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NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS.(U)
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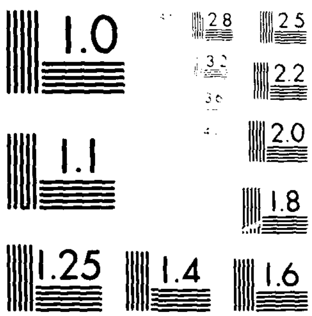

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NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS

AD A110302

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Resume of Recent Research

L. Ridgway Scott

L^∞ Estimates for Finite Element Methods

Two projects have been completed in the 1980-81 academic year. The first, a collaboration with C. Goldstein begun several years ago, concerned estimates for various methods for the Dirichlet problem on a domain with a curved boundary. The paper developed a central body of estimates for the approximate Green's function for arbitrary methods satisfying certain assumptions. These were then augmented by special estimates for three specific methods, yielding optimal-order error estimates for each method in the maximum norm. A manuscript entitled "Optimal maximum norm error estimates for some finite element methods for treating the Dirichlet problem" has been submitted for publication to CALCOLO.

The second project was a collaboration with R. Rannacher begun in August, 1980. We were able to settle, to some extent, an issue concerning the rate of convergence, in various norms, of the finite element method using piecewise linear elements. We were able to show that the error is $O(h^2)$ (h = mesh size) in any $L^p, 1 < p < \infty$, and $O(h)$ in the Sobolev space W^1_p for any p in the range $1 < p \leq \infty$. The best results previously had included a factor of $|\log h|$ to a power depending on p. For example, the best rate previously known in W^1_∞ was $\mathcal{O}(h|\log h|)$. Our research left open the rate of convergence in L^∞; an example indicates that the known estimate $O(h^2|\log h|)$ may be best possible. A manuscript describing these results, entitled "Some optimal error estimates for piecewise linear finite element approximations" has been submitted to Mathematics of Computation and is available as MRC Technical Summary Report #2191.

Numerical Simulation of Nonlinear Dispersive Waves

A numerical method for solving the equation

$$(*) \qquad u_t + \alpha u_x + \beta u\, u_x - \gamma u_{xxt} - \delta\, u_{xx} = 0$$

was developed, analyzed rigorously and used to compare the predictions of the model equation (*) with laboratory experiments, jointly with W.G. Pritchard and J.L. Bona. A paper describing this work, entitled "An evaluation of a model equation for water waves" will have appeared in Phil. Trans. Roy. Soc. Lond. A vol. 302, pp. 457-510.

The previously mentioned numerical method was also used to compare solutions of (*) to those of the so-called Korteweg-de Vries equation. The numerical scheme was used to integrate (*), starting with initial data for which the exact solution to the Korteweg-de Vries equation was known. This work will be described in a manuscript currently in preparation, jointly with Bona and Pritchard, which studies more generally the similarities between (*) and the Korteweg-de Vries equation.

Numerical Models for Plasticity

A finite element method using discontinuous piecewise polynomials was developed and proved to be convergent, jointly with C. Johnson. A report on this work, "A finite element method for problems in perfect plasticity using discontinuous trial functions," has appeared in Nonlinear Finite Element Analysis in Structural Mechanics, W. Wunderlich et al. eds., Springer-Verlag, Berlin, 1981, pp. 307-324. Research concerning rates of convergence of the method has been done recently by Johnson.

Numerical Methods for the Transport Equation

A collaboration with J. Pitkäranta is in progress concerning rates of convergence of the so-called discrete ordinates method for the transport equation. The simplest form of this occurs in the so-called "slab" geometry, namely,

$$(**)\quad \begin{cases} \mu \dfrac{\partial\psi}{\partial x}(x,\mu) + \psi(x,\mu) = \alpha \displaystyle\int_{-1}^{1} \psi(x,\nu)d\nu + f(x) \\ \psi(0,\mu) = 0 \quad \text{for} \quad \mu > 0 \ , \ \psi(1,\mu) = 0 \quad \text{for} \quad \mu < 0 \ . \end{cases}$$

Here $x \in [0,1]$ and $\mu \in [-1,1]$. The discrete ordinates method consists in discretizing the integral in (**) via numerical quadrature and the remaining spatial differential operator by a finite difference or finite element method. A key feature of the problem is the lack of smoothness of ψ near $\mu = 0$. A manuscript is currently in preparation that deals with the problem (**), giving rates of convergence in terms of both the μ-mesh and the x-mesh approximation parameters. In particular, results are given for Gaussian quadrature, which is the most widely used method for the μ-integral discretization. These results are based on estimates for Gaussian quadrature recently obtained jointly with R.A. DeVore. The latter work is described in "Error estimates for Gaussian quadrature and weighted-L^1 polynomial approximation," which will have appeared as an MRC Technical Summary Report and will be submitted to a journal.

Computation of Fluid-Flow Problems

During the '80-'81 academic year, a numerical computer code has been under development at MRC, jointly with W.G. Pritchard and Y.Y. Renardi. One of the ultimate goals is a code for a problem with a free surface. Various hurdles have been leapt toward this goal, but a working code for the free boundary problem is not yet available. Subroutines for calculating the free surface from certain quantities and for computing the exact solution to the Jeffery-Hamel flow problem have been written and tested. (The Jeffery-Hamel flow is to be used as a test problem; it has the property that the nonlinear term does not vanish identically.) Also, a program for flow in a pipe has been written and tested. The latter has been written in modular form, so the extension to more complicated flow regions, including a free surface, should be relatively easy.

Resume of Recent Research

Mitchell B. Luskin

Research has been undertaken in four areas during the past year. A manuscript has been prepared with Professor William Martin and Mr. Leonard Lorence of the Department of Nuclear Engineering at the University of Michigan on a finite element discretization of the neutron transport equation. The iterative solution of the resulting linear system by a block Gauss-Seidel method has also been analyzed. This procedure has been shown to require less storage and fewer arithmetic operations than the direct solution by Gaussian elimination.

A manuscript has been prepared with Professor Rolf Rannacher of the University of Erlangen-Nürnberg (West Germany) which demonstrates that for parabolic equations with rough initial data a $O(k^2/t^2)$ convergence rate for the Crank-Nicolson time discretization scheme can be obtained unconditionally if an appropriate smoothing procedure is performed (such as adding 4 backward Euler steps). Further, a report has been prepared with Professor Ivo Babuska of the University of Maryland on an adaptive time discretization procedure for parabolic problems which has been published in the Proceedings of the Fourth IMACS International Symposium on Computer Methods for Partial Differential Equations. A more extensive paper on this subject is currently being prepared.

Finally, a paper has been written with Professor Blake Temple of Rockefeller University which proves the existence of global weak solutions to an initial-boundary value problem for a nonlinear hyperbolic system which models fluid flow in a pipe. The effect of friction has been modeled by adding a quadratic zero order term to the system of conservation laws for compressible, frictionless flow. A priori bounds have been obtained by means of a nonincreasing functional that is compatible with the friction effects. The boundary values for this problem cannot be imposed weakly, so new results on the regularity of the solution at the boundary have been given.

Technical details of the research may be found in the following publications of Mitchell B. Luskin:

On the smoothing property of the Galerkin method for parabolic equations (with R. Rannacher), SIAM J. Numer. Anal., to appear.

On the existence of global smooth solutions for a model equation for fluid flow in a pipe, J. Math. Anal. Appl., to appear.

On a finite element method to solve the criticality eigenvalue problem for the transport equation (with J. Descloux), SIAM J. Numer. Anal., to appear.

An adaptive time discretization procedure for parabolic problems (with I. Babuska), Proceedings of the Fourth IMACS International Symposium on Computer *Methods for* Partial Differential Equations, 1981.

The existence of a global weak solution to the nonlinear waterhammer problem (with B. Temple), Comm. Pure Appl. Math., submitted.

An analysis of the Crank-Nicolson scheme for parabolic problems (with R. Rannacher), in preparation.

An iterative procedure for the finite element solution of the multidimensional transport equation (with W. Martin and L. Lorence), in preparation.

LIST OF PROFESSIONAL PERSONNEL ASSOCIATED WITH THE RESEARCH EFFORT:

1. Faculty Investigators at the University of Michigan

   L. Ridgway Scott
   M. Luskin

2. Professor Rolf Rannacher, University of Bonn, West Germany (partially supported by the University of Michigan as a Visiting Associate Professor during the 1979-80 academic year).

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Applications of mechanics; partial differential equations; numerical techniques; heat condution; fluid mechanics; finite element methods; nonlinear dispersive waves; plasticity; and transport equations.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The report summarizes progress made in several areas, including: estimates for finite element methods, numerical simulation of nonlinear dispersive waves, numerical models for plasticity, numerical methods for the transport equation, and computation of fluid-flow problems.

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