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Analytical Prediction of Turbulent Heat Transfer Parameters:

The First Annual Report

Adrian Bejan

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The objective of this research effort is to construct a purely theoretical foundation for the phenomenon of turbulent heat transfer. In this first annual report, the current methodology of theoretical research is reexamined critically. The report outlines in three self-standing units (papers) the new theoretical viewpoint that turbulence is the result of inviscid flow buckling. This viewpoint leads to first examples of theoretical prediction of turbulence parameters, for instance, the transition to turbulence, the flapping motion of turbulent jets, the frequency of vortex shedding and the meandering and mixing mechanism of thermal plumes.

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Report CUMER 81-3

ANALYTICAL PREDICTION OF TURBULENT HEAT TRANSFER PARAMETERS:

THE FIRST ANNUAL REPORT

December 1981

by

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Philosophy of Theoretical Research in Turbulence

The objective of this research ef'ort is to construct a purely theoretical foundation for the phenomenon of turbulent fluid flow and heat transfer. The starting point in this effort is the Principal Investigator's view that, as an unexplained physical phenomenon, turbulence represents a new physics, and that a new phsycis demands a new theory which must be judged based on entirely new experiments. This view recognizes that, historically, the classical Navier-Stokes equations have been shown to be inadequate as a theory of turbulence. One hundred years of consistent inability to avoid empiricism while proposing "theories" with the Navier-Stokes equations as starting point, is the best indicator that an entirely new theoretical viewpoint is needed. Indeed, based on its record of accomplishments, the theory embodied in the classical Navier-Stokes equations* emerges as only a theory of laminar flow.

The fundamental theoretical question

For a purely theoretical viewpoint to be successful in explaining and predicting the occurence of "turbulence", it must be able to account for the most basic and elementary feature of turbulent flow. This feature is unquestionably the "eddy", i.e., that finite-size portion of the flow field which rotates and - as a unit - llows the bulk motion of the field. The fundamental, zeroth-order question to be answered is then:

> What "makes" the FIRST EDDY, in a field which originally contains absolutely no sign (trace) of turbulent motion?

*The principle of mechanical equilibrium of a point-size fluid packet in communication with neighboring point-size packets.

It is worth pointing out that this fundamental question is totally disregarded by "theories" which proceed from the time-averaged Navier-Stokes equations, a procedure pioneered by Osborne Reynolds himself 100 years ago. Such analyses take the presence (birth) of eddies for granted, hence, they are by definition empirical (empiricism is the thinking process which has as starting point an unexplained physical observation). It is for this reason that the current search for "generally applicable models" for time-averaged eddy transport is destined to fail: to search for such models is the philosophical equivalent of claiming to understand the behavior of a society while negating (disregarding the importance of understanding) the individual.

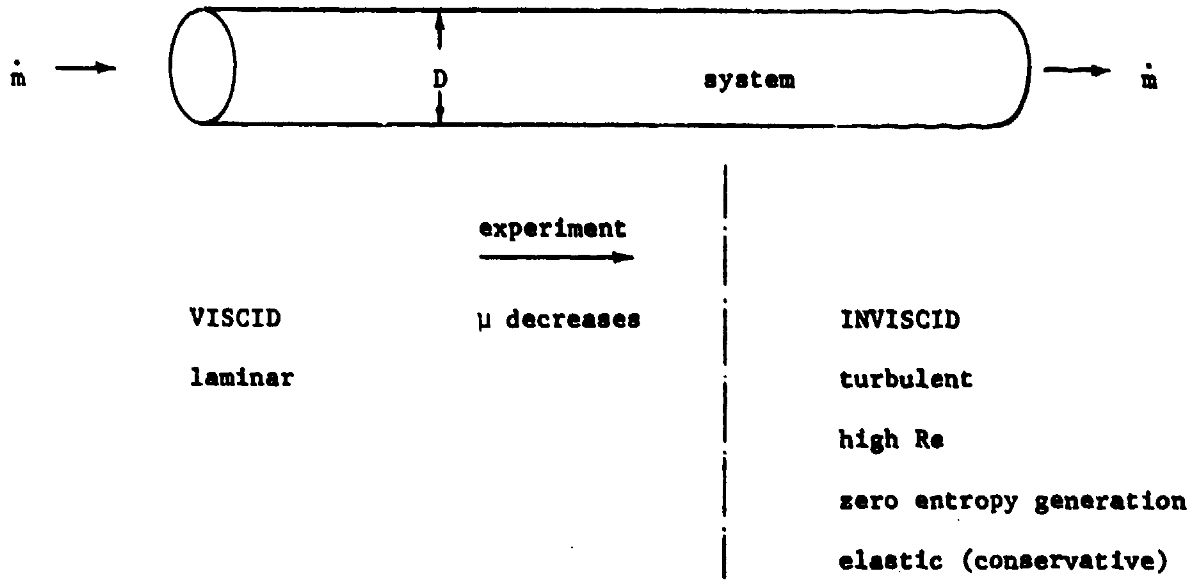
The fundamental question formulated above appears to have been avoided by turbulence theorists over the years. Of course, there are explanations for this unfortunate record. For example, it is generally acknowledged that the interface between the understood (laminar) and the not understood (turbulent) behavior of fluid flow is described satisfactorily by the Theory of Hydrodynamic Stability. Yet, the starting point in any hydrodynamic stability analysis is the postulate that "disturbances" of known size exist, superimposed on the "undisturbed" (laminar) field. With regard to explaining the origin of turbulence, the "theory" of Hydrodynamic Stability is nothing but an empirical exercise*, the game of watching what happens (in time) to turbulence which is already present. Hydrodynamic stability studies negate the importance of answering theoretically the fundamental question concerning the natural origin of disturbances. In this way, the field of Hydrodynamic Stability illustrates the intrinsic inability of the Navier-Stokes theory to explain turbulence.

*An experiment which, historically, was first pursued analytically and, more recently, numerically and in the laboratory (e.g., the loudspeaker harmonic forcing of an "undisturbed" flow).

There are other, equally important reasons for why the fundamental question of theoretical research in turbulence has not been asked. One reason is the fact that, traditionally, old theories and old theoreticians hide their inability in the face of a new physics by inventing and disseminating a "new" terminology which is misleading and self-serving. Note the postulate of "disturbances in a laminar field" employed in Hydrodynamic Stability studies which go on to claim that they shed light on the "transition" from laminar to turbulent flow. In reality, the "transition" has already taken place, before the start of the stability experiment. The initial condition, interestingly described as "disturbances superimposed on a laminar flow field", is nothing but "unexplained turbulence". The lack of emphasis science educators place on seeing through the language coined by any theory, old and new, is the major contributor to lack of progress in theoretical research, hence, to the conversion of science to religion.

The elasticity of turbulent streams.

Turbulence is a characteristic behavior of inviscid flow. In order to see this, consider the simple experiment shown in the figure. A stream of fixed flowrate $\dot{m}$ flows through a pipe of fixed diameter D. If, chemically, the experimentalist always decreases the fluid viscosity μ, the flow will eventually shift from the predictable laminar flow to the unpredictable turbulent behavior. At the same time, it can be said that the flow shifts from viscid to the inviscid flow limit. To be sure, nothing is viscid (or inviscid) everywhere and forever; however, it is clear that the viscidity of the stream as a whole (as a finite size system) decreases as μ decreases.



Another important aspect of the inviscid flow limit, the limit of turbulent behavior, is the fact that the entropy generation rate steadily decreases as μ decreases. The pipe becomes a thermodynamic system filled with conservative material incapable of irreversibility. Such a system possesses elastic properties. Indeed, if the pipe used in the experiment is highly flexible, then it is possible to store work in the system (the pipe) by temporarily curving it. Just like a garden hose delivering a high (turbulent) flowrate, the elastic stream-pipe system has the natural tendency to spring back.

The thinking experiment illustrated in the figure leads to a fundamental conclusion in the search for the origin of turbulence:

> Turbulence is the behavior of
> inviscid (elastic) streams.

The sinusoidal shape of inviscid (elastic) streams.

We learn more about turbulence, as the natural behavior of inviscid streams*, by taking an unbiased look at the tendencies of highly inviscid streams we see in nature. The first place to look is the flow of rivers where we recognize "meanders" as a characteristic of rivers of all sizes. Meanders represent a highly regular geometric pattern resembling a sinusoid with exaggerated elbows. But, most importantly, there appears to be a universal proportionality between the wavelength of the sinuscidal course and the width of the river.

Meanders are a characteristic of all streams in the turbulent flow limit. Just think of the snake-like shape of all turbulent plumes, the "buckled" shape of lightning and the meanders of rivulets (water tricklings on a shower curtain, car windshield, etc.). We conclude that:

> Natural inviscid streams which are not constrained by rigid walls have the property to meander, so that
>
> $$\frac{\text{the meander wavelength}}{\text{the stream width}} = \text{universal constant}$$

*Also called "high Reynolds number" flows.

The buckling property of inviscid streams.

To summarize the preceding discussion, we found that finite-size inviscid streams are elastic (springy) and that, if unconstrained, inviscid streams assume a regular, sinusoidal shape. If we think only about the finite-size space (column) occupied by the inviscid stream, and not about the fluid, the above conclusions can be restated as follows:

> The column contains springy material and,
> if free to deform laterally, assumes a
> regular sinusoidal shape.

This is precisely the physical observation which leads to the Buckling Theory of slender columns in the field of Strength of Materials. Indeed, the stream column possesses the previously unknown property of buckling, as shown in the remaining sections of this report.

The newly discovered buckling property of inviscid streams (fluid fibers of finite thickness) accounts for the "turbulent" motion of fluids. First examples of how this property explains turbulence are given in the 1st and 3rd papers in this report. From the outset, it is important to recognize that the buckling property of inviscid flow is not another "model" of the kind proposed frequently in contemporary turbulence research. The buckling property of inviscid streams is a purely theoretical discovery, an integral part of newtonian fluid mechanics which has eluded researchers until now. A reexamination of the 1st paper which develops the buckling theory of inviscid fibers of finite thickness, reveals that the buckling property is an immediate consequence of the classical Bernoulli equations. The new feature of the Buckling Theory is the fact that the system of interest is of finite size. This represents a drastic departure from previous theoretical research in fluid mechanics which, starting with the Navier-Stokes equations, focuses on the equilibrium of a point-size system.

Built into the Buckling Theory is the new philosophy that a finite-size portion of the flow possesses a global property (buckling). Consequently, the fluid contained in a fiber evolves as a finite-size entity, not as a point size fluid packet in laminar flow. Inviscid flow is a fibrous flow: fluid fibers are distinct portions of the flow field, defined by surfaces of relative motion (e.g., jets, wakes, shear layers). Fluid fibers buckle and the elbows of the buckled shape mix with the ambient giving birth to eddies. The new eddies thicken the old fiber into a new fiber which also buckles.

Published Papers.

No. 1. A. Bejan, "On the Buckling Property of Inviscid Jets and the Origin of Turbulence," Letters in Heat and Mass Transfer, Vol. 8, pp. 187-194, May-June 1981.

No. 2. A. Bejan, "Comments on Viscous Buckling of Thin Fluid Layers," Physics of Fluids, Vol. 24, No. 9, pp. 1764,1765, September 1981.

No. 3. A. Bejan, "Theory of Instantaneous Sinuous Structure in Turbulent Buoyant Plumes," Wärme-und Stoffübertragung, Vol. 16, March 1982 (to appear).

ON THE BUCKLING PROPERTY OF INVISCID JETS AND THE ORIGIN OF TURBULENCE*

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Abstract

This paper outlines the analogy which exists between inviscid jets and elastic columns in axial compression. It is shown that straight inviscid jet columns possess the property of sinusoidal infinitesimal buckling. The buckling wavelength scales with the transversal dimension of the jet. The repeated buckling and breakup of the jet column is responsible for the observed whiplash motion of turbulent jets. The buckling theory predicts correctly the natural frequency of the whiplash motion and the Reynolds number for the laminar-turbulent transition in free jet flow.

* published in Letters in Heat and Mass Transfer, Vol. 8, pp. 187-194, 1981.

Introduction

One aspect of jet turbulence receiving increased attention is the large scale "orderly" structure with a length scale of the same order as the jet diameter. Crow and Champagne [1] showed that as the Reynolds number increases from 10^2 to 10^3, the whiplash motion of jets evolves from a sinusoid to a helix and eventually to a train of axisymmetric waves. Similar observations have been reported by Reynolds [2]. The characteristic snake-like shape of a turbulent round jet is shown very clearly in Fig. 1.

The purpose of this paper is to offer a theoretical explanation for the observed large-scale periodic structure of turbulent jets. The explanation is founded on a very interesting analogy which exists between jet flows and slender elastic columns in axial compression. It may be recalled that Euler's theory of infinitesimal buckling (indifferent equilibrium) in axially compressed columns rests on only two premises [4-6]:

(i) the slender column is straight and in axial compression;

(ii) if subjected to a separate bending test, the column develops in its cross-section a resistive bending moment which is proportional to the induced curvature.

Slender elastic columns, of course, meet these two conditions. However, it is shown in the next section that exactly the same conditions are met by inviscid streams discharging freely into larger reservoirs. Consequently, the column (control volume) occupied by an inviscid stream buckles sinusoidally, and the stream mixes periodically with the stagnant ambient. From this result, we conclude that the natural property of inviscid streams is to follow a sinusoidal (meandering) path as they travel through a

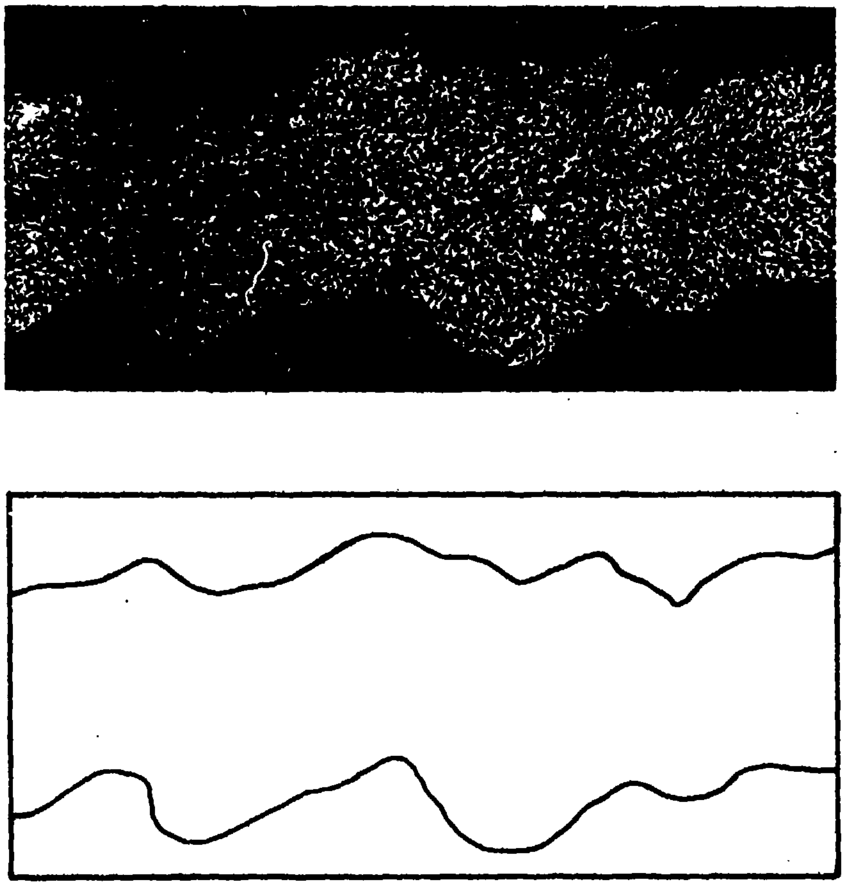

FIG. 1

The characteristic meandering path of a turbulent jet (after Yih [3]).

stagnant ambient. This natural property of inviscid flow is the basis for the unexplained turbulent behavior of fluids.

The Static Equilibrium of the Jet Envelope

Consider a straight inviscid jet of density ρ, uniform velocity V and cross-sectional area A, as shown in Fig. 2. The static pressure inside the jet and in the ambient fluid is P_o. Imagine now a stationary envelope (control surface) which surrounds a certain length of the jet. This envelope and two transveral end-cuts define a stationary control volume.

In the spirit of the thermodynamics of open flow systems [7], the only forces which act on this control volume are the inlet and outlet compressive forces

$$C = \rho A V^2 \tag{1}$$

Forces C are shown schematically in the lower half of Fig. 2 where the control volume is symbolized by the solid line. At this point we conclude that the fluid-filled column represented by the control volume satisfies condition (i) for buckling.

When the control volume is slightly curved, each face of the transversal cut is exposed not only to a compressive force but also to a bending moment. Consider the separate bending test in which the jet is held (forced to flow) in a slightly curved duct. The radius of the curvature of the duct, R_∞, is infinitely greater than the transversal dimension of the jet, D. Bernoulli's equation for a streamline [8] dictates

$$\frac{1}{2} \rho V^2 + P_o = \frac{1}{2} \rho v^2(z) + P(z) \tag{2}$$

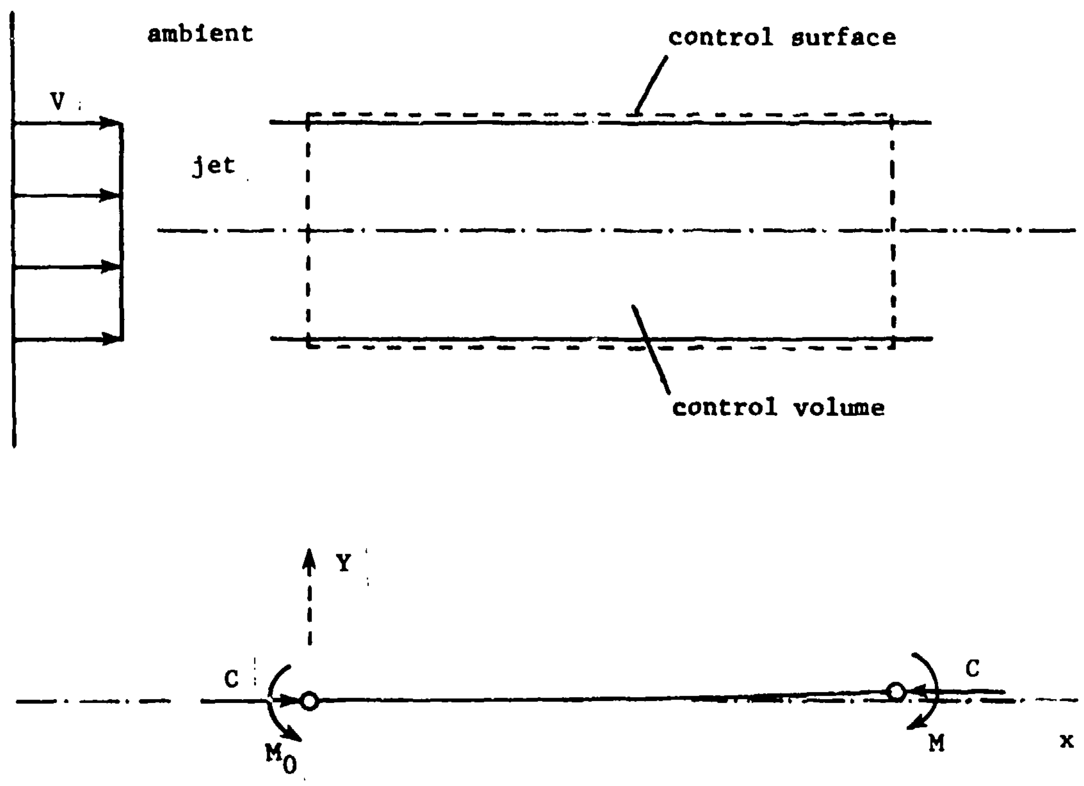

FIG. 2

The static equilibrium of the envelope surrounding a straight jet.

where the right-hand-side of the equation corresponds to the curved duct. Coordinate z is measured radially from the jet centerline towards the center of curvature. Radial equilibrium of the jet fluid in the curved duct requires also [9]

$$- \frac{\rho v^2(z)}{R_\infty} = \frac{\partial P}{\partial z} \tag{3}$$

In the limit of vanishingly small curvature, $D/R_\infty \to 0$, Equations (2) and (3) yield

$$v(z) = V(1 + \frac{z}{R_\infty}) \tag{4}$$

$$P(z) = P_o - \frac{\rho V^2 z}{R_\infty} \tag{5}$$

The bending moment acting over the cross-section is

$$M = \iint_A (\rho v^2 + P)\, z\, dA = \frac{\rho V^2 I}{R_\infty} \tag{6}$$

where $I = \iint_A z^2 dA$ is the area moment of inertia of the jet cross-section.

In conclusion, the cross-sectional bending moment M of a nearly straight jet column is proportional to the curvature of the column, $1/R_\infty$. This means that the inviscid jet column satisfies condition (ii) for the infinitesimal buckling of a straight column.

Consider now the static equilibrium of the straight jet envelope shown in Fig. 2. Clearly, the axial compressive forces C balance each other. In addition, the excentricity bending moment CY -- however small -- must be balanced at all times by the cross-sectional bending moment [10]

$$CY + M_o - M(x) = 0 \tag{7}$$

Noting that $1/R_\infty = -d^2Y/dx^2$, the rotational equilibrium condition (7) constitutes a differential equation for the equilibrium centerline $Y(x)$,

$$\rho V^2 I \frac{d^2Y}{dx^2} + \rho V^2 AY + M_o = 0 \tag{8}$$

The solution satisfying the nozzle conditions $Y = Y' = 0$ at $x = 0$ is

$$Y(x) = K[\cos(x\sqrt{A/I}) - 1] \tag{9}$$

where the amplitude K is indeterminate and infinitely small compared with the transversal length scale $\sqrt{I/A}$ or D.

We arrive at the important conclusion that the equilibrium centerline of the jet envelope is a sinusoid of infinitely small amplitude. The wavelength of this trajectory scales only with the transversal dimension of the jet,

$$\lambda = 2\pi\sqrt{I/A} \tag{10}$$

For example, $\lambda/D = \pi/2$ for a round jet of diameter D, and $\lambda/D = \pi/\sqrt{3}$ for a flat (two-dimensional) jet of thickness D. An analysis of the meandering contour of the round jet of Fig. 1 yields $\lambda/D \cong 1.2$ which is in good agreement with the theoretical value of 1.57 (it is very likely that the 24% discrepancy between the two values is caused by the fact that the counter shown in Fig. 1 overestimates the real diameter of the turbulent jet).

Formation of Large-Scale Vortices

The analogy between the infinitesimal buckling of elastic columns and inviscid jet envelopes terminates with the equilibrium centerline given by Equation (9). Whereas in slender elastic columns the equilibrium of the small-amplitude sinusoid is indifferent [10], in an inviscid jet the equilibrium is unstable. The unstable equilibrium of the inviscid column is described very well by the classical theory of hydrodynamic instability [11]. The slightest deviation of the jet from its rectilinear shape leads to the formation of lateral "lift" forces which consistently tend to amplify the deformation. As a result, the inviscid jet breaks up periodically as its overextended elbows penetrate and mix with the stagnant ambient. The degenerated elbow region becomes a "large scale turbulent structure" [1] which continues to move downstream with a speed of order $V/2$.

The repeated buckling and breakup phenomenon accounts for the observed whiplash and fluctuating motion of turbulent jets. The period of this fluctuation scales with the buckling time $t_B = \lambda/V$, i.e., with the time of fluid travel between successive elbows (breaks) in the jet column. The natural frequency of jet fluctuation can be expressed in dimensionless form as a Strouhal number

$$St = \frac{D}{t_B V} = \frac{D}{\lambda} \qquad (11)$$

which, as shown in the preceeding section, has a value of order 0.5. This order of magnitude estimate agrees very well with experimental measurements of the natural frequency of turbulent jets exposed to a range of external excitation frequencies. For example, Bechert and Pfizenmaier [12] reported St = 0.5 for maximum amplification of broadband jet noise. Most recently, Acton [13] conducted a computational simulation of a round turbulent jet by modeling the shear layer as a succession of discrete vortex ring elements. Acton showed that the natural periodicity of the round jet was St ≈ 0.47, and that the jet was most sensitive when forced at approximately the same frequency (St = 0.5).

The Transition to Turbulence

The buckling property and fluctuating nature of inviscid jets provide a theoretical basis for predicting the transition to turbulence. The transition from laminar jet flow to turbulent flow occurs when the stationary ambient is no longer capable of viscously communicating with the jet. The viscous communication time between the jet-ambient interface and the jet centerline follows from Stokes' first problem [14]

$$t_v = \frac{D^2}{16\nu} . \tag{12}$$

The jet is free to buckle, i.e., to get out of hand, if the viscous diffusion time t_v is longer than the buckling and breakup time t_B. Defining the buckling number N_B as the ratio of these two characteristic times of the flow configuration,

$$N_B = \frac{t_v}{t_B} > 1 , \tag{13}$$

we have a criterion which predicts the fluctuating (turbulent) behavior of the jet. Using Equation (12) and $t_B = \lambda/V$, the buckling number can be written also as

$$N_B = \frac{VD}{\nu} \quad \frac{D/\lambda}{16} \tag{14}$$

For a round jet, the transition criterion $N_B > 1$ becomes

$$\frac{VD}{\nu} > 25 \tag{15}$$

It should be noted that this order-of-magnitude estimate of the transition Reynolds number agrees very well with experimental observations. For example, Viilu [15] found the value of 11.2 for the Reynolds number of breakdown of the steady laminar jet. Viilu's observations were later confirmed by Reynolds [2] who reported the range $10 < Re < 30$ for the transition Reynolds number.

The buckling number criterion of transition to turbulence, Equation (13), explains also why the observed transition Re is a number considerably greater than unity. For a dimensionless group to truly delineate the transition from one mechanism to another, it must have a value of order one which, after all, reflects the balance between the competing mechanisms. The Reynolds number is not the correct dimensionless group to describe transition to turbulence. The Reynolds number is the experimental (measurable) reflection of the N_B transition criterion.

Concluding Remarks

This paper unveiled a theoretical basis for predicting the large scale structure and fluctuating behavior of inviscid jets. The analogy between jet envelopes and elastic columns in axial compression showed that the natural tendency of inviscid jets is to buckle over a precise wavelength which scales only with the jet diameter. The repeated buckling and breakup of the jet column is responsible for the sinusoidal, river-like, path of turbulent jets and also for their natural whiplash motion. The predicted natural frequency of the jet agrees very well with measurements from harmonic excitation experiments. Finally, we learned that the transition from laminar to turbulent jet flow occurs when the buckling and breakup time is shorter than the time of viscous diffusion across the jet. This last conclusion is supported strongly by experimental observations.

The buckling property described in this paper emerges as the fundamental property serving as origin for turbulent motion in inviscid flows.

Acknowledgement

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Paper no. 2

The buckling property of viscid and inviscid fluid layers *

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Suleiman and Munson[1] recently presented a sequence of interesting experiments concerning the buckling of thin viscous fluid layers. The authors referred in their paper to Taylor's observations on the buckling of slender viscous filaments[2]. Taylor's early observations and the Suleiman and Munson experiments point towards an important analogy between the buckling of elastic columns and the buckling of viscous filaments in longitudinal compression.

The object of this letter is two-fold. First, it brings to the readers' attention the fact that a buckling theory of thin viscous layers already exists. Second, it points out that the buckling of slender columns in axial compression is not a property of only elastic solids and highly viscous fluids, but also a property of inviscid columns (streams).

The buckling of a thin viscous layer was considered theoretically by Buckmaster, Nachman and Ting[3]. They showed that viscous layers in longitudinal compression satisfy two basic requirements:

(i) the viscous layer is in axial compression and the compressive force is proportional to the relative velocity between the two ends of the layer;

(ii) if curved, the viscous layer develops in its cross-section a bending moment which is proportional to the time-rate of change in the local curvature.

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*published in Physics of Fluids, Vol. 24, pp. 1764, 1765, 1981.

The momentum equations integrated over the viscous layer led Buckmaster, et al.[3] to a global equation for the evolution of the layer centerline. This equation was solved by assuming various initial disturbances as the starting shape in the evolution of the viscous layer. In a subsequent paper, Buckmaster and Nachman[4] have extended this theory to the case where surface tension effects play an important role.

Of fundamental interest are the similarities between the buckling of a viscous layer (the viscida problem) and the buckling of a slender elastic column (the elastica problem). A close examination of the Euler theory for slender elastic columns reveals that, for infinitesimal buckling, elastic columns must satisfy only two conditions[5]:

(iii) the column is straight and in axial compression;

(iv) if subjected to a separate bending test, the column develops in its cross-section a resistive bending moment which is proportional to the local curvature.

The infinitesimal buckling of the straight column follows from invoking static equilibrium for the system sketched in Fig. 1. For a straight or nearly straight column there are two equilibrium conditions to consider. The first is the obvious balance of compressive forces C in the longitudinal direction. The second condition is one of rotational equilibrium which states that the excentricity bending moment Cy -- however small -- must be balanced at all times by the cross-sectional bending moment M. In the case of a column containing elastic material, a special bending test of prescribed curvature[6] combined with knowledge of the elastic properties of individual fibers in the slender column leads to the notion of a cross-section bending moment proportional to the local curvature (iv). Combining this notion with the rotational equilibrium condition yields the equilibrium shape of the straight column, namely, a sinusoid of infinitely small amplitude. The infinitesimal amplitude is indeterminate, hence, the equilibrium of the nearly straight elastic column is indifferent. If present, the slightest lateral force is able to push the column away from the straight equilibrium shape into a nearly straight (sinusoidal) equilibrium shape[5].

It is important to include in this discussion the case of inviscid fluid columns: do such columns conform to buckling of type (i, ii) or to buckling of type (iii, iv)? Thermodynamic reasoning alone suggests that inviscid columns should buckle according to model (iii, iv) because, like elastic solids and unlike highly viscous fluids, inviscid fluids are free of entropy generation[7].

Referring again to Fig. 1, we note that a column containing inviscid fluid is none other than the imaginary control surface drawn around an inviscid jet flowing through an inviscid fluid at rest. Since the column is stationary, its equilibrium is described by the two static conditions (force and moment) discussed in connection with the Euler buckling of elastic columns. The remaining problem is to determine what special forms C and M take in the case of inviscid stream columns. For the compressive force on the control volume, C, we know from the thermodynamics of open flow systems that[8]

$$C = \rho A V^2 \tag{1}$$

where ρ, A, V are the density, cross-sectional area and velocity of the stream. In order to determine the cross-sectional bending moment M we conduct a special bending experiment where the inviscid stream is held in a duct of known radius of curvature, R_∞. In the limit of vanishingly small curvature, the inviscid flow equations show that the resistive bending moment in the cross-section is[9]

$$M = \rho\ V^2 I/R_\infty \qquad (2)$$

where I is the area moment of inertia of the cross-section[6]. Equations (1,2) show that inviscid fluid columns obey conditions (iii,iv) for infinitesimal buckling. The corresponding equilibrium shape of the column is a sinusoid of infinitely small amplitude[9], whose wavelength is

$$\lambda = 2\pi\sqrt{I/A} \qquad (3)$$

Inviscid streams, like elastic rods and viscous layers, possess the natural property of buckling. The wavelength of the buckled shape scales only with the transversal dimension of stream: for example, equation (3) for a jet of round cross-section yields $\lambda/D = \pi/2$. Unlike buckled elastic columns, whose equilibrium is indifferent[5], buckled inviscid columns are unstable. The post-buckling evolution of the inviscid column is well understood, forming the subject of the classical theory of hydrodynamic stability[10]. However, the buckling property is to be recognized as responsible for the wave-like "disturbance" assumed routinely (empirically) as starting point in any hydrodynamic stability analysis.

In essence, the buckling property of inviscid streams guarantees that such streams cannot flow straight through another fluid or through a flexible duct. This new property of inviscid fluids stands at the very root of the phenomenon of turbulence. For example, the interaction between the elbows of the buckled stream and the stagnant medium leads the periodic formation of large eddies. These large-scale turbulent structures are responsible for the river-like shape of turbulent jets, wakes and plumes: it is widely observed that the wavelength of this large-scale meandering path scales with the diameter of the stream, as predicted in equation (3). The natural buckling of inviscid streams is particularly visible in the case of streams flowing within flexible boundaries, like rivers and capillary rivulets at high Reynolds numbers. The buckling property serves as theoretical basis for predicting other turbulence parameters, for example, the natural frequency of turbulent jets (the Strouhal number) and the critical Reynolds number for transition to turbulence[9].

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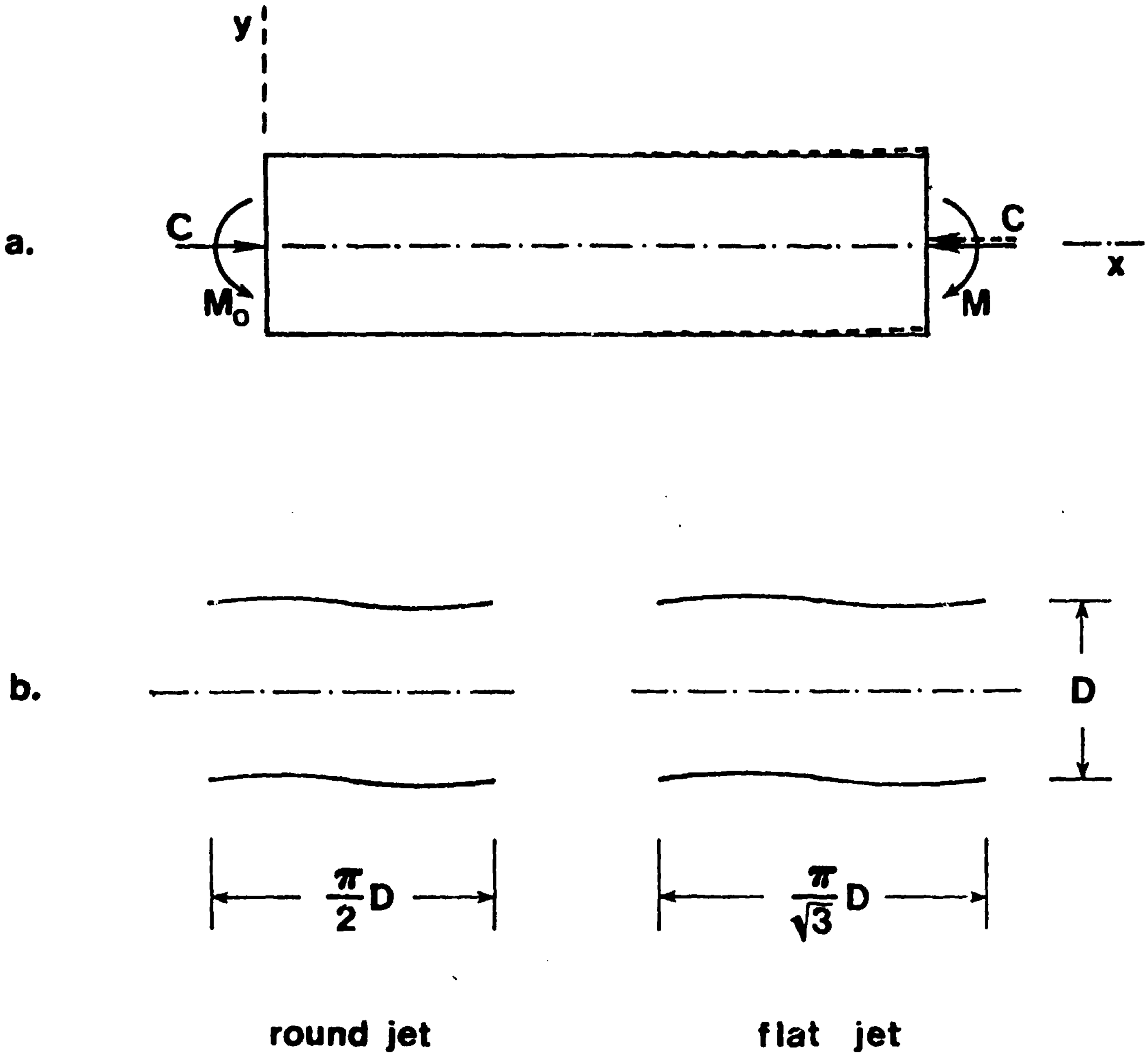

Figure 1. (a) Translational and rotational equilibrium of a slender culumn (control volume) in axial compression. (b) The natural buckling wavelength of circular and two-dimensional inviscid jets.

Paper no. 3

THEORY OF INSTANTANEOUS SINUOUS STRUCTURE

IN TURBULENT BUOYANT PLUMES *

by

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This paper reports a theory which explains the flickering motion of turbulent plumes as well as their large-scale sinuous structure. The theory is based on the fact that the inviscid region of the plume (the "plume column") possesses elastic properties analogous to those of elastic rods subjected to longitudinal compression. It is shown that the straight plume column is not stable and buckles. The distance between two consecutive elbows is proportional to the local plume diameter, in other words, the shapes of all buckled plumes are geometrically similar. A buckled plume collapses periodically due to the interaction of its lateral elbows with the stagnant ambient. This interaction is responsible for the intermittent formation of large-scale buoyant eddies on the periphery of the turbulent plume.

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1. Introduction

Turbulent plumes represent an extremely frequent natural phenomenon, in the atmosphere as well as in the hydrosphere. Furthermore, in heat transfer and environmental engineering, the turbulent plume constitutes one of the most effective mixing mechanisms known to man. The state of knowledge on turbulent plumes has been summarized by Turner [1] and, as part of a comprehensive monograph on buoyancy-driven flows, by Turner [2]. Another extensive overview of the field was published most recently by Fischer, List, Koh, Imberger and Brooks [3]. A critical examination of these review works leads to the conclusion that the favored approach in turbulent plume research is one where the "complications" of the turbulent flow field can be smoothed out by the classic method of time-averaging (Townsend, [4]). Consequently, most of the theoretical work on buoyant jets relies on the axisymmetric flow model which lends itself very well to boundary layer-type analysis.

Although mathematically attractive, the time-averaged turbulent plume concept does gross injustice to the physics of the phenomenon. A turbulent exhaust plume does not rise straight up into the air, with a fixed shape resembling that of an inverted cone or funnel. A real plume executes a periodic lateral movement; the plume is not straight, rather, it has recognizable bends separated by distances of the same order as the plume diameter. The smoke rising from a barbecue or camp fire "snakes" into the air. This large scale motion, which is so effectively blacked out by time-averaging, can be seen clearly in Fig. 1.

The object of this paper is to present a theory which explains for the first time the instantaneous structure and the "flickering" nature of turbulent plumes.

2. The Concept of Elastic Plume Column

Consider a fluid of density ρ rising vertically through a heavier fluid of density $\rho + \Delta\rho$. We model the starting section of this buoyant stream as inviscid. This means that for a certain height at the base of the plume we are neglecting the shear interaction between the plume fluid and the stagnant ambient. This is permissible for a height of the order of 4-6 times the plume diameter, where, as shown by Crow and Champagne [5], the plume-ambient shear layer is thin compared with the plume diameter. Having neglected the interaction at the plume-ambient interface, it is helpful to imagine the plume fluid as being surrounded by a thin flexible sleeve (control surface) which is stationary with respect to the stagnant ambient.

Imagine now a cut perpendicular to the plume axis. If the column is straight, in the cross-section generated by the cut we distinguish only the compressive force (impulse [6])

$$C = \iint_A \rho V^2 \, dA \; , \tag{2.1}$$

where V and A are the plume velocity and cross-sectional area. If the plume column is slightly curved, we distinguish also a cross-sectional bending moment M. This bending moment is due primarily to the fact that the plume fluid travels relatively faster through regions of the cut which are located on the inside bank of the bend. It is easy to show that the cross-sectional bending moment M is present as soon as the column acquires curvature. Consider for this purpose a bending test in which the uniform stream (ρ, V, A) passes through a slightly curved duct: the radius of curvature of the duct centerline

R_∞, is infinitely greater than the plume diameter, D. The Bernoulli equation applied along a streamline requires (Prandtl [7]),

$$\frac{1}{2}\rho V^2 = \frac{1}{2}\rho v^2 + P \tag{2.2}$$

where the left hand side applies to the straight section (P = 0) while the right hand side corresponds to the curved section. Equilibrium in the radial direction requires also (Prandtl [8]),

$$\frac{\rho v^2}{R_\infty} = -\frac{dP}{dy} \tag{2.3}$$

where y is the radial coordinate measured in the plane of curvature away from the plume centerline, toward the center of curvature. Combining Equation (2.2) and (2.3), we obtain

$$v = V\left(1 + \frac{y}{R_\infty}\right) \tag{2.4}$$

The cross-sectional bending moment M can now be calculated by writing

$$M = \iint_A (\rho v^2 + P)\, y\, dA \tag{2.5}$$

which, using Equations (2.4) and (2.2), yields

$$M = \frac{\rho V^2}{R_\infty} \iint_A y^2 dA\,. \tag{2.6}$$

Note that the area integral appearing in Equation (2.6) is the area moment of inertia ($I = \pi D^4/64$) employed routinely in the study of elastic beam flexure (Den Hartog [9]). The group ρV^2 plays the role of "modulus of elasticity": this is why it becomes increasingly difficult to manually bend a hose as the turbulent (inviscid) flow rate through it increases.

At this point we conclude that the plume column is in axial compression and that the cross-sectional bending moment is proportional to the curvature of the centerline ($1/R_\infty$). With such properties, the straight plume column becomes analogous to an elastic column subjected to axial compression (Den Hartog [10]). This analogy guarantees that if the plume is tall enough, it will buckle like an elastic rod compressed between the ends. In the next section we develop the "buckling theory" of plumes viewed as elastic columns.

3. The Natural Buckling of a Straight Plume Rising from Rest

Consider the stationary control volume defined by two cuts normal to the plume column, at z and $z + dz$ (Fig. 2). The balance of vertical forces acting on this volume dictates

$$\frac{dC}{dz} = \Delta\rho A g \quad . \tag{3.1}$$

The conservation of mass through the control volume requires also

$$V(z)A(z) = Q\,, \quad \text{constant}\,, \tag{3.2}$$

where Q is the volumetric flowrate through the plume column. Combining these conservation statements we obtain

$$V(z) = \left(2\,\frac{\Delta\rho}{\rho}\,gz\right)^{1/2}, \quad A(z) = Q/V(z)\,, \tag{3.3}$$

where we made the additional assumption that the plume fluid originates from rest, $V(0) = 0$.

As in the buckling theory of a vertical flagpole (Timoshenko and Gere [11]; Den Hartog [12]), the rotational equilibrium of the plume element dz requires

$$CdY + M - (M + dM) = 0 \tag{3.4}$$

or, using Equations (2.1) and (2.6),

$$\rho A V^2 \frac{dY}{dz} + \frac{d}{dz}\left(\rho V^2 I \frac{d^2 Y}{dz^2}\right) = 0 \tag{3.5}$$

Note that in the limit of vanishingly small curvature, $1/R_\infty = -d^2Y/dz^2$. Taking into account the z-dependence of V and A, Equations (3.3), the equati n of flexure (3.5) becomes

$$\frac{d^3Y}{dz^3} + \frac{4\pi(2\ g\ \Delta\rho/\rho)^{\frac{1}{2}}}{Q} z^{\frac{1}{2}} \frac{dY}{dz} = 0 \tag{3.6}$$

The first important conclusion of this analysis is that the plume column develops a "structure" whose characteristic vertical dimension scales with

$$z_o = \left(\frac{Q^2}{32\pi^2\ g\ \Delta\rho/\rho}\right)^{1/5} \tag{3.7}$$

The vertical length scale can also be written as

$$z_o = \left(\frac{zD^4}{256}\right)^{1/5},\ \text{constant} \tag{3.7a}$$

where D(z) is the plume diameter at level z (note that $D(z) \sim z^{-\frac{1}{4}}$). Introducing the dimensionless coordinate $x = z/z_o$, the equation of flexure becomes

$$\frac{d^3Y}{dz^3} + x^{1/2} \frac{dY}{dx} = 0 \quad . \tag{3.8}$$

The general solution to this equation is expressible in terms of Bessel functions (Watson [13])

$$\frac{dY}{dx} = K_1 \; x^{1/2} \; J_{2/5}(4/5 \; x^{5/4}) + K_2 \; x^{1/2} \; J_{-2/5}(4/5 \; x^{5/4}) \tag{3.9}$$

where K_1 and K_2 are arbitrary constants. Noting that

$$\lim_{x \to 0} \frac{dY}{dx} = K_1 x + K_2 \quad , \tag{3.10}$$

we set $K_2 = 0$, which accounts for the fact that in the very beginning the buoyant fluid rises vertically. The shape of the plume column, $Y(x)$, is obtained by integrating (3.9),

$$Y(x) = K \int_0^{\xi} m^{1/5} \; J_{2/5}(m) \; dm \tag{3.11}$$

where

$$\xi = 4/5 \; x^{5/4} \; . \tag{3.12}$$

The integral appearing in (3.11) was calculated with sixth-digit accuracy, by first expanding $J_{2/5}(m)$ in a power series. The principal values of this integral are reported here in Table 1, because they are not available in the literature.

The magnified shape of the buckled plume column is shown in Fig. 3 as the function $Y(x)/K$. It should be remembered that Equation (3.11) is the result of a small amplitude analysis ($|dY/dz| << 1$), hence, the amplitude K is negligible when compared with the vertical length scale

z_o. The static equilibrium of the straight plume column is indifferent [10]; however, given the slightest lateral disturbance, the plume buckles in accordance with the vertical periodicity shown in Fig. 3. The buckled shape is a succession of elbows whose spatial frequency increases with height. The first four elbows are located at

$$x_1 = 2.878,\ x_2 = 5.101,\ x_3 = 7.101,\ x_4 = 8.967 .$$

According to the classical arguments of Hydrodynamic Stability Theory [14], the sinuous shape will be amplified due to the dynamic interaction of the elbows with the stagnant ambient. It is this interaction which leads to the formation of large turbulent eddies visible in most atmospheric plumes (Fig. 1).

4. Formation and Evolution of Large Eddies

The mechanism of elbow turbulence formation is shown schematically in Fig. 4. Speaking only qualitatively, in the first phase of the process the segment of plume column located in the vicinity of a natural elbow is pushed to the side by the horizontal resultant of the two axial compressive forces. The lateral movement of the elbow is eventually blocked by the pressure build-up associated with either the stagnation of some plume fluid into the still ambient, or with accelerating some of the ambient fluid. This pressure build-up is consistent with the fact that in a column of finite curvature the pressure increases in the radial direction [see Equation (2.3)]. At the end of the first phase of the process, the elbow has degenerated into a buoyant eddy of mixed fluid; this eddy rotates as shown in the figure, and rises slower than the unmixed plume fluid found at the same altitude.

The second phase of the mechanism is triggered by the effect of high pressure nodules already present on both sides of the plume column: the plume buckles in a new mode, feeding its stream through the structures generated by the old set of elbows. At the end of the second phase, the new column gives birth to a new set of large eddies which continue to rise. In general, the eddies produced by the upper elbows interfere with older eddies rising from lower levels. This interaction is partly responsible for lateral growth of the turbulent plume, as x increases.

The most regular feature in the evolution of the turbulent plume column is the root section, $0 < x < x_1$, which precedes the formation of the first elbow. The root of the plume swings back and forth, not necessarily in the same plane, with a characteristic frequency, $f_1 = 1/(2\Delta t)$. The half period Δt of this flickering motion can be calculated by integrating the first of equations (3.3): Δt is the time interval needed by plume fluid to rise from $z = 0$ to the first elbow, $z_1 = x_1 z_o$. Combining this result with Equations (3.7) and (3.3), we discover that the Strouhal number based on the plume velocity and diameter at the first elbow, V_1 and D_1, is a universal constant for all inviscid plumes

$$St_1 = \frac{f_1 D_1}{V_1} = 0.534 . \qquad (4.1)$$

Another interesting result of the buckling theory is that the large elbow structure (the distance between elbows) scales only with the plume diameter, which means that the buckled shapes of all inviscid plumes are <u>geometrically similar</u>. This prediction agrees with the observed structure of atmospheric plumes. According to Equation (3.7a), the plume diameters corresponding to the first four elbows are,

$$\frac{D_i}{z_o} = \frac{4}{x_i^{1/4}} = 3.071,\ 2.662,\ 2.45,\ 2.332\ . \qquad (4.2)$$

Figure 5 shows a scale drawing of a round plume and the relative location of the first four elbows. This universal geometry agrees very well with visual observations of turbulent plumes rising above barbecues and camp fires. It also agrees with the plume photographed in Fig. 1.

5. Laminar Plumes vs. Turbulent Plumes

The instantaneous sinuous structure and the eddy formation mechanism described so far, apply only to large Reynolds number (turbulent) plumes which can be modeled as inviscid. The present theory does not apply to laminar plumes. It is important to note, however, that the universality of the sinuous structure (Fig. 5) provides a theoretical explanation for the phenomenon of transition from laminar to turbulent plume flow.

According to Equation (4.1), the base of the plume column fluctuates within a characteristic time interval which is proportional to the diameter and inversely proportional to the velocity. This local characteristic time,

$$t_{fluctuation} \sim \frac{D_1/V_1}{St_1}\ , \qquad (5.1)$$

represents the "natural heartbeat" of the plume as an inviscid stream. If the plume flow is highly viscous (laminar), the plume column has a different characteristic time: the viscous local time is associated with the travel of information by viscous diffusion from the plume-ambient interface to the plume centerline (total distance D/2). The viscous communication time is given by the solution to Stokes' first problem [15],

$$\frac{D_1/2}{2(\nu t_{viscous})^{\frac{1}{2}}} \sim 1 \quad \text{or} \quad t_{viscous} \sim \frac{D_1^{\,2}}{16\nu} \qquad (5.2)$$

The plume flow remains laminar if the ambient can communicate viscously with the stream faster than the stream can fluctuate. If the column can fluctuate faster than the viscous diffusion time, then, of course, the flow opts for the inviscid (turbulent) regime. At transition, the viscid and inviscid time scales are of the same order of magnitude; combining Equations (5.1) and (5.2) we conclude that the laminar-turbulent transition is marked by

$$\frac{V_1 D_1}{\nu} \sim 30 \ . \qquad (5.3)$$

This theoretical estimate of the transition Reynolds number agrees very well with experimental observations and with estimates based on hydrodynamic stability analysis. Experimentally, it is well known that the transition in free jet and wake flow occurs in the vicinity of $Re \sim 30$ (see, for example, Schlichting [16]). The hydrodynamic stability of buoyant plumes and wall layers has been studied extensively, as summarized by Gebhart [17]. For example, the lowest Reynolds number where instability has been detected in vertical laminar boundary layer flow over a constant-flux wall is of order 67 (see Fig. 8-21 in Ref. [17]). The difference between this estimate and Equation (5.3) is explained by the fact that in Ref. [17] the local Reynolds number (G*) is based on the thickness of the laminar boundary layer. If, instead, the Reynolds number is expressed based on the displacement or momentum thickness (more consistent with the slug flow model employed in the present theory) then the transition Re is a number of the same order as in Equation (5.3).

6. Concluding Remarks

The demonstrated ability to correctly predict the transition to turbulence is additional evidence that the sinuous structure (Fig. 5) and the characteristic time scale (Eq. 4.1) are real properties of all turbulent buoyant plumes. It is important to recognize also that the large-scale buckling phenomenon described in this paper is not observed only in buoyant plumes as in Fig. 1. The same phenomenon has been observed and described as "orderly structure", "corkscrew shape", and "whiplash motion" in the starting section of low speed turbulent jets (Crow and Champagne [5]; Reynolds [18]). Others have recognized this snake-like shape as the cause of the intermittency phenomenon (Yih [19], Fig. 8b, and pp. 545, 546). In the field of boiling heat transfer we are familiar with the occurrence of tall S-shaped vapor bubbles (buckled vapor plumes) on intensely heated horizontal surfaces: this phenomenon is recognized as the "continuous vapor column regime" (see Figs. 2(d) and 2(j) in Moissis and Berenson [20]).

The reader can easily reproduce the natural buckled shape by experimenting with the continuous water column falling from the kitchen faucet. This water column is a "sinking" plume defined not by a flexible plume-ambient interface as in Fig. 1, but by the flexible hose provided by capillary forces. Placing his finger about 1-2 cm under the faucet, the reader can buckle this sinking plume into a shape which resembles very closely the shape shown in Fig. 5, rotated by 180°. In the kitchen faucet experiment the buckled column does not break up, since the lateral growth of its elbows is suppressed by the effect of surface tension. The same stabilizing effect is present in the S-shaped tall bubbles photographed during intense boiling by Moissis and Berenson [20].

From the point of view of theoretical research in fluid mechanics, the Buckling Theory reported in this paper represents a substitute for Hydrodynamic Stability Theory [17], for determining the instability of large Reynolds number plume flow. This conclusion is not surprising, since the Navier-Stokes equations used in Hydrodynamic Stability Theory, and Equations (2.3), (3.1) and (3.4) used in the Buckling Theory, are statements of the same (classical) principles of mechanical equilibrium. The transition to turbulent plume flow, discussed in Section 5, indicates that the natural buckled shape identified by the Buckling Theory corresponds to the least stable asymmetric disturbance determined via Hydrodynamic Stability Theory.

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ξ	$\int_0^{\xi} m^{1/5} J_{2/5}(m)\, dm$
0.5	0.172638
1	0.492870
1.5	0.852135
2	1.168617
2.5	1.380308
3	1.452918
3.5	1.384381
4	1.203166
4.5	0.960384
5	0.717376
5.5	0.531599
6	0.443935
6.5	0.470241
7	0.598886
7.5	0.794658
8	1.007952
8.5	1.186949
9	1.289889
9.5	1.294589
10	1.203059
10.5	1.040267
11	0.847522
11.5	0.672151
12	0.556040
12.5	0.525758
13	0.586530

Table 1.

List of Captions

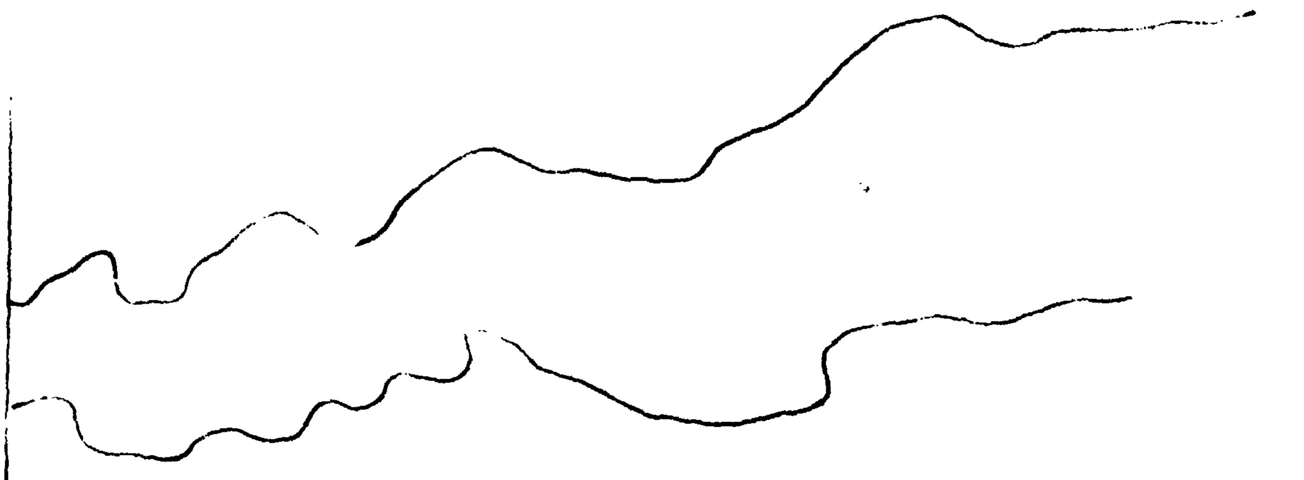


fig. 1

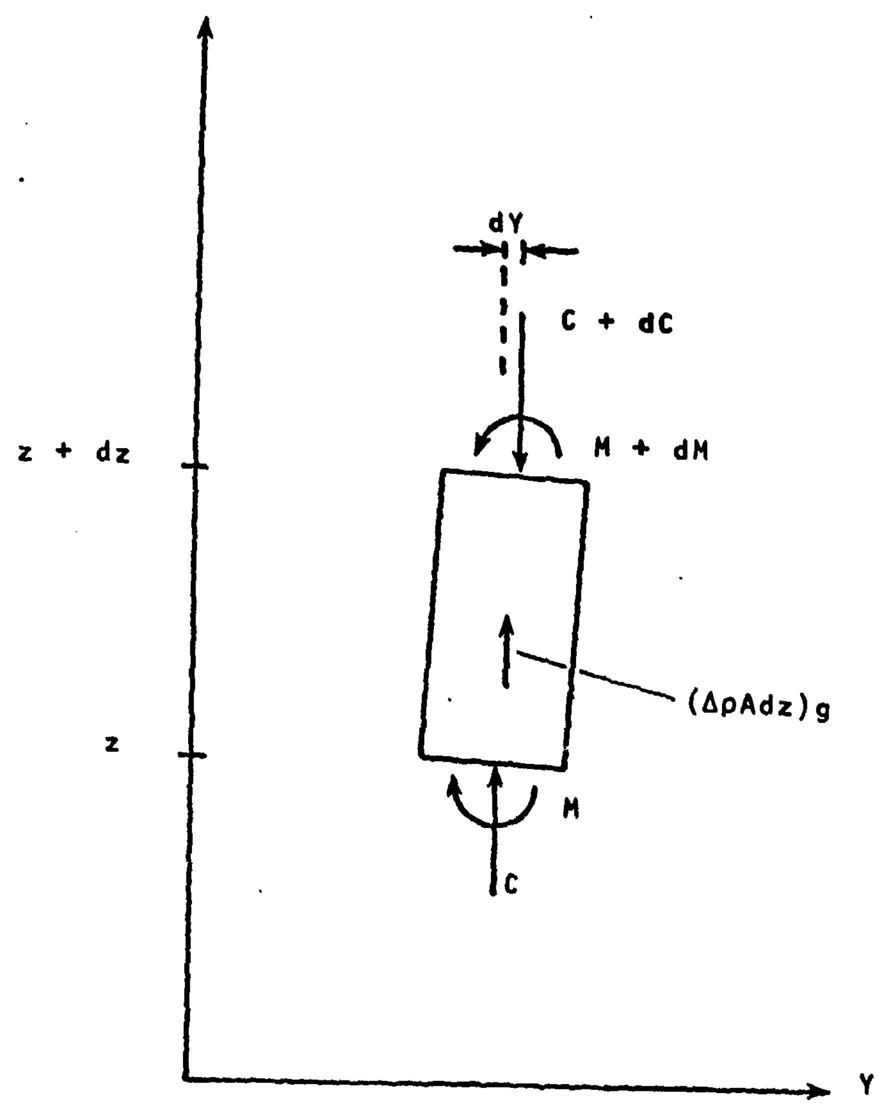
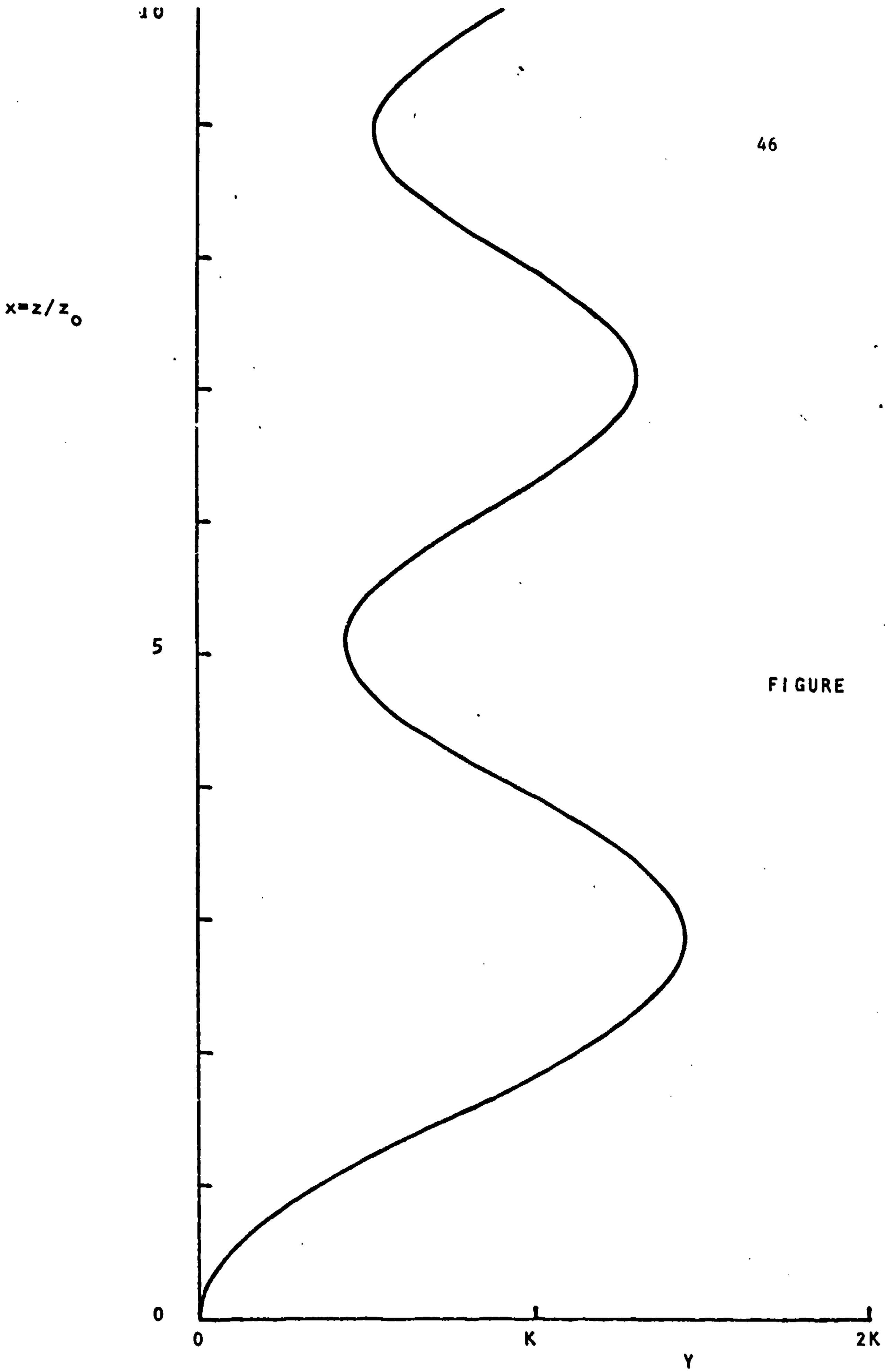


FIGURE 2



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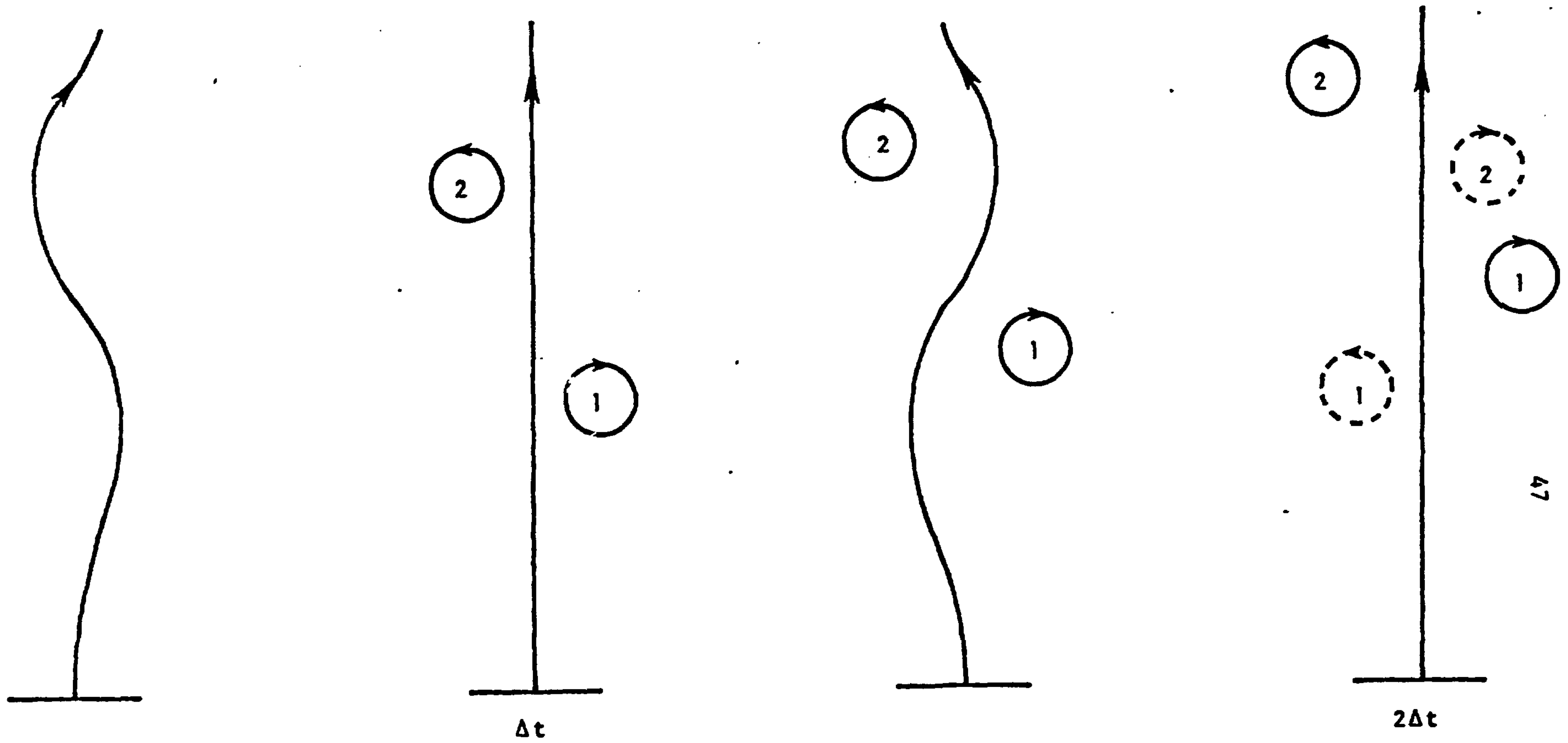

FIGURE 4

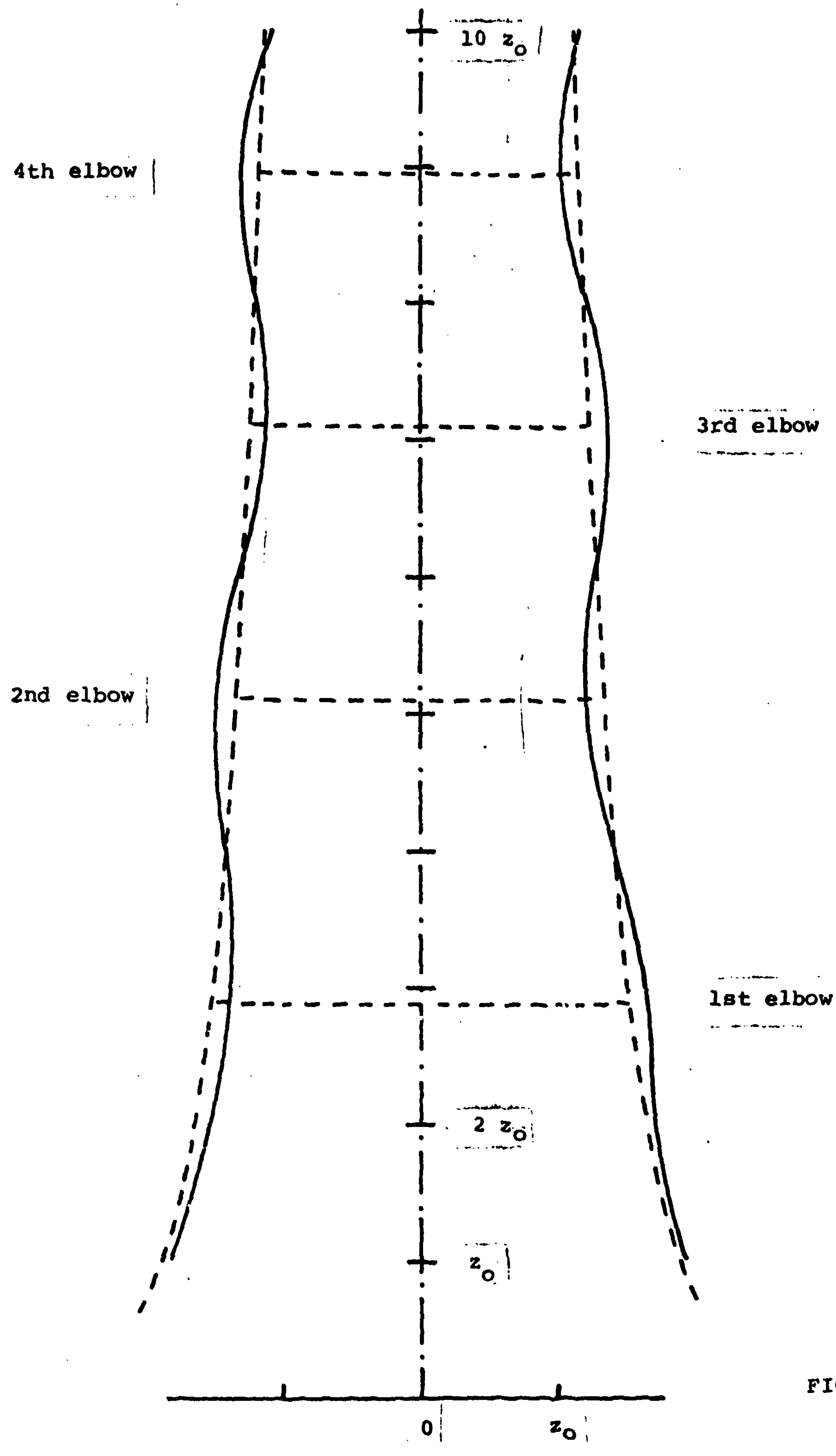

FIGURE 5

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