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COMPUTING SOLUTIONS OF THE REDUCED WAVE EQUATION.(U)
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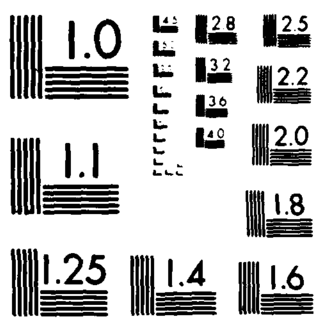

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FINAL REPORT

AIR FORCE GRANT NO. F-49620-79-C-0193

The final report for Air Force Grant No. F-49620-79-C-0193 consists of the annual progress reports dated November 21, 1979 and August 25, 1980, as well as the following report for 1980-81:

During 1980-81 various numerical experiments were made on one-dimensional model inverse problems modelled by a method that is extendable to higher dimensions. The underlying problem is to recover the speed of propagation or the shape of an object from a scattered field.

The one dimensional problem that was investigated was based on the wave equation with potential

$$u_{tt} - u_{xx} + q(x)u = 0$$

where the function to be recovered is the potential $q(x)$. The model initial conditions were $u = 0$, $u_t = -2\delta'(x)$. The model boundary condition was $u = 0$ on $x = 0$. And the "extra" condition which determines $q(x)$ is either (a) $\partial u/\partial x$ on $x = 0$ for all time or (b) the scattered field $u \sim \delta(t-x) + R(t-x)$ at $x = +\infty$. The free space $(q=0)$ solution is $u = \delta(x-t) - \delta(x+t)$. The distorted plane wave $P(x,t)$ is defined by the condition that it is a solution of the equation and at $x = -\infty$, $t \to +\infty$, it becomes $\delta(t-x)$ (exactly if q has compact support) and P is written as $\delta(t-x) + P_{scat}$.

The linearized equation based on the assumption that $q \ll 1$ and that terms of order q^2 may be neglected, yields for (a)

$$q = +2 \frac{d}{dx} \left[\mathcal{S}(2x) - P_{scat}(0,-2x) \right]$$

with $\int_{\infty}^{t} \frac{\partial u}{\partial x} dt \Big|_{x=0_{\infty}} = \mathcal{S}(t)$ for $t > 0$;

$$q = -2 \frac{d}{dx} R(2x)$$

for (b) .

The numerical method is based on the well-known formula that follows from the propagation of singularities

$$q = 2 \frac{d}{dx} \lim_{t \to x+} u(x,x)$$

and then representing $u(x,x)$ by means of distorted plane waves. (All these have analogous formulas in higher dimensions.)

The analogues of the above equation are for (b)

$$q = -2 \frac{d}{dx} R(2x) + \frac{d}{dx} \int_{0}^{\infty} (P_{scat}(x,x-s) - P_{scat}(-x,x-s))R(s)\, ds$$

and for (a) we insert in this formula

$$R(s) = -\mathscr{F}(s) + P_{scat}(0,-s) + \mathscr{F}(0)\int_{\infty}^{s} P_{scat}(0,-s')ds'$$

$$+ \int_{0}^{\infty} \mathscr{F}(t)\, P_{scat}(0,t-s)dt$$

The idea is to use one of these formulas as part of an iteration. We guess q and P and recompute q using the above formulas for cases (a) or (b).

Note that if the given data vanishes in case (b) we recover $q \equiv 0$ but in case (a) we donot.

We have been unable to implement (a) as planned. The reason is that we must operate in a finite region and use a radiation condition at a finite distance. In our case for example with q a simple quadratic with support in $|x| < 2$, over $-8 \leq x \leq 8$ and $0 \leq t \leq \ldots$ The errors produced are compounded in case (a). The potential could only be recovered to 40% of its original value with an error of about 5% of the maximum potential. However, the potentials were well beyond the range of the linear theory.

In case (b) even with a coarse mesh the results are surprisingly worse.

The results are being written up as a technical report to be issued at the Courant Institute.

BIBLIOGRAPHY

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[2] Morawetz, C. S., A formulation for higher dimensional inverse problems for the wave equation, Comp. & Math. with Appls., Vol. 7, 1981, pp. 319-331.

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This report summarizes progress made in various numerical experiments made on one-dimensional model inverse problems.

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