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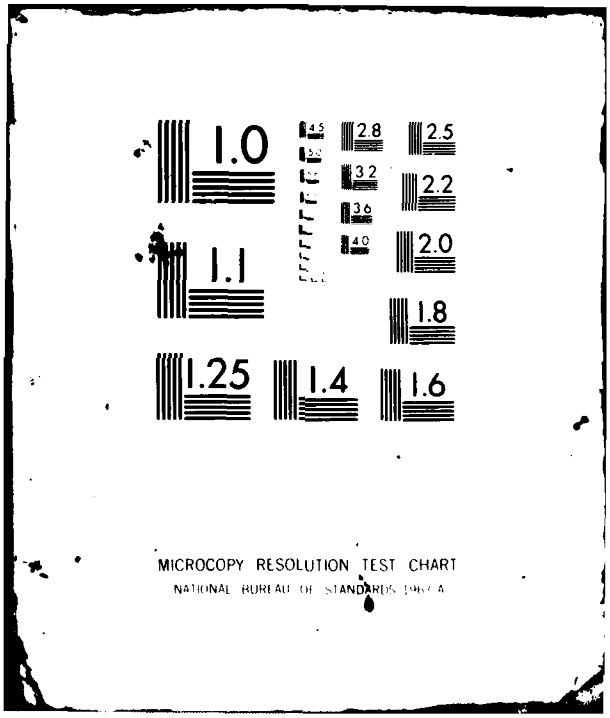
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QUALITATIVE TEXTURE DISCRIMINATION

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ABSTRACT

This paper investigates a set of features termed higher-order crossings for texture analysis. A general discrimination procedure is proposed based on statistics derived from sequential linear filters followed by a clipping operator, and the usefulness of this procedure in discrimination of texture fields is demonstrated.

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1. Introduction

Given a texture field, what are the more conspicuous features which help us in recognition and discrimination among textures? This is a fundamental question for which there are numerous ad hoc suggestions and some conjectures but not yet a universally accepted answer. For example, the famous conjecture of Julesz (1962) says that second order probability distributions defined over a field are sufficient for discrimination. However the recent work by Gagalowicz (1981) seems to indicate that this conjecture is incorrect and a different conjecture is made there as to the sufficiency of spatial averages. We believe that our ability to perceive is much more complex and consequently this paper does not suggest any set of statistics sufficient for discrimination in general. On the other hand, even though such statistics are difficult to come by, still it is very probable that some statistics are more useful than others and some are not useful at all, being either redundant or else degenerate in some sense, the idea being that our perception consumes or filters out the "useful" pictorial information and leaves unnoticed degenerate or redundant information. However, as our analysis of discrimination is carried out via mathematical means it probably yields necessary rather than sufficient conclusions.

The purpose of this work is to investigate the usefulness of some features termed higher order crossings in texture discrimination. Although we are not sure as to how the human perception operates in its fine and complex details and hence cannot model it precisely, certain coarse observations can nontheless be made. First, the visual system is capable of noticing simple changes or marked shifts in texture fields. For example, as the intensity of grey levels varies across a field, the human observer is capable of observing certain simple cyclical patterns or movements of rising and falling intensities caused by clusters of like grey levels. Second, discrimination is done on a qualitative basis in the sense that recognition is done by detecting relative changes of magnitude rather than by actual measurements. Third, texture is shift invariant. That is, our ability to recognize a texture field is not affected, or nearly so, by rigid shifts of this field.

Taking these observations into account, it is possible to construct a crude discrimination procedure as follows. If $\{Z\}$ is our texture field, then the shift invariance property suggests that the assumption that $\{Z\}$ is a stationary random field is reasonable. The cyclical movements and changes observed in a field can in many cases be detected by a linear filter or the successive application of linear filters. The qualitative information in a random field can very well be detected by applying to the filtered field the nonlinear clipping operator. At the final stage of this program comes the construction of powerful discrimination measures in the form of test statistics. If S_i is the i^{th} filter applied to the field $\{Z\}$, U the clipping operator, and φ the discrimination statistic, our coarse scheme is symbolically represented by

$$\varphi(U \circ S_{k-1} \circ \cdots \circ S_1 \circ S_0\{Z\}), \quad k = 1,2,\ldots .$$

The crossings of level 0 by $S_{k-1} \circ \cdots \circ S_0\{Z\}$ are called the higher order crossings of order k. Obviously this information is qualitative and is retained by the clipped filtered field. Our business then is to investigate the usefulness of the higher order crossings so defined in the discrimination of texture fields.

We note from the outset that apart from some rigorous results, this work is more of a suggestion for future work rather than a conclusive and finished work. In fact we shall give only one concrete example of φ and even this is incomplete.

2. The Effect of Clipping on Sequential Linear Filtering: The One Dimensional Case

Let $\{Z_t\}_{t=-\infty}^{\infty}$ be a zero mean stationary process with spectral representation

$$Z_t = \int_{-\pi}^{\pi} e^{it\lambda} d\xi(\lambda),$$

where $E|d\xi(\lambda)|^2 = f(\lambda)d\lambda$, f being the spectral density function of the process. Let B be the backwards shift operator,

$$BZ_t = Z_{t-1}$$

and let U be the clipping operator,

$$U Z_t = \begin{cases} 1, & Z_t \geq 0 \\ 0, & Z_t < 0 \end{cases} .$$

As a polynomial in B is well defined, we shall be interested in the operation

$$U(1-B)^m(1+B)^n, \tag{1}$$

applied to $\{Z_t\}$ for $m,n = 0,1,2,...,$ such that $m/n = c$ constant. Observe that U is insensitive to scale and that the effect of (1) on $\{Z_t\}$ is equivalently recorded by applying U to

$$Y_t^{(n)} = (1-B)^{cn}(1+B)^n\{Z_t/\sigma_n\}$$

where $\sigma_n > 0$ is defined so that $\text{Var}(Y_t^{(n)}) = 1$. Strictly speaking $\{Y_t^{(n)}\}$ depends on c but this is suppresed for the sake of simplified notation. Let the transfer function of

$(1-B)^{cn}(1+B)^n$ be denoted by $H_n(\lambda)$, again suppressing c. Then

$$H_n(\lambda) = (1-e^{-i\lambda})^{cn}(1+e^{-i\lambda})^n,$$

and it follows that the spectral density of $\{Y_t^{(n)}\}$ is given by

$$f_n(\lambda) = \frac{|H_n(\lambda)|^2 f(\lambda)}{\int_{-\pi}^{\pi} |H_n(\omega)|^2 f(\omega)d\omega}, \quad -\pi \le \lambda \le \pi .$$

We note that the squared gain $|H_n(\lambda)|^2$ is symmetric and

(i) $$|H_n(\lambda)|^2 = 2^{cn+n}(1-\cos\lambda)^{cn}(1+\cos\lambda)^n$$

(ii) For $0 \le \lambda \le \pi$, the squared gain is unimodal with a peak occurring at λ_c, say. We have

$$\max_{0\le\lambda\le\pi} |H_n(\lambda)|^2 = H_n(\lambda_c)$$

where $\lambda_c = \cos^{-1}\left(\frac{1-c}{1+c}\right)$.

(iii) For sufficiently small $\varepsilon > 0$

$$\frac{|H_n(\lambda_c - \varepsilon)|^2}{|H_n(\lambda_c - \varepsilon/2)|^2} \to 0, \quad n \to \infty,$$

and the same holds if a plus replaces the minus sign.

(iv) $$|H_n(\lambda_c)|^2 < |H_{n+1}(\lambda_c)|^2.$$

Associated with f_n is a spectral measure $\nu_n(\cdot)$ defined on $(-\pi,\pi]$ by

$$\nu_n(\Lambda) = \int_\Lambda f_n(\lambda)d\lambda, \quad \Lambda \subset (-\pi,\pi],$$

Λ being a Borel set. We first study the limit of ν_n.

Proposition 1: Assume $f(\lambda)$, the spectral density of $\{Z_t\}$, does *not* vanish at $\lambda = \lambda_c$. Then

$$\nu_n \Rightarrow \tfrac{1}{2}\delta_{-\lambda_c} + \tfrac{1}{2}\delta_{\lambda_c}$$

where λ_c is given in (ii) above, and δ_u is the unit point mass at u.

Proof: First note that ν_n is symmetric on $(-\pi,\pi]$. Now for $\varepsilon > 0$

$$\nu_n[0,\lambda_c-\varepsilon) = \frac{\int_0^{\lambda_c-\varepsilon} |H_n(\lambda)|^2 f(\lambda)d\lambda}{\int_{-\pi}^{\pi} |H_n(\lambda)|^2 f(\lambda)d\lambda}$$

$$\le \frac{\int_0^{\lambda_c-\varepsilon} |H_n(\lambda)|^2 f(\lambda)d\lambda}{\int_{\lambda_c-\varepsilon/2}^{\lambda_c} |H_n(\lambda)|^2 f(\lambda)d\lambda}$$

$$\le \frac{|H_n(\lambda_c-\varepsilon)|^2}{|H_n(\lambda_c-\varepsilon/2)|^2} \cdot \frac{\int_0^{\lambda_c-\varepsilon} f(\lambda)d\lambda}{\int_{\lambda_c-\varepsilon/2}^{\lambda_c} f(\lambda)d\lambda} \to 0, \quad n\to\infty .$$

Similarly

$$\nu_n(\lambda_c+\varepsilon,\pi] \le \frac{\int_{\lambda_c+\varepsilon}^{\pi} |H_n(\lambda)|^2 f(\lambda)d\lambda}{\int_{\lambda_c}^{\lambda_c+\varepsilon/2} |H_n(\lambda)|^2 f(\lambda)d\lambda}$$

$$\le \frac{|H_n(\lambda_c+\varepsilon)|^2}{|H_n(\lambda_c+\varepsilon/2)|^2} \cdot \frac{\int_{\lambda_c+\varepsilon}^{\pi} f(\lambda)d\lambda}{\int_{\lambda_c}^{\lambda_c+\varepsilon/2} f(\lambda)d\lambda} \to 0, \quad n \to \infty.$$

By symmetry then

$$\nu_n([-\lambda_c-\varepsilon,-\lambda_c+\varepsilon] \cup [\lambda_c-\varepsilon,\lambda_c+\varepsilon]) \to 1, \quad n \to \infty$$

and

$$\nu_n \Rightarrow \tfrac{1}{2}\delta_{-\lambda_c} + \tfrac{1}{2}\delta_{\lambda_c}, \quad n \to \infty. \qquad \text{Q.E.D.}$$

<u>Corollary</u>: $Y_t^{(n)}$ converges in mean square to a cosinusoid with frequency λ_c.

<u>Proof</u>: By the above argument,

$$E\left|\int_{\lambda \neq \pm\lambda_c} e^{it\lambda} \left\{\frac{H_n(\lambda)}{\left(\int_{-\pi}^{\pi} |H_n(\omega)|^2 f(\omega)d\omega\right)^{\frac{1}{2}}}\right\} d\xi(\lambda)\right|^2 \to 0, \quad n \to 0.$$

Q.E.D.

Now, define a binary process $\{X_t^{(n)}\}_{t=-\infty}^{\infty}$ by

$$X_t^{(n)} = \cup Y_t^{(n)}.$$

Proposition 2: Let $j \geq 1$ and define

$$c = \frac{1 - \cos(\pi/j)}{1 + \cos(\pi/j)} .$$

Without loss of generality let κ be an odd positive integer. Then if $f(\lambda)$ does not vanish at λ_c

$$(X_t^{(n)}, X_{t+j}^{(n)}, X_{t+2j}^{(n)}, \ldots, X_{t+\kappa j}^{(n)})$$

$$\xrightarrow{P} \begin{cases} 1\ 0\ 1\ 0\ \cdots\ 1\ 0 & \text{w.p. } \tfrac{1}{2} \\ 0\ 1\ 0\ 1\ \cdots\ 0\ 1 & \text{w.p. } \tfrac{1}{2} \end{cases}, \quad n \to \infty .$$

Proof: From Proposition 1, since the complex exponential is bounded, we have for a positive integer j as $n \to \infty$

$$\mathrm{Corr}(Y_t^{(n)}, Y_{t+j}^{(n)}) = \int_{-\pi}^{\pi} e^{ij\lambda} \nu_n(d\lambda)$$

$$\to \frac{1}{2}(e^{-ij\lambda_c} + e^{ij\lambda_c}) = \cos(j\lambda_c) = -1,$$

because

$$\lambda_c = \cos^{-1}\left(\frac{1-c}{1+c}\right) = \frac{\pi}{j} .$$

Therefore

$$\Pr(X_t^{(n)} \neq X_{t+j}^{(n)}) \to 1, \quad n \to \infty,$$

and invoke stationarity. Q.E.D.

It is instructive to consider some special cases of this last proposition.

Example 2.1: For $c = 1$ we have as $n \to \infty$

$$\begin{aligned} \mathrm{Corr}(Y_t^{(n)}, Y_{t+j}^{(n)}) &\to \cos(j\pi/2) \\ &= 0, \quad j = 1 \\ &= -1, \quad j = 2 \\ &= 0, \quad j = 3 \\ &= 1, \quad j = 4. \end{aligned}$$

It follows that the limiting binary series has the general form

1 - 0 - 1 - 0 - 1 - 0 - 1 - 0

or

0 - 1 - 0 - 1 - 0 - 1 - 0 - 1

where "-" stands for a single space which may contain either 0 or 1. In particular if $\{Z_t\}$ is Gaussian the "empty space" is independent of its neighbors. This procedure leads to the following table where an empty space contains either 0 or 1.

c	Limiting form of a finite series from $\{X_t^{(n)}\}$
0	1 1 1 1 · · · 1 w.p. ½ or 0 0 0 0 · · · 0 w.p. ½
1/3	1 - - 0 - - 1 - - 0 - - 1 - - 0 w.p. ½ or 0 - - 1 - - 0 - - 1 - - 0 - - 1 w.p. ½
1	1 - 0 - 1 - 0 - 1 - 0 - 1 - 0 w.p. ½ or 0 - 1 - 0 - 1 - 0 - 1 - 0 - 1 w.p. ½
∞	1 0 1 0 1 0 1 0 w.p. ½ or 0 1 0 1 0 1 0 1 w.p. ½

3. Higher Order Crossings

In this section we specialize the transformation (1) to the case $n = 0$. As this means that repeated differencing is applied to $\{Z_t\}$ this corresponds in the limit to $c = \infty$. So, with a slight change of notation, we consider the sequence of binary processes $\{X_t^{(k)}\}$ defined by

$$X_t^{(k)} = U\,(1-B)^{k-1}Z_t, \qquad t = 0,\pm 1,\ldots, \qquad k = 1,2,\ldots\ .$$

Then $\{X_t^{(k)}\}$ retains the axis-crossings information of $\{(1-B)^{k-1}Z_t\}$. These axis-crossings are referred to as the higher order crossings of order k, and their number, $D_{k,N}$, in a series of length N is given by

$$\begin{aligned} D_{k,N} &\equiv \sum_{t=1}^{N-1} \chi_{[X_{t+1}^{(k)} \neq X_t^{(k)}]} \\ &= 2\sum_{t=1}^{N} X_t^{(k)} - 2\sum_{t=2}^{N} X_t^{(k)}X_{t-1}^{(k)} - (X_1^{(k)} + X_N^{(k)}). \end{aligned} \tag{2}$$

Proposition 2 suggests that the sequence $\{D_{k,N}\}_k$ eventually increases. That this is indeed the case under suitable conditions is due in part to the fact that

$$D_{k+1,N} \geq D_{k,N} - 1 \qquad \text{with probability 1,}$$

and in part to the sequential application of a high pass filter. This fact is most conveniently proved for the Gaussian case and we shall do so. We shall also determine the approximate rate of increase in $D_{k,N}$ for large k,N.

<u>Proposition 3</u>: Assume $\{Z_t\}$ is a zero mean Gaussian stationary process with correlation function $\{\rho_k\}$ and spectral density f which does not vanish at π. If $\sum |\rho_k| < \infty$ then

$$\text{(i)} \qquad \lim_{N\to\infty} \frac{D_{k,N}}{N-1} > \lim_{N\to\infty} \frac{D_{k-1,N}}{N-1} \qquad \text{w.p. 1.}$$

Let

$$g(x) = \frac{1}{2} + \frac{1}{\pi}\sin^{-1}\left(\frac{-x}{x+1}\right), \qquad x \geq 0.$$

Then for sufficiently large N and k the rate of increase of $D_{k,N}/(N-1)$ is approximately determined by

$$\text{(ii)} \qquad \frac{ED_{k+1}}{N-1} \simeq 1 - g(0+)\exp\left[-\int_{0+}^{k} r(s)ds\right]$$

where

$$r(s) = \frac{\frac{1}{\pi}\frac{d}{ds}\left(\frac{-s}{s+1}\right)}{\left(1-\left(\frac{s}{s+1}\right)^2\right)^{\frac{1}{2}}\left\{\frac{1}{2}+\frac{1}{\pi}\sin^{-1}\left(\frac{-s}{s+1}\right)\right\}}, \qquad s \geq 0.$$

<u>Proof</u>: To prove (i) it is sufficient to consider the case $k = 2$. Let $\rho_\nabla(1)$ be the first correlation of $\{(1-B)Z_t\}_{t=-\infty}^{\infty}$. Now by stationarity

$$1 - 2\rho_1^2 + \rho_2 = \begin{vmatrix} 1 & \rho_1 & \rho_2 \\ \rho_1 & 1 & \rho_1 \\ \rho_2 & \rho_1 & 1 \end{vmatrix} \Big/ (1-\rho_2) > 0,$$

and so

$$\rho_1 - \rho_\nabla(1) = \frac{1 - 2\rho_1^2 + \rho_2}{2(1-\rho_1)} > 0,$$

or

$$\rho_\nabla(1) < \rho_1.$$

(This last inequality reconfirms Proposition 2 for the case $c = \infty$.) Since $\{Z_t\}$ is Gaussian with mean 0

$$\Pr(Z_t > 0 \mid Z_{t-1} > 0) = \frac{1}{2} + \frac{1}{\pi}\sin^{-1}(\rho_1).$$

From this and (2)

$$ED_{1,N} = (N-1)(\tfrac{1}{2} - \tfrac{1}{\pi}\sin^{-1}(\rho_1))$$
$$ED_{2,N} = (N-1)(\tfrac{1}{2} - \tfrac{1}{\pi}\sin^{-1}(\rho_\nabla(1))) .$$

(Note that the minus sign is correct.) But the arcsine is monotone increasing which implies

$$ED_{2,N} > ED_{1,N} ,$$

and in general

$$ED_{k,N} > ED_{k-1,N} .$$

This, (2), and the fact that the absolute summability of $\{\rho_k\}$ implies the ergodicity of $\{X_t^{(k)}\}$ [3 , p. 205] implies (i).

To prove (ii), observe that the density of $\{Y_t^{(k)}\}$ has the form

$$b_k|1 - e^{-i\lambda}|^{2k} f(\lambda)$$

where b_k is a normalizing constant. As in this expression $f(\lambda)$ is independent of k it plays no role as k increases. Thus for sufficiently large k if we replace $f(\lambda)$ by a positive constant the rate of convergence to $\{\cdots\ 0\ 1\ 0\ 1\ \cdots\}$ as k increases will not be greatly affected. This means that the rate of increase in $D_{k,N}$ for large k is approximately the same as that for white noise. Therefore from (2)

$$\frac{ED_{k+1,N}}{N-1} \simeq 1 - g(k), \qquad k = 0,1,2,\ldots .$$

Now, formally replace k in $g(k)$ by real positive x and note that $g(x)$, $x > 0$, is continuous and differentiable. We can now define

$$r(x) = -\, g'(x)/g(x), \qquad x > 0,$$

and this is explicitly given in (ii). It follows that

$$g(x) = g(0+)\exp[-\int_{0+}^{x} r(s)ds]. \qquad \text{Q.E.D.}$$

It should be noted that (ii) has been verified experimentally as well in numerous simulations.

4. A Two Dimensional Higher Order Crossings Theorem

In this section we extend the notion of higher order crossings to two dimensional stationary random fields. We only deal with the transformation

$$U\ (1-B_{t_1})^k(1-B_{t_2})^\ell\ Z(t_1,t_2) \tag{3}$$

where $(1-B_{t_1})Z(t_1,t_2) = Z(t_1,t_2) - Z(t_1-1,t_2)$ and similarly for $(1-B_{t_2})$. By two dimensional higher order crossings we mean the horizontal and vertical symbol changes in the infinite binary array (3).

Let $\{Z(t_1,t_2),\ t_1,t_2 = 0,\pm1,\ldots\}$ be a stationary zero mean random field. Then [3] the field admits the spectral representation

$$Z(t_1,t_2) = \int_{I^2} e^{i(t_1\lambda_1 + t_2\lambda_2)} d\xi(\lambda_1,\lambda_2), \quad I = (-\pi,\pi],$$

where $E|d\xi(\lambda_1,\lambda_2)|^2 = f(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2$, $f(\lambda_1,\lambda_2)$ being the spectral density of the field. Above it was shown that the number of higher order crossings in the one dimensional case increases for long time series, and also that repeated sequential differencing and clipping yields realizations of the form $\{\cdots\ 0\ 1\ 0\ 1\ 0\ 1\ \cdots\}$. The extension of these results to two dimensional random fields is not automatic due to the geometry of two dimensional arrays.

We shall now make the last assertion precise via an example. For convenience we introduce the notation

$$\nabla = 1-B.$$

Define a binary random field $\{X^{(k)}(t_1,t_2)\}$ by

$$X^{(k)}(t_1,t_2) = U\,\nabla_{t_1}^{k-1}\nabla_{t_2}^{k-1}Z(t_1,t_2), \quad t_1,t_2 = 0,\pm 1,\ldots .$$

Let $D_{k,MN}$ denote the number of symbol changes (higher order crossings) in a finite two dimensional time series from $\{X^{(k)}(t_1,t_2)\}$ such that $1 \le t_1 \le M$, $1 \le t_2 \le N$. Recall that in the one dimensional case $D_{k,N} \ge D_{k-1,N} - 1$ with probability one and in fact surely! This is due to the geometry of the series and not to any probabilistic argument. In two dimensional arrays the geometry is more complex and the number of higher order crossings may actually decrease sharply.

Example 4.1.

(a)

$$t_2 \quad \overset{Z}{\begin{array}{c|ccc|}
\cline{2-4}
4 & 0 & 2 & 4 \\
8 & -3 & 10 & 5 \\
6 & 4 & 1 & -2 \\
\cline{2-4}
1 & -2 & 5 & 7
\end{array}} \quad \rightarrow \quad \overset{\nabla_{t_1}\nabla_{t_2}Z}{\begin{array}{|ccc|}
\hline
7 & -11 & 7 \\
-9 & 16 & -2 \\
1 & -10 & -5 \\
\hline
\end{array}}$$

$$t_1$$

$$\underset{D_{1,3\times 3} = 5}{\overset{U\,Z}{\begin{array}{|ccc|}
\hline
1 & 1 & 1 \\
0 & 1 & 1 \\
1 & 1 & 0 \\
\hline
\end{array}}} \quad \rightarrow \quad \underset{D_{2,3\times 3} = 10}{\overset{U\,\nabla_{t_1}\nabla_{t_2}Z}{\begin{array}{|ccc|}
\hline
1 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
\hline
\end{array}}}$$

(b)

Z

t_2				
	-4	-15	2	4
	16	- 3	10	5
	25	4	1	-5
	40	12	7	0
		t_1		

→

$\nabla_{t_1}\nabla_{t_2} Z$

t_2			
	8	4	7
	2	16	1
	7	2	1
		t_1	

$U\ Z$

0	1	1
0	1	1
1	1	0

$D_{1,3\times 3} = 5$

→

$U\ \nabla_{t_1}\nabla_{t_2} Z$

1	1	1
1	1	1
1	1	1

$D_{2,3\times 3} = 0$

From the example it is evident that $D_{k,MN}$ may either decrease or increase with k. It is seen that by differencing a clear peak (trough) the number of higher order crossings increases while a crest or a ridge may sharply decrease this number for given M,N. Thus we conclude that the increase or decrease in the $D_{k,MN}$ is largely due to the <u>shape</u> or <u>appearance</u> of the field. However, we shall show that under appropriate probabilistic conditions the behavior of higher order crossings in two dimensional fields mimics the one dimensional case.

Theorem 1. Let $\{Z(t_1,t_2)\}$ be a non-constant zero mean stationary random field with spectral density $f(\lambda_1,\lambda_2)$ which does not vanish at (π,π). Then

$$\{U\, \nabla_{t_1}^{k} \nabla_{t_2}^{\ell} Z(t_1,t_2)\} \Rightarrow \{\underset{\sim}{a},\underset{\sim}{a}'\}, \quad k,\ell \to \infty,$$

where "$\Rightarrow$" means weak convergence,

$$\underset{\sim}{a} = \begin{matrix} & & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & & \\ & & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & & \\ & & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & 1 & 0 & 1 & 0 & 1 & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 0 & 1 & 0 & 1 & 0 & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & 0 & 1 & 0 & 1 & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 0 & 1 & 0 & 1 & 0 & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & 0 & 1 & 0 & 1 & 0 & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & & \\ & & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & & \\ & & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & & \end{matrix}, \quad \text{with probability } 1/2,$$

and $\underset{\sim}{a}'$ is the same array as $\underset{\sim}{a}$ shifted to the right once.

Proof: The proof is entirely analogous to the one dimensional case. First let $j = \min(k,\ell)$ and define the symmetric kernel

$$H(\lambda) = |1 - e^{-i\lambda}|^2, \quad -\pi < \lambda \le \pi.$$

From 0 to π it is monotonically increasing. Since we are just interested in knowing whether $\nabla_{t_1}^{k} \nabla_{t_2}^{k} Z(t_1,t_2)$ is above or below 0 we can normalize this process by a positive quantity with no loss of generality. Define

$$Z^{(k)}(t_1,t_2) = \frac{\nabla_{t_1}^{k} \nabla_{t_2}^{k} Z(t_1,t_2)}{(\operatorname{Var} \nabla_{t_1}^{k} \nabla_{t_2}^{k} Z(t_1,t_2))^{\frac{1}{2}}}.$$

The corresponding spectral density is

$$f_k(\lambda_1,\lambda_2) = \frac{h^k(\lambda_1)h^k(\lambda_2)f(\lambda_1,\lambda_2)}{\int_{I^2} h^k(\omega_1)h^k(\omega_2)f(\omega_1,\omega_2)d\omega_1 d\omega_2},$$

with spectral measure $\mu_k(\Lambda)$. As in the one dimensional case we shall show that the spectral mass is "pushed" to the point (π,π). We have for $\varepsilon > 0$

$$\mu_k\{[-\pi+\varepsilon,\pi-\varepsilon]\times I\} \le \frac{h^k(\pi-\varepsilon)\int_I \int_{-\pi+\varepsilon}^{\pi-\varepsilon} h^k(\lambda_2)f(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2}{\int_I [\int_{-\pi}^{-\pi-\varepsilon/2} + \int_{\pi-\varepsilon/2}^{\pi}]h^k(\lambda_2)f(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2}$$

$$\le \left(\frac{h(\pi-\varepsilon)}{h(\pi-\varepsilon/2)}\right)^k \to 0, \quad k\to\infty,$$

since h is monotone increasing on $[0,\pi]$. Similarly

$$\mu_k\{I\times[-\pi+\varepsilon,\pi-\varepsilon]\} \to 0, \quad k\to\infty.$$

There are now four corners left intact. Consider the first near $(\pi,-\pi)$.

$$\mu_k\{[\pi-\varepsilon,\pi]\times(-\pi,-\pi+\varepsilon]\} \le \frac{h^k(-\pi^+)h^k(\pi)}{h^{2k}(\pi)f(\pi,\pi)(d\pi)^2}\int_{-\pi^+}^{-\pi+\varepsilon}\int_{\pi-\varepsilon}^{\pi} f(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2$$

$$\to 0, \quad k\to\infty,$$

where by "$d\pi$" we mean a sufficiently small quantity. Similarly for the corners $(-\pi,-\pi)$ and $(-\pi,\pi)$. It follows that

$$\mu_k \Rightarrow \delta_{(\pi,\pi)}, \quad k\to\infty,$$

where $\delta_{(\pi,\pi)}$ is the unit mass at (π,π).

This implies

$$\mathrm{Corr}(Z^{(k)}(t_1,t_2),Z^{(k)}(t_1+r_1,t_2+r_2))$$

$$= \int_{I^2} e^{i(r_1\lambda_1+r_2\lambda_2)} \underset{k}{\mu}(d\lambda_1,d\lambda_2) \to e^{i\pi(r_1+r_2)} = (-1)^{r_1+r_2},$$

as $k\to\infty$. Therefore

$\{X^{(k)}(t_1,t_2),\ 1\le t_1\le M,\ 1\le t_2\le N\} \xrightarrow{P}$ M N binary array in which each column and each row is of the form $\{\cdots 0\,1\,0\,1\,0\,1\cdots\}$.

Now consider $\Omega = \{0,1\}^{\mathbb{Z}^2}$, the space of all infinite binary arrays. In this space we introduce the product topology in which a neighborhood of $\underset{\sim}{Y}\in\Omega$ has the form

$$V_M(\underset{\sim}{Y}) = \{\underset{\sim}{X}\in\Omega \mid X(t_1,t_2)=Y(t_1,t_2) \ \text{ if } \ |t_1|,|t_2|\le M\}.$$

Then the last statement means that for each M,

$$\Pr(\{X^{(k)}(t_1,t_2),t_1,t_2=0,\pm 1,\ldots\} \notin V_M(\underset{\sim}{a}) \cup V_M(\underset{\sim}{a}')) \to 0, \quad k\to\infty.$$

It follows that the probability law of $\{X^k(t_1,t_2)\}$ converges to a measure supported on $\underset{\sim}{a}$ and $\underset{\sim}{a}'$ only:

$$L(\{X^{(k)}\}) \to \alpha\delta_{\underset{\sim}{a}} + (1-\alpha)\delta_{\underset{\sim}{a}'}, \qquad 0\le\alpha\le 1.$$

But as this measure is stationary and is invariant under the shift which carries $\underset{\sim}{a}$ into $\underset{\sim}{a}'$, $\alpha = 1/2$ and the proof is complete. Q.E.D.

From the proof it is clear that this theorem can be proved in exactly the same way for higher order random fields. In particular we have under some conditions

$$\mathrm{Corr}\left(\prod_{j=1}^{m} \nabla_{t_j}^{k} Z(\underset{\sim}{t}),\ \prod_{j=1}^{m} \nabla_{t_j}^{k} Z(\underset{\sim}{t}+\underset{\sim}{r})\right) \to (-1)^{\underset{\sim}{1}'\underset{\sim}{r}}, \quad k \to \infty,$$

where $\prod_{j=1}^{m} \nabla_{t_j}^{k} = \nabla_{t_1}^{k} \cdots \nabla_{t_m}^{k}$, $\underset{\sim}{t}, \underset{\sim}{r}$ are $m \times 1$ vectors of integers. The one dimensional version of Theorem 1 has been first proved in [6] and referred to as the higher order crossings theorem. We call Theorem 1 the two dimensional higher order crossings theorem.

Finally, the $D_{k,MN}$ increase with k for large M,N. To show this, it is sufficient to consider only one row in $\{X^{(k)}(t_1,t_2)\}$, $1 \le t_1 \le M$, $1 \le t_2 \le N$, $k = 1,2,\ldots$. Let $D_k(M;t_2^*)$ be the number of symbol changes in row t_2^* of $\{X^{(k)}(t_1,t_2)\}$, $1 \le t_2 \le N$, $1 \le t_1 \le M$. Then we have for the Gaussian case

Proposition 4: Let $\{Z(t_1,t_2)\}$ be a zero mean stationary Gaussian random field, with correlation function $\{\rho(s,t)\}$. If $\sum_{s,t} |\rho(s,t)| < \infty$, and if $\rho_{\nabla\nabla Z}(1) < \rho(1)$ then

$$\lim_{N\to\infty} \frac{D_k(M,t_2^*)}{M-1} > \lim_{N\to\infty} \frac{D_{k-1}(M,t_2^*)}{M-1}.$$

Proof: Follow the proof of Proposition 3, part (i).

5. Discrimination by Higher Order Crossings

Guided by our three observations in the introduction and by our theoretical results in the preceding sections we shall develop a discrimination procedure based on higher order crossings. In this section we apply the results of Theorem 1 and Proposition 4.

Recall that the higher order crossings are defined as the axis crossings by

$$S_{k-1} \circ \cdots \circ S_1 \circ S_0\{Z\}.$$

In this section we specialize the sequential filter to

$$\nabla_{t_1}^{k-1} \nabla_{t_2}^{k-1} Z(t_1, t_2), \quad k = 1, 2, \ldots$$

where $\{Z(t_1,t_2)\}$ is a stationary texture field and $\nabla_{t_1}\nabla_{t_2}$ can be thought of as a convolution operation applied to the field sequentially. For example

$$\nabla_{t_1}\nabla_{t_2} = (1-B_{t_1})(1-B_{t_2})$$

is equivalent to the convolution operator

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

and $\nabla_{t_1}^2 \nabla_{t_2}^2$ is equivalent to the Laplacian convolution operator

$$\begin{pmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{pmatrix}.$$

Recall that the higher order crossings are the same as the symbol changes in

$$\{X^{(k)}\} = U\nabla_{t_1}^{k-1}\nabla_{t_2}^{k-1}\{Z\}$$

and that $\{D_{k,MN}\}_k$ counts the number of symbol changes in the binary array $\{X^{(k)}(t_1,t_2),\ 1 \le t_1 \le M,\ 1 \le t_2 \le N\}$. We suggest the use of the $\{D_{k,MN}\}$ as useful discrimination features. Before using these features in discrimination, consider some examples which display the general behavior of these features as recorded by Theorem 1 and Proposition 4.

Example 5.1. Consider the stationary wave model

$$Z(t_1,t_2) = \alpha[Z(t_1-1,t_2) + Z(t_1,t_2-1)] + u(t_1,t_2),$$

$u(t_1,t_2)$ normal white noise with mean 0 and variance 1 and $|\alpha| < 1/2$. The $\{D_{k,15\times 15}\}_{k=1}^{11}$ are given below for various values of α.

α \ k	$D_{k,15\times15}$, $k = 1,2,\ldots,11$										
	1	2	3	4	5	6	7	8	9	10	11
0.40	111	229	282	305	315	326	333	340	348	348	360
0.25	154	245	282	305	317	326	333	340	352	350	360
0.00	173	268	296	312	324	327	337	348	352	353	359
-0.25	199	296	314	324	331	335	342	352	353	361	364
-0.40	265	326	333	342	342	353	355	361	367	369	372

Example 5.2. In this example we derived the higher order crossings for texture fields obtained from stained paper, quartz and grass, where the textures are coded as grey levels ranged from 0 to 63 [1].

Texture	$D_{k,15\times15}$, $k=1,2,\ldots,10$									
Paper	43	207	270	316	323	322	328	327	334	336
Quartz	70	227	296	324	336	343	349	361	362	367
Grass	128	266	297	310	327	330	348	361	361	375

Both of these examples portray vividly the results of the preceding section and in particular the monotone property of the $\{D_k\}$. It is seen that for low order k, different texture fields give rise to different $\{D_k\}$, but that as k increases, the higher order crossings give redundant information which results in similar $\{D_k\}$. A closer look at the $\{D_k\}$ reveals that D_1 corresponds to axis-crossings, D_2 corresponds to peaks and troughs, D_3 corresponds to inflection points, etc. This can be best seen in the one dimensional case. Now, the above theoretical and experimental results suggest that these visual features are more useful relative to some features (for which we do not have names) corresponding to $\{D_k\}$ for large k. In this sense low order higher crossings contain discrimination information not found in high order higher crossings. Thus, for discrimination purposes it is sufficient to consider $\{D_k\}$ for $k=1,2,\ldots,8$ as in most cases more than 80% of possible symbol changes are already recorded by D_8. Another important point is that from the low order $\{D_k\}$ it is possible to perceive a rough skeleton of the texture field.

The preceding argument suggests that the vector

$$\underset{\sim}{D} = (D_{1,MN}, D_{2,MN}, \ldots, D_{K,MN}),$$

$K = 8$, say, contains information useful for discrimination. We have thus reduced the texture discrimination problem to a multivariate analysis problem where the vectors of observations correspond to visual features. This notion will be extended in the following section.

Now, our next goal is to construct a discrimination statistic from $\underset{\sim}{D}$. Based on our experience in the one dimensional case [5] we suggest here a fast and useful statistic. Define

$$\Delta_{j,MN} = \begin{cases} D_{1,MN}, & j = 1 \\ D_{j,MN} - D_{j-1,MN}, & j = 2, \ldots, K-1 \\ [M(N-1) + N(M-1)] - D_{K-1,MN}, & j = K. \end{cases}$$

Clearly

$$\sum_{j=1}^{K} \Delta_{j,MN} = N(M-1) + M(N-1) \equiv n,$$

and for sufficiently large M,N

$$\Delta_{j,MN} > 0.$$

Let $m_{j,MN} = E(\Delta_{j,MN})$. We form the well known quadratic form

$$\psi^2 = \sum_{j=1}^{K} \frac{(\Delta_{j,MN} - m_{j,MN})^2}{m_{j,MN}}.$$

This is by no means the only statistic we have in mind, but its advantage is that it takes into account the monotonicity property of the $\{D_k\}$. Many experimental results show that a critical value of 65 corresponds approximately to a 0.05 significance level.

<u>Example 5.3</u>. The estimated $m_{j,15\times15}$ for white noise are

	$j = 1,2,\ldots,9$								
$\hat{m}_{j,15\times15}$	210.44	70.39	26.85	15.82	11.61	6.9	5.73	4.03	68.23

From Example 5.2 we have

Texture	$\Delta_{j,15\times15}$, $j = 1,2,\ldots,9$								
Stained paper	43	164	63	46	7	0	6	0	93
Quartz	70	157	69	28	12	7	6	12	59
Grass	113	138	31	13	17	3	18	13	59
White noise	208	69	19	26	14	18	3	7	56
Wave model, $\alpha = 0.4$	111	118	53	23	10	11	7	7	80

In testing the hypothesis that the above textures are realizations of white noise we have:

Texture	ψ^2
Stained paper	$\psi^2 = 133.22 + 124.48 + 48.67 + \ldots + 8.99 > 65$
Quartz	$\psi^2 = 93.72 + 106.56 + 66.16 + \ldots + 1.24 > 65$
Grass	$\psi^2 = 32.29 + 64.93 + 0.64 + \ldots + 1.24 > 65$
White noise	$\psi^2 = 0.03 + 0.03 + 2.29 + 6.55 + 0.49 + 17.85$
	$+ 1.30 + 2.18 + 2.19 = 32.91 < 65$
Wave model, $\alpha = .4$	$\psi^2 = 46.98 + 32.20 + 25.46 + \ldots + 2.03 > 65$

We see that only in one case we accepted the hypothesis of white noise as we should. It is also seen that stained white paper is far from being white (noise).... Thus, our ψ^2 performs rather well in texture discrimination. In the last case of the wave model the power was found to be over 90%. Many similar results were obtained for the one dimensional case. The distribution problem of ψ^2 is an open question although we know the approximate tail behavior via many simulations.

6. A Suggestion for a General Procedure for Texture Discrimination

The higher order crossings corresponding to the sequential filter $\nabla_{t_1}^{k-1}\nabla_{t_2}^{k-1}$ were shown to be useful in discrimination of texture fields. This leads naturally to the examination of the higher crossings by

$$(1-B_{t_1})^p(1+B_{t_1})^q(1-B_{t_2})^r(1+B_{t_2})^s\{Z\}$$

for rather low order p,q,r,s. Denote the numbers of higher crossings in an $M\times N$ field by

$$\{D_{pqrs,MN}\} .$$

Then we suggest a very general discrimination procedure based on these quantities:

$$\underset{\sim}{D} = (D_{p_1q_1r_1s_1,MN},\dots,D_{p_Kq_Kr_Ks_K,MN}).$$

If $\underset{\sim}{D}$ is approximately distributed as $N(\underset{\sim}{\mu},\underset{\sim}{V})$ then

$$(\underset{\sim}{D}-\underset{\sim}{\mu})'\underset{\sim}{\hat{V}}^{-1}(\underset{\sim}{D}-\underset{\sim}{\mu})$$

is a reasonable test statistic.

References

[1] Brodatz, P. (1966). Textures, Dover, London.

[2] Gagalowicz, A. (1981). A new method for texture fields synthesis: Some applications to the study of human vision. IEEE Trans. Pattern Analysis and Machine Intelligence, PAMI-3, 520-533.

[3] Hannan, E. J. (1970). Multiple Time Series, Wiley, N. Y.

[4] Julesz, B. (1962). Visual pattern discrimination, IRE Trans. Inform. Theory, IT-8, 84-92.

[5] Kedem, B. and Slud, E. (1981). On goodness of fit of time series models: An application of higher order crossings, Biometrika, Vol. 68, 551-555.

[6] Kedem, B. and Slud, E. (1979). The signature problem for stationary time series. Submitted.

[7] Laws, K. L. (1980). Texture image segmentation, Dept. of EE, Univ. of Southern Calif., USCIPI Report # 940.

[8] Slutzky, E. (1927). The summation of random causes as the source of cyclic processes, Problems of Economic Conditions, Vol. 3, No. 1.(English translation in Econometrica, Vol. 5, 105-146.)