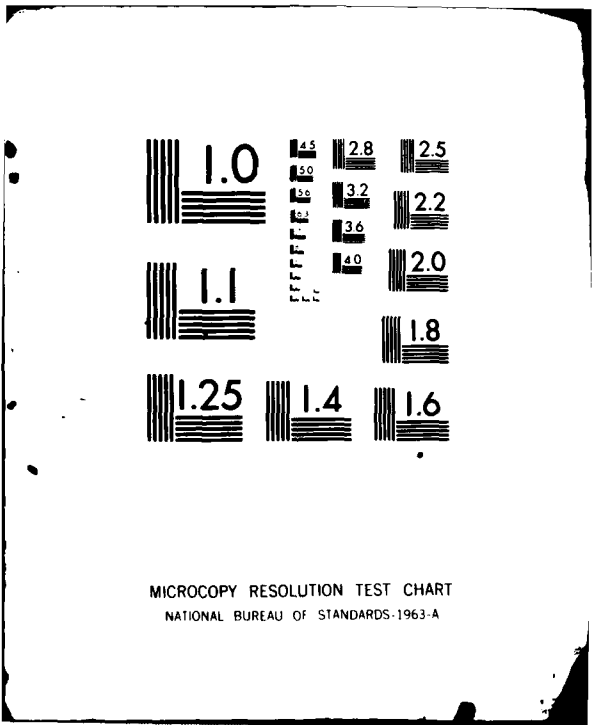


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GAMES WITH UNCERTAIN MODELS

by

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ABSTRACT

We present a normative decision model for hierarchical organizations whose levels operate at different tempos. We describe the mathematical methodology whereby each level forms internal models of the detailed state process, corresponding to its tempo of operation. Based on these models, we describe a rational aggregate strategy for the hierarchical organization.

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## 1. Introduction

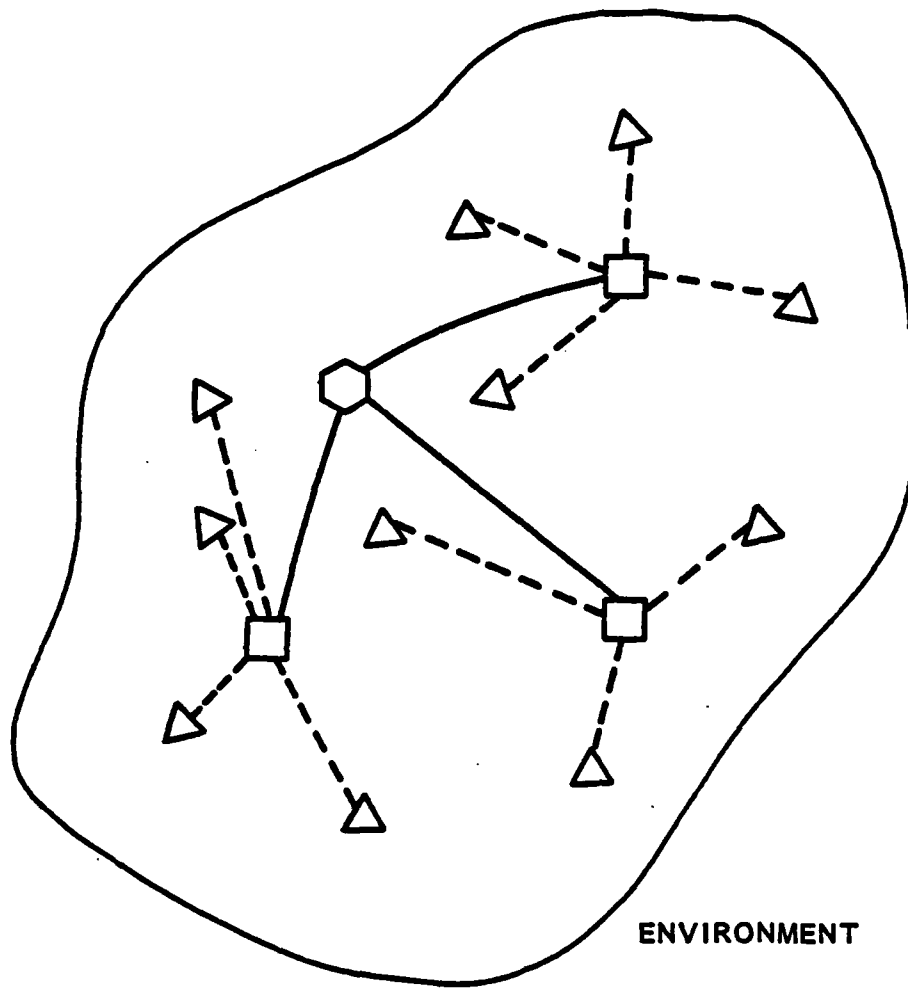
In many of the presentations included in the proceedings of this MIT-ONR C<sup>3</sup> workshop, a distinction has been made between the physical (electronic, human) system used to gather and distribute information and commands, (the C<sup>3</sup>I system) and the processing of this information to select appropriate courses of action (the C<sup>2</sup> process). It is the contention of many researchers that any evaluation of a C<sup>3</sup>I system is influenced by the C<sup>2</sup> process which it supports. This, in turn, requires proper understanding of how decisions are made in the complex environment in which C<sup>3</sup> systems operate.

One of the most interesting aspects of the C<sup>2</sup> decision processes in the C<sup>3</sup> environment is that decisions must take into account the presence of other decision makers in the environment. Furthermore, these decision makers are often arranged in a hierarchical fashion, distributed throughout the environment, as illustrated in figure 1.

Although figure 1 represents a simplistic view of a C<sup>3</sup> organization, it serves to highlight several important points: First, the presence of a hierarchical structure indicates that different perspectives of the same environment are needed at each decision level. Second, the amount of interaction between different nodes at the same level is often small, since they are typically interacting with different parts of the environment.

In this paper, we study the problem of developing a normative, rational model of a decision-maker operating as part of the C<sup>2</sup> process in the C<sup>3</sup> environment. This model reflects several important issues: the presence of many decision makers in a system, the hierarchical nature of the decision structure, and the existence of different perspectives of the environment at different decision levels. The model is developed as a precise set of mathematical axioms. The consequences of these axioms are then explored in the context of some simple dynamic decision problems.

The paper is divided into two main parts. The first part discusses in some detail the problem of developing mutually consistent mathematical models which represent perceptions of the environment at different levels of aggregation. The basic concept used in this part is that of tempo, or time scale, of operation. Based on an underlying accurate global description, consistent aggregate models are given, which represent the evolution of the system at a specific tempo of operation. The accuracy of these models is characterized, and conditions are given which specify when such models can be obtained. This part of the paper is a brief summary of the results of [1], consisting of joint work with Mr. Marcel Coderch, Prof. A. Willksy and Prof. S. Sastry of MIT/LIDS.



- △ FIRST LEVEL DECISION NODES
- SECOND LEVEL DECISION NODES
- ⬡ THIRD LEVEL DECISION NODES

FIGURE 1

The second part of the paper proposes a normative model for each decision maker in the  $C^3$  environment which uses the aggregate models obtained in the first part of the paper. The concept of aggregate rationality is proposed as a desirable solution to the overall decision problem, using concepts from game theory and hierarchical structures.

The development of consistent aggregate models which are accurate at a specific time scale is a problem which has received considerable attention in recent years. The works of Kokotovic et.al. [2], Korolyuk [3], Papanicolaou [4], and other are typical of the results obtained. Most of these authors developed approximations which were only valid at one specific time scale. An extension of this work to a hierarchy of models at many time scales was developed in [1]. The work on aggregation which we report here is a summary of our results in [1].

Dynamical decision problems with different cost criteria and a hierarchical structure have been studied extensively in the literature, under the name of Stackelberg games. The first authors to study these class of decision problems were Chan and Cruz [5]. Their work was extended by many authors, [6], [7]. In particular, Basar and his co-workers [8]-[9] have studied the solution of Stackelberg decision problems using incentive strategies to achieve goal coordination between different decision levels.

The results in this paper combine the concepts of aggregate modeling and hierarchical decision theory to produce a mathematical theory of rational behavior which captures some important features of realistic decision situations. First of all, decision makers involved at the higher (more "global") levels of the decision hierarchy will use coarser, aggregate models, and operate at a slower tempo than their low-level counterparts. Their decisions are interpreted as global directives, to be amplified and interpreted by their subordinates. This model of rational behavior is different from the classical game-theoretic models of hierarchical decision structures, where the higher levels are required to have more detailed information about the system than their lower level counterparts.

The second important aspect of this theory is that it proposes an analytical way for developing internal models (for each decision maker) at a specified tempo of operations, starting from a microscopic reality. This may not seem as an important advantage in real-life, on-line analysis of events, because a microscopic model of reality is not readily available. However, this result is useful for off-line, analytical studies of the value of  $C^3I$  equipment supporting a set of decision makers, where the scenarios are pre-specified and controlled.

Finally, it should be stressed that all of the results presented in this paper are strictly part of an analytical methodology, which needs to be tested to decide its actual value in the study of  $C^3$  systems. The concept of aggregate rationality is speculative, and needs to be investigated further. In addition, the methodology has been developed to the level where simple models can be studied. It remains to be seen whether additional extensions will make it sufficiently general to have practical significance.

## 2. Hierarchical Aggregation of Linear Systems

This section is a brief overview of the results in [1]. For a complete proof of the main theorem, as well as precise mathematical statements, the reader is referred to [1].

We begin by postulating a dynamic model which describes the entire evolution of the system when no decisions are applied. We assume that the system state, of finite dimension, evolves according to a differential equation of the form

$$\frac{dx}{dt} = A(\epsilon)x \quad (2.1)$$

where  $x$  represents the state vector, and  $A(\epsilon)$  is a matrix which depends on a small parameter  $\epsilon$ , as

$$A(\epsilon) = \sum_{i=0}^{\infty} A_i \epsilon^i \quad (2.2)$$

The presence of the parameter  $\epsilon$  is used to model the different tempos at which events occur in the evolution of the system. For instance,  $\epsilon$  reflects the strength of the coupling between events occurring distant corners of the environment, or the ratio between the average delay for decisions at a high level in a hierarchy, and the average delay for decisions at a lower level. The formulation of a system model with a properly identified structure of the form (2.1) and (2.2) is a difficult engineering task which we do not address here. We assume that the model (2.1) is given, and that  $\epsilon$  has been chosen appropriately on physical grounds.

Based on the model of equation (2.1), there are two important questions to answer: Given a specified tempo of operation (time-scale), does there exist a simple aggregate model of the system which accurately reflects the evolution of the model in equation 2.1? If so, what is that model? The purpose of this section is to make these questions mathematically precise, and to provide an answer in some cases.

We begin with some mathematical preliminaries. The solution of equation 2.1, considered as a time function on the semi-infinite interval  $[0, \infty)$ , can be written as

$$x^\epsilon(t) = e^{A(\epsilon)t} x_0, \quad t \in [0, \infty) \quad (2.3)$$

for arbitrary initial conditions  $x_0$ .

An aggregation  $y(t)$  of the trajectory  $x(t)$  is a mapping

$$G: X \rightarrow Y$$

where  $X$  is the state space of  $x(t)$ , and  $Y \rightarrow X$  is a proper subspace of  $X$ . Usually,  $Y$  is defined through an equivalence relationship which identifies equivalent states in  $X$ .

Our objective in this section is to establish conditions on the system 2.1 such that, when the system is viewed at a specific tempo or time scale  $\tau = t/f(\epsilon)$ , the evolution of the system state can be approximated by an aggregate model, operating at the  $\tau$  time scale. Mathematically, we want to define an aggregate trajectory  $y^\epsilon(\tau)$ , with its own generating model

$$\frac{dy^\epsilon}{d\tau}(\tau) = A^{(1)}(\epsilon) y^\epsilon(\tau) \quad (2.4)$$

such that

$$\lim_{\epsilon \rightarrow 0} \sup_{0 < \tau < \infty} \|y^\epsilon(\tau) - G(x^\epsilon(t/f(\epsilon)))\| = 0 \quad (2.5)$$

Equation 2.5 demands that the aggregation must provide an accurate representation of the time trajectory, depending on the physical parameter  $\epsilon$ .

The primary result in [1] can be stated as follows:

Let  $A_0(\epsilon)$ , for  $\epsilon$  in  $[0, \epsilon_0]$ , be a matrix-valued function, with constant rank  $d$  for  $0 < \epsilon < \epsilon_0$ . Assume furthermore that  $A_0(\epsilon)$  has semi-simple null structure (a full complement of zero eigenvector for all  $\epsilon$ ).

Construct the sequence of matrices

$$A^{(0)} = A_0$$

$$P_i = \lim_{t \rightarrow \infty} e^{A^{(i)}t}, \quad i=0,1$$

$$A^{(1)} = P_0 A_1 P_0$$

$$A^{(2)} = P_1 P_0 (A_2 - A_1 A_0^+ A_1) P_0 P_1$$

where  $A_0^+$  = any pseudo-inverse of  $A_0$ .

The exact construction of the rest of the sequence  $A^{(i)}$ ,  $i > 2$  is described in [1] and [10]. For the purposes of the theorem statement, we need to define one more term. A matrix  $A$  is semistable if and only if

$$\sup_t \|e^{At}\| < \infty$$

Theorem [1]: Let  $A_0(\epsilon)$  be a matrix with semisimple null structure.

If  $A^{(0)}, \dots, A^{(m)}$  are semistable matrices with

$$\sum_{i=0}^n \text{rank } A^{(i)} = d, \text{ then}$$

$$\lim_{\epsilon \rightarrow 0} \sup_{t \geq 0} \|e^{A(\epsilon)t} - \phi(\epsilon, t)\| = 0$$

where

$$\phi(\epsilon, t) = \prod_{k=0}^m e^{A^{(k)} \epsilon^k t}.$$

Furthermore, for any specific time scale  $\tau = t/\epsilon^k$ . let

$$\Pi_k = \prod_{i=0}^{k-1} P_i. \quad \text{Then,}$$

$$\lim_{\epsilon \rightarrow 0} \sup_{0 \leq t \leq T} \|x^\epsilon(t/\epsilon^k) - \pi_k e^{A^{(k)}t} x_0\| = 0.$$

Also, the only non-trivial time scales are integer powers of  $\epsilon$  less than or equal to  $m$ .

The last formula in the theorem defines our aggregate process. Define the equivalence relation  $x_1 \sim x_2$  if and only if

$$\pi_k x_1 = \pi_k x_2. \quad \text{Let } Y = X/\sim \text{ be the factor space of equivalence classes}$$

of  $X$  under  $\sim$ . Then, there is an aggregate model for the evolution of  $y(\tau)$ , given by the restriction of the flow

$$x(\tau) = \pi_k e^{A^{(k)}\tau} x_0$$

to the subspace  $\text{Range } \pi_k$ . The exact construction of this aggregate model is a question of simple algebra, described in [1].

Although the conditions of semistability required for the existence of accurate aggregate models at specific tempos of operation appear stringent, there are numerous examples of systems which satisfy these conditions. An important class of systems is that of positive systems, where the trajectory of  $x(t)$  lies in the positive orthant. An example of such a system is the evolution of the probabilities of a stochastic finite state Markov process. Further development of these results will appear in a forthcoming paper.

There are many issues concerning aggregate models which have not been addressed in this section. We have analyzed only the free, uncontrolled evolution of the system 2.1. The presence of control actions has not been modeled; these actions can change the time structure of the system. An example of such a change in structure consists of a supreme commander taking direct control of a low-level operation. In this case, new informational links, much faster than the standard channels, have to be set up in order to provide relevant information to the supreme level, about the details of the operation. What we have developed in this section is the concept that, if there are natural tempos of operation

for different decision nodes monitoring a specific environment, simplified aggregate models can be obtained which approximate the system at those tempos of operation. The next section will expand upon these ideas by studying the problem of decision making in such an environment.

### 3. Games with Uncertain Models

Informally, game theory can be viewed as the scientific study of decision making in situations with possible conflicts of interest. As discussed in standard textbooks (e.g. [11]), a game can be analyzed in terms of a three components (U,J,S), as follows:

The set U is the set of all possible decision strategies which can be used by all decision-makers (players) in a game. Typically, U can be decomposed in product form as

$$\underline{U} = \underline{U}_1 \times \underline{U}_2 \times \dots \times \underline{U}_m \quad (3.1)$$

where  $m$  is the number of players in the game, indicating that the choice of strategy is unconstrained by other player's strategies.

The element J, the performance function, is an  $R^m$ -valued real function, which assigns a set of utilities to each player for each choice of admissible strategy  $\underline{u} \in \underline{U}$ . Mathematically,

$$\underline{J}: \underline{U} \rightarrow R^m$$

The third element S is the controversial part of game theory. The set S is called a solution concept, or a selection rule, and its purpose is to identify sets of strategies, based on the performance function, which can be accepted as rational outcomes for the play of the game. Hence, we specify S as a subset of U completely described by a condition in terms of J. That is,

$$S = \{u^* \in \underline{U}: C(J(u^*)) \text{ is satisfied}\} \quad (3.2)$$

As an example of a typical selection rule, consider the problem of minimizing a specific cost functional,  $H(u)$ . The condition becomes

$$H(u^*) \leq H(u) \text{ for all } u \in \underline{U}.$$



FIGURE 2

Once the parameters of the game are established, and become known to all players, each player is in an individual position to compute all elements of the set S. Typically, S will consist of a single element, thereby defining uniquely the rational course of play for the game. Each player is assumed to follow his rational course of action, as established by the selection rule S.

There are several implicit assumptions which need to be considered carefully when one studies the concept of rational solutions of games, as described above. First of all, the assumption is made that all players have access to the complete detailed model  $(\underline{U}, \underline{J}, \underline{S})$ , so that each player can individually compute the rational strategy set S. Second, the situations where S has many elements have to be resolved by an additional, unspecified mechanism to identify a single rational play of the game.

In terms of the discussion on aggregate models specified in Section 2, the classical game assumptions would require each level to have a complete microscopic model of the situation, in order to be able to evaluate accurately all of the possible payoffs corresponding to individual plays of the game. This contradicts the philosophy of using simpler models, valid at specific tempos of operation, which was proposed in section 2. In the rest of this section, we propose a new normative model for determining rational solutions to game problems, which we denote as L, aggregate rationality.

Our first assumption is that each decision maker in the game has an intrinsic tempo of operation, which cannot be changed by the strategies of other decision makers. Basically, this implies that the aggregate, internal models used for decision purposes will not change dramatically with the choices of strategies. We will make this concept mathematically precise in this section.

For the present, we will restrict ourselves to discussing games with a strict hierarchical structure. That is, decision makers are arranged in a structure, as in Figure 2. To each level in the hierarchy, we assign tempo of operation, together with an internal model, which describes the evolution of the system state at that tempo. In addition, we define a "meta-game" at that tempo which involves an aggregate model of all other decision levels' objectives and actions. Based on this perceived game, each level is able to compute its rational aggregate strategy.

Formally, assume that the state of the system evolves according to the differential equation

$$\frac{dx}{dt} = A(\epsilon)x(t) + \sum_{i=1}^m B_i(\epsilon)u_i(t) \quad (3.3)$$

where  $m$  is the number of levels present in the hierarchical structure. We assume that  $A(\epsilon)$  can be factored into a form which explicitly displays a multiple time scale structure, as

$$A(\epsilon) = \text{diag}\{I, \epsilon I, \dots, \epsilon^{m-1} I\} \cdot \begin{pmatrix} A_{11}(\epsilon) & \dots & A_{1m}(\epsilon) \\ \vdots & & \vdots \\ A_{m1}(\epsilon) & \dots & A_{mm}(\epsilon) \end{pmatrix} \quad (3.4)$$

where the block matrices  $A_{ij}(\epsilon)$  are also analytically dependent on  $\epsilon$ . Partition the state vector  $x(t)$  conformally as

$$x(t) = \begin{pmatrix} x_1(t) \\ \vdots \\ x_m(t) \end{pmatrix} \quad (3.5)$$

Notice that this partition creates a causal structure in terms of aggregate models. At the fastest time scale,  $x_1(t)$  describes the evolution of the relevant variables, influenced by the levels set by the slower states  $x_2(t), \dots, x_n(t)$ . At the next time scale,  $\tau = \epsilon t$ , the fast transients of  $x_1(t)$  have died out, leading to a reduced model, whose principal evolution is described in terms of  $x_2(t)$ .

The control laws  $u_1(t), u_2(t), \dots, u_m(t)$  are assumed to vary at the natural time scale of the decision level, or slower. Hence,

$$\begin{aligned} u_2(t) &\triangleq u_2(\epsilon t) \\ u_3(t) &= u_3(\epsilon^2 t) \\ u_k(t) &= u_k(\epsilon^{k-1} t) \end{aligned} \quad (3.6)$$

The strategies are selected in a hierarchical order, starting from the slowest, or most aggregate, level. At the fastest, most detailed level, all of the higher levels have announced their strategy, so that the choice of strategy at this level is reduced to solving for a single decision maker's strategy. This strategy is parametrized by the higher level's decisions, which vary in a slower time scale.

Rational play in the game is determined as follows: At the most detailed level, with a tempo  $\tau$  of order  $t$ , the performance criteria is given as

$$J_1 = \int_0^{\infty} (x' Q_1 x + \sum_{i=1}^m u_i' R_i u_i) dt \quad (3.7)$$

where the evolution of the vector process  $x$  is governed by equation (3.3). Since level 1 is the lowest level, we assume that, from the hierarchical structure of the game, that the strategies from players 2 through  $m$  are known, so that player 1 can determine his optimal strategy in a straightforward optimal control problem.

For level 2, the natural time scale is  $\tau = t/\epsilon$ . At this time scale, the evolutions at the tempo  $\tau=t$  have achieved their steady state. That is,

$$\begin{aligned} \dot{x}_1 = 0 = & A_{11}(\epsilon)x_1^s + \dots + A_{1m}(\epsilon)x_m^s + \sum_{i=2}^m B_{i1}u_i(\epsilon^i t) \\ & + B_1 u_1^{ss}(\epsilon t) \end{aligned} \quad (3.8)$$

where we have assumed that

$$u_1(t) = u_1^f(t) + u_1^{ss}(\epsilon t) \quad (3.9)$$

so that the local decisions of level 1 can be divided into short term and long term strategies.

Hence, the decision problem at level 2 can be described in terms of an aggregate game. The cost function, in the  $\tau = t/\epsilon$  scale, is

$$J_2 = \int_0^{\infty} x'(\tau)Q_2x(\tau) + \sum_{j=1}^m u_j'(\tau)R_j^2u_j(\tau)d\tau, \quad (3.10)$$

where the evolution of  $x_1$  is assumed to have reached steady state, so that  $x_1$  is no longer a dynamic variable. In addition, in order to completely define a well-posed decision problem for player 2, we must specify an internal model of player 1's long-term strategy to be used by player 2. We use our aggregation philosophy, to specify a perceived decision model for level 1.

In level 2's tempo, level 1's decision problem becomes

$$\text{Minimize } J_1 = \int_0^{\infty} (x'Q_1x + u_1^{ss'}R_1' u_1^{ss} + \sum_{j=2}^m u_j'R_j'u_j)d\tau \quad (3.11)$$

subject to the same dynamic model used at level 2. Note that this perceived problem neglects the faster evolution terms present in level 1's original cost. Hence, it is only the perceived decision problem, at level 2's tempo, rather than the actual decision problem. This perception is obtained by using our philosophy of defining aggregate models based on tempo.

Following this structure, we endow level  $i$  with its own cost function, of the form

$$J_i = \frac{1}{e^{i-1}} \int_0^{\infty} x'(t/e^{i-1}) Q_i x(t/e^{i-1}) + \sum_{j=1}^m u_j'(t/e^{i-1}) R_j^i u_j(t/e^{i-1}) dt \quad (3.12)$$

subject to

$$\dot{x}_1 = \dots = \dot{x}_{i-1} = 0, \quad (3.13)$$

and define perceived decision problems at this tempo, consistent with the original decision problem specified at faster tempos.

We call the set of strategies which are optimal under this decision structures the aggregate rational strategies. In the next section, we illustrate the implications of this concept in the context of a simple example.

#### 4. Example

For the sake of simplicity, we consider an example with two levels of decision making. Furthermore, to illustrate the different time scales of the system, we assume that one component of the state vector operates in discrete time, while the other evolves continuously.

Assume that the system state is described by the evolution equation

$$x_{t+1} = x_t + 2y_t + u_t - v_t \quad (4.1)$$

$$e \frac{dy}{dt} = y_t - x_t + u_t + v_t \quad (4.2)$$

The system has two natural time scales of evolution:  $\tau = \epsilon t$ , corresponding to the evolution of the  $y$  process, and  $t$ , for the discrete part. We assume that there are two decision levels, with  $v_t$  corresponding to the fast level, and  $u_t$  to the slower level. The performance measure associated with  $v_t$  is

$$J_2 = \int_0^{\infty} (x_{\tau}^2 + y_{\tau}^2 + 2v_{\tau}^2) d\tau \quad (4.3)$$

while the performance measure associated with  $u_t$  is

$$J_1 = \sum_{t=0}^{\infty} (x_t^2 + u_t^2 + v_t^2) \quad (4.4)$$

It is assumed that  $x_t, u_t$  are constant on  $[t, t+1)$

We begin with the problem seen by  $u$ . At the  $t$  time scale,  $\dot{y}=0$ . Hence

$$y_t^s = x_t - u_t - v_t^s \quad (4.5)$$

Hence, the dynamical system seen by  $u_t$  is

$$x_{t+1} = 3x_t - u_t - 3v_t^s \quad (4.6)$$

Furthermore, his perception of  $J_2$  is

$$J_2^p = \sum_{t=0}^{\infty} x_t^2 + (x_t - u_t - v_t^s)^2 + 2v_t^{s2} \quad (4.7)$$

In this dynamical problem, player  $U$  is seen as a leader in a hierarchical problem. In most cases, he can use a memory-dependent strategy which achieves the team-optimal performance, by using goal coordination. We compute such a strategy in the following discussion.

Assume that  $u$  and  $v$  were a team, with common goal (4.4). Then, the optimal strategies would be

$$u_t^* = \frac{-1}{30} + \frac{\sqrt{.91}}{3} \quad x_t = a^* x_t \quad (4.8)$$

$$v_t^* = \frac{-1}{10} + \sqrt{.91} x_t = b^* x_t \quad (4.9)$$

$$x_{t+1}^* = 3x_t - u_t^* - 3v_t^* = \frac{10}{3} (1 - \sqrt{.91}) x_t^* \quad (4.10)$$

In order to achieve this optimal trajectory, level 2 can use a goal-coordination memory strategy, of the form

$$u_t^* = a^* x_t + g(x_t - (3 - a^* - 3b^*) x_{t-1}) \quad (4.11)$$

The exact value of  $g$  depends on the cost criterion used at level 1. According to (4.7), when (4.11) is used as a strategy, the resulting optimization problem for  $v$  is

$$\min J_2^D = \sum_{t=0}^{\infty} x_t^2 + (x_t - u_t^* - v_t^s)^2 + v_t^2 \quad (4.12)$$

The exact value of  $g$  can thus be calculated as the value for which  $b^* x_t$  is the optimal strategy which minimizes (4.12). We have not taken the time to compute this value, because it involves tedious algebraic manipulations better left to machines.

Once level 2's rational strategy (4.11) is decided, the next step is to solve for the rational strategy of level 1. This involves looking at the detailed performance index (4.3). Due to the consistency between (4.3) and (4.7), the effect of  $u^*$  is to force the slow part of  $v_t$  to match the optimal slow strategy (4.9). However,

$$v_t = v_t^s + v(\tau) \quad (4.13)$$

where

$$\lim_{\tau \rightarrow \infty} v(\tau) = 0, \quad \tau = t/\epsilon.$$

Note that the choice of  $v(\tau)$  is not constrained by the strategy  $u_t^*$ , because it is varying much faster than the tempo used for decisions at the discrete time scale. The strategy  $v(\tau)$  can be determined from the reduced problem on the  $\tau$  time scale.

5. Conclusion

The concept of aggregate rationality introduced in section 3 and illustrated in section 4 provides an interesting decision model for hierarchical organizations. By prescribing a sequence of perceived games, rational strategies can be defined at each level of aggregation. Some of our preliminary investigations hint that the concept of aggregate rationality is very robust; that is, there is little benefit to be gained from using more detailed models for decision making. We are in the process of developing a rigorous mathematical theory which will expand on the concepts presented here. In particular, there are several key questions which must be addressed, concerning well-posedness of the solution problem, and canonical forms for the decision models under study. These studies will be reported at future C<sup>3</sup> workshops.

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