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HANDBOOK FOR THE ANALYSIS OF ENGAGEMENTS WITH MOBILE TARGETS

RDA Staff

R & D Associates

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Marina del Rey, California 90291

1 June 1980

Handbook

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I. INTRODUCTION AND SUMMARY

Current targeting manuals such as the Staff Officer's Field Manual, Nuclear Weapons Employment Doctrine and Procedures, FM-101-31-1 (Ref. 1), do not include a means of analyzing the engagement of targets that move. This is reflected by the fact that no parameters which represent motion (for instance, speed or dwell time) are used in the manual's methodology. By implication, all targets are fixed targets.

→ This document describes a methodology and provides the data required for a realistic analysis of a mobile target engagement. For this report a mobile target is defined both as a target that moves nearly continuously (such as a tank company) and as one that moves only occasionally (such as an artillery battery or command post). Specifically, this manual provides a means of estimating the probability that a target is still present at an observed location as a function of time from the observation where the time the target stopped is unknown. With this methodology targets can then be evaluated not only on the basis of expected fractional coverage as in the manuals, but also on the basis of whether there is an adequate likelihood that they will still be present when a weapon arrives. ↗

Results are summarized in Figure 1. This chart shows the probability of a target being present at an observed location as a function of the expected target dwell time (τ) and the acquisition/engagement time (t). Its use is best illustrated by an example. Let us assume an expected target dwell time (τ) of 12 hours and that the time (t) necessary to acquire and process the target information, to communicate it to required elements, to make decisions, to plan and prepare weapon use,

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1. Staff Officer's Field Manual, Nuclear Weapons Employment Doctrine and Procedures, Department of the Army, FM-101-31-1, March 1977.

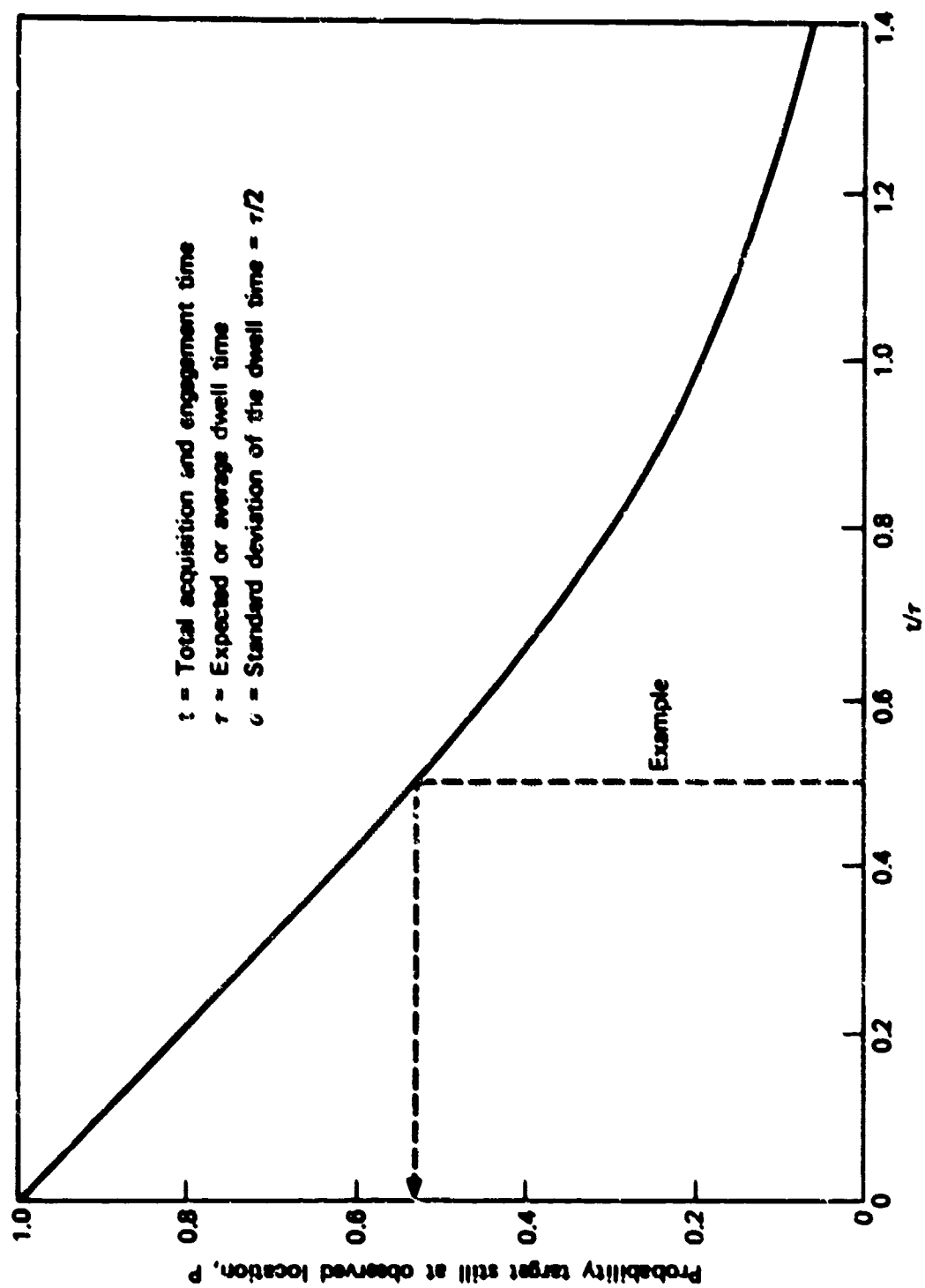


Figure 1. Probability of target presence.

and to employ the weapon is 6 hours. The ratio of t to r is therefore 0.5. The resultant expected probability is about 0.53. Thus, there is slightly better than a 50/50 chance of the target still being present when the weapon actually arrives for the example.

Figure 1 is based upon a particular assumption concerning the dwell time statistics,* but it is a representative curve with general applicability. The remainder of this report describes the methodology in detail and presents results for other cases in greater detail.

Specifically, the sections of this report present a brief description of the methodology (with a detailed development of the equations in an appendix); tabular data for target analysis of specific cases; an expanded generalized curve; and sample cases. Appendix A presents the derivation of the equations while Appendix B details a TI-59 calculator code for calculating target presence probabilities.

*The standard deviation of the expected dwell time is assumed to be one-half of the expected dwell time. Section IV of this report presents generalized data that permit evaluation of other values for standard deviation of the dwell time.

II. METHODOLOGY

In this section the basic methodology for the evaluation of mobile targets is described. Appendix A presents a detailed development of the equations. The problem addressed by the methodology is:

What is the probability of a target being at an observed location at some time (Δt) later? Factors to be considered include how long the target can be expected to stay fixed, when did it stop relative to the observation, and how much time is required to respond (i.e., to place a weapon on the target).

The methodology begins with the assumption that any target can be modeled as having a characteristic average dwell time (τ) at a given location. The value of τ is, of course, strongly dependent upon the particular scenario and situation but, in any case, the value can be estimated. The dwell time has some expected deviation (σ) about τ ; i.e., not all targets of the given type in that situation would move precisely at time τ .

A reasonable assumption is that the actual distribution of dwell times for a given target type and situation will be Gaussian, i.e., the bell-shaped curve as shown in Figure 2.* The curve is symmetrical about the mean dwell time τ with the degree of spread of the curve determined by the standard deviation σ . The physical meaning of the curve is that the most probable move time is τ ; however, some units move earlier and some move later. If the spread (σ) is large, then fewer units move at τ and more move earlier and later than τ .

*It does not appear that the actual mathematical form of the dwell time distribution is a critical assumption. The results are most sensitive to the parameter τ regardless of the distribution model.

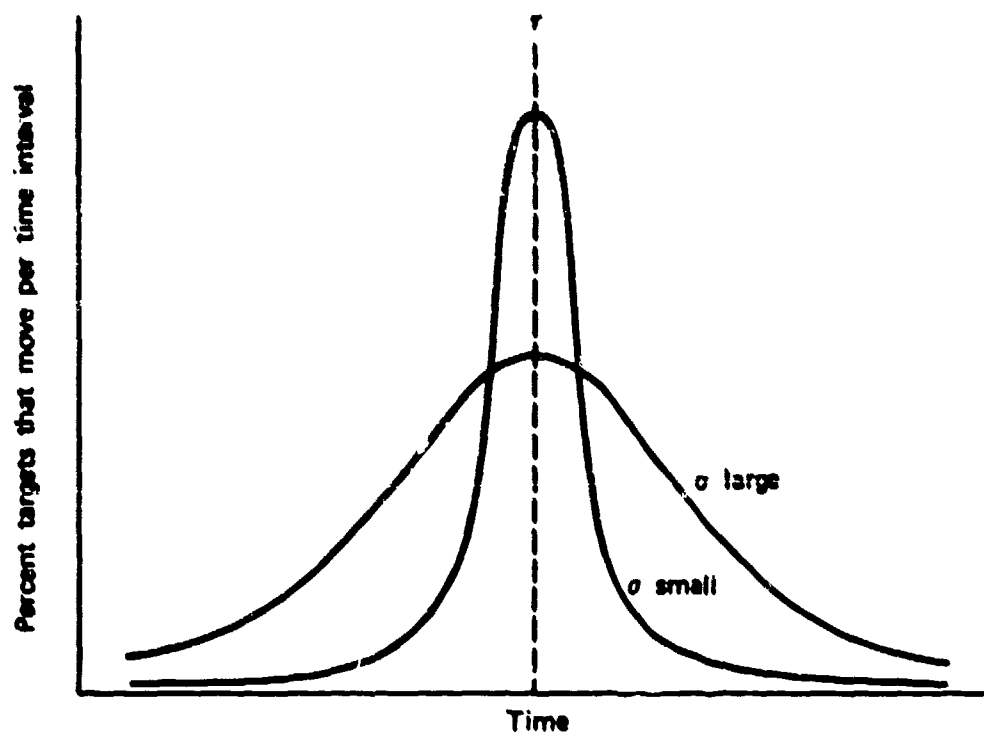


Figure 2. Target dwell time distribution model.

Selection of a dwell time model (τ and σ) permits calculation of the probability of a target being present as a function of time. The probability that a target moves before a given time is equal to the area under the dwell time distribution curve up to that time. This probably is expressed mathematically as

$$P_M = \int_0^t f(t) dt$$

where $f(t) \equiv$ Gaussian distribution function.

The probability of a target being present (P_p) at a given time is one minus the probability that it has moved before that time:

$$P_p = 1 - P_M$$

These manipulations result in a curve as illustrated in Figure 3.

Not knowing at what time the detection occurred relative to the time the target actually stopped complicates the problem. The mathematics of this complication are addressed in detail in Appendix A. The final result, i.e., the probability that the target will still be present at its original position at a time t after it was detected, is

$$P(t) = \frac{\sigma \sqrt{\frac{2}{\pi}} e^{-\frac{(t-\tau)^2}{2\sigma^2}} - (t-\tau) \left\{ 1 - \operatorname{erf} \left[\frac{(t-\tau)}{\sigma\sqrt{2}} \right] \right\}}{\sigma \sqrt{\frac{2}{\pi}} e^{-\frac{\tau^2}{2\sigma^2}} + \tau \left[1 + \operatorname{erf} \left(\frac{\tau}{\sigma\sqrt{2}} \right) \right]}$$

This equation, when normalized to present the probability in terms of the ratio t/τ and with the assumption that $\sigma = \tau/2$, results in the curve presented in Figure 1. Note the difference between Figures 1 (stop time not known) and Figure 3

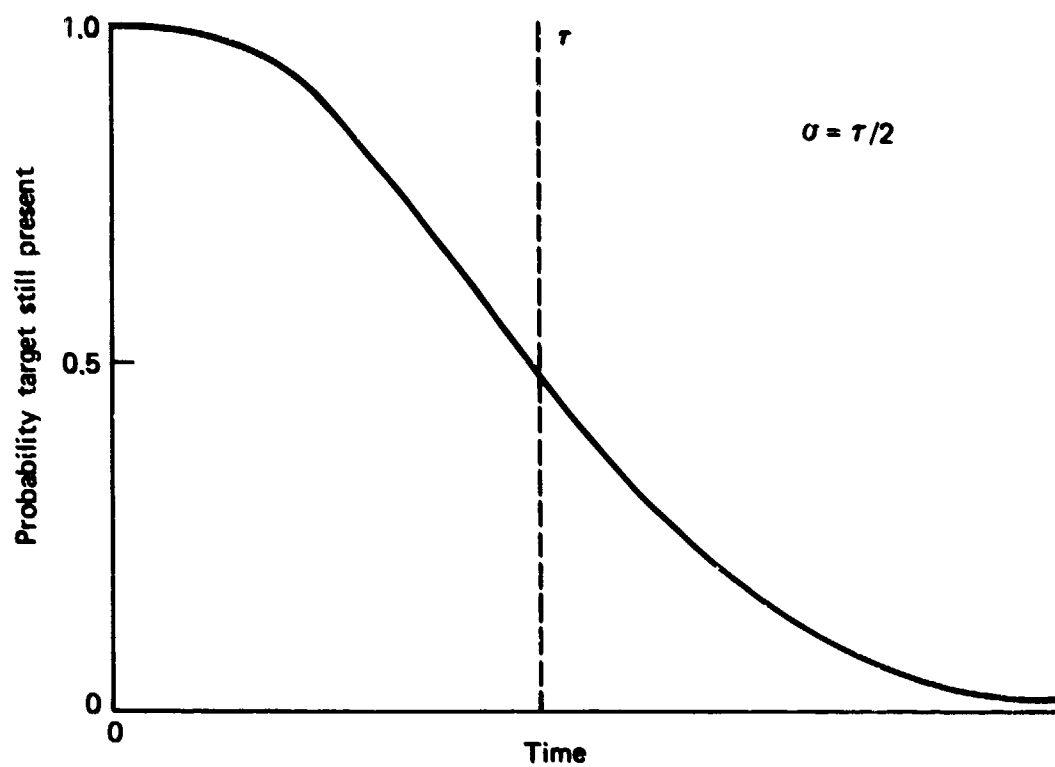


Figure 3. Probability target is still present with the time the target stops known.

(stop time known). For $t = \tau$, the former gives a probability of the target still being present of less than 20 percent while the latter, as expected, yields a probability of 50 percent. Alternatively, for a desired probability (e.g., 50 percent) t must be about 50 percent of τ with the stop time unknown or about equal to τ with the stop time known.

The remainder of this report will treat only the case where the stop time is unknown. This is probably the most realistic assumption when the demands on the target acquisition system in wartime are considered, especially with limited resources and degraded capabilities due to enemy actions.

In summary, the aforementioned methodology provides a means of estimating the probability of target presence as a function of time. The presence probability alone could be used as a criterion for selecting targets or it could be combined with the expected fractional coverage for a given weapon calculated from the field manuals as follows:

$$F' = F \times P(t)$$

where F' = revised expected coverage

F = expected coverage using weapon W assuming
static target (FM-101-31 or AP 550)

$P(t)$ = probability target at observed location.

Figure 4 shows an example result using this approach. The target is assumed to remain in one place an average time (τ) of 4 hours with $\sigma = 1$ hour. The static fraction coverages are 0.29 and 0.83 for 600 m and 200-m target location errors, respectively. The figure illustrates that if the desired expected coverage is 0.30, then it is unachievable with a target location error of 600 m. If the target were acquired with a system providing 200-m accuracy, then up to 2.6 hours can elapse between the observation and the attack execution while maintaining an expected coverage of 0.30. Note that in any

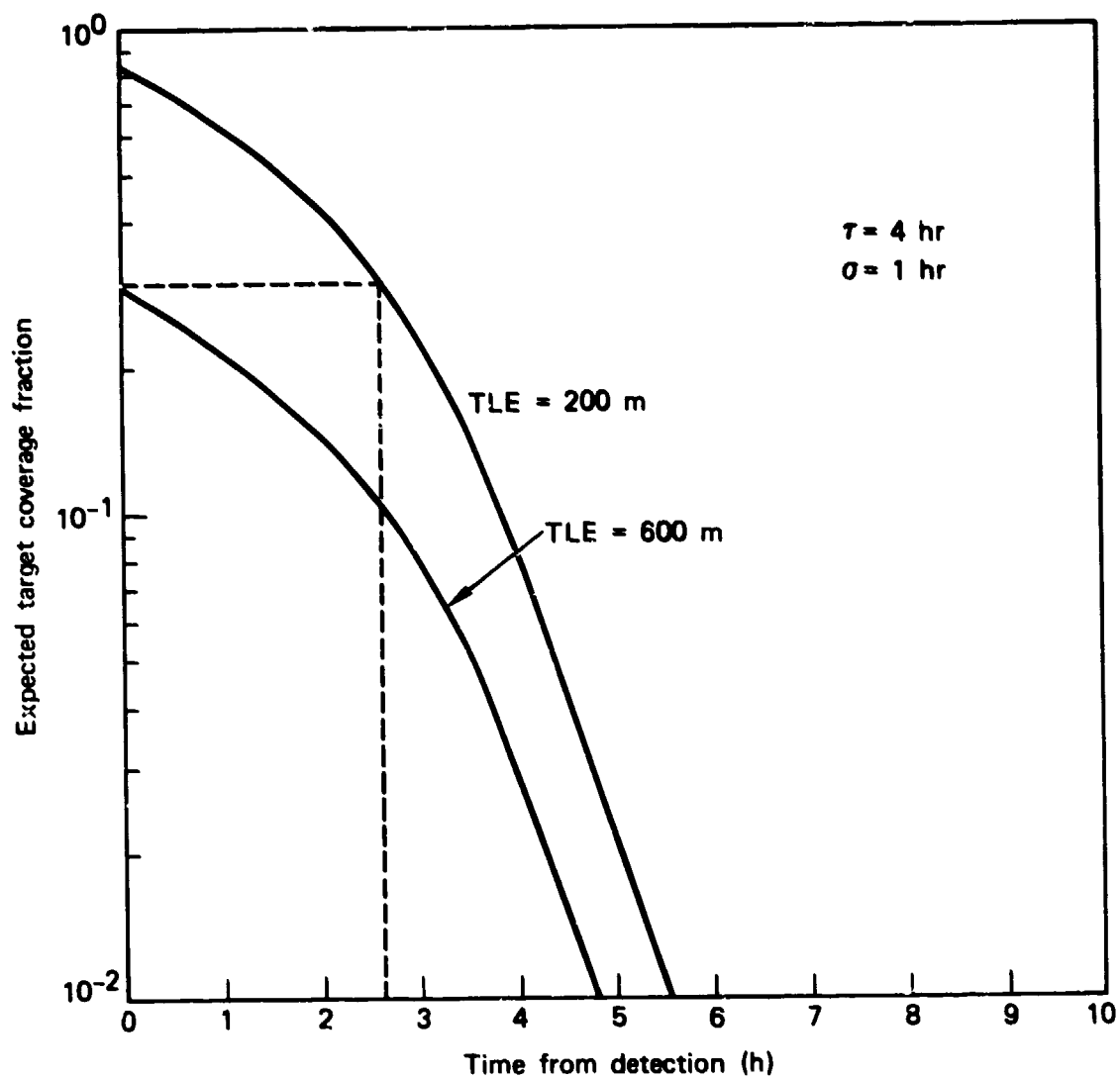


Figure 4. Effect of target movement on expected coverage (target stop time unknown).

individual case, a target will receive either 0.83 coverage or zero. Over a large number of targets the coverage to the targets considered together will be 0.30.

III. TABULAR DATA FOR TARGET ANALYSIS

Data are presented in this section for the estimation of target presence probabilities for selected values of the dwell time (τ) and standard deviation of the dwell time (σ) as noted in Table 1. Results for these cases are presented in Tables 2 through 23. All results assume the time the target stopped is unknown. See Section IV if other values of σ are desired.

TABLE 1. SPECIFIC VALUES OF τ AND σ FOR WHICH TABLES GENERATED

Case	Average dwell time, τ (h)	Standard deviation, σ (h)	
		Set A ($\sigma = \tau/3$)	Set B ($\sigma = \tau$)
1	0.1	0.033	0.1
2	0.2	0.067	0.2
3	0.5	0.167	0.5
4	1	0.333	1
5	2	0.667	2
6	4	1.333	4
7	6	2	6
8	12	4	12
9	24	8	24
10	48	12	48
11	96	32	96

TABLE 2. PROBABILITY OF TARGET PRESENCE, CASE 1A

AVERAGE RESIDENCE TIME (HOURS) = .1
 DEVIATION OF AVERAGE RESIDENCE TIME (HOURS) = .03333

TIME (HOURS)	TIME (MIN)	PROB TARGET PRESENT
0.000	0.0	1.000
.005	.3	0.950
.010	.6	0.900
.015	.9	0.850
.020	1.2	0.801
.025	1.5	0.751
.030	1.8	0.702
.035	2.1	0.653
.040	2.4	0.605
.045	2.7	0.557
.050	3.0	0.510
.055	3.3	0.464
.060	3.6	0.419
.065	3.9	0.375
.070	4.2	0.333
.075	4.5	0.294
.080	4.8	0.256
.085	5.1	0.221
.090	5.4	0.189
.095	5.7	0.159
.100	6.0	0.133
.105	6.3	0.109
.110	6.6	0.089
.115	6.9	0.071
.120	7.2	0.056
.125	7.5	0.044
.130	7.8	0.033

TABLE 3. PROBABILITY OF TARGET PRESENCE, CASE 1B

AVERAGE RESIDENCE TIME (HOURS) = .1
 DEVIATION OF AVERAGE RESIDENCE TIME (HOURS) = .1

TIME (HOURS)	TIME (MIN)	PROB TARGET PRESENT
0.000	0.0	1.000
.005	.3	0.961
.010	.6	0.923
.015	.9	0.886
.020	1.2	0.849
.025	1.5	0.813
.030	1.8	0.778
.035	2.1	0.743
.040	2.4	0.710
.045	2.7	0.676
.050	3.0	0.644
.055	3.3	0.612
.060	3.6	0.582
.065	3.9	0.552
.070	4.2	0.523
.075	4.5	0.495
.080	4.8	0.468
.085	5.1	0.442
.090	5.4	0.416
.095	5.7	0.392
.100	6.0	0.368
.105	6.3	0.346
.110	6.6	0.324
.115	6.9	0.302
.120	7.2	0.283
.125	7.5	0.264
.130	7.8	0.246
.135	8.1	0.229
.140	8.4	0.213
.145	8.7	0.197
.150	9.0	0.182
.155	9.3	0.169
.160	9.6	0.156
.165	9.9	0.143
.170	10.2	0.132
.175	10.5	0.121
.180	10.8	0.111
.185	11.1	0.102
.190	11.4	0.093
.195	11.7	0.085
.200	12.0	0.077
.205	12.3	0.070
.210	12.6	0.063
.215	12.9	0.057
.220	13.2	0.052
.225	13.5	0.047
.230	13.8	0.042
.235	14.1	0.038
.240	14.4	0.034
.245	14.7	0.030

TABLE 4. PROBABILITY OF TARGET PRESENCE, CASE 2A

AVERAGE RESIDENCE TIME (HOURS) = .2
 DEVIATION OF AVERAGE RESIDENCE TIME (HOURS) = .05667

TIME (HOURS)	TIME (MIN)	PROB TARGET PRESENT
0.000	0.0	1.000
.010	.6	0.950
.020	1.2	0.900
.030	1.8	0.850
.040	2.4	0.801
.050	3.0	0.751
.060	3.6	0.702
.070	4.2	0.653
.080	4.8	0.605
.090	5.4	0.557
.100	6.0	0.510
.110	6.6	0.464
.120	7.2	0.419
.130	7.8	0.375
.140	8.4	0.333
.150	9.0	0.294
.160	9.6	0.256
.170	10.2	0.221
.180	10.8	0.189
.190	11.4	0.159
.200	12.0	0.133
.210	12.6	0.109
.220	13.2	0.089
.230	13.8	0.071
.240	14.4	0.056
.250	15.0	0.044
.260	15.6	0.033

TABLE 5. PROBABILITY OF TARGET PRESENCE, CASE 2B

AVERAGE RESIDENCE TIME-HOURS: = .2
 DEVIATION OF AVERAGE RESIDENCE TIME-HOURS: = .2

TIME-HOURS	TIME-MIN	PROB TARGET PRESENT
0.000	0.0	1.000
.010	.6	0.961
.020	1.2	0.923
.030	1.8	0.886
.040	2.4	0.849
.050	3.0	0.813
.060	3.6	0.776
.070	4.2	0.740
.080	4.8	0.710
.090	5.4	0.676
.100	6.0	0.644
.110	6.6	0.613
.120	7.2	0.582
.130	7.8	0.552
.140	8.4	0.523
.150	9.0	0.495
.160	9.6	0.468
.170	10.2	0.442
.180	10.8	0.416
.190	11.4	0.392
.200	12.0	0.368
.210	12.6	0.346
.220	13.2	0.324
.230	13.8	0.303
.240	14.4	0.283
.250	15.0	0.264
.260	15.6	0.246
.270	16.2	0.229
.280	16.8	0.213
.290	17.4	0.197
.300	18.0	0.183
.310	18.6	0.169
.320	19.2	0.156
.330	19.8	0.143
.340	20.4	0.132
.350	21.0	0.121
.360	21.6	0.111
.370	22.2	0.102
.380	22.8	0.093
.390	23.4	0.085
.400	24.0	0.077
.410	24.6	0.070
.420	25.2	0.063
.430	25.8	0.057
.440	26.4	0.052
.450	27.0	0.047
.460	27.6	0.042
.470	28.2	0.038
.480	28.8	0.034
.490	29.4	0.030

TABLE 6. PROBABILITY OF TARGET PRESENCE, CASE 3A

AVERAGE RESIDENCE TIME (HOURS) = .5
 DEVIATION OF AVERAGE RESIDENCE TIME (HOURS) = .16667

TIME (HOURS)	TIME (MIN)	PROB TARGET PRESENT
0.000	0.0	1.000
.025	1.5	0.998
.050	3.0	0.990
.075	4.5	0.975
.100	6.0	0.951
.125	7.5	0.919
.150	9.0	0.882
.175	10.5	0.839
.200	12.0	0.792
.225	13.5	0.741
.250	15.0	0.687
.275	16.5	0.631
.300	18.0	0.573
.325	19.5	0.514
.350	21.0	0.454
.375	22.5	0.394
.400	24.0	0.335
.425	25.5	0.276
.450	27.0	0.218
.475	28.5	0.161
.500	30.0	0.105
.525	31.5	0.051
.550	33.0	0.009
.575	34.5	0.001
.600	36.0	0.000
.625	37.5	0.000
.650	39.0	0.000

TABLE 7. PROBABILITY OF TARGET PRESENCE, CASE 3B

AVERAGE RESIDENCE TIME-HOURS = .5
 DEVIATION OF AVERAGE RESIDENCE TIME-HOURS = .5

TIME-HOURS	TIME-MIN.	PROB TARGET PRESENT
0.000	0.	1.000
.025	2.	0.961
.050	3.	0.923
.075	5.	0.886
.100	6.	0.849
.125	8.	0.813
.150	9.	0.776
.175	11.	0.743
.200	12.	0.710
.225	14.	0.676
.250	15.	0.644
.275	17.	0.613
.300	18.	0.582
.325	20.	0.552
.350	21.	0.523
.375	23.	0.493
.400	24.	0.466
.425	26.	0.442
.450	27.	0.416
.475	29.	0.391
.500	30.	0.366
.525	32.	0.340
.550	33.	0.324
.575	35.	0.303
.600	36.	0.286
.625	38.	0.264
.650	39.	0.246
.675	41.	0.229
.700	42.	0.213
.725	44.	0.197
.750	45.	0.183
.775	47.	0.169
.800	49.	0.156
.825	50.	0.143
.850	51.	0.132
.875	53.	0.121
.900	54.	0.111
.925	56.	0.102
.950	57.	0.093
.975	59.	0.085
1.000	60.	0.077
1.025	62.	0.070
1.050	63.	0.063
1.075	65.	0.057
1.100	66.	0.052
1.125	68.	0.047
1.150	69.	0.042
1.175	71.	0.036
1.200	72.	0.034
1.225	74.	0.030

TABLE 8. PROBABILITY OF TARGET PRESENCE, CASE 4A

AVERAGE RESIDENCE TIME (HOURS) = 1
 DEVIATION OF AVERAGE RESIDENCE TIME (HOURS) = .3333

TIME (HOURS)	TIME (MIN)	PROB TARGET PRESENT
0.000	0.	1.000
.050	3.	0.950
.100	6.	0.900
.150	9.	0.850
.200	12.	0.801
.250	15.	0.751
.300	18.	0.702
.350	21.	0.653
.400	24.	0.605
.450	27.	0.557
.500	30.	0.510
.550	33.	0.464
.600	36.	0.419
.650	39.	0.375
.700	42.	0.333
.750	45.	0.294
.800	48.	0.256
.850	51.	0.221
.900	54.	0.189
.950	57.	0.159
1.000	60.	0.133
1.050	63.	0.109
1.100	66.	0.089
1.150	69.	0.071
1.200	72.	0.056
1.250	75.	0.044
1.300	78.	0.033

TABLE 9. PROBABILITY OF TARGET PRESENCE, CASE 4B

AVERAGE RESIDENCE TIME-HOURS: " 1
 DEVIATION OF AVERAGE RESIDENCE TIME-HOURS: " 1

TIME-HOURS	TIME-MIN	PROB TARGET PRESENT
0.000	0.	1.000
.030	3.	0.961
.100	6.	0.923
.150	9.	0.886
.200	12.	0.849
.250	15.	0.813
.300	18.	0.778
.350	21.	0.743
.400	24.	0.710
.450	27.	0.676
.500	30.	0.644
.550	33.	0.613
.600	36.	0.582
.650	39.	0.552
.700	42.	0.523
.750	45.	0.495
.800	48.	0.468
.850	51.	0.442
.900	54.	0.416
.950	57.	0.392
1.000	60.	0.368
1.050	63.	0.346
1.100	66.	0.324
1.150	69.	0.303
1.200	72.	0.283
1.250	75.	0.264
1.300	78.	0.246
1.350	81.	0.228
1.400	84.	0.213
1.450	87.	0.197
1.500	90.	0.183
1.550	93.	0.169
1.600	96.	0.156
1.650	99.	0.143
1.700	102.	0.132
1.750	105.	0.121
1.800	108.	0.111
1.850	111.	0.102
1.900	114.	0.093
1.950	117.	0.085
2.000	120.	0.077
2.050	123.	0.070
2.100	126.	0.063
2.150	129.	0.057
2.200	132.	0.052
2.250	135.	0.047
2.300	138.	0.042
2.350	141.	0.038
2.400	144.	0.034
2.450	147.	0.030

TABLE 10. PROBABILITY OF TARGET PRESENCE, CASE 5A

AVERAGE RESIDENCE TIME(HOURS) ≈ 2
 DEVIATION OF AVERAGE RESIDENCE TIME(HOURS) = .66667

TIME(HOURS)	TIME(MIN)	PROB TARGET PRESENT
0.000	0.	1.000
.100	6.	0.950
.200	12.	0.900
.300	18.	0.850
.400	24.	0.801
.500	30.	0.751
.600	36.	0.702
.700	42.	0.653
.800	48.	0.605
.900	54.	0.557
1.000	60.	0.510
1.100	66.	0.464
1.200	72.	0.419
1.300	78.	0.375
1.400	84.	0.333
1.500	90.	0.294
1.600	96.	0.256
1.700	102.	0.221
1.800	108.	0.189
1.900	114.	0.159
2.000	120.	0.133
2.100	126.	0.109
2.200	132.	0.089
2.300	138.	0.071
2.400	144.	0.056
2.500	150.	0.044
2.600	156.	0.033

TABLE 11. PROBABILITY OF TARGET PRESENCE, CASE 5B

AVERAGE RESIDENCE TIME (HOURS) = 2
 DEVIATION OF AVERAGE RESIDENCE TIME (HOURS) = 2

TIME (HOURS)	TIME (MIN)	PROB TARGET PRESENT
0.000	0.	1.000
.100	6.	0.961
.200	12.	0.923
.300	18.	0.886
.400	24.	0.849
.500	30.	0.813
.600	36.	0.778
.700	42.	0.743
.800	48.	0.710
.900	54.	0.676
1.000	60.	0.644
1.100	66.	0.613
1.200	72.	0.582
1.300	78.	0.552
1.400	84.	0.523
1.500	90.	0.495
1.600	96.	0.468
1.700	102.	0.442
1.800	108.	0.416
1.900	114.	0.392
2.000	120.	0.368
2.100	126.	0.346
2.200	132.	0.324
2.300	138.	0.303
2.400	144.	0.283
2.500	150.	0.264
2.600	156.	0.246
2.700	162.	0.229
2.800	168.	0.213
2.900	174.	0.197
3.000	180.	0.183
3.100	186.	0.169
3.200	192.	0.156
3.300	198.	0.143
3.400	204.	0.132
3.500	210.	0.121
3.600	216.	0.111
3.700	222.	0.102
3.800	228.	0.093
3.900	234.	0.085
4.000	240.	0.077
4.100	246.	0.070
4.200	252.	0.063
4.300	258.	0.057
4.400	264.	0.052
4.500	270.	0.047
4.600	276.	0.042
4.700	282.	0.038
4.800	288.	0.034
4.900	294.	0.030

TABLE 12. PROBABILITY OF TARGET PRESENCE, CASE 6A

AVERAGE RESIDENCE TIME(HOURS) = 4
 DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= 1.333

TIME(HOURS)	TIME(MIN)	PROB TARGET PRESENT
0.000	0.	1.000
.200	12.	0.950
.400	24.	0.900
.600	36.	0.850
.800	48.	0.801
1.000	60.	0.751
1.200	72.	0.702
1.400	84.	0.653
1.600	96.	0.605
1.800	108.	0.557
2.000	120.	0.510
2.200	132.	0.464
2.400	144.	0.419
2.600	156.	0.375
2.800	168.	0.333
3.000	180.	0.294
3.200	192.	0.256
3.400	204.	0.221
3.600	216.	0.189
3.800	228.	0.159
4.000	240.	0.133
4.200	252.	0.109
4.400	264.	0.089
4.600	276.	0.071
4.800	288.	0.056
5.000	300.	0.044
5.200	312.	0.033

TABLE 13. PROBABILITY OF TARGET PRESENCE, CASE 6B

AVERAGE RESIDENCE TIME (HOURS) = 4
 DEVIATION OF AVERAGE RESIDENCE TIME (HOURS) = 4

TIME (HOURS)	TIME (MIN)	PROB TARGET PRESENT
0.000	0.	1.000
.200	12.	0.961
.400	24.	0.923
.600	36.	0.886
.800	48.	0.849
1.000	60.	0.813
1.200	72.	0.778
1.400	84.	0.743
1.600	96.	0.710
1.800	108.	0.676
2.000	120.	0.644
2.200	132.	0.613
2.400	144.	0.582
2.600	156.	0.552
2.800	168.	0.523
3.000	180.	0.495
3.200	192.	0.468
3.400	204.	0.442
3.600	216.	0.416
3.800	228.	0.392
4.000	240.	0.368
4.200	252.	0.346
4.400	264.	0.324
4.600	276.	0.303
4.800	288.	0.283
5.000	300.	0.264
5.200	312.	0.246
5.400	324.	0.229
5.600	336.	0.213
5.800	348.	0.197
6.000	360.	0.183
6.200	372.	0.169
6.400	384.	0.156
6.600	396.	0.143
6.800	408.	0.132
7.000	420.	0.121
7.200	432.	0.111
7.400	444.	0.102
7.600	456.	0.093
7.800	468.	0.085
8.000	480.	0.077
8.200	492.	0.070
8.400	504.	0.063
8.600	516.	0.057
8.800	528.	0.052
9.000	540.	0.047
9.200	552.	0.042
9.400	564.	0.038
9.600	576.	0.034
9.800	588.	0.030

TABLE 14. PROBABILITY OF TARGET PRESENCE, CASE 7A

AVERAGE RESIDENCE TIME(HOURS) = 6
 DEVIATION OF AVERAGE RESIDENCE TIME(HOURS) = 2

TIME(HOURS)	TIME(MIN)	PROB TARGET PRESENT
0.000	0.	1.000
.200	12.	0.967
.400	24.	0.933
.600	36.	0.900
.800	48.	0.867
1.000	60.	0.834
1.200	72.	0.801
1.400	84.	0.768
1.600	96.	0.735
1.800	108.	0.702
2.000	120.	0.669
2.200	132.	0.637
2.400	144.	0.605
2.600	156.	0.573
2.800	168.	0.541
3.000	180.	0.510
3.200	192.	0.479
3.400	204.	0.448
3.600	216.	0.419
3.800	228.	0.389
4.000	240.	0.361
4.200	252.	0.333
4.400	264.	0.307
4.600	276.	0.281
4.800	288.	0.256
5.000	300.	0.233
5.200	312.	0.210
5.400	324.	0.189
5.600	336.	0.169
5.800	348.	0.150
6.000	360.	0.133
6.200	372.	0.117
6.400	384.	0.102
6.600	396.	0.089
6.800	408.	0.077
7.000	420.	0.066
7.200	432.	0.056
7.400	444.	0.048
7.600	456.	0.040
7.800	468.	0.033

TABLE 15. PROBABILITY OF TARGET PRESENCE, CASE 7B

AVERAGE RESIDENCE TIME (HOURS) = 6
 DEVIATION OF AVERAGE RESIDENCE TIME (HOURS) = 6

TIME (HOURS)	TIME (MIN)	PROB TARGET PRESENT
0.000	0.	1.000
.200	12.	0.974
.400	24.	0.949
.600	36.	0.923
.800	48.	0.899
1.000	60.	0.874
1.200	72.	0.849
1.400	84.	0.825
1.600	96.	0.802
1.800	108.	0.778
2.000	120.	0.755
2.200	132.	0.732
2.400	144.	0.710
2.600	156.	0.687
2.800	168.	0.666
3.000	180.	0.644
3.200	192.	0.623
3.400	204.	0.602
3.600	216.	0.582
3.800	228.	0.562
4.000	240.	0.542
4.200	252.	0.523
4.400	264.	0.504
4.600	276.	0.486
4.800	288.	0.468
5.000	300.	0.450
5.200	312.	0.433
5.400	324.	0.416
5.600	336.	0.400
5.800	348.	0.384
6.000	360.	0.368
6.200	372.	0.353
6.400	384.	0.338
6.600	396.	0.324
6.800	408.	0.310
7.000	420.	0.296
7.200	432.	0.283
7.400	444.	0.271
7.600	456.	0.258
7.800	468.	0.246
8.000	480.	0.235
8.200	492.	0.224
8.400	504.	0.213
8.600	516.	0.202
8.800	528.	0.192
9.000	540.	0.183
9.200	552.	0.173
9.400	564.	0.164
9.600	576.	0.156
9.800	588.	0.147

TABLE 16. PROBABILITY OF TARGET PRESENCE, CASE 8A

AVERAGE RESIDENCE TIME (HOURS) = 12
 DEVIATION OF AVERAGE RESIDENCE TIME (HOURS) = 4

TIME (HOURS)	TIME (MIN)	PROB TARGET PRESENT
0.000	0.	1.000
.500	30.	0.958
1.000	60.	0.917
1.500	90.	0.875
2.000	120.	0.834
2.500	150.	0.793
3.000	180.	0.751
3.500	210.	0.710
4.000	240.	0.669
4.500	270.	0.629
5.000	300.	0.589
5.500	330.	0.549
6.000	360.	0.510
6.500	390.	0.471
7.000	420.	0.433
7.500	450.	0.397
8.000	480.	0.361
8.500	510.	0.327
9.000	540.	0.294
9.500	570.	0.262
10.000	600.	0.233
10.500	630.	0.205
11.000	660.	0.179
11.500	690.	0.155
12.000	720.	0.133
12.500	750.	0.113
13.000	780.	0.095
13.500	810.	0.080
14.000	840.	0.066
14.500	870.	0.054
15.000	900.	0.044
15.500	930.	0.035

TABLE 17. PROBABILITY OF TARGET PRESENCE, CASE 8B

AVERAGE RESIDENCE TIME (HOURS) = 12
 DEVIATION OF AVERAGE RESIDENCE TIME (HOURS) = 12

TIME (HOURS)	TIME (MIN)	PROB TARGET PRESENT
0.000	0.	1.000
.500	30.	0.968
1.000	60.	0.936
1.500	90.	0.905
2.000	120.	0.874
2.500	150.	0.843
3.000	180.	0.813
3.500	210.	0.784
4.000	240.	0.755
4.500	270.	0.726
5.000	300.	0.698
5.500	330.	0.671
6.000	360.	0.644
6.500	390.	0.618
7.000	420.	0.592
7.500	450.	0.567
8.000	480.	0.542
8.500	510.	0.518
9.000	540.	0.495
9.500	570.	0.472
10.000	600.	0.450
10.500	630.	0.429
11.000	660.	0.408
11.500	690.	0.388
12.000	720.	0.368
12.500	750.	0.349
13.000	780.	0.331
13.500	810.	0.313
14.000	840.	0.296
14.500	870.	0.280
15.000	900.	0.264
15.500	930.	0.249
16.000	960.	0.235
16.500	990.	0.221
17.000	1020.	0.207
17.500	1050.	0.195
18.000	1080.	0.183
18.500	1110.	0.171
19.000	1140.	0.160
19.500	1170.	0.149
20.000	1200.	0.139
20.500	1230.	0.130
21.000	1260.	0.121
21.500	1290.	0.113
22.000	1320.	0.105
22.500	1350.	0.097
23.000	1380.	0.090
23.500	1410.	0.083
24.000	1440.	0.077
24.500	1470.	0.071

TABLE 18. PROBABILITY OF TARGET PRESENCE, CASE 9A

AVERAGE RESIDENCE TIME (HOURS) = 24
 DEVIATION OF AVERAGE RESIDENCE TIME (HOURS) = 8

TIME (HOURS)	TIME (MIN)	PROB TARGET PRESENT
0.000	0.	1.000
1.000	60.	0.958
2.000	120.	0.917
3.000	180.	0.875
4.000	240.	0.834
5.000	300.	0.793
6.000	360.	0.751
7.000	420.	0.710
8.000	480.	0.669
9.000	540.	0.629
10.000	600.	0.589
11.000	660.	0.549
12.000	720.	0.510
13.000	780.	0.471
14.000	840.	0.433
15.000	900.	0.397
16.000	960.	0.361
17.000	1020.	0.327
18.000	1080.	0.294
19.000	1140.	0.262
20.000	1200.	0.233
21.000	1260.	0.205
22.000	1320.	0.179
23.000	1380.	0.155
24.000	1440.	0.133
25.000	1500.	0.113
26.000	1560.	0.095
27.000	1620.	0.080
28.000	1680.	0.066
29.000	1740.	0.054
30.000	1800.	0.044
31.000	1860.	0.035

TABLE 19. PROBABILITY OF TARGET PRESENCE, CASE 9B

AVERAGE RESIDENCE TIME HOURS = 24
 DEVIATION OF AVERAGE RESIDENCE TIME HOURS = 24

TIME HOURS	TIME MIN.	PROB. TARGET PRESENT
0.000	0.	1.000
1.000	60.	0.968
2.000	120.	0.936
3.000	180.	0.905
4.000	240.	0.874
5.000	300.	0.843
6.000	360.	0.813
7.000	420.	0.784
8.000	480.	0.755
9.000	540.	0.726
10.000	600.	0.698
11.000	660.	0.671
12.000	720.	0.644
13.000	780.	0.618
14.000	840.	0.592
15.000	900.	0.567
16.000	960.	0.542
17.000	1020.	0.518
18.000	1080.	0.495
19.000	1140.	0.472
20.000	1200.	0.450
21.000	1260.	0.429
22.000	1320.	0.408
23.000	1380.	0.388
24.000	1440.	0.368
25.000	1500.	0.349
26.000	1560.	0.331
27.000	1620.	0.313
28.000	1680.	0.296
29.000	1740.	0.280
30.000	1800.	0.264
31.000	1860.	0.249
32.000	1920.	0.235
33.000	1980.	0.221
34.000	2040.	0.207
35.000	2100.	0.195
36.000	2160.	0.183
37.000	2220.	0.171
38.000	2280.	0.160
39.000	2340.	0.149
40.000	2400.	0.139
41.000	2460.	0.130
42.000	2520.	0.121
43.000	2580.	0.113
44.000	2640.	0.105
45.000	2700.	0.097
46.000	2760.	0.090
47.000	2820.	0.083
48.000	2880.	0.077
49.000	2940.	0.071

TABLE 20. PROBABILITY OF TARGET PRESENCE, CASE 10A

AVERAGE RESIDENCE TIME (HOURS) = 48
 DEVIATION OF AVERAGE RESIDENCE TIME (HOURS) = 12

TIME (HOURS)	TIME (MIN)	PROB TARGET PRESENT
0.000	0.	1.000
2.000	120.	0.958
4.000	240.	0.917
6.000	360.	0.875
8.000	480.	0.833
10.000	600.	0.792
12.000	720.	0.750
14.000	840.	0.709
16.000	960.	0.667
18.000	1080.	0.625
20.000	1200.	0.584
22.000	1320.	0.543
24.000	1440.	0.502
26.000	1560.	0.462
28.000	1680.	0.422
30.000	1800.	0.382
32.000	1920.	0.344
34.000	2040.	0.307
36.000	2160.	0.271
38.000	2280.	0.237
40.000	2400.	0.204
42.000	2520.	0.174
44.000	2640.	0.147
46.000	2760.	0.122
48.000	2880.	0.100
50.000	3000.	0.080
52.000	3120.	0.064
54.000	3240.	0.049
56.000	3360.	0.038

TABLE 21. PROBABILITY OF TARGET PRESENCE, CASE 10B

AVERAGE RESIDENCE TIME (HOURS) = 48
 DEVIATION OF AVERAGE RESIDENCE TIME (HOURS) = 48

TIME (HOURS)	TIME (MIN)	PROB TARGET PRESENT
0.000	0.	1.000
2.000	120.	0.969
4.000	240.	0.936
6.000	360.	0.905
8.000	480.	0.874
10.000	600.	0.843
12.000	720.	0.813
14.000	840.	0.784
16.000	960.	0.755
18.000	1080.	0.726
20.000	1200.	0.698
22.000	1320.	0.671
24.000	1440.	0.644
26.000	1560.	0.618
28.000	1680.	0.592
30.000	1800.	0.567
32.000	1920.	0.542
34.000	2040.	0.518
36.000	2160.	0.495
38.000	2280.	0.472
40.000	2400.	0.450
42.000	2520.	0.429
44.000	2640.	0.408
46.000	2760.	0.388
48.000	2880.	0.368
50.000	3000.	0.349
52.000	3120.	0.331
54.000	3240.	0.313
56.000	3360.	0.296
58.000	3480.	0.280
60.000	3600.	0.264
62.000	3720.	0.249
64.000	3840.	0.235
66.000	3960.	0.221
68.000	4080.	0.207
70.000	4200.	0.195
72.000	4320.	0.183
74.000	4440.	0.171
76.000	4560.	0.160
78.000	4680.	0.149
80.000	4800.	0.139
82.000	4920.	0.130
84.000	5040.	0.121
86.000	5160.	0.113
88.000	5280.	0.105
90.000	5400.	0.097
92.000	5520.	0.090
94.000	5640.	0.083
96.000	5760.	0.077
98.000	5880.	0.071

TABLE 22. PROBABILITY OF TARGET PRESENCE, CASE 11A

AVERAGE RESIDENCE TIME (HOURS) = 96
 DEVIATION OF AVERAGE RESIDENCE TIME (HOURS) = 32

TIME (HOURS)	TIME (MIN)	PROB TARGET PRESENT
0.000	0.	1.000
5.000	300.	0.948
10.000	600.	0.896
15.000	900.	0.844
20.000	1200.	0.793
25.000	1500.	0.741
30.000	1800.	0.690
35.000	2100.	0.639
40.000	2400.	0.589
45.000	2700.	0.539
50.000	3000.	0.490
55.000	3300.	0.443
60.000	3600.	0.397
65.000	3900.	0.352
70.000	4200.	0.310
75.000	4500.	0.270
80.000	4800.	0.233
85.000	5100.	0.198
90.000	5400.	0.167
95.000	5700.	0.138
100.000	6000.	0.113
105.000	6300.	0.091
110.000	6600.	0.073
115.000	6900.	0.057
120.000	7200.	0.044
125.000	7500.	0.033

TABLE 23. PROBABILITY OF TARGET PRESENCE, CASE 11B

AVERAGE RESIDENCE TIME HOURS: $\bar{t} = 96$
 DEVIATION OF AVERAGE RESIDENCE TIME HOURS: $\sigma = 96$

TIME HOURS	TIME MIN	PROB TARGET PRESENT
0.000	0.	1.000
5.000	300.	0.960
10.000	600.	0.920
15.000	900.	0.882
20.000	1200.	0.843
25.000	1500.	0.806
30.000	1800.	0.769
35.000	2100.	0.733
40.000	2400.	0.698
45.000	2700.	0.664
50.000	3000.	0.631
55.000	3300.	0.598
60.000	3600.	0.567
65.000	3900.	0.536
70.000	4200.	0.507
75.000	4500.	0.478
80.000	4800.	0.450
85.000	5100.	0.424
90.000	5400.	0.398
95.000	5700.	0.373
100.000	6000.	0.349
105.000	6300.	0.327
110.000	6600.	0.305
115.000	6900.	0.284
120.000	7200.	0.264
125.000	7500.	0.246
130.000	7800.	0.228
135.000	8100.	0.211
140.000	8400.	0.195
145.000	8700.	0.180
150.000	9000.	0.165
155.000	9300.	0.152
160.000	9600.	0.139
165.000	9900.	0.128
170.000	10200.	0.117
175.000	10500.	0.107
180.000	10800.	0.097
185.000	11100.	0.088
190.000	11400.	0.080
195.000	11700.	0.072
200.000	12000.	0.065
205.000	12300.	0.059
210.000	12600.	0.053
215.000	12900.	0.048
220.000	13200.	0.043
225.000	13500.	0.038
230.000	13800.	0.034
235.000	14100.	0.030

IV. GENERALIZED DATA FOR TARGET ANALYSIS

Figure 5 presents the generalized curve for the probability of a target being present as a function of time from detection. The probability is presented as a function of two parameters, τ/σ and t/τ , where

τ = average target dwell time

σ = standard deviation of the dwell time

t = time.

This curve is applicable to all values of τ and σ and may be used for those cases for which the tabular data in Section III are inadequate. Examples illustrating its use are presented in Section V. Coding for calculating these data on the TI-59 is presented in Appendix B.

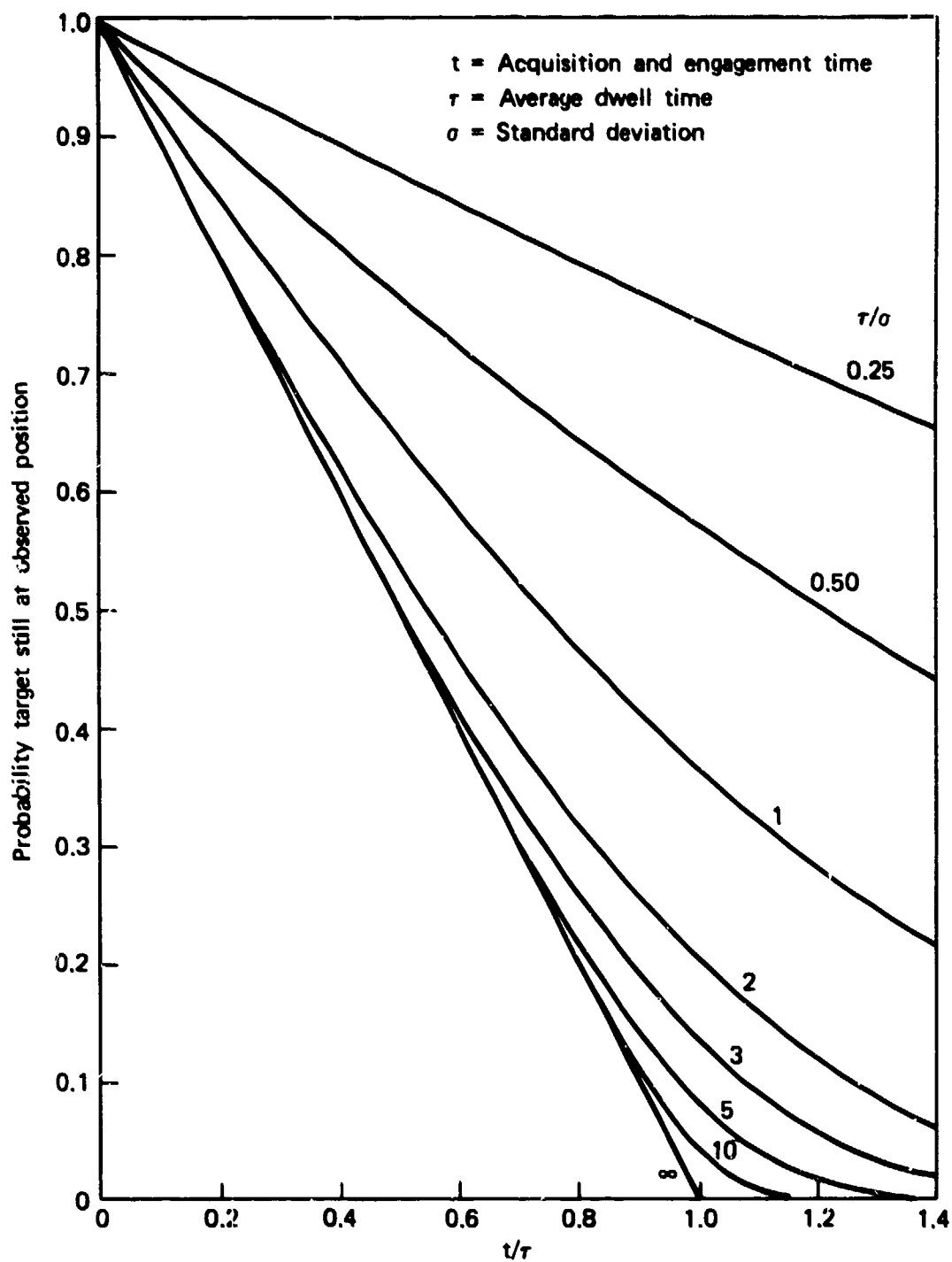


Figure 5. Generalized curve for estimating probability target is still at observed location.

V. SAMPLE CASES

Case 1

Compute the probability that a target is still at an observed position 3 hours after the observation if the target is assumed to have an average dwell time (τ) of 4 hours with a deviation (σ) of 1.33 hours.

Solution: Table 12, p. 26 applicable.
Answer is $P = 0.294$.

Case 2

For the above case, what is the revised expected fractional coverage with a 1-KT weapon with a CEP of 140 m, a 200-m target location error, and a 200-m target radius?

Solution: The damage radius is about 660 m with a criterion of latent lethality. FM-101-31 estimates the static fractional coverage to be 0.98. The revised expected fractional coverage is $0.95 \times 0.294 = 0.29$.

Case 3

For a target represented by a dwell time (τ) of 6 hours and a standard deviation (σ) of 6 hours, what is the allowable response time for a desired probability of target presence of 0.30?

Solution: Table 15, p. 29 is applicable.
By interpolation, $T = 6.9$ hours.

Case 4

What is the probability that a target is present after 3 hours if it is represented by the parameters $\tau = 3.2$ hours and $\sigma = 1.5$?

Solution: Generalized curve, p. 34.

$$\frac{\tau}{\sigma} = \frac{3.2}{1.5} = 2.13$$

$$\frac{t}{\tau} = \frac{3.0}{3.2} = 0.94$$

By interpolation, $P = 0.22$.

Case 5

For the values of τ and σ assumed in Case 4, what is the allowable response time if the desired expected coverage is 0.50?

Solution: From the generalized curve with:

$$\frac{\tau}{\sigma} = 2.13, P = 0.5$$

$$\frac{t}{\tau} = 0.54$$

$$t = (0.54)(3.2) = 1.73 \text{ hours.}$$

APPENDIX A. EQUATIONS FOR TARGET PERMANENCE

We begin by assuming that the probability of the target leaving (i.e., beginning to move from) its original position between t and $t + dt$ is

$$P_1(t)dt = \frac{1}{C\sigma\sqrt{2\pi}} e^{-(t-\tau)^2/2\sigma^2} dt$$

where $t = 0$ is the time at which the target originally settled into the given position, τ is the average time that the target remains in place, σ^2 is the variance in the distribution, and the normalization constant

$$C = \frac{1}{2} + \frac{1}{2} \operatorname{erf} \left(\frac{\tau}{\sigma\sqrt{2}} \right)$$

is chosen such that

$$\int_0^{\infty} P_1(t)dt = 1$$

Sixty-eight percent of the targets will leave between $\tau - \sigma$ and $\tau + \sigma$. Ninety-five percent of the targets will leave between $\tau - 2\sigma$ and $\tau + 2\sigma$.

We now assume that the target is detected at some arbitrary time $t = t_1 > 0$ which is completely uncorrelated to the movements of the target, and we wish to know the probability density $P_2(t_2)$ of the time t_2 between detection and the departure of the target.

This turns out to be one of the main problems of a branch of probability theory called renewal theory. The random variable t_2 is called the residual waiting time or the excess lifetime. Using the results of renewal theory (Ref. A1) it can be shown that the probability that the target will leave at a time t_2 after it is detected is

$$P_2(t_2)dt_2 = \frac{1 - F_1(t_2)}{\mu} dt_2$$

where

$$F_1(t_2) = \int_0^{t_2} P_1(t) dt$$

and

$$\mu = \int_0^{\infty} t P_1(t) dt$$

or, integrating by parts and using $F_1(\infty) = 1$,

$$\mu = \int_0^{\infty} [1 - F_1(t)] dt$$

so that

$$P_2(t_2)dt_2 = \frac{[1 - F_1(t_2)]dt_2}{\int_0^{\infty} [1 - F_1(t)]dt}$$

Using the original expression for $P_1(t)$ we now have

$$\begin{aligned} 1 - F_1(t_2) &= \int_{t_2}^{\infty} P_1(t)dt \\ &= \frac{1}{C \sigma \sqrt{2\pi}} \int_{t_2}^{\infty} e^{-(t-\tau)^2/2\sigma^2} dt \end{aligned}$$

Setting $t-\tau = \sigma\sqrt{2}x$, this becomes

$$1 - F_1(t_2) = \frac{1}{C \sqrt{\pi}} \int_{\frac{t_2-\tau}{\sigma\sqrt{2}}}^{\infty} e^{-x^2} dx$$

which gives (Ref. A2)

$$1 - F_1(t_2) = \frac{1}{2C} \left\{ 1 - \operatorname{erf} \left[\frac{(t_2-\tau)}{\sigma\sqrt{2}} \right] \right\}$$

where $\text{erf}(x)$ is the error function, so that the expression for $P_2(t_2)dt_2$ becomes

$$P_2(t_2)dt_2 = \frac{\left| 1 - \text{erf} \left[\frac{(t_2 - \tau)}{\sigma\sqrt{2}} \right] \right| dt_2}{\int_0^\infty \left| 1 - \text{erf} \left[\frac{(t-\tau)}{\sigma\sqrt{2}} \right] \right| dt}$$

This can be further simplified. Using the formulas (Ref. A3)

$$\int \text{erf}(ax) dx = x \text{erf}(ax) + \frac{e^{-a^2 x^2}}{a\sqrt{\pi}}$$

and

$$\int_0^\infty [1 - \text{erf}(ax)] dx = \frac{1}{a\sqrt{\pi}}$$

we obtain

$$\int_y^\infty [1 - \text{erf}(ax)] dx = \frac{e^{-a^2 y^2}}{a\sqrt{\pi}} - y[1 - \text{erf}(ay)]$$

Using this we have

$$\begin{aligned} \int_0^\infty \left| 1 - \text{erf} \left[\frac{(t-\tau)}{\sigma\sqrt{2}} \right] \right| dt &= \sigma\sqrt{\frac{2}{\pi}} e^{-\tau^2/2\sigma^2} \\ &+ \tau \left| 1 + \text{erf} \left(\frac{\tau}{\sigma\sqrt{2}} \right) \right| \end{aligned}$$

so that the final result for $P_2(t_2)dt_2$ is

$$P_2(t_2)dt_2 = \frac{\left| 1 - \operatorname{erf} \left[\frac{(t_2 - \tau)}{\sigma\sqrt{2}} \right] \right| dt_2}{\sigma\sqrt{\frac{2}{\pi}} e^{-\tau^2/2\sigma^2} + \tau \left[1 + \operatorname{erf} \left(\frac{\tau}{\sigma\sqrt{2}} \right) \right]}$$

The probability that the target will still be present at a time t_3 after it was detected is now given by

$$P_3(t_3) = 1 - \int_0^{t_3} P_2(t_2)dt_2$$

or, since $\int_0^{\infty} P_2(t_2)dt_2 = 1$,

$$P_3(t_3) = \int_{t_3}^{\infty} P_2(t_2)dt_2$$

Using the expression for $P_2(t_2)$, we obtain after integrating

$$P_3(t_3) = \frac{\sigma\sqrt{\frac{2}{\pi}} e^{-(t_3 - \tau)^2/2\sigma^2} - (t_3 - \tau) \left| 1 - \operatorname{erf} \left[\frac{(t_3 - \tau)}{\sigma\sqrt{2}} \right] \right|}{\sigma\sqrt{\frac{2}{\pi}} e^{-\tau^2/2\sigma^2} + \tau \left[1 + \operatorname{erf} \left(\frac{\tau}{\sigma\sqrt{2}} \right) \right]}$$

REFERENCES TO APPENDIX A

- A1. Feller, W., An Introduction to Probability Theory and Its Applications, Vol. 2, Second Edition, John Wiley & Sons, New York, 1971, p. 370.
- A2. Abramowitz, M., and Stegun, I. A., Handbook of Mathematical Functions, Dover Publications, Inc., New York, 1965, p. 297.
- A3. Gradshteyn, I. W., and Ryzhik, I. M., Table of Integrals, Series, and Products, Academic Press, New York, 1965, pp. 633 and 648.

APPENDIX B. TI-59 CODE FOR MOBILE TARGET CALCULATIONS

The equation for the probability of a target being present at an observed location as a function of time where it is not known when the observation took place relative to the time the target stopped is

$$P(t) = \frac{\sigma \sqrt{\frac{2}{\pi}} e^{-\frac{(t-\tau)^2}{2\sigma^2}} - (t-\tau) \left[1 - \operatorname{erf}\left(\frac{t-\tau}{\sigma\sqrt{2}}\right) \right]}{\sigma \sqrt{\frac{2}{\pi}} e^{-\frac{\tau^2}{2\sigma^2}} + \tau \left[1 + \operatorname{erf}\left(\frac{\tau}{\sigma\sqrt{2}}\right) \right]}$$

where erf = error function

τ = mean dwell time

σ = standard deviation in dwell time.

The derivation of this equation is presented in Appendix A. An approximation to the error function suitable for use in the TI-59 is

$$\text{If } |X| < 1.18: \operatorname{erf}(X) = \frac{2}{\sqrt{\pi}} \left(X - \frac{X^3}{3} + \frac{X^5}{10} - \frac{X^7}{42} \right)$$

$$\text{If } |X| \geq 1.18: \operatorname{erf}(X) = 1 - \frac{1}{\sqrt{\pi}} \frac{e}{X}$$

With this approximation the maximum error in the error function (about 5 percent) occurs at $X = 1.18$. The resultant error in the calculated probability of target presence is less than 1 percent for cases resulting in probabilities greater than 5 percent.

The following pages present TI-59 coding of the above equations and directions for running the program.

INSTRUCTIONS

The program uses standard partitioning (479/59) and will fit on two magnetic cards (four "sides"). After entering the program, the procedures are as follows:

<u>Step</u>	<u>Instruction</u>	<u>Data*</u>	<u>Keys</u>
1	Set initial time	0.	STO 5
2	Set stop criteria**	.001	STO 16
3	Initialize		INV 2nd FIX
4	Enter average dwell time	6	STO 3
5	Enter standard deviation	2	STO 2
6	Enter time increment	1	STO 1
7	Begin run		A

*Data for sample case illustrated.

**Program stops when the probability of a target being present is less than this value.

The output from the printer is illustrated on the next page.
The quantities are:

ADP = average dwell time (input)

DEV = standard deviation of the dwell time (input)

TIME = time

PROB = probability of target being present at given time.

RESULTS FOR SAMPLE CASE

ADT
DEV
6.
2.

TIME
PROB
0.000
1.000

TIME
PROB
1.000
0.833

TIME
PROB
2.000
0.667

TIME
PROB
3.000
0.508

TIME
PROB
4.000
0.361

TIME
PROB
5.000
0.233

TIME
PROB
6.000
0.133

TIME
PROB
7.000
0.066

TIME
PROB
8.000
0.028

TIME
PROB
9.000
0.008

TIME
PROB
10.000
0.000

PROGRAM CODING

000	76	LBL	
001	11	R	
002	02	2	
003	34	FX	
004	65	X	
005	43	RCL	$\sigma\sqrt{2}$
006	02	02	
007	95	=	
008	42	STD	
009	06	06	
010	89	1	
011	34	FX	$\sqrt{\pi}$
012	42	STD	
013	07	07	
014	43	RCL	
015	06	06	
016	55	-	
017	43	RCL	$A/\sqrt{\pi}$
018	07	07	
019	95	=	
020	42	STD	
021	08	08	
022	43	RCL	
023	02	02	
024	33	13	
025	65		$2\sigma^2$
026	01	0	
027	95	=	
028	42	STD	
029	04	04	
030	43	RCL	
031	03	03	
032	55	-	
033	43	RCL	
034	06	06	$x = \tau/A$
035	95	=	
036	42	STD	
037	12	12	
038	42	STD	
039	10	10	
040	01	1	
041	93	.	
042	01	1	1.18
043	08	8	
044	42	STD	
045	04	04	
046	41	STD	
047	03	03	
048	44	42	
049	61	STD	Print τ and σ
050	03	03	
051	68	68	
052	58	FIN	
053	03	03	

PROGRAM CODING (CONTINUED)

054	43	RCL	$E_1 = \text{erf}(x)$
055	12	12	
056	71	SBR	
057	12	B	
058	42	STD	
059	20	20	Initialize time
060	98	ADV	
061	43	RCL	
062	05	05	
063	42	STD	
064	15	15	$x^1 = \frac{t-r}{\sigma\sqrt{2}}$
065	98	ADV	
066	43	RCL	
067	15	15	
068	75	-	
069	43	RCL	$E_2 = \text{erf}(x^1)$
070	03	03	
071	95	=	
072	55	+	
073	43	RCL	
074	06	06	E_2 sign check
075	95	=	
076	42	STD	
077	10	10	
078	50	I×I	
079	42	STD	t/r
080	12	12	
081	71	SBR	
082	12	B	
083	42	STD	
084	21	21	Print time
085	71	SBR	
086	13	C	
087	43	RCL	
088	15	15	
089	55	+	$r[1+E_1] = d_1$
090	43	RCL	
091	03	03	
092	95	=	
093	68	NOP	
094	61	GTO	
095	03	03	
096	87	87	
097	01	1	
098	85	+	
099	43	RCL	
100	20	20	
101	95	=	
102	65	×	
103	43	RCL	
104	03	03	
105	95	=	
106	42	STD	
107	23	23	

PROGRAM CODING (CONTINUED)

108	43	RCL	
109	03	03	
110	33	X ²	
111	55	+	
112	43	RCL	
113	09	09	
114	95	=	$\sigma\sqrt{2/\pi} e^{-\tau^2/2\sigma^2} = d_2$
115	71	SBR	
116	14	D	
117	65	X	
118	43	RCL	
119	08	08	
120	95	=	
121	65	+	
122	43	RCL	
123	23	23	
124	95	=	$d_1 + d_2$
125	42	STD	
126	23	23	
127	43	RCL	
128	15	15	
129	75	-	
130	43	RCL	
131	03	03	
132	95	=	
133	33	X ²	
134	55	+	
135	43	RCL	
136	09	09	
137	95	=	$\sigma\sqrt{2/\pi} e^{\frac{-(t-\tau)^2}{2\sigma^2}} = n_1$
138	71	SBR	
139	14	D	
140	65	X	
141	43	RCL	
142	08	08	
143	95	=	
144	42	STD	
145	24	24	
146	01	1	
147	75	-	
148	43	RCL	
149	21	21	
150	95	=	
151	65	X	
152	53	(	
153	43	RCL	
154	15	15	$(t-\tau)(1-E_2) = n_2$
155	75	-	
156	43	RCL	
157	03	03	
158	54	)	
159	95	=	
160	94	+/-	
161	42	STD	
162	25	25	

PROGRAM CODING (CONTINUED)

```

163 43 RCL
164 24 24
165 85 +
166 43 RCL
167 25 25
168 95 =
169 55 +
170 43 RCL
171 23 23
172 95 =
173 42 STO
174 29 29
175 61 GTO
176 04 04
177 08 08
178 32 XRT
179 43 RCL
180 16 16
181 77 GE
182 01 01
183 97 97
184 98 ADV
185 68 NOP
186 43 RCL
187 15 15
188 85 +
189 43 RCL
190 01 01
191 95 =
192 42 STO
193 15 15
194 61 GTO
195 00 00
196 66 66
197 91 R/S

```

$$P(t) = \frac{(n_1 + n_2)}{(d_1 + d_2)}$$

Stop check

Increment time

PROGRAM CODING (CONTINUED)

198	76	LBL	
199	12	B	
200	43	RCL	
201	04	04	
202	89	NIT	
203	43	RCL	
204	12	12	
205	77	GE	Check for x > or < 1.18
206	02	02	
207	59	59	
208	43	RCL	
209	12	12	
210	45	YX	
211	05	5	
212	95	=	
213	55	-	$x^5/10$
214	01	1	
215	00	0	
216	95	=	
217	42	STD	
218	13	13	
219	43	RCL	
220	12	12	
221	45	YX	
222	01	1	
223	95	=	
224	50	-	
225	04	4	
226	02	2	$x^7/42$
227	95	=	
228	32	+ -	
229	95	=	
230	43	RCL	
231	12	12	
232	95	=	
233	42	STD	
234	13	13	
235	43	RCL	
236	12	12	
237	45	YX	
238	03	3	
239	95	=	
240	55	+ -	
241	03	3	$x^3/3$
242	95	=	
243	34	+ -	
244	95	=	
245	43	RCL	
246	12	12	
247	95	=	
248	43	RCL	
249	12	12	
250	95	=	
251	95	=	
252	02	2	$\text{erf}(x) = 2/\sqrt{\pi} (x - x^3/3 + x^5/10 + x^7/42)$
253	95	=	
254	95	=	
255	43	RCL	
256	07	07	
257	95	=	
258	92	RTN	

PROGRAM CODING (CONTINUED)

259	03	3	
260	32	X:T	
261	43	RCL	x > 3?
262	12	12	
263	77	GE	
264	02	02	
265	88	88	
266	43	RCL	
267	12	12	
268	33	X2	
269	94	+/-	
270	27	INV	
271	23	LNX	
272	55	-	
273	43	RCL	
274	12	12	
275	95	=	
276	55	+	
277	43	RCL	
278	07	07	
279	95	=	
280	42	STD	
281	13	13	
282	01	1	
283	75	-	
284	43	RCL	
285	13	13	
286	95	=	
287	92	RTN	
288	01	1	
289	92	RTN	
290	00	0	
291	76	LEL	
292	13	0	
293	00	0	
294	32	X:T	x = 0?
295	43	RCL	
296	12	12	
297	67	EQ	
298	03	03	
299	12	12	
300	43	RCL	
301	12	12	
302	55	-	
303	43	RCL	
304	10	10	
305	95	=	
306	65	X	Sign check for E
307	43	RCL	
308	21	21	
309	95	=	
310	42	STD	
311	21	21	
312	43	RCL	
313	21	21	
314	92	RTN	

$\text{erf}(x) = 1 - \frac{e^{-x^2}}{\sqrt{\pi} x}$

PROGRAM CODING (CONTINUED)

315	76	LBL	
316	14	D	
317	42	STD	
318	26	26	
319	50	I×I	
320	42	STD	
321	27	27	Limit exponent to 99
322	09	9	
323	09	9	
324	32	X: T	
325	43	RCL	
326	27	27	
327	77	GE	
328	03	03	
329	36	36	
330	43	RCL	
331	26	26	
332	94	+/-	
333	22	INV	
334	23	LN X	
335	92	RTN	
336	43	RCL	
337	27	27	
338	55	-	
339	43	RCL	
340	26	26	e ^Y , Y < 99
341	95	=	
342	65	×	
343	09	9	
344	09	9	
345	95	=	
346	61	GTO	
347	03	03	
348	32	32	
349	25	CLR	
350	69	DP	
351	00	00	
352	01	1	
353	03	3	
354	01	1	
355	06	6	
356	03	3	
357	07	7	
358	69	DP	
359	01	01	Label "ADT"
360	69	DP	
361	05	05	
362	43	RCL	
363	03	03	
364	99	PRT	
365	61	GTO	
366	00	00	
367	49	49	

PROGRAM CODING (CONCLUDED)

[illegible]

Label "DEV"

Label "Time"

Label "Prob"

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U.S. Naval Forces, Europe
ATTN: NS4, Nuc Warfare Off

DEPARTMENT OF THE AIR FORCE

Air University Library
Department of the Air Force
ATTN: AUL/LSE

Assistant Chief of Staff
Intelligence
Department of the Air Force
ATTN: INA

Assistant Chief of Staff
Studies & Analyses
Department of the Air Force
ATTN: AF/SAG, H. Ziemer
ATTN: AF/SASB, R. Mathis
ATTN: AF/SASF
ATTN: AF/SASM
ATTN: AF/SAHI
ATTN: AF/SASM, W. Adams

Headquarters Space Div
Air Force Systems Command
ATTN: YKD

Commander-in-Chief
U.S. Air Forces in Europe
ATTN: USAFE/INA

Commander-in-Chief
U.S. Air Forces in Europe
ATTN: USAFE/XPX

OTHER GOVERNMENT AGENCY

Central Intelligence Agency
ATTN: OSMR/NED

DEPARTMENT OF ENERGY CONTRACTORS

Sandia Labs
Livermore Lab
ATTN: L. Hostetler
ATTN: A. Kernstein
ATTN: T. Gold

DEPARTMENT OF DEFENSE CONTRACTORS

BDM Corp
ATTN: J. Braddock
ATTN: F. Conant

Boeing Co
ATTN: J. Russel

DEPARTMENT OF DEFENSE CONTRACTORS (Continued)

66th MI Group
4 cy ATTN: D. Welch

General Research Corp
ATTN: Tac Warfare Ops

Hudson Institute, Inc
ATTN: C. Gray
ATTN: H. Kahn

IRT Corp
ATTN: J. Hengle
ATTN: W. Macklin

JAYCOR
ATTN: R. Sullivan

Kaman Sciences Corp
ATTN: F. Shelton

Kaman Sciences Corp
ATTN: E. Daugs

Kaman Tempc
ATTN: DASIAC

Martin Marietta Corp
ATTN: F. Marion

Martin Marietta Corp
ATTN: J. Donathan

Pacific-Sierra Research Corp
ATTN: H. Brode
ATTN: G. Lang

R & D Associates
ATTN: P. Haas
2 cy ATTN: J. Hurley

DEPARTMENT OF DEFENSE CONTRACTORS (Continued)

Rand Corp
ATTN: V. Jackson

Santa Fe Corp
ATTN: D. Paolucci

Science Applications, Inc
ATTN: C. Whittenbury/W. Yengst
ATTN: M. Drake
ATTN: J. Martin

Science Applications, Inc
ATTN: R. Craver
ATTN: W. Layson
ATTN: J. Shannon

SRI International
ATTN: J. Scholz
ATTN: J. Sloss

System Planning Corp
ATTN: J. Douglas

Systems, Science & Software, Inc
ATTN: K. Pyatt

Tetra Tech, Inc
ATTN: F. Bothwell

TRW Defense & Space Sys Group
ATTN: N. Lipner
ATTN: D. Scally
ATTN: R. Burnett

TRW Defense & Space Sys Group
ATTN: P. Dai