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July 1981

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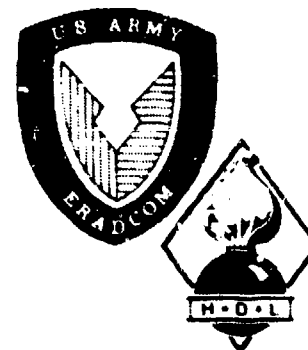
**Numerical Deconvolution to Obtain Cloud Signatures
from Scattered Light Pulses**

AD A108806

Prepared by James M. Supplee
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Obtain Cloud Signatures from Scattered Light Pulses, July 1981

TO: Recipients of HDL-CR-81-100-1

Please show the following corrections in subject report.

Page 8, paragraph 2, line 9

$[C(x_i) = 0.15 \text{ m}]$ should read $C(x_i = 0.15 \text{ m})$

Page 8, paragraph 2, line 11

$C(x)$ should read $C(x_i)$

Page 9, line 3

$C(x_{100}) = 15 \text{ m}$ should read $C(x_{100} = 15 \text{ m})$

FOR THE COMMANDER:

Nelda H. McNeil

NELDA H. MCNEIL

Acting Chief,

Technical Reports Branch

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20. ABSTRACT (Cont'd)

the return pulse by the range response. (This relatively good resolution is available in this way because the transmitted pulse is fairly short, being about 6 ns FWHM.) It is shown that modestly improved resolution is possible if the signal to noise ratio in the return pulse is greater than about 100 to 1.

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1. PROBLEM

Optical pulses that result from the reflection of GaAs laser pulses from water clouds have been measured. The amplitude of the return pulse as a function of time, $V(t)$, depends on the transmitted laser pulse, $P(t)$; the overall system sensitivity as a function of distance (range response) $R(x)$; and the optical properties of the cloud. For the present purpose, the properties of the cloud along the pencil beam path of the optical probe can be described by a single function, $C(x)$, the cloud signature.^{1,2} Light arriving at the detector at time t must have been transmitted at an earlier time, $t - \tau$ ($\tau > 0$). τ is the time that it takes light at speed c to travel from the transmitter to the point of reflection (distance $= x$) and then back to the detector, which is near the transmitter. Therefore,

$$\tau = 2x/c \quad (1)$$

All pairs of τ and x such that $\tau \geq 0$ and equation (1) holds will contribute to the return at time t , with the contribution weighted according to the system range response at the distance x and the cloud signature at that location. Therefore, the total return is given by the convolution

$$V(t) = K \int_0^\infty P(t - \tau) R(x) C(x) d\tau, \quad x = c\tau/2, \quad (2)$$

where K is for normalization.

The purpose of this study is to investigate a method for finding $C(x)$ given $V(t)$, $P(t)$, and $R(x)$. $P(t)$ and $R(x)$ are constant functions of the system. $V(t)$ has been measured for a large number of pulses.

One method for solving for $C(x)$ in equation (2) is as follows.² Using the notation of McGuire,² let

$$h(t) = K \cdot C(ct/2) \cdot R(ct/2) \quad (3)$$

Now, to consider equation (2) numerically, let

$$t = n \cdot \Delta t, \quad \tau = i \cdot \Delta t$$

¹H. H. Burroughs, *Computation of Cloud Backscatter Power as a Function of Time for an Active Optical Radar (U)*, Naval Weapons Center NWC TP 5090 (April 1971). (CONFIDENTIAL)

²Dennis McGuire and Michael Conner, *The Deconvolution of Aerosol Backscattered Optical Pulses to Obtain System-Independent Aerosol Signatures*, Harry Diamond Laboratories HDL-TR-1944 (June 1981).

The integral then becomes a sum:

$$V(n\Delta t) = \Delta t \sum_{i=1}^n P((n-i)\Delta t) \cdot h(i\Delta t) \quad (4)$$

The upper limit equals n because $P = 0$ for $t \leq 0$. Thinking of V and h as vectors, the notation becomes

$$\vec{V} = \Delta t \cdot P \vec{h} \quad (5)$$

where

$$P = \begin{pmatrix} P(\Delta t) & 0 & 0 & \dots & 0 \\ P(2\Delta t) & P(\Delta t) & 0 & \dots & 0 \\ P(3\Delta t) & P(2\Delta t) & P(\Delta t) & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ P(n\Delta t) & P((n-1)\Delta t) & \dots & \dots & P(\Delta t) \end{pmatrix}$$

The problem of solving for $C(x)$ (by here solving for h) then becomes one of matrix inversion. The solution to equation (5) is

$$\vec{h} = (1/\Delta t) P^{-1} \vec{V} \quad (6)$$

Equation (6) does not give satisfactory results² if V contains noise. This failure is probably because the form of $P(t)$ causes some large elements to appear in P^{-1} , giving too much significance to some small variation in V due to noise. It is not surprising that there is some difficulty related to dealing with P^{-1} . This difficulty is because the determinant of P is $[P(\Delta t)]^n$, which is very small since $P(\Delta t)$ is the first nonzero point of the transmitted pulse. One might easily expect this problem from another point of view; namely, since the convolution of C (eq. 2) will smooth out small bumps in C , the deconvolution of V using the same equation will badly exaggerate small bumps (noise).

²Dennis McGuire and Michael Conner, *The Deconvolution of Aerosol Backscattered Optical Pulses to Obtain System-Independent Aerosol Signatures*, Harry Diamond Laboratories HDL-TR-1944 (June 1981).

A way to avoid all such difficulty is to make the approximation that $P(t) = A \cdot \delta(t - t_0)$, where A is a normalization constant and t_0 is the center of the transmitted pulse. Then equation (2) immediately yields

$$C \left[\frac{c(t - t_0)}{2} \right] = \frac{V(t)}{K \cdot A \cdot R \left[\frac{c(t - t_0)}{2} \right]} \quad (7)$$

This discards all information about the shape of the transmitted pulse, thus reducing the resolution in $C(x)$ to about 1 to 2 m since the transmitted pulse has a full width at half maximum (FWHM) of about 6 ns.

2. DESCRIPTION OF METHOD

In this investigation, equation (7) is used as a first approximation to $C(x)$. Since one desires a cloud signature with better than the 1- to 2-m resolution achieved by using equation (7), some effort is then made to modify $C(x)$ to find a more accurate cloud signature. In this discussion, a more accurate (or "better") cloud signature refers to a signature with a smaller error, where error is defined as follows: Insert the cloud signature currently being considered into equation (2) to calculate the return pulse (V_c , subscript c for calculated) that would result from that signature. Compare that with the measured return pulse (V_m) defining

$$\text{error} = \int [V_m(t) - V_c(t)]^2 dt \quad (8)$$

Various functions (as discussed later) are tried for $C(x)$, and the $C(x)$ that gives the smallest error is recorded as the extracted cloud signature.

Now, equation (6) immediately gives the cloud signature with error = 0, but that signature is noise dominated nonsense.² Here, then, one is not seeking the absolute minimum in the error (which would be zero), but rather one seeks a relative minimum in error by varying $C(x)$ in some gentle way about the δ (delta) extracted (eq. 7) first guess.

The following measures are available to prevent the cloud signature from becoming wildly bumpy:

- a. Restrict how far the successive estimates of $C(x)$ can vary from the original δ extracted $C(x)$.

²Dennis McGuire and Michael Conner, *The Deconvolution of Aerosol Backscattered Optical Pulses to Obtain System-Independent Aerosol Signatures*, Harry Diamond Laboratories HDL-TR-1944 (June 1981).

- b. Subject the return pulse, $V_m(t)$, to a low-pass filter.
- c. Subject the final answer for $C(x)$ to a low-pass filter.

The severity of each of these restrictions is easily varied. It was hoped that with these restrictions a relative minimum in the error could be found associated with some reasonable cloud signature that would have the same gross properties as the delta extracted signature, but with sharper features. Before sample calculations were carried out, it was not clear whether restrictions and filtering (measures a to c) adequate to smooth out the noise would simultaneously doom resolution to worse than 1- to 2-m resolution of the delta function case. If filtering alone (b and c) can successfully suppress the noise in a given data bank while still allowing improved resolution (this would depend chiefly on the signal to noise ratio for that data), then one may consider using equation (6) after all, with the appropriate filtering of $V(t)$ and $C(x)$. On the other hand, if the a priori assumption that the desired cloud signature closely resembles the delta extracted signature is essential, then the method described below may be useful.

In the present method, the calculations are done by computer. The cloud signature, $C(x)$, is in digitized form with 100 values for C . These values are at 0.15-m intervals in distance (x), covering a total of 15 m. The first estimate of $C(x)$ is provided by the delta function method; that is, equation (7) is used for 100 values of t (corresponding to 100 values of x with $x = ct/2$). The various values of $C(x)$ are then varied in an effort to find a $C(x)$ with a smaller error (as defined above). The variation of C proceeds as follows: For a chosen initial value of Q ($Q > 1$), begin with the first point [$C(x_1) = 0.15$ m] and consider the following five possibilities:

- a. Leave $C(x)$ unaltered.
- b. Multiply $C(x_1)$ by Q .
- c. Divide $C(x_1)$ by Q .
- d. Multiply $C(x_1)$ by Q and decrease the next value, $C(x_{1+1})$, by the amount by which $C(x_1)$ increased.
- e. Divide $C(x_1)$ by Q and increase $C(x_{1+1})$ by the amount by which $C(x_1)$ decreased.

(Choices d and e are motivated by the consideration that they leave the integrated return less changed; since the original guess had approximately the correct total energy, these possibilities are probably desirable.) One then takes whichever of the five resulting cloud signatures that has the lowest associated error and considers it to be the

current (or "new and improved") cloud signature. This variational method is then repeated for the next point and successive points through $C(x_{100}) = 15$ m. This constitutes one pass. If after any pass the error is smaller than it was at the beginning of the pass, it is deemed worthwhile to make another pass by using a smaller value of Q ,

$$Q_{\text{new}} = (nQ + 1)/(n + 1), \quad n > 0,$$

to achieve finer variations in $C(x)$. The flow chart and coding for the computer program that carries out this procedure are shown in appendices A and B, respectively. The values for n and the initial Q can be specified as desired for each computer run. The particular values of Q_1 ($= Q_{\text{initial}}$) and n that were used to generate the examples in this report are given in appendix C. The cloud signature is constrained to vary only within a certain region because no value of C could be multiplied or divided by more than approximately $Q_1 \cdot Q_2 \cdot Q_3 \cdot \dots \cdot Q_n$, and this product has a finite value depending on Q_1 ($= Q_{\text{initial}}$) and n (see app D).

In this way, $C(x)$ varies until some relative minimum in the error is found. When a pass is executed (one Q , all 100 points) with no decrease in the error, the process is stopped and the cloud signature is recorded.

3. RESULTS

The characteristics of the signature found by this method depend critically on the signal to noise ratio of the return signal, $V(t)$, as the following examples show.

3.1 Case I

An idealized cloud with the backscatter coefficient proportional to the extinction coefficient³ and extinction coefficient profile as shown in figure 1 results in the cloud signature shown in figure 2. Here one assumes^{1,2}

$$C(x) = \mu(x) e^{-2 \int_0^x \sigma(s) ds}.$$

¹H. H. Burroughs, *Computation of Cloud Backscatter Power as a Function of Time for an Active Optical Radar (U)*, Naval Weapons Center NWC TP 5090 (April 1971). (CONFIDENTIAL)

²Dennis McGuire and Michael Conner, *The Deconvolution of Aerosol Backscattered Optical Pulses to Obtain System-Independent Aerosol Signatures*, Harry Diamond Laboratories HDL-TR-1944 (June 1981).

³D. Diermendjian, *Electromagnetic Scattering on Spherical Polydispersions*, American Elsevier, New York (1969). The ratio of backscatter coefficient to extinction coefficient depends on the wavelength of the radiation and the aerosol particle size distribution. If the latter is a locally homogeneous property of the cloud, μ/σ (backscatter/extinction coefficients) will be constant.

The return pulse (calculated by a straightforward convolution, eq 2) that would result from such a cloud was subjected to the present signature extraction algorithm. The algorithm includes filtering out high frequency bumps in the return pulse and signature. The resulting extracted signature (fig. 3) is relatively close to the actual signature and provides better resolution of the sharp peak than the delta function extracted signature (fig. 4).

Noise in the return signal becomes exaggerated on deconvolution. This exaggeration means that the signature extraction becomes less reliable as the signal to noise ratio decreases. To demonstrate this effect, the return pulse from the idealized cloud has been corrupted with noise and the signature extraction has been attempted again. The results for a signal to noise ratio of about 100 to 1 are shown in figures 5 and 6 and for a signal to noise ratio of about 40 to 1 are shown in figures 7 and 8. For a signal to noise ratio of 100 to 1, the initial rise and sharp peak are just slightly more accurate than the delta function was, but a large amount of noise has shown up in the exponentially decaying tail. For a signal to noise ratio of about 40 (fig. 6), the situation is worse.

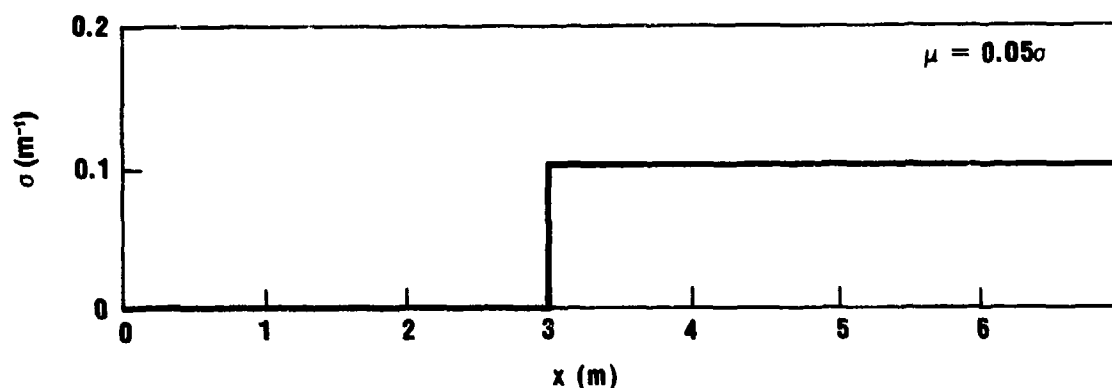


Figure 1. Case I: extinction coefficient.

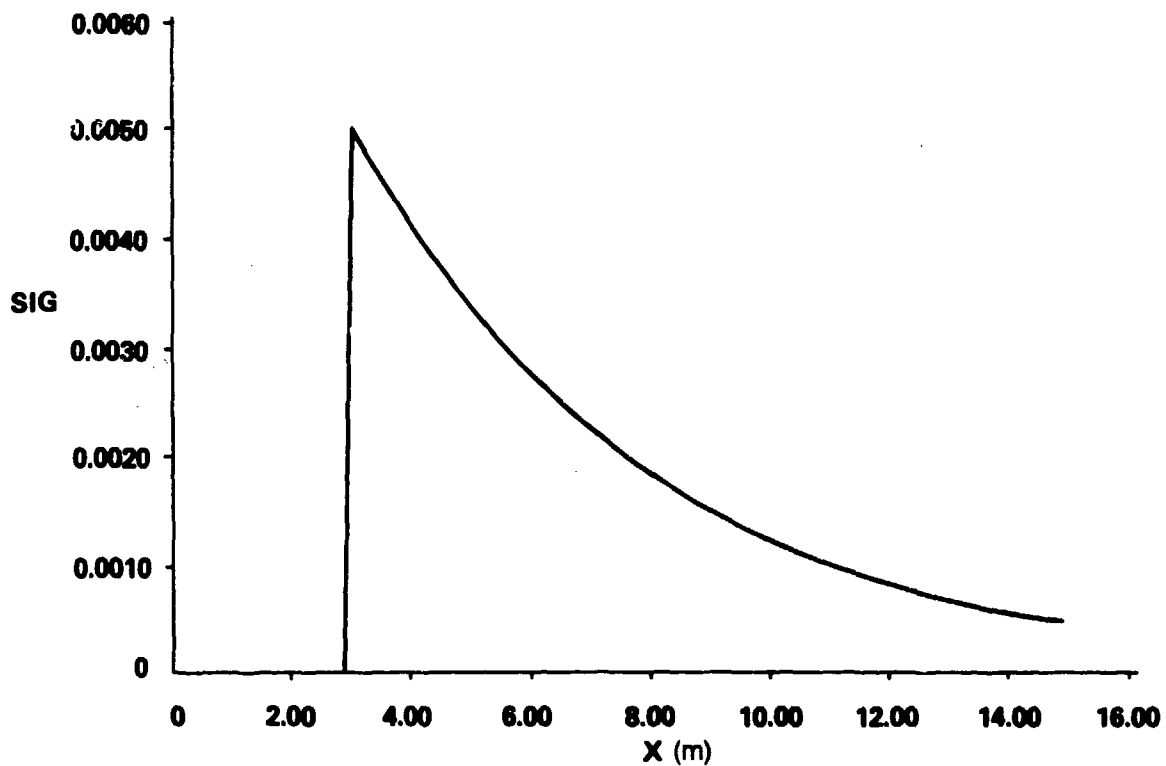


Figure 2. Case I: cloud signature.

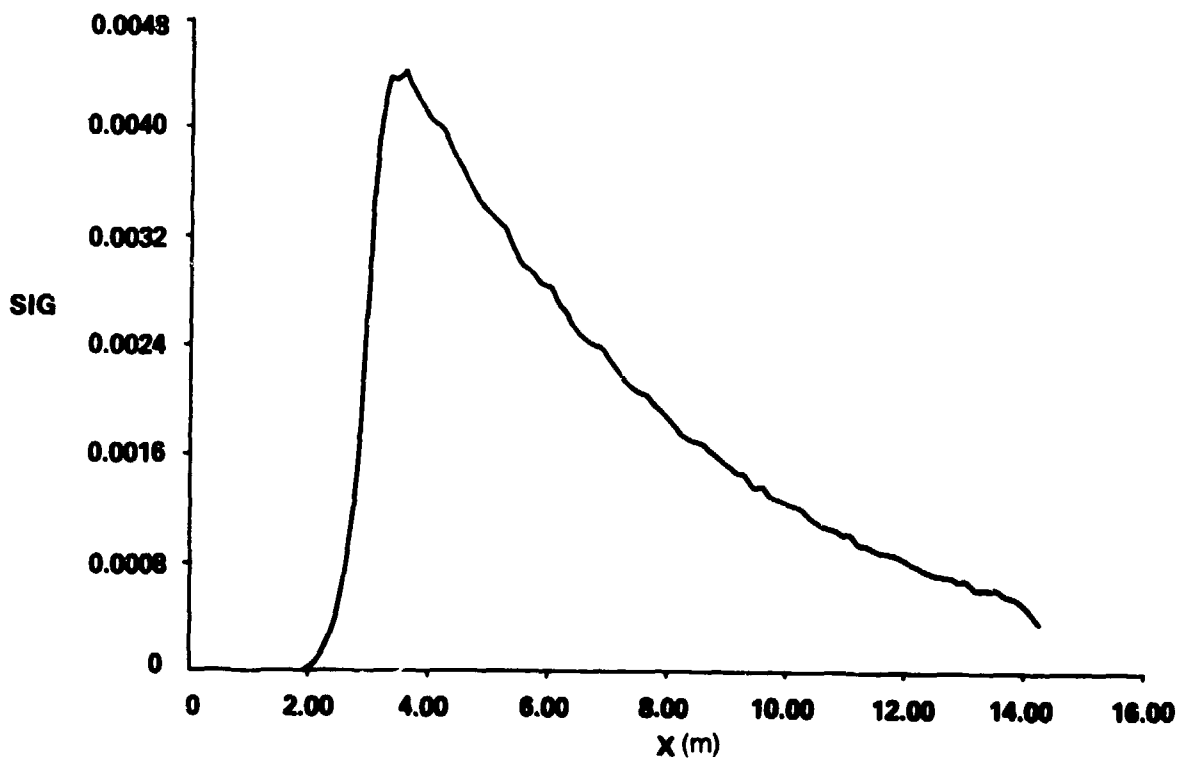


Figure 3. Case I: extracted cloud signature (noiseless return pulse).

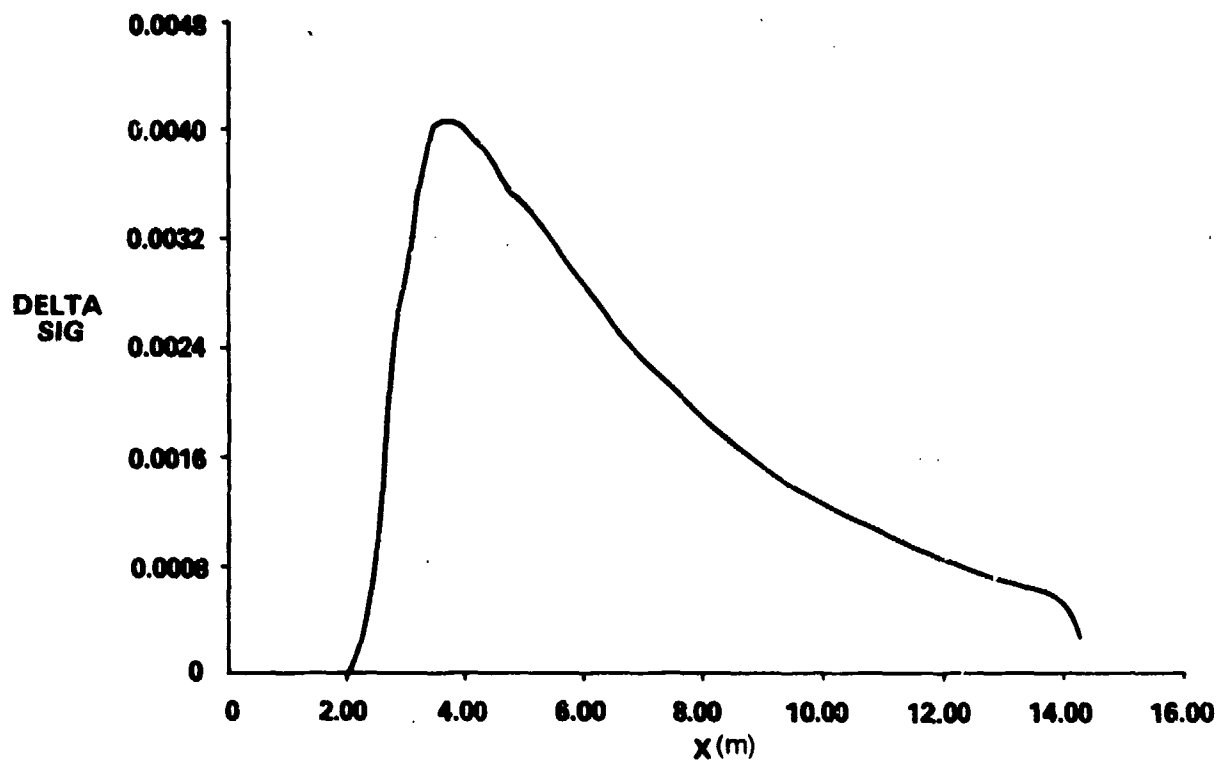


Figure 4. Case I: delta extracted cloud signature (noiseless return pulse).

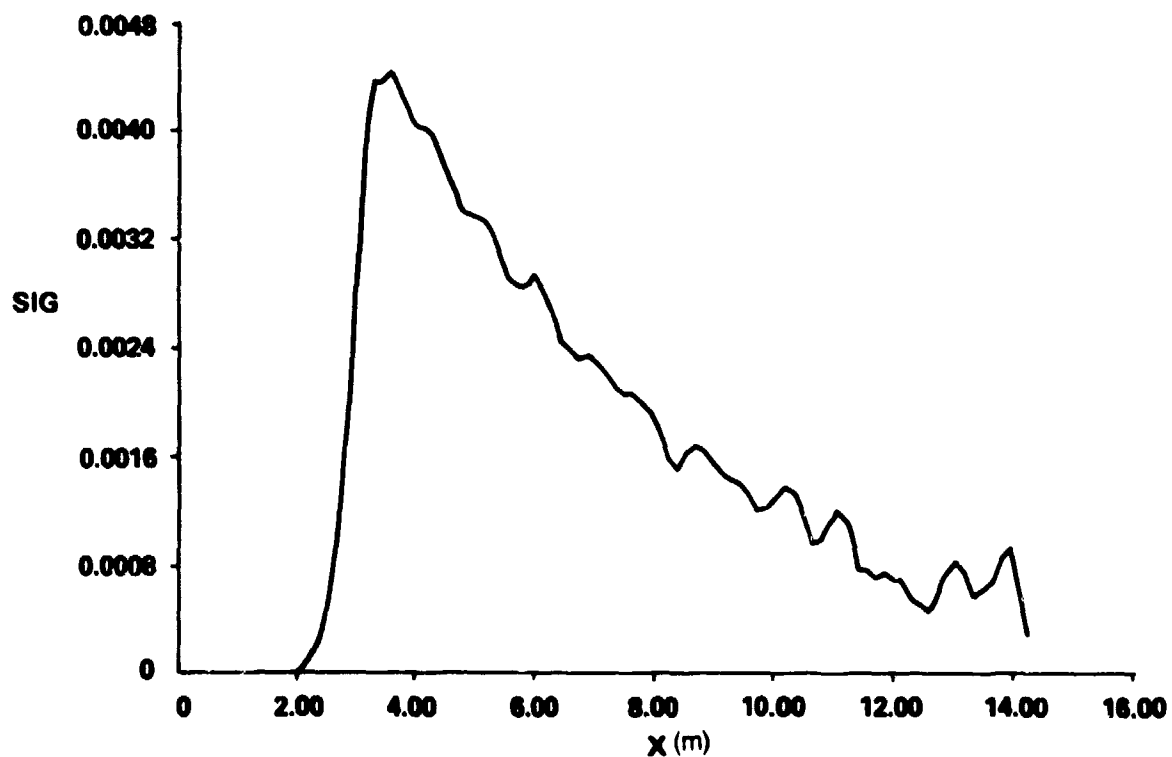


Figure 5. Case I: extracted cloud signature (return pulse signal to noise ratio ~ 100).

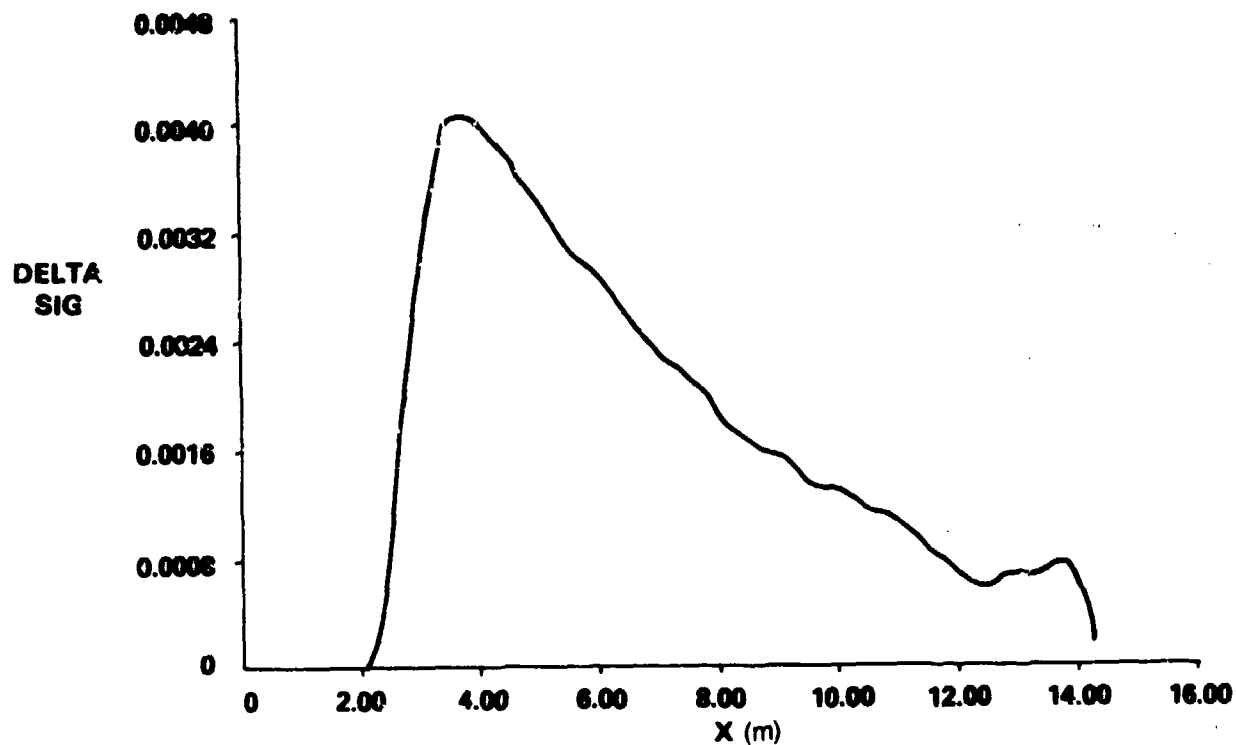


Figure 6. Case I: delta extracted cloud signature
(return pulse signal to noise ratio ~ 100).

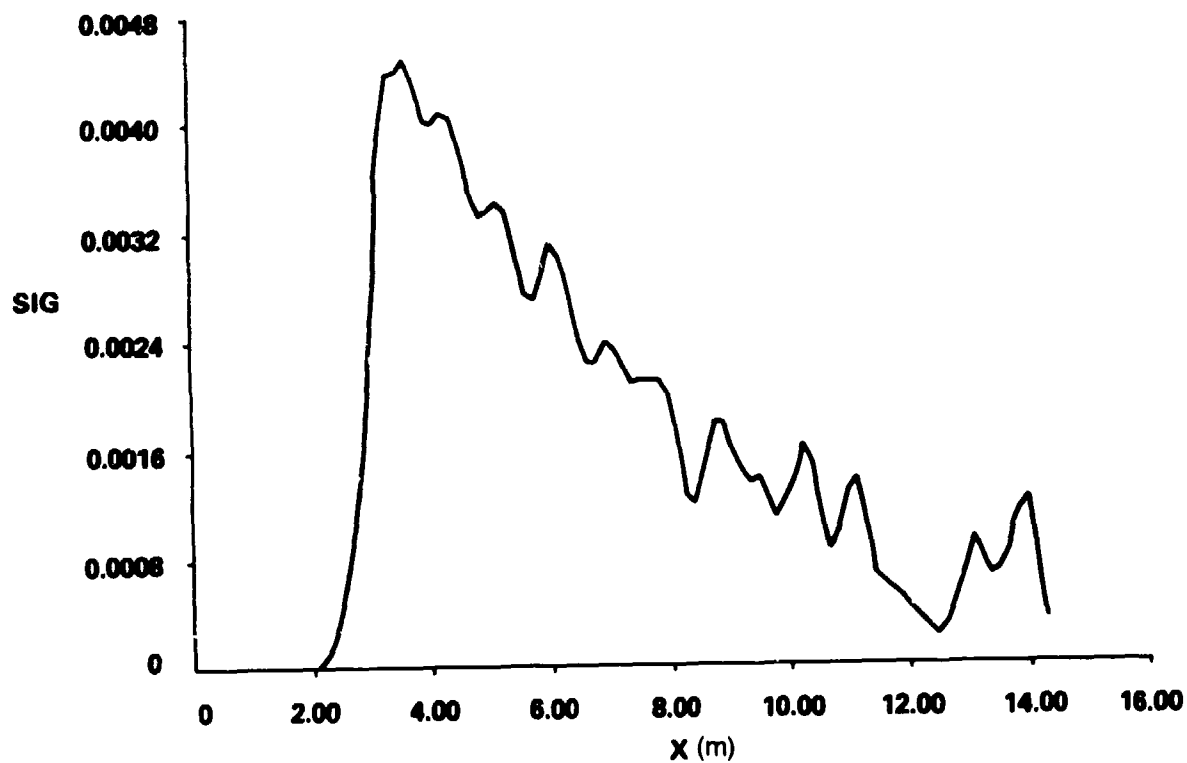


Figure 7. Case I: extracted cloud signature
(return pulse signal to noise ratio ~ 40).

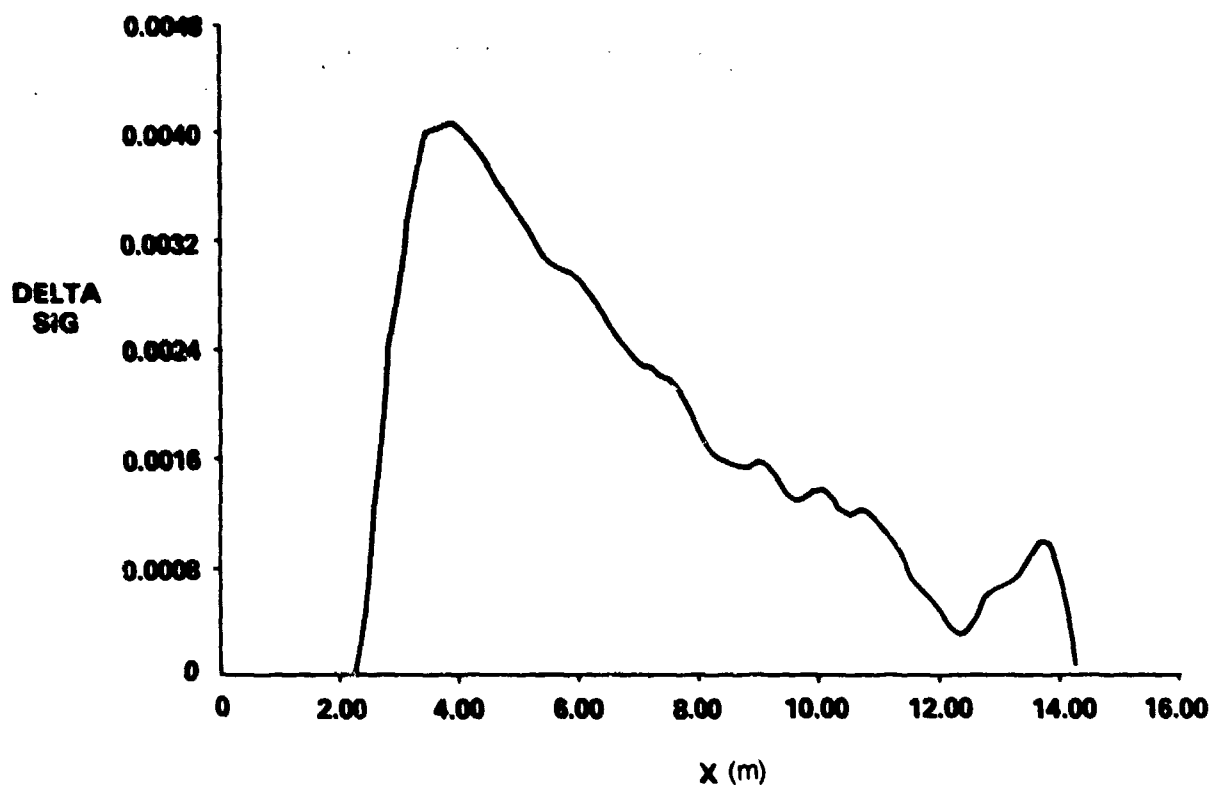


Figure 8. Case I: delta extracted cloud signature (return pulse signal to noise ratio ~ 40).

3.2 Case II

Figures 9 to 16 follow the same format as figures 1 to 8, but for a different idealized cloud, case II. Here the improvement is less substantial because the case II cloud has a softer edge, so the delta extracted signature is closer to the correct signature.

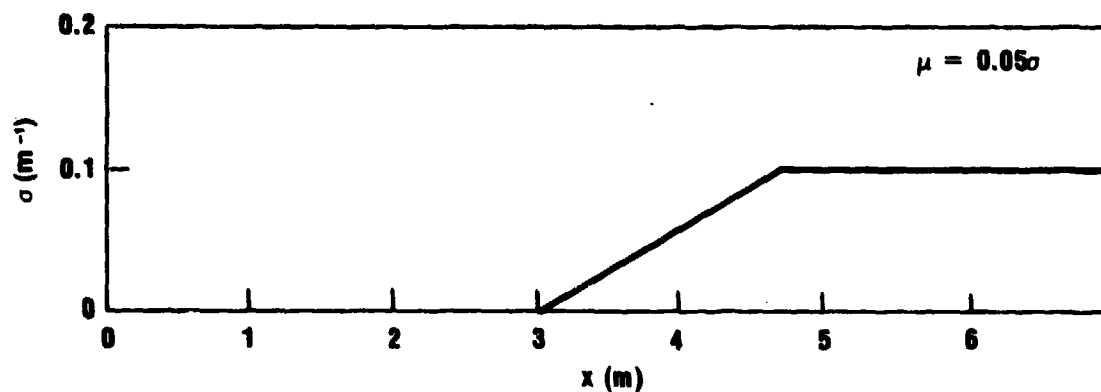


Figure 9. Case II: extinction coefficient.

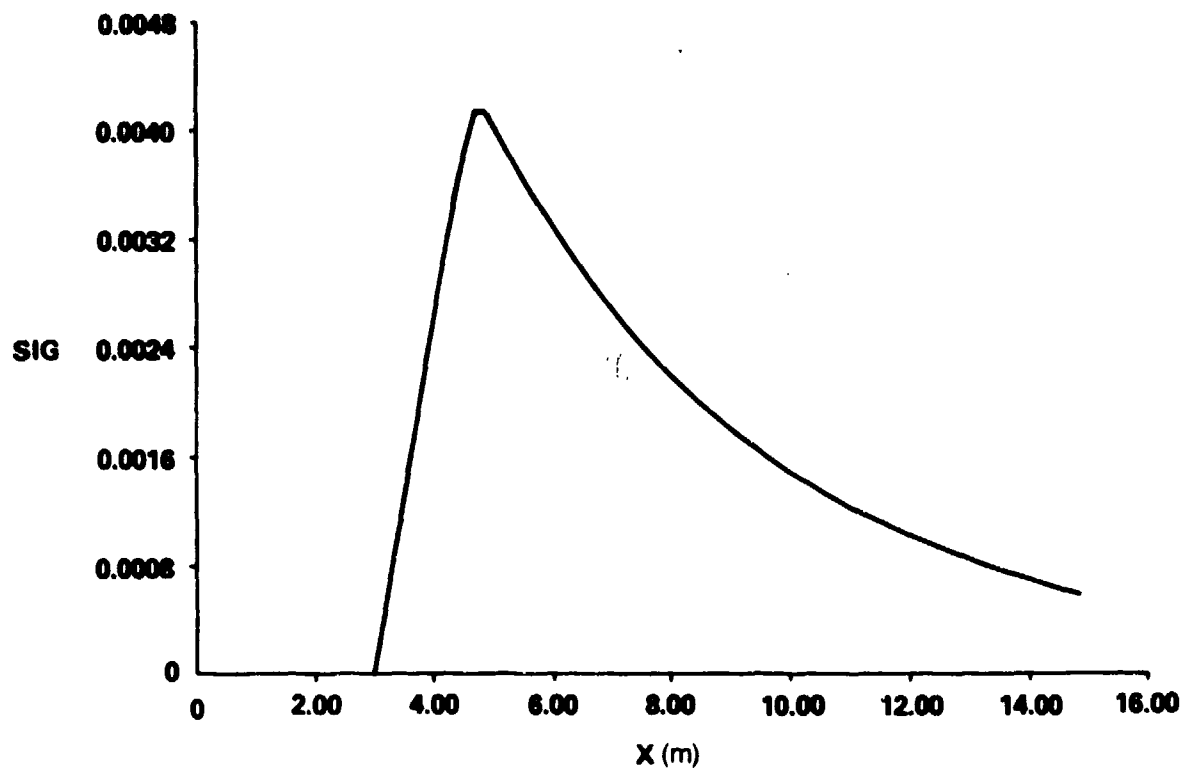


Figure 10. Case II: cloud signature.

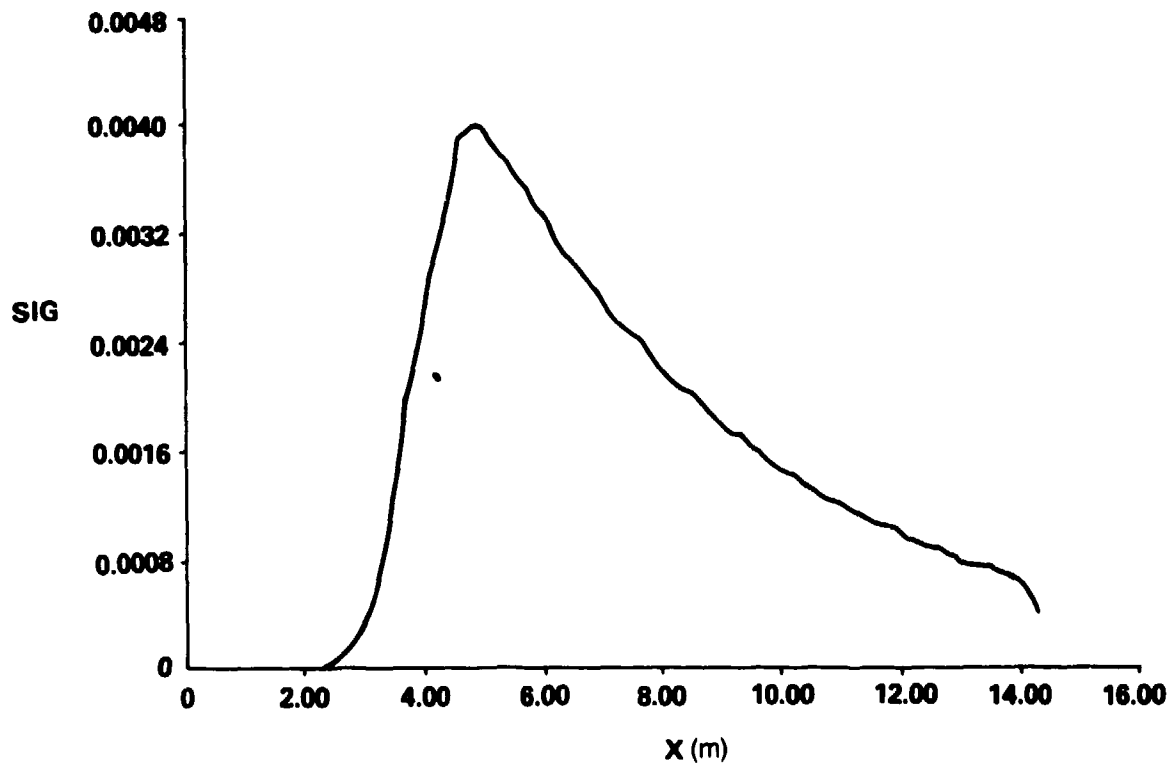


Figure 11. Case II: extracted cloud signature (noiseless return pulse).

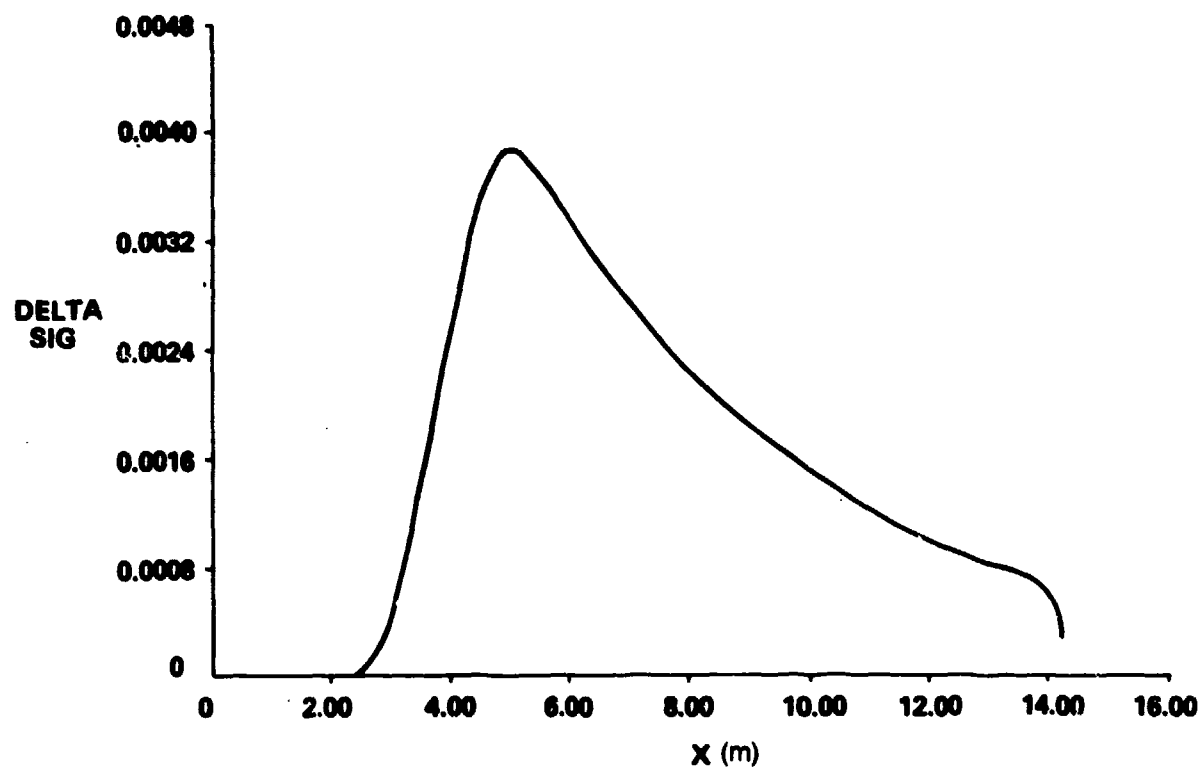


Figure 12. Case II: delta extracted cloud signature (noiseless return pulse).

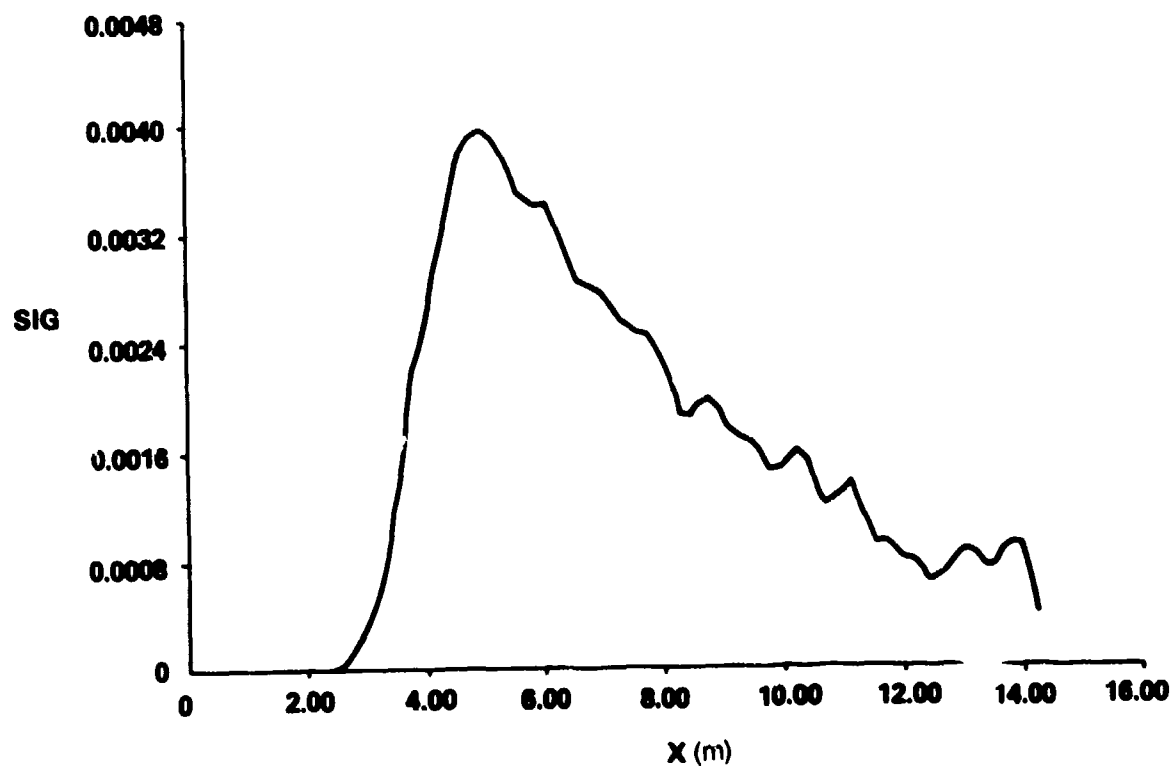


Figure 13. Case II: extracted cloud signature (return pulse signal to noise ratio ~ 100).

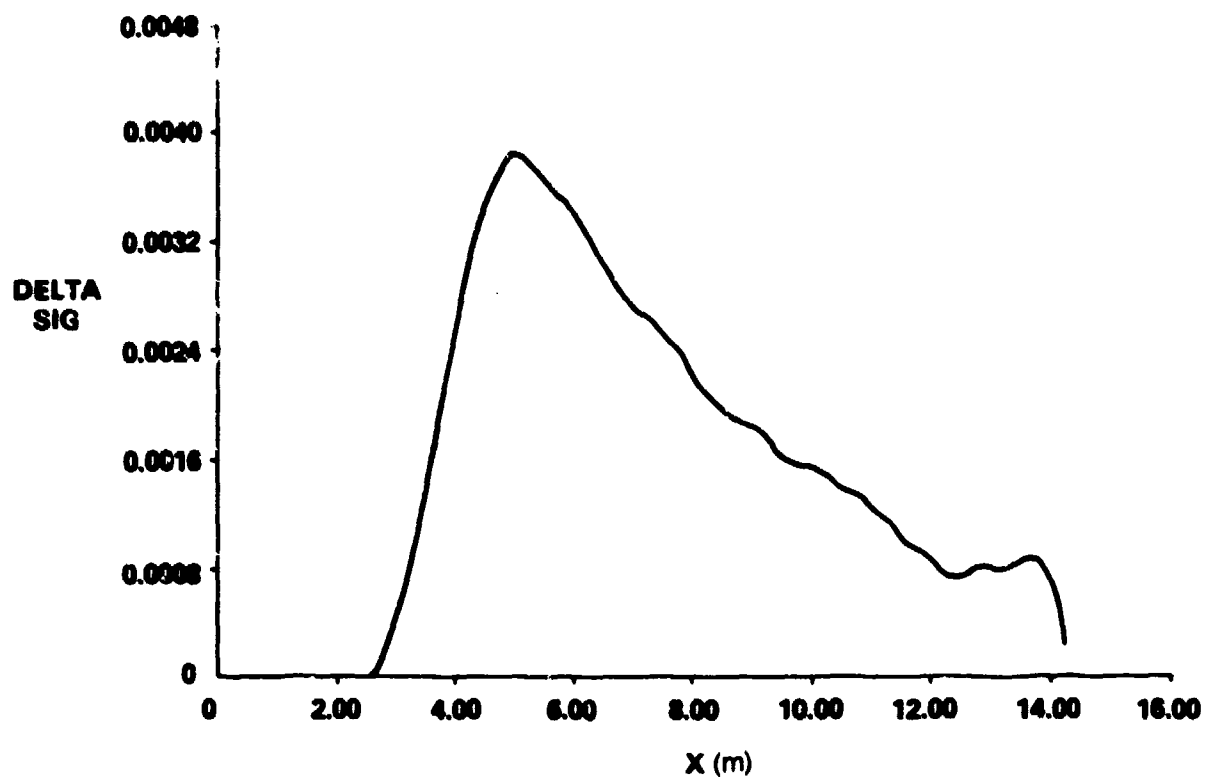


Figure 14. Case II: delta extracted cloud signature
(return pulse signal to noise ratio ~ 100).

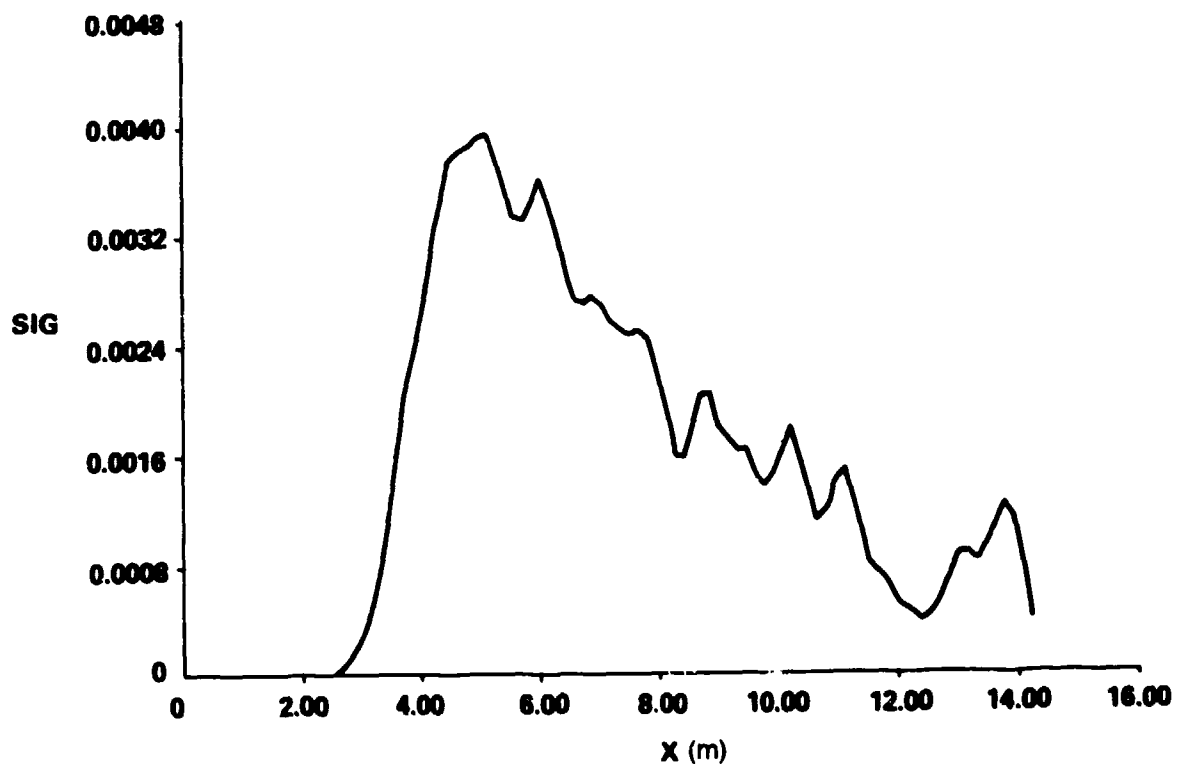


Figure 15. Case II: extracted cloud signature
(return pulse signal to noise ratio ~ 40).

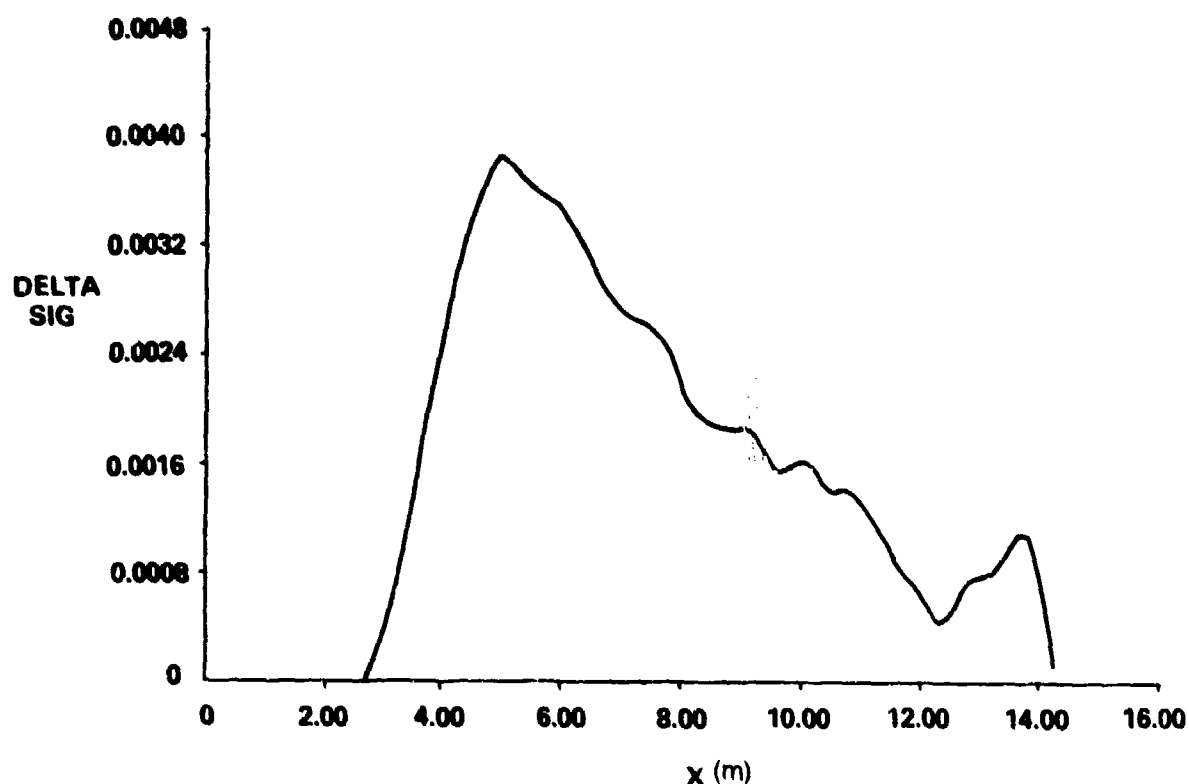


Figure 16. Case II: delta extracted cloud signature (return pulse signal to noise ratio ~ 40).

4. CONCLUSIONS

If the data have a signal to noise ratio of about 100 to 1 or better, it is possible to achieve slightly better resolution in the cloud signature than the resolution that would be available from equation (7), dividing by the range response (delta function extraction). The extent of the improvement in the resolution depends on the signal to noise ratio. There is no evidence that restriction a (sect. 2) (limiting how far estimates of the cloud signature can vary from the delta function estimate) is essential to the reasonable behavior of the cloud signature. Low-pass filtering of the return pulse or resulting cloud signature probably is necessary. This necessity indicates that, with a signal to noise ratio greater than 100 to 1 and adequate filtering, perhaps equation (6) should be reconsidered as a method for solving this problem. It may be worthwhile to consider using a hybrid method by which equation (6) could be used to treat that portion of the return where the signal is high, while settling for the delta extracted signature where the signal is lower.

It was considered desirable to analyze a select portion of the measured data by using the present method. Because of a computer malfunction, this analysis has not been done. It is suggested that for each return pulse two estimates of the cloud signature be recorded, one using the algorithm discussed in this report and, for comparison, one using the delta function extraction. Unlike the test cases shown in the figures, the real data provide no answer key and, hence, it may not be clear which signature is more accurate. However, it may prove instructive to generate graphs of the two estimates for various pulses and to note the similarities and differences in the two.

Interestingly, McGuire² predicts that the signal to noise ratio should decrease by a factor of about 0.14 on deconvolution, with filtering of the return pulse as was done in the test cases. This compares the signal to noise ratio of the filtered return with that of the cloud signature. In the present study, the signal to noise ratio in the unfiltered return signal is compared with that of the cloud signature. Also, the cloud signature was smoothed here. For these two reasons, one expects the apparent increase in noise to be less than that predicted by McGuire.² The two test cases here show an increase in noise (compared with signal) on deconvolution by a factor of about three, roughly consistent with this expectation.

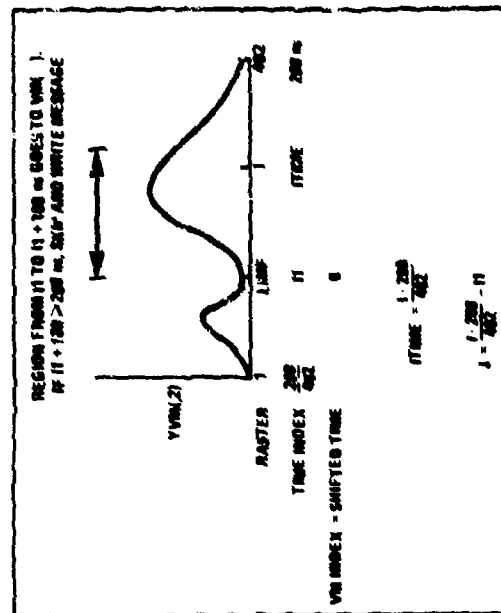
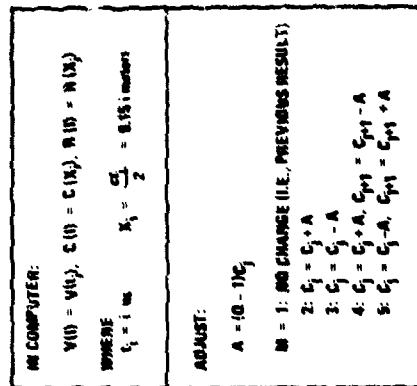
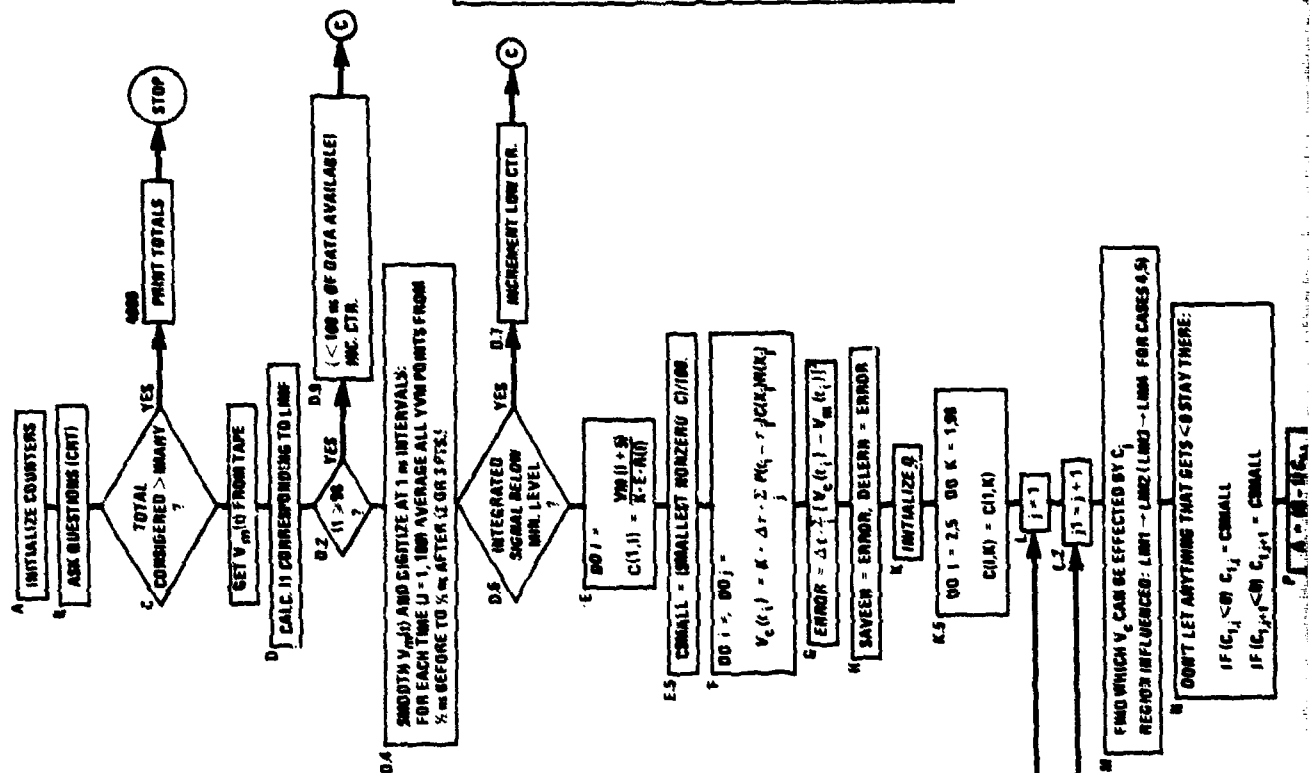
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²Dennis McGuire and Michael Conner, *The Deconvolution of Aerosol Backscattered Optical Pulses to Obtain System-Independent Aerosol Signatures*, Harry Diamond Laboratories HDL-TR-1944 (June 1981).

**APPENDIX A.--FLOW CHART OF COMPUTER PROGRAM FOR CLOUD
SIGNATURE EXTRACTION**

A computer program was developed to deconvolve an optical return pulse in such a way as to minimize the effect of return signal noise. The flow chart for this computer program follows (fig. A-1).



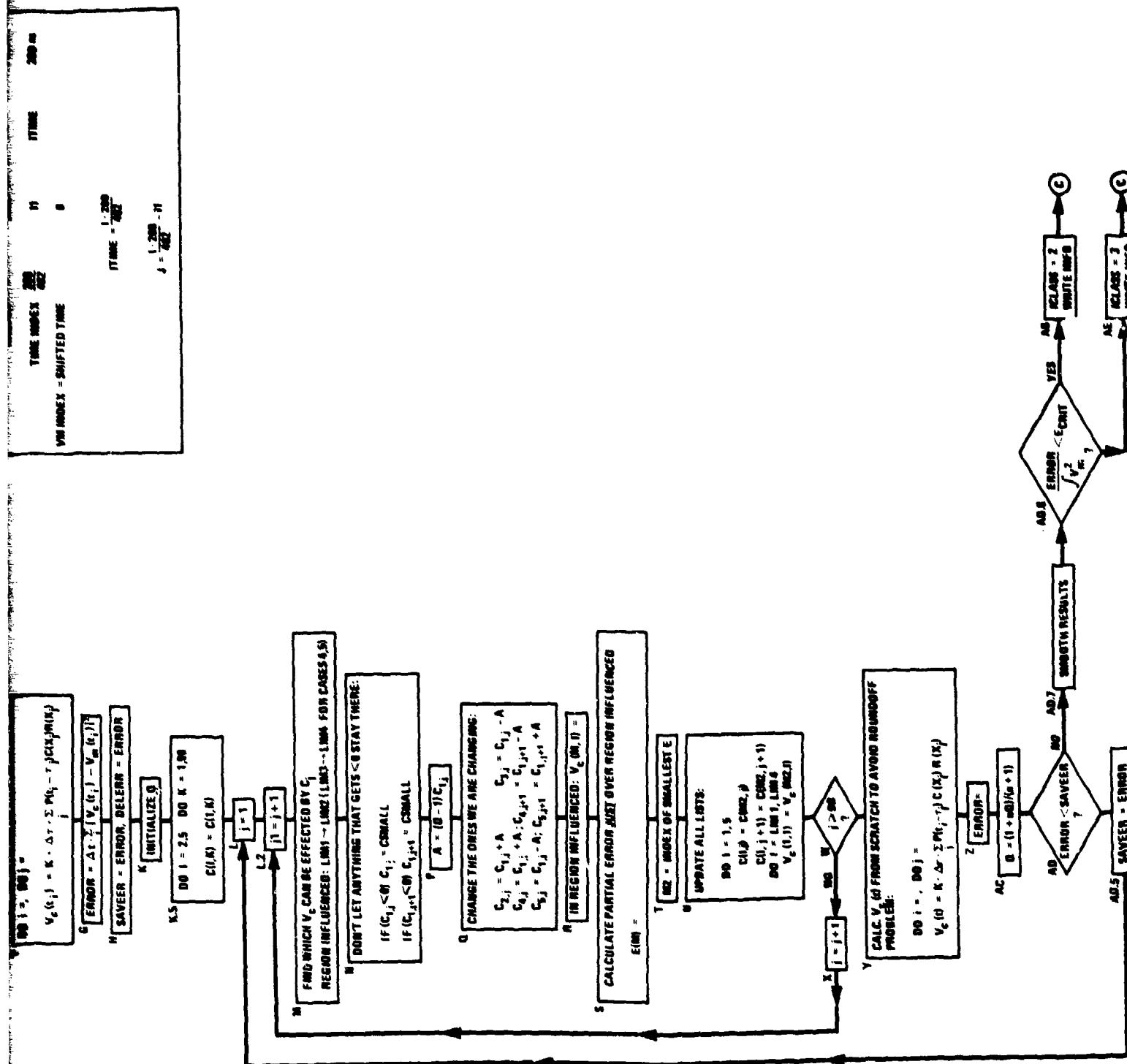


Figure A-1. Flow chart of computer program for cloud signature extraction.

APPENDIX B.--COMPUTER CODE FOR CLOUD SIGNATURE EXTRACTION

A computer program was developed to deconvolve an optical return pulse in such a way as to minimize the effect of return signal noise. A FORTRAN listing of this program follows.

APPENDIX B

```

1  SBATCH
2  C
3  C
4  C
5  C      VERSION: AUGUST 20, 1980
6  C      CLOUD SIGNATURE EXTRACTION
7  C
8  C
9  C
10 C
11 C
12 C*****
13 C
14 C      ASK JIM GRIFFIN IF CALL
15 C      FOR "GETFR" HAS CHANGED.
16 C
17 C*****
18 C
19 C      WARNING:
20 C      THIS PROGRAM ASSUMES P=0 FOR T GE 13 NS (TO SAVE CPU TIME). IF THIS
21 C      PROGRAM IS USED WITH A TRANSMITTER PULSE THAT EXCEEDS 13 NS, THE SUM
22 C      LIMITS NEED TO BE MODIFIED.
23 C
24 C
25 C      SOME VARIABLES:
26 C      UM(I): (INPUT) MEASURED RETURN PULSE AT EACH NSEC (SHOULD BE SMOOTHED
27 C              OVER ABOUT 1 OR 2 NSEC)
28 C      R(I): RANGE RESPONSE. X=IX.15 M (=CXT/2)
29 C      P(I): TRANSMITTER PULSE (EACH NSEC)
30 C      C(M,I): CLOUD SIGNATURE (VARIATION M) X=IX.15 M
31 C              CLOUD SIGNATURE PRINTED IS FOR I=11 TO 98.
32 C      UC(M,I): RETURN PULSE AT EACH NSEC THAT WOULD RESULT FROM THIS SIGNATURE.
33 C              (VARIATION M)
34 C              COMPARE IT WITH UM(I) TO SEE HOW WELL WE DID.
35 C      VARIATION M:
36 C          A=(Q-1.)XC(J)
37 C          M=1: NO CHANGE (I.E. PREVIOUS RESULT)
38 C              2: C(J)=C(J)+A
39 C              3: C(J)=C(J)-A
40 C              4: C(J)=C(J)+A, C(J+1)=C(J+1)-A
41 C              5: C(J)=C(J)-A, C(J+1)=C(J+1)+A
42 C
43 C      COMMON /BR/TRASH(20),LIMN,LIMF
44 C      COMMON /FRAME/YUM(482,4),IFRAM1,IFRAM2
45 C      COMMON /UNITS/LUOUT
46 C      INTEGER*2 NAM1(9)
47 C      DIMENSION UC(5,100),UM(100),C(5,99),R(99),P(12),E(5),
48 C      CTMATRIX(100),RASTER(482),CSHIFT(85),
49 C      CKSHIFT(85),CTEMP(85),UCPLOT(100),VPASS(482)
50 C      EQUIVALENCE (YUM(1,2),VPASS(1))
51 C      DATA P/0.44,3.60,4.96,5.68,5.88,5.50,4.58,3.20,1.70,0.62,0.27,0.10
52 C
53 C      DATA R/1.,4.,10.,15.,24.,30.,31.,32.,33.,35.,55.,80.,105.,140.,
54 C      C175.,210.,250.,300.,350.,430.,480.,520.,550.,585.,605.,615.,
55 C      C622.,625.,618.,612.,608.,595.,579.,565.,550.,535.,520.,505.,
56 C      C485.,470.,455.,443.,431.,418.,402.,390.,377.,362.,350.,338.,
57 C      C325.,313.,300.,288.,280.,268.,258.,243.,235.,245.,238.,230.

```

APPENDIX B

```

52      C235..210..210..204..198..191..187..182..177..173..171..168..
53      C161..157..152..147..142..140..137..132..128..125..120..117..
54      C113..111..108..104..101..99..98..95..92..90..88..84./
55      C
56      C *****
57      CALL SETPDG
58      CALL INITG(1.0,.8)
59      C
60      C NOT NORMALIZED NOW
61      C FKKKK=1.
62      C
63      IOUTDU=2
64      INDU=1
65      U OUTPUT DEVICE 8:
66      LUOUT=8
67      C *****
68      C
69      C FLOW CHART BOX A
70      C
71      LOWCTR=0
72      ICTR1=0
73      ICTR2=0
74      ICTR3=0
75      ITOTCT=0
76      DO 125 I=1,100
77          FI=I
78          TMRX(I)=FI
79      125 CONTINUE
80      DO 125 I=1,85
81          K=I+10
82          FK=K
83          XSHIFT(I)=FK*.15
84      125 CONTINUE
85      DO 130 I=1,488
86          FI=I
87          RASTER(I)=FI
88      130 CONTINUE
89      C
90      C FLOW CHART BOX B
91      C
92      WRITE(IOUTDU,90113)
93      READ(INDU,90103)SIGMIN
94      WRITE(IOUTDU,90118)
95      WRITE(IOUTDU,90502)
96      READ(INDU,90505)FMANY
97      IMANY=FMANY
98      WRITE(IOUTDU,90101)
99      READ(INDU,90103)ECRIT
100      WRITE(IOUTDU,90105)
101      WRITE(IOUTDU,90502)
102      READ(INDU,90505)Q1
103      WRITE(IOUTDU,90107)
104      WRITE(IOUTDU,90502)
105      READ(INDU,90505)FN
106      WRITE(IOUTDU,90221)
107      WRITE(IOUTDU,90502)
108      READ(INDU,90505)XTIMEI

```

APPENDIX B

```

115      WRITE(IOUTDU,90523)
116      WRITE(IOUTDU,90524)
117      READ(INDU,90525)ETIMEO
118      WRITE(IOUTDU,90526)
119      WRITE(IOUTDU,90527)
120      READ(INDU,90528)PLOTFL
121      IPLOTFL=PLOTFL+.01
122      WRITE(IOUTDU,90108)
123      READ(INDU,90529)(NAM1(I),I=1,9)
124      CALL OPENU(LUOUT,NAM1,3,0,0,IST)
125      DELT=1.E-8
126
127      C
128      C      FLOW CHART BOX C
129      C
130      300      ITOTCT=ITOTCT+1
131      IF(ITOTCT.GT.IPMNY)GO TO 4000
132      CALL GETFR(ISTAT,ITEMP,122..1,0,95,95)
133      IF(ITEMP.GT.0.AND.ITEMP.LE.12)GO TO 4000
134      IF(ISTAT.NE.0)GO TO 5000
135      305      IF(IPLOTFL.EQ.1)CALL SUPPLY(RASTER,422,'RASTER',0,
136      CYPASS,'RAW PULSE',0,IFRAN1)
137      MAKE PULSE RIGHT SIDE UP!
138      C
139      DO 310 I=1,422
140      YUM(I,2)=255.-YUM(I,2)
141      310      CONTINUE
142      C
143      C      FLOW CHART BOX D
144      C
145      FLIMF=LIHF
146      FI1=FLIMF*200./422.
147      FI1=FI1+.5
148      I1=FI1
149      C
150      C      FLOW CHART BOX D.2
151      C
152      IF(I1.GE.95)GO TO 400
153      C
154      C      FLOW CHART BOX D.4
155      C
156      FI1=I1
157      C      BELOW:
158      C      J IS UM INDEX = SHIFTED TIME IN MSEC.
159      C      K IS RASTER INDEX
160      C
161      DO 457 J=1,100
162      FJ=J
163      TEMP1=FJ-2.*ETIMEI
164      TEMP2=FJ+2.*ETIMEI
165      FLIM1=(TEMP1+FI1)*422./200.
166      FLIM2=(TEMP2+FI1)*422./200.
167      LIM1=FLIM1
168      LIM2=FLIM2
169      IF(LIM1.LT.LIMF)LIM1=LIMF
170      IF(LIM2.GT.422)LIM2=422
171      SUM=0.
172      FNORM=0.
173
174      TEMP4=FJ+FI1
175      DO 445 K=LIM1,LIM2
176      FK=K
177      TEMP3=ABS((TEMP4-FK*200./422.)/ETIMEI)
178      TEMP7=EXP(-TEMP3)
179      SUM=SUM+YUM(K,2)*TEMP7
180      FNORM=FNORM+TEMP7
181      445      CONTINUE
182      UM(J)=SUM/FNORM
183      457      CONTINUE
184      C

```

APPENDIX B

```

183 C
184 C FLOW CHART BOX D.6
185 C
186 C JIM GRIFFIN SUGGESTS AVOIDING THE LAST 14 RASTER
187 C POINTS TO ESTABLISH BASELINE.
188 C FIND BASELINE:
189 C TEMP=0.
190 DO 462 I=448,467
191 TEMP=TEMP+VM(I,2)
192 462 CONTINUE
193 BASE=TEMP/20.
194 C SUBTRACT BASELINE
195 DO 464 I=1,100
196 UM(I)=UM(I)-BASE
197 IF(UM(I).LT.0.)UM(I)=0.
198 464 CONTINUE
199 SUM=0.
200 DO 465 I=1,100
201 SUM=SUM+UM(I)
202 465 CONTINUE
203 IF(SUM.GE.SIGMIN)GO TO 500
204 C
205 C FLOW CHART BOX D.7
206 C
207 C LOWCTR=LOWCTR+1
208 GO TO 300
209 C FLOW CHART BOX E
210 C
211 DO 520 I=1,95
212 J=I+5
213 C(1,I)=UM(J)/(FKKKK*3.657E-08R(I))
214 520 CONTINUE
215 DO 530 I=96,99
216 C LACKING ANY BETTER INFORMATION, TRY, FOR INITIAL VALUES:
217 C(1,I)=C(1,95)
218 530 CONTINUE
219 C
220 C
221 C*****
222 C*****
223 C***** JIM GRIFFIN:
224 C***** HERE, WRITE TWO COPIES OF THE FRAME CODE
225 C***** (IFRAM1, IFRAME) ON THE OUTPUT TAPE.
226 C***** ALSO, WRITE TO TAPE: C(1,I), I=11,95
227 C***** (THIS IS THE CLOUD SIGNATURE OBTAINED
228 C***** BY DIVIDING BY THE RANGE RESPONSE.)
229 C*****
230 C*****
231 C
232 C
233 C
234 C FLOW CHART BOX E.6
235 C
236 DO 552 I=1,95
237 IF(C(1,I).NE.0.)GO TO 553
238 552 CONTINUE
239 553 TEMP=C(1,I)

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APPENDIX B

```

240      DO 500 I=1,50
241      IF(C(1,I).NE.0...AND.C(1,I).LT.TEMP)TEMP=C(1,I)

242      500 CONTINUE
243      CSMALL=TEMP/100.
244      C
245      C FLOW CHART BOX F
246      C
247      UC(1,1)=0.
248      DO 620 I=2,100
249      UC(1,I)=0.
250      ITEM=I-12
251      IF(ITEM.LT.1)ITEM=1
252      LIM2=I-1
253      DO 620 J=ITEM,LIM2
254      M=I-J
255      UC(1,I)=UC(1,I)+P(M)SC(1,J)SR(J)
256      620 CONTINUE
257      UC(1,I)=FKKKK*DELLT*UC(1,I)
258      630 CONTINUE
259      C
260      C FLOW CHART BOX G
261      C
262      SUM=0.
263      DO 730 I=1,100
264      TEMP2=UC(1,I)-UM(I)
265      SUM=SUM+TEMP2*TEMP2
266      730 CONTINUE
267      ERROR=DELLT*SUM
268      C

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APPENDIX B

```

269 C      FLOW CHART BOX H
270 C
271      SAVEEN=ERROR
272      DELEEN=ERROR
273 C
274      IF(IPLTF.NE.1)GO TO 280
275      DO 240 I=1,25
276          K=I+10
277          CTEMP(I)=C(1,K)
278      240 CONTINUE
279      CALL SUBPLT(XSHIFT,25,'X',1,CTEMP,'DELTA SIG',9,IFRAME1)
280      250 CONTINUE
281 C
282 C      FLOW CHART BOX K
283 C
284      Q=Q1
285 C
286 C      FLOW CHART BOX K.E
287 C
288      DO 1155 I=2,5
289          DO 1155 K=1,20
290              C(I,K)=C(1,K)
291      1155 CONTINUE
292 C
293 C      FLOW CHART BOX L
294 C
295      1200 J=1
296 C
297 C      FLOW CHART BOX L.E
298 C
299      1220 J1=J+1
300 C
301 C      FLOW CHART BOX M
302 C
303      LIM1=J+1
304      LIM2=J+12
305      LIM3=.1+2
306      LIM4=J+13
307      IF(LIM2.GT.100)LIM2=100
308      IF(LIM4.GT.100)LIM4=100
309 C
310 C      FLOW CHART BOX N
311 C
312      IF(C(1,J).LE.0.)C(1,J)=CSMALL
313      IF(C(1,J1).LE.0.)C(1,J1)=CSMALL
314 C
315 C      FLOW CHART BOX P
316 C
317      A=(Q-1.)XC(1,J)
318 C
319 C      FLOW CHART BOX Q
320 C
321      C(2,J)=C(1,J)+A
322      C(3,J)=C(1,J)-A
323      C(4,J)=C(1,J)+A
324      C(4,J1)=C(1,J1)-A
325      C(5,J)=C(1,J)-A

```

APPENDIX B

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326      C(I,J1)=C(I,J1)+A
327      C
328      C      FLOW CHART BOX R
329      C
330      DO 1820 M=2,3
331          DO 1815 I=LIM1,LIM2
332              N=I-J
333              UC(M,I)=UC(I,I)+FKKKK*DELTAP(N)*R(J)*C(M,J)-C(I,J))
334      1815      CONTINUE
335              UC(M,LIM4)=UC(I,LIM4)
336      1820      CONTINUE
337      DO 1840 M=4,5
338          UC(M,LIM1)=UC(I,LIM1)+FKKKK*DELTAP(1)*R(J)*C(M,J)-C(I,J))
339          DO 1835 I=LIM3,LIM2
340              N=I-J
341              N1=I-J1
342              UC(M,I)=UC(I,I)+FKKKK*DELT
343              C(I(N)*R(J)*C(M,J)-C(I,J))+P(N1)*R(J1)*C(M,J1)-C(I,J1))
344      1835      CONTINUE
345              UC(M,LIM4)=UC(I,LIM4)+FKKKK*DELTAP(18)*R(J1)*C(M,J1)-C(I,J1))
346      1840      CONTINUE
347      C
348      C      FLOW CHART BOX S
349      C
350      DO 1940 M=1,5
351          SUM=0.
352          DO 1935 I=LIM1,LIM4
353              TEMPR=UC(M,I)-UM(I)
354              SUM=SUM+TEMPR*TEMPR
355      1935      CONTINUE
356              E(M)=DELT*SUM
357      1940      CONTINUE
358      C
359      C      FLOW CHART BOX T
360      C
361      L=2
362      M2=1
363      2010 DO 2020 I=L,5
364          IF(E(I).LT.E(M2))GO TO 2025
365      2020      CONTINUE
366          GO TO 2030
367      2025      M2=I
368          L=L+1
369          IF(L.GT.5)GO TO 2030
370          GO TO 2010
371      2030      CONTINUE
372      C
373      C      FLOW CHART BOX U
374      C
375      DO 2120 I=1,5
376          C(I,J)=C(M2,J)
377          C(I,J1)=C(M2,J1)
378      2120      CONTINUE
379      DO 2130 I=LIM1,LIM4
380          UC(I,I)=UC(M2,I)
381      2130      CONTINUE
382      C

```

APPENDIX B

```

383 C      FLOW CHART BOX U
384 C
385 2300 IF(J.GE.99)GO TO 2800
386 C
387 C      FLOW CHART BOX X
388 C
389 J=J+1
390 GO TO 1820
391 C
392 C      FLOW CHART BOX Y
393 C
394 2500 UC(1,1)=0.
395 DO 2530 I=1,100
396 UC(1,I)=0.
397 ITEM=I-12
398 IF(ITEM.LT.1)ITEM=1
399 LIMIT=I-1
400 DO 2520 J=ITEM,LIMIT
401 M=I-J
402 UC(1,I)=UC(1,I)+P(M)SC(1,J)WR(J)
403 2520 CONTINUE
404 UC(1,I)=FROCKKRDDELLTUC(1,I)
405 2530 CONTINUE
406 C
407 C      FLOW CHART BOX Z
408 C
409 SUM=0.
410 DO 2600 I=1,100
411 TEMPE=UC(1,I)-UN(I)
412 SUM=SUM+TEMPE*TEMPE
413 2600 CONTINUE
414 ERROR=DELLT*SUM
415 GO TO 2800
416 C
417 C      FLOW CHART BOX AB
418 C
419 2800 ICLASS=2
420 ICTRE=ICTRE+1
421 GO TO 3100
422 C
423 C
424 C      FLOW CHART BOX AC
425 C
426 2900 Q=(1.+FNEG)/(FN+1.)
427 C
428 C      FLOW CHART BOX AD
429 C
430 IF(ERROR.GE.SAUEIR)GO TO 3070
431 C
432 C      FLOW CHART BOX AD.5
433 C
434 SAUEIR=ERROR
435 GO TO 1800
436 C
437 C      FLOW CHART BOX AD.7
438 C
439 3070 DO 3075 I=1,99

```

APPENDIX B

```

440      YUN(I,4)=C(I,1)
441      3072 CONTINUE
442      C
443      C      BELOW:
444      C
445      C      I=UNINDEX-SHIFTED TIME
446      C
447      DO 3074 I=11,85
448      F1=1
449      TEMP1=F1-2.5*ETIMEO
450      TEMP2=F1-2.5*ETIMEO
451      LIM1=TEMP1
452      IF(LIM1.LT.1) LIM1=1
453      LINE=TEMP2
454      IF(LINE.GT.95) LINE=95
455      C(I,1)=0.
456      FNORM=0.
457      DO 3076 K=LIM1,LINE
458      FK=K
459      TEMP=ABS((F1-FK)/ETIMEO)
460      C(I,1)=C(I,1)+YUN(K,4)*EXP(-TEMP)
461      FNORM=FNORM+EXP(-TEMP)
462      3076 CONTINUE
463      C(I,1)=C(I,1)/FNORM
464      3074 CONTINUE
465      C
466      C      FLOW CHART BOX AD.B
467      C
468      SUM=0.
469      DO 3085 I=1,100
470      SUM=SUM+UN(I)*XUN(I)
471      3085 CONTINUE
472      ERRORA=ERROR/SUM
473      DELERA=DELERR/SUM
474      IF(ERRORA.LT.ECRIT) GO TO 2800
475      C
476      C      FLOW CHART BOX AE
477      C
478      ICLASS=3
479      ICTR3=ICTR3+1
480      3100 CONTINUE
481      IF(IPLTF.NE.1) GO TO 3090
482      DO 3103 I=1,85
483      J=I+10
484      CSHIFT(I)=C(I,J)
485      3103 CONTINUE
486      CALL SUSPLT(XSHIFT,85,1,CSHIFT,'S10',3,IFR
487      AM1)
488      3105 CONTINUE
489      DO 3107 I=1,100
490      UCPLT(I)=UC(I,1)
491      3107 CONTINUE
492      CALL SUSPLT(THATRX,100,1,UCPLT,'CALC.
493      RETURN',12,IFRAN1)
494      3110 CONTINUE
495      CALL SUSPLT(THATRX,100,1,UCPLT,'CALC.
496      G4,UN,'MEAS. RETURN',12,IFRAN1)

```

APPENDIX B

```

475 3000 CONTINUE
476 C
477 C
478 C*****
479 C*****
480 C***** JIM GRIFFIN:
481 C***** HERE, WRITE TO TAPE: G(1,1), 1-11,00
482 C***** (THIS IS THE MODIFIED VERSION OF THE
483 C***** CLOUD SIGNATURE.)
484 C*****
485 C*****
486 C
487 C
488 GO TO 300
489 C
490 END OF PROGRAM SUMMARY
491 C
492 C
493 4000 CALL SCREEN
494 WRITE(IOUTDU,90301)IMMY
495 WRITE(IOUTDU,90303)LOUCTR
496 WRITE(IOUTDU,90304)ICTR1
497 WRITE(IOUTDU,90305)ICTR2
498 WRITE(IOUTDU,90307)ICTR3
499 C
500 C
501 C*****
502 C*****
503 C***** INSERT A CALL FROM HERE.
504 C*****
505 C*****
506 C
507 C
508 STOP
509 490 ICTR1=ICTR1+1
510 GO TO 300
511 4990 WRITE(IOUTDU,90300)ITEMP
512 STOP
513 C
514 C IF WE GET A BAD FRAME:
515 C
516 5000 WRITE(IOUTDU,90301)ISTAT
517 C
518 C
519 C*****
520 C*****
521 C***** INSERT A CALL FROM HERE.
522 C*****
523 C*****
524 C
525 C
526 C
527 C
528 STOP
529 C
530 C
531 C
532 C
533 C
534 C
535 C
536 C
537 C
538 C
539 C
540 C*****
541 C*****
542 C***** INSERT A CALL FROM HERE.
543 C*****
544 C*****
545 C
546 C
547 C
548 C
549 STOP
550 C
551 C

```

APPENDIX B

```

552 90101 FORMAT(' ENTER CRITICAL ERROR.')
553 90103 FORMAT(E15.3)
554 90105 FORMAT(' ENTER INITIAL VALUE FOR Q.')
555 90107 FORMAT(' ENTER N (WEIGHT FACTOR FOR Q DECREASE).')
556 90109 FORMAT(' ENTER OUTPUT TAPE DEVICE.')
557 90113 FORMAT(' ENTER INTEGRATED PULSE HEIGHT BELOW WHICH ',
558 C'SIGNAL SHOULD NOT BE PROCESSED.')
559 90115 FORMAT(' ENTER NUMBER OF PULSES TO BE PROCESSED.')
560 90221 FORMAT(' FOR FILTERING OF RETURN PULSE, ENTER ',
561 C'1/E TIME (IN NSEC) DO NOT USE ZERO',
562 C'SHOULD USE AT LEAST .5 NSEC SO YOU AVG.',
563 C'OVER MORE THAN ONE RASTER POINT.')
564 90223 FORMAT(' FOR FILTERING OF CLOUD SIGNATURE, ENTER ',
565 C'1/E DISTANCE (IN UNITS OF .15 METERS) DO NOT USE ZERO')
566 90225 FORMAT(' ENTER 1. FOR PLOTS.')
567 90301 FORMAT(1X,15,' RETURN PULSES WERE EXAMINED.')
568 90303 FORMAT(1X,15,' OF THESE PULSES WERE BELOW THRESHOLD.')
569 90304 FORMAT(1X,15,' OF THESE PULSES HAS LESS THAN 100 NSEC',
570 C'OF USEABLE RETURN.')
571 90305 FORMAT(1X,15,' CLOUD SIGNATURES WERE SUCCESSFULLY EXTRACTED.')
572 90307 FORMAT(1X,15,' CASES FAILED.')
573 90502 FORMAT(' USE DECIMAL POINT')
574 90503 FORMAT(F15.3)
575 90509 FORMAT(9AS)
576 90509 FORMAT(' ASSIGNMENT ERROR',15,'FROM GETFR')
577 90551 FORMAT(' IF CODE = 136, FILE MARKER FOUND.',
578 C' ANY OTHER CODE MEANS TAPE ERROR.',
579 C' CODE = ',110)
580 END
581 SUBROUTINE SUBPLT(PASS1,IPASS2,IPASS3,IPASS4,
582 CPASS5,IPASS6,IPASS7,IFRAME)
583 DIMENSION PASS1(1),IPASS3(4),IPASS6(4),PASS6(1)
584 C
585 C VARIABLES PASSED:
586 C
587 C 1: X ARRAY NAME
588 C 2: 8 POINTS
589 C 3: 'X LABEL'
590 C 4: 8 CHARACTERS IN LABEL
591 C 5: Y ARRAY NAME
592 C 6: 'Y LABEL'
593 C 7: 8 CHARACTERS IN LABEL
594 C IFRAME: FRAME CODE
595 CALL SCREEN
596 WRITE(1,905)IFRAME
597 CALL SCALE(PASS1,IPASS2,8.,1.0)
598 CALL SCALE(PASS6,IPASS2,8.,1.1)
599 CALL ENTGRA
600 CALL XAXIS(IPASS3,IPASS4,8.)
601 CALL YAXIS(IPASS6,IPASS7,8.)
602 CALL DATAQ(PASS1,PASS5,IPASS2,1,1)
603 CALL EXITGR
604 READ(1,903)IFRAME
605 RETURN
606 903 FORMAT(11)
607 905 FORMAT(25X,'FRAME',15)
608 END

```

APPENDIX C.--VALUES OF PARAMETERS USED IN POINT
WEIGHTING ALGORITHM

APPENDIX C

The return pulse was filtered by replacing each digitized value, $V(t_i)$, by a weighted average of the points in its neighborhood. The weight factor decreases exponentially as t moves away from t_i . The time (in nanoseconds) by which the weight factor has decreased to $1/e$ is called t_0 . The filtering of the cloud signature is similar, except that the units for x_0 are 0.15 m (corresponding to 1 ns).

The filtering values and the values for Q_1 and n used to generate the figures in the main body of this report were these:

Case I

$$t_0 = 1.3 (= 1.3 \text{ ns}),$$

$$x_0 = 1.2 (= 0.195 \text{ m}),$$

$$Q_1 = 1.2,$$

$$n = 2.$$

Case II

$$t_0 = 1.3 (= 1.3 \text{ ns}),$$

$$x_0 = 1.3 (= 0.195 \text{ m}),$$

$$Q_1 = 1.1,$$

$$n = 2.$$

**APPENDIX D.--CLOUD SIGNATURE RESTRICTIONS PRODUCED BY PARAMETERS
USED IN POINT WEIGHTING ALGORITHM**

APPENDIX D

As discussed in the main body of this report, the extent of variation of the cloud signature depends on the product of the Q_i 's. The author has not been able to write this product in terms of Q_1 and n in closed form, but some discussion and numerical results are given below.

Note: $1 < Q < 2$, $n > 0$, $0 < Z < 1$, $0 < \alpha < 1$.

Find

$$\prod_{i=1}^{\infty} Q_i = Q_1 \cdot Q_2 \cdot Q_3 \cdot Q_4 \cdot \dots \quad (D-1)$$

where

$$Q_{i+1} = \frac{1 + nQ_i}{n + 1} \quad (D-2)$$

Let

$$Z_i = Q_i - 1 \quad (D-3)$$

Then from equation (D-2) it follows that

$$Z_{i+1} = \frac{n}{n + 1} (Z_i) = \alpha Z_i \quad (D-4)$$

$$\alpha \equiv \frac{n}{n + 1} \quad (D-5)$$

Using equation (D-4) repeatedly shows that

$$Z_i = \alpha^{i-1} Z_1 \quad (D-6)$$

Back to the task at hand,

$$\prod_{i=1}^{\infty} Q_i = \prod_{i=1}^{\infty} (Z_i + 1) \quad (D-7)$$

$$\ln \left(\prod Q_i \right) = \ln \left[\prod (Z_i + 1) \right] = \sum \ln (Z_i + 1) \quad (D-8)$$

$$\begin{aligned} \ln \left(\prod Q_i \right) &= \ln (1 + Z_1) + \ln (1 + \alpha Z_1) + \ln (1 + \alpha^2 Z_1) \\ &\quad + \ln (1 + \alpha^3 Z_1) + \dots \end{aligned} \quad (D-9)$$

Using

$$\ln (1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \cdot \dots \quad \text{for } x^2 < 1 \text{ and } x = 1$$

in each term in equation (D-9) yields

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$$\begin{aligned}
 \ln (\prod Q_i) = & \left[z_1 - \frac{z_1^2}{2} + \frac{z_1^3}{3} - \frac{z_1^4}{4} + \frac{z_1^5}{5} \dots \right] \\
 & + \left[\alpha z_1 - \frac{(\alpha z_1)^2}{2} + \frac{(\alpha z_1)^3}{3} - \frac{(\alpha z_1)^4}{4} + \frac{(\alpha z_1)^5}{5} \dots \right] \\
 & + \left[\alpha^2 z_1 - \frac{(\alpha^2 z_1)^2}{2} + \frac{(\alpha^2 z_1)^3}{3} - \frac{(\alpha^2 z_1)^4}{4} + \frac{(\alpha^2 z_1)^5}{5} \dots \right] \\
 & + \left[\alpha^3 z_1 - \frac{(\alpha^3 z_1)^2}{2} + \frac{(\alpha^3 z_1)^3}{3} - \frac{(\alpha^3 z_1)^4}{4} + \frac{(\alpha^3 z_1)^5}{5} \dots \right] \\
 & + \dots \dots \dots
 \end{aligned} \tag{D-10}$$

Collecting like powers of Z,

$$\begin{aligned}
 \ln (\prod Q_i) = & z_1 \left[1 + \alpha + \alpha^2 + \alpha^3 + \dots \right] \\
 & - \frac{z_1^2}{2} \left[1 + \alpha^2 + \alpha^4 + \alpha^6 + \dots \right] \\
 & + \frac{z_1^3}{3} \left[1 + \alpha^3 + \alpha^6 + \alpha^9 + \dots \right] \\
 & - \frac{z_1^4}{4} \left[1 + \alpha^4 + \alpha^8 + \alpha^{12} + \dots \right] \\
 & + \dots \dots \dots
 \end{aligned} \tag{D-11}$$

Summing the algebraic series in each square bracket yields

$$\begin{aligned}
 \ln (\prod Q_i) = & z_1 \left[\frac{1}{1 - \alpha} \right] - \frac{z_1^2}{2} \left[\frac{1}{1 - \alpha^2} \right] + \frac{z_1^3}{3} \left[\frac{1}{1 - \alpha^3} \right] \\
 & - \frac{z_1^4}{4} \left[\frac{1}{1 - \alpha^4} \right] + \dots \dots \dots
 \end{aligned} \tag{D-12}$$

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These terms get very small very fast. The sum therefore can be approximated by the first few terms. The exponential of the sum then gives

$$\prod_{i=1}^{\infty} Q_i .$$

Table D-1 was generated by computer. Using the first five terms of equation (D-12) gives the same results as the first 100 terms of equation (D-1) to the accuracy shown here.

TABLE D-1. NUMERICAL RESULTS FOR PRODUCT $\prod_{i=1}^{\infty} Q_i$

n	Q_1						
	1.4	1.3	1.2	1.1	1.05	1.02	1.01
0.5	1.70	1.50	1.32	1.16	1.08	1.03	1.02
1	2.04	1.73	1.46	1.21	1.10	1.04	1.02
2	2.95	2.29	1.76	1.34	1.16	1.06	1.03
3	4.25	3.04	2.14	1.48	1.22	1.08	1.04
4	6.13	4.02	2.58	1.63	1.28	1.10	1.05
5	8.84	5.32	3.13	1.79	1.34	1.13	1.06
10	55.1	21.6	8.12	2.92	1.72	1.24	1.12

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