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MODIFICATIONS TO COMPUTER PROGRAM FOR PARAMETER ESTIMATION FOR --ETC(U)
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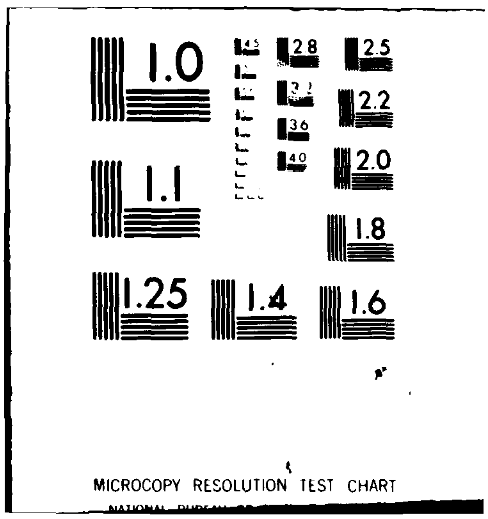
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MODIFICATIONS TO COMPUTER PROGRAM FOR PARAMETER ESTIMATION FOR THE GENERALIZED GAMMA DISTRIBUTION

by

Michel K. Ochi

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SHIP PERFORMANCE DEPARTMENT

May 1977

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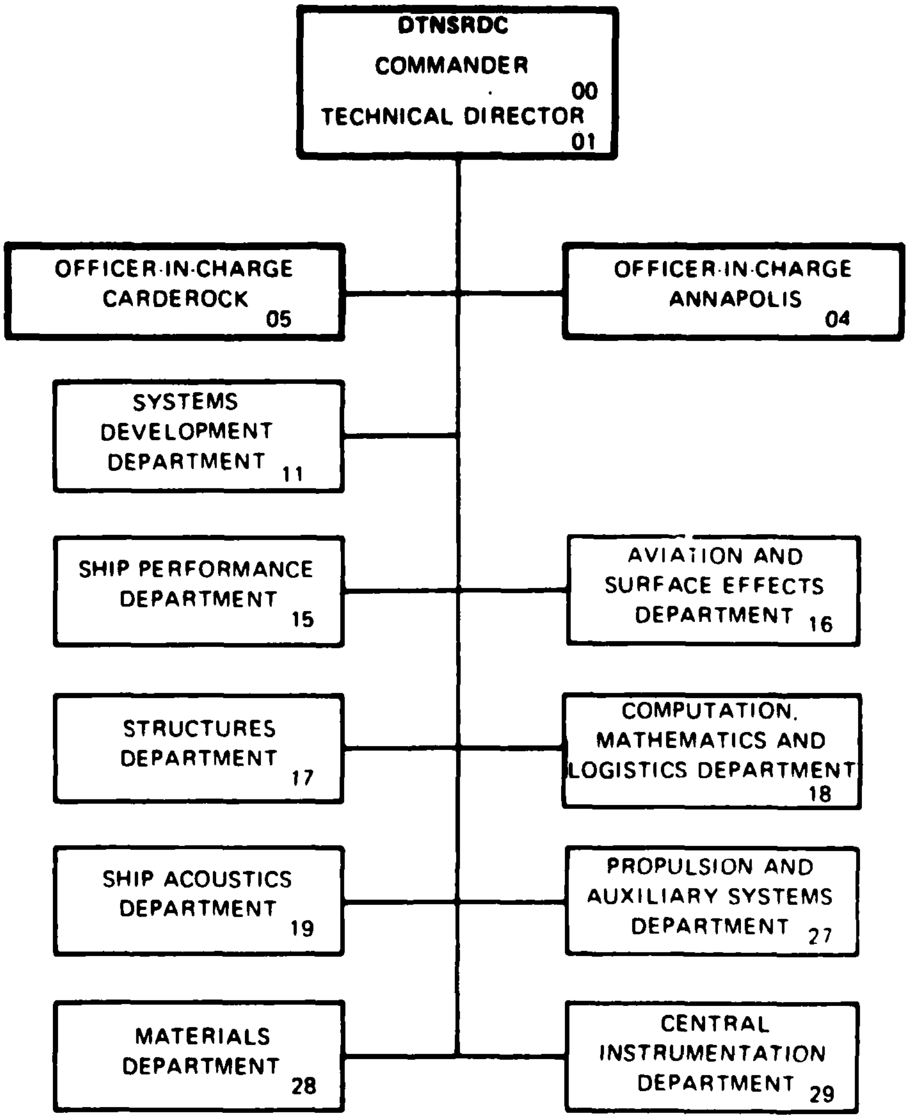
MODIFICATIONS TO COMPUTER PROGRAM FOR PARAMETER ESTIMA
FOR THE GENERALIZED GAMMA DISTRIBUTION

MAJOR DTNSRDC ORGANIZATIONAL COMPONENTS



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An extensive study of an existing digital computer program for determining parameters required to represent SES 100B trials data by a generalized gamma function is carried out. The purpose of the study is to resolve the difficulties encountered in earlier studies. In these earlier studies, approximately 5 percent of the cases analyzed produced unrealistic values for the distribution parameters. In order to resolve this problem, data sensitivity studies are carried out. Based on results of the study, criteria are developed for determining cases that should not be processed. Three other techniques besides

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ABSTRACT

An extensive study of an existing digital computer program for determining parameters required to represent SES 100B trials data by a generalized gamma function is carried out. The purpose of the study is to resolve the difficulties encountered in earlier studies. In these earlier studies, approximately 5 percent of the cases analyzed produced unrealistic values for the distribution parameters. In order to resolve this problem, data sensitivity studies are carried out. Based on results of the study, criteria are developed for determining cases that should not be processed. Three other techniques besides the Stacy-Mihram method are investigated. The most successful alternative method is the maximum likelihood method, which is incorporated into the new digital computer program. A program listing and an example of the generated output are included in this report.

ADMINISTRATIVE INFORMATION

This work was conducted at the David W. Taylor Naval Ship Research and Development Center and authorized by Naval Sea Systems Command, Surface Effect Ships Project Office, Program Element Number 63534N. It is identified as Work Unit Number 1-1506-012.

INTRODUCTION

A computerized procedure for evaluating the parameters of the generalized gamma distribution from a set of random data following the method proposed by Stacy and Mihram (see Reference 1) has been previously developed and extensively exercised (see Reference 2 and 3). During these earlier studies it was discovered that what appeared to be unrealistic values for the distribution parameters were obtained in roughly 5 percent of the cases analyzed. The initial purpose of the study, whose results are summarized in this report, was to explore in detail a representative sample of cases resulting in unrealistic values of the parameters, and if possible, to develop an alternate method for processing these cases.

The results of the present investigation can be summarized as follows:

a. In the many cases studied, it appears that the predicted extreme design values are most sensitive to experimental measurements and not the numerical procedure for determining the parameters. This is particularly true when one of the parameters is negative.

b. All four methods considered in this investigation yield probability distribution functions which represent very well the experimentally obtained histogram even though some of the parameter values were judged unrealisitic in previous studies. However, the Stacy-Mihram and maximum likelihood methods are by far the most efficient and are the preferred procedures based on probabilistic arguments.

c. In some cases the predicted extreme design values are unrealistically large whereas the significant values are realistic. Based on an analysis of over 100 cases, it can be concluded that cases having a nondimensional third logarithmic moment greater than -0.5 produce unrealistic extreme design values. Results of a data sensitivity study have revealed that in all these cases the data may not be considered as a sample taken from a steady-state random phenomenon, and hence it is recommended not to carry out statistical analysis for these cases.

MATHEMATICAL FORMULATION

OVERVIEW

The generalized gamma density function under consideration in this report has the form

$$f(x; \lambda, m, c) = \frac{c\lambda^{cm}}{\Gamma(m)} x^{cm-1} e^{(-(\lambda x)^c)} \quad (1)$$

where λ, m, and c are estimation parameters and Γ(m) is the gamma function. Many other distributions such as the exponential, gamma, chi-squared, Weibul, hydrograph, chi, truncated normal, and Rayleigh are special cases of the generalized gamma function. Due to the general nature of f(x; λ, m, c), it is the preferred distribution particularly when the exact form of the distribution function is unknown.

There are several techniques that can be used to estimate the three parameters. Four such techniques are considered in this effort. These are the Stacy-Mihram, the maximum likelihood, the nonlinear least squares, and grid search methods. The last two methods determine the parameters by searching for that set of parameters resulting in a minimum error between the experimental and theoretical histograms. Since statistical inference cannot be applied in a rigorous manner, the last two methods are not recommended. The Stacy-Mihram method determines the parameters by requiring that the first three theoretical logarithmic moments agree with the logarithmic moments determined from the experimental data. The maximum likelihood method determines parameters such that the joint probability density function is a maximum.

A detailed derivation of the Stacy-Mihram method is presented in Reference 2 pages 6 - 8. A derivation of the nonlinear least squares procedure is presented in Reference 4. The grid search procedure merely involves a systematic search of the λ, m, c space for parameters that minimize the error between the predicted distribution and the measured distribution. A derivation of the equations required in the maximum likelihood parametric estimation method is presented in the next section.

MAXIMUM LIKELIHOOD METHOD

Briefly, the maximum likelihood estimation technique involves finding the parameter (or parameters) which maximizes the joint probability density function of a random sample $X_1, \ldots, X_N$ from a distribution with $f(x, \theta)$ as its probability density function and θ as an element of the parameter space. The joint probability density function, referred to as the likelihood function, is given by

$$L = \prod_{i=1}^{N} f(x_i; \theta) \tag{2}$$

Assume one has N_i samples at random variable x_i. Then the maximum likelihood function will have the form

$$L = \prod_i f(x_i, \theta)^{N_i} \tag{3}$$

The natural logarithm of the likelihood function is used in maximizing since it is a maximum when the function itself is a maximum, and it proves more convenient to use. The logarithm is given by,

$$\ln L = \sum_{i=1}^{K} N_i \ln f(x_i) \tag{4}$$

The maximum of lnL is obtained by searching for parameters θ such that,

$$\left.\frac{\partial \ln L}{\partial \theta_i}\right|_{i=1,N_p} = 0.0 \tag{5}$$

where N_p = the number of parameters.

For the generalized gamma function given by Equation (1), the likelihood function becomes

$$L = \prod_{i=1}^{K} \left[\frac{c\lambda^{cm} x_i^{cm-1}}{\Gamma(m)} \quad e^{(-(\lambda x_i)^c)} \right]^{N_i} \tag{6}$$

and lnL becomes

$$\ln L = N \ln c + cmN \ln\lambda - N \ln\Gamma(m) + (cm-1)\sum_i N_i \ln x_i - \lambda^c \sum_i x_i^c N_i \tag{7}$$

In the above equation, c is assumed positive. If the above equation is differentiated with respect to cm, λ and m and set to zero, the following equations can be derived (see Reference 5).

$$\lambda = \left[(mN) / \sum_i x_i^c N_i \right]^{1/c} \tag{8}$$

$$\ln c + \ln\Big(\sum_i x_i^c N_i\Big) - \ln cm - \ln N + \psi(m) - c \sum_i N_i \ln x_i \Big/ N = 0 \tag{9}$$

$$1/m + \psi(m) - \ln cm + \ln c + \ln\Big(\sum_i x_i^c N_i\Big) - \ln N - c \sum_i \big(\ln x_i \,(x_i N_i)\big) \Big/ \sum_i x_i^c N_i = 0 \tag{10}$$

where the summation on i is from 1 to K. An expression for m can be obtained from the above equations:

$$1/m = c \left[\frac{\sum_i (\ln x_i)(x_i^c N_i)}{\sum_i x_i^c N_i} - \frac{\sum_i N_i \ln x_i}{N} \right] \tag{11}$$

The parameter c can be obtained by solving the following equation numerically.

$$\ln c + \ln\Big(\sum_i x_i^c N_i\Big) + \ln\left[\frac{\sum_i (\ln x_i)(x_i^c N_i)}{\sum_i x_i^c N_i} - \frac{\sum_i N_i \ln x_i}{N} \right] \tag{12}$$

$$- \ln N + \psi\left[\frac{1}{c\left(\frac{\sum_i (\ln x_i)(x_i^c N_i)}{\sum_i x_i^c N_i} - \frac{\sum_i N_i \ln x_i}{N} \right)} \right] - \frac{c \sum_i N_i \ln x_i}{N} = 0$$

Once a value of c is determined by solving Equation (12), values for λ and m can be obtained via Equations (8) and (11). The numerical method used to solve Equation (12) for c is due to Wegstein (see Reference 5).

ANALYSIS

During previous studies in which the Stacy-Mihram method was used to process SES 100B data, large design extreme values were predicted in approximately 5 percent of the cases although the most significant values agreed with the experimental measurements. This difficulty seemed to be associated with large values of the parameter m and with negative values of the parameter c. In order to resolve this difficulty, several cases were examined in detail. The results of this investigation follows.

The first case considered in this investigation is a set of pitching motion data measured during the SES 100B full scale trials program at 30 kts in a seastate of 3. This particular case, identified as AR1 3E 30, is presented in Reference 3. A comparison of the estimated probability density function with the experimental histogram is presented in Figure 1. As is demonstrated by the results in the figure, the agreement is excellent. All other cases examined show similar results. Assuming that an objective of the estimation procedure is to develop the best representation of the data that is possible, it is concluded that large values of $m (> 5)$ are not necessarily unreasonable.

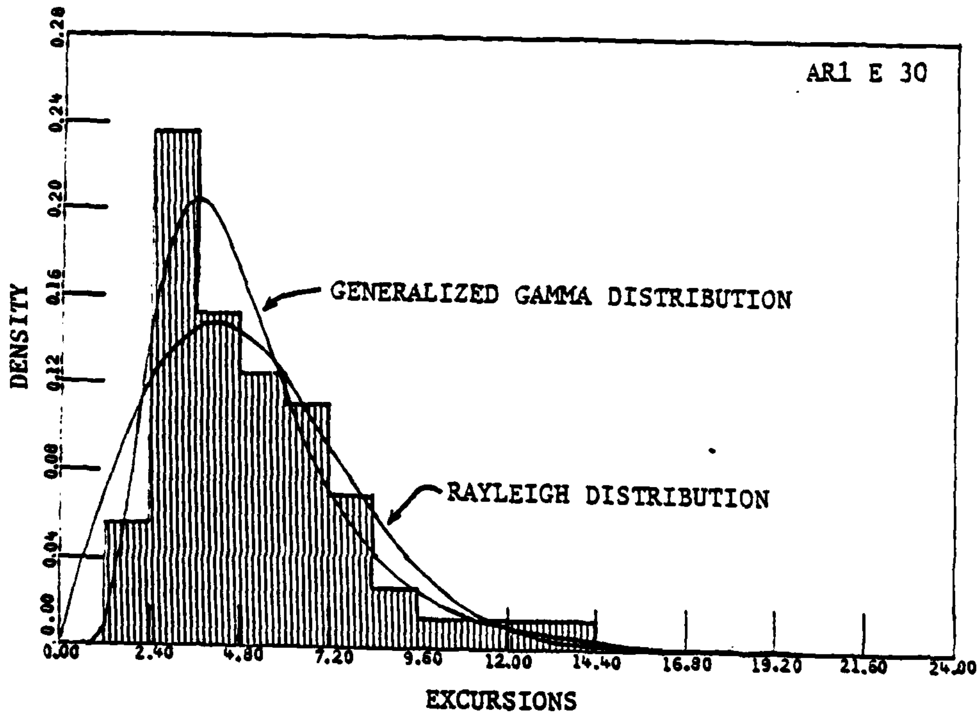

FIGURE 1. A COMPARISON OF THE GENERALIZED GAMMA DISTRIBUTION WITH EXPERIMENTAL DATA

Also, since the predicted representations always represent the data very well for both large and small m, the numerical procedure used to solve the transcendental equations for the estimation parameters is judged sufficiently accurate.

Further investigations revealed that in some of the cases the predicted design extreme values were unreasonably high as compared with the most significant values. In order to determine more precisely a reasonable value for the ratio of the design extreme value to the most significant value (X_D/X_s), an analysis of over 100 previously processed data cases were examined. The results indicated that unrealistically large ratios were obtained for those cases for which the normalized logarithmic moment (see Equation 2.4 in Reference 2) is greater than -0.5. Most significant values were realisitic as compared with those measured in trials. Consequently, in the current data base, cases for which $T > -0.5$ should not be processed to determine design extreme values using the generalized gamma procedure.

The experimental histogram of AR1 3E 30 has a substantial tail which may contribute to the large design extreme value prediction. The last four bins (out of 12) each have only one observation. A data sensitivity study was conducted in order to determine how sensitive the design values are to the tail portion of the measured data. The results of this study are presented in the next section.

SENSITIVITY OF RESULTS ON EXPERIMENTAL DATA

A data sensitivity study on cases AR1 3E 30, S14 3H 10, S14 3Q 10, and S82 3Q 10 was carried out in order to make an assessment of how sensitive predicted results are to the experimental data. Computations were carried out for several cases for which four observations from a rather distinct tail were dropped. The results are presented in Table 1.

The results indicate that elimination of a small percentage of the experimental data can result in large changes in the design extreme value, particularly when $T > -0.5$ and $c < 0.0$. Based on these results, it appears that a substantially large number of observations in the tail portion of the experimental data is desirable in order to predict results that are insensitive to minor errors in the observations.

It should be noted that the data sensitivity study as conducted above is rather severe, i.e., four points were removed in a systematic manner from the tail portion of the measured histogram. The probability that four points from the tail would be incorrectly measured is rather small. One way of determining how long an experiment should be conducted and consequently how many observations should be made is to monitor various statistics such as moments during any particular experiment. Once these statistical measures reach steady state, the sample can be judged adequate. Proceeding in this manner may have been impractical during a single prototype run. However, combining several runs under nearly identical experimental conditions may prove possible.

TABLE 1

RESULTS OF SENSITIVITY
STUDY ON FOUR REPRESENTATIVE CASES

CASE	m	c	λ	X_S	X_D	X_D/X_S	N	$\%\Delta X_D$	$\%\Delta X_S$
AR1 3E 30	105.	-0.21	0.2×10^{-10}	8.19	29.16	3.56	60		
	23.	0.50	123.3	6.96	16.80	2.41	56	-42	-15%
S14 3H 10	11.9	0.33	209.0	25.12	174.0	6.93	209		
	7.8	0.42	13.9	23.15	137.8	5.95	205	-21	-8%
S14 3Q 10	9.7	-0.37	0.1×10^{-2}	8.18	155.0	19.0	131		
	26.3	-0.24	0.5×10^{-6}	6.77	80.4	11.9	127	-93	-17%
S82 3Q 10	11.8	-0.31	0.3×10^{-3}	5.0	85.3	17.1	63		
	42.8	-0.18	0.6×10^{-9}	3.6	35.2	9.8	59	-59	-28%

EVALUATION OF THE MAXIMUM LIKELIHOOD METHOD

The method previously employed to determine estimation parameters for the generalized gamma distribution was based on the Stacy-Mihram method (see Reference 1 and 2). One of the advantages of the Stacy-Mihram method is that negative values of the parameter c are allowed. However, based on the current analysis of over 100 cases, it appears that cases for which c is negative result in unrealistically large values of the ratio of the design extreme value to the significant value regardless of the processing technique (Stacy-Mihram method, grid search, nonlinear least squares method). Consequently, development of representations of the experimental via the generalized gamma function should be restricted to cases for which the parameter $c > 0$. More detailed analysis (see comments at the beginning of this section) has revealed that processing should be restricted to those cases for which the normalized logarithmic moment is less than -0.5*. If the above restrictions are imposed, the preferred processing method is the maximum likelihood method as developed in a previous section of this report. One of the reasons the maximum likelihood method is preferred is that it provides unbiased and efficient parameter estimates in the asymptotic limit, i.e., as the number of samples becomes

*Examination of the data shows that it cannot be adequately represented by any of the standard distributions.

large, provided specified regularity conditions hold (see References 6 and 7 for details). Also, the method is applied to the actual experimental measurements, i.e., grouping the data into histograms is not necessary.

A comparison of the maximum likelihood results with experimental data is presented in Figure 2. As demonstrated by the figure, the agreement between the generalized gamma representation and the experimental data is excellent. The ratio of design extreme value to most significant value is 3.65 which is less than the value obtained by the Stacy-Mihram method is 5.58.

DIGITAL PROGRAM DESCRIPTION

The current digital program is based on a previous program that implemented the Stacy and Mihram method. The following modifications have been made:

1. The maximum likelihood method has been incorporated into the previous program. This is accomplished by replacing subroutine GETCML with a new version. The maximum likelihood method should be used only when the numbers of observations is greater than 100, or the number of bins is greater than 60. The Stacy-Mihram method should be used when the number of bins is less than 20 and the number of observations is greater than 100.

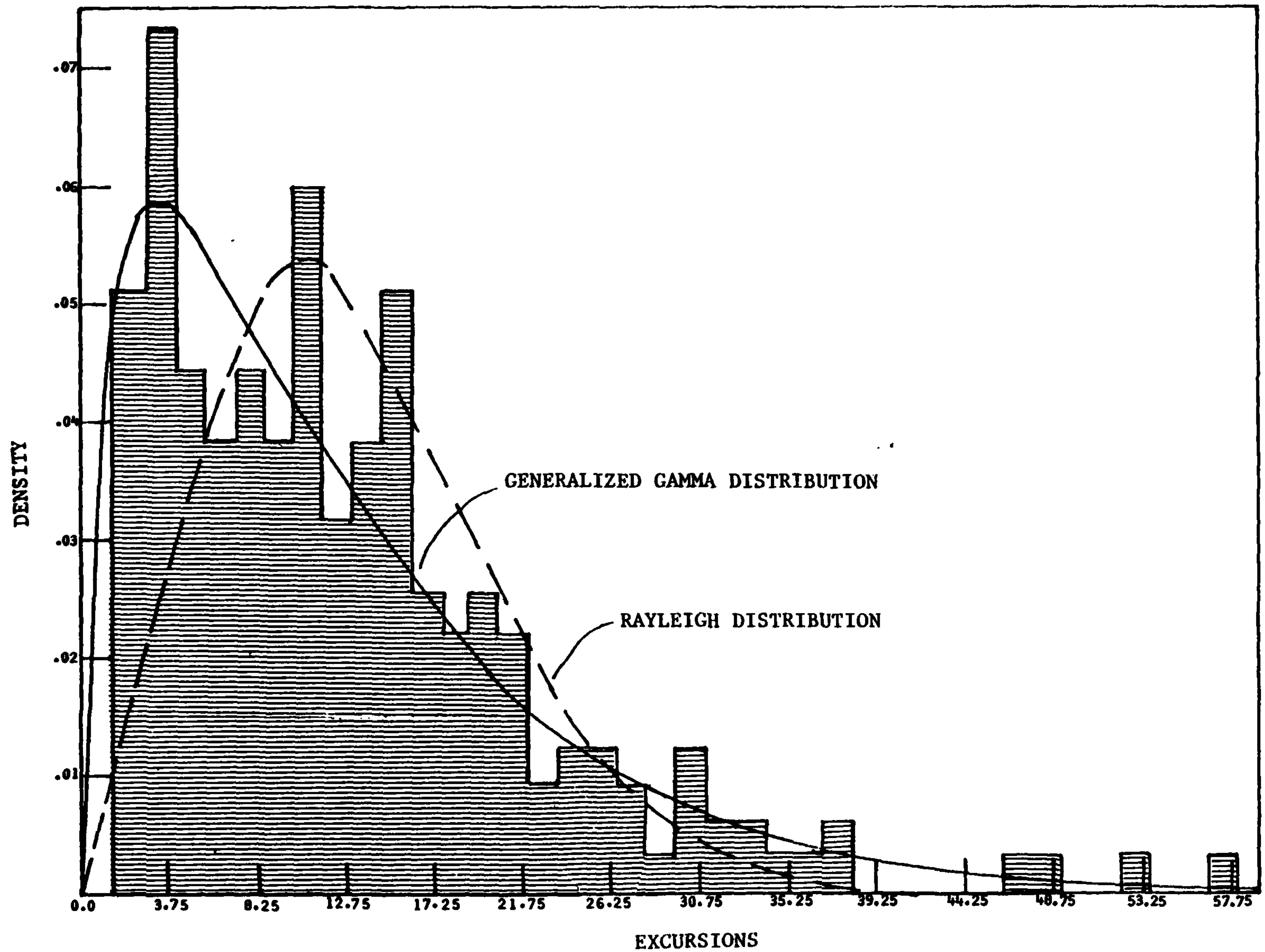

FIGURE 2. A COMPARISON OF THE GENERALIZED GAMMA DISTRIBUTION AS DETERMINED BY THE MAXIMUM LIKELIHOOD METHOD WITH EXPERIMENTAL DATA

2. As an added option, data can be read in the form of densities or counts per bin.

3. Printer plots of Rayleigh, generalized gamma function, and experimental data are generated.

4. Since the extreme values are a function of the number of observations (time), time effect plots are generated along with tabular results.

Three proprietary programs supplied by DTNSRDC are used in the present system. These are PLOTPR, MGAMMA, and MDGAM. PLOTPR is used to produce printer plots of data written on unit 3, MGAMMA computes the gamma function, and MDGAM computes the incomplete gamma function. Printer plot programs and programs for the gamma function are readily available on most computer systems. Hence, replacing these with programs available at different installations should present no major difficulties. The incomplete gamma function can be obtained from the gamma function via a simple integration procedure. However, due care must be exercised to assure sufficient numerical accuracy.

A complete description of input data, printed output, and subroutines follows. A listing of the program is presented in Appendix A. An example of print out is presented in Appendix B.

INPUT DATA

A description of input data follows. Card images of a typical run stream are illustrated in Figure 3.

Card	Variable	Cols.	Format	Description
1	IC(1)	1-2	I2	option to convert input data from frequencies to densities. $H_{NEW} = H_{OLD}/(N \times DEL)$ ON: IC(1) = 1
2	IC(2)	3-4	I2	option to convert L values from time to sample/run time $L_{NEW} = (L_{OLD}/\text{Run time}) \times N$ ON: IC(2) = 1
3	IC(3)	5-6	I2	option to change random variable values to midpoint values ON: IC(3) = 1
1	IC(4)	7-8	I2	option to convert input data by factor CAL ON: IC(4) = 1
1	IC(5)	9-10	I2	option to plot Histogram, Rayleigh and Gamma with "PLOTPR" ON: IC(5) = 1
1	IC(6)	11-12	I2	option to plot above in MKS units (change by factor CONVER) ON: IC(6) = 1
1	IC(7)	13-14	I2	option to plot L in hours vs. X_D, X_G (Time effect plots) ON: IC(7) = 1
1	IC(8)	15-16	I2	option to plot above in MKS units ON: IC(8) = 1
1	IC(9)	17-18	I2	option to print gamma functions for all L values OFF: IC(9) = 1 ON: otherwise

Card	Variable	Cols.	Format	Description
1	IC(10)	19-20	I2	option to print above in MKS units ON: IC(10) = 1
1	IC(11)	21-22	I2	scale on Time Effect Plots Each dot will be spaced by 10 ** (IC(11)). -9 < IC(11) < 99 Default: .05, 1-5 hrs.
1	IC(12)	23-24	I2	IC(12)=0 calculates design values and option for plots. IC(12)=1 no design values calculated and no plots printed IC(12)=2 no design values calculated, but option for plots.
1	IC(15)	29-30	I2	debug printout for Stacy-Mihram method
1	IC(16)	31-32	I2	number of double pages on plot Default: 1 double page minimum is 1 double page (2 pages)
1	IC(20)	39-40	I2	IC(20)=1: Use maximum likelihood method in estimating parameters Otherwise: Use Stacy-Mihram method
1	ISTOPD	41-50	I10	Number of increments in excess of 200 to be used in determining various statistical measures. Recommended value is zero or one.
1	IPLOT	51-55	I5	IF IPLOT ≠ 0, Calcomp plots will be generated
1	IRAY	56-60	I5	IF IRAY ≠ 0, Rayleigh distribution will be generated

Card	Variable	Cols.	Format	Description
1	CONVER	61-70	F10.0	Conversion factor to go from English to MKS units
1	CAL	71-80	F10.0	Conversion factor of input data to measurement desired
2	IOP(1)	1-2	I2	IOP(1)=0: c = 2.0 as initial guess for use in Wegstein iteration IOP(1)=1: Initial c value read in.
2	IOP(2)	3-4	I2	IOP(2)=1: Write initial c value
2	IOP(3)	5-6	I2	IOP(3)=1: Write c, m, λ estimates obtained by maximum likelihood estimation
2	IOP(5)	9-10	I2	IOP(5)=1: Debug printout
3	N	1-5	I5	Total number of samples
3	K	6-10	I5	Number of divisions
3	ALP	11-20	F10.0	Value of 1 - F(x) corresponding to the desired extreme value
3	DEL	21-30	F10.0	Division size
3	CMNT(1-5)	31-80	5A10	Description of the set of data
4	NL	1-10	I10	Number of L values to be considered. Up to five can be used (See Equation 2.15 in Reference 2.)
4	ELL(1-5)	11-60	5F10.0	up to five values of L
5	X(1), H(1)	1-80	8F10.0	K pairs of ordinate and experimental density values to be analyzed 4 pairs per card.

Multiple cases are processed by stacking the data sets with the last card as a blank to serve as a flag indicating the last case. However, the option cards (first two cards) are only read once. A run consisting of two sets of data is illustrated in Figure 3.

```
XXXXXXX.CM77000.T200.P4.
CHARGE,TTTT.NNNNNNNNNN.CC.K.
ATTACH.BINAR,BBBBBBBBBBB.ID=TTTT.
ATTACH,NSRDC.
ATTACH,IMSL.
LDSET(LIB=IMSL/NSRDC)
BINAR.
8
          1         1
  1 1
 311   15  0.01          12.0                 R22 H 30
        5   311.       833.       2499.      4998.      9996.
   6.        .0075    18.        .0147      30.        .0212      42.        .0174
  54.        .0102    66.        .0051      78.        .0039      90.        .0016
 102.        .0005   114.        .0005     126.        .0005     138.        .0
 150.        .0      162.        .0        174.        .0003
 262   14       0.01          3.0       R22 3F 10
        5       262.       546.       1638.      3275.      6550.
  1.5       .01527    4.5       .03817      7.5      .05089    10.5      .04962
 13.5       .05598   16.5       .03435     19.5      .04071    22.5      .01998
 25.5       .01272   28.5       .00763     31.5      .00382    34.5      .00127
 37.5       .00127   40.5       .00254
BLANK CARD
8
8
```

FIGURE 3. CARD IMAGES FOR A TYPICAL COMPUTER RUN

PRINTED OUTPUT

The printed output is dependent upon the options that are chosen. A general description of the printout follows. A sample run is shown in Appendix B.

The first page consists of the user options chosen. The options are only printed once for all the data sets. The initial guess for c, the value of c at each iteration used in finding c and the estimated parameters are next, if maximum likelihood method is chosen. If Stacy-Mihram method is chosen the iterations for finding m are given.

The second page identifies the data set being processed. The number of observations, number of bins, α, and bin size are next printed followed by the bin widths, the histogram data. Next the logarithmic moments of the data follows; $\overline{y}$, s, and T. The last items in the header are the calculated generalized gamma parameters m, c, λ as computed by the maximum likelihood method or Stacy-Mihram method.

Eight columns follow: the values of the random variable, the value of the random variable in MKS units if CONVER is given and Option 6 of the IC options is chosen, the corresponding Rayleigh predicted frequencies, the corresponding measured densities, the corresponding Rayleigh predicted densities and the corresponding generalized gamma predicted densities.

The values of L, the density function, the cumulative distributions, and corresponding random variables are printed for each value of L. These occupy eight columns of output arranged as follows:

Column 1 - running index

Column 2 - value of the random variable.

Column 3 - f(x), (Equation 1 in Reference 1)

Column 4 - f(x), (Equation 11 in Reference 1)

Column 5 - $f(x)^L$

Column 6 - G(x), (Equation 12 in Reference 1)

Column 7 - h(x), (Equation 14 in Reference 1)

Column 8 - H(x), (Equation 15 in Reference 1)

The following page gives the Rayleigh design values for all L values.

The fourth page gives the generalized gamma design values for one L value. These include x_M, x_O, x_S, x_G, x_{GA}, x_D, and x_{DA}. To the right of these values, x_O, x_G, x_{GA}, x_D, and x_{DA} as computed by a table look-up procedure are printed. This page is repeated for each L value.

After these pages there is a comparison of design values for all L values. The next pages consist of a printer plot which show the measured density, the density as predicted by the generalized gamma function, and the density predicted by the Rayleigh density function.

The following pages consist of a time effect printer plot which give L in hours vs. x_D and x_G. The CalComp plots generated replicate the printer plots.

PROGRAM DESCRIPTIONS

Main Program

The main program serves as a driver for the entire digital program. In addition, many of the required computations are performed in the main program. A description of program flow follows.

All data required for the program is read from unit 5. A description of this data is presented earlier in connection with the input data.

After reading the data, the moments of the logarithms of the random variables are computed along with the Rayleigh parameter and maximum histogram value. The normalized third logarithmic moment, T, is tested against the value -0.5. If T > -0.5, the case is not processed. The parameters m, c, and λ are computed by a call to subprogram GETCML via the maximum likelihood or Stacy-Mihram methods.

Once values for m, c, and λ are determined, the various statistical measures are computed for up to five values of L. The density functions and cumulative distributions are printed for each L value considered, followed by the extreme, most probable, and significant values. The extreme, most probable, and significant values are obtained by the Wegstein iterative method (see Subroutine WEG).

After the above computations are performed, CalComp and printer plots are generated. These consist of comparisons of the measured density function with the generalized Gamma density function and with the Rayleigh density function.

Descriptions of major FORTRAN symbols are presented in Table 2.

TABLE 2

MAJOR FORTRAN SYMBOLS USED IN MAIN PROGRAM

FORTRAN Symbol	Math Symbol	Description
N	N	total number of samples
K	K	number of divisions
ALP	α	value of 1-F(x) corresponding to the desired extreme value
DEL	Δx	bin size
NL	-	number of L values
ELL	L	array of L values
ISTOPD	-	number of increments in excess of 200 to be used in determining extreme values
X	x	array of ordinate values
H	f_i	experimental density values
S	S	value related to second logarithmic moment
YBAR	$\bar{y}$	first logarithmic moment
T	T	value related to third logarithmic moment
R	R	Rayleigh parameter
C	c	parameter c
M	m	parameter m, real
AMBDA	λ	parameter lambda
XO	x	ordinate value
FOXO	$f(x;\lambda,m,c)$	predicted density function
BFOXO	F(x)	cumulative distribution function
BFOXL	$F(x)^L$	cumulative distribution raised to the Lth power
GOFX	g(x)	density function corresponding to $F(x)^L$
HLO	h(x)	asymptotic density function
HHO	H(x)	asymptotic cumulative distribution
XSUBM*	$x\mu$	most probable value
XSUBO*	x_o	value at which $F(x) \doteq 2/3$
XSUBS*	x_s	significant value
XSUBG*	x_g	most probable exteme value

*These symbols are used as labels in the printed output generated by the program.

TABLE 2

MAJOR FORTRAN SYMBOLS USED IN MAIN PROGRAM (Cont.)

FORTRAN Symbol	Math Symbol	Description
XSUBGA*	x_{ga}	most probable asymptotic value
XSUBD*	x_D	design extreme value
XSUBDA*	x_{da}	extreme asymptotic value

* These symbols are used as labels in the printed output generated by the program.

Subroutine GETCML

Purpose: To determine the parameters of a generalized gamma distribution from statistical measures of input data by either Stacy-Mihram method or maximum likelihood method.

Usage: CALL GETCML(AT,U,E,S,YBAR,GAMMA,OOC,T)

COMMON/PARAMS/,/PRNT/,/HIST/,/INPUT/,/OPT/,/IOUNIT/

Subroutines and Subprograms Called:

MLHCML	Maximum likelihood estimation
WEG	Wegstein iteration
PSI	Digamma function
PSI1	Trigamma function
DIST	Evalutes measured and observed frequencies and densities
MGAMMA	Gamma function

Description of Parameters: See Table 3.

Remarks:

(1) If X_D does not converge, a check value is sent to calling routine.

TABLE 3

MAJOR FORTRAN SYMBOLS

Name	Location	Description
AMBDA	/PARAMS/	λ parameter of output distribution
C	/PARAMS/	c parameter of output distribution
M	/PARAMS/	m parameter of output distribution
X(I)	/INPUT/	random variable values
ICK1	/OPT/	Check value. If ICK1=1, c root was not found via maximum likelihood method
IOP(I)	/OPT/	options
YBAR	argument	logarithmic mean of input data
S	argument	second log moment of input data
T	argument	third log moment of data
CMNT(I)	/INPUT/	header label for GETCML output
FOBS, FTH	/PRNT/	computed values for observed and theoretical frequencies
DENO, DENT	/PRNT/	computed values for observed and theoretical densities
OOC	argument	reciprocal of c
ICK	argument	check value. If X_D does not converge, ICK=1 is sent back to calling routine.
U	argument	trigamma function of final m value
E	argument	digamma function of final m value

TABLE 3

MAJOR FORTRAN SYMBOLS (Cont.)

Name	Location	Description
AT	argument	absolute value of the third log moment of data
H(I)	/HIST/	densities corresponding to random variable values.

Subroutine MLHCML

Purpose: To estimate parameters λ, c, and m by the maximum likelihood method for the generalized gamma function.

Usage: CALL MLHCML

COMMON/PARAMS/,/HIST/,/INPUT/,/IOUNIT/,/PRNT/,/OPT/

Subroutines and Subprograms Called:

WEG	Wegstein iteration
PSI	Digamma function
F	Function of parameter c
G	Function used by WEG to determine c

Description of Parameters:

Name	Location	Description
AMBDA	/PARAMS/	λ parameter of output distribution
C	/PARAMS/	c parameter of output distribution
M	/PARAMS/	m parameter of output distribution
X(I)	/INPUT/	random variable values
O(I)	/HIST/	densities corresponding to random variable values
IOP(I)	/OPT/	options
ICK1	/OPT/	check value. If ICK1=1, c root was not found

Remarks:

(1) c is restricted to values greater than zero.

(2) If a root for F(c) is not found, a check value is sent to the calling routine.

(3) An initial guess for c for use in WEG may be read in. Otherwise a default value of 2.0 is used.

Subroutine DIST

<u>Purpose</u>: To calculate frequency and density distributions for Rayleigh and generalized gamma distributions.

<u>Usage</u>: CALL DIST (all parameters and arguments are passed in COMMON Blocks)

<u>Subroutines and Subprograms called</u>:

MDGAM	evaluates incomplete Gamma function

<u>Description of Variables</u>:

Variable	Location	Description
X(I)	/INPUT/	random variables values
H(I)	/INPUT/	array of histogram ordinates for input data
K	/HIST/	number of bins in input histogram
N	/INPUT/	number of observations in input histogram
FTH(I), FOBS(I)	/PRNT/	computed values for the observed frequency distribution and theoretical frequency distributions
DENO(I), DENTR(I)	/PRNT/	computed values for the observed density distribution and theoretical frequency distributions

<u>Remarks</u>: The input histogram need not be evenly spaced in the abscissa x.

<u>Method</u>: Generalized gamma cumulative distribution is evaluated by MDGAM. The density function is obtained by numerically differentiating the obtained cumulative. The Rayleigh density distribution is obtained in a like manner.

Subroutine WEG

<u>Purpose</u>: To determine root of x = f(x) by Wegstein iteration.

<u>Usage</u>: CALL WEG (I, X, J, N)

<u>Description of parameters</u>:

I - input, iteration count. Initialization action is taken when I = 1.

X - input, estimate of root of x = f(x). On output, refined estimate of root.

J - number of significant digits of accurary desired in x for solution -- used only in initialization.

N - output - completion flag

0 convergence not obtained

1 convergence is within given accuracy

<u>Remarks</u>: The function whose root is to be found is computed externally. The calling sequence is as follows.

(1) Initialize by issuing CALL WEG (1, XG, J, N) where XG is the initial guess for the root and J is the tolerance described above. N is irrelevant.

(2) Set up a loop to calculate XN=F(X) using output X from WEG

(3) The simplest calling sequence would then be

```
      X=XG                          Initialize X to guess value
      DO 1  I=1,ITERCT
      CALL WEG(I,X,ITOL,N)          First pass will inialize WEG
      IF (N.EQ.1)  GO TO 2
      X = F(X)
 1    CONTINUE
 C    MAXIMUM NUMBER OF ITERATIONS REACHED WITHOUT CONVERGENCE
      STOP
 2    CONTINUE
 C    ROOT FOUND WITHIN TOLERANCE
```

Method: The Wegstein method refines the root of the equation $x = f(x)$ by calculating an improved estimate x_{I+1} which is the intersection point of the line $y = x$ and the secant line of $f(x)$ based on the previous two evaluations of $f(x)$. This method requires only one function evaluation per iteration. The equation for the intersection point is given by

$$X_{I+1} = \frac{F(X_I) * X_{I-1} - F(X_{I-1}) * X_I}{F(X_I) - F(X_{I-1}) - X_I + X_{I-1}}$$

Completion code N is set 1 when

$$|X_{I+1} - X_I| < |10^J * X_{I+1}|,$$

i.e., when the change in X between iterations is less than one part in 10^J.

Function PSI1

Purpose: To evaluate the trigamma function, $\Psi'(x)$

Usage: PSI1(x)

Description of Parameters:

PSI1(x) - value of the trigamma function at argument X (Output)

X - argument of the function (Input)

Remarks:

(1) Argument x must be greater than zero.

(2) The trigamma function is the second derivative of the natural logarithm of the gamma function.

Method: For arguments greater than 13 an asymptotic expansion,

$$\psi'(x) \simeq \frac{1}{x} + \frac{1}{2x^2} + \frac{1}{6x^3} - \frac{1}{30x^5} + \frac{1}{42x^7} - \frac{1}{30x^9}$$

is used. The truncation error associated with the above expansion is less than 1×10^{-10} for $x > 13$. For arguments between 0 and 13, the recursion relation

$$\psi'(x) = \psi'(m+x) + \sum_{i=1}^{k} \frac{1}{(x+i-1)^2}$$

is used, where m is chosen as the smallest integer for which $x + m \geq 13$. $\Psi'(m + x)$ is evaluated by the previous formula. The formulas are obtained from Reference 8.

Function PSI2

Purpose: To evaluate the tetragamma function, $\Psi''(x)$

Usage: PSI2(x)

Description of Parameters:

PSI2(x) - value of tetragamma function at argument X (Output).

X - argument of the function (Input).

Remarks:

(1) argument must be greater than zero.

(2) the tetragamma function is the third derivative of the natural logarithm of the gamma function.

Method: For arguments greater than 13, the asymptotic expansion,

$$\psi''(x) \simeq \frac{1}{x^2} - \frac{1}{x^3} - \frac{1}{2x^4} - \frac{1}{6x^6} + \frac{1}{6x^8} - \frac{1}{10x^{10}} - \frac{1}{6x^{12}}$$

is used. The truncation error associated with the above expansion is less than 1×10^{-10} for $x > 13$.

For arguments between zero and 13, the recursion relation

$$\psi''(x) = \psi''(x+m) - \sum_{i=1}^{k} \frac{2}{(x+i-1)^3}$$

is used, where m is the smallest integer which satisfies $x + m \geq 13$, and $\Psi''(x + m)$ is evaluated by the previous formula. The above formulas are obtained from Reference 8.

Function PSI

Purpose: To evaluate the digamma function, $\Psi(x)$.

Usage: PSI(x)

Description of Parameters:

PSI(x) - value of digamma function at argument X (Output)

X - argument of function (Input)

Remarks:

(1) Argument X must be greater than zero.

(2) The digamma function is the derivative of the natural logarithm of the gamma function.

Method: For arguments greater than 13 an asymptotic expansion is used:

$$\psi(x) \simeq \ln x - \frac{1}{2x} + \frac{1}{12x^2} + \frac{1}{120x^4} - \frac{1}{252x^6}$$

The truncation error associated with the above expansion is than 1 x 10^{-10} for x > 13. For arguments less than 13, the recursion relation

$$\psi(x) = \psi(x+m) - \sum_{i=1}^{k} \frac{1}{x+i-1}$$

is used, where m is the smallest integer which satisfies $x + m \geq 13$, and $\Psi(x + m)$ is evaluated by the previous formula. The above formulas are obtained from Reference 8.

Subroutine MGAMMA

Purpose: To evaluate Gamma function

Usage: CALL MGAMMA (X, GAMMA, IER)

Description of Parameters:

X - input, argument of Gamma function

GAMMA - output value of Gamma function at X

IER - error code

Remarks: This is a library routine supplied by DTNSRDC.

User must attach library (IMSL).

Error code meaning is unknown.

Subroutine MDGAM

<u>Purpose</u>: To evaluate incomplete gamma function

<u>Usage</u>: Call MDGAM(T, X, PR, IE)

<u>Description of Parameters</u>:

T input, limit of integral

X input, exponent in integral

PR output, value of incomplete gamma function

IE output, error code

<u>Remarks</u>:

(1) This is a library provided by DTNSRDC

(2) User must attach IMSL library

(3) Meaning of error code unknown

(4) Function evaluated is given by,

$$P(x,T) = \frac{1}{\Gamma(x)} \int_0^T u^{x-1} e^{-u} \, du$$

Subroutine HISTO

<u>Purpose</u>: To set up labels and plot the experimentally measured density function.

<u>Usage</u>: Call HISTO

<u>Subprograms and function called</u>: SCALE, AXIS, SYMBOL, LINE

<u>Remarks</u>: This program takes the measured density, which is passed in COMMON and uses these values to generate data for a CalComp plot.

Function G

<u>Purpose</u>: To be used by WEG in evaluating the c parameter by the maximum likelihood method.

<u>Usage</u>: G(c)

<u>Subroutines and Subprograms Called</u>: PSI

<u>Description of Parameters</u>:

G(c) - value of function (Output)

c - argument of the function (Input)

<u>Remarks</u>: c must be greater than zero.

<u>Method</u>: To implement the Wegstein technique the equation $f(x) = 0$ must be put into the form $x = g(x)$. The equation used is given by

$$c = \frac{\sum_{i=1}^{k} o_i}{\sum_{i=1}^{k} o_i \ln x_i} \Bigg[\ln c + \ln\left(\sum_{i=1}^{k} x_i^c o_i\right) + \ln\left[\frac{\sum_{i=1}^{k} \ln x_i (x_i^c o_i)}{\sum_{i=1}^{k} x_i^c o_i} - \frac{\sum_{i=1}^{k} o_i \ln x_i}{\sum_{i=1}^{k} o_i} \right] - \ln\left(\sum_{i=1}^{k} o_i\right)$$

$$+ \psi\left[\frac{1}{c\left(\frac{\sum_{i=1}^{k} \ln x_i (x_i^c o_i)}{\sum_{i=1}^{k} x_i^c o_i} - \frac{\sum_{i=1}^{k} o_i \ln x_i}{\sum_{i=1}^{k} o_i} \right)} \right] \Bigg]$$

O_i is the density

X_i is the value of random variable

Function F

Purpose: To evaluate the function used to find the c parameter by the maximum likelihood method.

Usage: F(c)

Subroutines and Subprograms Called: PSI

Description of Parameters:

F(c) - value of the function used to find the parameter c (Output)

c - argument of the function (Input)

Remarks: c must be greater than zero

Method: The equation used is given by

$$F(c) = \ln c + \ln\left(\sum_{i=1}^{k} x_i^c o_i\right)$$

$$+ \ln\left[\frac{\sum_{i=1}^{k} \ln x_i (x_i^c o_i)}{\sum_{i=1}^{k} x_i^c o_i} - \frac{\sum_{i=1}^{k} o_i \ln x_i}{\sum_{i=1}^{k} o_i}\right] - \ln \sum_{i=1}^{k} o_i$$

$$+ \psi\left[\frac{1}{c\left(\frac{\sum_{i=1}^{k} \ln x_i (x_i^c o_i)}{\sum_{i=1}^{k} x_i^c o_i} - \frac{\sum_{i=1}^{k} o_i \ln x_i}{\sum_{i=1}^{k} o_i}\right)}\right]$$

$$- \frac{c \sum_{i=1}^{k} o_i \ln x_i}{\sum_{i=1}^{k} o_i} = 0$$

O_i is the density

X_i is the value of the random variable

CONCLUSIONS

An extensive study of an existing digital computer program for determining parameters required to represent SES 100B trials data by a generalized gamma function was carried out. Difficulties encountered in earlier studies in which approximately 5 percent of the cases analyzed produced unrealistic values for the distribution parameters have been resolved. In order to resolve this problem, data sensitivity studies were carried out and three other techniques besides the Stacy-Mihram method were investigated. Based on results of the study, criteria are developed for determining cases that should not be processed. A summary of major conclusions follows.

a. The Stacy-Mihram, maximum likelihood, grid search, and nonlinear least squares methods all yield estimated probability distribution functions which represent very well the experimentally measured histograms.

b. Design extreme values are more sensitive to the experimental measurements than to the numerical procedure used in determining the estimation parameters.

c. In some cases the predicted extreme design values are unrealistically large whereas the significant values are realistic. Based on an analysis of over 100 cases, it is concluded that cases having a nondimensional third logarithmic moment greater than -0.5 produce unrealistic extreme design values. Results of a data sensitivity study have revealed that in all these cases the data may not be considered as a sample taken from a steady-state random phenomenon, and hence it is recommended that statistical analyses for these cases not be carried out.

d. The Stacy-Mihram method should be used when the number of bins (number of divisions used in the analyses) is small (< 20) and when the number of samples is large (> 100).

e. The maximum likelihood method should be used when the number of bins is large (> 60) or when the sample size is large (> 100).

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APPENDIX A

COMPUTER PROGRAM LISTING

```
      PROGRAM ORI1(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7,TAPE3)
      REAL MAXY
      REAL M
      DIMENSION XN(500),Z(500),FN(500),PN(500),IVAR(4)
      DIMENSION XNMS(500),RNMS(500),FNMS(500),ZM(500)
      DIMENSION XOMS(200),FOXOMS(200),-FO(200),BFOL(200),
     $OFXM(200),HLOMS(200),HHOMS(200)
      DIMENSION PDX(7),ELT(7),FSC(7),PDXM(7),FGX(7),RGXM(7)
      COMMON/INPUT/X(100),ELL(7),N,ALP,D(100),ISTOPD,IPLOT,IRAY,IC(20)
      COMMON/OPT/IOP(20),ICK1
      COMMON/PLO/NRUN,NPLOT,ITP(6),ITY(6),ITX(6),
     $NUMPAG,MAXSCA,SCA(10),FROM(10)
      COMMON/PRNT/FOBS(100),FTH(100),DENO(100),DENT(100)
     $,FTH2(100),DENT2(100),CONVER,CAL
      COMMON/HIST2/FIRSTX,DELX,FIRSTY,DELY
      COMMON/HIST/PINLO,K,PINV(100),H(100),YMAX,XLEN,YLEN,CMNT(5)
      COMMON/PARAMS/M,AMBDA,C,P
      COMMON/IOUNIT/IDR,IRD,IWRT,IPCH
      DATA IDR,IRD,IWRT,IPCH/6,5,6,6/
      DATA XLEN,YLEN/10.,8./
      TTH=2./3.
      CALL PLOTS(0,0,7)
C     READ OPTIONS
      READ(5,15) (IC(I),I=1,20),ISTOPD,IPLOT,IRAY,CONVER,CAL
      WRITE(6,1413) IPLOT,IRAY
 1413 FORMAT(1H1,39H IC OPTIONS USED IN THIS RUN SET IPLOT=,I5,3X,
     $5HIRAY=,I5)
      DO 14 I=1,20
   14 IF(IC(I).GT.0) WRITE(6,1414) I,IC(I)
 1414 FORMAT(1H ,9H OPTION ,I2,1H=,I2)
      WRITE(6,2013)
 2013 FORMAT(1H0,32HIOP OPTIONS USED IN THIS RUN SET)
      READ(5,115) (IOP(I),I=1,20)
  115 FORMAT(20I2)
      DO 979 I=1,20
      IF(IOP(I).GT.0) WRITE(6,1414) (I,IOP(I))
  979 CONTINUE
      IF(CONVER.EQ.0.) CONVER=1.0
15    FORMAT(20I2,I10,2I5,2F10.0)
C
   99 CONTINUE
      XN(1)=0.
      FN(1)=0.
      IP=1
      PN(1)=0.
      KLL=XLEN+2
C
C              *****  READ INPUT AND PRINT  *****
C
      READ (5,100) N,K,ALP,DEL,CMNT
      IF(K.EQ.0) GO TO 10003
 100  FORMAT(2I5,2F10.0,5A10)
C
C     READ L VALUES
      READ(5,102) NL,(ELL(I),I=1,NL)
102   FORMAT(I10,7F10.0)
      READ(5,101)(X(I),H(I),I=1,K)
```

```
  101 FORMAT( 8F10.0 )
  200 FORMAT( 1X, 8E10.4 )
      OMA = 1.-ALP
C
C   IF UNEQUAL INTERVALS DEL=0
C   DEL READ IN FROM X ARRAY
C   IF STARTING POINT N.E. 0.0 - SIGN WILL PRECEED
C     THE STARTING VALUE GIVEN AS DEL.
      DMX=DEL
      IF(DEL.GT.0.0) GO TO 199
      ST=-DEL
      D(1)=X(1)
      DSUM=D(1)
      X(1)=ST
      DMX=D(1)
      DO 196 I=2,K
      D(I)=X(I)
      IF(D(I).GE.D(I-1)) DMX=D(I)
      X(I)=X(I-1)+D(I-1)
      BINW(I)=D(I)
      DSUM=DSUM+D(I)
196   CONTINUE
      IF(IC(2).EQ.0) IFLG=1
      IF(IFLG.EQ.1) GO TO 10007
      GO TO 197
  199 DO 198 I=1,K
  198 D(I)=DEL
  197 CONTINUE
C   OPTION 1: CONVERT INPUT TO DENSITY IF NECESSARY
      IF(IC(1).NE.1) GO TO 2
      DO 11 I=1,K
11    H(I) = H(I) / (FLOAT(N)*D(I))
C
C   OPTION 2: MODIFY L VALUES IF NECESSARY
2     IF(IC(2).NE.1) GO TO 3
      RTIME = ELL(1)
      DO 12 I=1,NL
      ELT(I) = ELL(I)
12    ELL(I) =(ELL(I)/RTIME)*M
C
C   OPTION 3: ADJUST INPUT X(I) TO MIDPOINT OF INTERVALS IF NECESSARY
3     IF(IC(3).NE.1) GO TO 4
      DO 13 I= 1, K
13    X(I) = X(I) + D(I)/2.0
C   OPTION 4: CONVERT INPUT TO MEASUREMENT DESIRED
4     IF(IC(4).NE.1) GO TO 5
      DO 7 I = 1,K
      H(I) = H(I) / CAL
      D(I)=D(I)*CAL
7     X(I) = X(I) * CAL
5     CONTINUE
C   IF L VALUES ARE READ IN AS SAMPLES PER TIME INTERVAL
C   COMPUTE TIME VALUES FOR PLOT
      IF(IC(2).EQ.1) GO TO 6
      RTIME=ELL(1)/M
      DO 57 I = 1,NL
57    ELT(I) = ELL(I) * RTIME/M
```

```
6     CONTINUE
      R=0.
      YMAX=0.
      SH=0.
      S1 = 0.
      S2 = 0.
      S3 = 0.
C
C          *****  COMPUTE R, YI  *****
C
      DO 1 I=1,K
      T1 =H(I)*P(I)
      IF(T1.GT.YMAX)YMAX=T1
      R=R+T1*X(I)**2
      YI=ALOG(X(I))
      SH=SH+T1
      YI2=YI*YI
      YI3=YI*YI2
      S1=S1+T1*YI
      S2=S2+T1*YI2
      S3=S3+T1*YI3
    1 CONTINUE
C
C          *****  COMPUTE YBAR, S, T  *****
C
      EN=N
      YBAR=S1
      T1=EN/(EN-1.)
      T2=T1*EN/(EN-2.)
      T3=S2
      T4=YBAR*YBAR
      T5=T4*SH
      S=T1*(T3-2.*T4+T5)
      T= T2*(S3-3.*YBAR*T3+YBAR*(3.*T4-T5))/S**1.5
      IF(T.GT.(-.5)) WRITE(6,1237)
 1237 FORMAT(1H0,45HT IS GREATER THAN -.5  GO ON TO NEXT DATA SET)
      IF (T.GT.(-.5)) GO TO 99
      AT=ABS(T)
C
C          *****  COMPUTE M,C,LAMBDA, AND GAMMA  *****
C
      ICK=0
      CALL GETCML(AT,U,E,S,YBAR,GAMMA,CCC,T,ICK)
      IF(ICK.EQ.1) GO TO 99
      IF(ICK1.EQ.1) GO TO 99
      WRITE(6,8014)
8014  FORMAT(1H1,22HRAYLEIGH DESIGN VALUES)
      DO 8012 J=1,NL
      XRD=SQRT(R*ALOG(FLL(J)/FLP))
      XRDMS=XRD*CONVER
      WRITE(6,8013) FLL(J),XRD,XRDMS
8012  CONTINUE
8013  FORMAT(1H ,3HL= ,F10.4,5X,8HXRSUBD= ,F10.4,5X,13HXRSUBD(MKS)= ,
     $F10.4)
      IF(IC(12).EQ.1) GO TO 10004
C
C          *****  COMPUTE C,M,LAMBDA PRODUCTS FOR LATER USE  *****
```

```
C
      T1 = C*M
      T1M = T1-1.
      T2 = (C*AMBDA**T1)/GAMMA
      T9 = ABS(T2)
      CMMOC=T1M*OOC
C
C           *****  10000 LOOP---OVER THE L VALUES  *****
      DO 10000 IL=1,NL
C                  **  INITIAL GUESSES FOR X  AND X    **
C                                            D       G
      TXD = 1000.
      EXD = 0.
      EXG = 0.
      TXG = 0.
      TXO=1000.
      EXO=0.
      TXGA=0.
      EXGA=0.
      TXDA=1000.
      EXDA=0.
      EL = ELL(IL)
      WRITE (6,10001) EL
10001 FORMAT(1H1,3HL= ,F7.1/)
      IF (EL .GT. 300)  GO TO 8001
      EML = EXP(-EL)
      GO TO 8002
 8001 EML = 0.
 8002 CONTINUE
      IF(IC(9).EQ.0) WRITE(6,304)
  304 FORMAT (3X,*I*,9X,*X*,12X,*LF(X)*,12X,*F(X)*,10X,*F(X)L*,10X,
     1     *G(X)*,10X,*LH(X)*,10X,*H(X)*)
  700 FORMAT (//* EXTRAPOLATED VALUES *//)
      ISTOP = 200*ISTOPD
C                  **  SET UP HISTOGRAM PARAMETERS  **
      I = 0
      J=0
      BINLO=X(1)-D(1)/2.
      MAXX=2*DMX*XLEN
      X0= X(1)-D(1)
      X0D=X0+D(1)/2.
      IF(X0D .LE. .01) X0=0.
  701 IRES=I/2
      RES=(FLOAT(I)/2.)-IRES
      IF((RES.EQ.0).AND.(J.LT.K)) J=J+1
      CELL=D(J)/2.0
      DFINC=D(J)/10.0
      X0=X0+DFINC
      I = I + 1
      IF (I .GT. ISTOP )  GO TO 11002
      XT0 = X0
C                  **  11000 LOOP  **
      DO 11000  IF=1,5
C
C           *****  COMPUTE SMALL F(X), PROBABILITY DENSITY FN  *****
C
      X0 = XT0+DFINC*(IF-1)
```

```
      T4=X0**T1M
      T45=(AMBDA*X0)**C
      T5 = EXP(-T45)
      FOX0 = T3*T4*T5
C
      IF(IL.GT.1) GO TO 10002
      IF(X0.GT.MAXX) GO TO 10002
      IF(IP.EQ.500) GO TO 10002
C
      IP=IP+1
      FM(IP)=FOX0
      XM(IP)=X0
      XT1=X0/R
      XT2=X0*XT1
      RNX=2.*XT1*EXP(-XT2)
      RM(IP)=RNX
      IF(RNX.GT.YMAX)YMAX=RNX
      IF(FOX0.GT.YMAX)YMAX=FOX0
10002 CONTINUE
C
C             *****  COMPUTE CAPITAL F(X), PROBABILITY DISTRIBUTION FM  **
C
      CALL MDGAM(T45,M,PR,IE)
      IF (IE.EQ.129.OR.IE.EQ.130)  WRITE (6,501) IE
 501  FORMAT(59H ERROR IN GAMMA DISTRIBUTION FUNCTION COMPUTATION.  ERRO
     .R= ,I8)
      IF (C .LT. 0.) PR = 1.-PR
      BFOX0 = PR
C
      TTT = FL*ALOG(BFOX0)
      IF (TTT.LT.-300.)  GO TO 8003
      BFOXL = BFOX0**EL
      GO TO 8004
 8003 BFOXL = 0.
 8004 CONTINUE
C
C             *****  COMPUTE SMALL G(X), SMALL H(X), CAPITAL H(X) *****
C
      IF(IC(12).EQ.2) GO TO 11000
      GOFX = EL*(BFOXL/BFOX0)*FOX0
      EX = EL*(PR-1.)
      IF (EX.LT.-300.)  GO TO 8010
      EX = EXP(EX)
      HLO = EL*FOX0*EX
      HH0 = EX-EML
      GO TO 8011
 8010 HLO = 0.
      HH0 =-EML
 8011 CONTINUE
      IF (IF .GT. 1)  GO TO 11001
      X0MS(I) = X0 * CONVER
      FOX0MS(I) = FOX0 / CONVER
      BFO(I) = BFOX0
      BFOL(I) = BFOXL
      GOFXM(I) = GOFX / CONVER
      HLOMS(I) = HLO/CONVER
      HH0MS(I) = HH0
```

```
      ICON = I
      IF(IC(9).EQ.1) GO TO 11001
      WRITE (6,710) I,X0,FOX0,BFOX0,BFOXL,GOFX,HLO,HHO
 710  FORMAT(1X,I5,7E15.6)
11001 CONTINUE
C
C             *****  INITIALIZATION  *****
C
      TX=ABS(BFOXL-OMA)
      IF (TX-TXD)  9000,9001,9001
 9000 TXD = TX
      EXD = X0
 9001 CONTINUE
      IF (TXG.GE.GOFX)  GO TO 9002
      TXG=GOFX
      EXG=X0
 9002 CONTINUE
      TX=ABS(BFOX0-TTH)
      IF (TX-TX0)  9003,9004,9004
 9003 TX0 = TX
      EX0 = X0
 9004 CONTINUE
      IF (TXGA .GE. HLO)  GO TO 9005
      TXGA = HLO
      EXGA = X0
 9005 CONTINUE
      TX = ABS(HHO-OMA)
      IF (TX-TXDA)  9006,9007,9007
 9006 TXDA = TX
      EXDA = X0
 9007 CONTINUE
11000 CONTINUE
      X0=XT0+DELL
      IF (BFOX0.LE..999999)  GO TO 701
      GO TO 11003
11002 WRITE (6,11004) ISTOP
11004 FORMAT (1X,*NO. OF RECORDS GT*1X,I4)
11003 CONTINUE
      IF(IC(10).NE.1) GO TO 17
      WRITE(6,711)
711   FORMAT(//1X,T40,13HM.K.S. SYSTEM//)
      WRITE(6,304)
      DO 16 J= 1,ICON
      WRITE(6,710) J,X0MS(J),FOX0MS(J),BFO(J),BFOL(J),GOFXM(J),
     * HLOMS(J),HHOMS(J)
16    CONTINUE
17    CONTINUE
C
      IF(IC(12).EQ.2) GO TO 970
C
C
C             *****  SOLVE FOR X   *****
C                               M
C
      T5=(M-COC)**COC
      XSUBM = T5/AMBDA
      XSUBMS = XSUBM * CONVER
```

```
      WRITE(6,207)  XSUBM,XSUBMS
207   FORMAT(/2X,7HXSUBM= ,F12.4,3X,12HXSUBM(MKS)= ,F12.4)
C
C             *****  SOLVE FOR X  USING WEGSTEIN ITERATIVE METHOD  *****
C                               D
C
      T3=OMA**(1./EL)
      TT=ALP/EL
      U = EXC
      NE=0
      CALL WEG(1,U,4,0)
C                    **  START ITERATION LOOP  **
   33 T45 = (AMBDA*U)**C
      CALL MDGAM(T45,M,PR,IE)
      IF (C .LT. 0.) PR = 1.-PR
      U = U+PR-T3
C     U=U+TT*ALOG(PR)
      CALL WEG(2,U,0,NC)
C     WRITE (6,205) U,PR,T3,TT
  205 FORMAT (1X,*U= *,G20.8,5X,*PR*,G20.8,5X,*T3*,G20.8,5X,*TT*,G20.8)
      IF(U .LT. 0.) U=.0001
      NE = NE + 1
      IF (NE .LT. 50)  GO TO 3000
      WRITE (6,206)
 206  FORMAT(34H NO CONVERGENCE OBTAINED FOR XSUBD)
      U = 0.
      GO TO 32
 3000 CONTINUE
      IF(NC.EQ.0) GO TO 33
C                    **  END ITERATION FOR X   **
C                                           D
   32 XSUBD = U
C
C             *****  EVALUATE GAMMA DISTRIBUTION FUNCTION  *****
C
C     TT=(AMBDA*U)**C
C     CALL MDGAM( TT,M,PR,IE)
C     WRITE (6,702) PR
  702 FORMAT (* PR *,G20.8)
C
C             *****  SOLVE FOR X  USING WEGSTEIN ITERATIVE METHOD  *****
C                               G
C
      CA = C*AMBDA
      OMCM=1.-C*M
      CM3 = C*M-3.
      ELM = EL-1.
      U0 = EXG
      U = U0
      NE = 0
 435  CALL WEG(1,U,4,0)
C                    **  START ITERATION LOOP  **
      J=0
   43 AU = AMBDA*U
      AUC = AU**C
      IF(AUC .LT. 20.) GO TO 6001
      U=1.E-10
```

```
      GO TO 6002
 6001 CONTINUE
      EXPT = EXP(-AUC)
      F=ELM*(TP*EXPT*U**T1M)**2
      CALL MDGAM(AUC,M,PR,IE)
      IF (C .LT. 0.) PR = 1.-PR
      U=F/((PR*U**CM3*EXPT*(OMCM*C*AUC))*TP)
 6002 CONTINUE
      CALL WEG(2,U,0,NC)
C     WRITE (6,703) U,PR,F,CM3,EXPT,OMCM,AUC,AU,T2
  703 FORMAT (* DEBUG *,5G20.8)
      IF (U .GE. 0.)  GO TO 6003
      U0=U0+D(1)*2
      U = U0
      GO TO 435
 6003 CONTINUE
      NE = NE + 1
      IF (NE .LT. 50)  GO TO 4000
      WRITE (6,208)
  208 FORMAT(34H NO CONVERGENCE OBTAINED FOR XSUBG)
      U = 0
      GO TO 42
 4000 CONTINUE
      IF(NC.EQ.0) GO TO 43
C                     **  END ITERATION FOR X    **
C                                               G
   42 XSUBG = U
  437 CONTINUE
C
C             *****  SOLVE FOR X  USING WEGSTEIN ITERATIVE METHOD  *****
C                                0
C
C     U0 = X(K)
      KK=K/3
      U0= X(KK)
      U = U0
      NE = 0
      CALL WEG (1,U,4,0)
C                     **  START ITERATION LOOP  **
 5007 AU = AMBDA*U
      AUC = AU**C
      CALL MDGAM(AUC,M,PR,IE)
      IF (C .LT. 0.) PR = 1.-PR
      U = U+TTH-PR
      CALL WEG(2,U,0,NC)
C     WRITE(6,703) U,PR
      IF (U.GE.0)  GO TO 6004
      U=U0+D(1)
C     U = U0
      U=1.E-10
 6004 CONTINUE
      NE = NE + 1
      IF (NE.LT.50)  GO TO 6005
      WRITE (6,2080)
 2080 FORMAT(34H NO CONVERGENCE OBTAINED FOR XSUBO)
      U = 0.
      GO TO 6006
```

```
 6005 CONTINUE
      IF(NC.EQ.0) GO TO 6007
C                    **  END ITERATION FOR X    **
C                                             O
 6006 CONTINUE
      UMS = U * CONVER
      EXOMKS = EXO * CONVER
      WRITE(6,310) U,UMS,EXO,EXOMKS
 310  FORMAT(2X,*XSUBO= *,F12.4,3X,*XSUBO(MKS)= *,F12.4,3X,
     $ *TXSUBO= *,F12.4,3X,*TXSUBO(MKS)= *,F12.4)
C
C            *****  COMPUTE SIGNIFICANT VALUE X    *****
C                                             S
C
      IF (U .EQ. 0.)  U=EXO
      EMT=M+COC
      CALL MGAMMA(EMT,GAM,IER)
      IF(IER.EQ.129.OR.IER.EQ.130) WRITE(6,500)IER
  500 FORMAT(46H ERROR IN GAMMA FUNCTION COMPUTATION.  ERROR= ,I8)
      AU = AMBDA*U
      AUC = AU**C
      CALL MDGAM(AUC,EMT,PR,IE)
      IF (C .LT. 0.) PR = 1.-PR
      U=3.*(1.-PR)*GAM/(GAMMA*AMBDA)
      UMS = U * CONVER
      WRITE(6,312) U,UMS
 312  FORMAT(2X,*XSUBS= *,F12.4,3X,*XSUBS(MKS)= *,F12.4/)
 6050 CONTINUE
      XSUBGM = XSUBG * CONVER
      RGX(IL) = XSUBG
      RGXM(IL) = XSUBGM
      EXGM = EXG * CONVER
      WRITE(6,308) XSUBG,XSUBGM,EXG,EXGM
 308  FORMAT(2X,7HXSUBG= ,F12.4,3X,12HXSUBG(MKS)= ,F12.4,3X,
     $ 8HTXSUBG= ,F12.4,3X,13HTXSUBG(MKS)= ,F12.4)
C
C            *****  SOLVE FOR ASYMPTOTIC VALUE OF X  USING WEGSTEIN
C                   METHOD                       G                   *****
C
      ELOG = EL/GAMMA
      IF (C .LT. 0.) ELOG=-ELOG
      UC = EXGA
      U = (UC*AMBDA)**C
      NE = 0
      CALL WEG(1,U,4,0)
C                    **  START ITERATION LOOP **
 6010 IF (U .LT. 300)  GO TO 8501
      UT = 0.
      GO TO 8502
 8501 UT = U**M*EXP(-U)
 8502 U=CMMOC+ELOG*UT
      CALL WEG(2,U,0,NC)
      IF (U.GE.0.)  GOTO 6011
      U0=U0+C(1)
      U = U0
      U=1.E-10
 6011 CONTINUE
```

```
      NE = NE + 1
      IF (NE.LT.50)  GO TO 6012
      WRITE (6,2085)
 2085 FORMAT(35H NO CONVERGENCE OBTAINED FOR XSUBGA)
      U = 0.
      WRITE(6,703) U,DOC,AMBDA,C
      GO TO 6013
 6012 CONTINUE
      IF(NC.EQ.0) GO TO 6010
C                   **  END ITERATION FOR X      **
C                                          GA
 6013 U=U**DOC/AMBDA
      UMS = U * CONVER
      EXGAM = EXGA * CONVER
      WRITE(6,6014) U,UMS,EXG,EXGA
6014  FORMAT(2X,*XSUBGA= *,F12.4,2X,*XSUBGA(MKS)= *,F12.4,2X,
     $ *TXSUBGA= *,F12.4,2X,*TXSUBGA(MKS)= *,F12.4/)
6030  CONTINUE
      XSUBDM = XSUBD * CONVER
      RDX(IL) = XSUBD
      RDXM(IL) = XSUBDM
      EXDM = EXD * CONVER
      WRITE(6,6015) XSUBD,XSUBDM,EXD,EXDM
6015  FORMAT(2X,*XSUBD= *,F12.4,3X,*XSUBD(MKS)= *,F12.4,
     $ 3X,*TXSUBD= *,F12.4,3X,*TXSUBD(MKS)= *,F12.4)
C
C             *****  SOLVE FOR ASYMPTOTIC VALUE OF X  USING WEGSTEIN
C                    METHOD                            D              *****
C
      UU =EL+ALOG(CMA)
      U0 = EXDA
      U = U0
      NE = 0
      CALL WEG(1,U,4,0)
C                   **  START ITERATION LOOP  **
 6040 AU = AMBDA*U
      AUC = AU**C
      CALL MDGAM(AUC,M,PR,IE)
      IF (C .LT. 0.) PR = 1.-PR
      U = U+LU
      ELPR = EL+PR
      UUU = ELPR
      IF(ELPR.LE.300.) UUU=ALOG(EXP(ELPR)-1.)
 8020 U = U-LUU
      CALL WEG(2,U,0,NC)
      IF (U.GE.0.)  GO TO 6043
      UU=U0+D(1)
      U = U0
 6043 CONTINUE
      NE = NE+1
      IF(NE.LT.50)  GO TO 6044
      WRITE (6,2089)
 2089 FORMAT(35H NO CONVERGENCE OBTAINED FOR XSUBDA)
      U = 0.
      GO TO 6045
 6044 IF(NC.EQ.0) GO TO 6040
C                   **  END ITERATION FOR X      **
```

```
C                                                    DA
      UM = U * CONVER
      EXDAM = EXDA * CONVER
6045  WRITE(6,6046) U,UM,EXDA,EXDAM
6046  FORMAT(2X,*XSUBDA= *,F12.4,2X,*XSUBDA(MKS)= *,F12.4,2X,
     $ *TXSUBDA= *,F12.4,2X,*TXSUBDA(MKS)= *,F12.4///)
10000 CONTINUE
      WRITE(6,6047)
6047  FORMAT(1H1,T3,9HTIME(MIN),T20,9HTIME(HRS),T35,7HL VALUE,T52,
     $5HXSUBE,T67,5HXSUBG,T77,10HXSUBD(MKS),T92,10HXSUBG(MKS))
      DO 98 I=1,NL
      ESC(I) = ELT(I) / 60.0
      WRITE(6,6048) ELT(I),ESC(I),ELL(I),RDX(I),RGX(I),RDXM(I),RGXM(I)
98    CONTINUE
6048  FORMAT(1H ,T2,F10.4,T19,F10.4,T32,F10.4,T47,F10.4,T62,F10.4,
     $T77,F10.4,T92,F10.4)
C
C   OPTION 5:  PLOT HISTOGRAM WITH PLOTPR PACKAGE
  970 IF(IC(5).LT.1) GO TO 97
C
C    COMPUTE HISTOGRAM VALUES
C
      DIV=1.0
      ICNT = 0
      IF(IC(18).EQ.1) DIV=D(1)
      DO 85 I=1,11
      ICNT = ICNT + 1
85    Z(ICNT) = DENO(1)/DIV
      DO 94 I = 2,K
      IF(IC(18).EQ.1) DIV=D(I)
      DO 93 J = 1,10
      ICNT = ICNT + 1
93    Z(ICNT) = DENO(I)/DIV
94    CONTINUE
      RM = 0.0001
      IPP = 20
      K10 = 10*K
84    IPP = IPP + 10
      IF(RN(IPP).LT.RM.AND.FN(IPP).LT.RM.AND.Z(IPP).EQ.0.) GO TO 83
      IF(IPP.LT.K10) GO TO  84
83    IP = IPP
      IF(IP.LT.K10) GO TO 92
      K11 = K10 + 1
      DO 90 I = K11,IP
90    Z(I) = 0.0
92    CONTINUE
      CALL INITPLO
C    OPTION 15: NUMBER OF PAGES NEEDED FOR PLOT(DEFAULT=1 DOUBLE PG)
      NUMPAG = 1
      IF(IC(15).NE.0) NUMPAG=IC(15)
      MAXSCA = 2
      IVAR(1) = 1HX
      IVAR(2) = 5HYHIST
      IVAR(3)=5HYGAMM
      IVAR(4) = 4HYRAY
      ITP(1) = 6HOBSERV
      ITP(2) = 6HED  ,GA
```

```
      ITP(3) = 6HMMA AN
      ITP(4) = 6HD RAYL
      ITP(5) = 6HEIGH P
      ITP(6) = 6HLOTS
      ITX(1) = 6HVALUE
      ITX(2) = 6HOF RAN
      ITX(3) = 6HDOM VA
      ITX(4) = 6HRIABLE
      ITX(5) = 6H
      ITX(6) = 6H
      ITY(1) = 6HDENSIT
      ITY(2) = 6HY
      ITY(3) = 6H(NUM./
      ITY(4) = 6HBIN)/N
      ITY(5) = 6H*BIN-W
      ITY(6) = 6HIDTH
C
C    IC(5)=2 WRITE VALUES BEFORE PLOTTING
      IF(IC(5).EQ.2) WRITE(6,9190)(IVAR(I),I=1,4)
 9190 FORMAT(1H1,1X,A1,10X,A5,6X,A5,6X,A4)
C
C    OPTION 6: DISTRIBUTIONS IN MKS UNITS
      IF(IC(6).NE.1) GO TO 80
      ITX(5) = 6H MKS
      DO 81 I = 1,IP
      XNMS(I) = XN(I)*CONVER
      ZM(I) = Z(I) / CONVER
      FNMS(I) = FN(I) /CONVER
      RNMS(I) = RN(I) /CONVER
81    WRITE(3) (XNMS(I),ZM(I),FNMS(I),RNMS(I))
      GO TO 70
80    CONTINUE
      DO 91 I = 1,IP
      IF(IC(5).EQ.2) WRITE(6,9191)  XN(I),Z(I),FN(I),RN(I)
 9191 FORMAT(1H ,4F13.4)
91    WRITE(3) (XN(I),Z(I),FN(I),RN(I))
70    CONTINUE
      CALL PLOTPR(3,4,IVAR)
97    CONTINUE
C
C    OPTION 7: TIME EFFECT PLOTS
C
      IF(IC(7).NE.1) GO TO 95
C
C    TEST TO SEE IF TIME EFFECT PLOTS ARE GOOD
C
      IFG1=0
      IFG2=0
      IF(NL.EQ.1) GO TO 95
      DO 90909 II=2,NL
      IF(RGX(II).NE.RGX(II-1)) IFG1=1
90909 IF(RDX(II).NE.RDX(II-1)) IFG2=1
      IF(IFG1.EQ.0.OR.IFG2.EQ.0) GO TO 95
C
      CALL INITPLO
C    OPTION 16: NUMBER OF PAGES NEEDED FOR PLOT(DEFAULT=1 DOUBLE PG)
      NUMPAG = 1
```

```
      IF(IC(16).NE.0) NUMPAG=IC(16)
      NPLOT= 2
      FROM(1) = 0.0
C   OPTION 11: SCALE FOR INDEPENDENT VARIABLE ON PLOT(DEFAULT=.1)
      SCA(1) = 0.05
      IF(IC(11).NE.0) SCA(1)=10**(IC(11))
      MAXSCA = 2
      ITP(1) = 6HTIME E
      ITP(2) = 6HFFECT
      ITP(3) = 6HPLOTS
      ITX(1) = 6HTIME I
      ITX(2) = 6HN HOUR
      ITX(3) = 6HS
      ITY(1) = 6HDESIGN
      ITY(2) = 6H VALUE
      ITY(3) = 6H
      ITY(4) = 6H
      IVAR(1) = 4HTIME
      IVAR(2) = 6HXSURD
      IVAR(3) = 6HXSURG
      IVAR(4) = 6H
C   OPTION 8: TIME EFFECT PLOTS IN MKS UNITS
C
C
      IF(IC(8).NE.1) GO TO 60
      IVAR(2) = 6HXD-MKS
      IVAR(3) = 6HXG-MKS
      ITY(3) = 6H(MKS)
      DO 61 J = 1,NL
61    WRITE(3) (ESC(J),RDXM(J),RGXM(J))
      GO TO 65
60    CONTINUE
      DO 96 J = 1,NL
96    WRITE(3) (ESC(J),RDX(J),RGX(J))
65    CONTINUE
      CALL PLOTPR(3,3,IVAR)
95    CONTINUE
      IF(IPLOT.EQ.0) GO TO 99
C            *****  END 10000 LOOP  *****
C
C
C            *****  PLOT RESULTS  *****
C
      CALL PLOT(XLL,0.,-3)
      CALL HISTO
      XN(IP+1)=FIRSTX
      XN(IP+2)=DELX
      FN(IP+1)=FIRSTY
      FN(IP+2)=DELY
      RN(IP+1)=FIRSTY
      RN(IP+2)=DELY
      CALL LINE(XN,FN,IP,1,0,0)
      IF(IRAY.EQ.0) GO TO 10004
      CALL LINE(XN,RN,IP,1,5,3)
10004 CONTINUE
      GO TO 99
10003 IF(IPLOT.EQ.0) STOP
      CALL PLOT(XLL,0.,999)
      STOP
      END
```

```
      SUBROUTINE GETCML(AT,U,F,S,YBAR,GAMMA,OOC,T,ICK)
      COMMON/PARAMS/M,AMBDA,C,R
      COMMON/OPT/IOP(20),ICK1
      COMMON/INPUT/X(100),ELL(7),N,ALP,D(100),ISTOPD,IPLOT,IRAY,IC(20)
      COMMON/PRNT/FOBS(100),FTH(100),DENO(100),DENT(100)
     $,FTHR(100),DENTR(100),CONVER,CAL
      COMMON/HIST/BINLO,K,BINW(100),H(100),YMAX,XLEN,YLEN,CMNT(5)
      DIMENSION XMKS(100),DELMKS(100)
      DIMENSION DHEAD(100)
      REAL M
      DATA DHEAD/100*2HD(/
      IF(IC(20).EQ.1) GO TO 5
C
C           *****  SOLVE FOR M USING WEGSTEIN ITERATIVE METHOD  *****
C
      M=5.
      NE = 0
      CALL WEG(1,M,4,0)
C
C           *****  START ITERATION LOOP  *****
C
    3 F=PSI1(M)
      U = PSI2(M)
      M=M-AT-U/F**1.5
      CALL WEG(2,M,0,NC)
      WRITE(6,461) M
  461 FORMAT(1H0,2HM=,E10.4)
      NE = NE+1
      IF(NE.LT.50) GO TO 2000
      WRITE(6,206)
  206 FORMAT(34H NO CONVERGENCE OBTAINED FOR XSUBD)
      ICK=1
      RETURN
 2000 IF(M.GE.0.) GO TO 2001
      M = .0001
 2001 CONTINUE
      IF(NC.EQ.0) GO TO 3
C
C        *****  END ITERATION FOR M  *****
C
C           *****  COMPUTE C  *****
C
    2 CONTINUE
      U1 = PSI(M)
      U2 = PSI1(M)
      C = SQRT(U2/S)
      IF(T.GE.0.) C=-C
      OOC = 1./C
C
C           *****  COMPUTE LAMBDA  *****
C
    4 AMBDA = EXP(-YBAR+U1*OOC)
    5 CONTINUE
      ICK1=0
      IF(IC(20).EQ.1) CALL MLHCML
      IF(ICK1.EQ.1) RETURN
      WRITE(6,101) CMNT
```

```
101   FORMAT(1H1,5A10)
      OMA = 1.-ALP
      WRITE(6,180) CONVER ,CAL
180   FORMAT(1H ,8HCONVER= ,F10.3,5X,20HCALIBRATION FACTOR= ,F10.3)
C     COMPUTE X(MKS) AND DEL(MKS)
      DO 17 JI=1,K
      DELMKS(JI)=D(JI)*CONVER
17    XMKS(JI) = X(JI) * CONVER
C     FIND MKS VALUE OF YBAR
      YBARMK = YBAR +      K*ALOG(CONVER)
      WRITE(6,300) N,K,ALP,           OMA
300   FORMAT(//1X,3HN= ,I5,2X,3HK= ,I5,2X,7HALPHA= ,G10.4,2X,
     $                                    2X,9H1-ALPHA= ,G10.4)
      WRITE(6,297)
  297 FORMAT(/1X,*DEL ARRAY*)
      WRITE(6,298) (DHEAD(I),I,D(I),I=1,K)
  298 FORMAT(1H ,9(A2,I2,1H),F7.4,2X))
      IF(CONVER.NE.1) WRITE(6,296)
  296 FORMAT(/1X,*DEL ARRAY(MKS)*)
      IF(CONVER.NE.1) WRITE(6,298) (DHEAD(I),I,DELMKS(I),I=1,K)
      WRITE(6,301) YBAR,YBARMK,S,T
301   FORMAT(/1X,6HYBAR= ,G12.4,2X,11HYBAR(MKS)= ,G12.4,2X,
     $ 3HS= ,G12.4,2X,3HT= ,G12.4)
  203 FORMAT(1X,3HM= ,G12.4,3X,8HLAMBDA= ,G12.4,3X,3HC= ,G12.4/)
C
C           *****  EVALUATE GAMMA FUNCTION  *****
C
      IF(IER.EQ.129.OR.IER.EQ.130) WRITE(6,500)IER
  500 FORMAT(46H ERROR IN GAMMA FUNCTION COMPUTATION.  ERROR= ,I3)
      IF(IC(15).EQ.1.AND.IC(20).NE.1) WRITE(6,250) AMBDA,M,C
  250 FORMAT(1H ,25HSTACY-MIHRAM PAR.  AMBDA=,F16.5,4H  M=,F16.5,
     $4H  C=,F16.5)
      CALL DIST
      OMA=1.-ALP
      OOC=1./C
      CALL MGAMMA(M,GAMMA,IER)
      AMKS=AMBDA/CONVER
      BMKS = B * (CONVER**2)
      MET=10H   STACY/M
      HOD=9HIHRAM
      IF(IC(20).EQ.1) MET=10HMAXIMUM LI
      IF(IC(20).EQ.1) HOD=8HKELIHOOD
      WRITE(6,302) M,C,AMBDA,AMKS,B,BMKS,MET,HOD
302   FORMAT(/1X,3HM= ,G12.4,2X,3HC= ,G12.4,2X,8HLAMBDA= ,
     $ G12.4,2X,13HLAMBDA(MKS)= ,G12.4,,3HB= ,G12.4,8HB(MKS)= ,G12.4,
     $/,7H , VIA ,A10,A8,7H METHOD)
      WORD=8HRAYLEIGH
      WRITE(6,110) WORD,WORD
110   FORMAT(T3,8HVALUE OF,T19,8HVALUE OF,T35,8HMEASURED,T51,
     $A8          ,T66,5HGAMMA,T82,8HMEASURED,T98,A8          ,T114,5HGAMMA
     $,/,T4,6HRANDOM,T19,6HRANDOM,T35,9HFREQUENCY,T51,9HPREDICTED,T66,
     $9HPREDICTED,T82,7HDENSITY,T98,9HPREDICTED,T114,9HPREDICTED,/,
     $T3,8HVARIABLE,T19,8HVARIABLE,T51,9HFREQUENCY,T66,9HFREQUENCY,
     $T82,7H=/BIN/N,T98,7HDENSITY,T114,7HDENSITY,/,T20,5H(MKS))
      WRITE(6,120) (X(I),XMKS(I),FOBS(I),FTHR(I),FTH(I),DENO(I),
     $DENTR(I),DENT(I),I=1,K)
120   FORMAT(T3,F8.4,T19,F8.4,T32,F12.4,T48,F12.4,T63,F12.4,
     $T79,F12.4,T96,F12.4,T111,F12.4)
      RETURN
      END
```

```
      SUBROUTINE HISTO
      COMMON/HIST/BINLO,NOBINS,BINW(100),ORD(100),YMAX,XLEN,YLEN,CMNT(5)
      COMMON/HIST2/FIRSTX,DELX,FIRSTY,DELY
      DIMENSION X(200),Y(200),RAY(5)
      N2=NOBINS+1
      ORD(N2)=YMAX
C       COMPUTE SCALE FOR Y-AXIS
C         ORD IS THE ARRAY OF HEIGTHS FOR THE BINS
C         YLEN IS THE LENGTH IN INCHES OF THE Y-AXIS DEFINED IN
C              A DATA STATEMENT IN THE MAIN PROGRAM TO BE 8 INCHES
C         N2 IS THE NUMBER OF POINTS  IN ORD AND +1 SAYS EVERY POINT IS
C            TO BE PLOTTED
C         FIRSTY THE FIRST Y VALUE ANNOTATED ON THE Y-AXIS IS RETURNED
C                IN THE N2+1 LOCATION OF ORD
C         DELY THE SCALE VALUE - AMOUNT OF DATA PER INCH- IS RETURNED
C              BY SCALE IN THE N2+2 LOCATION OF ORD
      CALL SCALE(ORD,YLEN,N2,+1)
      FIRSTY=ORD(N2+1)
      DELY=ORD(N2+2)
      TITLX=10HEXCURSIONS
      TITLY=7HDENSITY
      ORD(N2)=0.
C         SCALES SET UP BY AXIS TO BE USED BY LINE TO PLOT
C         POINTS - X-AXIS LENGTH IS CONSTANT AND IS DEFINED IN A
C                DATA STATEMENT IN THE MAIN PROGRAM TO BE 10 INCHES
C         THE AXIS ANNOTATION WILL BE DONE WITHOUT THE  USE OF
C         AXIS  SO THAT THE NUMBERS LINE UP WITH THE BINS DRAWN
      KOUNT = 1
      N = 1
      X(N) = 0.0
      N = N+1
      IF (BINLO.EQ.0.0) GO TO 5
      N = N+1
      X(N) = BINLO
      N2 = N2+1
    5 DO 10 I=1,N2
      X(I) = X(I-1)+BINW(KOUNT)
      KOUNT = KOUNT+1
   10 CONTINUE
         CALL SCALE(X,XLEN,N2,+1)
      FIRSTX = X(N2+1)
      DELX = X(N2+2)
      UNIT = 1./DELX
C         PUT PEN AT ORIGIN
      CALL PLOT(0.,0.,3)
      DO 20 I=1,N2
      STEP = X(I)*UNIT
C         DRAW AXIS PORTION FROM CURRENT LOCATION TO END OF NEXT BIN
      CALL PLOT(STEP,0.,2)
C         DRAW TICK MARK
      CALL PLOT(STEP,-.1,2)
C         ANNOTATE AXIS WITH NUMERALS
      CALL NUMBER(STEP-.21,-.21,.1,X(I),0.0,2)
C         RETURN PEN TO AXIS LINE WITHOUT DRAWING LINE
C         CONTINUE DRAWING AXIS AND ANNOTATING IT FOR THE REMAINDER
C         OF THE VALUES
   20 CONTINUE
```

```
C        TITLE THE AXIS
      CALL SYMBOL(4.25,-.32,.14,TITLX,0.0,10)
      CALL AXIS(0.,0.,TITLY,+7,YLEN,90.,FIRSTY,DELY)
      RAY(1)=10H+RAYLEIGH
      RAY(2) = 10HDIST.
      RAY(3)=10H-COMPUTED
      RAY(4) = 10HVALUE
      RAY(5) = 10H
      XPAGE=XLEN-2.8
      YPAGE=YLEN-.5
      CALL SYMBOL(XPAGE,YPAGE,.14,RAY(1),0.0,10)
      XPAGE=XPAGE+1.4
      CALL SYMBOL(XPAGE,YPAGE,.14,RAY(2),0.0,10)
      XPAGE=XLEN-2.8
      YPAGE=YLEN-1.
      CALL SYMBOL(XPAGE,YPAGE,.14,RAY(3),0.0,10)
      XPAGE=XPAGE+1.4
      CALL SYMBOL(XPAGE,YPAGE,.14,RAY(4),0.0,10)
      XPAGE=XPAGE+1.4
      CALL SYMBOL(XPAGE,YPAGE,.14,RAY(5),0.0,10)
      XPAGE=XLEN-2.8
      YPAGE=YPAGE-.5
      CALL SYMBOL(XPAGE,YPAGE,.14,CMNT,0.0,50)
      X(1)=BINLO
      Y(1)=0.
      X(2)=BINLO
      Y(2)=ORD(1)
      J=2
      DO 1 I=1,NOBINS
      J=J+1
      XT = X(J-1)+BINW(I)
      X(J)=XT
      Y(J)=ORD(I)
      J=J+1
      X(J)=XT
      Y(J)=ORD(I+1)
    1 CONTINUE
      X(J+1)=FIRSTX
      X(J+2)=DELX
      Y(J+1)=FIRSTY
      Y(J+2)=DELY
      CALL LINE(X,Y,J,1,0,0)
      RETURN
      END
```

```
      FUNCTION PSI(M)
C
C              FUNCTION PSI EVALUATES THE DIGAMMA FUNCTION PSI(M) FOR
C           ARGUMENT M. PSI(M) IS THE DERIVATIVE WRT M OF THE LN OF
C           THE GAMMA FUNCTION.
C
      REAL M
      DATA C16,C1126/.166666667,.007936508/
C
      T1 = 0.
      W = M
      IF(W.GT.13.) GO TO 2
C                M.LE.13
      N = 14. - W
      W = M + N
      D = M - 1
      DO 1 I = 1,N
    1 T1 = T1 + 1./(D + I)
C                M.GT.13
    2 RCW = 1./W
      RCW2 = RCW**2
      PSI = ALOG(W) + .5*RCW*(-1. + RCW*(-C16 + RCW2*(.1*C16 - C1126*
     .   RCW2))) - T1
      RETURN
      END
```

```
      FUNCTION PSI1(M)
C
C          FUNCTION PSI1 EVALUATES THE TRIGAMMA FUNCTION PSI (M)
C          FOR ARGUMENT M. PSI1(M) IS THE DERIVATIVE WRT M OF THE
C          DIGAMMA FUNCTION.
C
      REAL M
      DATA C16,C130,C142/.166666667,.033333333,.023809524/
C
      T1 = 0.
      W = M
      IF(W.GT.13.) GO TO 2
C                M.LE.13
      N = 14. - W
      W = M + N
      D = M - 1
      DO 1 I=1,N
    1 T1 = T1 + 1./(D+I)**2
C                M.GT.13
    2 RCW = 1./W
      RCW2 = RCW**2
      PSI1 = RCW*(1.+RCW*(.5+RCW*(C16  +RCW2*(-C130  +RCW2*(C142
     *     -RCW2*C130)))))    +T1
      RETURN
      END
```

```
      FUNCTION PSI2(M)
C
C                                                                   ..
C          FUNCTION PSI2 EVALUATES THE TETRAGAMMA FUNCTION PSI  (M)
C          FOR ARGUMENT M. PSI2(M) IS THE 2ND DERIVATIVE WRT M OF
C          THE DIGAMMA FUNCTION.
C
      REAL M
      DATA C16,C56/.166666667,.833333333/
C
      T1 = 0.
      W = M
      IF(W.GT.13.) GO TO 2
C                   M.LE.13
      N = 14.-W
      W = M + N
      D = M - 1
      DO 1 I=1,N
 1    T1 = T1 + 2./(D + I)**3
C                   M.GT.13
 2    RCW=1./W
      RCW2 = RCW**2
      PSI2 = RCW2*(-1. +RCW*( -1. +RCW*(-.5 +RCW2*(C16  +RCW2*( -C16
     *     +RCW2*(.3  -C56*RCW2)))))) -T1
      RETURN
      END
```

```
      SUBROUTINE WEG(I,XNP1,J,N)
C                     WEGSTEIN ITERATIVE SOLUTION METHOD FOR X     =F(X )
C                                                             N+1      N
C
C
C                               INITIALIZATION
      IF(I.NE.1) GO TO 2
    1 K = 1
      XBN = XNP1
      XTEMP = 10.**(-J)
      XP = XNP1
      RETURN
C
C                               FIRST ITERATION
 2    IF(K.NE.1) GO TO 4
    3 IF(ABS(XP-XNP1)-XTEMP*ABS(XP))  5,5,8
    8 XP = XNP1
      XBNM1 = XBN
      XBN = XNP1
      K = 2
    7 XN = XNP1
      XNP1 = XBN
      N = 0
      RETURN
C
C                               SUCCEEDING ITERATIONS
    4 XBNP1 = ((XNP1*XBNM1)-(XN*XBN))/(XNP1+XBNM1-XN-XBN)
      IF((ABS(XP-XBNP1))-(ABS(XTEMP*XP)))  5,5,6
    6 XP = XBNP1
      XBNM1 = XBN
      XBN = XBNP1
      GO TO 7
    5 N = 1
      RETURN
      END
```

```
SUBROUTINE AXIS
ENTRY LINE
ENTRY NUMBER
ENTRY PLOT
ENTRY SYMBOL
ENTRY PLOTS
RETURN
END
```

```
      FUNCTION G(C)
      COMMON/HIST/BINLO,K,BINW(100),O(100),YMAX,XLEN,YLEN,CMNT(5)
      COMMON/INPUT/X(100),ELL(7),N,ALP,D(25),ISTOPD,IPLOT,IRAY,IC(20)
      OLNSUM=0.
      SUMO=0.
      XOSUM=0.
      XLOSUM=0.
      DO 10 I=1,K
      SUMO= SUMO+O(I)
      OLNSUM=OLNSUM+ O(I)*ALOG(X(I))
      XLOSUM= XLOSUM + ALOG(X(I))*(X(I)**C)*O(I)
      XOSUM= XOSUM+(X(I)**C)*O(I)
   10 CONTINUE
      ARG=  (XLOSUM/XOSUM)-(OLNSUM/SUMO)
      G= (SUMO/OLNSUM)* (ALOG(C)+ ALOG(XOSUM)+ALOG(ARG)-ALOG(SUMO)+
     $   PSI(1./(C*ARG)))
      RETURN
      END
```

```
      FUNCTION F(C)
      COMMON/HIST/BINLO,K,BINW(100),O(100),YMAX,XLEN,YLEN,CMNT(5)
      COMMON/INPUT/X(100),FLL(7),N,ALP,D(25),ISTOPD,IPLOT,IRAY,IC(20)
      XOSUM=0.
      XLOSUM=0.
      OLNSUM=0.
      SUMO=0.
      DO 10 I=1,K
      XOSUM= XOSUM+ (X(I)**C)*O(I)
      OLNSUM=OLNSUM+ O(I)*ALOG(X(I))
      SUMO=SUMO+O(I)
      XLOSUM=XLOSUM+ ALOG(X(I)) *(X(I)**C)*O(I)
   10 CONTINUE
      ARG= (XLOSUM/XOSUM)-(OLNSUM/SUMO)
      F= ALOG(C)+ALOG(XOSUM) + ALOG(ARG)-ALOG(SUMO)+PSI(1./(C*ARG))
     *   -(C*OLNSUM)/SUMO
      RETURN
      END
```

```
      SUBROUTINE MLHCML
C
C    MAXIMUM LIKELYHOOD ESTIMATION
C
      COMMON/HIST/BINLO,K,BINW(100),O(100),YMAX,XLEN,YLEN,CMNT(5)
      COMMON/INPUT/X(100),FLL(7),N,ALP,D(25),ISTOPP,IPLOT,IRAY,IC(20)
      COMMON/IOUNIT/IO5,IP5,IP6,IPCH
      COMMON/PARAMS/M,AMBDA,C,P
      COMMON/OPT/IOP(20),ICK1
      REAL M
      IF(IOP(1).NE.0)GO TO 10
C   RAYLEIGH PARAMETERS
      C=2.
      GO TO 30
   10 READ(IP5,101) C
  101 FORMAT(F10.0)
   30 IF(IOP(2).EQ.1) WRITE(IP6,102)  C
  102 FORMAT(1H0,20HINITIAL GUESS FOR C=,F10.5)
      WRITE(IP6,555)
  555 FORMAT(1H ,29HITERATIONS FOR FINDING C ROOT)
      WRITE(IP6,551)
  551 FORMAT(1H ,8X,1HC,10X,4HF(C))
      KE=0
      DO 1 I=1,50
      CALL WEG(I,C,4,KE)
      IF(C.LT.0.) C=ABS(C)
      IF(C.EQ.0.) C=.1
      ZERO=F(C)
      WRITE(IP6,211) I,C,ZERO
  211 FORMAT(1H ,I2,F10.5,F10.6)
      IF(KE.EQ.1)GO TO 22
      C=G(C)
    1 CONTINUE
C   MAXIMUM NUMBER OF ITERATIONS REACHED WITHOUT CONVERGENCE
      ICK1=1
      WRITE(IP6,444)
  444 FORMAT(1H0,16HC ROOT NOT FOUND)
      RETURN
   22 CONTINUE
C   C ROOT FOUND WITHIN TOLERANCE
C   FIND M
      XLOSUM=0.
      XOSUM=0.
      OLNSUM=0.
      SUMO =0.
      DO 33 I=1,K
      XLOSUM=XLOSUM + ALOG(X(I))*(X(I)**C)*O(I)
      XOSUM=XOSUM + (X(I)**C)*O(I)
      OLNSUM=OLNSUM+ O(I)*ALOG(X(I))
      SUMO= O(I)+SUMO
   33 CONTINUE
      IF(IOP(5).EQ.1) WRITE(IP6,111) XLOSUM,XOSUM,OLNSUM,SUMO
  111 FORMAT(1H0,7HXLOSUM=,F10.4,2X,6HXOSUM=,F10.4,2X,7HOLNSUM=,F10.4,
     $2X,5HSUMO=,F10.4)
      M=1./(C*((XLOSUM/XOSUM)-(OLNSUM/SUMO)))
C   FOUND M--SOLVE FOR LAMBDA
      AMBDA=((XOSUM/(M*SUMO))**(-1./C))
      IF(IOP(3).EQ.1) WRITE(IP6,100)AMBDA,M,C
  100 FORMAT(1H0,42HPARAMETERS ESTIMATED BY MAXIMUM LIKELYHOOD,
     $11H ESTIMATION//1H ,7HLAMBDA=,F20.6,5X,2HM=,F10.6,5X,2HC=,F10.6)
      RETURN
      END
```

```
      SUBROUTINE DIST
      REAL M
      COMMON/INPUT/X(100),ELL(7),N,ALP,D(100),ISTOPD,IPLOT,IRAY,IC(20)
      COMMON/PARAMS/M,AMBDA,C,R
      COMMON/PRNT/FOBS(100),FTH(100),DENO(100),DENT(100),FTHR(100)
     $,DENTR(100),CONVER,CAL
      COMMON/HIST/BINLO,K,BINW(100),H(100),YMAX,XLEN,YLEN,CMNT(5)
      BOTTOM = 0.0
      RMIN1 = 1.0
      DIV=1.0
      D2=1.0
      DO 10 I =1,K
C  OPTION 18: IF ON, DENSITY =(NO./BIN)/N
C             IF OFF,DENSITY=(NO./BIN)/(N*D(I))
      IF(IC(18).EQ.0) DIV=D(I)
      IF(IC(18).EQ.1) D2=D(I)
      ARGR=(X(I)+D(I)/2.)**2./R
      R1 = EXP(-ARGR)
      ARG=(AMBDA*(X(I)+D(I)/2.))**C
      CALL MDGAM(ARG,M,TOP,IER)
C
C   OBSERVED FREQUENCY
      FOBS(I)=D(I)*H(I)*N
C
C   OBSERVED DENSITY
      DENO(I)=H(I)*D2
C
C   THEORETICAL RAYLEIGH(EXPONENTIAL) FREQUENCY
      FTHR(I) = (RMIN1 - R1 ) * N
C   RAYLEIGH(EXPONENTIAL) DENSITY
      DENTR(I) =(RMIN1 - R1)/DIV
C
C   THEORETICAL GAMMA FREQUENCY
      IF(IER.NE.0) GO TO 10
      FTH(I) = (TOP - BOTTOM) * N
C   GAMMA PREDICTED DENSITY
      DENT(I) =(TOP- BOTTOM)/DIV
C
      RMIN1 = R1
      BOTTOM = TOP
10    CONTINUE
      RETURN
      END
```

APPENDIX B

SAMPLE OUTPUT

```
 IC OPTIONS USED IN THIS RUN SET IPLOT=    0   IRAY=    0
 OPTION  5= 1
 OPTION  9= 1
 OPTION 20= 1

IOP OPTIONS USED IN THIS RUN SET
 OPTION  2= 1
 OPTION  3= 1

INITIAL GUESS FOR C=   2.00000
ITERATIONS FOR FINDING C ROOT
         C           F(C)
 1   2.00000  -.004530
 2   1.93812  -.004446
 3   1.89815  -.000400
 4   1.88827  -.000042
 5   1.88712  -.000000
 6   1.88712  -.000000

PARAMETERS ESTIMATED BY MAXIMUM LIKELYHOOD ESTIMATION

LAMBDA=              .064866     M=   .978837     C=  1.887116
```

P22 3F 10
CONVER= 1.000 CALIBRATION FACTOR= 0.000

N= 262 K= 14 ALPHA= .1000E-01 1-ALPHA= .9900

DEL ARRAY
D(1) 3.0000 D(2) 3.0000 D(3) 3.0000 D(4) 3.0000 D(5) 3.0000 D(6) 3.0000 D(7) 3.0000 D(8) 3.0000
(9) 3.0000 D(10) 3.0000 D(11) 3.0000 D(12) 3.0000 D(13) 3.0000 D(14) 3.0000

YBAR= 2.411 YBAR(MKS)= 2.411 S= .4751 T= -1.030

M= .9788 C= 1.887 LAMBDA= .6487E-01 LAMBDA(MKS)= .6487E-01R= 238.7 R(MKS)= 238.7
. VIA MAXIMUM LIKELIHOOD METHOD

VALUE OF RANDOM VARIABLE	VALUE OF RANDOM VARIABLE (MKS)	MEASURED FREQUENCY	RAYLEIGH PREDICTED FREQUENCY	GAMMA PREDICTED FREQUENCY	MEASURED DENSITY =/BIN/N	RAYLEIGH PREDICTED DENSITY	GAMMA PREDICTED DENSITY
1.5000	1.5000	.1200E+02	.9696E+01	.1257E+02	.1527E-01	.1234E-01	.1599E-01
4.5000	4.5000	.3000E+02	.2699E+02	.3003E+02	.3817E-01	.3433E-01	.3821E-01
7.5000	7.5000	.4000E+02	.3872E+02	.3961E+02	.5089E-01	.4926E-01	.5040E-01
10.5000	10.5000	.3900E+02	.4329E+02	.4204E+02	.4962E-01	.5508E-01	.5348E-01
13.5000	13.5000	.4400E+02	.4124E+02	.3884E+02	.5598E-01	.5247E-01	.4942E-01
16.5000	16.5000	.2700E+02	.3465E+02	.3224E+02	.3435E-01	.4409E-01	.4102E-01
19.5000	19.5000	.3200E+02	.2612E+02	.2443E+02	.4071E-01	.3323E-01	.3108E-01
22.5000	22.5000	.1500E+02	.1784E+02	.1706E+02	.1908E-01	.2269E-01	.2170E-01
25.5000	25.5000	.9998E+01	.1110E+02	.1104E+02	.1272E-01	.1412E-01	.1405E-01
28.5000	28.5000	.5997E+01	.6319E+01	.6655E+01	.7630E-02	.8039E-02	.8467E-02
31.5000	31.5000	.3003E+01	.3301E+01	.3746E+01	.3820E-02	.4199E-02	.4766E-02
34.5000	34.5000	.9982E+00	.1585E+01	.1974E+01	.1270E-02	.2017E-02	.2511E-02
37.5000	37.5000	.9982E+00	.7009E+00	.9754E+00	.1270E-02	.8918E-03	.1241E-02
40.5000	40.5000	.1996E+01	.2857E+00	.4528E+00	.2540E-02	.3635E-03	.5761E-03

```
RAYLEIGH DESIGN VALUES
L=    262.0000      XRSUBD=      49.2759      XRSUBD(PKS)=      49.2759
L=    546.0000      XRSUBD=      51.0232      XRSUBD(PKS)=      51.0232
L=   1638.0000      XRSUBD=      53.5310      XRSUBD(PKS)=      53.5310
```

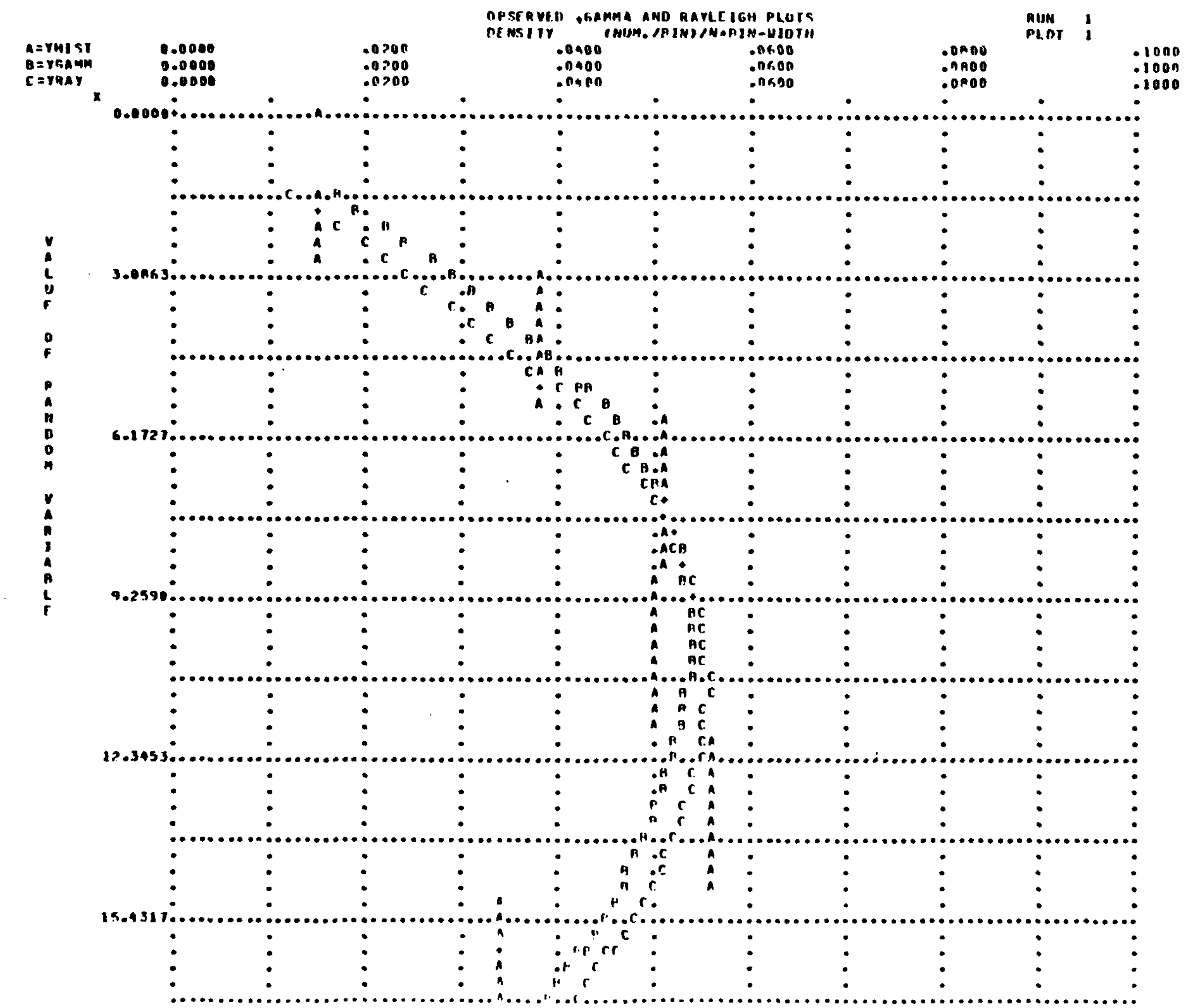
```
L=   262.0

XSUBM=       10.0849   XSUBM(MKS)=      10.0849
XSUBO=       15.9986   XSUBO(MKS)=      15.9986   TXSUBO=       15.9000   TXSUBO(MKS)=       15.9000
XSUBS=       22.1135   XSUBS(MKS)=      22.1135

XSUBG=       38.4161   XSUBG(MKS)=      38.4161   TXSUBG=       38.4000   TXSUBG(MKS)=       38.4000
XSUBGA=       38.4206  XSUBGA(MKS)=      38.4206  TXSUBGA=       38.4000  TXSUBGA(MKS)=       38.4000

XSUBD=       52.5000   XSUBD(MKS)=      52.5000   TXSUBD=       52.5000   TXSUBD(MKS)=       52.5000
XSUBDA=       52.5001  XSUBDA(MKS)=      52.5001  TXSUBDA=       52.5000  TXSUBDA(MKS)=       52.5000
```

```
L=   546.0

XSUBM=     10.0849   XSUBM(MKS)=     10.0849
XSUBO=     15.9986   XSUBO(MKS)=     15.9986   TXSUBO=      15.9000   TXSUBO(MKS)=     15.9000
XSUBS=     22.1135   XSUBS(MKS)=     22.1135

XSUBG=     40.9344   XSUBG(MKS)=     40.9344   TXSUBG=      41.1000   TXSUBG(MKS)=     41.1000
XSUBGA=     40.9670  XSUBGA(MKS)=     40.9670  TXSUBGA=      41.1000  TXSUBGA(MKS)=     41.1000

XSUBD=     54.6000   XSUBD(MKS)=     54.6000   TXSUBD=      54.6000   TXSUBD(MKS)=     54.6000
XSUBDA=     54.5996  XSUBDA(MKS)=     54.5996  TXSUBDA=      54.6000  TXSUBDA(MKS)=     54.6000
```

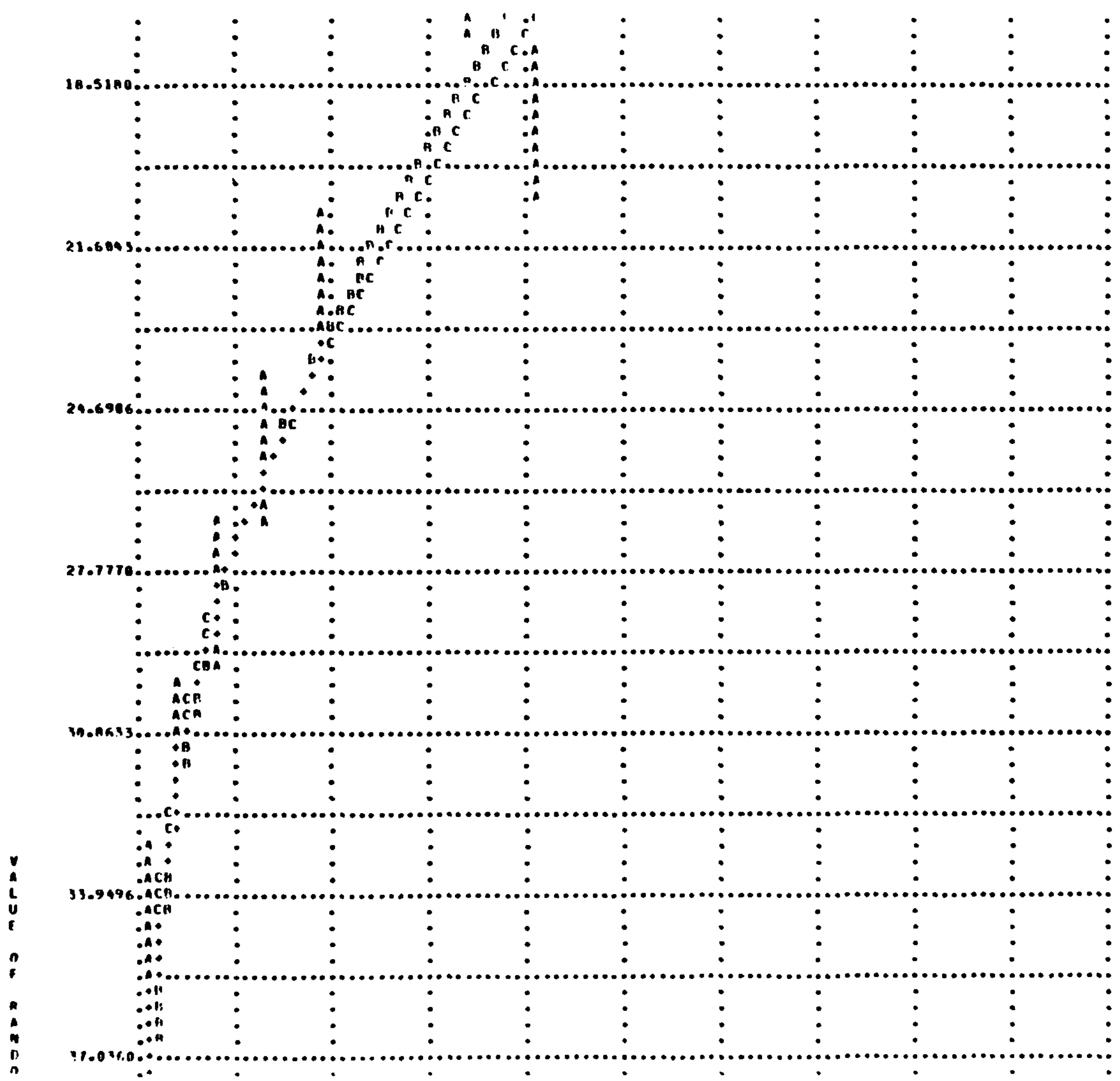
L= 1638.0

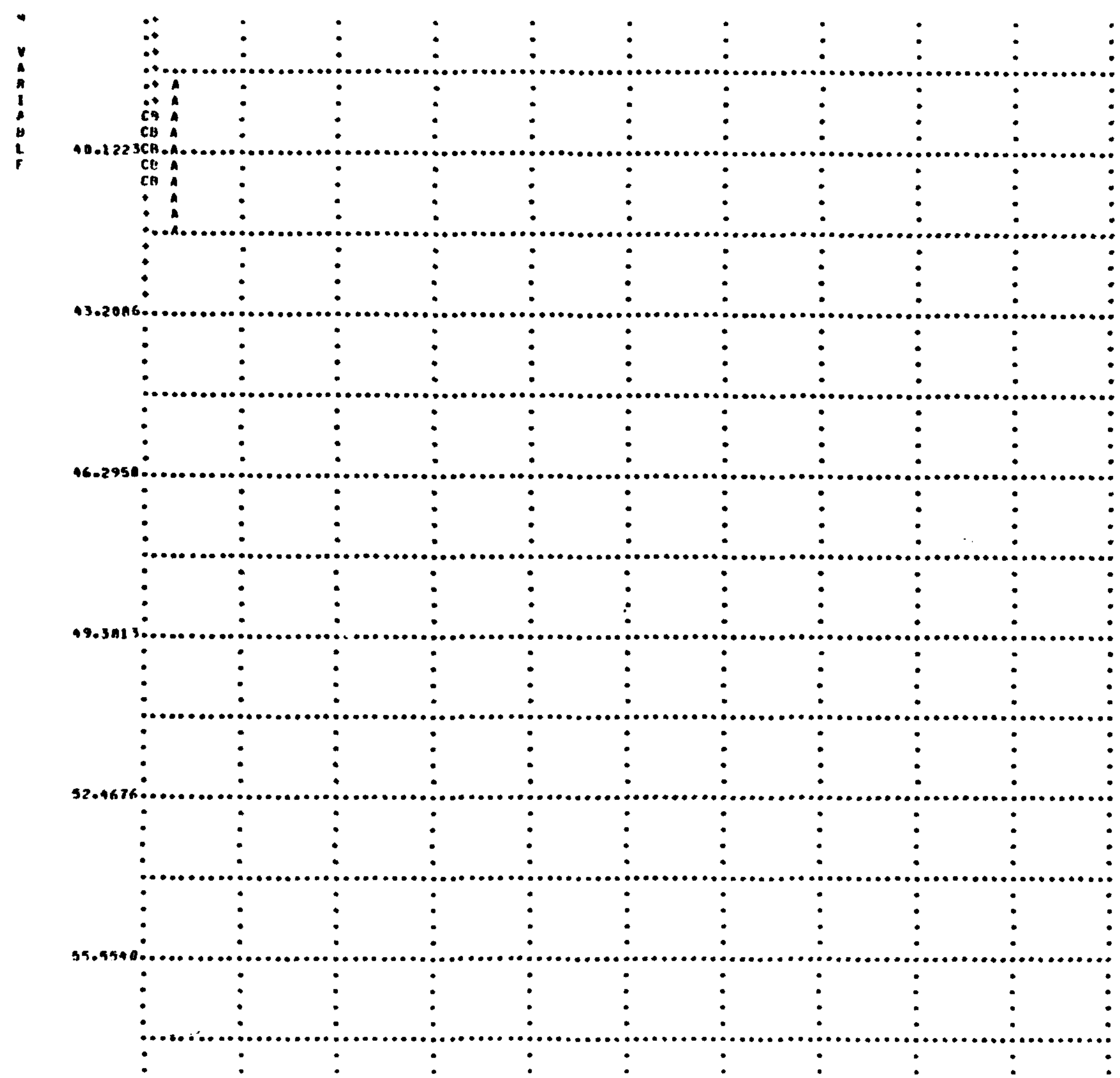
XSUBM=	10.0849	XSUBM(MKS)=	10.0849				
XSUBO=	15.9986	XSUBO(MKS)=	15.9986	TXSUBO=	15.9000	TXSUBO(MKS)=	15.9000
XSUBS=	22.1135	XSUBS(MKS)=	22.1135				
XSUBG=	44.5527	XSUBG(MKS)=	44.5527	TXSUBG=	44.7000	TXSUBG(MKS)=	44.7000
XSUBGA=	44.5440	XSUBGA(MKS)=	44.5440	TXSUBGA=	44.7000	TXSUBGA(MKS)=	44.7000
XSUBD=	57.3000	XSUBD(MKS)=	57.3000	TXSUBD=	57.3000	TXSUBD(MKS)=	57.3000
XSUBDA=	57.3002	XSUBDA(MKS)=	57.3002	TXSUBDA=	57.3000	TXSUBDA(MKS)=	57.3000

TIME(MIN)	TIME(HRS)	L VALUE	XSURD	XSURG	XSURD(MKS)	XSURG(MKS)
1.0000	.0167	272.0000	52.5000	38.4161	52.5000	38.4161
2.0840	.0347	546.0000	54.6000	40.9344	54.6000	40.9344
6.2519	.1042	1638.0000	57.3000	44.5527	57.3000	44.5527



OBSERVED ,GAMMA AND RAYLEIGH PLOTS
DENSITY (NUM./BIN)/N*BIN-WIDTH
RUN 1
PLOT 1
A=YHIST
B=YGAMM
C=YRAY
0.0000
.0200
.0400
.0600
.0800
.1000
X
0.0000
3.0863
6.1727
9.2590
12.3453
15.4317
VALUE OF RANDOM VARIABLE


18.5180
21.6843
24.6986
27.7778
30.8633
33.9496
37.0360
VALUE OF RANDO



VARIABLE
40.1223
43.2086
46.2958
49.3813
52.4676
55.5540

INITIAL GUESS FOR C=   2.00000

ITERATIONS FOR FINDING C ROOT

	C	F(C)
1	2.00000	-.056611
2	1.98360	-.054346
3	1.59004	-.014936
4	1.44087	-.006558
5	1.32413	-.002033
6	1.27169	-.000508
7	1.25423	-.000064
8	1.25173	-.000003
9	1.25173	-.000002

PARAMETERS ESTIMATED BY MAXIMUM LIKELYHOOD ESTIMATION

LAMBDA=   .038833   M= 1.758994   C= 1.251726

P22 H 30

CONVER= 1.000 CALIBRATION FACTOR= 0.000

N= 311 K= 15 ALPHA= .1000E-01 1-ALPHA= .9900

DEL ARRAY

D(1)12.0000 D(2)12.0000 D(3)12.0000 D(4)12.0000 D(5)12.0000 D(6)12.0000 D(7)12.0000 D(8)12.0000
D(9)12.0000 D(10)12.0000 D(11)12.0000 D(12)12.0000 D(13)12.0000 D(14)12.0000 D(15)12.0000

YBAR= 3.450 YBAR(MKS)= 3.450 S= .4901 T= -.8245

M= 1.759 C= 1.252 LAMPDA= .3883E-01 LAMBDA(MKS)= .3883E-01R= 2059. R(MKS)= 2059.

. VIA MAXIMUM LIKELIHOOD METHOD

VALUE OF RANDOM VARIABLE	VALUE OF RANDOM VARIABLE (MKS)	MEASURED FREQUENCY	RAYLEIGH PREDICTED FREQUENCY	GAMMA PREDICTED FREQUENCY	MEASURED DENSITY =/BIN/N	RAYLEIGH PREDICTED DENSITY	GAMMA PREDICTED DENSITY
6.0000	6.0000	.2799E+02	.2101E+02	.2809E+02	.7500E-02	.5630E-02	.7527E-02
18.0000	18.0000	.5486E+02	.5489E+02	.6607E+02	.1470E-01	.1471E-01	.1770E-01
30.0000	30.0000	.7912E+02	.6938E+02	.6981E+02	.2120E-01	.1859E-01	.1871E-01
42.0000	42.0000	.6494E+02	.6416E+02	.5593E+02	.1740E-01	.1719E-01	.1499E-01
54.0000	54.0000	.3807E+02	.4744E+02	.3847E+02	.1020E-01	.1271E-01	.1031E-01
66.0000	66.0000	.1903E+02	.2905E+02	.2390E+02	.5100E-02	.7783E-02	.6405E-02
78.0000	78.0000	.1418E+02	.1497E+02	.1378E+02	.3800E-02	.4012E-02	.3691E-02
90.0000	90.0000	.5971E+01	.6563E+01	.7483E+01	.1600E-02	.1758E-02	.2005E-02
102.0000	102.0000	.1866E+01	.2460E+01	.3871E+01	.5000E-03	.6592E-03	.1037E-02
114.0000	114.0000	.1866E+01	.7920E+00	.1921E+01	.5000E-03	.2122E-03	.5148E-03
126.0000	126.0000	.1866E+01	.2195E+00	.9197E+00	.5000E-03	.5882E-04	.2464E-03
138.0000	138.0000	0.	.5251E-01	.4264E+00	0.	.1407E-04	.1142E-03
150.0000	150.0000	0.	.1085E-01	.1921E+00	0.	.2908E-05	.5146E-04
162.0000	162.0000	0.	.1940E-02	.8428E-01	0.	.5199E-06	.2258E-04
174.0000	174.0000	.1120E+01	.3004E-03	.3611E-01	.3000E-03	.8050E-07	.9675E-05

```
RAYLEIGH DESIGN VALUES
L=    311.0000      XRSUBD=    145.9384      XRSUBD(MKS)=    145.9384
L=    833.0000      XRSUBD=    152.7299      XRSUPD(MKS)=    152.7299
L=   2499.0000      XRSUBD=    159.9632      XRSUPD(MKS)=    159.9632
```

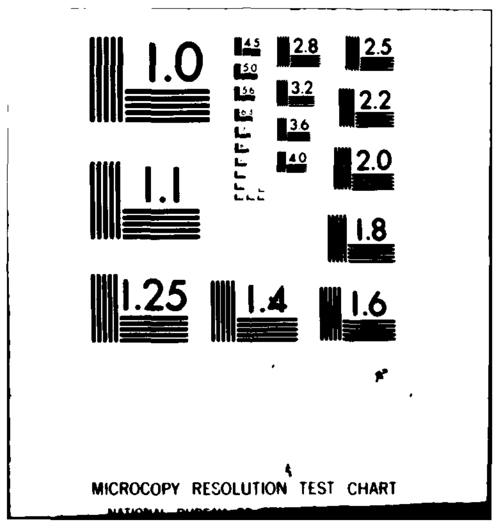
```
L=   311.0

XSUBM=      24.9272   XSUBM(MKS)=      24.9272
XSUBO=      44.9450   XSUBO(MKS)=      44.9450   TXSUBO=       44.4000   TXSUBO(MKS)=       44.4000
XSUBS=      65.4169   XSUBS(MKS)=      65.4169

XSUBG=     128.5468   XSUBG(MKS)=     128.5468   TXSUBG=      128.4000   TXSUBG(MKS)=      128.4000
XSUBGA=     128.5892  XSUBGA(MKS)=     128.5892  TXSUBGA=      128.4000  TXSUBGA(MKS)=      128.4000

XSUBD=     192.0000   XSUBD(MKS)=     192.0000   TXSUBD=      192.0000   TXSUBD(MKS)=      192.0000
XSUBDA=     192.0003  XSUBDA(MKS)=     192.0003  TXSUBDA=      192.0000  TXSUBDA(MKS)=      192.0000
```

```
L=   833.0

XSUBM=      24.9272   XSUBM(MKS)=     24.9272
XSUBO=      44.9450   XSUBO(MKS)=     44.9450   TXSUBO=     44.4000   TXSUBO(MKS)=     44.4000
XSUBS=      65.4169   XSUBS(MKS)=     65.4169

XSUBG=     143.0865   XSUBG(MKS)=    143.0865   TXSUBG=    142.8000   TXSUBG(MKS)=    142.8000
XSUBGA=     143.0789  XSUBGA(MKS)=    143.0789  TXSUBGA=    142.8000  TXSUBGA(MKS)=    142.8000

XSUBD=     205.2000   XSUBD(MKS)=    205.2000   TXSUBD=    205.2000   TXSUBD(MKS)=    205.2000
XSUBDA=     205.2000  XSUBDA(MKS)=    205.2000  TXSUBDA=    205.2000  TXSUBDA(MKS)=    205.2000
```

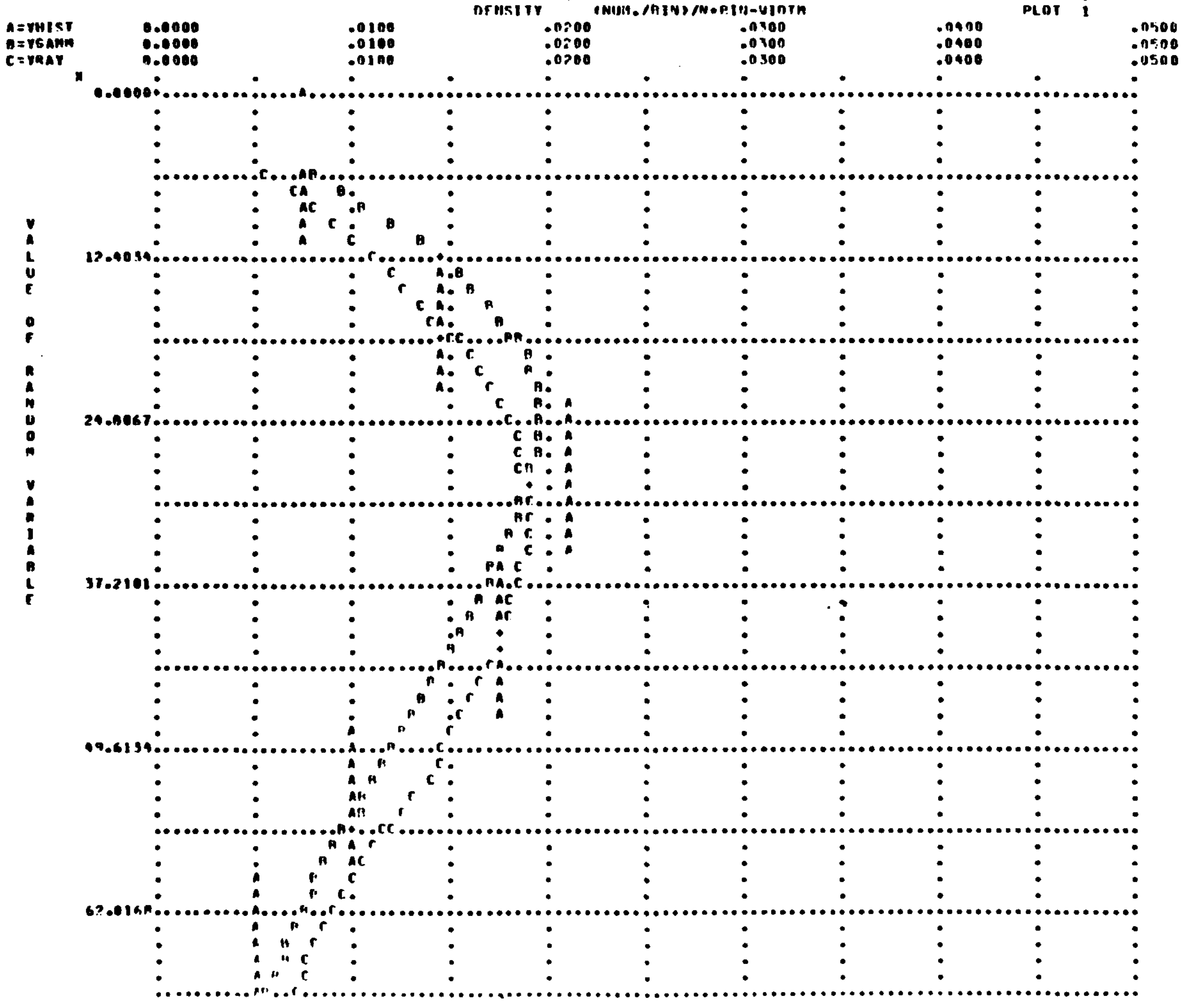
L= 2499.0

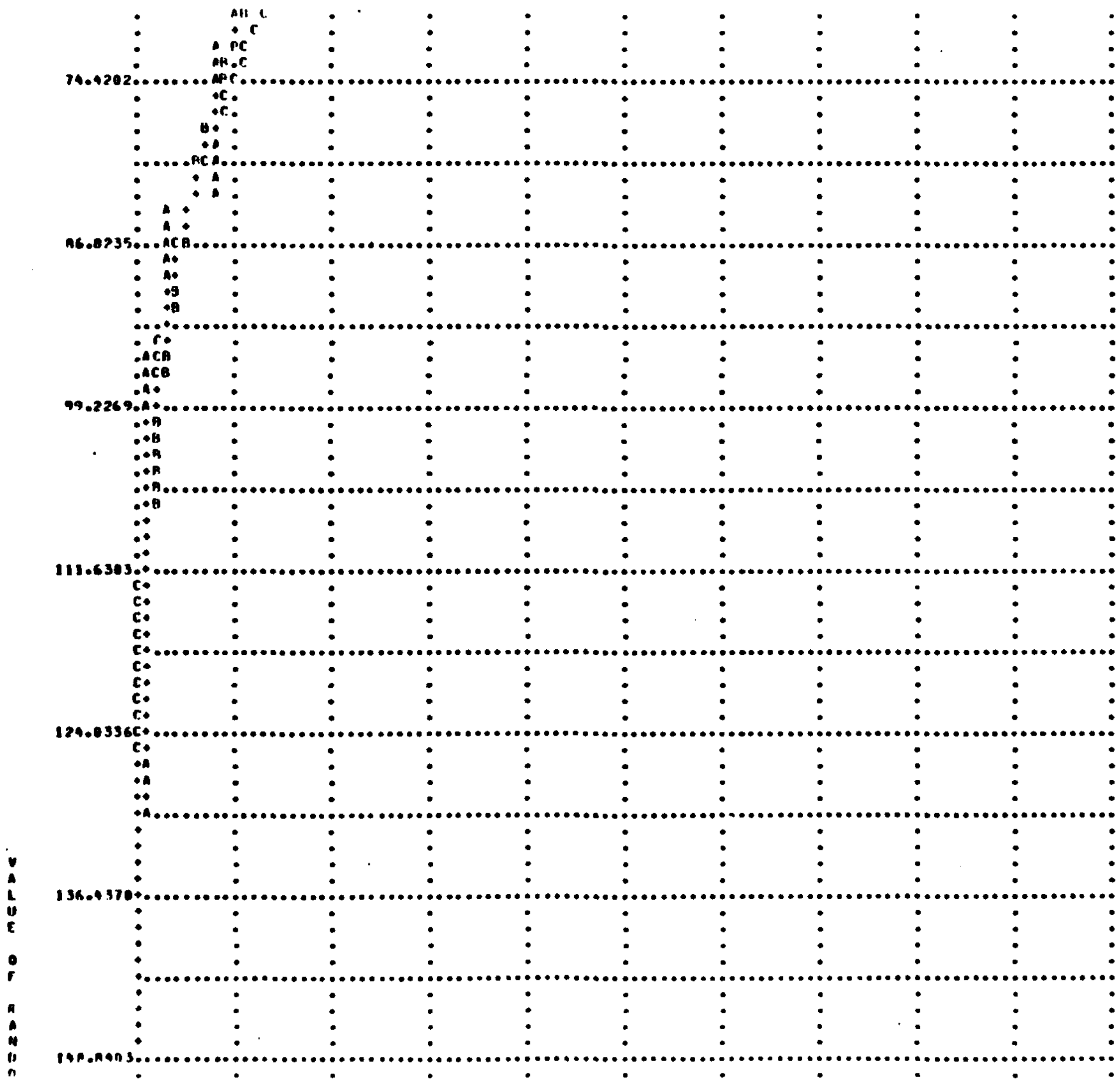
XSUBM=	24.9272	XSUBM(MKS)=	24.9272				
XSUBO=	44.9450	XSUBO(MKS)=	44.9450	TXSUBO=	44.4000	TXSUBO(MKS)=	44.4000
XSUPS=	65.4169	XSUBS(MKS)=	65.4169				
XSUBG=	158.6699	XSUBG(MKS)=	158.6699	TXSUBG=	158.4000	TXSUBG(MKS)=	158.4000
XSUBGA=	158.6588	XSUBGA(MKS)=	158.6588	TXSUBGA=	158.4000	TXSUBGA(MKS)=	158.4000
XSUBD=	219.6000	XSUBD(MKS)=	219.6000	TXSUBD=	219.6000	TXSUBD(MKS)=	219.6000
XSUBDA=	219.5997	XSUBDA(MKS)=	219.5997	TXSUBDA=	219.6000	TXSUBDA(MKS)=	219.6000

TIME(MIN)	TIME(HRS)	L VALUE	XSUBD	XSUBG	XSUBD(MKS)	XSUBG(MKS)
1.0000	.0167	311.0000	192.0000	128.5468	192.0000	128.5468
2.6785	.0446	873.0000	205.2000	143.0865	205.2000	143.0865
8.0354	.1339	2499.0000	219.6000	158.6699	219.6000	158.6699

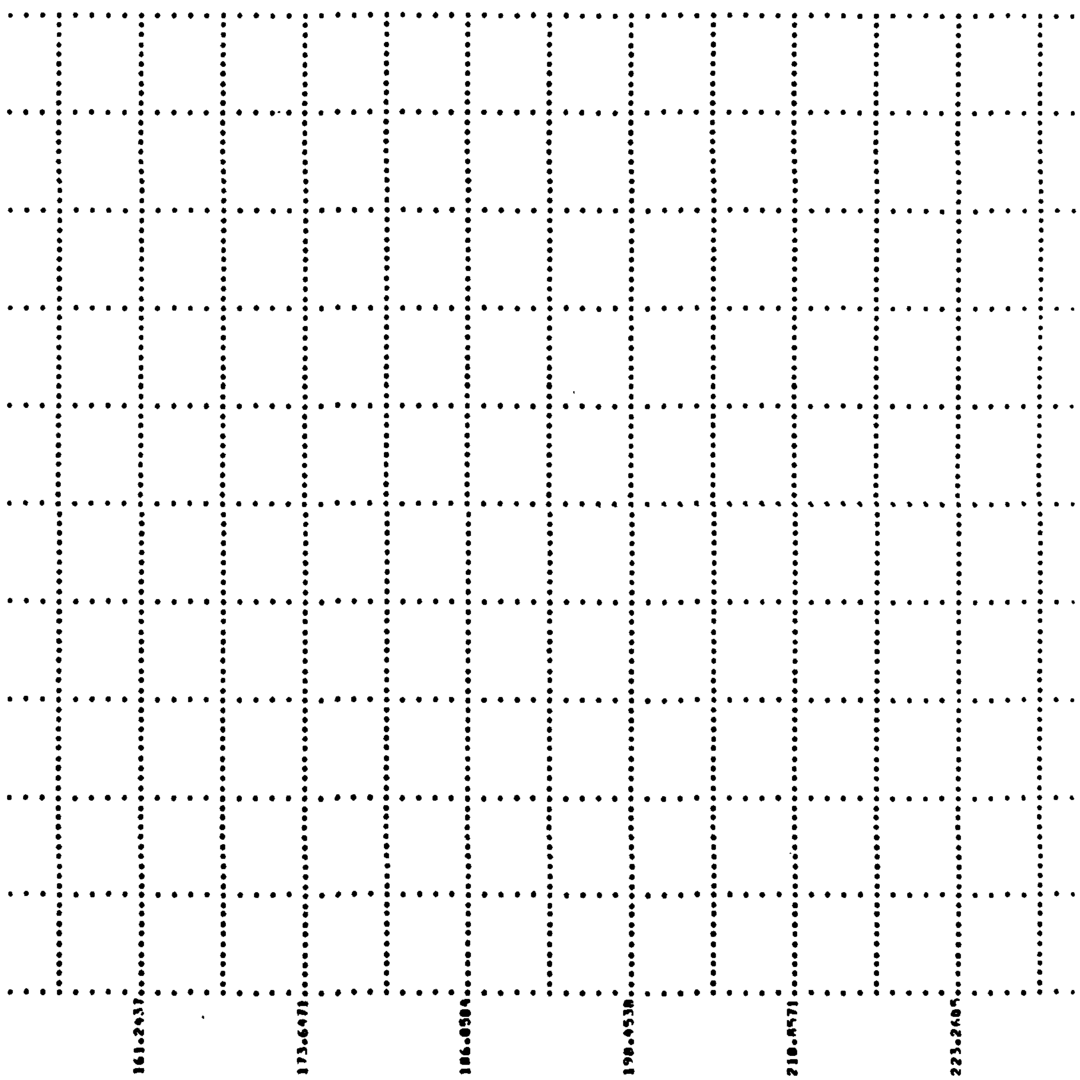

1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6
MICROCOPY RESOLUTION TEST CHART

OBSERVED ,GAMMA AND RAYLEIGH PLOTS





74.4202
86.8235
99.2269
111.6303
124.0336
136.4370
148.8403
VALUE OF RANDO


161.2437
173.6471
186.0504
198.4538
210.8571
223.2605

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