

ETL-0270

LEVEL

12

Scattering from a vegetation
layer with an irregular
vegetation soil boundary

AD A109138

Richard A. Hevenor

OCTOBER 1981

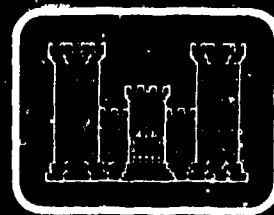
DTIC FILE COPY

**DTIC
SELECTED
DEC 31 1981
H**

U.S. ARMY CORPS OF ENGINEERS
ENGINEER TOPOGRAPHIC LABORATORIES
FORT BELVOIR, VIRGINIA 22060

81 12 31 009

APPROVED FOR PUBLIC RELEASE DISTRIBUTION UNLIMITED



E

T

L



Destroy this report when no longer needed.
Do not return it to the originator.

The findings in this report are not to be construed as an official
Department of the Army position unless so designated by other
authorized documents.

The citation in this report of trade names of commercially available
products does not constitute official endorsement or approval of the
use of such products.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
1. REPORT NUMBER ETL - 0270	2. GOVT ACCESSION NO. AD-A109 238	3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) SCATTERING FROM A VEGETATION LAYER WITH AN IRREGULAR VEGETATION SOIL BOUNDARY		5. TYPE OF REPORT & PERIOD COVERED Technical Report June 1979 - June 1980	
		6. PERFORMING ORG. REPORT NUMBER	
7. AUTHOR(s) Richard A. Hevenor		8. CONTRACT OR GRANT NUMBER(s)	
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Army Engineer Topographic Laboratories Fort Belvoir, Virginia 22060		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 4A161102B52C	
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Engineer Topographic Laboratories Fort Belvoir, Virginia 22060		12. REPORT DATE October 1981	
		13. NUMBER OF PAGES 113	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified	
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report) Approved for Public Release; Distribution Unlimited.			
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)			
18. SUPPLEMENTARY NOTES			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Electromagnetic Waves Vegetation Scattering			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A theoretical model is computed for the backscattering of electromagnetic waves from a layer of vegetation by using a first-order renormalization technique to determine volume scattering. The vegetation soil interface is assumed rough according to the tangent plane approximation and the scattering from this boundary is added incoherently to the volume scattering result. The mean wave in the vegetation is obtained using a bilocal approximation of the Dyson's equation. A free space dyadic Green's function is used, along with a correlation function of the dielectric fluctuations that are exponential in form and that also possess different correlation lengths l_x , l_y , and			

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

403192

20. Continued

② in the x, y, and z, directions. Effective propagation constants are obtained for both horizontal and vertical polarization. The scattered wave is solved for by using a two-dimensional Fourier transform technique, and the boundary conditions at either end of the vegetation layer are matched. The far field backscatter coefficients are computed for both horizontal and vertical polarizations. The mean and variance of the dielectric fluctuations are calculated with the aid of Peake's model for the dielectric constant of vegetation. The theory is matched to experimental data taken from a corn field. The resulting values for the correlation parameters are then used to monitor the growth pattern of the corn field over a period of time. Comparisons between the theoretical and experimental results over this time period are shown. The theory is also matched to experimental data from spring and fall deciduous trees. <

PREFACE The authority for performing the work described in this report is contained in Project 4A161102B52C, "Research in Geodetic, Cartographic, and Geographic Sciences."

The theory described is the result of in-house work and represents an application of the renormalization technique for studying scattering from certain types of vegetation. A solution for the mean wave is obtained by using the bilocal approximation of the Dyson's equation. A Fourier transform of the dyadic Green's function is used to compute a solution utilizing an anisotropic correlation function for the random dielectric fluctuations. The scattered waves are computed from the mean wave and finally the radar backscatter coefficient is calculated. The influence of a rough surface under the vegetation is considered by using a noncoherent technique.

This task was performed under the supervision of Dr. Frederick Rohde, Team Leader, Center for Theoretical and Applied Physical Sciences; Mr. Melvin Crowell, Jr., Director, Research Institute.

COL Daniel L. Lycan, CE and COL Edward K. Wintz, CE were Commanders and Directors and Mr. Robert P. Macchia was Technical Director of the Engineer Topographic Laboratories during the study period.

Accession For	
NTIS GRA&I	☒
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A	

CONTENTS

TITLE	PAGE
PREFACE	1
ILLUSTRATIONS	3
INTRODUCTION	4
Purpose	4
Background	4
ANALYSIS	9
Mean Wave Solution	9
Scattered Wave Solution	19
Modifying the Volume Scattering Results to Incorporate the Influence of An Irregular Vegetation-Soil Boundary	34
Development of a Vegetation Permittivity Model	38
DISCUSSION OF RESULTS	40
CONCLUSIONS	75
APPENDIXES	76
A. Definition of Terms Involved in Computing $\langle A_y A_y^* \rangle$, $\langle A_x A_x^* \rangle$, $\langle A_z A_z^* \rangle$, $\langle A_x A_z^* \rangle$, $\langle A_x A_y^* \rangle$	76
B. Computer Programs for Calculating the Backscattering Coefficients	82
LIST OF SYMBOLS	111

ILLUSTRATIONS

FIGURE	TITLE	PAGE
1	Scattering Geometry	6
2	Corn Ground Truth, 1974	41
3	Data Record of Soil Moisture, Plant Moisture, Plant Height, and Precipitation as Measured During Observation Period	42
4	Representative Dielectric Constant Value as a Function of Volumetric Water Content	43
5 - 15	Comparison of Theory with Experimental Data	45 - 56
16	Study of the Experimental Variations of σ° for Alfalfa	57
17	Study of the Sensor Look Direction	58
18	Scattering Coefficient σ° as a Function of Incidence Angle at (a) 2.75GHz, (b) 5.25GHz, and (c) 7.25GHz Data Set #1, July 16, 1974	59
19	Study of the Variation of σ° with Layer Thickness	60
20	Study of the Skin Depth of the Mean Wave Versus Incidence Angle	61
21	Comparison of Half Space, Plane Layer, and Layer with Rough Surface Solutions	62
22	Study of σ° Variations with F	64
23	Study of σ° Variations with R_v	65
24	Study of σ° Variations with L	66
25	Study of σ° Variations with Soil Moisture	68
26	Study of σ° Variations with Frequency	69
27	Study of σ° Variations with m_s	70
28	Study of σ° Variations with ℓ_x	71
29	Study of σ° Variations with ℓ_y	72
30	Study of σ° Variations with ℓ_z	73

SCATTERING FROM A VEGETATION LAYER WITH AN IRREGULAR VEGETATION SOIL BOUNDARY

INTRODUCTION

PURPOSE This research report presents a theory for analyzing the nature of radar wave scattering from certain types of vegetation. The vegetation is simulated by a continuous random medium, and use is made of a first-order renormalization technique to calculate the radar backscatter coefficient. The influence of an irregular vegetation soil interface has also been considered, using a noncoherent approach.

BACKGROUND In a previous report,¹ a derivation was presented of the radar backscatter coefficient from a half space of random medium using a first-order renormalization solution for the scattered wave and an isotropic correlation function for the random dielectric fluctuations. Recently, Fung solved the problem of scattering from a vegetation layer by using a scalar first-order renormalization approach.² In his solution, however, he did not consider the existence of a rough vegetation soil boundary. He did consider an anisotropic correlation function in which the horizontal variation is different from the vertical.

Tsang and Kong solved the problem of volume scattering from a half space random medium that contains lateral and vertical fluctuations.³ A radiative transfer approach was used to calculate the backscattering cross sections up to second order in approximation. This enabled the cross polarized terms to be obtained.

There are two important practical applications for developing and analyzing various radar scattering theories. The first application is radar image simulation of terrain features. In this problem, the radar system parameters and terrain parameters are known and used to calculate a radar response in the form of a gray tone or density.

¹R.A. Hevenor, *Backscattering of Radar Waves By Vegetated Terrain*, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA. ETL-0105, June 1977, AD-A047 669.

²A.K. Fung, "Scattering From a Vegetation Layer," *IEEE Transactions on Geoscience Electronics*, Vol. GE-17, No. 1, January 1979.

³L. Tsang and J.A. Kong, "Radiative Transfer Theory for Active Remote Sensing of Half Space Random Media," *Radio Science*, Vol. 13, No. 5, September-October 1978.

The scattering theories can be used to compute the radar backscatter coefficient, which in turn is used to calculate gray tone. Using scattering theories in this type of application is straightforward, even though a solution for any one particular scattering problem may be extremely complicated.

The second application is in the field of remote sensing of terrain in which the sensor responses must be used to determine various terrain parameters. Using scattering theories for this application is not straightforward.

However, there are two important uses of scattering theories that bear directly on remote sensing. The first use is a parameter sensitivity study. The theory can be used to analyze the influence of various vegetation, terrain, and radar parameters upon the sensor response. Such parameters as surface roughness, soil moisture, vegetation height, and density could be varied one at a time to determine the influence on the sensor response. This type of analysis should lead to determining what radar parameters are most sensitive to certain terrain parameter changes. This type of analysis assumes the existence of scattering theories that have been developed and compared with existing experimental data.

The second use is to analyze the radar response for two different types of terrain features to see if the two features could be distinguished from each other on an image. Once again, this would assume the existence of scattering theories that have been developed and tested against experimental data. These applications provide the incentive for developing, analyzing, and testing various scattering theories.

In this report, the geometry of the scattering problem to be solved and the basic technique used for the solution will be discussed. In the analysis section, the derivation of the necessary equations will be provided. In the results section, the resulting theory will be compared with existing experimental data, and a study on the sensitivity of the input parameters will be provided.

In this report, the rationalized MKS system of units is used. A line under a symbol will be used to represent a vector quantity. A double line under a symbol will be used to represent a dyadic. A list of the most important symbols is provided at the end of this report.

In figure 1, the scattering geometry of the vegetation problem is shown. A plane wave with a time harmonic of $\exp(j\omega t)$ is incident from free space at an angle θ_i onto a layer of vegetation. The mean thickness of the vegetation is L . The vegetation-soil boundary is considered to be randomly rough according to the tangent plane approximation. The vegetation is simulated by a continuous random medium in which $\epsilon(\underline{r})$ and $\sigma(\underline{r})$ represent the three-dimensional random dielectric and conductivity fluctuations, respectively. These fluctuations consist of the sum of an average and a fluctuating component. The standard deviations of the fluctuations are represented by η_1 and η_2 . The angle of refraction of the mean wave in the random medium is θ_e .

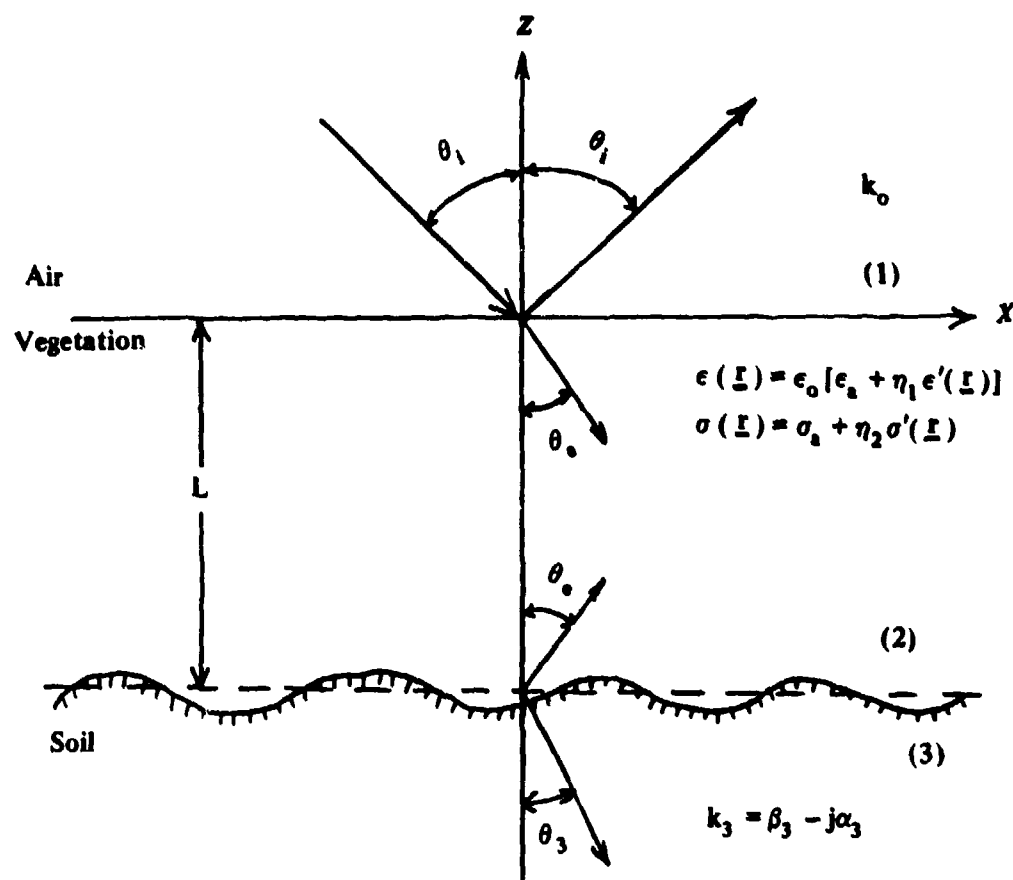


FIGURE 1. Scattering Geometry.

The soil below the vegetation represented by medium 3 is assumed homogeneous with a complex propagation constant k_3 . The magnetic permeability for all three media is assumed to be that of free space. The electric field ($\underline{E}_i$) incident onto the vegetation layer can be written as follows:

$$\underline{E}_i = \{a_1 \underline{a}_x + a_2 \underline{a}_y + a_3 \underline{a}_z\} e^{-jk_0(x \sin \theta_i - z \cos \theta_i)}$$

where $\underline{a}_x$, $\underline{a}_y$, and $\underline{a}_z$ are unit vectors in the x, y, and z directions, respectively. The constants a_1 , a_2 , and a_3 are arbitrary, allowing for the consideration of both horizontal and vertical polarizations. A first-order renormalization method will be used to calculate the mean and scattered waves in the random medium. A solution will be developed first for the case where the vegetation soil boundary is a plane interface. The irregular boundary will be considered in a noncoherent manner afterwards. The dielectric and conductivity fluctuation terms ($\epsilon'(\underline{r})$ and $\sigma'(\underline{r})$) are considered as being generated by statistically homogeneous random processes. The means and correlation functions of the random processes are defined as follows:

$$\langle \epsilon'(\underline{r}) \rangle = \langle \sigma'(\underline{r}) \rangle = 0$$

$$\langle \epsilon'(\underline{r}) \epsilon'(\underline{r}') \rangle = \langle \sigma'(\underline{r}) \sigma'(\underline{r}') \rangle = e^{-|x-x'|/\ell_x} e^{-|y-y'|/\ell_y} e^{-|z-z'|/\ell_z}$$

where ℓ_x , ℓ_y , and ℓ_z are the correlation distances in the x, y, and z directions, respectively. The correlation functions have been chosen to be anisotropic. This representation, with unequal correlation distances, is believed to be closer to reality than an isotropic correlation function. This is because the size of vegetation scatterers in a horizontal plane is not the same as the size of the scatterers in a vertical plane. The mean wave in the random medium is determined from the bilocal approximation of the Dyson's equation:

$$[\nabla \times \nabla \times - k_0^2] \langle \underline{E}(\underline{r}) \rangle - \int_{V'} \langle \xi(\underline{r}) \xi(\underline{r}') \rangle \langle \underline{E}(\underline{r}') \rangle \cdot \underline{\Gamma}(\underline{r}, \underline{r}') d\underline{r}' = 0$$

where $\langle \underline{E}(\underline{r}) \rangle$ is the mean wave in the random medium.

$$\xi(\underline{r}) = -j\omega\mu_0\eta_2\sigma'(\underline{r}) + \omega^2\mu_0\epsilon_0\eta_1\epsilon'(\underline{r}).$$

$$k_a^2 = -j\omega\mu_0\sigma_a + \omega^2\mu_0\epsilon_0\epsilon_a.$$

$\underline{\Gamma}(\underline{r}, \underline{r}')$ is the dyadic Green's function.

V' is the volume of the random medium.

In the next section, plane wave solutions will be sought to the Dyson equation using an infinite space dyadic Green's function. It should be noted that the mean wave is located in the integrand, making any solution very difficult. Once the mean wave has been calculated and the appropriate boundary conditions have been matched for the mean waves in all three media, then the scattered wave in the vegetation layer can be calculated from the following equation:

$$[\nabla \times \nabla \times - k_a^2] \underline{E}_s(\underline{r}) = \xi(\underline{r}) <\underline{E}(\underline{r})>$$

where $\underline{E}_s(\underline{r})$ is the scattered wave.

The mean wave acts as a source term for the scattered wave, which will be computed using a Fourier transform technique. This in turn will enable the scattered waves in air to be determined. The necessary boundary conditions will be matched, and the backscatter coefficient will be calculated for horizontal and vertical polarizations. The influence of the rough boundary between the vegetation and the soil will be considered apart from the volume scattering solution, using the tangent plane method. The backscatter coefficient for rough surface scattering will be modified by the attenuation through the vegetation. This result will then be added to the volume scattering solution to obtain a final answer for the backscatter coefficient. An elementary permittivity model will be developed that will relate certain parameters of the random media to the parameters of actual vegetation. A discussion of results section will follow in which the theoretical results are compared with actual experimental data. Also, a parameter sensitivity study will be conducted on the theory to determine the influence of various parameter changes upon the final result.

ANALYSIS

In this section, the necessary mathematical derivations will be provided to enable a scattering model for vegetation to be obtained. First, a solution for the mean wave will be presented and then the scattered wave will be calculated.

MEAN WAVE SOLUTION

The first step in obtaining a solution to the Dyson equation is to write the dyadic Green's function:

$$\underline{\Gamma}(\underline{r}, \underline{r}') = \frac{\nabla \nabla'}{k_a^2} - \underline{I} G_o(\underline{r}, \underline{r}') \quad (1)$$

where $\underline{I}$ is the unit dyadic, and $G_o(\underline{r}, \underline{r}')$ is the scalar Green's function that satisfies the following equation:

$$(\nabla^2 + k_a^2) G_o(\underline{r}, \underline{r}') = \delta(\underline{r} - \underline{r}') \quad (2)$$

The solution for equation (2) is usually expressed in terms of $R = |\underline{r} - \underline{r}'|$. However, this particular form is not useful when working with an anisotropic correlation function that is expressed in rectangular coordinates. We shall therefore seek a solution to (2) that uses rectangular coordinates. Let $G_o(\underline{r}, \underline{r}')$ take the following form:

$$G_o(\underline{r}, \underline{r}') = \frac{1}{(2\pi)^3} \int d\underline{k} \widetilde{G}_o(\underline{k}) e^{i\underline{k} \cdot (\underline{r} - \underline{r}')} \quad (3)$$

where

$$\underline{k} = k_x \underline{a}_x + k_y \underline{a}_y + k_z \underline{a}_z$$

Substituting (3) into (2) results in a solution for $\widetilde{G}_o(\underline{k})$.

$$\widetilde{G}_o(\underline{k}) = \frac{1}{k_a^2 - k_x^2 - k_y^2 - k_z^2} \quad (4)$$

When (4) is placed in (3) and integration is performed in the complex k_z plane, $G_o(\underline{r}, \underline{r}')$ becomes

$$G_o(\underline{r}, \underline{r}') = \frac{j}{8\pi^2} \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \cdot \frac{\exp[j\{k_x(x-x') + k_y(y-y') - k'_z |z-z'| \}]}{k'_z} \quad (5)$$

$$k'_z = \sqrt{k_a^2 - k_x^2 - k_y^2}$$

Transforming (5) into polar form results in

$$G_o(\underline{r}, \underline{r}') = \frac{j}{8\pi^2} \int_0^{\infty} dk \int_0^{2\pi} d\theta \cdot k \frac{\exp[j\{k(x-x')\cos\theta + k(y-y')\sin\theta - \sqrt{k_a^2 - k^2} |z-z'| \}]}{\sqrt{k_a^2 - k^2}} \quad (6)$$

$$k = \sqrt{k_x^2 + k_y^2} \quad k_x = k\cos\theta \quad k_y = k\sin\theta$$

When (6) is used in (1), the dyadic Green's function becomes

$$\underline{\underline{\Gamma}}(\underline{r}, \underline{r}') = \frac{j}{8\pi^2} \int_0^{\infty} dk \int_0^{2\pi} d\theta \frac{k}{\sqrt{k_a^2 - k^2}} \{ \underline{\underline{C}}(k, \theta) \underline{\underline{B}}(k, \theta) / k_a^2 - \underline{\underline{I}} \} \cdot \exp[j\{k(x-x')\cos\theta + k(y-y')\sin\theta - \sqrt{k_a^2 - k^2} |z-z'| \}] \quad (7)$$

where

$$\underline{\underline{C}}(k, \theta) = \underline{\underline{a}}_x (jk\cos\theta) + \underline{\underline{a}}_y (jk\sin\theta) + \underline{\underline{a}}_z f_2(k)$$

$$\underline{\underline{B}}(k, \theta) = \underline{\underline{a}}_x (-jk\cos\theta) + \underline{\underline{a}}_y (-jk\sin\theta) + \underline{\underline{a}}_z f_1(k)$$

$$f_1(k) = \begin{cases} j \sqrt{k_a^2 - k^2} & \text{when } z > z' \\ -j \sqrt{k_a^2 - k^2} & \text{when } z < z' \end{cases}$$

and

$$f_2(k) = \begin{cases} -j \sqrt{k_a^2 - k^2} & \text{when } z > z' \\ j \sqrt{k_a^2 - k^2} & \text{when } z < z' \end{cases}$$

Plane wave solutions to the Dyson equation take the following form:

$$\langle \underline{E}(\underline{r}) \rangle = \underline{A} e^{-j \underline{k}_e \cdot \underline{r}}$$

The vector $\underline{A}$ can be obtained by matching boundary conditions. The mean wave is seen to propagate with an effective propagation constant $\underline{k}_e$, which must be determined. When the above equation is placed in the Dyson equation along with the dyadic Green's function given by (7), the following equation is obtained when the cross correlation terms between the dielectric and conductivity are ignored:

$$[\nabla \times \nabla \times - k_0^2 \hat{\underline{\epsilon}}] \underline{A} e^{-j \underline{k}_e \cdot \underline{r}} = 0 \quad (8)$$

$$\hat{\underline{\epsilon}} = \frac{k_a^2}{k_0^2} \underline{I} - \frac{1}{(2\pi)^3} \underline{M} \quad (9)$$

$$\begin{aligned} \underline{M} = & j\pi(\eta^2 \eta_2^2 - k_0^2 \eta_1^2) \int d\underline{r}' \int_0^\infty dk \int_0^\pi d\theta \\ & \cdot d(k) e^{-|x-x'|/l_x} e^{-|y-y'|/l_y} e^{-|z-z'|/l_z} \\ & \cdot e^{+j \underline{k}_e \cdot (\underline{r} - \underline{r}')} [\underline{C}(k, \theta) \underline{B}(k, \theta) / k_a^2 - \underline{I}] \exp [j \{ k(x-x') \cos \theta \\ & + k(y-y') \sin \theta - \sqrt{k_a^2 - k^2} |z-z'| \}] \end{aligned} \quad (10)$$

$$d(k) = k / \sqrt{k_a^2 - k^2}$$

$$\eta = \sqrt{\mu_o / \epsilon_o}$$

The integral in $\underline{r}'$ is allowed to be over all space. For the case where ℓ_z is very small this should be a good approximation except for the points extremely close to either boundary. By carrying out the integration in $\underline{r}'$, one obtains a result for M_{ij} that represents the ij^{th} element of the dyadic $\underline{\underline{M}}$.

$$\begin{aligned} M_{ij} = & 4K\ell_x\ell_y\ell_z \int_0^\infty dk \int_0^{2\pi} d\theta d(k) \{ g(k) \\ & \cdot [F_1(k, \theta)F_{1j}(k, \theta) + k_a^2 \delta_{ij}] + h(k) [F_{21}(k, \theta)F_{2j}(k, \theta) \\ & + k_a^2 \delta_{ij}] \} / \{ 1 + \ell_x^2(k_{ex} + k\cos\theta)^2 \} \\ & \cdot [1 + \ell_y^2(k_{ey} + k\sin\theta)^2] \} \end{aligned} \quad (11)$$

where

$$g(k) = \frac{-1}{1 + j\ell_z(k_{ez} + \sqrt{k_a^2 - k^2})} \quad h(k) = \frac{1}{j\ell_z(k_{ez} - \sqrt{k_a^2 - k^2}) - 1}$$

$$F_{11}(k, \theta) = jk\cos\theta \quad F_{12}(k, \theta) = jk\sin\theta \quad F_{13}(k, \theta) = j\sqrt{k_a^2 - k^2}$$

$$F_{21}(k, \theta) = jk\cos\theta \quad F_{22}(k, \theta) = jk\sin\theta \quad F_{23}(k, \theta) = -j\sqrt{k_a^2 - k^2}$$

$$\delta_{ij} = 1 \text{ when } i = j$$

$$\delta_{ij} = 0 \text{ when } i \neq j$$

$$K = j\pi(\eta^2\eta_2^2 - k_o^2\eta_1^2) / k_a^2$$

The form of the incident wave dictates that $k_{ey} = 0$ and $k_{ex} = k_o \sin \theta_i$. This result makes the following elements of the $\hat{\epsilon}$ dielectric tensor become equal to zero:

$$\hat{\epsilon}_{12} = \hat{\epsilon}_{21} = \hat{\epsilon}_{23} = \hat{\epsilon}_{32} = 0$$

The above result is easily shown by considering a transformation of (11) back to rectangular coordinates and recognizing that the integrand is even in k_y . Carrying out the indicated differentiation in equation (8) and writing the result in matrix form produces

$$\begin{bmatrix} k_{ez}^2 - k_o^2 \hat{\epsilon}_{11} & 0 & -k_{ez} k_o \sin \theta_i - k_o^2 \hat{\epsilon}_{13} \\ 0 & k_{ez}^2 + k_o^2 \sin^2 \theta_i - k_o^2 \hat{\epsilon}_{22} & 0 \\ k_{ez} k_o \sin \theta_i - k_o^2 \hat{\epsilon}_{13} & 0 & k_o^2 \sin^2 \theta_i - k_o^2 \hat{\epsilon}_{33} \end{bmatrix} \begin{bmatrix} A_x e^{-jk_e \cdot \mathbf{r}} \\ A_y e^{-jk_e \cdot \mathbf{r}} \\ A_z e^{-jk_e \cdot \mathbf{r}} \end{bmatrix} = 0 \quad (12)$$

In forming the above matrix, use has been made of the fact that $k_{ey} = 0$ and $k_{ex} = k_o \sin \theta_i$. Now, only solutions for k_{ez} are needed. Two solutions can be developed for k_{ez} , one for a horizontally polarized wave and one for a vertically polarized wave. For a horizontally polarized wave, one has $A_x = A_z = 0$ and $A_y \neq 0$. The following equation can be used to determine k_{ez} for this case:

$$\{ k_{ez}^2 + k_o^2 \sin^2 \theta_i - k_o^2 \hat{\epsilon}_{22} \} A_y e^{-jk_e \cdot \mathbf{r}} = 0 \quad (13)$$

Since the term outside the brackets is not zero, this means that the quantity inside the brackets must be zero.

$$k_{ez} = \pm k_o \sqrt{\sin^2 \theta_i - \hat{\epsilon}_{22}} \quad (14)$$

The above result is not an explicit solution for k_{ez} since this quantity also appears in the integral of M_{22} . A first approximation for k_{ez} can be obtained by letting $\eta_1 = \eta_2 = 0$. This represents the case where there are no random fluctuations at all.

$$k_{ez}^{(0)} = -\sqrt{k_o^2 - k_o^2 \sin^2 \theta_i}$$

The above value can be used to compute M_{22} , which in turn can be used to calculate a new value of k_{ez} that will be called k_h . The minus sign on the square root is chosen over the plus sign to consider waves propagating in the minus z direction.

$$k_h = -k_o \sqrt{\sin^2 \theta_i - \hat{\epsilon}_{22}} \quad (15)$$

When computing k_h in (15), the expression for $k_{ez}^{(0)}$ is used to calculate M_{22} . It is interesting to see that even when k_h is real, k_h still comes out complex so that the mean wave decays as it propagates into the medium. This decay has been explained as resulting from multiple scattering. It is not clear, however, how much multiple scattering is being considered. For a vertical polarized wave, $A_x \neq 0$, $A_y = 0$, and $A_z \neq 0$. This leads to the following determinant:

$$\begin{vmatrix} k_{ez}^2 - k_o^2 \hat{\epsilon}_{11} & -k_{ez} k_o \sin \theta_i - k_o^2 \hat{\epsilon}_{13} \\ -k_o k_{ez} \sin \theta_i - k_o^2 \hat{\epsilon}_{13} & k_o^2 (\sin^2 \theta_i - \hat{\epsilon}_{33}) \end{vmatrix} = 0$$

The above determinant provides another solution for k_{ez} .

$$k_{ez} = k_o \left\{ \frac{-\hat{\epsilon}_{13} \sin \theta_i \pm \sqrt{\hat{\epsilon}_{13}^2 \sin^2 \theta_i - \hat{\epsilon}_{33} (\hat{\epsilon}_{11} \sin^2 \theta_i - \hat{\epsilon}_{11} \hat{\epsilon}_{33} + \hat{\epsilon}_{13}^2)}}{\hat{\epsilon}_{33}} \right\} \quad (16)$$

Once again a first approximation for k_{ez} can be obtained for the case where the random dielectric and conductivity fluctuations disappear ($\eta_1 = \eta_2 = 0$).

$$k_{ez}^{(0)} = -\sqrt{k_a^2 - k_o^2 \sin^2 \theta_i}$$

This value for k_{ez} can be used to calculate the elements of the dielectric tensor. These elements are used in (16) to compute a new value for k_{ez} , which will be called k_v .

$$k_v = -k_o \left\{ \frac{\hat{\epsilon}_{13} \sin \theta_i + \sqrt{\hat{\epsilon}_{13}^2 \sin^2 \theta_i - \hat{\epsilon}_{33} (\hat{\epsilon}_{11} \sin^2 \theta_i - \hat{\epsilon}_{11} \hat{\epsilon}_{33} + \hat{\epsilon}_{13}^2)}}{\hat{\epsilon}_{33}} \right\}$$

The sign associated with the square root has been chosen as minus in order to consider waves propagating in the minus z direction. The effective propagation constant for the mean wave has been determined, and now the amplitude must be calculated by matching appropriate boundary conditions. The total mean electric field in air can be written as:

$$\begin{aligned} \underline{E}_1(r) = & \{ [a_1 \underline{a}_x + a_2 \underline{a}_y + a_3 \underline{a}_z] e^{jk_0 z \cos \theta_1} \\ & + [R_1 \underline{a}_x + R_2 \underline{a}_y + R_3 \underline{a}_z] e^{-jk_0 z \cos \theta_1} \} e^{-jk_0 x \sin \theta_1} \\ & z \geq 0 \quad (17) \end{aligned}$$

The first bracketed term in (17) is the incident wave, and the second bracketed term is the reflected wave. The unknowns in (17) are represented by R_1 , R_2 , and R_3 . However, it will not be necessary to obtain an explicit solution for them since they are not needed in determining the scattered waves. The total mean electric field in the vegetation ($\underline{E}_2(r)$) can be written in the following form, using previous results:

$$\begin{aligned} \underline{E}_2(r) = & \{ T_2 \underline{a}_y e^{p_1 z} e^{jq_1 z} + (T_1 \underline{a}_x + T_3 \underline{a}_z) e^{p_2 z} e^{jq_2 z} + V_2 \underline{a}_y e^{-p_2 z} e^{-jq_2 z} \\ & + (V_1 \underline{a}_x + V_3 \underline{a}_z) e^{-p_2 z} e^{-jq_2 z} \} e^{-jk_0 x \sin \theta_1} \\ & -L \leq z \leq 0 \quad (18) \end{aligned}$$

where

$$\begin{aligned} p_1 + jq_1 &= -jk_h \\ p_1 &= \text{Im}(k_h) & q_1 &= -\text{Re}(k_h) \end{aligned}$$

also

$$\begin{aligned} p_2 + jq_2 &= -jk_v \\ p_2 &= \text{Im}(k_v) & q_2 &= -\text{Re}(k_v) \end{aligned}$$

In (18), both upward and downward waves have been considered. The unknowns are represented by the amplitudes T_1 , T_2 , T_3 , V_1 , V_2 , and V_3 . An explicit solution for each of these is required in terms of propagation constants, medium characteristics, and layer thickness. The explicit solution is needed to compute the scattered waves. An expression for the mean wave in the homogeneous soil medium can be written as

$$\underline{E}_3(z) = [W_1 \underline{a}_x + W_2 \underline{a}_y + W_3 \underline{a}_z] e^{jk_0 x \sin \theta_1} e^{jk_3 z \cos \theta_3} \quad z \leq -L \quad (19)$$

In the soil, the mean wave propagates in the minus z direction and the constants W_1 , W_2 , and W_3 are the unknown amplitudes. Once again, no explicit solution will be required for these amplitudes since they are not needed to compute the scattered waves. To compute the six amplitudes of the mean wave in the vegetation, the following boundary conditions are used:

$$E_{1x} = E_{2x} \text{ at } z = 0$$

$$E_{1y} = E_{2y} \text{ at } z = 0$$

$$\frac{\partial E_{1z}}{\partial y} - \frac{\partial E_{1y}}{\partial z} = \frac{\partial E_{2z}}{\partial y} - \frac{\partial E_{2y}}{\partial z} \text{ at } z = 0$$

$$\frac{\partial E_{1x}}{\partial z} - \frac{\partial E_{1z}}{\partial x} = \frac{\partial E_{2x}}{\partial z} - \frac{\partial E_{2z}}{\partial x} \text{ at } z = 0$$

$$E_{2x} = E_{3x} \text{ at } z = -L$$

$$E_{2y} = E_{3y} \text{ at } z = -L$$

$$\frac{\partial E_{2z}}{\partial y} - \frac{\partial E_{2y}}{\partial z} = \frac{\partial E_{3z}}{\partial y} - \frac{\partial E_{3y}}{\partial z} \text{ at } z = -L$$

$$\frac{\partial E_{2x}}{\partial z} - \frac{\partial E_{2z}}{\partial x} = \frac{\partial E_{3x}}{\partial z} - \frac{\partial E_{3z}}{\partial x} \text{ at } z = -L$$

$$D_{1z} = D_{2z} \text{ at } z = 0$$

$$D_{2z} = D_{3z} \text{ at } z = -L$$

$$\underline{D}_1 = \epsilon_0 \underline{E}_1$$

$$\underline{D}_2 = \epsilon_v E_{2x} \underline{a}_x + \epsilon_h E_{2y} \underline{a}_y + \epsilon_v E_{2z} \underline{a}_z$$

where

$$\epsilon_h = \frac{q_1^2 - p_1^2}{\omega^2 \mu_0} \text{ evaluated at } \theta_1 = 0^\circ$$

$$\epsilon_v = \frac{q_2^2 - p_2^2}{\omega^2 \mu_0} \text{ evaluated at } \theta_1 = 0^\circ$$

$$\underline{D}_3 = \epsilon_3 \underline{E}_3$$

where ϵ_3 is the dielectric constant of the soil. Two divergence conditions will also be used, along with the above boundary conditions.

$$\nabla \cdot \underline{E}_1 = 0 \quad \text{and} \quad \nabla \cdot \underline{E}_3 = 0$$

When the equations for the mean fields given by (17) through (19) are placed in the boundary conditions, the result is 12 equations and 12 unknowns. Explicit solutions are only required for the six amplitudes associated with the mean wave in the random medium. Solving for these six values yields the following results:

$$T_2 = \frac{2jk_0 a_{12} a_2 \cos \theta_1}{a_{21} a_{12} - a_{11} a_{22}} \quad (20)$$

$$V_2 = \frac{-2jk_0 a_2 a_{11} \cos \theta_1}{a_{21} a_{12} - a_{11} a_{22}} \quad (21)$$

$$T_1 = \frac{jk_0 b_{12} \{ 2a_1 \cos \theta_1 + \sin \theta_1 (1 - \epsilon_0 / \epsilon_v) [a_3 + a_1 \tan \theta_1] \}}{b_{21} b_{12} - b_{11} b_{22}} \quad (22)$$

$$V_1 = \frac{-jk_0 b_{11} \{ 2a_1 \cos \theta_1 + \sin \theta_1 (1 - \epsilon_0 / \epsilon_v) [a_3 + a_1 \tan \theta_1] \}}{b_{21} b_{12} - b_{11} b_{22}} \quad (23)$$

$$T_3 = \frac{1}{\epsilon_v [e^{-p_2 L} e^{-jq_2 L} - e^{p_2 L} e^{jq_2 L}]} \left\{ \frac{\epsilon_3 k_0 \sin \theta_1}{k_3 \cos \theta_3} \right. \\ \cdot [T_1 e^{-p_2 L} e^{-jq_2 L} + V_1 e^{p_2 L} e^{jq_2 L}] - \epsilon_0 e^{p_2 L} e^{jq_2 L} \\ \cdot [a_3 - \tan \theta_1 (T_1 + V_1 - a_1)] \left. \right\} \quad (24)$$

$$V_3 = \epsilon_0 / \epsilon_v [a_3 - \tan \theta_1 (T_1 + V_1 - a_1)] - T_3 \quad (25)$$

The parameters used in the above equations are defined below:

$$a_{11} = (jk_3 \cos \theta_3 - p_1 - jq_1) e^{-p_1 L} e^{-jq_1 L}$$

$$a_{12} = (jk_3 \cos \theta_3 + p_1 + jq_1) e^{p_1 L} e^{jq_1 L}$$

$$a_{21} = jk_0 \cos \theta_1 + p_1 + jq_1$$

$$a_{22} = jk_0 \cos \theta_1 - p_1 - jq_1$$

$$b_{11} = e^{-p_2 L} e^{-jq_2 L} (p_2 + jq_2 - \alpha)$$

$$b_{12} = -e^{p_2 L} e^{jq_2 L} (p_2 + jq_2 + \alpha)$$

$$b_{21} = p_2 + jq_2 + jk_0 [\cos\theta_1 + \sin\theta_1 \tan\theta_1 (1 - \epsilon_0/\epsilon_v)]$$

$$b_{22} = - \{ p_2 + jq_2 - jk_0 [\cos\theta_1 + \sin\theta_1 \tan\theta_1 (1 - \epsilon_0/\epsilon_v)] \}$$

where

$$\alpha = j \{ k_3 \cos\theta_3 + k_0^2 \sin^2\theta_1 (1 - \epsilon_3/\epsilon_v) / (k_3 \cos\theta_3) \}$$

Now that the mean waves have been fully determined, the scattered waves can be calculated. The scattered waves in the upper medium (air) will then be used to compute the backscatter coefficient.

SCATTERED WAVE SOLUTION

In the random medium, the scattered or incoherent field is calculated from the following equation:

$$[\nabla_x \nabla_x - k_0^2] \underline{E}_s^{(2)}(\underline{r}) = \xi(\underline{r}) < \underline{E}(\underline{r}) > \quad (26)$$

For our problem, the mean wave $< \underline{E}(\underline{r}) >$ is given by $\underline{E}_2(\underline{r})$ as shown in (18). A superscript 2 is used with $\underline{E}_s^{(2)}(\underline{r})$ to indicate clearly the scattered field in the random medium. A solution for this scattered field can be obtained by using a two-dimensional Fourier transform.

$$\underline{E}_s^{(2)}(\underline{r}) = \frac{1}{(2\pi)^2} \int \underline{dk}_t \underline{G}_s(k_t, z) e^{jk_t \cdot \underline{r}_t} \quad (27)$$

where

$$\underline{k}_t = k_x \underline{a}_x + k_y \underline{a}_y \quad \underline{r}_t = x \underline{a}_x + y \underline{a}_y$$

$$\underline{G}_s(k_t, z) = G_{sx}(k_t, z) \underline{a}_x + G_{sy}(k_t, z) \underline{a}_y + G_{sz}(k_t, z) \underline{a}_z$$

When the complete expression for the mean wave in the random medium as given by (18) is placed in (26), a term of the form $\xi(\underline{r}) \exp(-jk_0 x \sin\theta_1)$ on the right side results. This term will be written as a two-dimensional Fourier transform.

$$S(\underline{k}_t, z) = \int d\underline{r}_t \xi(\underline{r}_t) e^{-jk_0 x \sin \theta_i} e^{-j\underline{k}_t \cdot \underline{r}_t}$$

$$\xi(\underline{r}_t) e^{-jk_0 x \sin \theta_i} = \frac{1}{(2\pi)^2} \int d\underline{k}_t S(\underline{k}_t, z) e^{j\underline{k}_t \cdot \underline{r}_t} \quad (28)$$

When (27) and (28) are placed in (26) and the result is put in matrix form,

$$\begin{bmatrix} (k_y^2 - k_a^2 - D_z^2) & -k_x k_y & jk_x D_z \\ -k_x k_y & (k_x^2 - k_a^2 - D_z^2) & jk_y D_z \\ jk_x D_z & jk_y D_z & (k_x^2 + k_y^2 - k_a^2) \end{bmatrix} \begin{bmatrix} G_{sx} \\ G_{sy} \\ G_{sz} \end{bmatrix} = \begin{bmatrix} f_x(z) \\ f_y(z) \\ f_z(z) \end{bmatrix} \quad (29)$$

where D_z represents the differential operator d/dz . The quantities on the right side of (29) are defined below:

$$f_x(z) = S(\underline{k}_t, z) [T_1 e^{p_2 z} e^{jq_2 z} + V_1 e^{-p_2 z} e^{-jq_2 z}]$$

$$f_y(z) = S(\underline{k}_t, z) [T_2 e^{p_1 z} e^{jq_1 z} + V_2 e^{-p_1 z} e^{-jq_1 z}]$$

$$f_z(z) = S(\underline{k}_t, z) [T_3 e^{p_2 z} e^{jq_2 z} + V_3 e^{-p_2 z} e^{-jq_2 z}]$$

Solutions for the quantities G_{sx} , G_{sy} , and G_{sz} can be obtained by solving the three differential equations in (29) using the method of variation of parameters.

$$G_{sx}(\underline{k}_t, z) = A_1 e^{-jk'_z z} + A_2 e^{jk'_z z} + \frac{1}{2jk'_z k_a^2} \left[\int_{-L}^z f(z) e^{jk'_z z} dz \right]$$

$$\cdot e^{-jk'_z z} - \frac{1}{2jk'_z k_a^2} \left[\int_L^z f(z) e^{-jk'_z z} dz \right] e^{jk'_z z} \quad (30)$$

$$G_{xy}(k_1, z) = B_1 e^{-jk'_z z} + B_2 e^{jk'_z z} - \frac{1}{2jk_z k_a^2} \left[\int_{-L}^z h(z) e^{jk'_z z} dz \right] \\ \cdot e^{-jk'_z z} + \frac{1}{2jk_z k_a^2} \left[\int_{-L}^z h(z) e^{-jk'_z z} dz \right] e^{jk'_z z} \quad (31)$$

$$G_{zz}(k_1, z) = \frac{(k_x A_1 + k_y B_1)}{k'_z} e^{jk'_z z} - \frac{(k_x A_2 + k_y B_2)}{k'_z} \\ \cdot e^{jk'_z z} + \frac{e^{-jk'_z z}}{2jk_z'^2 k_a^2} \int_{-L}^z [k_x f(z) - k_y h(z)] e^{jk'_z z} dz \\ + \frac{e^{jk'_z z}}{2jk_z'^2 k_a^2} \int_{-L}^z [k_x f(z) - k_y h(z)] e^{-jk'_z z} dz - \frac{f_z(z)}{k_z'^2} \quad (32)$$

$$k'_z = \sqrt{k_a^2 - k_x^2 - k_y^2}$$

$$f(z) = f_x(z)[k_a^2 - k_x^2] - k_y k_x f_y(z) + jk_x D_z f_z(z)$$

$$h(z) = \frac{k_y(k_a^2 - k_y^2)}{k_x} f_x(z) - (k_a^2 - k_y^2) f_y(z) - jk_y D_z f_z(z) \\ + \frac{k_y}{k_x} (k_x^2 + k_y^2 - k_a^2) f_x(z)$$

The quantities A_1 , A_2 , B_1 , and B_2 are not functions of z and at present, are unknown. Although the solutions for G_{xx} , G_{xy} , and G_{zz} are rather formidable in appearance, it will be found after some mathematical manipulations that a solution will emerge. The three components of the scattered electric field in the upper medium ($z > 0$) can be written in the following form:

$$E_{sx}^{(1)}(r) = \frac{1}{(2\pi)^2} \int dk_t A_x(k_t) e^{j(k_x x + k_y y - k_{1z} z)} \quad (33)$$

$$E_{sy}^{(1)}(r) = \frac{1}{(2\pi)^2} \int dk_t A_y(k_t) e^{j(k_x x + k_y y - k_{1z} z)} \quad (34)$$

$$E_{sz}^{(1)}(r) = \frac{1}{(2\pi)^2} \int dk_t A_z(k_t) e^{j(k_x x + k_y y - k_{1z} z)} \quad (35)$$

where

$$k_{1z} = \sqrt{k_0^2 - k_x^2 - k_y^2}$$

The parameter k_{1z} is obtained by taking any one of the field components given above and putting it in the free space scalar wave equation. The superscript 1 is used to refer to the field in the upper medium, which is air. Therefore, $E_{sx}^{(1)}(r)$ would indicate the x component of the scattered electric field in air. The components of the scattered electric field in the soil ($z < -L$) can be written as follows:

$$E_{sx}^{(3)}(r) = \frac{1}{(2\pi)^2} \int dk_t C_x(k_t) e^{j(k_x x + k_y y + k_{3z} z)} \quad (36)$$

$$E_{sy}^{(3)}(r) = \frac{1}{(2\pi)^2} \int dk_t C_y(k_t) e^{j(k_x x + k_y y + k_{3z} z)} \quad (37)$$

$$E_{sz}^{(3)}(r) = \frac{1}{(2\pi)^2} \int dk_t C_z(k_t) e^{j(k_x x + k_y y + k_{3z} z)} \quad (38)$$

where

$$k_{3z} = \sqrt{k_3^2 - k_x^2 - k_y^2}$$

The superscript 3 refers to the computation of the scattered fields in the soil. The expression for k_{3z} is obtained by putting any one of the field components into the

scalar wave equation, which has a propagation constant k_3 . The unknowns associated with the scattered waves are represented by A_x , A_y , A_z , A_1 , A_2 , B_1 , B_2 , C_x , C_y , and C_z . The only unknowns for which an explicit solution is needed are A_x , A_y , and A_z . Since the only interest is in computing the backscattered far field in air, complete solutions for the scattered fields in the other mediums are not needed. The boundary conditions that the scattered waves must satisfy are provided below:

$$E_{sx}^{(1)} = E_{sx}^{(2)} \text{ at } z = 0$$

$$E_{sy}^{(1)} = E_{sy}^{(2)} \text{ at } z = 0$$

$$\frac{\partial E_{sz}^{(1)}}{\partial y} - \frac{\partial E_{sy}^{(1)}}{\partial z} = \frac{\partial E_{sz}^{(2)}}{\partial y} - \frac{\partial E_{sy}^{(2)}}{\partial z} \text{ at } z = 0$$

$$\frac{\partial E_{sx}^{(1)}}{\partial z} - \frac{\partial E_{sz}^{(1)}}{\partial x} = \frac{\partial E_{sx}^{(2)}}{\partial z} - \frac{\partial E_{sz}^{(2)}}{\partial x} \text{ at } z = 0$$

$$E_{sx}^{(2)} = E_{sx}^{(3)} = -L$$

$$E_{sy}^{(2)} = E_{sy}^{(3)} \text{ at } z = L$$

$$\frac{\partial E_{sz}^{(2)}}{\partial y} - \frac{\partial E_{sy}^{(2)}}{\partial z} = \frac{\partial E_{sz}^{(3)}}{\partial y} - \frac{\partial E_{sy}^{(3)}}{\partial z} \text{ at } z = -L$$

$$\frac{\partial E_{sx}^{(2)}}{\partial z} - \frac{\partial E_{sz}^{(2)}}{\partial x} = \frac{\partial E_{sx}^{(3)}}{\partial z} - \frac{\partial E_{sz}^{(3)}}{\partial x} \text{ at } z = -L$$

The boundary conditions given above, along with the divergence equations in the two homogeneous media allow ten independent equations to be formulated.

$$k_x A_x + k_y A_y = k_{1z} A_z \quad (39)$$

$$k_x C_x + k_y C_y = -k_{3z} C_z \quad (40)$$

$$A_x = A_1 + A_2 + \frac{1}{2jk'_z k_s^2} \int_{-L}^0 f(z) [e^{jk'_z z} - e^{-jk'_z z}] dz \quad (41)$$

$$A_y = B_1 + B_2 + \frac{1}{2jk'_z k_s^2} \int_{-L}^0 h(z) [e^{-jk'_z z} - e^{jk'_z z}] dz \quad (42)$$

$$C_x = e^{jk_{3z} L} \{ A_1 e^{jk'_z L} + A_2 e^{-jk'_z L} \} \quad (43)$$

$$C_y = e^{jk_{3z} L} \{ B_1 e^{jk'_z L} + B_2 e^{-jk'_z L} \} \quad (44)$$

$$jk_y A_z + jk_{1z} A_y = jk_y G_{sz}(k_t, 0) - \left. \frac{\partial G_{sy}}{\partial z} \right|_{z=0} \quad (45)$$

$$k_{1z} A_x + k_x A_z = j \left. \frac{\partial G_{sx}}{\partial z} \right|_{z=0} + k_x G_{sz}(k_t, 0) \quad (46)$$

$$jk_y G_{sz}(k_t, -L) - \left. \frac{\partial G_{sy}}{\partial z} \right|_{z=-L} = jk_y C_z e^{-jk_{3z} L} - jk_{3z} C_y e^{jk_{3z} L} \quad (47)$$

$$\left. \frac{\partial G_x}{\partial z} \right|_{z=-L} - jk_x G_{sx}(k_t, -L) = jk_{3z} C_x e^{-jk_{3z} L} - jk_x C_z e^{jk_{3z} L} \quad (48)$$

A solution for A_x , A_y , and A_z using the above equations would be quite difficult for arbitrary values of k_x and k_y . However, since we are only interested in the backscattered far field in air, a basic expression for the far field can be obtained by using the Stratton-Chu integral as modified by Silver.⁴ When equations (33) through (35) are evaluated on the surface ($z = 0$) and placed into the Stratton-Chu integral, the following equation for the scattered far field ($\underline{E}_{sf}$) will result:

⁴S. Silver, *Microwave Antenna Theory and Design*, McGraw Hill, New York, 1947.

$$\begin{aligned} \underline{E}_{sf} = & \frac{2 \cos \theta_1 (jk_0) e^{-jk_0 R}}{4\pi R} \left[\underline{a}_x A_x(k_0 \sin \theta_1, 0) + \underline{a}_y A_y(k_0 \sin \theta_1, 0) \right. \\ & \left. + \underline{a}_z A_z(k_0 \sin \theta_1, 0) \right] \end{aligned} \quad (49)$$

Where R is the distance from the origin of the coordinate system to the field point where $\underline{E}_{sf}$ is required. It can now be seen that solutions for A_x , A_y , and A_z are only needed for values of $k_x = k_0 \sin \theta_1$ and $k_y = 0$. When the appropriate expressions for the derivatives are substituted into equations (45) through (48) and we let $k_y = 0$, the 10 equations (39) through (48) become.

$$k_x A_x = k_{1z} A_z \quad (50)$$

$$k_x C_x = k_{3z} C_z \quad (51)$$

$$A_x = A_1 + A_2 + \frac{I_1}{2jk'_z k_a^2} \quad (52)$$

$$A_y = B_1 + B_2 + \frac{I_2}{2jk'_z k_a^2} \quad (53)$$

$$C_x = e^{jk_{3z}L} \left[A_1 e^{jk'_z L} + A_2 e^{-jk'_z L} \right] \quad (54)$$

$$C_y = e^{jk_{3z}L} \left[B_1 e^{jk'_z L} + B_2 e^{-jk'_z L} \right] \quad (55)$$

$$jk_{1z} A_y = jk'_z B_1 - jk'_z B_2 - \frac{I_3}{2k_a^2} \quad (56)$$

$$\begin{aligned} k_{1z} A_x + k_x A_z = & k'_z A_1 - k'_z A_2 + \frac{1}{2jk_a^2} (1 + k_x^2/k_z'^2) I_4 \\ & + \frac{k_x^2}{k_z'} (A_1 - A_2) - \frac{k_x f_z(0)}{k_z'^2} \end{aligned} \quad (57)$$

$$-k'_z B_1 e^{jk'_z L} + k'_z B_2 e^{-jk'_z L} = k_{3z} C_y e^{-jk_{3z} L} \quad (58)$$

$$\begin{aligned} A_2(k'_z{}^3 + k_x^2 k'_z) e^{-jk'_z L} - A_1(k'_z{}^3 + k_x^2 k'_z) e^{jk'_z L} + k_x f_z(-L) \\ = k'_z{}^2 e^{-jk_{3z} L} (k_{3z} C_x - k_x C_z) \end{aligned} \quad (59)$$

The quantities I_1 , I_2 , I_3 , and I_4 used in the above equations are integrals in z and are defined below:

$$I_1 = \int_{-L}^0 f(z) \left[e^{jk'_z z} - e^{-jk'_z z} \right] dz$$

$$I_2 = \int_{-L}^0 h(z) \left[e^{-jk'_z z} - e^{jk'_z z} \right] dz$$

$$I_3 = \int_{-L}^0 h(z) \left[e^{jk'_z z} + e^{-jk'_z z} \right] dz$$

$$I_4 = \int_{-L}^0 f(z) \left[e^{jk'_z z} + e^{-jk'_z z} \right] dz$$

The functions $f(z)$, $h(z)$ and k'_z can be evaluated at $k_y = 0$, before integration takes place. When all the above equations are used to solve for A_x , A_y , and A_z , the following results are derived:

$$\begin{aligned} A_x(k_x, 0) = b_1 f_z(-L) + b_2 f_z(0) + \int_{-L}^0 f(z) \{ b_5 e^{jk'_z z} \\ + b_6 e^{-jk'_z z} \} dz \end{aligned} \quad (60)$$

$$A_y(k_x, 0) = \frac{-1}{jk_a^2 k_{1z}(1 + \tilde{a}_1) + k'_z(1 - \tilde{a}_1)} \int_{-L}^0 h(z) \cdot \{ \tilde{a}_1 e^{-jk'_z z} + e^{jk'_z z} \} dz \quad (61)$$

$$A_z(k_x, 0) = k_x \left\{ b_1 f_z(-L) + b_2 f_z(0) + \int_{-L}^0 f(z) \cdot \left[b_5 e^{jk'_z z} + b_6 e^{-jk'_z z} \right] dz \right\} / k_{1z} \quad (62)$$

In the above three equations, k_x is to be evaluated at $k_0 \sin \theta_i$. The new quantities introduced into the above equations are defined below:

$$\tilde{a}_1 = (k'_z - k_{3z}) e^{-2jk'_z L} / (k'_z + k_{3z})$$

$$\tilde{a}_2 = \frac{\left[k_{3z}(k'_z{}^3 + k_x^2 k'_z) - k'_z{}^2(k_{3z}^2 + k_x^2) \right] e^{-2jk'_z L}}{\left[k_{3z}(k'_z{}^3 + k_x^2 k'_z) + k'_z{}^2(k_{3z}^2 + k_x^2) \right]}$$

$$\tilde{a}_3 = \frac{k_{3z} k_x e^{-jk'_z L}}{\left[k_{3z}(k'_z{}^3 + k_x^2 k'_z) + k'_z{}^2(k_{3z}^2 + k_x^2) \right]}$$

$$\tilde{a}_4 = k'_z{}^2(k_{1z}^2 + k_x^2)$$

$$\tilde{a}_5 = k_{1z}(k'_z{}^3 + k'_z k_x^2) (1 - \tilde{a}_2)$$

$$\tilde{a}_6 = k_{1z}(k'_z{}^3 + k'_z k_x^2) \tilde{a}_3$$

$$\tilde{a}_7 = k_{1z}(k'_z{}^2 + k_x^2) / (2jk_z^2)$$

$$\tilde{a}_8 = \tilde{a}_5 + \tilde{a}_4 (\tilde{a}_2 + 1)$$

$$b_1 = 2k_{1z}(k_z^3 + k_z'k_x^2)\tilde{a}_3 / \tilde{a}_8$$

$$b_2 = -k_{1z}k_x(1 + \tilde{a}_2) / \tilde{a}_8$$

$$b_3 = \tilde{a}_7(1 + \tilde{a}_2) / \tilde{a}_8$$

$$b_4 = \tilde{a}_5 / (2j\tilde{a}_8 k_z'k_x^2)$$

$$b_5 = b_3 + b_4$$

$$b_6 = b_3 - b_4$$

If the receiver in the far field is sensitive to a unit polarization vector $\underline{e}_r$, then the received field (E_R) will be

$$E_R = \underline{e}_r \cdot \underline{E}_{sf}$$

where $\underline{e}_r$ in general has three components ($\underline{e}_r = a_x \underline{e}_{rx} + a_y \underline{e}_{ry} + a_z \underline{e}_{rz}$). The next step in calculating the backscatter coefficient is to determine the statistical average of $E_R E_R^*$, using (49).

$$\begin{aligned} \langle E_R E_R^* \rangle = & \frac{4k_o^2 \cos^2 \theta_i}{16\pi^2 R^2} \left\{ e_{rx} e_{rx}^* \langle A_x A_x^* \rangle + 2\text{Re}[e_{rx} e_{ry} \right. \\ & \cdot \langle A_x A_y^* \rangle + 2\text{Re}[e_{rx} e_{rz} \langle A_x A_z^* \rangle + e_{ry}^2 \\ & \cdot \langle A_y A_y^* \rangle + 2\text{Re}[e_{ry} e_{rz} \langle A_y A_z^* \rangle + e_{rz} e_{rz}^* \\ & \cdot \langle A_z A_z^* \rangle \left. \right\} \end{aligned} \quad (63)$$

The brackets $\langle \dots \rangle$ around $E_R E_R^*$ are used to indicate the calculation of the statistical average. The possibility of e_{rx} and e_{rz} being complex is anticipated, with e_{ry} always being real. What is required now is the computation of each of the six terms inside the brackets of (63). The determination of $\langle A_y A_y^* \rangle$ will be considered first:

$$\begin{aligned} \langle A_y A_y^* \rangle &= k_x^2 k_z^2 M_0 M_0^* \int_{-L}^0 dz \int_{-L}^0 dz \int_{-\infty}^{\infty} dx \\ &\quad \cdot \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dy' \langle \xi(\underline{r}) \xi^*(\underline{r}') \rangle \\ &\quad \cdot \hat{f}(z, z') \exp[-2jk_0(x - x') \sin \theta_1] \end{aligned} \quad (64)$$

where

$$\langle \xi(\underline{r}) \xi^*(\underline{r}') \rangle = (\omega^2 \mu_0^2 \eta_2^2 + k_0^4 \eta_1^2) e^{-|x - x'|/l_x} e^{-|y - y'|/l_y} e^{-|z - z'|/l_z}$$

$$\begin{aligned} \hat{f}(z, z') &= \tilde{a}_1 \tilde{a}_1^* T_2 T_2^* e^{D_1 z} e^{D_1^* z'} + \tilde{a}_1 T_2 a_1^* V_2^* e^{D_1 z} e^{D_2^* z'} \\ &\quad + \tilde{a}_1 T_2 T_2^* e^{D_1 z} e^{D_2^* z'} + \tilde{a}_1 T_2 V_2^* e^{D_1 z} e^{D_1^* z'} \\ &\quad + \tilde{a}_1 V_2 \tilde{a}_1^* T_2^* e^{D_2 z} e^{D_1^* z'} + \tilde{a}_1 \tilde{a}_1^* V_2 V_2^* e^{D_2 z} e^{D_2^* z'} \\ &\quad + \tilde{a}_1 V_2 T_2^* e^{D_2 z} e^{D_2^* z'} + \tilde{a}_1 V_2 V_2^* e^{D_2 z} e^{D_1^* z'} \\ &\quad + \tilde{a}_1^* T_2 T_2^* e^{D_2 z} e^{D_1^* z'} + \tilde{a}_1^* T_2 V_2^* e^{D_2 z} e^{D_2^* z'} \\ &\quad + T_2 T_2^* e^{D_2 z} e^{D_2^* z'} + T_2 V_2^* e^{D_2 z} e^{D_1^* z'} \\ &\quad + V_2 \tilde{a}_1^* T_2^* e^{D_1 z} e^{D_1^* z'} + \tilde{a}_1^* V_2 V_2^* e^{D_1 z} e^{D_2^* z'} \\ &\quad + V_2 T_2^* e^{D_1 z} e^{D_2^* z'} + V_2 V_2^* e^{D_1 z} e^{D_1^* z'} \end{aligned}$$

where

$$D_1 = p_1 + j(q_1 - k'_z) \quad \text{and} \quad D_2 = -(p_1 + jq_1 + jk'_z)$$

$$M_o = \frac{-1}{jk_a^2 k_{1z}(1 + a_1) + k'_z(1 - a_1)}$$

The form of $\hat{f}(z, z')$ given above appears to be very complicated. However, each of the 16 terms in $\hat{f}(z, z')$ consist of simple exponentials in z and z' and therefore can be integrated easily. It should be remember when computing D_1 and D_2 that p_1 and q_1 are real, but k'_z will be complex. Making the substitution that $u = x - x'$ and $v = y - y'$ and transforming the x and y integrals into integrals in u and v produces

$$\begin{aligned} \langle A_y A_y^* \rangle &= (\omega^2 \mu_o^2 \eta_2^2 + k_o^4 \eta_1^2) k_a^2 k_a^{*2} M_o M_o^* \int_{-L}^0 dz \\ &\quad \int_{-L}^0 dz' \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \\ &\quad f(z, z') e^{-2jk_o u \sin \theta_i} e^{-|u|/\ell_x} e^{-|v|/\ell_y} e^{-|z - z'|/\ell_z} \end{aligned} \quad (65)$$

The integrals in x' and y' appear somewhat meaningless. These integrals actually represent the illuminated area in the xy plane, since it is physically unrealistic to have backscattered energy from a portion of the surface that is not illuminated. Considering the integrals in x' and y' to form the illuminated area (A_1) and carrying out the integrations in u and v will yield the following result:

$$\begin{aligned} \langle A_y A_y^* \rangle &= \frac{4\ell_x \ell_y A_1 (\omega^2 \mu_o^2 \eta_2^2 + k_o^4 \eta_1^2) k_a^2 k_a^{*2} M_o M_o^*}{(1 + 4k_o^2 \ell_x^2 \sin^2 \theta_i)} \\ &\quad \int_{-L}^0 dz \int_{-L}^0 dz' \hat{f}(z, z') e^{-|z - z'|/\ell_z} \end{aligned} \quad (66)$$

Consider now a typical term of $\hat{f}(z, z')$ which is of the form $Ae^{az}e^{bz'}$ where A, a , and b are not functions of z or z' , and make a transformation of variables from z and z' to $z_z = z - z'$ and $z'' = z'$. Then, the results of carrying out the integration for this one term becomes

$$\int_{-L}^0 dz \int_{-L}^0 dz' Ae^{az} e^{bz'} e^{-|z-z'|/\ell_z} =$$

$$\frac{A\ell_z}{(a+b)} \left\{ \frac{1 - \ell_z b + \ell_z(a+b)e^{-L(a+1/\ell_z)} - (1 + \ell_z a)e^{-L(a+b)}}{(1 + \ell_z a)(1 - \ell_z b)} \right.$$

$$+ \left. \frac{1 - \ell_z a + \ell_z(a+b)e^{-L(1/\ell_z+b)} - e^{-L(a+b)}(1 + b\ell_z)}{(1 + \ell_z b)(1 - \ell_z a)} \right\}$$

When the answer for the integration in z and z' given above is used for each of the 16 terms in $\hat{f}(z, z')$, then the final result for $\langle A_y A_y^* \rangle$ can be written.

$$\langle A_y A_y^* \rangle = \frac{4\ell_x \ell_y \ell_z A_1 (\omega^2 \mu_0^2 \eta_2^2 + k_c^4 \eta_1^2) k_a^2 k_a^{*2} M_0 M_0^*}{(1 + 4k_0^2 \rho_x^2 \sin^2 \theta_1)}$$

$$\cdot \sum_{n=1}^{16} \frac{A_n}{(c_n + d_n)} \left\{ \frac{1 - \ell_z d_n + \ell_z(c_n + d_n)e^{-L(c_n + 1/\ell_z)} - (1 + \ell_z c_n)e^{-L(c_n + d_n)}}{(1 + \ell_z c_n)(1 - \ell_z d_n)} \right.$$

$$+ \left. \frac{1 - \ell_z c_n + \ell_z(c_n + d_n)e^{-L(d_n + 1/\ell_z)} - e^{-L(c_n + d_n)}(1 + d_n \ell_z)}{(1 + \ell_z d_n)(1 - \ell_z c_n)} \right\} \quad (67)$$

The values for the A_n 's, the c_n 's, and d_n 's are provided in appendix A and simply come from the expression for $\hat{f}(z, z')$. Using the methodology for computing $\langle A_y A_y^* \rangle$, one can calculate all the remaining terms in (63). All of these other terms are given in appendix A. An expression for the radar backscatter coefficient (σ_v^0) can be written in terms of $\langle E_R E_R^* \rangle$

$$\sigma_v^o = \frac{4\pi R^2}{A_1} \frac{\langle E_R E_R^* \rangle}{E_1 \cdot E_1^*} \quad (68)$$

The subscript v on σ_v^o is used to indicate a volume scattering result from a plane layer of random media. No consideration for rough surface scattering is given in σ_v^o . Using (63) and the expression for the incident wave given previously, one can write a final result for σ_v^o :

$$\begin{aligned} \sigma_v^o = & \frac{k_o^2 \cos^2 \theta_i}{\pi(a_1 a_1^* + a_2 a_2^* + a_3 a_3^*)} \\ & \cdot \left\{ e_{rx} e_{rx}^* \alpha_{xx} + 2 \operatorname{Re} [e_{rx} e_{ry} \alpha_{xy}] \right. \\ & + 2 \operatorname{Re} [e_{rx} e_{rz} \alpha_{xz}] + e_{ry}^2 \alpha_{yy} + 2 k_o \sin \theta_i \\ & \cdot \left. \operatorname{Re} [e_{ry} e_{rz} \alpha_{yz} / k_{1z}] + e_{rz} e_{rz}^* \alpha_{zz} \right\} \quad (69) \end{aligned}$$

where

$$\alpha_{xx} = \langle A_x A_x^* \rangle / A_1$$

$$\alpha_{xy} = \langle A_x A_y^* \rangle / A_1$$

$$\alpha_{xz} = \langle A_x A_z^* \rangle / A_1$$

$$\alpha_{yy} = \langle A_y A_y^* \rangle / A_1$$

$$\alpha_{zz} = \langle A_z A_z^* \rangle / A_1$$

Consider now the form of σ_v^o for horizontal and vertical polarizations. The following parameters are used to describe a wave that is transmitted with horizontal polarization and horizontal polarization is received.

$$\begin{aligned}
a_1 &= 0 & e_{rx} &= 0 \\
a_2 &= 1 & e_{ry} &= 1 \\
a_3 &= 0 & e_{rz} &= 0
\end{aligned}$$

For these parameters, the backscatter coefficient can be given the additional subscripts of HH to indicate horizontal polarization transmit, and horizontal polarization receive.

$$\sigma_{HH}^o = k_o^2 \alpha_{yy} \cos^2 \theta_i / \pi \quad ((70))$$

The case of vertical polarization transmit, vertical polarization receive can be characterized as follows:

$$\begin{aligned}
a_1 &= \cos \theta_i & e_{rx} &= \cos \theta_i \\
a_2 &= 0 & e_{ry} &= 0 \\
a_3 &= \sin \theta_i & e_{rz} &= \sin \theta_i
\end{aligned}$$

The backscatter coefficient associated with these parameters can be given the additional subscripts VV.

$$\begin{aligned}
\sigma_{VV}^o &= \frac{k_o^2 \cos^2 \theta_i}{\pi} \left\{ \alpha_{xx} \cos^2 \theta_i + 2 \operatorname{Re} [\alpha_{xz} \cos \theta_i \sin \theta_i] \right. \\
&\quad \left. + \alpha_{zz} \sin^2 \theta_i \right\} \quad (71)
\end{aligned}$$

If a result is computed for the cross-polarized backscatter coefficient (HV or VH), the term will disappear. The reason for this is that the particular elements of the dyadic M , which would yield cross-polarized terms in the mean wave, are all zero. Next, let's consider the influence of an irregular vegetation-soil boundary.

**MODIFYING THE VOLUME
SCATTERING RESULTS TO IN-
CORPORATE THE INFLUENCE
OF AN IRREGULAR VEGETA-
TION - SOIL BOUNDARY**

In this section, we will consider what must be done to equations (70) and (71) to include the influence of a rough ground surface. It is expected that the influence of the rough surface would be greater when the angle of incidence is small. Also, as the vegetation height or density gets larger, less scattering is expected from the ground surface below. In what follows, the horizontal and vertical polarizations will be considered separately.

Consider a horizontally polarized wave incident from free space onto a layer of vegetation that has an average thickness L . The interface between the vegetation and soil will be considered as randomly rough in such a way that the tangent plane approximation is applicable. The radar backscatter coefficient (σ_{HH}^0) will be considered as the sum of a term resulting from surface scattering and a term resulting from volume scattering.

$$\sigma_{HH}^0 = \sigma_{HHS}^0 \exp[-4\alpha_{e1}L \sec\psi_{e1}] + \sigma_{HHV}^0 \quad (72)$$

In equation (72), σ_{HHS}^0 represents the backscatter coefficient for a randomly rough surface with a gaussian distribution of surface heights. The subscript s indicates surface scattering. The quantity σ_{HHS}^0 is multiplied by a decaying exponential in which α_{e1} is the imaginary part of the effective propagation constant and ψ_{e1} is the true angle of refraction for the mean wave in the vegetation. The second subscript 1 on α_{e1} and ψ_{e1} is used to indicate horizontal polarization since these parameters will have different values for vertical polarization. The effective propagation constant (k_{e1}) can be obtained from k_h .

$$k_{e1}^2 = k_{ez}^2 + k_{ex}^2 = k_h^2 + k_o^2 \sin^2 \theta_1$$

If we now let $k_{e1} = \beta_{e1} - j\alpha_{e1}$ and in place of k_h we put $jp_1 - q_1$, then we can solve for β_{e1} and α_{e1} .

$$\beta_{e1} = \rho_{e1}^{1/2} \cos(\phi_{e1}/2)$$

$$\alpha_{e1} = \rho_{e1}^{1/2} \sin(\phi_{e1}/2)$$

Where ρ_{e1} and ϕ_{e1} are defined below:

$$\rho_{e1} = \left\{ 4p_1^2 q_1^2 + (q_1^2 + k_o^2 \sin^2 \theta_1 - p_1^2)^2 \right\}^{1/2}$$

$$\phi_{e1} = \tan^{-1} \left\{ \frac{2p_1 q_1}{q_1^2 + k_o^2 \sin^2 \theta_1 - p_1^2} \right\}$$

The planes of constant phase for the mean wave in the random medium are used to calculate an expression for $\sec \psi_{e1}$.

$$\sec \psi_{e1} = \frac{\sqrt{k_o^2 \sin^2 \theta_1 + \rho_1^2 (\beta_{e1} \cos \gamma_1 - \alpha_{e1} \sin \gamma_1)}}{\rho_1 (\beta_{e1} \cos \gamma_1 - \alpha_{e1} \sin \gamma_1)}$$

Where ρ_1 and γ_1 are given below:

$$\rho_1 = \left\{ 4a_1^2 b_1^2 \sin^4 \theta_1 + [1 - (a_1^2 - b_1^2) \sin^2 \theta_1]^2 \right\}^{1/2}$$

$$\gamma_1 = \frac{1}{2} \tan^{-1} \left[\frac{2a_1 b_1 \sin^2 \theta_1}{1 - (a_1^2 - b_1^2) \sin^2 \theta_1} \right]$$

$$a_1 = \frac{k_o \beta_{e1}}{\beta_{e1}^2 + \alpha_{e1}^2} \quad b_1 = \frac{k_o \alpha_{e1}}{\beta_{e1}^2 + \alpha_{e1}^2}$$

Many derived expressions are available for the backscatter coefficient from a randomly rough surface using the tangent plane method. The following equation will be used:⁵

$$\alpha_{HHS}^o = \frac{R_o^2}{4m_s^2 \cos^4 \psi_{e1}} \left\{ 1 - \sin^2 \psi_{e1} [2(1 - g_o \cos^2 \psi_{e1}) - \sin^2 \psi_{e1} (1 + g_o^2)] \right\} \exp [-\tan^2 \psi_{e1} / (4m_s^2)] \quad (73)$$

⁵ R. A. Hevenor, *Backscattering of Electromagnetic Waves From a Surface Composed of Two Types of Surface Roughness*, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia, FTL-TR-71-4, October 1971, AD-737 675.

The quantity R_o in (73) represents the Fresnel reflection coefficient and can be computed as follows:

$$R_o = \frac{\beta_{e1} \cos \psi_{e1} - \sqrt{\beta_3^2 - \beta_{e1}^2 \sin^2 \psi_{e1}}}{\beta_{e1} \cos \psi_{e1} + \sqrt{\beta_3^2 - \beta_{e1}^2 \sin^2 \psi_{e1}}}$$

In calculating R_o , we have neglected the effect of the imaginary parts of the propagation constants. The term m_s represents the ratio of the standard deviation of the surface fluctuations to the correlation distance. The quantity g_o is defined by the following expression:

$$g_o = 1 - \frac{2\beta_{e1} \cos \psi_{e1}}{\sqrt{\beta_3^2 - \beta_{e1}^2 \sin^2 \psi_{e1}}}$$

A final equation can now be written for σ_{HH}^o that considers both the volume-scattering and surface-scattering effects.

$$\begin{aligned} \sigma_{HH}^o &= \frac{R_o^2}{4m_s^2 \cos^4 \psi_{e1}} \left\{ 1 - \sin^2 \psi_{e1} [2(1 - g_o \cos^2 \psi_{e1}) \right. \\ &\quad \left. - \sin^2 \psi_{e1} (1 + g_o^2)] \right\} \exp[-\tan^2 \psi_{e1} / (4m_s^2)] \\ &\quad \cdot \exp[-4\alpha_{e1} L \sec \psi_{e1}] + k_o^2 \alpha_{yy} \cos^2 \theta_1 / \pi \end{aligned} \quad (74)$$

In the same manner, a complete solution for vertical polarization can be obtained:

$$\begin{aligned} \sigma_{VV}^o &= \frac{r_o^2}{4m_s^2 \cos^4 \psi_{e2}} \left\{ 1 + \frac{r_o' \sin^2 \psi_{e2} \cos \psi_{e2}}{\gamma_o^2} [2r_o \sin \psi_{e2} \right. \\ &\quad \left. + r_o' \cos \psi_{e2}] + \frac{2r_o'}{r_o} \sin \psi_{e2} \cos^3 \psi_{e2} \right\} \end{aligned}$$

$$\begin{aligned}
& \cdot \exp [-\tan^2 \psi_{e2} / (4m_s^2)] \exp [-4 \alpha_{e2} L \sec \psi_{e2}] \\
& + \frac{k_o^2 \cos^2 \theta_i}{\pi} \left\{ \alpha_{xx} \cos^2 \theta_i + 2 \operatorname{Re} [\cos \theta_i \sin \theta_i \alpha_{xz}] + \alpha_{zz} \sin^2 \theta_i \right\}
\end{aligned} \tag{75}$$

The new quantities introduced into the above equation are defined below:

$$\beta_{e2} = \rho_{e2}^{1/2} \cos (\phi_{e2}/2)$$

$$\alpha_{e2} = \rho_{e2}^{1/2} \sin (\phi_{e2}/2)$$

The quantities ρ_{e2} and ϕ_{e2} are given as follows:

$$\rho_{e2} = \left\{ 4p_2^2 q_2^2 + (q_2^2 + k_o^2 \sin^2 \theta_i - p_2^2)^2 \right\}^{1/2}$$

$$\phi_{e2} = \tan^{-1} \left\{ \frac{2p_2 q_2}{q_2^2 + k_o^2 \sin^2 \theta_i - p_2^2} \right\}$$

$$\sec \psi_{e2} = \frac{\sqrt{k_o^2 \sin^2 \theta_i + \rho_{e2}^2 (\beta_{e2}^2 \cos \gamma_2 - \alpha_{e2} \sin \gamma_2)^2}}{\rho_{e2} (\beta_{e2} \cos \gamma_2 - \alpha_{e2} \sin \gamma_2)}$$

$$\rho_2 = \left\{ 4a_2^2 b_2^2 \sin^4 \theta_i + [1 - (a_2^2 - b_2^2) \sin^2 \theta_i]^2 \right\}^{1/2}$$

$$\gamma_2 = \frac{1}{2} \tan^{-1} \left[\frac{2a_2 b_2 \sin^2 \theta_i}{1 - (a_2^2 - b_2^2) \sin^2 \theta_i} \right]$$

where a_2 and b_2 are

$$a_2 = \frac{k_o \beta_{e2}}{\beta_{e2}^2 + \alpha_{e2}^2} \quad b_2 = \frac{k_o \alpha_{e2}}{\beta_{e2}^2 + \alpha_{e2}^2}$$

$$r_2 = \frac{\beta_3^2 \cos \psi_{e2} - \beta_{e2} \sqrt{\beta_3^2 - \beta_{e2}^2 \sin^2 \psi_{e2}}}{\beta_3^2 \cos \psi_{e2} + \beta_{e2} \sqrt{\beta_3^2 - \beta_{e2}^2 \sin^2 \psi_{e2}}}$$

$$r'_o = \frac{2\beta_3^2 \beta_{e2} \sin \psi_{e2} (\beta_3^2 - \beta_{e2}^2)}{\sqrt{\beta_3^2 - \beta_{e2}^2 \sin^2 \psi_{e2}} \left\{ \beta_3^2 \cos \psi_{e2} + \beta_{e2} \sqrt{\beta_3^2 - \beta_{e2}^2 \sin^2 \psi_{e2}} \right\}}$$

Equations (74) and (75) are the final results for the radar backscatter coefficient for horizontal and vertical polarizations. Before the results of computing equations (74) and (75) are shown, an elementary vegetation permittivity model must be developed that relates some of the model input parameters to the complex dielectric constants of vegetation and water.

DEVELOPMENT OF A VEGETATION PERMITTIVITY MODEL

To determine the influence of various vegetation parameters (such as moisture content) upon the calculation of the backscatter coefficient, one must relate some of the permittivity parameters in the scattering model to the physical parameters of the vegetation. Peake and Oliver's model⁶ will be used to calculate the relative complex dielectric constant of vegetation ($\hat{\epsilon}_v$):

$$\epsilon_v = (F/2) \text{Re}[\hat{\epsilon}_w] + j(F/3) \text{Im}[\hat{\epsilon}_w]$$

⁶W.H. Peake and T.L. Oliver, *The Response of Terrestrial Surfaces at Microwave Frequencies*, Technical Report AFAL-TR-70-301, The Ohio State University, Electroscience Laboratory, AD-884 106.

where F is the fraction of water by weight in the vegetation; $\text{Re}[\hat{\epsilon}_w]$ and $\text{Im}[\hat{\epsilon}_w]$ are the real and imaginary parts of the relative complex dielectric constant of water ($\hat{\epsilon}_w$), which can be written as

$$\hat{\epsilon}_w = 5 + \frac{75}{1 + j(1.85/\lambda)}$$

where λ is the wavelength in centimeters. For particular values of λ and F , we can now compute $\hat{\epsilon}_v$. With a knowledge of $\hat{\epsilon}_v$, one can estimate the average relative complex dielectric constant ($\hat{\epsilon}_a$) by using the following

$$\hat{\epsilon}_a = (V_v \hat{\epsilon}_v + \epsilon_A V_A) / V_T$$

$$\hat{\epsilon}_a = \text{Re}[\hat{\epsilon}_a] \quad \sigma_a = -\omega\epsilon_0 \left(\frac{V_v}{V_T} \right) \text{Im}(\hat{\epsilon}_v)$$

where V_v is the volume occupied by the vegetation; V_A is the volume occupied by air; V_T is the total volume equal to $V_v + V_A$; ϵ_A is the relative dielectric constant of air, assumed equal to 1. The variances η_1^2 and η_2^2 can be computed by using the following formulas:

$$\eta_1^2 = \frac{V_v (\epsilon'_v - \epsilon_a)^2 + V_A (\epsilon_A - \epsilon_a)^2}{V_T}$$

$$\eta_2^2 = \frac{V_v (\sigma_v - \sigma_a)^2 + V_A \sigma_a^2}{V_T}$$

where

$$\sigma_v = -\omega\epsilon_0 F \text{Im}[\epsilon_w] / 3 \quad \text{and} \quad \epsilon'_v = \text{Re}[\hat{\epsilon}_v]$$

The symbol R_v shall be used to designate the volume ratio V_v / V_T .

The developed model for the radar backscatter coefficient is now complete and calculations can be made. In the next section, computed results will be shown, and the theory will be compared with some existing experimental data.

DISCUSSION OF RESULTS

In this section, some numerical calculations will be shown for the theory derived in the previous section, and a study will be presented of the influence of the various input parameters on the backscatter coefficient. Two computer programs were developed for solving equations (74) and (75). One program solves for equation (74) and the second solves for (75). The solutions to the half-space and plane layer problems are also generated for comparison. A listing of the computer program for solving equation (74) is given in appendix B. The 10 input parameters to the programs are

1. Fraction of water by weight in the vegetation (F)
2. Volume of vegetation divided by the total volume (R_V)
3. Correlation distance in the x direction (ℓ_x)
4. Correlation distance in the y direction (ℓ_y)
5. Correlation distance in the z direction (ℓ_z)
6. Mean thickness of the vegetation layer (L)
7. Relative dielectric constant of the soil below the vegetation (ϵ_g)
8. Conductivity of the soil below the vegetation (σ_3)
9. Frequency (f)
10. Ratio of the standard deviation of the rough surface fluctuations to the correlation distance of the fluctuations (m_s).

The output of the computer programs is the backscatter coefficient in decibels as a function of incidence angle. The backscatter coefficient in decibels is related to the backscatter coefficient as follows:

$$\sigma^0 \text{ (in decibels)} = 10 \log_{10} \sigma^0$$

The backscatter coefficient on the right side of the above equation is computed by (74) or (75). The following discussion centers on figures 2 through 30, which show the results of computing equations (74) and (75). Figures 2 and 3 come from Ulaby and Bush and provide pertinent ground truth data associated with the experimental measurements. Figure 4 comes from Cihlar and Ulaby⁷ and provides a relationship between soil moisture and relative complex dielectric constant. Figures 5 through 15 provide a comparison of the developed theory with experimental data taken from a cornfield by Ulaby and Bush.⁸

⁷F.T. Ulaby and T.F. Bush, *Corn Growth as Monitored by Radar*, The University of Kansas Center for Research, Inc. RSL Technical Report 117-57, November 1975.

⁸F.T. Ulaby and J. Cihlar, *Dielectric Properties of Soils as a Function of Moisture Content*, The University of Kansas Center for Research, Inc., RSL Technical Report 177-47, November 1974.

FIGURE 2. Corn Ground Truth, 1974.

Date	Soil Moisture (g/cm ³)			% Plant Moisture	Normalized Plant Water Content (g/m)	Plant Height (m)
	N	M	F			
May 20	.19	.19	.24	89.5	77	0.30
May 24	.28	.26	.26	86.5	167	0.40
May 30	.12	.10	.12	87.1	252	0.58
June 5	.10	.13	.10	88.5	262	0.88
June 13	.34	.34	.34	89.9	552	1.25
June 20	.30	.31	.32	89.9	442	1.90
June 26	.06	.09	.08	83.9	383	2.30
July 1	.05	.05	.07	82.4	425	2.60
July 8	.21	.27	.30	84.7	440	2.60
July 11	.17	.15	.15	81.5	490	2.60
July 16	.04	.07	.06	81.5	374	2.60
July 22	.04	.04	.04	73.4	482	2.60
August 5	.06	.07	.07	62.4	236	2.60
August 15	.26	.27	.26	52.9	99	2.70
September 5	.31	.30	.29	47.5	151	2.70
September 19	.12	.12	.10	16.5	58	0.23
July 30*	.26	.26	.26	74.8	413	2.60

* = irrigated corn field N = near range sample M = medium range sample F = far range sample

SOURCE : F.T. Ulaby and T.F. Bush, *Corn Growth as Monitored by Radar*, The University of Kansas Center for Research, Inc. RSL Technical Report 117-57, November 1975.

Figure 2. Corn Ground Truth, 1974.

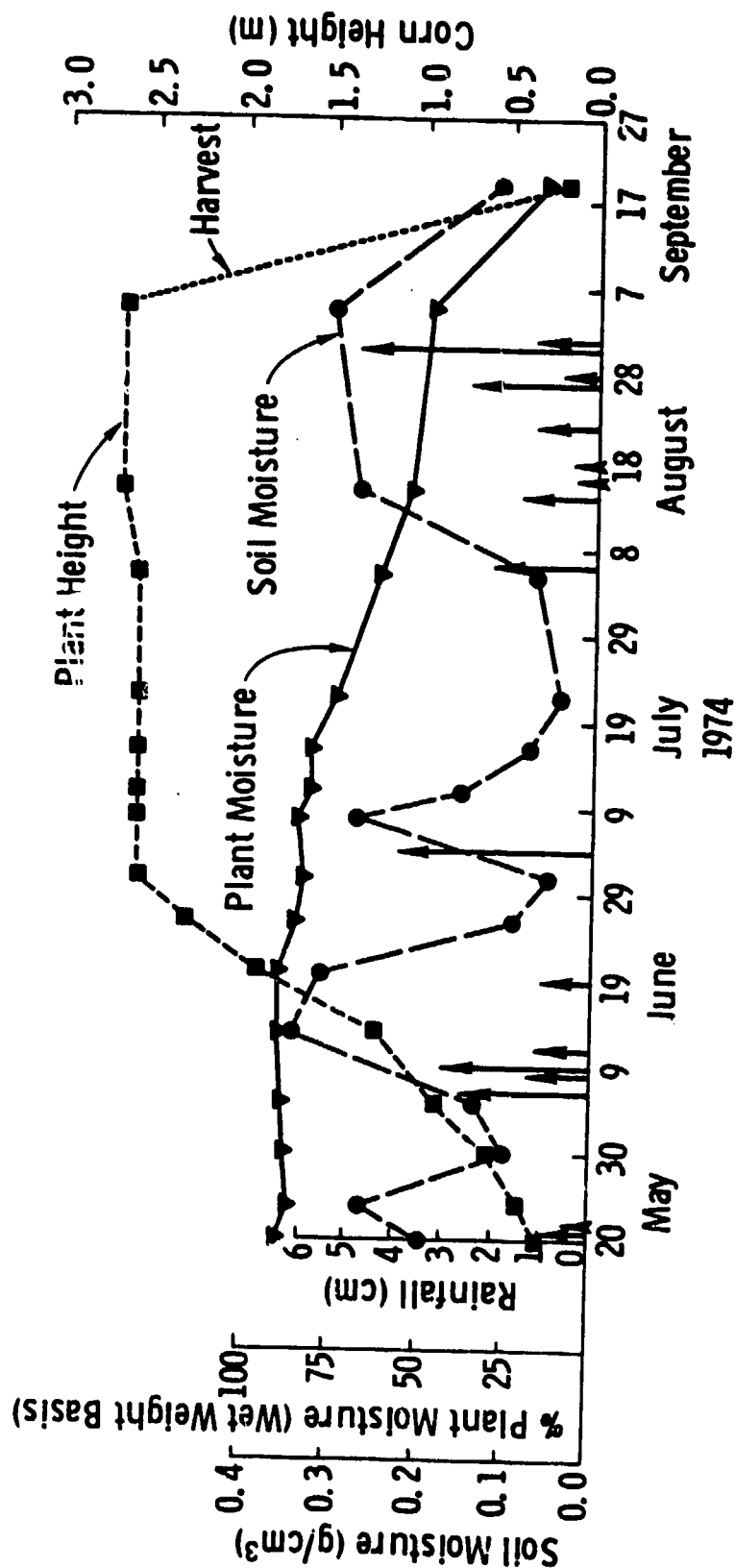
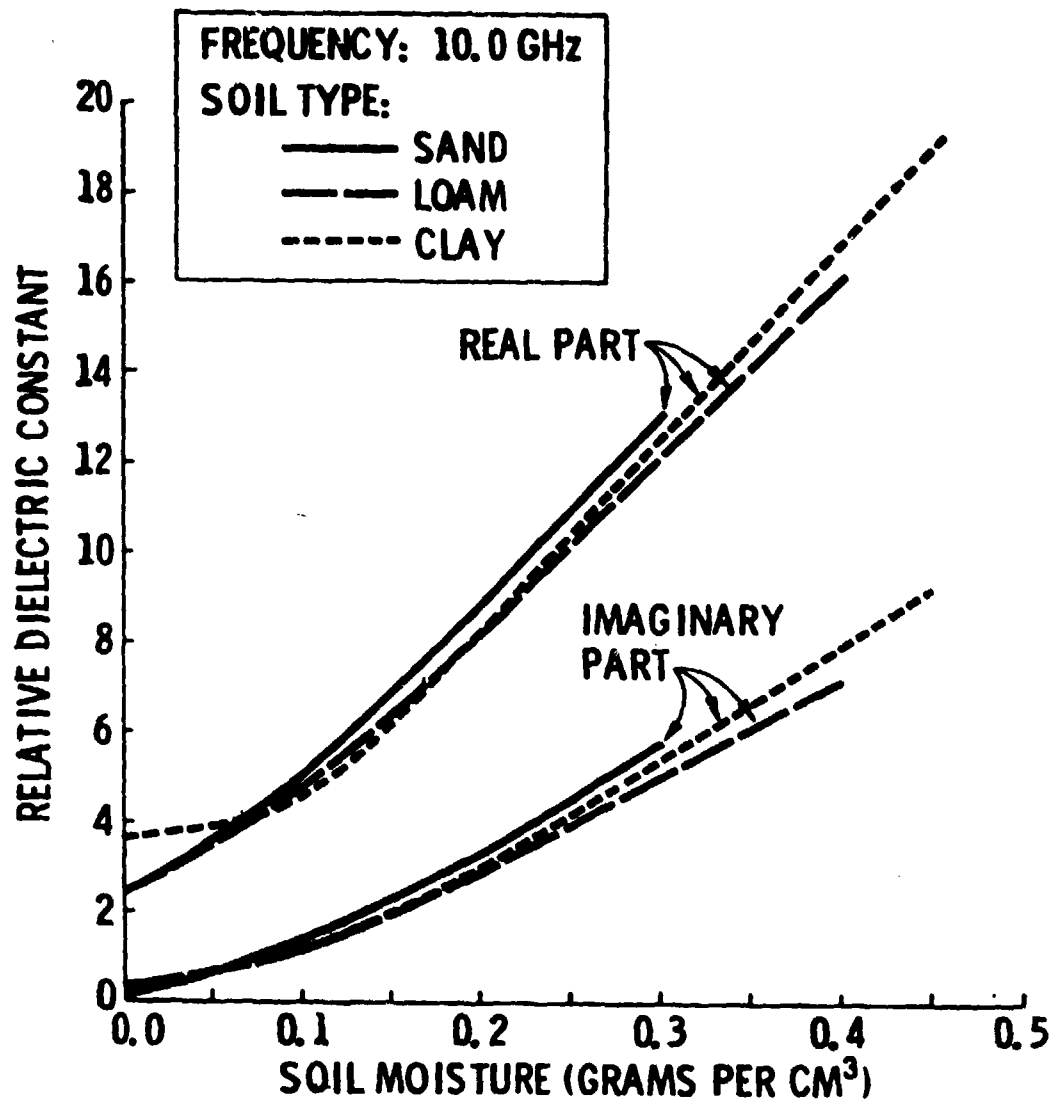


Figure 3. Data Record of Soil Moisture, Plant Moisture, Plant Height, and Precipitation as Measured During the Observation Period.

SOURCE : F.T. Ulaby and T.F. Bush, *Corn Growth as Monitored by Radar*, The University of Kansas Center for Research, Inc. RSL Technical Report 117-57, November 1975.



Source : F.T. Ulaby and J. Cihlar, *Dielectric Properties of Soils as a Function of Moisture Content*, The University of Kansas Center for Research, Inc., RSL Technical Report 177-47, November 1974.

Figure 4. Representative Dielectric Constant Values as a Function of Volumetric Water Content.

The soil moisture is obtained from figure 2 for a particular set of measurements performed on a given date. This soil moisture is used along with the curves of figure 4 to determine the relative dielectric constant and the conductivity of the soil. Throughout all comparisons of theory with experiment, it has been assumed that the soil type is a loam. In comparing theory with experiment, remember that certain input parameters to the theoretical model were not known and had to be estimated; whereas, other input parameters were known from the ground truth data collected during the experiment. The unknown input parameters are R_v , ℓ_x , ℓ_y , ℓ_z and m_s .

In figure 5, the theory is matched to the experimental data for corn that is at a height of 30 centimeters. The large rise that occurs in σ° as θ_i goes from 10° to 0° is indicative of a rough surface effect. In this case, the rough surface is quasi-specular since m_s is given such a small value. It can be seen that to match the theory with the experimental data, it was necessary to let ℓ_x be different from ℓ_y and to let ℓ_z be much smaller than ℓ_x or ℓ_y . The fact that ℓ_x is different from ℓ_y shows an anisotropic effect in the horizontal plane, which probably arises from the corn being planted in rows.

Figure 6 matches the theory to experimental data for corn that is 2.3 meters high. It can be seen that a good match is obtained for ℓ_x equal to ℓ_y , indicating that the anisotropic effect in the horizontal plane has essentially disappeared for 8.6 GHz. The values of the parameters used for R_v , ℓ_x , ℓ_y , ℓ_z , and m_s in figure 6 are also used in figures 7 through 10 to determine whether the model could provide a correct prediction of σ° for different values of F , soil moisture, and vegetation height.

Figures 7 through 9 show an excellent agreement between theory and experiment. Figure 10 shows an excellent agreement between theory and experiment for angles of incidence equal to and greater than 30° . For angles of incidence less than 30° , the agreement is poor. A possible reason for this poor agreement may be due to the rainfall that came prior to the August 15th measurements. The rainfall could have disturbed the soil surface in both a physical and an electrical manner such that its scattering behavior is no longer predictable from prior values.

Figures 11 and 12 show an attempt to match the theory to the experimental data for frequencies of 11 GHz and 13 GHz. It can be seen that to obtain a good match, the values of ℓ_x and ℓ_y must be altered from the values used at 8.6 GHz. This seems to indicate that as the frequency goes higher, the vegetation medium becomes more complicated and the anisotropic behavior becomes more pronounced.

Polarization: Horizontal
 $F = 0.895$ $f = 8.6 \text{ GHz}$
 $R_v = 0.0008$
 $\ell_x = 4\text{mm}$ $\ell_y = 2.5\text{mm}$ $\ell_z = 0.05\text{mm}$
 Layer Thickness = 0.3 meter
 Relative Dielectric Constant of Soil = 9.0
 Conductivity of Soil = 1.44
 $m_i = 0.04$

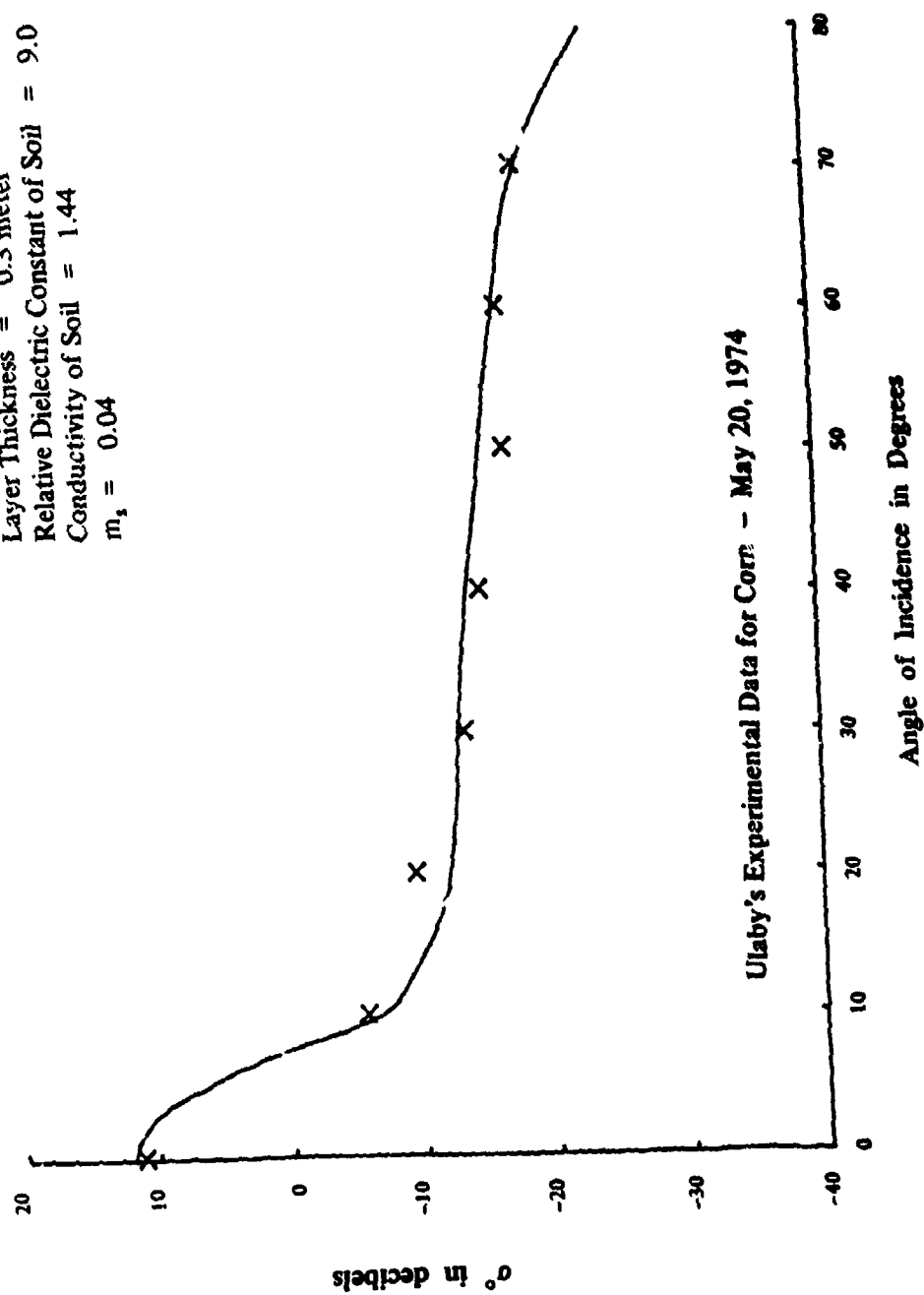


Figure 5. Comparison of Theory with Experimental Data.

Polarization: Horizontal
 $F = 0.839$ $f = 8.6 \text{ GHz}$
 $R_v = 0.0002$
 $\rho_x = \rho_y = 5 \text{ mm}$ $\rho_z = 0.095 \text{ mm}$
 Layer Thickness = 2.3 meters
 Relative Dielectric Constant of Soil = 4.0
 Conductivity of soil = 0.278
 $m_s = 0.035$

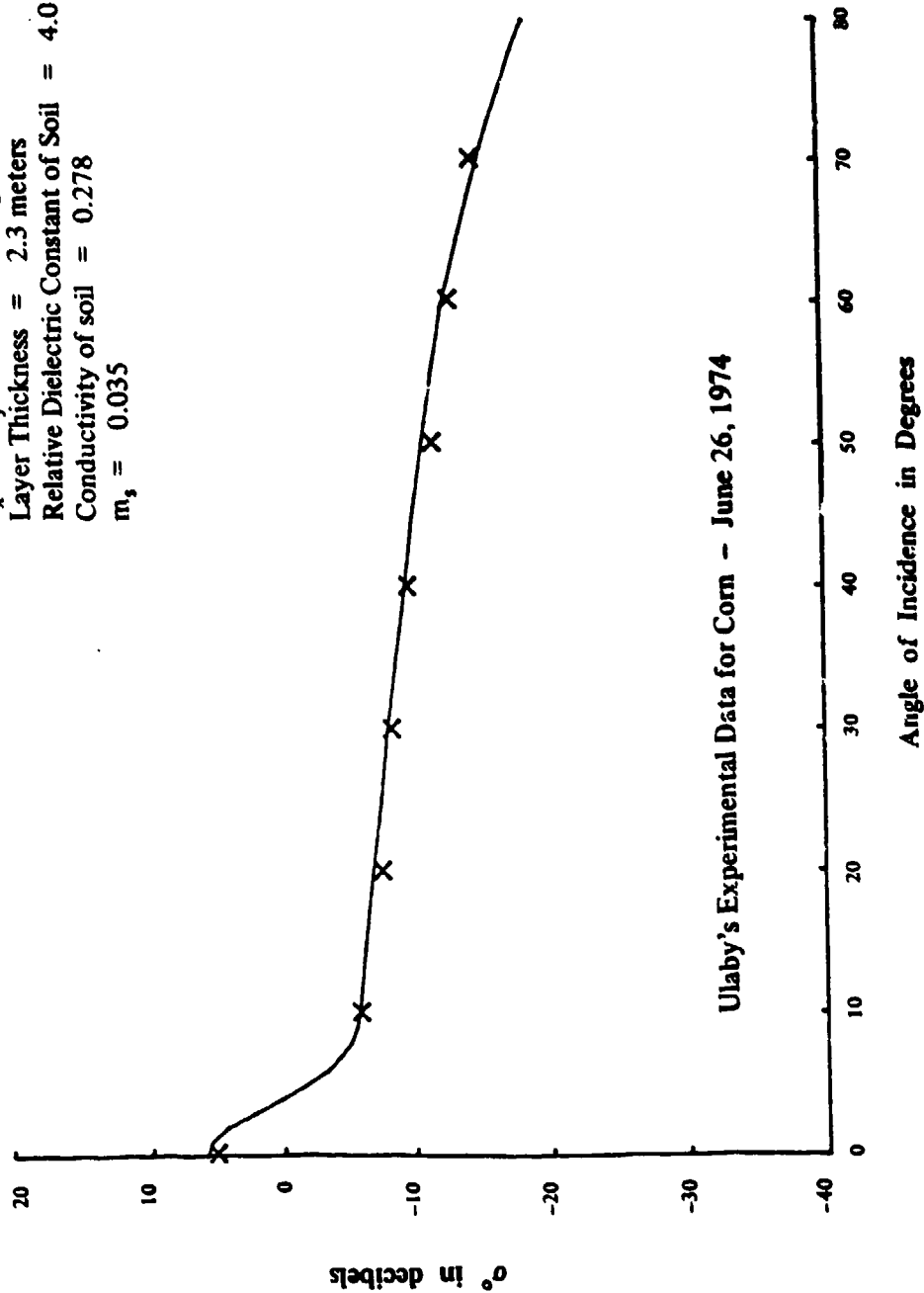


Figure 6. Comparison of Theory with Experimental Data.

Polarization: Horizontal
 $F = 0.815$ $f = 8.6 \text{ GHz}$
 $R_v = 0.0002$
 $\rho_x = \rho_y = 5 \text{ mm}$ $\rho_z = 0.095 \text{ mm}$
 Layer Thickness = 2.6 meters
 Relative Dielectric Constant of Soil = 3.5
 Conductivity of Soil = 0.389
 $m_s = 0.035$

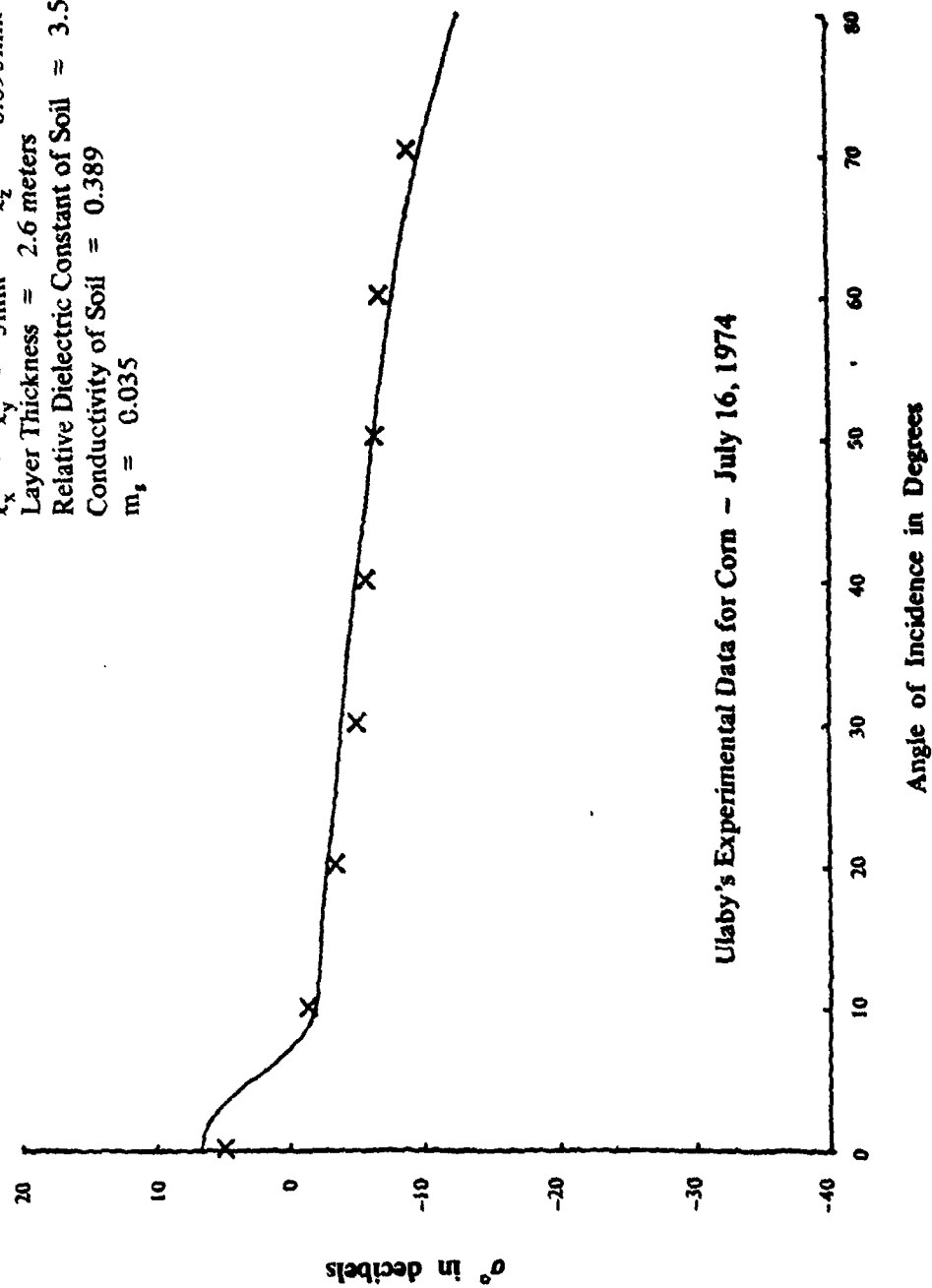


Figure 7. Comparison of Theory with Experimental Data.

Polarization: Horizontal
 $F = 0.734$ $f = 8.6 \text{ GHz}$
 $R_v = 0.0002$
 $q_x = q_y = 5 \text{ mm}$ $l_z = 0.095 \text{ mm}$
 Layer Thickness = 2.6 meters
 Dielectric Constant of Soil = 3.0
 Conductivity of Soil = 0.278
 $m_s = 0.035$

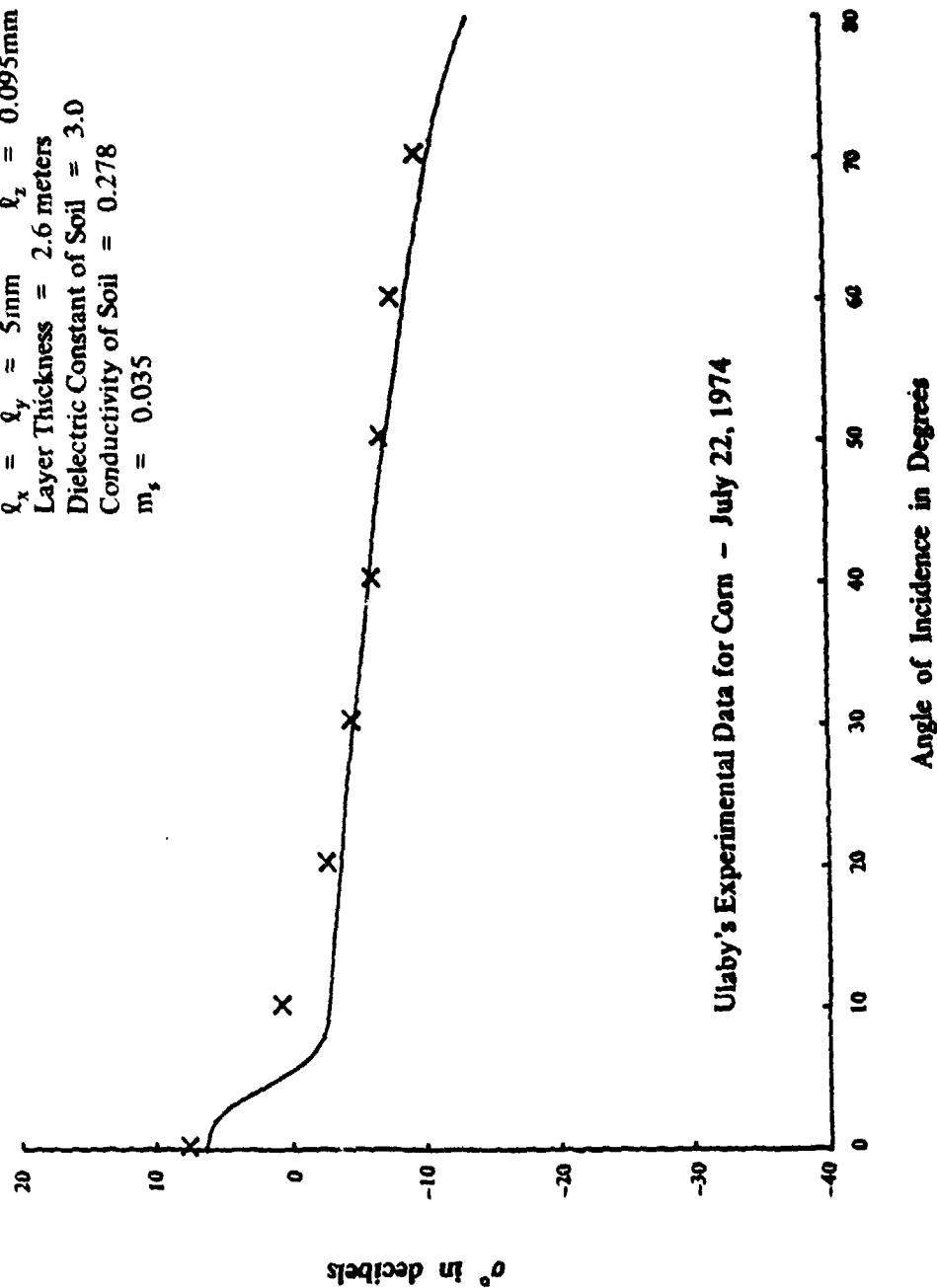


Figure 8. Comparison of Theory with Experimental Data.

Polarization: Horizontal
 $F = 0.624$ $f = 8.6 \text{ GHz}$
 $R_v = 0.0002$
 $\ell_x = \ell_y = 5 \text{ mm}$ $\ell_z = 0.095 \text{ mm}$
 Layer Thickness = 2.6 meters
 Dielectric Constant of Soil = 4.0
 Conductivity of Soil = 0.278
 $m_s = 0.035$

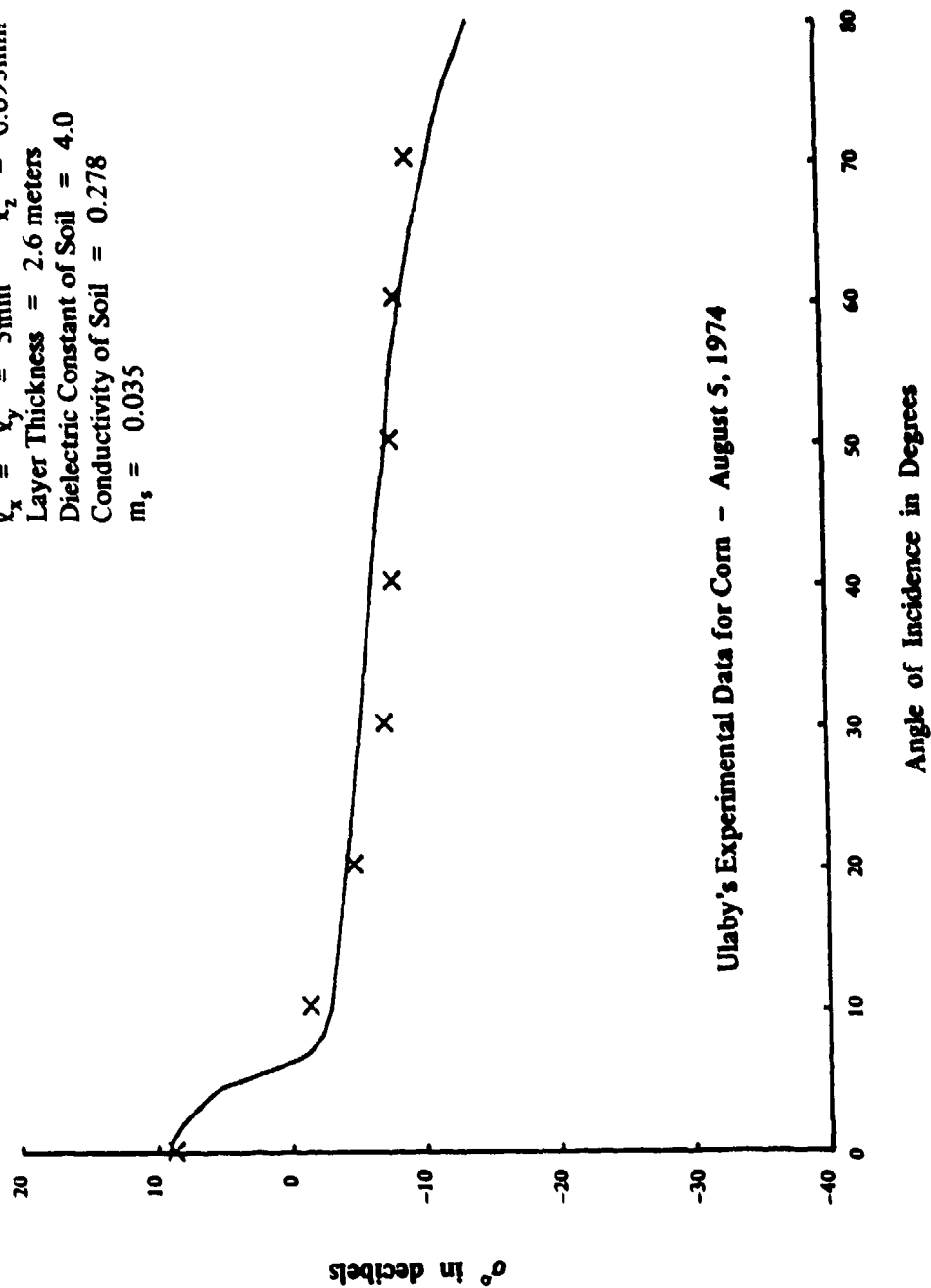


Figure 9. Comparison of Theory with Experimental Data.

Polarization: Horizontal
 $F = 0.529$ $f = 8.6 \text{ GHz}$
 $R_v = 0.0002$
 $\ell_x = \ell_y = 5 \text{ mm}$ $\ell_z = 0.005 \text{ mm}$
 Layer Thickness = 2.7 meters
 Relative Dielectric Constant of Soil = 10.5
 Conductivity of Soil = 2.34
 $m_s = 0.035$

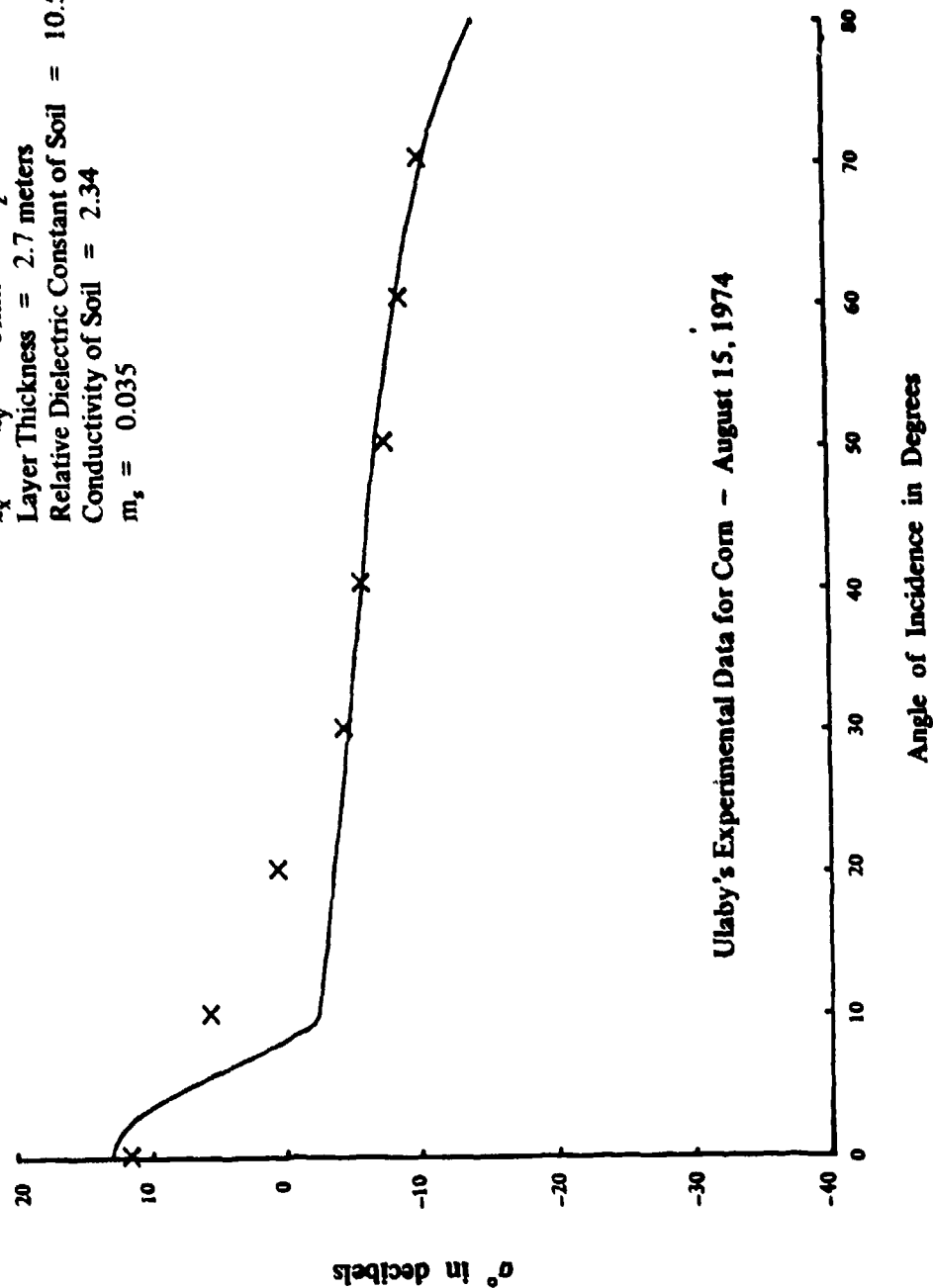


Figure 10. Comparison of Theory with Experimental Data.

Polarization: Horizontal
 $F = 0.839$ $f = 11 \text{ GHz}$
 $R_v = 0.0002$
 $\ell_x = 4 \text{ mm}$ $\ell_y = 6 \text{ mm}$ $\ell_z = 0.095 \text{ mm}$
 Relative Dielectric Constant of Soil = 4.0
 Conductivity of Soil = 0.278
 $m_s = 0.035$

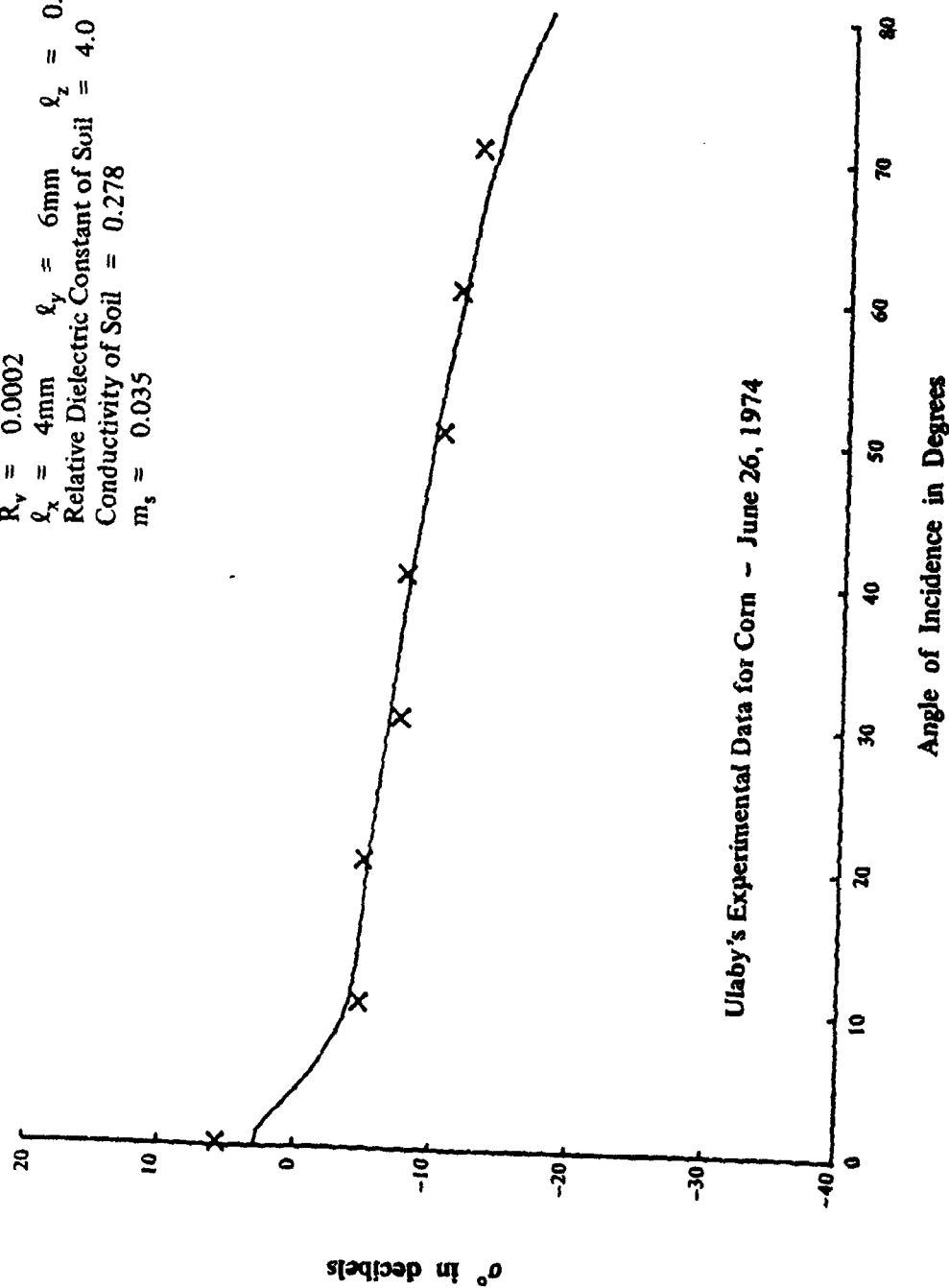


Figure 11. Comparison of Theory with Experimental Data.

Polarization: Horizontal
 $F = 0.839$ $f = 13 \text{ GHz}$
 $R_v = 0.0002$
 $\rho_x = 2 \text{ mm}$ $\rho_y = 8 \text{ mm}$ $\rho_z = 0.0001$
 Layer Thickness = 2.3 meters
 Relative Dielectric Constant of Soil = 4.0
 Conductivity of Soil = 0.278
 $m_s = 0.035$

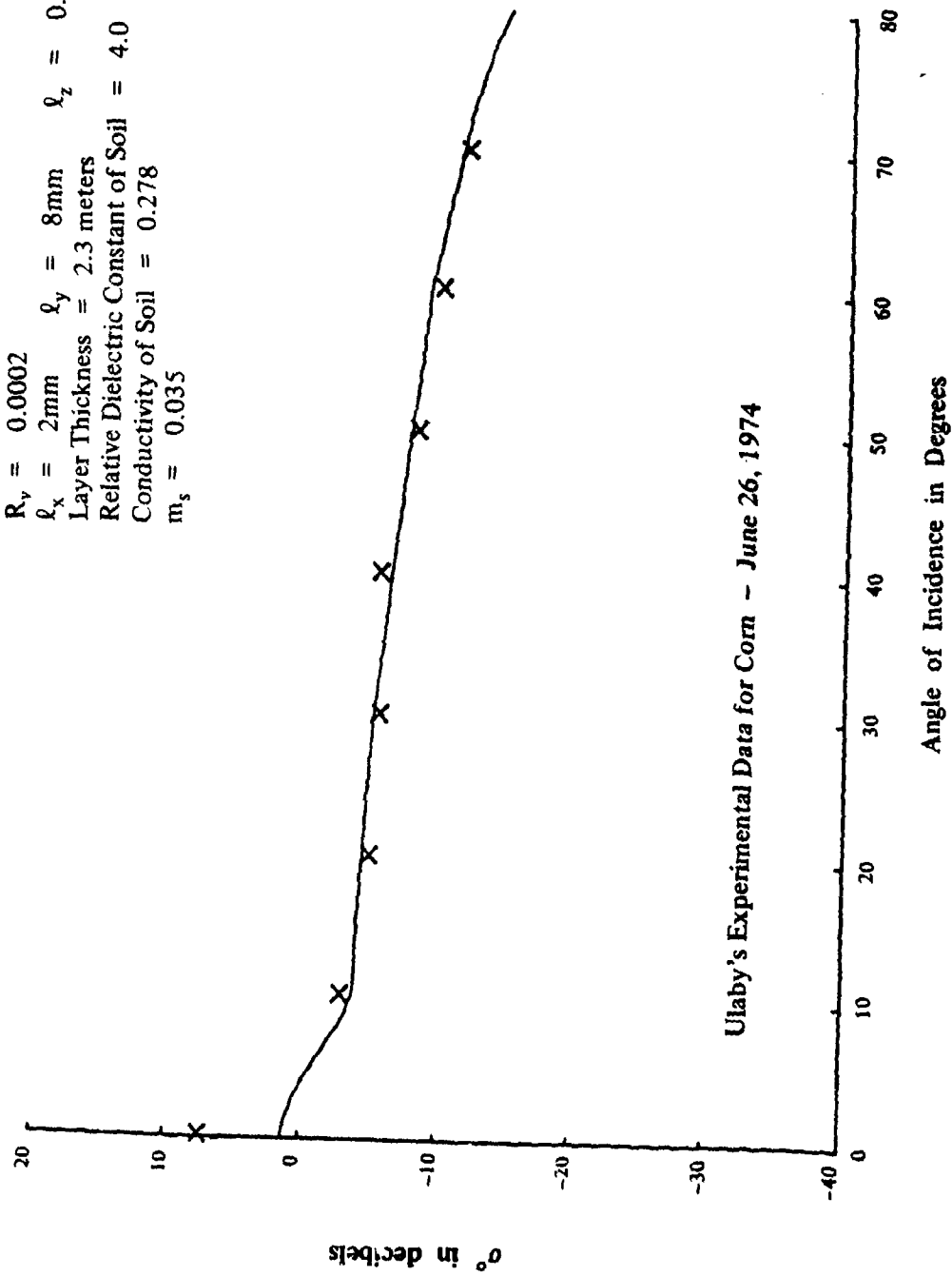


Figure 12. Comparison of Theory with Experimental Data.

Polarization: Vertical
 $F = 0.839$ $f = 8.6 \text{ GHz}$
 $R_v = 0.0002$
 $\ell_x = 4\text{mm}$ $\ell_y = 7\text{mm}$ $\ell_z = 0.095\text{mm}$
 Layer Thickness = 2.3 meters
 Relative Dielectric Constant of Soil = 4.0
 Conductivity of Soil = 0.278
 $m_s = 0.035$

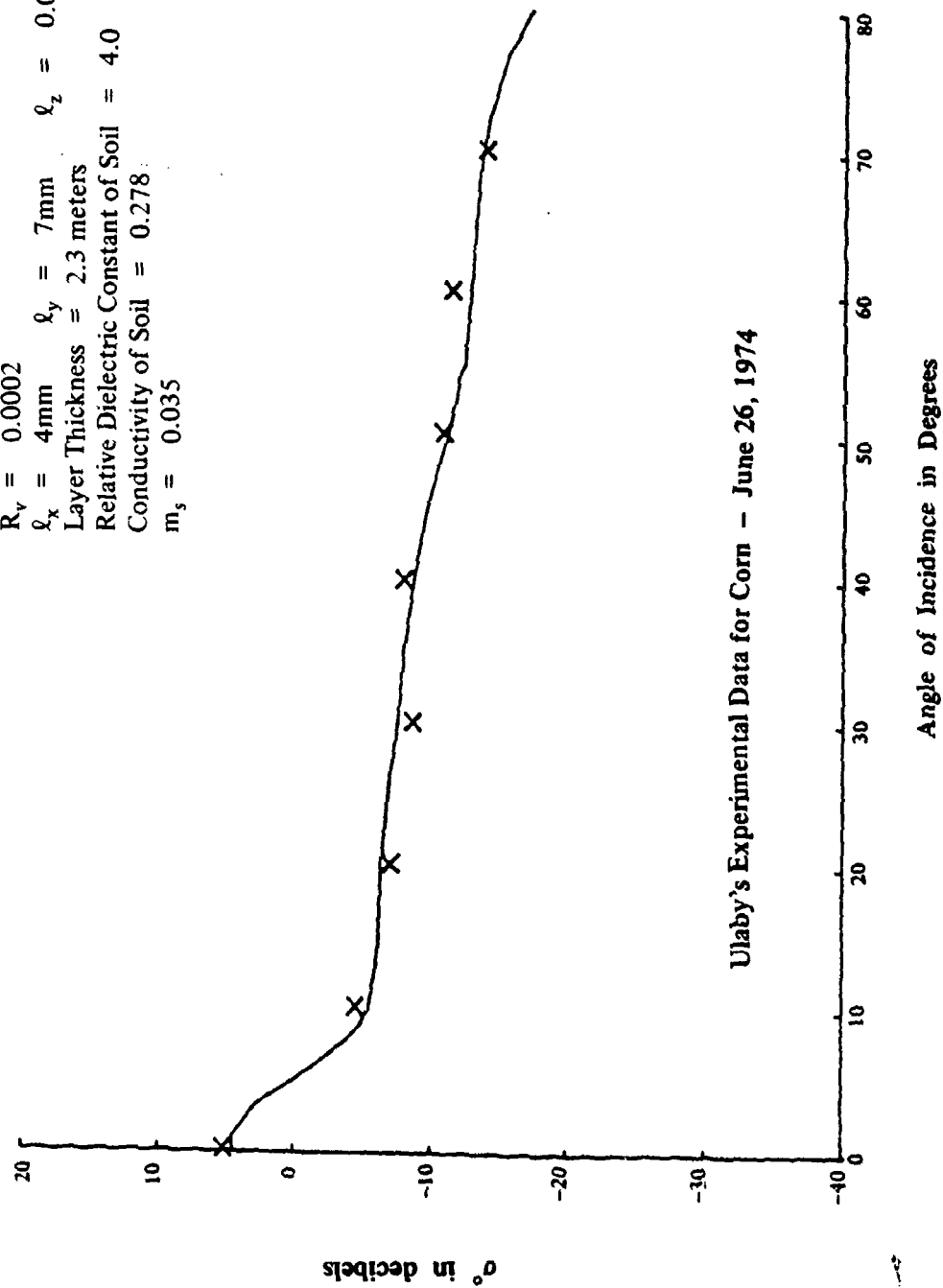


Figure 13. Comparison of Theory with Experimental Data.

In figures 13 through 15 a match is shown of the theory with experimental data for vertical polarization. For all three curves, ℓ_x must be made unequal to ℓ_y to obtain a good match. In figure 16, the variations are shown in the experimental measurements of σ° , which can occur throughout the spring and summer for alfalfa. Large variations in the backscatter coefficient occur prior to and after harvesting.

Figure 17 presents a study of sensor-look direction with respect to vegetation planted in rows. The two parameters $\sigma_\perp^\circ$ and $\sigma_\parallel^\circ$ represent the backscatter coefficient when the look direction is perpendicular and parallel to the rows, respectively. We see that when θ_i equals zero degrees, $\sigma_\perp^\circ$ and $\sigma_\parallel^\circ$ are equal. However, for θ_i greater than zero degrees, $\sigma_\perp^\circ$ is greater than $\sigma_\parallel^\circ$. The theoretical results agree only partially with the experimental results given in figure 18, which comes from Batlivala and Ulaby.⁹

Figure 19 provides a study of backscatter coefficient versus layer thickness for two angles of incidence. For the θ_i equal to zero curve, the solution for σ° with a rough layer differs from the half-space solution by approximately 16dB (decibels) for a layer thickness of 0.5 meters. As the layer thickness is increased, the solution for σ° at θ_i equal to zero, approaches the half-space solution. The θ_i equal to 20° curve also approaches the half-space solution when the layer thickness is increased. In this case, σ° differs from the half-space solution by only about 3.5dB when the layer thickness is 0.5 meters.

Figure 20 presents a study of the skin depth of the mean wave versus incidence angle for three different frequencies. For a horizontally polarized wave, the skin depth is taken to be the reciprocal of p_1 , which was derived earlier as part of the solution to the Dyson's equation. It should be remembered that the mean wave decays for two reasons, absorption and scattering. For a frequency of 8.6 GHz, the skin depth goes from approximately 5 meters at θ_i equal to 0° down to 1 meter at θ_i equal to 80°. When the frequency is increased, the overall level of the curve is lowered considerably, but it does not drop off as fast with increasing incidence angle.

In figure 21, the solutions are compared for the half space, the plane layer, and the layer with a rough surface. For angles of incidence between 0° and 30°, the rough interface at the vegetation soil boundary can have a dramatic effect on σ° . In this case, it is clearly not sufficient to use a plane layer model.

⁹ P.P. Batlivala and F.T. Ulaby, *The Effect of Look Direction on the Radar Return From a Row Crop*, National Aeronautics and Space Administration, Lyndon B. Johnson Space Center, Houston, Texas, RSL Technical Report 264-3, May 1975.

Polarization: Vertical
 $F = 0.815$ $f = 8.6 \text{ GHz}$
 $R_v = 0.0002$
 $\ell_x = 4 \text{ mm}$ $\ell_y = 7 \text{ mm}$ $\ell_z = 0.095 \text{ mm}$
 Layer Thickness = 2.6 meters
 Relative Dielectric Constant of Soil = 3.5
 Conductivity of Soil = 0.389
 $m_s = 0.035$

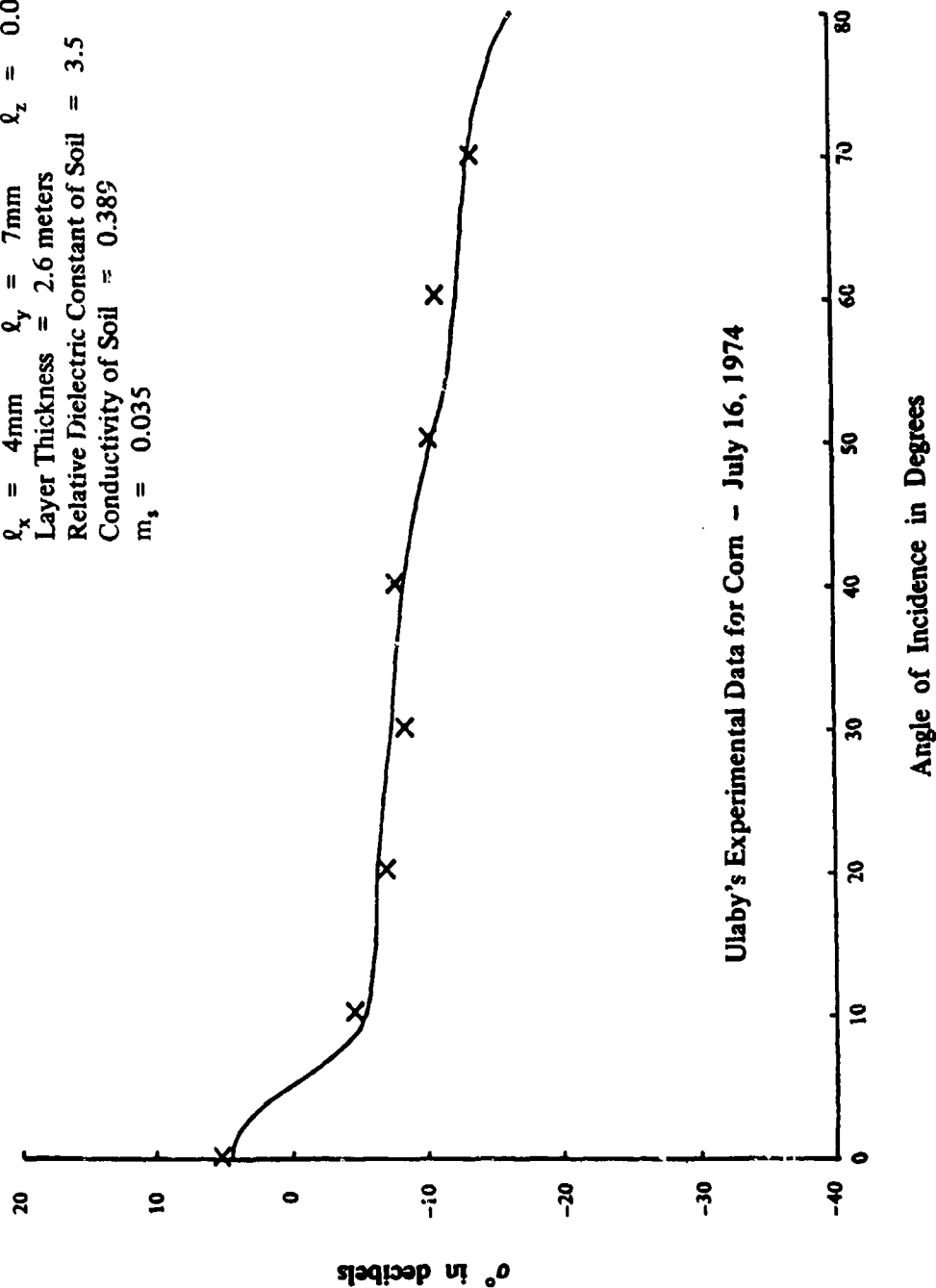


Figure 14. Comparison of Theory with Experimental Data.

Polarization: Vertical
 $F = 0.734$ $f = 8.6 \text{ GHz}$
 $R_v = 0.0002$
 $\ell_x = 4 \text{ mm}$ $\ell_y = 7 \text{ mm}$ $\ell_z = 0.095 \text{ mm}$
 Layer Thickness = 2.6 meters
 Relative Dielectric Constant of Soil = 3.0
 Conductivity of Soil = 0.278
 $m_s = 0.035$

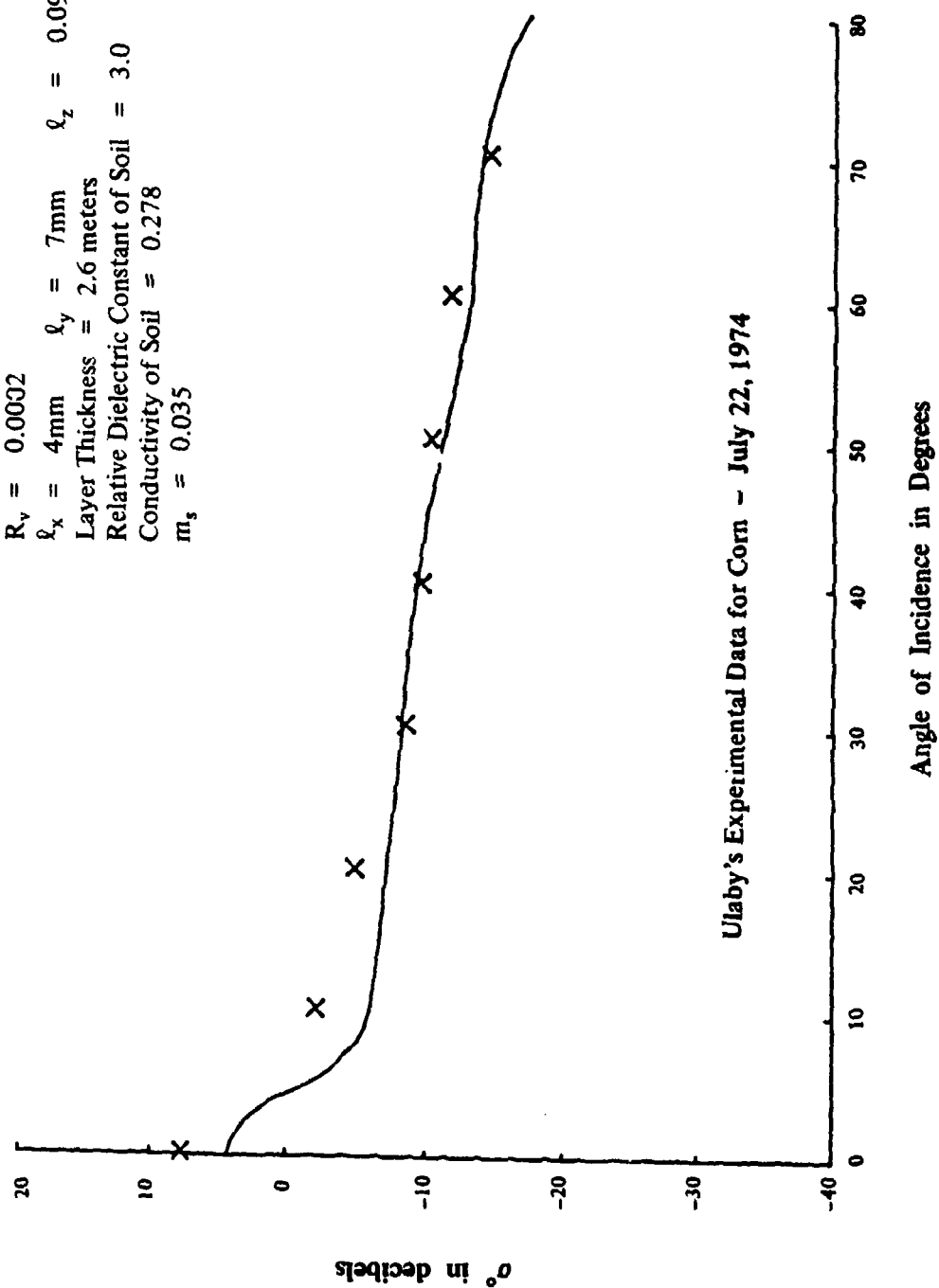


Figure 15. Comparison of Theory with Experimental Data.

Polarization: Horizontal
 $\theta_i = 0^\circ$

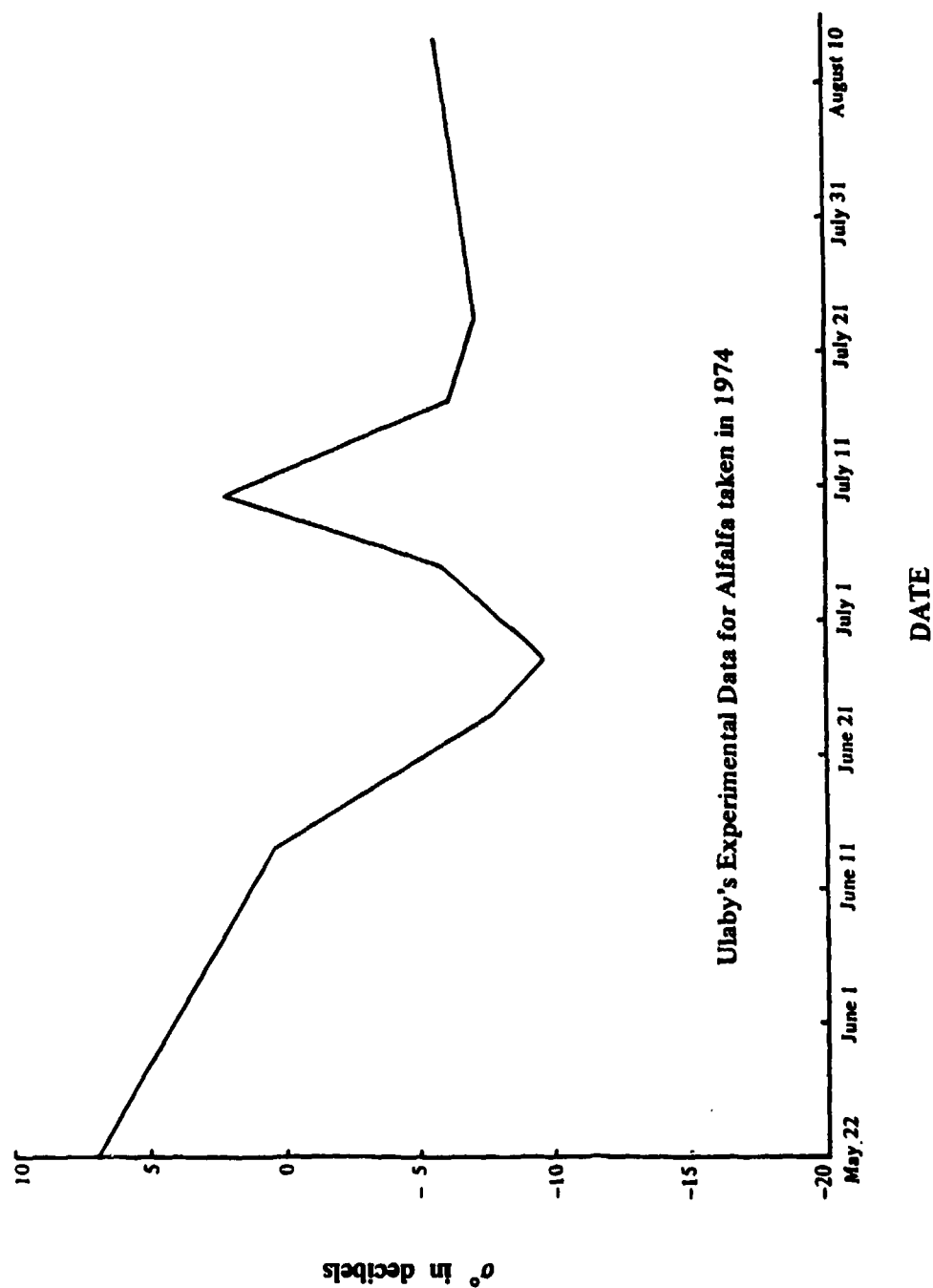


Figure 16. Study of the Experimental Variations of the σ° for Alfalfa.

Polarization: Horizontal
 $F = 0.8$ $f = 7.25 \text{ GHz}$
 $R_v = 0.001$
 $\ell_z = 0.2 \text{ mm}$
 Layer Thickness = 2.5 meters
 Dielectric Constant of Soil = 7.5
 Conductivity of Soil = 1.39
 $m_s = 0.09$

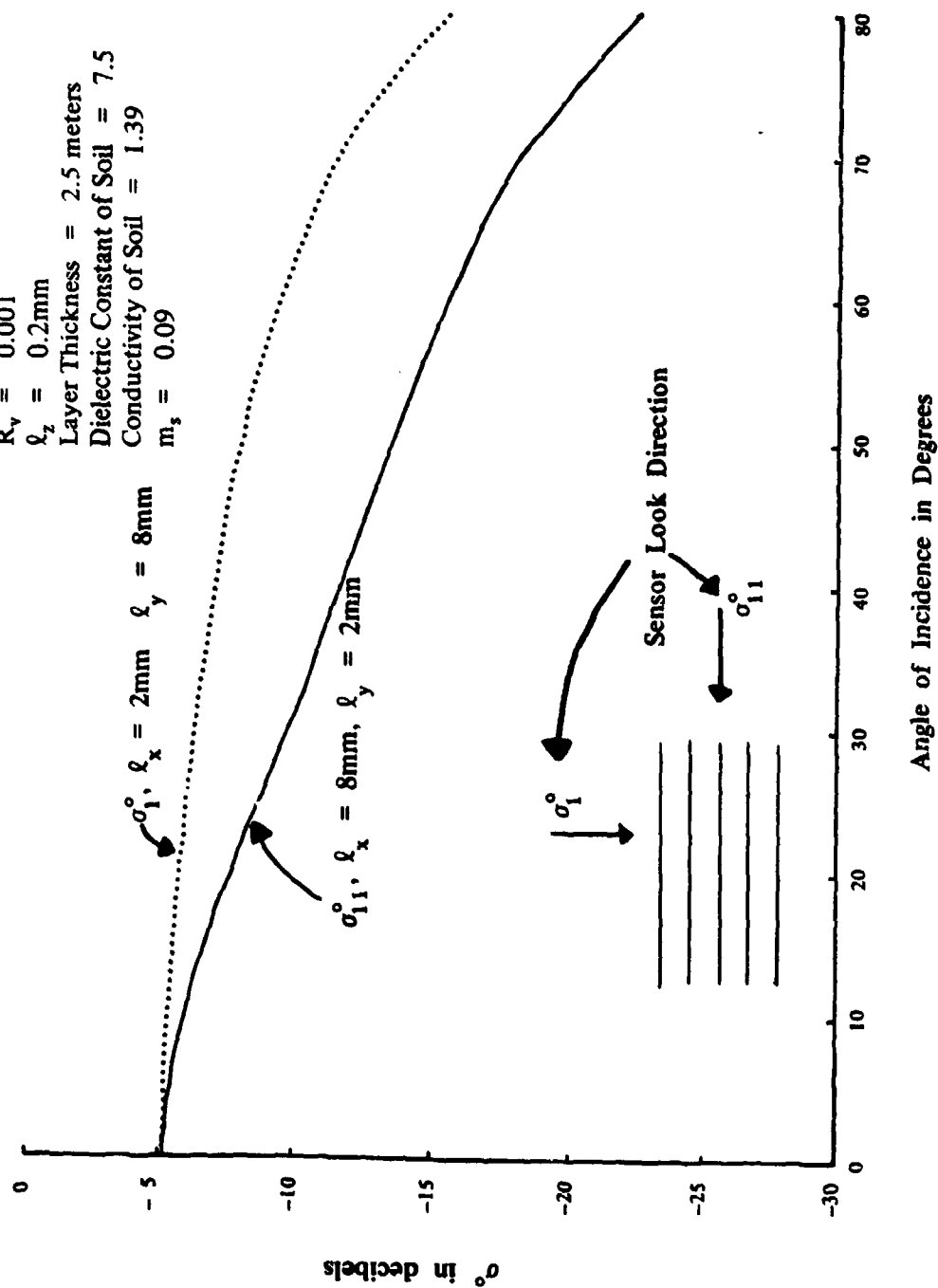
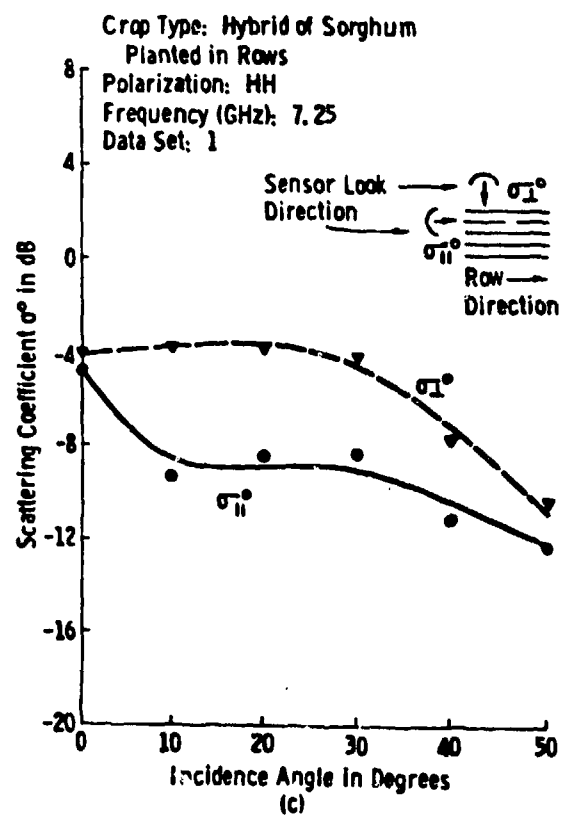
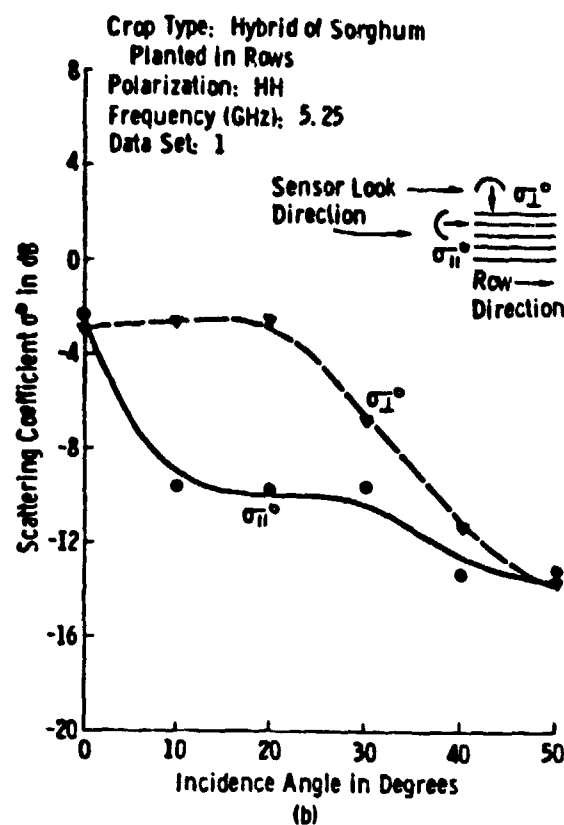
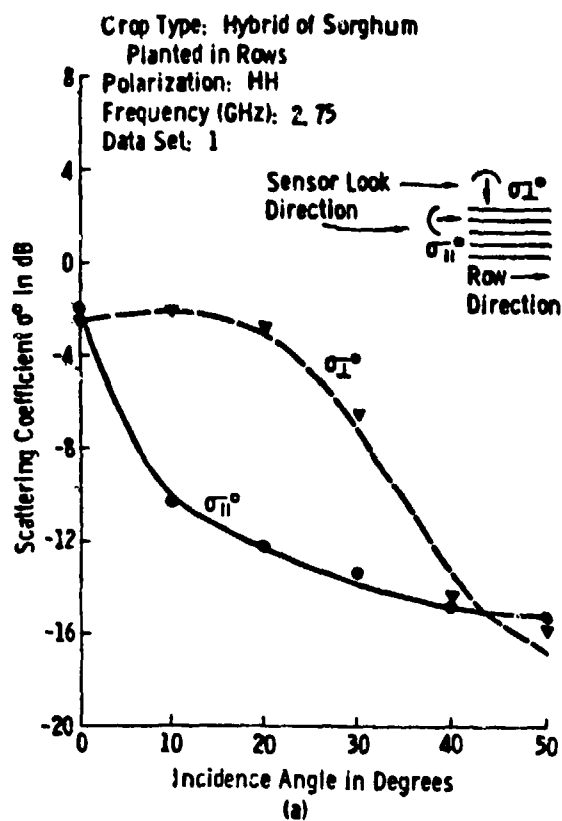


Figure 17. Study of the Sensor Look Direction.



SOURCE : P.P. Battivala and F.T. Ulaby, *The Effect of Look Direction on the Radar Return From a Row Crop*, National Aeronautics and Space Administration, Lyndon B. Johnson Space Center, Houston, Texas RSL Technical Report 264-3, May 1975.

Figure 18. Scattering Coefficient σ^0 as a Function of Incidence Angle at (a) 2.75GHz, (b) 5.25GHz, and (c) 7.25GHz. Data set $\neq 1$, July 16, 1974.

Polarization: Horizontal
 $F = 0.839$ $f = 8.6 \text{ GHz}$
 $R_v = 0.0002$
 $\ell_x = \ell_y = 5 \text{ mm}$ $\ell_z = 0.095 \text{ mm}$
 Dielectric Constant of Soil = 4.0
 Conductivity of Soil = $0.27\mathcal{E}$
 $m_s = 0.035$

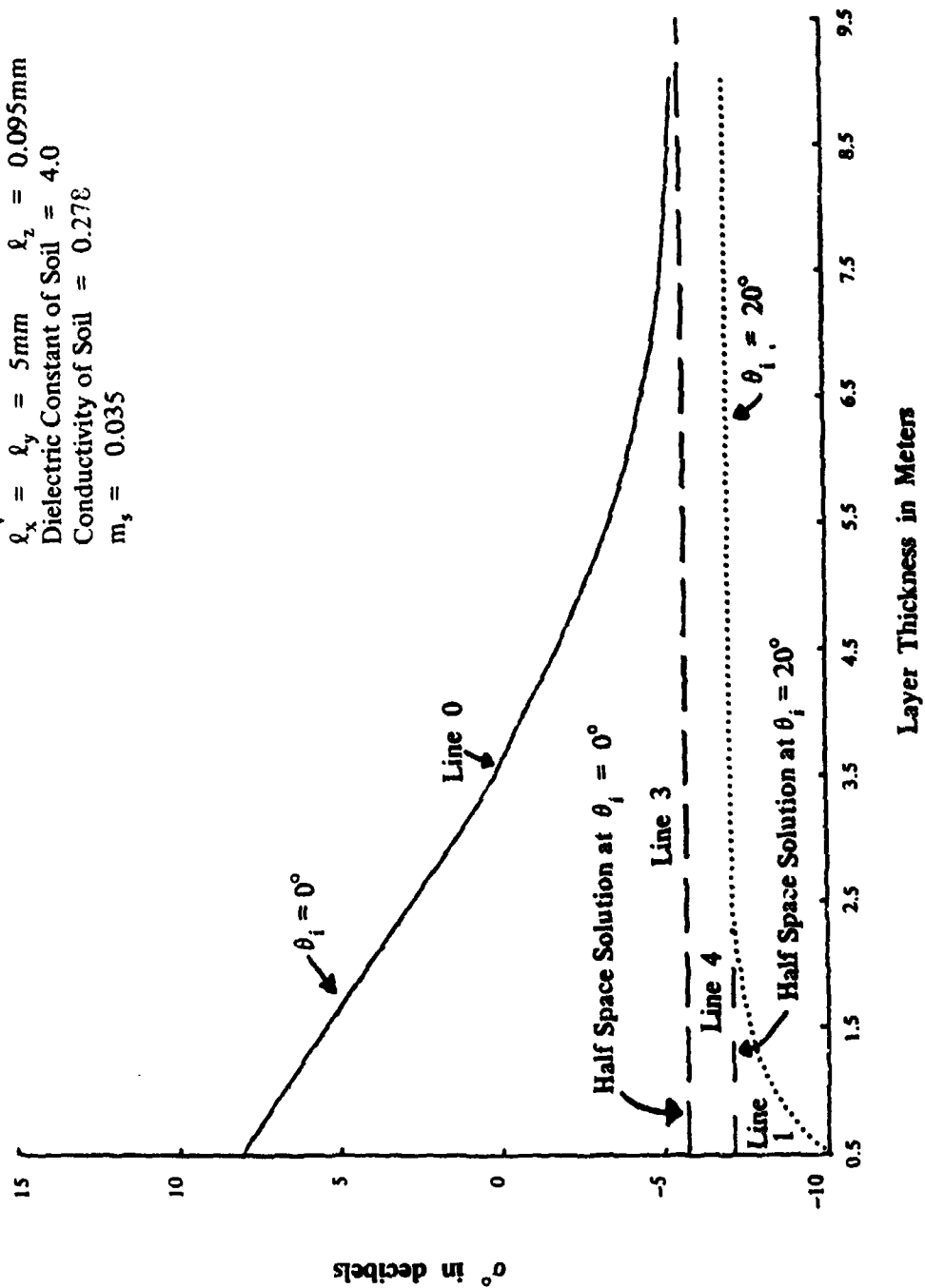


Figure 19. Study of the Variation of σ^o with Layer Thickness.

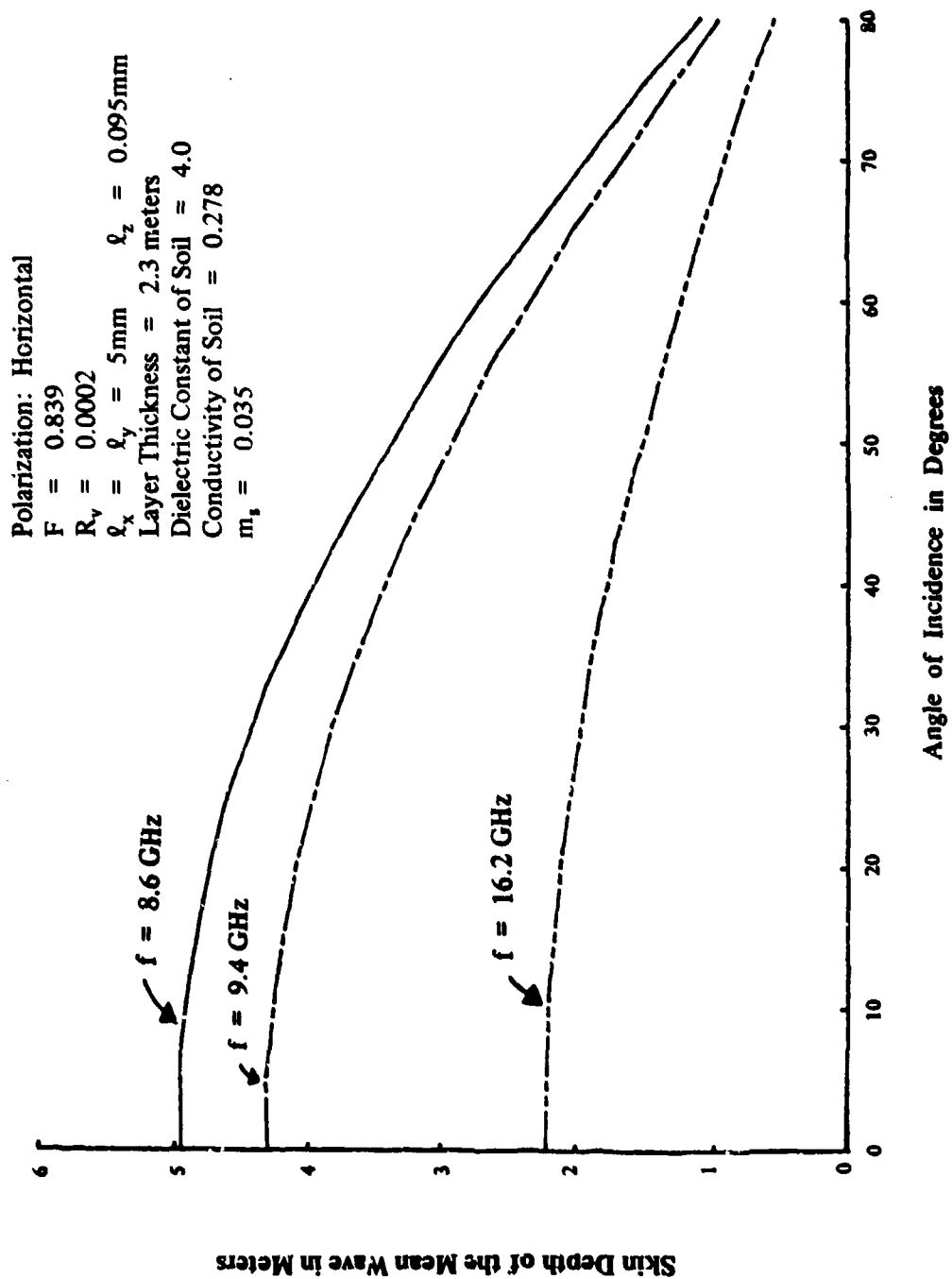


Figure 20. Study of the Skin Depth of the Mean Wave Versus Incidence Angle.

Polarization: Horizontal
 $F = 0.6$ $f = 8.6 \text{ GHz}$
 $R_v = 0.0005$
 $\ell_x = \ell_y = 1.11 \text{ mm}$ $\ell_z = 0.194 \text{ mm}$
 Layer Thickness = 0.3 meters
 Relative Dielectric Constant of Soil = 9.0
 Conductivity of Soil = 0.05
 $m_s = 0.1$

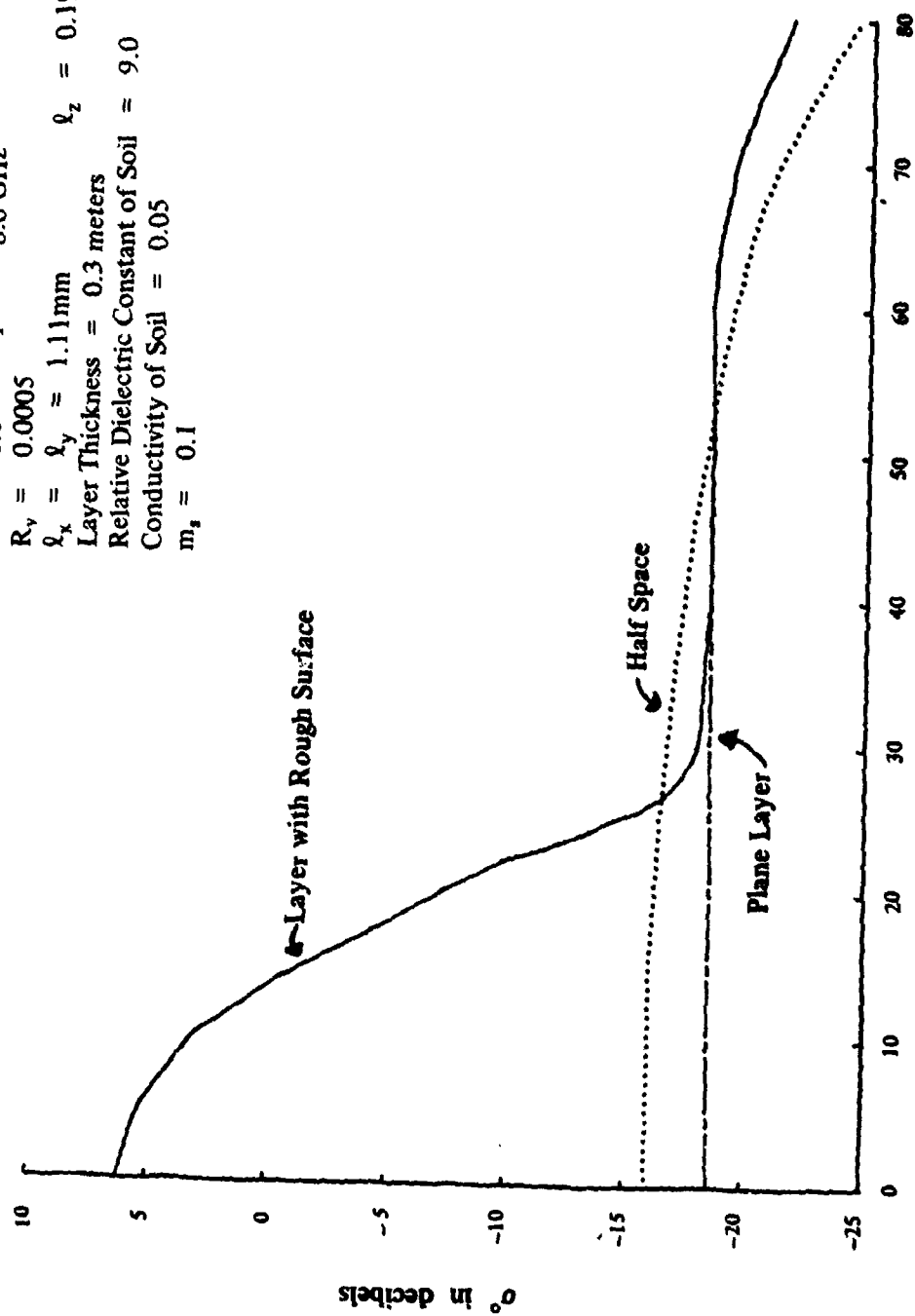


Figure 21. Comparison of Half Space, Plane Layer and Layer with Rough Surface Solutions.

In figures 22 through 30, a sensitivity-of-parameters study is presented. In this study, the input parameters are varied individually to determine the overall effect on the scattering coefficient. In figure 22, a study is provided of σ° variations with F (the vegetation moisture content). When θ_i is greater than 20° , the larger values of F produce higher levels of σ° . The shape of the σ° versus θ_i curve does not change much, but the overall level is significantly different. At approximately $\theta_i = 10^\circ$, a crossover of the curves exists such that the curve with the highest moisture content now yields the lowest value of σ° . The reason for the crossover is that two entirely different mechanisms are responsible for scattering. For θ_i greater than 20° , the volume-scattering mechanism dominates so that higher moisture in the vegetation results in larger backscatter values. However, for angles of incidence θ_i less than 10° when the mechanism for scattering is dominated by the rough surface under the vegetation, the higher moisture values result in lower σ° values. This is because the higher moisture values provide more attenuation of the mean wave, which means that less is available for scattering from the rough surface.

Figure 23 presents a study of σ° variations with the parameter R_V . The smaller value of R_V yields a larger σ° value for small angles of incidence. This is again because rough surface scattering dominates for small angles, and smaller values of R_V mean that more energy gets down to the surface. A crossover occurs at approximately $\theta_i = 15^\circ$ where the volume scattering result begins to dominate. Another crossover occurs between $\theta_i = 40^\circ$ and $\theta_i = 50^\circ$ such that for angles larger than 50° , σ° falls off faster for the larger value of R_V . The reason for this second crossover is possibly because that for the larger value of R_V , the lower interface between the vegetation and soil no longer provides a contribution for backscattering.

Figure 24 presents a study of σ° variations with L (the mean thickness of the vegetation layer). For small angles of incidence, the smaller value of L yields larger values of σ° . The rough surface below the vegetation is dominating the return and the smaller value of L provides a lower attenuation, thus making more energy available for scattering from the surface. A crossover point occurs around $\theta_i = 12^\circ$, which indicates that volume scattering is now beginning to dominate and so the thicker layer will yield a larger value of σ° . Another crossover point occurs at approximately $\theta_i = 67^\circ$. This crossover point possibly indicates that for $L = 2$ meters, the lower interface is having no influence, but for $L = 0.5$ meters, the lower interface still provides a contribution.

Polarization: Horizontal
 $R_v = 0.0002$ $f = 8 \text{ GHz}$
 $\ell_x = \ell_y = 2 \text{ mm}$ $\ell_z = 0.02 \text{ mm}$
 Layer Thickness = 1 meter
 Relative Dielectric Constant of Soil = 8.2
 Conductivity of Soil = 1.67
 $m_s = 0.04$

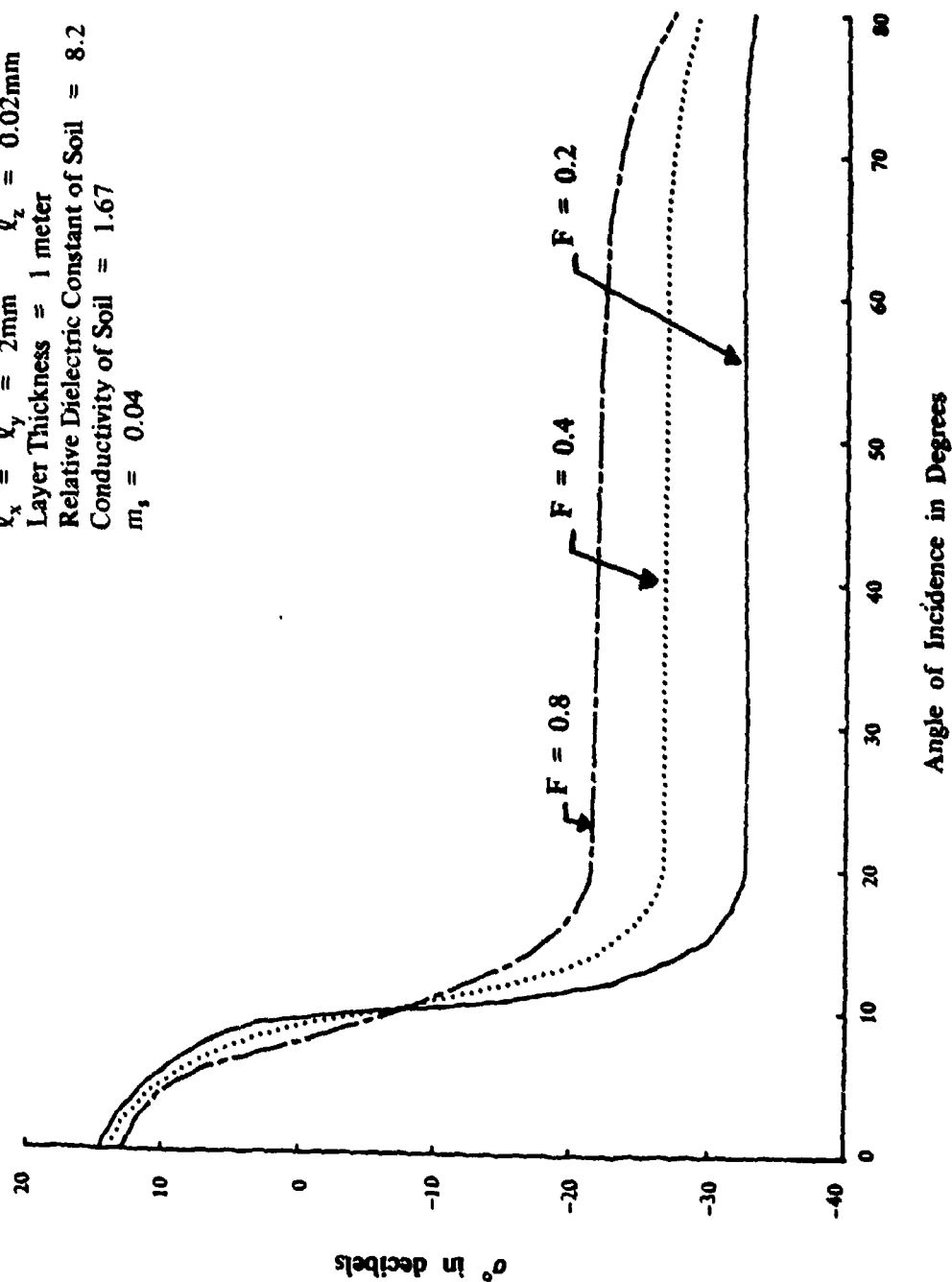


Figure 22. Study of σ^o Variations with F .

Polarization: Horizontal
 $F = 0.8$ $f = 8 \text{ GHz}$
 $\ell_x = \ell_y = 2 \text{ mm}$ $\ell_z = 0.02 \text{ mm}$
 Layer Thickness = 1 meter
 Relative Dielectric Constant of Soil = 8.2
 Conductivity of Soil = 1.67
 $m_s = 0.04$

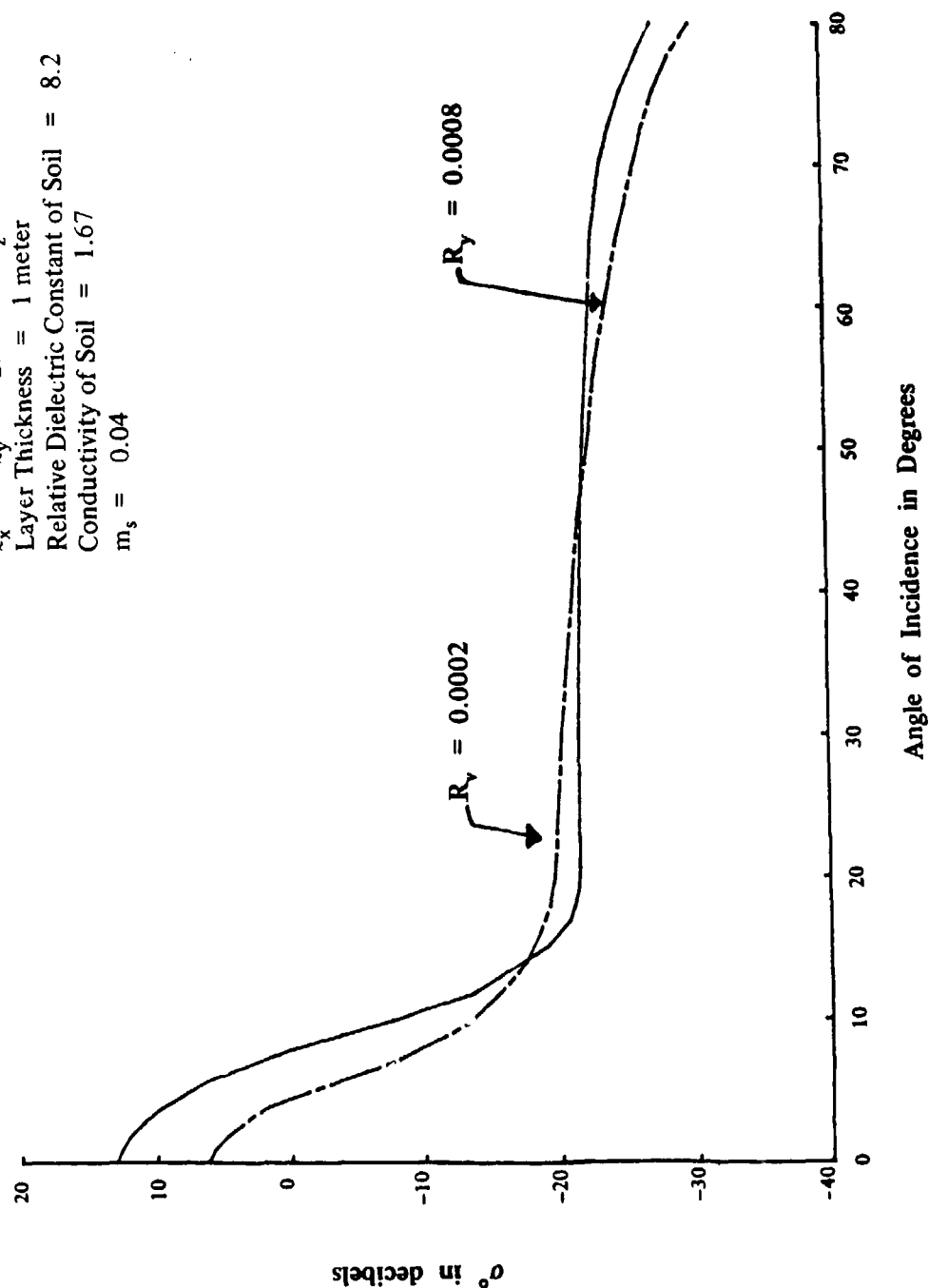


Figure 23. Study of σ^o Variations with R_v .

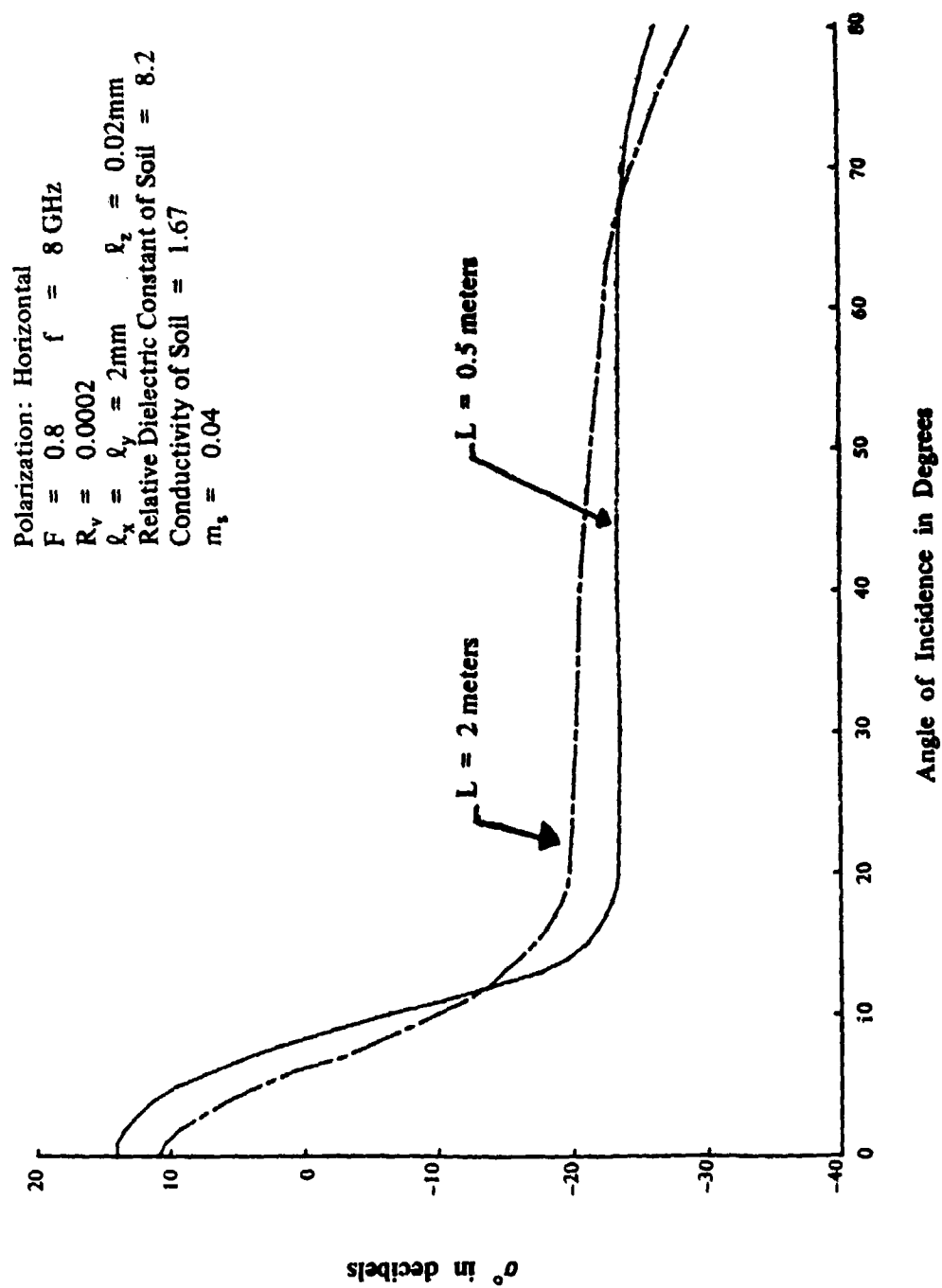


Figure 24. Study of σ° Variations with L .

Figure 25 shows a study of σ° variations with soil moisture. The soil moisture influences both the dielectric constant and the conductivity. The effect is shown of doubling the soil moisture underneath a 1-meter layer of vegetation. Thus, doubling the soil moisture content increases the level of σ° slightly over all angles of incidence.

In figure 26, a study is presented of σ° variations with frequency for two angles of incidence. We see only a slight frequency dependence at $\theta_i = 10^\circ$, because the rough surface scattering is independent of frequency except for the attenuation portion. At $\theta_i = 30^\circ$ where volume scattering is playing a more important role, we see a very significant frequency dependence.

In figure 27, σ° variations with the parameter m_s are shown. This parameter represents the ratio of the standard deviation of the rough surface undulations to the correlation distance. The most specular surface ($m_s = 0.04$) yields the highest value of σ° at $\theta_i = 0^\circ$. However, for this surface, σ° falls off very fast with increasing incidence angle so that at $\theta_i = 20^\circ$, the rough surface effect has disappeared. When m_s is allowed to increase, the value of σ° at $\theta_i = 0^\circ$ decreases. Also, as m_s increases, the rough surface influences σ° over a larger range of incidence angles.

In figure 28, σ° variations with ℓ_x are shown. Notice first that a change in ℓ_x yields virtually no influence upon σ° for angles of incidence less than 10° . For angles of incidence greater than 10° , the curve associated with the larger value of ℓ_x is higher until a crossover point is reached around $\theta_i = 48^\circ$. For angles of incidence greater than 48° , the curve associated with the larger value of ℓ_x falls off much faster than the curve associated with the smaller value of ℓ_x . Increasing the value of ℓ_x will then move the curve upward for angles of incidence less than about 50° , but will lower the curve for angles of incidence greater than about 50° .

In figure 29, σ° variations with ℓ_y are shown. Once again, notice that a change in ℓ_y has virtually no influence on σ° for angles less than 10° . For angles of incidence greater than or equal to 20° , an increase in ℓ_y results in an increase in σ° . Therefore, increasing ℓ_y simply increases the level of the curve for angles equal to and greater than 20° .

In figure 30, σ° variations with ℓ_z are shown. For angles of incidence less than 10° , changes in ℓ_z have no influence on σ° owing to the dominance of the rough surface. For angles of incidence greater than 20° , increasing the value of ℓ_z simply increases the overall level of the curve without changing the shape. It can be seen that the σ° curve is sensitive to slight changes in ℓ_z . A change in ℓ_z of only a fraction of a millimeter produces a significant change in σ° . This sensitivity may make any attempt to determine ℓ_z in a rigorous experimental manner very difficult.

Polarization: Horizontal
 $F = 0.8$ $f = 8 \text{ GHz}$
 $R_v = 0.0002$
 $\ell_x = \ell_y = 2 \text{ mm}$ $\ell_z = 0.02 \text{ m}$
 Layer Thickness = 1 meter
 $m_s = 0.04$

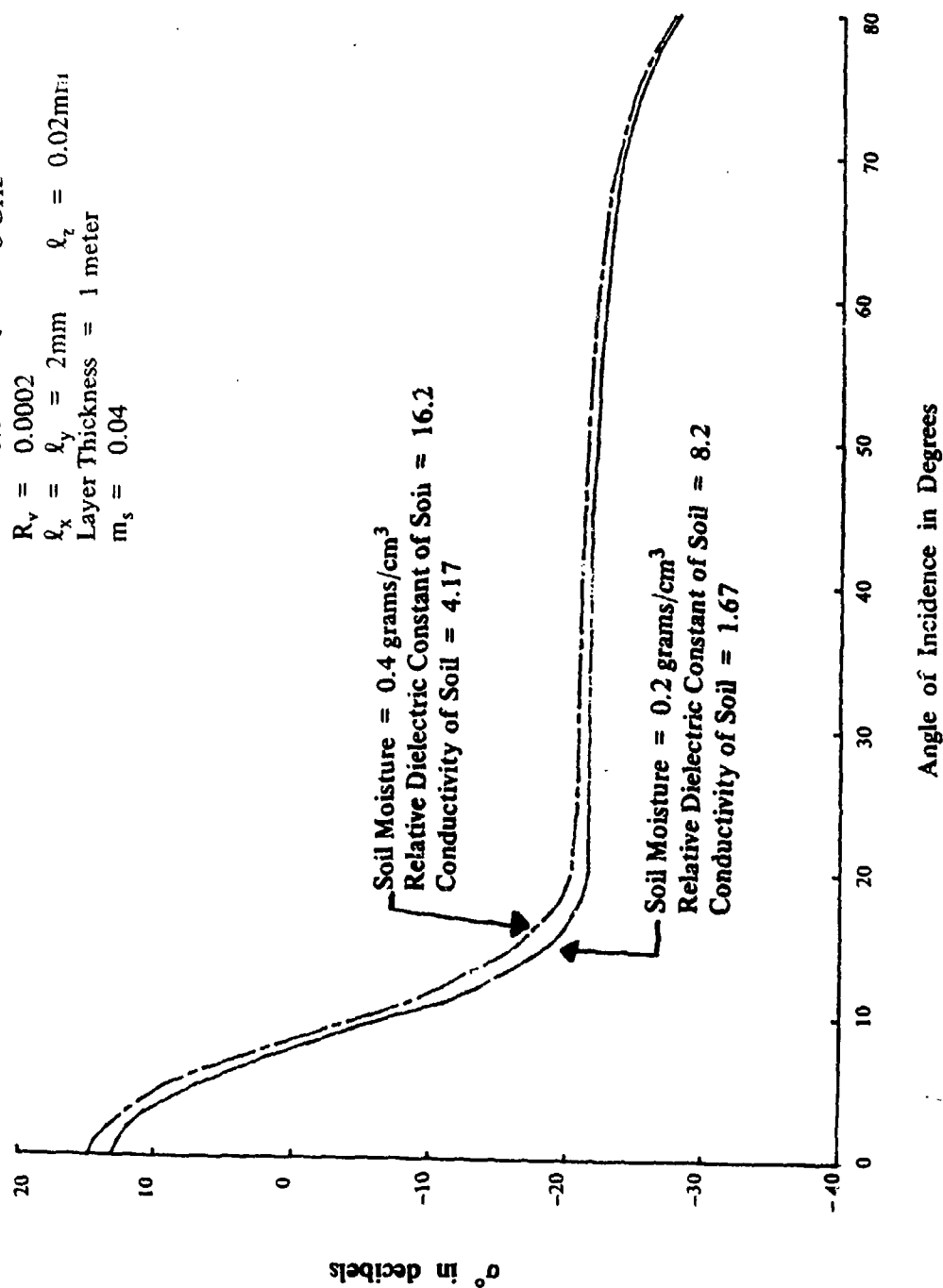


Figure 25. Study of σ^o Variations with Soil Moisture.

Polarization: Horizontal
 $F = 0.08$ $m_s = 0.04$
 $R_v = 0.0002$
 $\rho_x = \rho_y = 2\text{mm}$ $\rho_z = 0.02\text{mm}$
 Layer Thickness = 1 meter
 Relative Dielectric Constant of Soil = 8.2
 Conductivity of Soil = 1.67

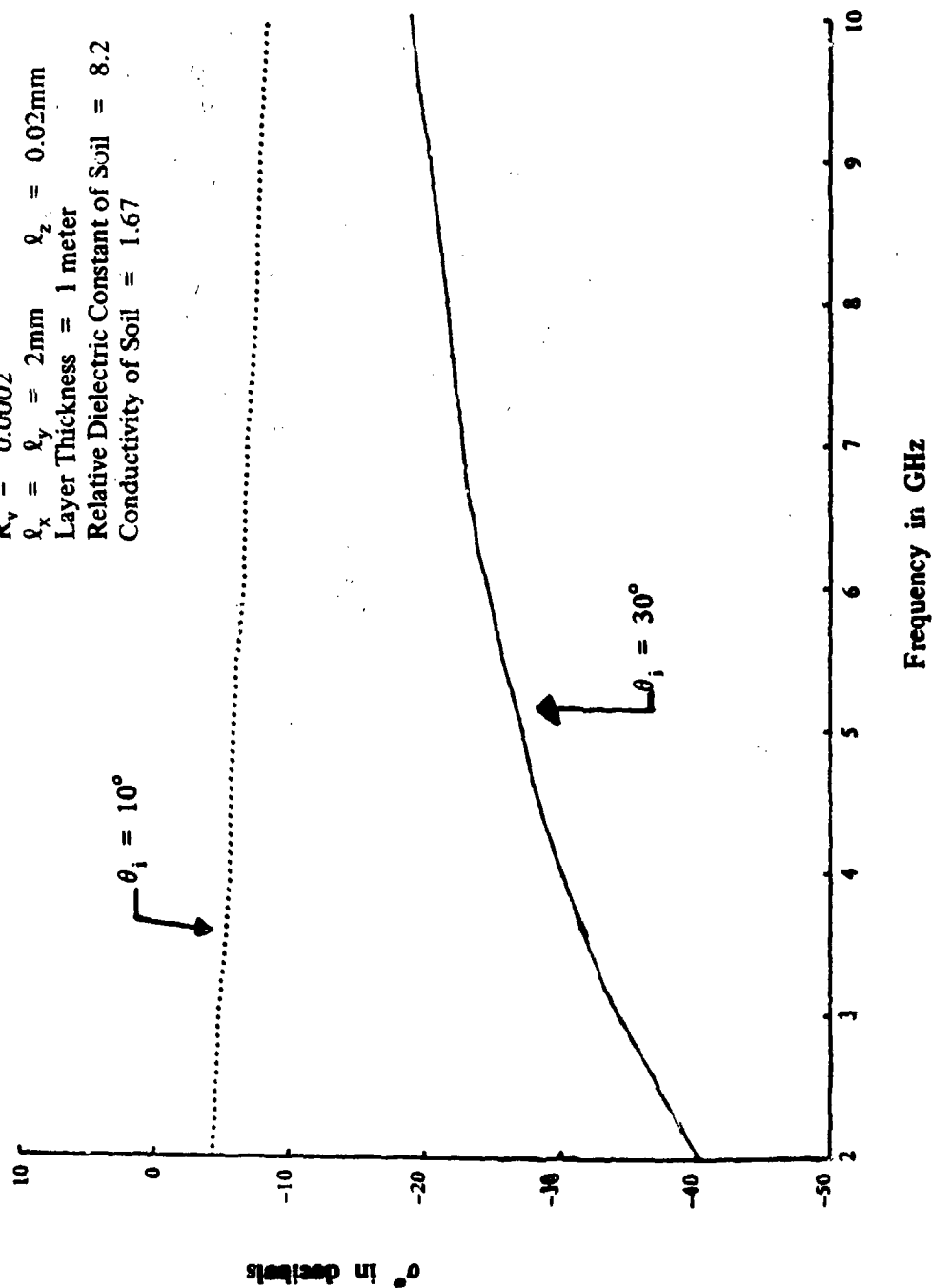


Figure 26. Study of σ° Variations with Frequency.

Polarization: Horizontal
 $F = 0.8$ $f = 8 \text{ GHz}$
 $R_y = 0.0002$
 $\rho_x = \rho_y = 2 \text{ mm}$ $\rho_z = 0.02 \text{ mm}$
 Layer Thickness = 1 meter
 Relative Dielectric Constant of Soil = 8.2
 Conductivity of Soil = 1.67

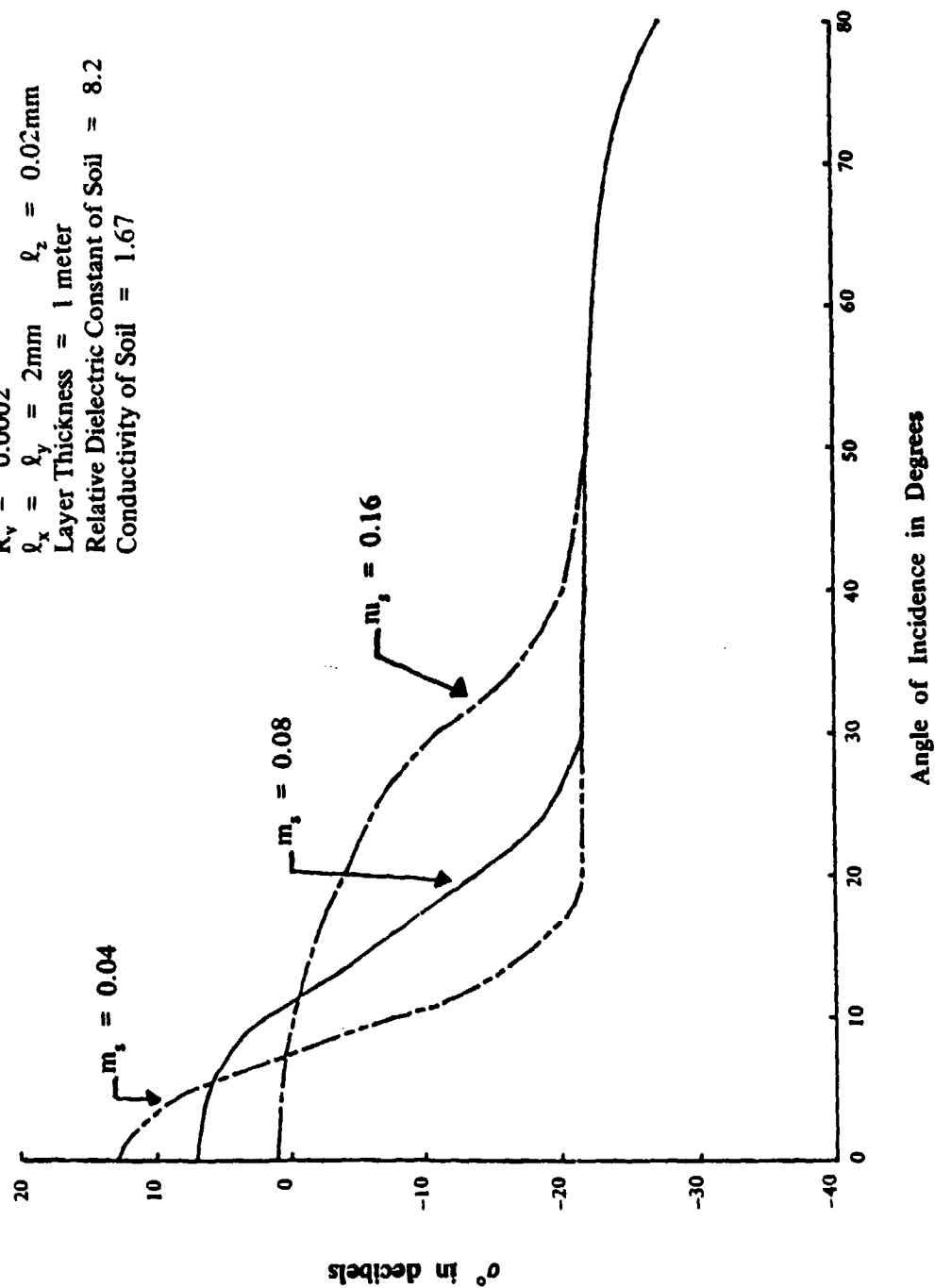


Figure 27. Study of σ^o Variations with m_s .

Polarization: Horizontal
 $F = 0.8$ $f = 8 \text{ GHz}$
 $R_v = 0.0002$
 $\ell_y = 2 \text{ mm}$ $\ell_z = 0.02 \text{ mm}$
 Layer Thickness = 1 meter
 Relative Dielectric Constant of Soil = 8.2
 Conductivity of Soil = 1.67
 $m_s = 0.04$

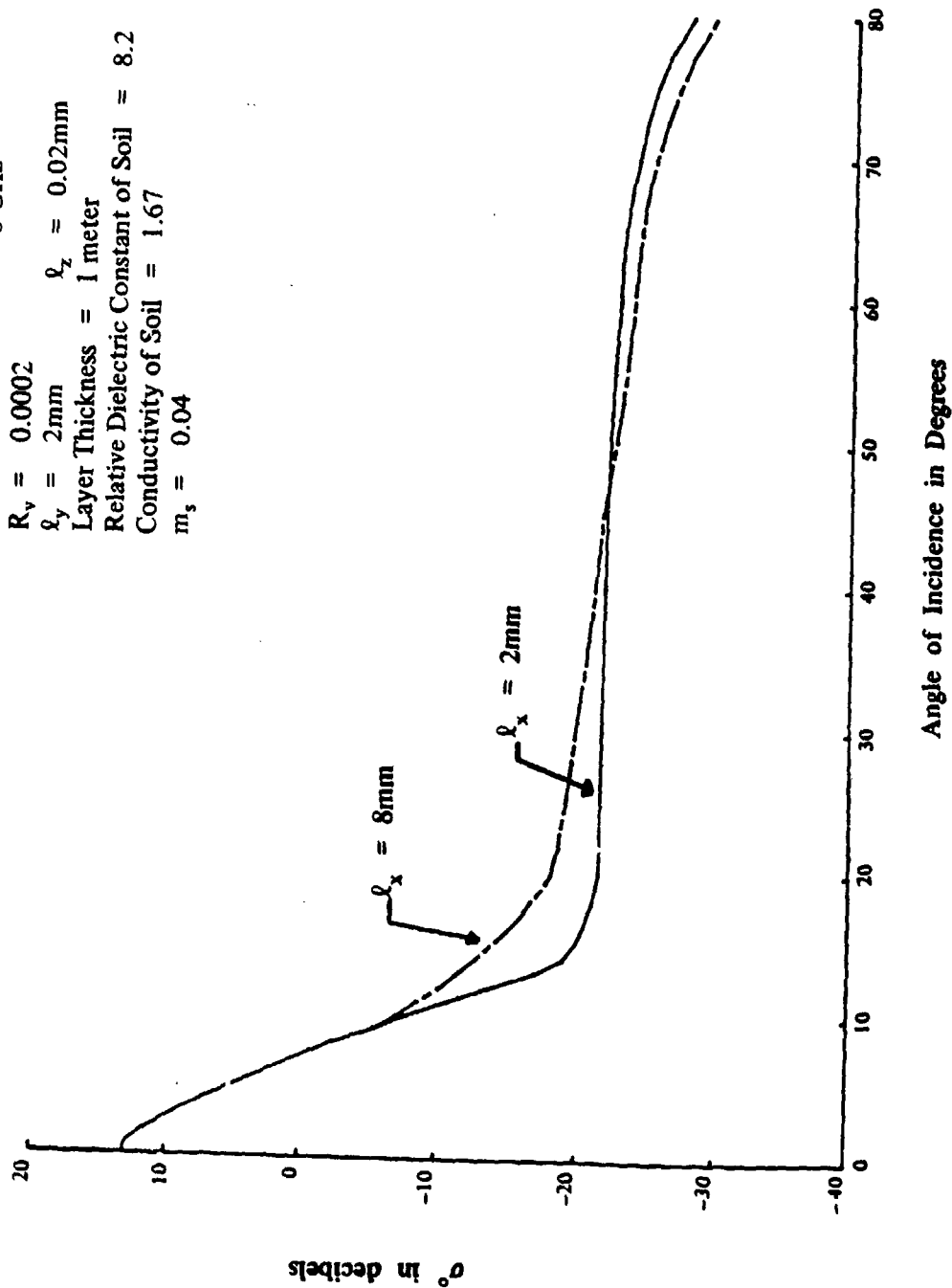
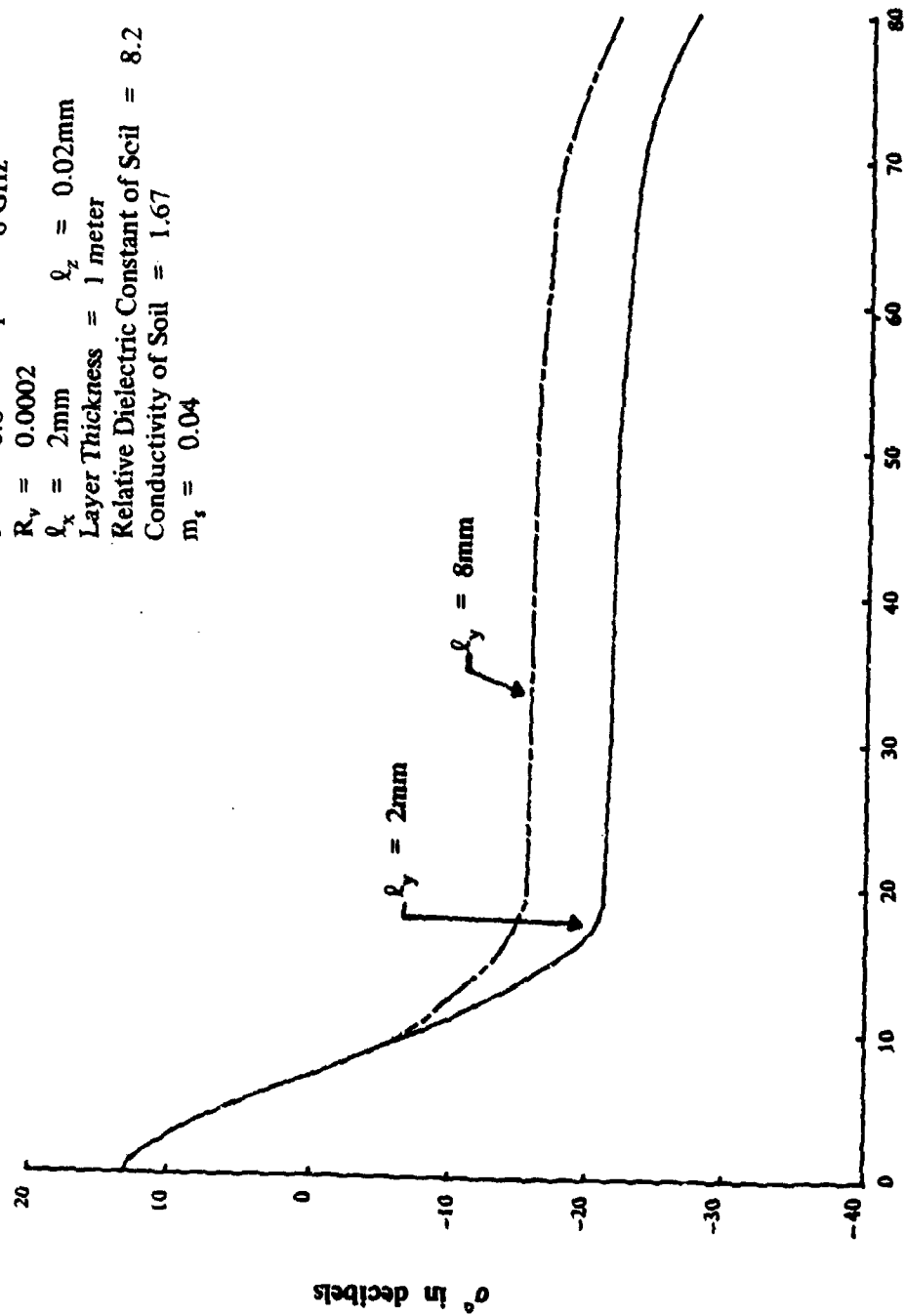


Figure 28. Study of σ^o Variations with ℓ_x .

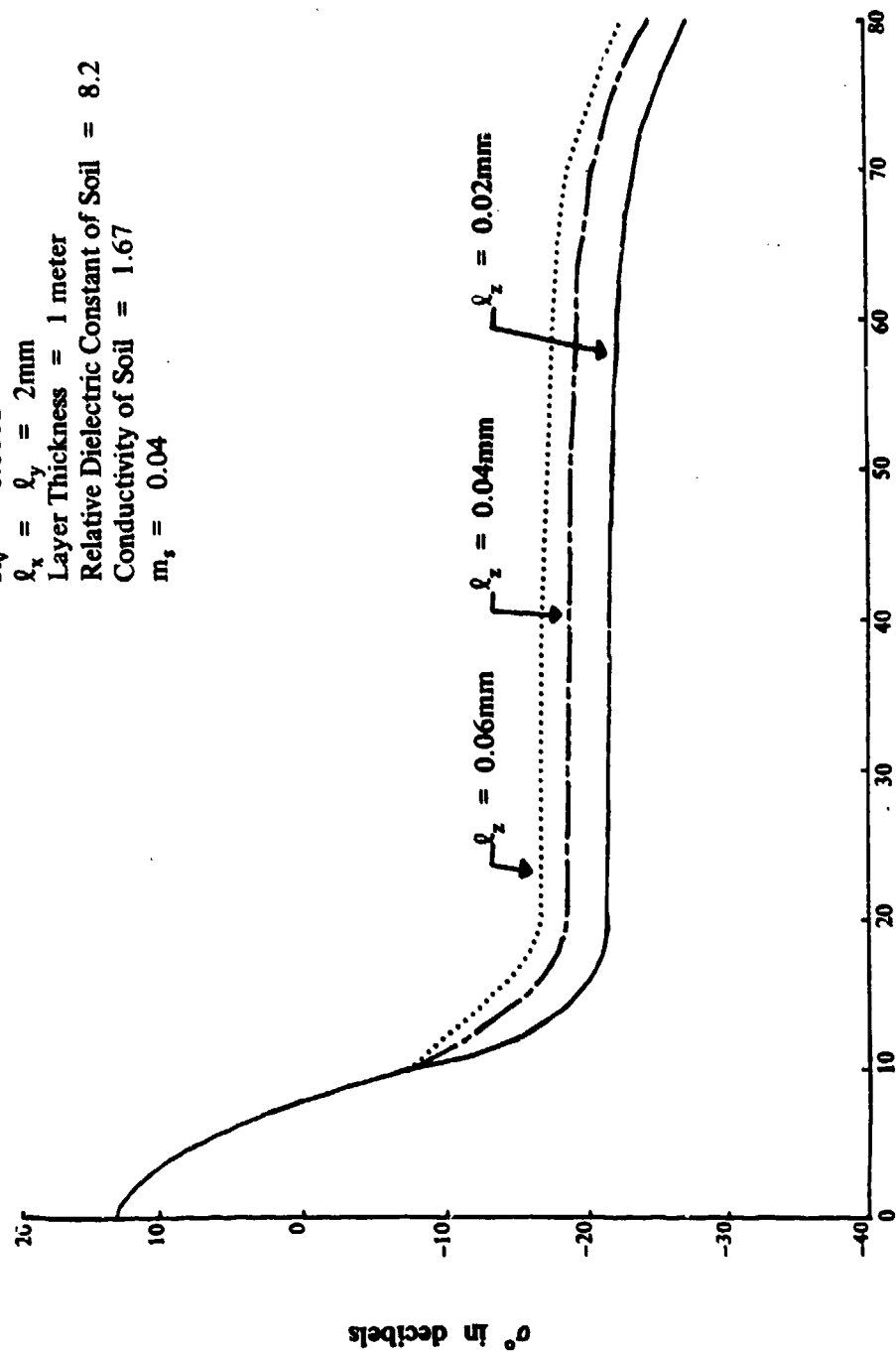
Polarization: Horizontal
 $F = 0.8$ $f = 8 \text{ GHz}$
 $R_v = 0.0002$
 $\ell_x = 2 \text{ mm}$ $\ell_z = 0.02 \text{ mm}$
 Layer Thickness = 1 meter
 Relative Dielectric Constant of Soil = 8.2
 Conductivity of Soil = 1.67
 $m_s = 0.04$



Angle of Incidence in Degrees

Figure 29. Study of σ^o Variations with ℓ_y .

Polarization: Horizontal
 $F = 0.8$ $f = 8 \text{ GHz}$
 $R_v = 0.0002$
 $\ell_x = \ell_y = 2 \text{ mm}$
 Layer Thickness = 1 meter
 Relative Dielectric Constant of Soil = 8.2
 Conductivity of Soil = 1.67
 $m_s = 0.04$



Angle of Incidence in Degrees

Figure 30. Study of σ^o Variations with ℓ_z .

This section concludes with a brief discussion on the limitations and difficulties encountered in developing the theory presented in this report. It appears valid to simulate a region of vegetation with a continuous random medium, although it is not certain as to how well the first-order renormalization technique does in solving the problem. It is not clear how much multiple scattering is being considered, and it is not even clear as to how much multiple scattering must be considered. A free space dyadic Green's function was used in solving the Dyson's equation; however, what should have been used was a Green's function applicable to a layered problem. Such Green's functions are very complicated to develop and work with. Also, it is not clear how much the final result will change if a more complicated Green's function is used.

An equation for the radar backscatter coefficient σ^0 was developed by first obtaining plane wave solutions to the Dyson's equation. Expressions were found for the z component of the effective propagation constant for both horizontal and vertical polarizations. The mean wave was then used to calculate the scattered wave, which in turn was used to compute σ^0 . The final result for σ^0 indicated the necessity to develop a permittivity model that would relate some of the permittivity parameters in the scattering model to the physical parameters of the vegetation. This was accomplished with only limited success since a rather elementary permittivity model was used. The final result for σ^0 still contained five input parameters that were unknown. The five parameters are ϵ_x , ϵ_y , ϵ_z , R_V , and m_s .

Further work in this area should attempt to determine correlation functions and distances, as well as R_V and m_s to relate clearly theory and experiment. An anisotropic correlation function was used; however, this did not result in a depolarization term. The existence of depolarization is clearly evident from the experimental data. The reason for this depolarization is, as yet, unknown. A depolarization term could be obtained by computing the scattered field to a second-order approximation. It could also be obtained by initially allowing for an anisotropic random medium. At this time, it is unclear which approach is correct.

CONCLUSIONS

1. For certain types of vegetation, such as corn, the irregular vegetation soil boundary dominates the backscattering results for angles of incidence between 0° and 20° . Any remote sensing of surface phenomena beneath vegetation should be done in this angular range.
2. The effect of the rough surface boundary between the vegetation and soil increases with decreases in frequency, vegetation moisture content, vegetation volume, and layer thickness.
3. Increasing the soil moisture content increases the level of the σ° curve slightly.
4. Using different correlation distances in x and y does not completely explain the effect of look direction on scattering from a row crop.
5. The σ° curve versus incidence angle curve is sensitive to very slight changes in the correlation distance in z .
6. The predictability of the σ° curve is dependent on meteorological phenomena, such as rain.
7. The correlation distances and the vegetation volume ratio do not stay constant throughout the entire growth cycle of the corn crop. However, once the crop matured, these parameters remained fairly constant.
8. Because different correlation distances were required to match the experimental data for horizontal and vertical polarizations, a more correct model for the vegetation may be an anisotropic random media model.
9. The predictability of the σ° versus incidence angle curve depends upon a very detailed knowledge of the dielectric fluctuations of the vegetation and the surface roughness properties of the soil below. However, such knowledge for particular vegetation features does not exist at the present time. This detailed understanding should be obtained if theoretical models are to have ultimate usefulness in predicting scattering from vegetation features.

APPENDIX A. Definition of Terms Involved in Computing

$$\langle A_y A_y^* \rangle, \langle A_x A_x^* \rangle, \langle A_z A_z^* \rangle, \langle A_x A_z^* \rangle, \langle A_x A_y^* \rangle$$

Repeating equation (67) produces

$$\langle A_y A_y^* \rangle = \frac{4\ell_x \ell_y \ell_z A_1 (\omega^2 \mu_o^2 \eta_2^2 + k_o^4 \eta_1^2) k_a^2 k_a^{*2} M_o M_o^*}{(1 + 4k_o^2 \ell_x^2 \sin^2 \theta_1)}$$

$$\cdot \sum_{n=1}^{16} \frac{A_n}{(c_n + d_n)} \left\{ \frac{1 - \ell_z d_n + \ell_z (c_n + d_n) e^{-L(c_n + 1/\ell_z)} - (1 + \ell_z c_n) e^{-L(c_n + d_n)}}{(1 + \ell_z c_n) (1 - \ell_z d_n)} \right.$$

$$\left. + \frac{1 - \ell_z c_n + \ell_z (c_n + d_n) e^{-L(d_n + 1/\ell_z)} - e^{-L(c_n + d_n)} (1 + d_n \ell_z)}{(1 + \ell_z d_n) (1 - \ell_z c_n)} \right\}$$

The values for A_n , c_n , and d_n are defined below:

$A_1 = \tilde{a}_1 \tilde{a}_1^* T_2 T_2^*$	$c_1 = D_1$	$d_1 = D_1^*$
$A_2 = \tilde{a}_1 T_2 \tilde{a}_1^* V_2^*$	$c_2 = D_1$	$d_2 = D_2^*$
$A_3 = \tilde{a}_1 T_2 T_2^*$	$c_3 = D_1$	$d_3 = -D_2^*$
$A_4 = \tilde{a}_1 T_2 V_2^*$	$c_4 = D_1$	$d_4 = -D_1^*$
$A_5 = \tilde{a}_1 V_2 \tilde{a}_1^* T_2^*$	$c_5 = D_2$	$d_5 = D_1^*$
$A_6 = \tilde{a}_1 \tilde{a}_1^* V_2 V_2^*$	$c_6 = D_2$	$d_6 = D_2^*$
$A_7 = \tilde{a}_1 V_2 T_2^*$	$c_7 = D_2$	$d_7 = -D_2^*$

$A_8 = \hat{a}_1 V_2 V_2^*$	$c_8 = D_2$	$d_8 = -D_1^*$
$A_9 = \hat{a}_1^* T_2 T_2^*$	$c_9 = -D_2$	$d_9 = D_1^*$
$A_{10} = \hat{a}_1^* T_2 V_2^*$	$c_{10} = -D_2$	$d_{10} = D_2^*$
$A_{11} = T_2 T_2^*$	$c_{11} = -D_2$	$d_{11} = -D_2^*$
$A_{12} = T_2 V_2^*$	$c_{12} = -D_2$	$d_{12} = -D_1^*$
$A_{13} = V_2 \hat{a}_1^* T_2^*$	$c_{13} = -D_1$	$d_{13} = D_1^*$
$A_{14} = \hat{a}_1^* V_2 V_2^*$	$c_{14} = -D_1$	$d_{14} = D_2^*$
$A_{15} = V_2 T_2^*$	$c_{15} = -D_1$	$d_{15} = -D_2^*$
$A_{16} = V_2 V_2^*$	$c_{16} = -D_1$	$d_{16} = -D_1^*$

A basic expression for $\langle A_x A_x^* \rangle$ can be written as follows:

$$\langle A_x A_x^* \rangle = \frac{4\ell_x \ell_y \ell_z A_1 (\omega^2 \mu_o^2 \eta_2^2 + k_o^4 \eta_1^2)}{(1 + 4k_o^2 \ell_x^2 \sin^2 \theta_1)}$$

$$\sum_{n=1}^{16} \frac{\Lambda_n}{(\beta_n + \gamma_n)} \left\{ \frac{1 - \ell_z \gamma_n + \ell_z (\beta_n + \gamma_n) e^{-L(\beta_n + 1/\ell_z)} - (1 + \ell_z \beta_n) e^{-L(\beta_n + \gamma_n)}}{(1 + \ell_z \beta_n) (1 - \ell_z \gamma_n)} + \frac{1 - \ell_z \beta_n + \ell_z (\beta_n + \gamma_n) e^{-L(\beta_n + \gamma_n)} (1 + \gamma_n \ell_z)}{(1 + \ell_z \gamma_n) (1 - \ell_z \beta_n)} \right\}$$

Before giving the values for Λ_n , β_n , and γ_n , the following parameters are defined:

$$D'_1 = p_2 + jq_2 - jk'_z$$

$$D'_2 = -(p_2 + jq_2 + jk'_z)$$

$$h_1 = b_6 [T_1(k_a^2 - k_o^2 \sin^2 \theta_1) - T_3 k_o k'_z \sin \theta_1]$$

$$h_2 = b_6 [V_1(k_a^2 - k_o^2 \sin^2 \theta_1) - V_3 k_o k'_z \sin \theta_1]$$

$$h_3 = b_5 [T_1(k_a^2 - k_o^2 \sin^2 \theta_1) + T_3 k_o k'_z \sin \theta_1]$$

$$h_4 = b_5 [V_1(k_a^2 - k_o^2 \sin^2 \theta_1) + V_3 k_o k'_z \sin \theta_1]$$

The values of Λ_n , β_n , and γ_n will now be written in terms of the above parameters:

$\Lambda_1 = h_1 h_1^*$	$\beta_1 = D'_1$	$\gamma_1 = D_1'^*$
$\Lambda_2 = h_1 h_2^*$	$\beta_2 = D'_1$	$\gamma_2 = D_2'^*$
$\Lambda_3 = h_1 h_3^*$	$\beta_3 = D'_1$	$\gamma_3 = D_2'^*$
$\Lambda_4 = h_1 h_4^*$	$\beta_4 = D'_1$	$\gamma_4 = D_1'^*$
$\Lambda_5 = h_2 h_1^*$	$\beta_5 = D'_2$	$\gamma_5 = D_1'^*$
$\Lambda_6 = h_2 h_2^*$	$\beta_6 = D'_2$	$\gamma_6 = D_2'^*$
$\Lambda_7 = h_2 h_3^*$	$\beta_7 = D'_2$	$\gamma_7 = -D_2'^*$

$\Lambda_8 = h_2 h_4^*$	$\beta_8 = D_2'$	$\gamma_8 = -D_1'^*$
$\Lambda_9 = h_3 h_1^*$	$\beta_9 = -D_2'$	$\gamma_9 = D_1'^*$
$\Lambda_{10} = h_3 h_2^*$	$\beta_{10} = -D_2'$	$\gamma_{10} = D_2'^*$
$\Lambda_{11} = h_3 h_3^*$	$\beta_{11} = -D_2'$	$\gamma_{11} = -D_2'^*$
$\Lambda_{12} = h_3 h_4^*$	$\beta_{12} = -D_2'$	$\gamma_{12} = -D_1'^*$
$\Lambda_{13} = h_4 h_1^*$	$\beta_{13} = -D_1'$	$\gamma_{13} = D_1'^*$
$\Lambda_{14} = h_4 h_2^*$	$\beta_{14} = -D_1'$	$\gamma_{14} = D_2'^*$
$\Lambda_{15} = h_4 h_3^*$	$\beta_{15} = -D_1'$	$\gamma_{15} = -D_2'^*$
$\Lambda_{16} = h_4 h_4^*$	$\beta_{16} = -D_1'$	$\gamma_{16} = -D_1'^*$

The basic equation for $\langle A_x A_y^* \rangle$ can be written as

$$\langle A_x A_y^* \rangle = \frac{-4\ell_x \ell_y \ell_z A_1 (\omega^2 \mu_0^2 \eta_2^2 + k_0^4 \eta_1^2) k_a^{*2} M_0^*}{(1 + 4k_0^2 \ell_x^2 \sin^2 \theta_1)}$$

$$\sum_{n=1}^{16} \frac{P_n}{(\nu_n + \rho_n)} \left\{ \frac{1 - \ell_z \rho_n + \ell_z (\nu_n + \rho_n) e^{-L(\nu_n + 1/\ell_z)} - (1 + \ell_z \nu_n) e^{-L(\nu_n + \rho_n)}}{(1 + \ell_z \nu_n) (1 - \ell_z \rho_n)} + \frac{1 - \ell_z \nu_n + \rho_n e^{-L(\rho_n + 1/\ell_z)} - e^{-L(\nu_n + \rho_n)} (1 + \rho_n \ell_z)}{(1 + \ell_z \rho_n) (1 - \ell_z \nu_n)} \right\}$$

The values of P_n , ρ_n , and ν_n can be defined in terms of previous parameters.

$$P_1 = h_1 \hat{a}_1^* T_2^*$$

$$P_2 = h_1 \hat{a}_1^* V_2^*$$

$$P_3 = h_1 T_2^*$$

$$P_4 = h_1 V_2^*$$

$$P_5 = h_2 \hat{a}_1^* T_2^*$$

$$P_6 = h_2 \hat{a}_1^* V_2^*$$

$$P_7 = h_2 T_2^*$$

$$P_8 = h_2 V_2^*$$

$$P_9 = h_3 \hat{a}_1^* T_2^*$$

$$P_{10} = h_3 \hat{a}_1^* V_2^*$$

$$P_{11} = h_3 T_2^*$$

$$P_{12} = h_3 V_2^*$$

$$P_{13} = h_4 \hat{a}_1^* T_2^*$$

$$P_{14} = h_4 \hat{a}_1^* V_2^*$$

$$\nu_1 = D_1'$$

$$\nu_2 = D_1'$$

$$\nu_3 = D_1'$$

$$\nu_4 = D_1'$$

$$\nu_5 = D_2'$$

$$\nu_6 = D_2'$$

$$\nu_7 = D_2'$$

$$\nu_8 = D_2'$$

$$\nu_9 = -D_2'$$

$$\nu_{10} = -D_2'$$

$$\nu_{11} = -D_2'$$

$$\nu_{12} = -D_2'$$

$$\nu_{13} = -D_1'$$

$$\nu_{14} = -D_1'$$

$$\rho_1 = D_1^*$$

$$\rho_2 = D_2^*$$

$$\rho_3 = -D_2^*$$

$$\rho_4 = -D_1^*$$

$$\rho_5 = D_1^*$$

$$\rho_6 = D_2^*$$

$$\rho_7 = -D_2^*$$

$$\rho_8 = -D_1^*$$

$$\rho_9 = D_1^*$$

$$\rho_{10} = D_2^*$$

$$\rho_{11} = -D_2^*$$

$$\rho_{12} = -D_1^*$$

$$\rho_{13} = D_1^*$$

$$\rho_{14} = D_2^*$$

$$P_{15} = h_4 T_2^*$$

$$\nu_{15} = -D_1'$$

$$\rho_{15} = -D_2^*$$

$$P_{16} = h_4 V_2^*$$

$$\nu_{16} = -D_1'$$

$$\rho_{16} = -D_1^*$$

Expressions for $\langle A_z A_z^* \rangle$, and $\langle A_x A_z^* \rangle$ can be obtained in terms of previous results.

$$\langle A_z A_z^* \rangle = \frac{k_o^2 \sin^2 \theta_1}{k_{1z} k_{1z}^*} \langle A_x A_x^* \rangle$$

$$\langle A_x A_z^* \rangle = \frac{k_o \sin \theta_1}{k_{1z}^*} \langle A_x A_x^* \rangle$$

APPENDIX B.

Computer Programs for Calculating the Backscattering Coefficients

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

UNCLASSIFIED
09/08/88 09-21-02

FTW 4-6-46

UNCLASSIFIED

UNCLASSIFIED
P-0000000000

7/7/7

DPT=1

UNCLASSIFIED

4127=7A21-0A12-0A11-0A22

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0A2N=2-0A2N-0A2N-0A2N

0001=0A14*J2
0002=0A11*CONJG(QNJ1)
0003=001
0004=001C
0005=0A14*J2
0006=0A11*CONJG(QNJ2)
0007=001
0008=002C
0009=0A14*CONJG(QNJ3)
0010=001
0011=002C
0012=0A14*CONJG(QNJ4)
0013=001
0014=002C
0015=0A14*CONJG(QNJ5)
0016=001
0017=002C
0018=0A14*CONJG(QNJ6)
0019=001
0020=002C
0021=0A14*CONJG(QNJ7)
0022=001
0023=002C
0024=0A14*CONJG(QNJ8)
0025=001
0026=002C
0027=0A14*CONJG(QNJ9)
0028=001
0029=002C
0030=0A14*CONJG(QNJ10)
0031=001
0032=002C
0033=0A14*CONJG(QNJ11)
0034=001
0035=002C
0036=0A14*CONJG(QNJ12)
0037=001
0038=002C
0039=0A14*CONJG(QNJ13)
0040=001
0041=002C
0042=0A14*CONJG(QNJ14)
0043=001
0044=002C
0045=0A14*CONJG(QNJ15)
0046=001
0047=002C
0048=0A14*CONJG(QNJ16)
0049=001
0050=002C
0051=0A14*CONJG(QNJ17)
0052=001
0053=002C
0054=0A14*CONJG(QNJ18)
0055=001
0056=002C
0057=0A14*CONJG(QNJ19)
0058=001
0059=002C
0060=0A14*CONJG(QNJ20)
0061=001
0062=002C
0063=0A14*CONJG(QNJ21)
0064=001
0065=002C
0066=0A14*CONJG(QNJ22)
0067=001
0068=002C
0069=0A14*CONJG(QNJ23)
0070=001
0071=002C
0072=0A14*CONJG(QNJ24)
0073=001
0074=002C
0075=0A14*CONJG(QNJ25)
0076=001
0077=002C
0078=0A14*CONJG(QNJ26)
0079=001
0080=002C
0081=0A14*CONJG(QNJ27)
0082=001
0083=002C
0084=0A14*CONJG(QNJ28)
0085=001
0086=002C
0087=0A14*CONJG(QNJ29)
0088=001
0089=002C
0090=0A14*CONJG(QNJ30)
0091=001
0092=002C
0093=0A14*CONJG(QNJ31)
0094=001
0095=002C
0096=0A14*CONJG(QNJ32)
0097=001
0098=002C
0099=0A14*CONJG(QNJ33)
0100=001
0101=002C
0102=0A14*CONJG(QNJ34)
0103=001
0104=002C
0105=0A14*CONJG(QNJ35)
0106=001
0107=002C
0108=0A14*CONJG(QNJ36)
0109=001
0110=002C
0111=0A14*CONJG(QNJ37)
0112=001
0113=002C
0114=0A14*CONJG(QNJ38)
0115=001
0116=002C
0117=0A14*CONJG(QNJ39)
0118=001
0119=002C
0120=0A14*CONJG(QNJ40)
0121=001
0122=002C
0123=0A14*CONJG(QNJ41)
0124=001
0125=002C
0126=0A14*CONJG(QNJ42)
0127=001
0128=002C
0129=0A14*CONJG(QNJ43)
0130=001
0131=002C
0132=0A14*CONJG(QNJ44)
0133=001
0134=002C
0135=0A14*CONJG(QNJ45)
0136=001
0137=002C
0138=0A14*CONJG(QNJ46)
0139=001
0140=002C
0141=0A14*CONJG(QNJ47)
0142=001
0143=002C
0144=0A14*CONJG(QNJ48)
0145=001
0146=002C
0147=0A14*CONJG(QNJ49)
0148=001
0149=002C
0150=0A14*CONJG(QNJ50)
0151=001
0152=002C
0153=0A14*CONJG(QNJ51)
0154=001
0155=002C
0156=0A14*CONJG(QNJ52)
0157=001
0158=002C
0159=0A14*CONJG(QNJ53)
0160=001
0161=002C
0162=0A14*CONJG(QNJ54)
0163=001
0164=002C
0165=0A14*CONJG(QNJ55)
0166=001
0167=002C
0168=0A14*CONJG(QNJ56)
0169=001
0170=002C
0171=0A14*CONJG(QNJ57)
0172=001
0173=002C
0174=0A14*CONJG(QNJ58)
0175=001
0176=002C
0177=0A14*CONJG(QNJ59)
0178=001
0179=002C
0180=0A14*CONJG(QNJ60)
0181=001
0182=002C
0183=0A14*CONJG(QNJ61)
0184=001
0185=002C
0186=0A14*CONJG(QNJ62)
0187=001
0188=002C
0189=0A14*CONJG(QNJ63)
0190=001
0191=002C
0192=0A14*CONJG(QNJ64)
0193=001
0194=002C
0195=0A14*CONJG(QNJ65)
0196=001
0197=002C
0198=0A14*CONJG(QNJ66)
0199=001
0200=002C
0201=0A14*CONJG(QNJ67)
0202=001
0203=002C
0204=0A14*CONJG(QNJ68)
0205=001
0206=002C
0207=0A14*CONJG(QNJ69)
0208=001
0209=002C
0210=0A14*CONJG(QNJ70)
0211=001
0212=002C
0213=0A14*CONJG(QNJ71)
0214=001
0215=002C
0216=0A14*CONJG(QNJ72)
0217=001
0218=002C
0219=0A14*CONJG(QNJ73)
0220=001
0221=002C
0222=0A14*CONJG(QNJ74)
0223=001
0224=002C
0225=0A14*CONJG(QNJ75)
0226=001
0227=002C
0228=0A14*CONJG(QNJ76)
0229=001
0230=002C
0231=0A14*CONJG(QNJ77)
0232=001
0233=002C
0234=0A14*CONJG(QNJ78)
0235=001
0236=002C
0237=0A14*CONJG(QNJ79)
0238=001
0239=002C
0240=0A14*CONJG(QNJ80)
0241=001
0242=002C
0243=0A14*CONJG(QNJ81)
0244=001
0245=002C
0246=0A14*CONJG(QNJ82)
0247=001
0248=002C
0249=0A14*CONJG(QNJ83)
0250=001
0251=002C
0252=0A14*CONJG(QNJ84)
0253=001
0254=002C
0255=0A14*CONJG(QNJ85)
0256=001
0257=002C
0258=0A14*CONJG(QNJ86)
0259=001
0260=002C
0261=0A14*CONJG(QNJ87)
0262=001
0263=002C
0264=0A14*CONJG(QNJ88)
0265=001
0266=002C
0267=0A14*CONJG(QNJ89)
0268=001
0269=002C
0270=0A14*CONJG(QNJ90)
0271=001
0272=002C
0273=0A14*CONJG(QNJ91)
0274=001
0275=002C
0276=0A14*CONJG(QNJ92)
0277=001
0278=002C
0279=0A14*CONJG(QNJ93)
0280=001
0281=002C
0282=0A14*CONJG(QNJ94)
0283=001
0284=002C
0285=0A14*CONJG(QNJ95)
0286=001
0287=002C
0288=0A14*CONJG(QNJ96)
0289=001
0290=002C
0291=0A14*CONJG(QNJ97)
0292=001
0293=002C
0294=0A14*CONJG(QNJ98)
0295=001
0296=002C
0297=0A14*CONJG(QNJ99)
0298=001
0299=002C
0300=0A14*CONJG(QNJ100)

403

405

479

475

480

485

490

495

503

505

510

UNCLASSIFIED
39/08/80 09.21.02
PAGE 13

UNCLASSIFIED
FTN 4-5446

UNCLASSIFIED
73/74 JP=1

UNCLASSIFIED
PROGRAM MODEL

```

4.5  SYGHR=RS*(HVF+IXX)*CS)+21/01
      DHH=10.*ALOG10(SIGHM)
      WETC(6,50)THETA
      601 FORMATTIN 16MTHEX=F5.2)
      50151/01
      WETC(6,715)PI,JO1
      715 FORMATTIN 13HPI=,E15.7,10X,3H015,E15.7)
      WETC(6,80)DS,TC
      602 FORMATTIN 10X,22M4LF SPACE SIGMA ZFRO=F15.9)
      WETC(6,80) DS,IGU
      603 FORMATTIN 10X,17HLAVER SIGMA 7ER0=F15.8)
      WETC(6,50)DS,DM
      604 FORMATTIN 10X,39HLAYER WITH ROUGH SURFACE SIGMA ZERO HH=F15.8//)
      IF (THETA=PC,1302,374,104
      503 THETA=THETA+1.
      GO TO 401
      704 IF (N3=4)DS,5,606,606
      END MC=4+1
      GO TO 5
      606 STOP
      END
  
```

95

SYMBOLIC REFERENCE MAP (P=1)

ENTRY POINTS
6136 MODEL

VIOLATES	SW	TYPE	RELOCATION
12213 ALD	FEAL	17 AER	FEAL
12212 ALA	FEAL	12242 ALE	FEAL
12151 AXI	FEAL	12221 AD	FEAL
12153 AVP	FEAL	12245 AVV	FEAL
12150 AZ	FEAL	12152 AZI	FEAL
12223 BL	FEAL	12277 AZ	FEAL
12176 BS	FEAL	12175 BR	FEAL
12075 CS	FEAL	12175 BR	FEAL
12247 DDM	FEAL	12175 BR	FEAL
12215 DS100	FEAL	12175 BR	FEAL
12230 FI	FEAL	12175 BR	FEAL
12067 FA	FEAL	12175 BR	FEAL
12202 FM	FEAL	12175 BR	FEAL
12061 CLFP	FEAL	12175 BR	FEAL
12171 FR	FEAL	12175 BR	FEAL
12162 EXV	FEAL	12175 BR	FEAL
12076 EY	FEAL	12175 BR	FEAL
12063 E-A11	FEAL	12175 BR	FEAL
1206F E162	FEAL	12175 BR	FEAL
12174 EY	FEAL	12175 BR	FEAL
12109 ENK	FEAL	12175 BR	FEAL

THIS PAGE IS BEST QUALITY PRACTICE
FROM COPY FURNISHED TO DDC

UNCLASSIFIED

UNCLASSIFIED

UNCLASSIFIED

UNCLASSIFIED
09/08/00 09.21.02 PAGE 14

FTN 4-54460

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDO

UNCLASSIFIED

7/17/76 OPT=1

UNCLASSIFIED
PROGRAM MODEL

VARIABLES	SM	TYPE	RELOCATION	UNCLASSIFIED	FTN 4-54460
12244	GT	REAL		12244	REAL
12242	GT1	REAL		12242	REAL
12240	G1	REAL		12240	REAL
12238	I	INTEGER		12238	INTEGER
12236	J	INTEGER		12236	INTEGER
12234	WQ	INTEGER		12234	INTEGER
12232	NKK	INTEGER		12232	INTEGER
12230	NTI	INTEGER		12230	INTEGER
12228	DME	REAL		12228	REAL
12226	OM3	REAL		12226	REAL
12224	PI	REAL		12224	REAL
12222	PM	REAL		12222	REAL
12220	P2	REAL		12220	REAL
12218	QA	COMPLEX		12218	COMPLEX
12216	QAL	COMPLEX		12216	COMPLEX
12214	QAX	COMPLEX		12214	COMPLEX
12212	QAV1	COMPLEX		12212	COMPLEX
12210	QAV2	COMPLEX		12210	COMPLEX
12208	QAV3	COMPLEX		12208	COMPLEX
12206	QAV4	COMPLEX		12206	COMPLEX
12204	QAV5	COMPLEX		12204	COMPLEX
12202	QAV6	COMPLEX		12202	COMPLEX
12200	QAV7	COMPLEX		12200	COMPLEX
12198	QAV8	COMPLEX		12198	COMPLEX
12196	QAV9	COMPLEX		12196	COMPLEX
12194	QAV10	COMPLEX		12194	COMPLEX
12192	QAV11	COMPLEX		12192	COMPLEX
12190	QAV12	COMPLEX		12190	COMPLEX
12188	QAV13	COMPLEX		12188	COMPLEX
12186	QAV14	COMPLEX		12186	COMPLEX
12184	QAV15	COMPLEX		12184	COMPLEX
12182	QAV16	COMPLEX		12182	COMPLEX
12180	QAV17	COMPLEX		12180	COMPLEX
12178	QAV18	COMPLEX		12178	COMPLEX
12176	QAV19	COMPLEX		12176	COMPLEX
12174	QAV20	COMPLEX		12174	COMPLEX
12172	QAV21	COMPLEX		12172	COMPLEX
12170	QAV22	COMPLEX		12170	COMPLEX
12168	QAV23	COMPLEX		12168	COMPLEX
12166	QAV24	COMPLEX		12166	COMPLEX
12164	QAV25	COMPLEX		12164	COMPLEX
12162	QAV26	COMPLEX		12162	COMPLEX
12160	QAV27	COMPLEX		12160	COMPLEX
12158	QAV28	COMPLEX		12158	COMPLEX
12156	QAV29	COMPLEX		12156	COMPLEX
12154	QAV30	COMPLEX		12154	COMPLEX
12152	QAV31	COMPLEX		12152	COMPLEX
12150	QAV32	COMPLEX		12150	COMPLEX
12148	QAV33	COMPLEX		12148	COMPLEX
12146	QAV34	COMPLEX		12146	COMPLEX
12144	QAV35	COMPLEX		12144	COMPLEX
12142	QAV36	COMPLEX		12142	COMPLEX
12140	QAV37	COMPLEX		12140	COMPLEX
12138	QAV38	COMPLEX		12138	COMPLEX
12136	QAV39	COMPLEX		12136	COMPLEX
12134	QAV40	COMPLEX		12134	COMPLEX
12132	QAV41	COMPLEX		12132	COMPLEX
12130	QAV42	COMPLEX		12130	COMPLEX
12128	QAV43	COMPLEX		12128	COMPLEX
12126	QAV44	COMPLEX		12126	COMPLEX
12124	QAV45	COMPLEX		12124	COMPLEX
12122	QAV46	COMPLEX		12122	COMPLEX
12120	QAV47	COMPLEX		12120	COMPLEX
12118	QAV48	COMPLEX		12118	COMPLEX
12116	QAV49	COMPLEX		12116	COMPLEX
12114	QAV50	COMPLEX		12114	COMPLEX
12112	QAV51	COMPLEX		12112	COMPLEX
12110	QAV52	COMPLEX		12110	COMPLEX
12108	QAV53	COMPLEX		12108	COMPLEX
12106	QAV54	COMPLEX		12106	COMPLEX
12104	QAV55	COMPLEX		12104	COMPLEX
12102	QAV56	COMPLEX		12102	COMPLEX
12100	QAV57	COMPLEX		12100	COMPLEX
12098	QAV58	COMPLEX		12098	COMPLEX
12096	QAV59	COMPLEX		12096	COMPLEX
12094	QAV60	COMPLEX		12094	COMPLEX
12092	QAV61	COMPLEX		12092	COMPLEX
12090	QAV62	COMPLEX		12090	COMPLEX
12088	QAV63	COMPLEX		12088	COMPLEX
12086	QAV64	COMPLEX		12086	COMPLEX
12084	QAV65	COMPLEX		12084	COMPLEX
12082	QAV66	COMPLEX		12082	COMPLEX
12080	QAV67	COMPLEX		12080	COMPLEX
12078	QAV68	COMPLEX		12078	COMPLEX
12076	QAV69	COMPLEX		12076	COMPLEX
12074	QAV70	COMPLEX		12074	COMPLEX
12072	QAV71	COMPLEX		12072	COMPLEX
12070	QAV72	COMPLEX		12070	COMPLEX
12068	QAV73	COMPLEX		12068	COMPLEX
12066	QAV74	COMPLEX		12066	COMPLEX
12064	QAV75	COMPLEX		12064	COMPLEX
12062	QAV76	COMPLEX		12062	COMPLEX
12060	QAV77	COMPLEX		12060	COMPLEX
12058	QAV78	COMPLEX		12058	COMPLEX
12056	QAV79	COMPLEX		12056	COMPLEX
12054	QAV80	COMPLEX		12054	COMPLEX
12052	QAV81	COMPLEX		12052	COMPLEX
12050	QAV82	COMPLEX		12050	COMPLEX
12048	QAV83	COMPLEX		12048	COMPLEX
12046	QAV84	COMPLEX		12046	COMPLEX
12044	QAV85	COMPLEX		12044	COMPLEX
12042	QAV86	COMPLEX		12042	COMPLEX
12040	QAV87	COMPLEX		12040	COMPLEX
12038	QAV88	COMPLEX		12038	COMPLEX
12036	QAV89	COMPLEX		12036	COMPLEX
12034	QAV90	COMPLEX		12034	COMPLEX
12032	QAV91	COMPLEX		12032	COMPLEX
12030	QAV92	COMPLEX		12030	COMPLEX
12028	QAV93	COMPLEX		12028	COMPLEX
12026	QAV94	COMPLEX		12026	COMPLEX
12024	QAV95	COMPLEX		12024	COMPLEX
12022	QAV96	COMPLEX		12022	COMPLEX
12020	QAV97	COMPLEX		12020	COMPLEX
12018	QAV98	COMPLEX		12018	COMPLEX
12016	QAV99	COMPLEX		12016	COMPLEX
12014	QAV100	COMPLEX		12014	COMPLEX
12012	QAV101	COMPLEX		12012	COMPLEX
12010	QAV102	COMPLEX		12010	COMPLEX
12008	QAV103	COMPLEX		12008	COMPLEX
12006	QAV104	COMPLEX		12006	COMPLEX
12004	QAV105	COMPLEX		12004	COMPLEX
12002	QAV106	COMPLEX		12002	COMPLEX
12000	QAV107	COMPLEX		12000	COMPLEX
11998	QAV108	COMPLEX		11998	COMPLEX
11996	QAV109	COMPLEX		11996	COMPLEX
11994	QAV110	COMPLEX		11994	COMPLEX
11992	QAV111	COMPLEX		11992	COMPLEX
11990	QAV112	COMPLEX		11990	COMPLEX
11988	QAV113	COMPLEX		11988	COMPLEX
11986	QAV114	COMPLEX		11986	COMPLEX
11984	QAV115	COMPLEX		11984	COMPLEX
11982	QAV116	COMPLEX		11982	COMPLEX
11980	QAV117	COMPLEX		11980	COMPLEX
11978	QAV118	COMPLEX		11978	COMPLEX
11976	QAV119	COMPLEX		11976	COMPLEX
11974	QAV120	COMPLEX		11974	COMPLEX
11972	QAV121	COMPLEX		11972	COMPLEX
11970	QAV122	COMPLEX		11970	COMPLEX
11968	QAV123	COMPLEX		11968	COMPLEX
11966	QAV124	COMPLEX		11966	COMPLEX
11964	QAV125	COMPLEX		11964	COMPLEX
11962	QAV126	COMPLEX		11962	COMPLEX
11960	QAV127	COMPLEX		11960	COMPLEX
11958	QAV128	COMPLEX		11958	COMPLEX
11956	QAV129	COMPLEX		11956	COMPLEX
11954	QAV130	COMPLEX		11954	COMPLEX
11952	QAV131	COMPLEX		11952	COMPLEX
11950	QAV132	COMPLEX		11950	COMPLEX
11948	QAV133	COMPLEX		11948	COMPLEX
11946	QAV134	COMPLEX		11946	COMPLEX
11944	QAV135	COMPLEX		11944	COMPLEX
11942	QAV136	COMPLEX		11942	COMPLEX
11940	QAV137	COMPLEX		11940	COMPLEX
11938	QAV138	COMPLEX		11938	COMPLEX
11936	QAV139	COMPLEX		11936	COMPLEX
11934	QAV140	COMPLEX		11934	COMPLEX
11932	QAV141	COMPLEX		11932	COMPLEX
11930	QAV142	COMPLEX		11930	COMPLEX
11928	QAV143	COMPLEX		11928	COMPLEX
11926	QAV144	COMPLEX		11926	COMPLEX
11924	QAV145	COMPLEX		11924	COMPLEX
11922	QAV146	COMPLEX		11922	COMPLEX
11920	QAV147	COMPLEX		11920	COMPLEX
11918	QAV148	COMPLEX		11918	COMPLEX
11916	QAV149	COMPLEX		11916	COMPLEX
11914	QAV150	COMPLEX		11914	COMPLEX
11912	QAV151	COMPLEX		11912	COMPLEX
11910	QAV152	COMPLEX		11910	COMPLEX
11908	QAV153	COMPLEX		11908	COMPLEX
11906	QAV154	COMPLEX		11906	COMPLEX
11904	QAV155	COMPLEX		11904	COMPLEX
11902	QAV156	COMPLEX		11902	COMPLEX
11900	QAV157	COMPLEX		11900	COMPLEX
11898	QAV158	COMPLEX		11898	COMPLEX
11896	QAV159	COMPLEX		11896	COMPLEX
11894	QAV160	COMPLEX		11894	COMPLEX
11892	QAV161	COMPLEX		11892	COMPLEX
11890	QAV162	COMPLEX		11890	COMPLEX
11888	QAV163	COMPLEX		11888	COMPLEX
11886	QAV164	COMPLEX		11886	COMPLEX
11884	QAV165	COMPLEX		11884	COMPLEX
11882	QAV166	COMPLEX		11882	COMPLEX
11880	QAV167	COMPLEX		11880	COMPLEX
11878	QAV168	COMPLEX		11878	COMPLEX
11876	QAV169	COMPLEX		11876	COMPLEX
11874	QAV170	COMPLEX		11874	COMPLEX
11872	QAV171	COMPLEX		11872	COMPLEX
11870	QAV172	COMPLEX		11870	COMPLEX
11868	QAV173	COMPLEX		11868	COMPLEX
11866	QAV174	COMPLEX		11866	COMPLEX
11864	QAV175	COMPLEX		11864	COMPLEX
11862	QAV176	COMPLEX		11862	COMPLEX
11860	QAV177	COMPLEX		11860	COMPLEX
11858	QAV178	COMPLEX		11858	COMPLEX
11856	QAV179	COMPLEX		11856	COMPLEX
11854	QAV180	COMPLEX		11854	COMPLEX
11852	QAV181	COMPLEX		11852	COMPLEX
11850	QAV182	COMPLEX		11850	COMPLEX
11848	QAV183	COMPLEX		11848	COMPLEX
11846	QAV184	COMPLEX		11846	COMPLEX
11844	QAV185	COMPLEX		11844	COMPLEX
11842	QAV186	COMPLEX		11842	COMPLEX
11840	QAV187	COMPLEX		11840	COMPLEX
11838	QAV188	COMPLEX		11838	COMPLEX
11836	QAV189	COMPLEX		11836	COMPLEX
11834	QAV190	COMPLEX		11834	COMPLEX
11832	QAV191	COMPLEX		11832	COMPLEX
11830	QAV192	COMPLEX		11830	COMPLEX
11828	QAV193	COMPLEX		11828	COMPLEX
11826	QAV194	COMPLEX		11826	COMPLEX
11824	QAV195	COMPLEX		11824	COMPLEX
11822	QAV196	COMPLEX		11822	COMPLEX
11820	QAV197	COMPLEX		11820	COMPLEX
11818	QAV198	COMPLEX		11818	COMPLEX
11816	QAV199	COMPLEX		11816	COMPLEX
11814	QAV200	COMPLEX		11814	COMPLEX
11812	QAV201	COMPLEX		11812	COMPLEX
11810	QAV202	COMPLEX		11810	COMPLEX
11808	QAV203	COMPLEX		11808	COMPLEX
11806	QAV204	COMPLEX		11806	COMPLEX
11804	QAV205	COMPLEX		11804	COMPLEX
11802	QAV206	COMPLEX		11802	COMPLEX
11800	QAV207	COMPLEX		11800	COMPLEX
11798	QAV208	COMPLEX		11798	COMPLEX
11796	QAV209	COMPLEX		11796	COMPLEX
11794	QAV210	COMPLEX		11794	COMPLEX
11792	QAV211	COMPLEX		11792	COMPLEX
11790	QAV212	COMPLEX		11790	COMPLEX
11788	QAV213	COMPLEX		11788	COMPLEX
11786	QAV214	COMPLEX		117	

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

PAGE 16

UNCLASSIFIED
09/10/80 09:21:02

PTN 4.0046

UNCLASSIFIED

00/00 00/00

FILE NAME	INPUT	TYPE	2054	OUTPUT	C	2054	TAPE6	FMT	FMT
11452	UT22	REAL							
11453	UT22	REAL							
11454	UT22	REAL							
11455	UT22	REAL							
11456	UT22	REAL							
11457	UT22	REAL							
11458	UT22	REAL							
11459	UT22	REAL							
11460	UT22	REAL							
11461	UT22	REAL							
11462	UT22	REAL							
11463	UT22	REAL							
11464	UT22	REAL							
11465	UT22	REAL							
11466	UT22	REAL							
11467	UT22	REAL							
11468	UT22	REAL							
11469	UT22	REAL							
11470	UT22	REAL							
11471	UT22	REAL							
11472	UT22	REAL							
11473	UT22	REAL							
11474	UT22	REAL							
11475	UT22	REAL							
11476	UT22	REAL							
11477	UT22	REAL							
11478	UT22	REAL							
11479	UT22	REAL							
11480	UT22	REAL							
11481	UT22	REAL							
11482	UT22	REAL							
11483	UT22	REAL							
11484	UT22	REAL							
11485	UT22	REAL							
11486	UT22	REAL							
11487	UT22	REAL							
11488	UT22	REAL							
11489	UT22	REAL							
11490	UT22	REAL							
11491	UT22	REAL							
11492	UT22	REAL							
11493	UT22	REAL							
11494	UT22	REAL							
11495	UT22	REAL							
11496	UT22	REAL							
11497	UT22	REAL							
11498	UT22	REAL							
11499	UT22	REAL							
11500	UT22	REAL							
11501	UT22	REAL							
11502	UT22	REAL							
11503	UT22	REAL							
11504	UT22	REAL							
11505	UT22	REAL							
11506	UT22	REAL							
11507	UT22	REAL							
11508	UT22	REAL							
11509	UT22	REAL							
11510	UT22	REAL							
11511	UT22	REAL							
11512	UT22	REAL							
11513	UT22	REAL							
11514	UT22	REAL							
11515	UT22	REAL							
11516	UT22	REAL							
11517	UT22	REAL							
11518	UT22	REAL							
11519	UT22	REAL							
11520	UT22	REAL							
11521	UT22	REAL							
11522	UT22	REAL							
11523	UT22	REAL							
11524	UT22	REAL							
11525	UT22	REAL							
11526	UT22	REAL							
11527	UT22	REAL							
11528	UT22	REAL							
11529	UT22	REAL							
11530	UT22	REAL							
11531	UT22	REAL							
11532	UT22	REAL							
11533	UT22	REAL							
11534	UT22	REAL							
11535	UT22	REAL							
11536	UT22	REAL							
11537	UT22	REAL							
11538	UT22	REAL							
11539	UT22	REAL							
11540	UT22	REAL							
11541	UT22	REAL							
11542	UT22	REAL							
11543	UT22	REAL							
11544	UT22	REAL							
11545	UT22	REAL							
11546	UT22	REAL							
11547	UT22	REAL							
11548	UT22	REAL							
11549	UT22	REAL							
11550	UT22	REAL							
11551	UT22	REAL							
11552	UT22	REAL							
11553	UT22	REAL							
11554	UT22	REAL							
11555	UT22	REAL							
11556	UT22	REAL							
11557	UT22	REAL							
11558	UT22	REAL							
11559	UT22	REAL							
11560	UT22	REAL							
11561	UT22	REAL							
11562	UT22	REAL							
11563	UT22	REAL							
11564	UT22	REAL							
11565	UT22	REAL							
11566	UT22	REAL							
11567	UT22	REAL							
11568	UT22	REAL							
11569	UT22	REAL							
11570	UT22	REAL							
11571	UT22	REAL							
11572	UT22	REAL							
11573	UT22	REAL							
11574	UT22	REAL							
11575	UT22	REAL							
11576	UT22	REAL							
11577	UT22	REAL							
11578	UT22	REAL							
11579	UT22	REAL							
11580	UT22	REAL							
11581	UT22	REAL							
11582	UT22	REAL							
11583	UT22	REAL							
11584	UT22	REAL							
11585	UT22	REAL							
11586	UT22	REAL							
11587	UT22	REAL							
11588	UT22	REAL							
11589	UT22	REAL							
11590	UT22	REAL							
11591	UT22	REAL							
11592	UT22	REAL							
11593	UT22	REAL							
11594	UT22	REAL							
11595	UT22	REAL							
11596	UT22	REAL							
11597	UT22	REAL							
11598	UT22	REAL							
11599	UT22	REAL							
11600	UT22	REAL							
11601	UT22	REAL							
11602	UT22	REAL							
11603	UT22	REAL							
11604	UT22	REAL							
11605	UT22	REAL							
11606	UT22	REAL							
11607	UT22	REAL							
11608	UT22	REAL							
11609	UT22	REAL							
11610	UT22	REAL							
11611	UT22	REAL							
11612	UT22	REAL							
11613	UT22	REAL							
11614	UT22	REAL							
11615	UT22	REAL							
11616	UT22	REAL							
11617	UT22	REAL							
11618	UT22	REAL							
11619	UT22	REAL							
11620	UT22	REAL							
11621	UT22	REAL							
11622	UT22	REAL							
11623	UT22	REAL							
11624	UT22	REAL							
11625	UT22	REAL							

UNCLASSIFIED
PROGRAM MODEL 7/1/74 SPT=1
INSTRAC LABEL
LABEL LABEL

UNCLASSIFIED
89/08/80 19-21-02
FTV 4-50446
CONJUG COMPLEX 1 INTRAV

UNCLASSIFIED
7/1/74 SPT=1
INSTRAC LABEL
LABEL LABEL

UNCLASSIFIED
PROGRAM MODEL 7/1/74 SPT=1
INSTRAC LABEL
LABEL LABEL

PAGE 17

STATEMENT LABELS
4114 7 110-3 10 FMT 11056 11 FMT
11066 11 10773 101 FMT 11032 102 FMT
11142 10 11050 110 FMT 11013 103 FMT
11022 11 1307 202 4527 205
0 202 0 202 0 303
10723 704 0 202 0 303
4357 404 0 303 0 303
4312 401 0 303 0 303
11226 601 0 303 0 303
11270 606 FMT 0270 312
11124 708 FMT 11257 603 FMT
11160 711 FMT 10727 605 FMT
11217 714 FMT 11147 710 FMT
J 802 INACTIVE 11234 713 FMT
11066 801

LOOPS LABEL INDEX FROM-TO LENGTH PROPERTIES
4426 203 J 111 179 273 OPT
4511 204 J 111 147 273 OPT
4770 207 I 178 181 273 OPT
4715 208 I 183 183 273 OPT
5074 511 I 225 173 023 OPT
5710 511 I 336 342 023 OPT

COMMON BLOCKS LENGTH
//

STATEMENTS
PROGRAM LENGTH 477453 10557
BUFFER LENGTH 41300 4186
ON BLANK COMMON LENGTH 200 20

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

4-1-85 (P.1-1)

10-1-85 (P.1-1)

10-1-85 (P.1-1)

SYMBOLIC REFERENCE MAP (S=1)

ENTRY POINT
3 SWOY

VARIABLES	SN	TYPE	DECLARATION
7 A	17	REAL	454
71 A	18	REAL	455
20 B	19	REAL	456
40 C	20	REAL	457
1 D	21	REAL	458
14 E	22	REAL	459
30 F	23	REAL	460
30 G	24	REAL	461
30 H	25	REAL	462
30 I	26	REAL	463
30 J	27	REAL	464
30 K	28	REAL	465
30 L	29	REAL	466
30 M	30	REAL	467
30 N	31	REAL	468
30 O	32	REAL	469
30 P	33	REAL	470
30 Q	34	REAL	471
30 R	35	REAL	472
30 S	36	REAL	473
30 T	37	REAL	474
30 U	38	REAL	475
30 V	39	REAL	476
30 W	40	REAL	477
30 X	41	REAL	478
30 Y	42	REAL	479
30 Z	43	REAL	480
30 AA	44	REAL	481
30 AB	45	REAL	482
30 AC	46	REAL	483
30 AD	47	REAL	484
30 AE	48	REAL	485
30 AF	49	REAL	486
30 AG	50	REAL	487
30 AH	51	REAL	488
30 AI	52	REAL	489
30 AJ	53	REAL	490
30 AK	54	REAL	491
30 AL	55	REAL	492
30 AM	56	REAL	493
30 AN	57	REAL	494
30 AO	58	REAL	495
30 AP	59	REAL	496
30 AQ	60	REAL	497
30 AR	61	REAL	498
30 AS	62	REAL	499
30 AT	63	REAL	500
30 AU	64	REAL	501
30 AV	65	REAL	502
30 AW	66	REAL	503
30 AX	67	REAL	504
30 AY	68	REAL	505
30 AZ	69	REAL	506
30 BA	70	REAL	507
30 BB	71	REAL	508
30 BC	72	REAL	509
30 BD	73	REAL	510
30 BE	74	REAL	511
30 BF	75	REAL	512
30 BG	76	REAL	513
30 BH	77	REAL	514
30 BI	78	REAL	515
30 BJ	79	REAL	516
30 BK	80	REAL	517
30 BL	81	REAL	518
30 BM	82	REAL	519
30 BN	83	REAL	520
30 BO	84	REAL	521
30 BP	85	REAL	522
30 BQ	86	REAL	523
30 BR	87	REAL	524
30 BS	88	REAL	525
30 BT	89	REAL	526
30 BU	90	REAL	527
30 BV	91	REAL	528
30 BW	92	REAL	529
30 BX	93	REAL	530
30 BY	94	REAL	531
30 BZ	95	REAL	532
30 CA	96	REAL	533
30 CB	97	REAL	534
30 CC	98	REAL	535
30 CD	99	REAL	536
30 CE	100	REAL	537
30 CF	101	REAL	538
30 CG	102	REAL	539
30 CH	103	REAL	540
30 CI	104	REAL	541
30 CJ	105	REAL	542
30 CK	106	REAL	543
30 CL	107	REAL	544
30 CM	108	REAL	545
30 CN	109	REAL	546
30 CO	110	REAL	547
30 CP	111	REAL	548
30 CQ	112	REAL	549
30 CR	113	REAL	550
30 CS	114	REAL	551
30 CT	115	REAL	552
30 CU	116	REAL	553
30 CV	117	REAL	554
30 CW	118	REAL	555
30 CX	119	REAL	556
30 CY	120	REAL	557
30 CZ	121	REAL	558
30 DA	122	REAL	559
30 DB	123	REAL	560
30 DC	124	REAL	561
30 DD	125	REAL	562
30 DE	126	REAL	563
30 DF	127	REAL	564
30 DG	128	REAL	565
30 DH	129	REAL	566
30 DI	130	REAL	567
30 DJ	131	REAL	568
30 DK	132	REAL	569
30 DL	133	REAL	570
30 DM	134	REAL	571
30 DN	135	REAL	572
30 DO	136	REAL	573
30 DP	137	REAL	574
30 DQ	138	REAL	575
30 DR	139	REAL	576
30 DS	140	REAL	577
30 DT	141	REAL	578
30 DU	142	REAL	579
30 DV	143	REAL	580
30 DW	144	REAL	581
30 DX	145	REAL	582
30 DY	146	REAL	583
30 DZ	147	REAL	584
30 EA	148	REAL	585
30 EB	149	REAL	586
30 EC	150	REAL	587
30 ED	151	REAL	588
30 EE	152	REAL	589
30 EF	153	REAL	590
30 EG	154	REAL	591
30 EH	155	REAL	592
30 EI	156	REAL	593
30 EJ	157	REAL	594
30 EK	158	REAL	595
30 EL	159	REAL	596
30 EM	160	REAL	597
30 EN	161	REAL	598
30 EO	162	REAL	599
30 EP	163	REAL	600
30 EQ	164	REAL	601
30 ER	165	REAL	602
30 ES	166	REAL	603
30 ET	167	REAL	604
30 EU	168	REAL	605
30 EV	169	REAL	606
30 EW	170	REAL	607
30 EX	171	REAL	608
30 EY	172	REAL	609
30 EZ	173	REAL	610
30 FA	174	REAL	611
30 FB	175	REAL	612
30 FC	176	REAL	613
30 FD	177	REAL	614
30 FE	178	REAL	615
30 FF	179	REAL	616
30 FG	180	REAL	617
30 FH	181	REAL	618
30 FI	182	REAL	619
30 FJ	183	REAL	620
30 FK	184	REAL	621
30 FL	185	REAL	622
30 FM	186	REAL	623
30 FN	187	REAL	624
30 FO	188	REAL	625
30 FP	189	REAL	626
30 FQ	190	REAL	627
30 FR	191	REAL	628
30 FS	192	REAL	629
30 FT	193	REAL	630
30 FU	194	REAL	631
30 FV	195	REAL	632
30 FW	196	REAL	633
30 FX	197	REAL	634
30 FY	198	REAL	635
30 FZ	199	REAL	636
30 GA	200	REAL	637
30 GB	201	REAL	638
30 GC	202	REAL	639
30 GD	203	REAL	640
30 GE	204	REAL	641
30 GF	205	REAL	642
30 GG	206	REAL	643
30 GH	207	REAL	644
30 GI	208	REAL	645
30 GJ	209	REAL	646
30 GK	210	REAL	647
30 GL	211	REAL	648
30 GM	212	REAL	649
30 GN	213	REAL	650
30 GO	214	REAL	651
30 GP	215	REAL	652
30 GQ	216	REAL	653
30 GR	217	REAL	654
30 GS	218	REAL	655
30 GT	219	REAL	656
30 GU	220	REAL	657
30 GV	221	REAL	658
30 GW	222	REAL	659
30 GX	223	REAL	660
30 GY	224	REAL	661
30 GZ	225	REAL	662
30 HA	226	REAL	663
30 HB	227	REAL	664
30 HC	228	REAL	665
30 HD	229	REAL	666
30 HE	230	REAL	667
30 HF	231	REAL	668
30 HG	232	REAL	669
30 HH	233	REAL	670
30 HI	234	REAL	671
30 HJ	235	REAL	672
30 HK	236	REAL	673
30 HL	237	REAL	674
30 HM	238	REAL	675
30 HN	239	REAL	676
30 HO	240	REAL	677
30 HP	241	REAL	678
30 HQ	242	REAL	679
30 HR	243	REAL	680
30 HS	244	REAL	681
30 HT	245	REAL	682
30 HU	246	REAL	683
30 HV	247	REAL	684
30 HW	248	REAL	685
30 HX	249	REAL	686
30 HY	250	REAL	687
30 HZ	251	REAL	688
30 IA	252	REAL	689
30 IB	253	REAL	690
30 IC	254	REAL	691
30 ID	255	REAL	692
30 IE	256	REAL	693
30 IF	257	REAL	694
30 IG	258	REAL	695
30 IH	259	REAL	696
30 II	260	REAL	697
30 IJ	261	REAL	698
30 IK	262	REAL	699
30 IL	263	REAL	700
30 IM	264	REAL	701
30 IN	265	REAL	702
30 IO	266	REAL	703
30 IP	267	REAL	704
30 IQ	268	REAL	705
30 IR	269	REAL	706
30 IS	270	REAL	707
30 IT	271	REAL	708
30 IU	272	REAL	709
30 IV	273	REAL	710
30 IW	274	REAL	711
30 IX	275	REAL	712
30 IY	276	REAL	713
30 IZ	277	REAL	714
30 JA	278	REAL	715
30 JB	279	REAL	716
30 JC	280	REAL	717
30 JD	281	REAL	718
30 JE	282	REAL	719
30 JF	283	REAL	720
30 JG	284	REAL	721
30 JH	285	REAL	722
30 JI	286	REAL	723
30 JJ	287	REAL	724
30 JK	288	REAL	725
30 JL	289	REAL	726
30 JM	290	REAL	727
30 JN	291	REAL	728
30 JO	292	REAL	729
30 JP	293	REAL	730
30 JQ	294	REAL	731
30 JR	295	REAL	732
30 JS	296	REAL	733
30 JT	297	REAL	734
30 JU	298	REAL	735
30 JV	299	REAL	736
30 JW	300	REAL	737
30 JX	301	REAL	738
30 JY	302	REAL	739
30 JZ	303	REAL	740
30 KA	304	REAL	741
30 KB	305	REAL	742
30 KC	306	REAL	743
30 KD	307	REAL	744
30 KE	308	REAL	745
30 KF	309	REAL	746
30 KG	310	REAL	747
30 KH	311	REAL	748
30 KI	312	REAL	749
30 KJ	313	REAL	750
30 KK	314	REAL	751
30 KL	315	REAL	752
30 KM	316	REAL	753
30 KN	317	REAL	754
30 KO	318	REAL	755
30 KP	319	REAL	756
30 KQ	320	REAL	757
30 KR	321	REAL	758
30 KS	322	REAL	759
30 KT	323	REAL	760
30 KU	324	REAL	761
30 KV	325	REAL	762
30 KW	326	REAL	763
30 KX	327	REAL	764
30 KY	328	REAL	765
30 KZ	329	REAL	766
30 LA	330	REAL	767
30 LB	331	REAL	768
30 LC	332	REAL	769
30 LD	333	REAL	770
30 LE	334	REAL	771
30 LF	335	REAL	772
30 LG	336	REAL	773
30 LH	337	REAL	774
30 LI	338	REAL	775
30 LJ	339	REAL	776
30 LK	340	REAL	777
30 LL	341	REAL	778
30 LM	342	REAL	779
30 LN	343	REAL	780
30 LO	344	REAL	781
30 LP	345	REAL	782
30 LQ	346	REAL	783
30 LR	347	REAL	784
30 LS	348	REAL	785
30 LT	349	REAL	786
30 LU	350	REAL	787
30 LV	351	REAL	788
30 LW	352	REAL	789
30 LX	353	REAL	790
30 LY	354	REAL	791
30 LZ	355	REAL	792
30 MA	356	REAL	793
30 MB	357	REAL	794
30 MC	358	REAL	795
30 MD	359	REAL	796
30 ME	360	REAL	797
30 MF	361	REAL	798
30 MG	362	REAL	799
30 MH	363	REAL	800
30 MI	364	REAL	801
30 MJ	365	REAL	802
30 MK	366	REAL	803
30 ML	367	REAL	804
30 MM	368	REAL	805
30 MN	369	REAL	806
30 MO	370	REAL	807

UNCLASSIFIED
FOUO/INT
STATISTICS
2000 11 15

7/74 0113

UNCLASSIFIED

FT 40000

UNCLASSIFIED
09/09/88 09.21.02

PAGE 3

THIS PAGE IS BEST QUALITY PRINT
FROM COM-1 FURNISHED TO LDU

UNCLASSIFIED

UNCLASSIFIED

UNCLASSIFIED

LAYER SIGMA ZERO= -1.51515E-06
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -1.6503560E

THETA=20.00
PI= .8295379E+01
J1= .1215493E+00
HALF SPACE SIGMA ZERO= -2.14151E-09
LAYER SIGMA ZERO= -2.16072E-09
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -2.15372E-09

THETA=70.00
PI= .015349E+01
J1= .1091286E+00
HALF SPACE SIGMA ZERO= -2.60136E-09
LAYER SIGMA ZERO= -2.60576E-09
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -2.60576E-09

THETA=40.00
PI= .1055435E+01
J1= .075032E-01
HALF SPACE SIGMA ZERO= -1.14451E-03
LAYER SIGMA ZERO= -1.11590E-03
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -3.11359E-03

THETA=50.00
PI= .1215979E+01
J1= .041505E-01
HALF SPACE SIGMA ZERO= -1.44535E-03
LAYER SIGMA ZERO= -1.4475E-03
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -3.54738E-03

THETA=60.00
PI= .155545E+02
J1= .040637E-01
HALF SPACE SIGMA ZERO= -3.54924E-03
LAYER SIGMA ZERO= -3.94710E-03
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -3.94710E-03

THETA=70.00
PI= .237163E+02
J1= .121655E-01
HALF SPACE SIGMA ZERO= -5.04035E-03
LAYER SIGMA ZERO= -5.07837E-03
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -5.03837E-03

THETA=80.00
PI= .347557E+02
J1= .285137E-01
HALF SPACE SIGMA ZERO= -9.53713E-03
LAYER SIGMA ZERO= -9.53735E-03
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -9.53735E-03

FRACTION OF WATER BY WEIGHT IN THE VEGETATION= .400
VOLUME OF VEGETATION CIVILIAN IN TOTAL VOLUME= .0910080
FREQUENCY= .72500E+10
RATIO OF THE STANDARD DEVIATION OF THE ROUGH SURFACE FLUCTUATIONS TO THE CORRELATION DISTANCE OF THE FLUCTUATIONS= .03000
X1= .0100000 X2= .0050000
LAYER THICKNESS IN METERS= 2.50000
DIELECTRIC CONSTANT OF THE SOIL= 7.50000
CONDUCTIVITY OF THE SOIL= 1.490000
THE AVERAGE DIELECTRIC CONSTANT= 1.020002
STD. DEV. OF DIELECTRIC FLUCTUATIONS= .8218577
STD. DEV. OF CONDUCTIVITY FLUCTUATIONS= .0349091
AD= .55979E+00
THETA= 0.00

THIS PAGE IS BEST QUALITY PRACTICE
FROM COPY FURNISHED TO DDG

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

P1= .47870720+01 J1= .4710444E+03
HALF SPACE SIGMA ZERO= -1.02257100
LAYER SIGMA ZERO= -1.02257100
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -1.07257100

THETA=10.00
P1= .4929444E+01 J1= .4431115E+00
HALF SPACE SIGMA ZERO= -2.13424332
LAYER SIGMA ZERO= -2.04511277
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -2.10511277

THETA=20.00
P1= .7311444E+01 J1= .4430875E+00
HALF SPACE SIGMA ZERO= -4.14902044
LAYER SIGMA ZERO= -4.17175220
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -4.17175220

THETA=30.00
P1= .4032144E+01 J1= .4244936E+00
HALF SPACE SIGMA ZERO= -5.98110841
LAYER SIGMA ZERO= -5.97614737
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -5.97614737

THETA=40.00
P1= .0114444E+01 J1= .4097543E+00
HALF SPACE SIGMA ZERO= -7.34902564
LAYER SIGMA ZERO= -7.32103667
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -7.32103667

THETA=50.00
P1= .1077284E+02 J1= .9242571E-01
HALF SPACE SIGMA ZERO= -8.27761762
LAYER SIGMA ZERO= -8.24943447
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -8.24943447

THETA=60.00
P1= .1357144E+02 J1= .7304202E-01
HALF SPACE SIGMA ZERO= -8.99010703
LAYER SIGMA ZERO= -8.98509082
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -8.98509082

THETA=70.00
P1= .1092412E+02 J1= .5284231E-01
HALF SPACE SIGMA ZERO= -10.13759989
LAYER SIGMA ZERO= -10.13592344
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -10.13592344

THETA=80.00
P1= .2070040E+02 J1= .3545672E-01
HALF SPACE SIGMA ZERO= -14.30669351
LAYER SIGMA ZERO= -14.30665782
LAYER WITH ROUGH SURFACE SIGMA ZERO MM= -14.30665782

FRACTION OF WATER BY WEIGHT IN THE VEGETATION= .003
VOLUME OF VEGETATION DIVIDED BY TOTAL VOLUME= .001000
FREQUENCY= .52000E+10
RATIO OF THE STANDARD DEVIATION OF THE ROUGH SURFACE FLUCTUATIONS TO THE CORRELATION DISTANCE OF THE FLUCTUATIONS= .10000

XLX= .0040000 XLV= .0100000 XLZ= .0080000
 LAYER THICKNESS IN METERS = 7.50000
 DIELECTRIC CONSTANT OF THE SOIL = 7.50000
 CONDUCTIVITY OF THE SOIL = 7.50000
 THE AVERAGE DIELECTRIC CONSTANT = 7.50000
 STD. DEV. OF DIELECTRIC FLUCTUATIONS = 1.00000
 A0= .3179245E+00
 TMEFA= .0000
 P1= .3622050E+01
 HALF SPACE SIGMA ZERO= .2715674E+00
 LAYER SIGMA ZERO= -1.35117420
 LAYER WITH ROUGH SURFACE SIGMA ZERO HME= -1.00289420

-1.00289420

THETA=10.00
 P1= .3752204E+01
 HALF SPACE SIGMA ZERO= .2505097E+00
 LAYER SIGMA ZERO= -1.4429795E
 LAYER WITH ROUGH SURFACE SIGMA ZERO HME= -1.2000047E

-1.2500047E

THETA=20.00
 P1= .3972397E+01
 HALF SPACE SIGMA ZERO= .4517378E+00
 LAYER SIGMA ZERO= -1.6742295E
 LAYER WITH ROUGH SURFACE SIGMA ZERO HME= -1.0000745E

-1.0000745E

THETA=30.00
 P1= .4375752E+01
 HALF SPACE SIGMA ZERO= .2295531E+00
 LAYER SIGMA ZERO= -1.9393434E
 LAYER WITH ROUGH SURFACE SIGMA ZERO HME= -1.0001103E

-1.0001103E

THETA=40.00
 P1= .5031155E+01
 HALF SPACE SIGMA ZERO= .1007595E+00
 LAYER SIGMA ZERO= -2.7005487E
 LAYER WITH ROUGH SURFACE SIGMA ZERO HME= -2.2830104E

-2.2830104E

THETA=50.00
 P1= .6037554E+01
 HALF SPACE SIGMA ZERO= .1002697E+00
 LAYER SIGMA ZERO= -2.7000012E
 LAYER WITH ROUGH SURFACE SIGMA ZERO HME= -2.0000000E

-2.0000000E

THETA=60.00
 P1= .7034055E+01
 HALF SPACE SIGMA ZERO= .1000000E+00
 LAYER SIGMA ZERO= -2.0000000E
 LAYER WITH ROUGH SURFACE SIGMA ZERO HME= -2.0000000E

-2.0000000E

THETA=70.00
 P1= .8127373E+01
 HALF SPACE SIGMA ZERO= .1000000E+00
 LAYER SIGMA ZERO= -2.0000000E
 LAYER WITH ROUGH SURFACE SIGMA ZERO HME= -2.0000000E

-2.0000000E

THETA=80.00
 P1= .9100000E+01
 HALF SPACE SIGMA ZERO= .1000000E+00
 LAYER SIGMA ZERO= -2.0000000E
 LAYER WITH ROUGH SURFACE SIGMA ZERO HME= -2.0000000E

-2.0000000E

THIS PAGE IS BEST QUALITY PRINTABLE
 FROM COPY FURNISHED TO DDC

HALE SPACE SIGMA Z=20= -6.07023184
 LAYER SIGMA Z=20= -8.94272473
 LAYER WITH ROUGH SURFACE SIGMA Z=20= -8.94272473

.03000

FRACTION OF WATER BY WEIGHT IN THE VEGETATION= .000
 VOLUME OF VEGETATION DIVIDED BY TOTAL VOLUME= .001000

FREQUENCY= .00000000
 RATIO OF THE STANDARD DEVIATION OF THE ROUGH SURFACE FLUCTUATIONS TO THE CORRELATION DISTANCE OF THE FLUCTUATIONS= .00000000

ALFA= .00000000 ALFA= .00000000 XL7= .00000000

LAYER THICKNESS IN METERS= 2.500000

ELECTRIC CONSTANT OF THE SOIL= 1.000000

THE AVERAGE DIELECTRIC CONSTANT OF THE SOIL= 1.000000

STD. DEV. OF DIELECTRIC FLUCTUATIONS= .000000

STD. DEV. OF CONDUCTIVITY FLUCTUATIONS= .000000

MO= .00000000

THEIR= .000000

PIE= .00000000

HALE SPACE SIGMA Z=20= -6.07023184

LAYER SIGMA Z=20= -8.94272473

LAYER WITH ROUGH SURFACE SIGMA Z=20= -8.94272473

.03000

FRACTION OF WATER BY WEIGHT IN THE VEGETATION= .000
 VOLUME OF VEGETATION DIVIDED BY TOTAL VOLUME= .001000

FREQUENCY= .00000000
 RATIO OF THE STANDARD DEVIATION OF THE ROUGH SURFACE FLUCTUATIONS TO THE CORRELATION DISTANCE OF THE FLUCTUATIONS= .00000000

ALFA= .00000000 ALFA= .00000000 XL7= .00000000

LAYER THICKNESS IN METERS= 2.500000

ELECTRIC CONSTANT OF THE SOIL= 1.000000

THE AVERAGE DIELECTRIC CONSTANT OF THE SOIL= 1.000000

STD. DEV. OF DIELECTRIC FLUCTUATIONS= .000000

STD. DEV. OF CONDUCTIVITY FLUCTUATIONS= .000000

MO= .00000000

THEIR= .000000

PIE= .00000000

HALE SPACE SIGMA Z=20= -6.07023184

LAYER SIGMA Z=20= -8.94272473

LAYER WITH ROUGH SURFACE SIGMA Z=20= -8.94272473

.03000

FRACTION OF WATER BY WEIGHT IN THE VEGETATION= .000
 VOLUME OF VEGETATION DIVIDED BY TOTAL VOLUME= .001000

FREQUENCY= .00000000
 RATIO OF THE STANDARD DEVIATION OF THE ROUGH SURFACE FLUCTUATIONS TO THE CORRELATION DISTANCE OF THE FLUCTUATIONS= .00000000

ALFA= .00000000 ALFA= .00000000 XL7= .00000000

LAYER THICKNESS IN METERS= 2.500000

ELECTRIC CONSTANT OF THE SOIL= 1.000000

THE AVERAGE DIELECTRIC CONSTANT OF THE SOIL= 1.000000

STD. DEV. OF DIELECTRIC FLUCTUATIONS= .000000

STD. DEV. OF CONDUCTIVITY FLUCTUATIONS= .000000

MO= .00000000

THIS PAGE IS BEST QUALITY PRACTICABLE
 FROM COPY FURNISHED TO DDC

LIST OF SYMBOLS

$\underline{r}$	Position vector
$\epsilon(\underline{r})$	Permittivity of the random medium
$\sigma(\underline{r})$	Conductivity of the random medium
ϵ_a	Average relative dielectric constant of the random medium
σ_a	Average conductivity of the random medium
η_1	Standard deviation of the dielectric fluctuations
η_2	Standard deviation of the conductivity fluctuations
L	Mean thickness of the vegetation layer
k_3	Complex propagation constant in the soil
θ_i	Angle of incidence
k_0	Free space propagation constant
$\underline{\underline{\Gamma}}(\underline{r}, \underline{r}')$	Infinite space dyadic Green's function
$\langle \underline{E}(\underline{r}) \rangle$	Mean wave in the random medium
$G_0(\underline{r}, \underline{r}')$	Infinite space scalar Green's function
$\underline{E}_s(\underline{r})$	Scattered electric field in the random medium
$\delta(\underline{r} - \underline{r}')$	Three-dimensional Dirac delta function equal to $\delta(x - x') \delta(y - y') \delta(z - z')$
$\underline{\underline{I}}$	Unit dyadic
$\underline{a}_x, \underline{a}_y, \underline{a}_z$	Unit vectors in x, y, and z

k_x, k_y

i

δ_{ij}

k_{ex}, k_{ey}, k_{ez}

k_h

k_v

$\underline{G}_s(\underline{k}_t, z)$

$A_x(k_x, k_y), A_y(k_x, k_y), A_z(k_x, k_y)$

A_I

ℓ_x, ℓ_y, ℓ_z

σ_{HHV}°

σ_{VVV}°

σ_{HHs}°

σ_{HH}°

Fourier transform variables

$\sqrt{-1}$

Kronecker Delta

Components of the effective propagation constant

Value of k_{ex} for horizontal polarization

Value of k_{ex} for vertical polarization

Two-dimensional Fourier transform of the scattered electric field in the random medium

Fourier transform of the amplitudes of the scattered electric field in air

Illuminated surface area

Correlation distances in x , y , and z , respectively

Backscatter coefficient for volume scattering and for the case of horizontal polarization transmit, horizontal polarization receive

Backscatter coefficient for volume scattering and for the case of vertical polarization transmit, vertical polarization receive

Backscatter coefficient for a randomly rough surface for the case of horizontal polarization transmit, horizontal polarization receive

Final backscatter coefficient result that includes both volume scattering and rough surface scattering for the case of horizontal polarization transmit, horizontal polarization receive

σ_{VV}^0

Final backscatter coefficient result that includes both volume scattering and rough surface scattering for the case of vertical polarization transmit, vertical polarization receive

F

Fraction of water by weight in the vegetation

R_V

Volume of vegetation divided by the total volume

m_s

Standard deviation of the rough surface fluctuations divided by the correlation distance

f

Frequency