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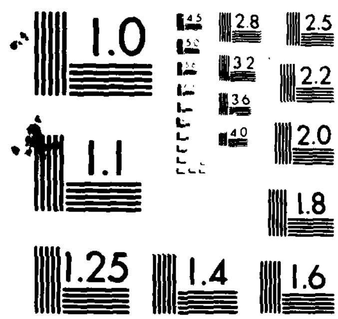

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PROPAGATION OF A RELATIVISTIC CHARGED PARTICLE BEAM THROUGH AN AIR MEDIUM WITH CONDUCTING BOUNDARIES

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May 1980

Final Report

Approved for public release; distribution unlimited.

AIR FORCE WEAPONS LABORATORY
Air Force Systems Command
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Many propagation experiments involve firing a relativistic charged particle beam into a column of air bounded by the conducting walls of a drift tube. In this investigation, the extent to which the magnitude of the resulting return current and, hence, the stability of the propagating beam is affected by the presence of conducting boundaries. Maxwell's equations are employed to derive the appropriate wave equation. This equation is transformed into a diffusion equation which is numerically solved for various boundary parameters.

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CONTENTS

INTRODUCTION

Some applications of relativistic electron beams (REB) necessitate that they stably propagate some distance through air. Whether the beam will propagate or not depends on the pressure of the air into which it is fired. The goal of many REB experiments is to determine the pressure most conducive to beam propagation.

Low pressure environments, into which an REB may be fired, can be created by various means. The most practical means, experimentally, is to use a drift tube whose interior pressure is adjustable. The beam is then fired into the tube and monitored as it passes various points along the length of the chamber.

The drift tubes used in propagation experiments generally have cylindrical symmetry, and are sometimes made of highly conductive metals. Hence, because of the conducting boundary, firing an REB into a metallic drift tube may not be the same as firing it into the unbounded atmosphere.

In this investigation, we endeavor to determine (by solving Maxwell's equations with appropriate boundary conditions) a minimum drift tube diameter such that conducting boundaries have a negligible affect on relativistic electron beam—plasma interactions.

ASSUMPTIONS AND EQUATIONS

As is well known, Maxwell's equations, in Gaussian units, for material media are:

$$\nabla \cdot \vec{E} = 4\pi\rho \tag{1}$$

$$\nabla \cdot \vec{B} = 0 \tag{2}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{3}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \tag{4}$$

To obtain the appropriate wave equation, apply the $\nabla\times$ operator to equation 3 and employ the identity

$$\nabla \times \nabla \times \vec{F} = \nabla (\nabla\cdot\vec{F}) - \nabla^2 \vec{F}$$

to get

$$\nabla^2 \vec{E} - \nabla (\nabla\cdot\vec{E}) = \frac{\partial}{\partial t} \nabla \times \vec{B} \tag{5}$$

Substituting into equation 5, the expressions for $\nabla \cdot \vec{E}$ and $\nabla \times \vec{B}$ yields

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c}\frac{\partial \vec{J}}{\partial t} \tag{6}$$

To investigate the problem, we need solve only the z component of equation 6. To do so, we make the assumption of cylindrical symmetry. That is, we assume that E_Θ, B_r, and B_z are zero, and that the remaining components of $\vec{E}$ and $\vec{B}$ are independent of the azimuthal coordinate Θ. In addition, we assume that Ohm's law, $\vec{J} = \sigma\vec{E}$, is valid in the plasma. With the above assumptions, the z component of equation 6 becomes

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} E_z + \frac{\partial^2}{\partial z^2} E_z - \frac{4\pi}{c^2}\frac{\partial}{\partial t}\sigma E_z - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} E_z$$

$$= 4\pi\frac{\partial\rho}{\partial z} + \frac{4\pi}{c^2}\frac{\partial}{\partial t} J'_z \tag{7}$$

where J'_z represents the REB current density.

We simplify equation 7 via the transformation

$$\tau = t - \frac{z}{c} \tag{8a}$$

$$Z = z \tag{8b}$$

Now

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial \tau} \tag{9a}$$

and

$$\frac{\partial}{\partial z} \rightarrow \frac{\partial}{\partial Z} - \frac{1}{c}\frac{\partial}{\partial \tau} \tag{9b}$$

We now make the so-called frozen beam approximation; i.e., we assume that the fields translate rigidly with the beam. For any given field component F, the frozen beam approximation is equivalent to the mathematical statement

$$\frac{DF}{Dt} = \frac{\partial F}{\partial t} + v\,\frac{\partial F}{\partial z} = 0 \tag{10}$$

or

$$\frac{\partial F}{\partial t} = -\,v\,\frac{\partial F}{\partial z} \tag{11}$$

where v is the velocity of the beam electrons. The energy of the beam electrons is generally greater than 1 MeV. Hence, we can further assume that $v = c$. Thus,

$$\frac{\partial E_z}{\partial t} = -\,c\,\frac{\partial E_z}{\partial z} \tag{12}$$

Substitution of the operator identities 9a and 9b into equation 12 yields

$$\frac{\partial E_z}{\partial \tau} = \frac{\partial E_z}{\partial \tau} - c\,\frac{\partial E_z}{\partial Z} \tag{13}$$

Equation 13 is valid only if

$$c\,\frac{\partial E_z}{\partial Z} \equiv 0 \tag{14}$$

That is, with respect to the coordinates Z and τ, the frozen beam approximation is equivalent to

$$\frac{\partial E_z}{\partial Z} = 0 \tag{15}$$

In view of equation 15, the identities 9a and 9b become

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial \tau} \tag{16a}$$

and

$$\frac{\partial}{\partial z} \rightarrow -\frac{1}{c}\frac{\partial}{\partial \tau} \tag{16b}$$

In terms of the new coordinates, equation 7 can be written as

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} E_z - \frac{4\pi}{c^2}\frac{\partial}{\partial \tau} \sigma E_z = -\frac{4\pi}{c}\frac{\partial \rho}{\partial \tau} + \frac{4\pi}{c^2}\frac{\partial J_z'}{\partial \tau} \tag{17}$$

An REB propagating through air generates a plasma, resulting in space charge neutralization (SCN). We account for SCN by letting

$$\rho = \frac{J_z'}{c} \exp(-4\pi\sigma\tau) \tag{18}$$

Substituting equation 18 into 17 yields

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} E_z - \frac{4\pi}{c^2}\frac{\partial}{\partial \tau} \sigma E_z = \frac{4\pi}{c^2}\frac{\partial}{\partial \tau} J_z'^{*} \tag{19}$$

where

$$J_z'^{*} = J_z' \left[1 - \exp(-4\pi\sigma\tau)\right] \tag{20}$$

We assume that the conductivity of the drift tube is large enough to be considered infinite. Continuity of E_z across the plasma—wall interface requires that

$$E_z\left(R_w, \tau\right) = 0 \tag{21}$$

where R_w represents the inner radius of the drift tube.

Since the beam electrons are highly relativistic, E_z is approximately zero at the head of the pulse. That is,

$$E_z(r, o) = 0 \tag{22}$$

The Θ component of equation 3, in the new coordinates, is

$$\frac{\partial}{\partial r} E_z (r,\tau) = - \frac{1}{c} \frac{\partial}{\partial \tau} B_\Theta (r,\tau) \tag{23}$$

Evaluating equation 23 at r = 0 yields

$$\frac{\partial}{\partial r} E_z (r,\tau) \Big|_{r=0} = 0 \tag{24}$$

Equations 19, 21, 22 and 24 uniquely determine E_z (r,τ) inside the drift tube chamber.

BEAM PROFILE AND PLASMA CONDUCTIVITY

The beam J'_z is assumed to have the following profile:

$$J'_z = \begin{cases} J_b^o\,(\tau/\tau_1)/(1 + r^2/a^2)^2 & \tau \le \tau_1 \quad (25a) \\ J_b^o\,/(1 + r^2/a^2)^2 & \tau_1 \le \tau \le \tau_2 \quad (25b) \\ 0 & \tau > \tau_2 \quad (25c) \end{cases}$$

where J_b^o and a represent the on-axis beam current density and Bennet radius, respectively. Also,

$$J_b^o = I_b^o/\pi a^2 \qquad (26)$$

where I_b^o is the on axis beam current.

We assume that the plasma conductivity (σ) is generated with a radial profile similar to that of the beam. That is, we let

$$\sigma = \sigma_0/(1 + r^2/a^2)^2 \qquad (27)$$

where σ_0 is the on-axis conductivity. We employed existing air chemistry models (Ref. 1)* to numerically obtain the on-axis conductivity. Figure 1 is a graph of on-axis conductivity versus distance (in seconds) back into the pulse. The beam parameters used are indicated at the lower right-hand corner of the graph. The rise length of the pulse is $\Delta L(= \tau_1 c)$ and its length is $L(= \tau_2 c)$.

1 Dreyer. ' A., *Electric and Magnetic Fields of An Intense Pulse of Relativistic* *.lect; .s Propagating Through Air*, AFIT/DS/PH/79-1, AFIT, Wright-Patterson AFB ..io, 1979.

* Johnston, R., private communication.

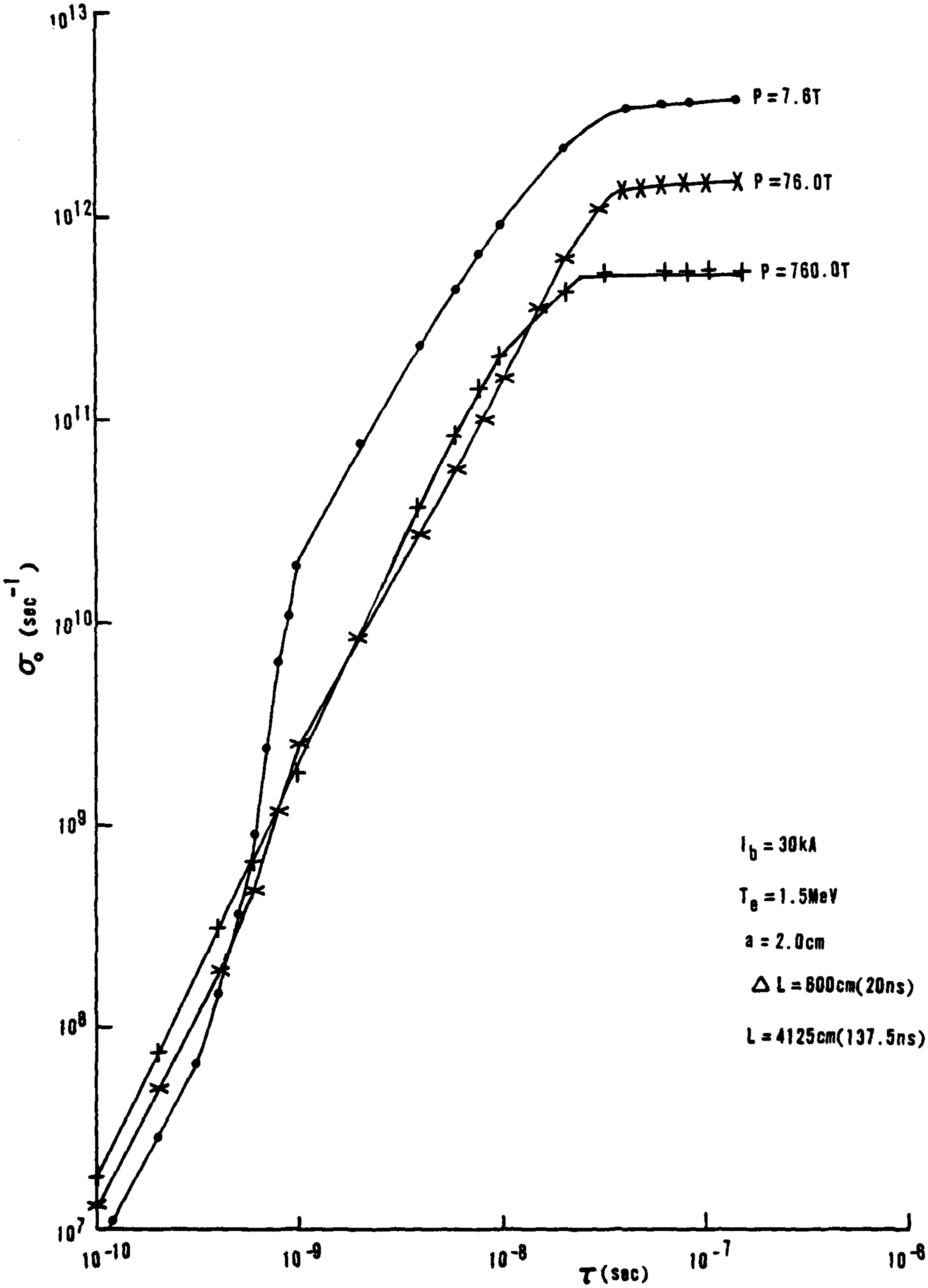

Figure 1. On-Axis conductivity versus distance.

NUMERICAL ALGORITHM

We solve equation 19 using an implicit finite difference scheme developed by Crank and Nicholson (Ref. 2). Following this scheme, equation 19 becomes

$$\frac{4\pi}{c^2}\frac{1}{\Delta\tau}\left[\left(\sigma E_z\right)_i^{n+1} - \left(\sigma E_z\right)_i^n\right] = \frac{1}{2}\frac{1}{r_i}\,\delta_r r_i \delta_r\, E_{zi}^{n+1} + \frac{1}{2}\frac{1}{r_i}\,\delta_r r_i \delta_r E_{zi}^n - \frac{4\pi}{c^2}\frac{1}{\Delta\tau}\left(J_{zi}^{'*n+1} - J_{zi}^{'*n}\right) \tag{28}$$

The boundary conditions, equations 21, 22 and 24, become

$$E_{zi}^{o} = 0 \tag{29}$$

$$E_{zI}^{n+1} = 0 \tag{30}$$

$$E_{zi+1}^{n+1} = E_{zi}^{n+1} \tag{31}$$

where

$$\tau = n\Delta\tau \tag{32}$$

$$r = i\Delta r \tag{33}$$

and

$$R_w = I\Delta r \tag{34}$$

The step sizes, $\Delta\tau$ and Δr, were assigned the values 5.00×10^{-12} s and 0.01 cm, respectively.

The z component of the plasma current density $\left(J_z\right)$ is calculated using Ohm's law. That is,

$$J_{zi}^{n+1} = \sigma_i^{n+1}\, E_{zi}^{n+1} \tag{35}$$

2. Carnahan, B., Luther, H. A., Wilkes, J. O., *Applied Numerical Methods*, J. Whiley & Sons, Inc., New York, 1969.

We integrate equation 35, using Simpson's rule, to obtain the plasma current $\left(I_p\right)$ at each time step. The numerical expression for I_p is

$$I_p^{n+1} = 2\pi \frac{\Delta r}{3} \left(r_o J_{zo}^{n+1} + 4r_1 \, J_{z1}^{n+1} + 2r_2 \, J_{z2}^{n+1} \right.$$

$$\left. + \; . \; . \; . \; . \; + 2 \, r_{I-3} \, J_{zI-3}^{n+1} + 4r_{I-2} \, J_{zI-2}^{n+1} + r_{I-1} \, J_{zI-1}^{n+1} \right) \qquad (36)$$

where $I = R_w/\Delta r$.

RESULTS AND CONCLUSIONS

The results of our computer runs are summarized in Table 1 and Figure 2. The beam parameters, with which equation 28 was solved, are given in the lower right-hand corner of this figure. The plasma currents in this table and figure were evaluated at the top of the current rise ($\tau=\tau_1$) for three different pressures, given in units of Torr.

TABLE 1. RESULTS OF COMPUTER RUNS

	P=760T	P=76T	P=7.6T
R_w (cm)	$\|I_p\|$ (KA)	$\|I_p\|$ (KA)	$\|I_p\|$ (KA)
3.0	3.3	3.8	8.5
5.0	6.5	7.1	14.0
8.0	9.5	10.0	17.9
10.0	10.7	11.2	19.3
12.0	11.7	12.1	20.3
15.0	12.7	13.0	21.3
20.0	13.9	14.1	22.3
35.0	15.7	15.9	23.8
50.0	16.7	16.8	24.5

The curves in Figure 2 indicate that, at each pressure, the plasma current increases significantly out to R_w = 20.0 cm. Thereafter, out to R_w = 50.0 cm, the plasma current increases only a few percentage points. That is, the conducting boundaries significantly influence the plasma current when the drift tube radius is less than 10 Bennet radii. Thus, for a beam with a 2 cm Bennet radius, the minimum drift tube radius beyond which wall effects are insignificant is 10 Bennet radii.

On the basis of our results, it is suggested that regardless of beam size wall affects are negligible when the ratio of drift tube radius to Bennet radius is at least 10. That is, drift tube propagation experiments may simulate propagation of beams through the air, provided that the ratio of beam radius to drift tube radius is greater than 10.

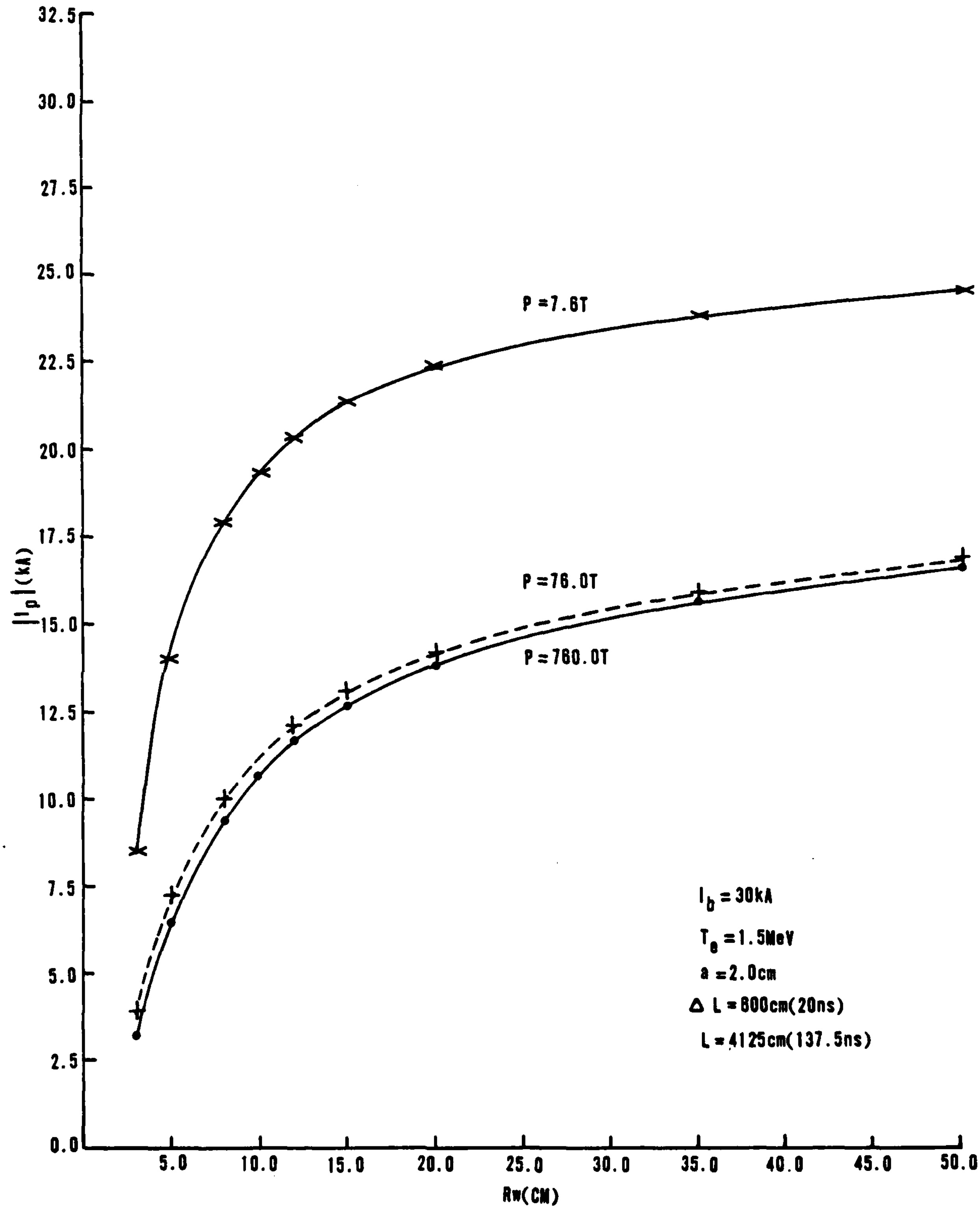

Figure 2. Results of computer runs.