.300	.599	.356E-03	3.000	.240E+02	.308E-04	.240E+02	1.0000000	.823E-05	.477E-05	0.	0.	.477E-05	.881E-71
.599	.300	.599	.416E-03	3.500	.280E+02	.419E-04	.280E+02	1.0000000	.960E-05	.556E-05	0.	0.	.556E-05	.192E-69
.599	.300	.599	.475E-03	4.000	.320E+02	.547E-04	.320E+02	1.0000001	.110E-04	.635E-05	0.	0.	.635E-05	.278E-68
.599	.300	.599	.535E-03	4.500	.360E+02	.693E-04	.360E+02	1.0000001	.123E-04	.715E-05	0.	0.	.715E-05	.293E-67
.599	.300	.599	.594E-03	5.000	.400E+02	.855E-04	.400E+02	1.0000001	.137E-04	.794E-05	0.	0.	.794E-05	.241E-66
.599	.300	.599	.654E-03	5.500	.440E+02	.103E-03	.440E+02	1.0000001	.151E-04	.874E-05	0.	0.	.874E-05	.162E-65
.599	.300	.599	.713E-03	6.000	.480E+02	.123E-03	.480E+02	1.0000001	.165E-04	.953E-05	0.	0.	.953E-05	.923E-65
.599	.300	.599	.772E-03	6.500	.520E+02	.145E-03	.520E+02	1.0000002	.178E-04	.103E-04	0.	0.	.103E-04	.458E-64
.599	.300	.599	.832E-03	7.000	.560E+02	.168E-03	.560E+02	1.0000002	.192E-04	.111E-04	0.	0.	.111E-04	.201E-63
.599	.300	.599	.891E-03	7.500	.600E+02	.192E-03	.600E+02	1.0000002	.206E-04	.119E-04	0.	0.	.119E-04	.801E-63

.599	.300	.599	.951E-03	8.000	.639E+02	.219E-03	.639E+02	1.00000002	.220E-04	.127E-04	0.	0.	.127E-04	.291E-62
.599	.300	.599	.101E-02	8.500	.679E+02	.247E-03	.679E+02	1.00000003	.233E-04	.135E-04	0.	0.	.135E-04	.979E-62
.599	.300	.599	.107E-02	9.000	.719E+02	.277E-03	.719E+02	1.00000003	.247E-04	.143E-04	0.	0.	.143E-04	.307E-61
.599	.300	.599	.113E-02	9.500	.759E+02	.309E-03	.759E+02	1.00000003	.261E-04	.151E-04	0.	0.	.151E-04	.905E-61

N = 20.00    A/B = .3503  
H1 = .2373    H2 = 3.2838    H3 = 2.3892

ALL RESULTS IN ENGLISH UNITS

A	A/W	AE	P/P0	LOAD	K	J	SQRT(EJ)	SQRT(J/JE)	CMOD	LDISP	EN	PN	EC	PC
.699	.350	.699	0.	0.000	0.	0.	0.	0.0000000	0.	0.	0.	0.	0.	0.
.699	.350	.699	.707E-04	.500	.455E+01	.111E-05	.455E+01	1.00000000	.164E-05	.101E-05	0.	0.	.101E-05	.327E-85
.699	.350	.699	.141E-03	1.000	.909E+01	.443E-05	.909E+01	1.00000000	.328E-05	.202E-05	0.	0.	.202E-05	.343E-79
.699	.350	.699	.212E-03	1.500	.136E+02	.996E-05	.136E+02	1.00000000	.492E-05	.303E-05	0.	0.	.303E-05	.114E-75
.699	.350	.699	.283E-03	2.000	.182E+02	.177E-04	.182E+02	1.00000000	.656E-05	.404E-05	0.	0.	.404E-05	.360E-73
.699	.350	.699	.354E-03	2.500	.227E+02	.277E-04	.227E+02	1.00000000	.820E-05	.505E-05	0.	0.	.505E-05	.312E-71
.699	.350	.699	.424E-03	3.000	.273E+02	.398E-04	.273E+02	1.00000000	.984E-05	.606E-05	0.	0.	.606E-05	.120E-69
.699	.350	.699	.495E-03	3.500	.318E+02	.542E-04	.318E+02	1.00000001	.115E-04	.707E-05	0.	0.	.707E-05	.261E-68
.699	.350	.699	.566E-03	4.000	.364E+02	.708E-04	.364E+02	1.00000001	.131E-04	.808E-05	0.	0.	.808E-05	.377E-67
.699	.350	.699	.637E-03	4.500	.409E+02	.896E-04	.409E+02	1.00000001	.148E-04	.909E-05	0.	0.	.909E-05	.398E-66
.699	.350	.699	.707E-03	5.000	.455E+02	.111E-03	.455E+02	1.00000001	.164E-04	.101E-04	0.	0.	.101E-04	.327E-65
.699	.350	.699	.778E-03	5.500	.500E+02	.134E-03	.500E+02	1.00000001	.180E-04	.111E-04	0.	0.	.111E-04	.220E-64
.699	.350	.699	.849E-03	6.000	.546E+02	.159E-03	.546E+02	1.00000002	.197E-04	.121E-04	0.	0.	.121E-04	.126E-63
.699	.350	.699	.919E-03	6.500	.591E+02	.187E-03	.591E+02	1.00000002	.213E-04	.131E-04	0.	0.	.131E-04	.622E-63
.699	.350	.699	.990E-03	7.000	.637E+02	.217E-03	.637E+02	1.00000002	.230E-04	.141E-04	0.	0.	.141E-04	.274E-62
.699	.350	.699	.106E-02	7.500	.682E+02	.249E-03	.682E+02	1.00000003	.246E-04	.151E-04	0.	0.	.151E-04	.109E-61
.699	.350	.699	.113E-02	8.000	.727E+02	.283E-03	.727E+02	1.00000003	.262E-04	.162E-04	0.	0.	.162E-04	.396E-61
.699	.350	.699	.120E-02	8.500	.773E+02	.320E-03	.773E+02	1.00000003	.279E-04	.172E-04	0.	0.	.172E-04	.133E-60
.699	.350	.699	.127E-02	9.000	.818E+02	.359E-03	.818E+02	1.00000004	.295E-04	.182E-04	0.	0.	.182E-04	.417E-60
.699	.350	.699	.134E-02	9.500	.864E+02	.399E-03	.864E+02	1.00000004	.312E-04	.192E-04	0.	0.	.192E-04	.123E-59

N = 20.00    A/B = .4004  
H1 = .0851    H2 = .3497    H3 = .2437

ALL RESULTS IN ENGLISH UNITS

A	A/W	AE	P/P0	LOAD	K	J	SQRT(EJ)	SQRT(J/JE)	CMOD	LDISP	EN	PN	EC	PC
.799	.400	.799	0.	0.000	0.	0.	0.	0.0000000	0.	0.	0.	0.	0.	0.
.799	.400	.799	.852E-04	.500	.518E+01	.144E-05	.518E+01	1.00000000	.197E-05	.128E-05	0.	0.	.128E-05	.159E-84
.799	.400	.799	.170E-03	1.000	.104E+02	.574E-05	.104E+02	1.00000000	.393E-05	.255E-05	0.	0.	.255E-05	.167E-78
.799	.400	.799	.256E-03	1.500	.155E+02	.129E-04	.155E+02	1.00000000	.590E-05	.383E-05	0.	0.	.383E-05	.555E-75
.799	.400	.799	.341E-03	2.000	.207E+02	.230E-04	.207E+02	1.00000000	.786E-05	.511E-05	0.	0.	.511E-05	.175E-72
.799	.400	.799	.426E-03	2.500	.259E+02	.359E-04	.259E+02	1.00000000	.983E-05	.639E-05	0.	0.	.639E-05	.152E-70
.799	.400	.799	.511E-03	3.000	.311E+02	.517E-04	.311E+02	1.00000001	.118E-04	.766E-05	0.	0.	.766E-05	.582E-69
.799	.400	.799	.597E-03	3.500	.363E+02	.704E-04	.363E+02	1.00000001	.138E-04	.894E-05	0.	0.	.894E-05	.127E-67
.799	.400	.799	.682E-03	4.000	.414E+02	.919E-04	.414E+02	1.00000001	.157E-04	.102E-04	0.	0.	.102E-04	.184E-66
.799	.400	.799	.767E-03	4.500	.466E+02	.116E-03	.466E+02	1.00000001	.177E-04	.115E-04	0.	0.	.115E-04	.194E-65
.799	.400	.799	.852E-03	5.000	.518E+02	.144E-03	.518E+02	1.00000002	.197E-04	.128E-04	0.	0.	.128E-04	.159E-64
.799	.400	.799	.938E-03	5.500	.570E+02	.174E-03	.570E+02	1.00000002	.216E-04	.140E-04	0.	0.	.140E-04	.107E-63
.799	.400	.799	.102E-02	6.000	.621E+02	.207E-03	.621E+02	1.00000002	.236E-04	.153E-04	0.	0.	.153E-04	.611E-63
.799	.400	.799	.111E-02	6.500	.673E+02	.243E-03	.673E+02	1.00000003	.256E-04	.166E-04	0.	0.	.166E-04	.303E-62
.799	.400	.799	.119E-02	7.000	.725E+02	.281E-03	.725E+02	1.00000003	.275E-04	.179E-04	0.	0.	.179E-04	.133E-61
.799	.400	.799	.128E-02	7.500	.777E+02	.323E-03	.777E+02	1.00000004	.295E-04	.192E-04	0.	0.	.192E-04	.530E-61
.799	.400	.799	.136E-02	8.000	.829E+02	.368E-03	.829E+02	1.00000004	.314E-04	.204E-04	0.	0.	.204E-04	.193E-60
.799	.400	.799	.145E-02	8.500	.880E+02	.415E-03	.880E+02	1.00000005	.334E-04	.217E-04	0.	0.	.217E-04	.647E-60
.799	.400	.799	.153E-02	9.000	.932E+02	.465E-03	.932E+02	1.00000005	.354E-04	.230E-04	0.	0.	.230E-04	.203E-59
.799	.400	.799	.162E-02	9.500	.984E+02	.518E-03	.984E+02	1.00000006	.373E-04	.243E-04	0.	0.	.243E-04	.599E-59

APPENDIX C

A SAMPLE INPUT LISTING OF  
THE INTERACTIVE COMPUTER  
PROGRAM EST FOR A MONOTONICALLY  
LOADED CT SPECIMEN

COMMAND-ATTACH,EST,ID=TUSIT,SN=AFML,CY=1  
COMMAND-EST.

ENTER SPECIMEN GEOMETRY  
(CCP,CT,DCP,STB,SET)==|CT  
PLANE STRESS (0) OR PLANE STRAIN (1) ?1  
MATERIAL ==|COPPER  
E(NGLISH) OR M(ETRIC) ?E

IF THE FOLLOWING VALUES ARE CORRECT,  
RESPOND WITH A "Y". IF THEY ARE INCORRECT,  
ANSWER "N", THEN ENTER THE CORRECT VALUE

MINIMUM LOAD (KIPS/N) = .0 (Y/N)?Y  
LOAD INCREMENT (KIPS/N) = .25000 (Y/N)?N  
0.5  
MAXIMUM LOAD(KIPS/N) = 5.3 (Y/N)?N  
10  
INITIAL A/B = .25000 (Y/N)?Y  
RAMBERG-OSGOOD EXPONENT - N = 36.000 (Y/N)?Y  
YIELD STRENGTH - SIG0 (KSI/PA) = 33000. (Y/N)?Y  
PLASTIC ZONE CORRECTION FACTOR - PZC= 1.0000 (Y/N)?Y  
ELASTIC MODULUS - E (KSI/PA) = .17000E+08 (Y/N)?Y  
HALF SPAN (IN/M) = 2.0000 (Y/N)?Y  
WIDTH - W (IN/M) = 1.9950 (Y/N)?Y  
THICKNESS - TH (IN/M) = .49800 (Y/N)?Y  
CRACK LENGTH INCREMENT (IN/M) = .10000 (Y/N)?Y  
CORR. FACTOR FOR LIMIT LOAD - LC = 1.0000 (Y/N)?Y  
MAXIMUM DISPLACEMENT (IN/M) = .10000E-01 (Y/N)?N  
0.1

STOP  
030400 MAXIMUM EXECUTION FL.  
.438 CP SECONDS EXECUTION TIME.