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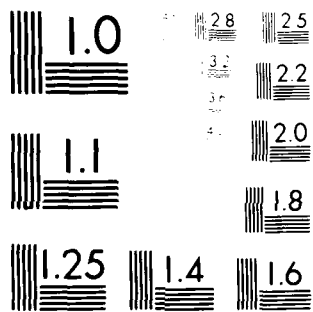
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Phase Report

October 1981

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**COMPUTER PROGRAM FOR ELECTRO-
MAGNETIC COUPLING TO A CONDUCT-
ING BODY OF REVOLUTION WITH A
HOMOGENEOUS MATERIAL REGION**

Syracuse University

Joseph R. Mautz

Roger F. Harrington

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A computer program is given to calculate electromagnetic coupling to a perfectly conducting body of revolution with a loss-free homogeneous material region. The material region is also a body of revolution. It is bounded by an aperture and part of the surface of the conducting body. The maximum dimensions of the conducting body and its associated material region are of the order of a few wavelengths. The electromagnetic excitation is an obliquely incident polarized plane wave.		

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Application of the equivalence principle and subsequent enforcement of the boundary conditions for tangential fields give a set of integral equations. These equations are then solved by means of the method of moments. The computer program calculates the electric current induced on the surface of the conducting body and the equivalent electric and magnetic currents in the aperture. These currents radiate the field scattered by the conducting body and its associated material region. They also radiate the field transmitted through the aperture into the material region. The computer program is described and listed along with sample input and output data.

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I. INTRODUCTION

The computer program presented here calculates the Fourier coefficients [1, Eqs. (87)-(89)] of the electric and magnetic currents on the object shown in [1, Fig. 1] excited by an obliquely incident θ -polarized plane wave. The I's and V's in [1, Eqs. (87)-(89)] are the elements [1, Eq. (68a)] of $\tilde{X}_n^\theta$ obtained by solving [1, Eq. (65a)] for non-negative values of n with $\tilde{B}_n^\theta$ given by [1, Eq. (101)].

The computer program consists of a main program and the subprograms YZA, BLOG, PLANE, DECOMP, SOLVE, and PRNT. The subroutine YZA calculates the matrix elements [1, Eqs. (35) and (36)] which appear in expression [1, Eq. (33)] for the moment matrix T_n . T_n is needed in [1, Eq. (65a)]. The subroutine YZA calls the function BLOG. The subroutine PLANE calculates $V_{ni}^{t\theta}$, $-V_{ni}^{\phi\theta}$, $-V_{ni}^{t\phi}$, and $V_{ni}^{\phi\phi}$, which, according to [1, Eq. (103)], determine $\tilde{B}_n^\theta$ of [1, Eq. (101)]. The subroutine DECOMP decomposes T_n into the product of a lower triangular matrix with an upper triangular matrix. The subroutine SOLVE uses these triangular matrices to calculate the solution $\tilde{X}_n^\theta$ to [1, Eq. (65a)]. The subroutine PRNT uses the elements of $\tilde{X}_n^\theta$ of [1, Eq. (68a)] to calculate the Fourier coefficients [1, Eqs. (87)-(89)]. The subroutine PRNT also prints out [1, Eqs. (87)-(89)].

The main program calls the subroutines YZA, PLANE, DECOMP, SOLVE, and PRNT in order to calculate and print out [1, Eqs. (87)-(89)]. The main program is rather long because it has to rearrange the Z's and Y's calculated by the subroutine YZA and the V's calculated by the subroutine PLANE. Unfortunately, the Z's and Y's come out of the subroutine YZA

arranged not as in [1, Eqs. (35) and (36)] but as on the right-hand sides of [1, Eqs. (47) and (50)] for consecutive values of i' and j' . Likewise, the V 's exit the subroutine PLANE arranged not as on the left-hand sides of [1, Eq. (103)] but as on the right-hand sides of [1, Eq. (103)] for consecutive values of i' . The main program has to rearrange these Z 's, Y 's, and V 's so as to realize the transition from $\underline{W}_{ni}^{r\pm}$, and $\underline{J}_{nj}^{s\pm}$, to $\underline{W}_{ni}^{p\pm}$ and $\underline{J}_{nj}^{q\pm}$.

II. THE SUBROUTINE YZA

The subroutine YZA(M1, M2, NP, NPHI, NT, IN, RH, ZH, X, A, XT, AT, Y, Z) calculates the elements of matrices Y'_n and Z_n defined by

$$Y'_n = \begin{bmatrix} Y_n^{tt} & Y_n^{t\phi} \\ Y_n^{\phi t} & Y_n^{\phi\phi} \end{bmatrix}, \quad n=M1, M1+1, \dots, M2 \quad (1)$$

$$Z_n = \begin{bmatrix} Z_n^{tt} & Z_n^{t\phi} \\ Z_n^{\phi t} & Z_n^{\phi\phi} \end{bmatrix}, \quad n=M1, M1+1, \dots, M2 \quad (2)$$

where the ij th elements of the submatrices on the right-hand sides of (1) and (2) are given by

$$\left. \begin{aligned} Y_{nij}^{rs} &= - \langle W_{ni}^r, H(J_{nj}^s, 0) \rangle \\ Z_{nij}^{rs} &= - \langle W_{ni}^r, \frac{1}{\eta} E(J_{nj}^s, 0) \rangle \end{aligned} \right\} \begin{aligned} r &= t, \phi \\ s &= t, \phi \end{aligned} \quad (3)$$

$$(4)$$

The quantities on the right-hand sides of (3) and (4) are those on the right-hand sides of [1, Eqs. (50) and (47)] with the $\pm$ notation omitted and with i' and j' replaced by i and j . The $\langle \rangle$ notation in (3) and (4) denotes the same symmetric product as in [1, Eqs. (50) and (47)]. The $\pm$ notation in [1] serves to distinguish the surface $(S^+ + A)$ from the surface $(S^- + A)$. The $\pm$ notation is not needed in (3) and (4) because the subroutine YZA deals with only one surface of revolution at a time. Later on in Section V, the main program calls YZA twice, once for $(S^+ + A)$ and once for $(S^- + A)$. For a particular value of n , Y'_n of (1) is stored by columns in Y and Z_n of (2) is stored by columns in Z . Y'_{n+1} is placed immediately after Y'_n in Y . Similarly, Z_{n+1} follows Z_n in Z . Y and Z are the only output arguments of YZA. The rest of the arguments of YZA are input arguments.

The input arguments of YZA have the same meaning as those of the subroutine YZ presented in [2, pages 65-79]. In comparison with YZA, the subroutine YZ of [2] puts Y_n in Y and Z_n of (2) in Z where

$$Y_n = \begin{bmatrix} Y_n^{tt} & Y_n^{t\phi} \\ Y_n^{\phi t} & Y_n^{\phi\phi} \end{bmatrix} \quad (5)$$

The ij th elements of the submatrices on the right-hand side of (5) are given by

$$Y_{nij}^{rs} = - \langle W_{ni}^r, \underline{n} \times \underline{H}(J_{-nj}^s, 0) \rangle \quad \begin{cases} r = t, \phi \\ s = t, \phi \end{cases} \quad (6)$$

where [2, Eq. (4)]

$$\underline{n} = \underline{u}_\phi \times \underline{u}_t \quad (7)$$

As defined by (7), $\underline{n}$ is a unit vector normal to the surface of revolution.

Although we have been omitting the vector designation from vectors inside the symmetric product, we decided to designate $\underline{n}$ and $\underline{H}$ as vectors in (6) to clearly indicate that the vector product $\underline{n} \times \underline{H}$ is intended there.

YZ of [2] allows for the Ampere's law contribution to the magnetic field $\underline{H}$ in (6). The Ampere's law contribution to the magnetic field is the contribution due to the value of the electric current at the field point. The Ampere's law contribution in the subroutine YZ of [2] is controlled by means of the input argument IN of YZ of [2]. The action of IN is described in [2, page 39] under the assumptions that

- 1) $\underline{n}$ is given by (7)
- 2) $\underline{n}$ points outward from the surface of revolution.

However, the action of IN can be described without recourse to assumption 2) in the following manner.

$$\left. \begin{array}{l} \text{IN} = 1, \text{ Magnetic field evaluation on the side of the} \\ \text{electric current sheet where the } \underline{\text{tail}} \text{ of } \underline{n} \text{ is.} \\ \text{IN} = 0, \text{ No Ampere's law contribution.} \\ \text{IN} = -1, \text{ Magnetic field evaluation on the side of the} \\ \text{electric current sheet where the } \underline{\text{head}} \text{ of } \underline{n} \text{ is.} \end{array} \right\} \quad (8)$$

In (8), $\underline{n}$ is visualized as piercing the sheet of electric current which produces the magnetic field. Apparently, assumption 2) for $\underline{n}$ can be eliminated from [2] by replacing the description [2, page 39] of the action of the input argument IN of YZ of [2] by (8).

Presumably, the magnetic field $\underline{H}$ in (3) contains no Ampere's law contribution because the magnetic field $\underline{H}^+$ in [1, Eq. (50)] had no Ampere's law contribution. However, we decided to allow for the Ampere's

law contribution associated with $\underline{H}$ in (3) in order to enhance the usefulness of YZA. This Ampere's law contribution is controlled by means of the input argument IN of YZA. The action of IN is described by (8) where it is assumed that $\underline{n}$ is given by (7). We do not require $\underline{n}$ to point outward in (8).

Most of the statements in YZA are exactly the same as those in YZ of [2]. In the remaining part of this section, the differences between YZA and YZ of [2] are pointed out and explained and then YZA is listed. Unless stated otherwise, all line numbers and statement numbers cited henceforth in this section refer to the listing of YZA.

In line 4, more space is allotted to Y and Z than in YZ of [2]. Nevertheless, the minimum allocations given in [2, pages 65-66] apply to YZA. Line 9 in YZ of [2] has no counterpart in YZA because UG and UH are not used in YZA. On the other hand, line 17 has no counterpart in YZ of [2]. Line 17 enables YZA to calculate the moment matrix for the E-field solution [3] for the two conducting bodies in [1, Fig. 13]. This E-field solution was obtained by connecting the generating curves ABC and DEF in [1, Fig. 13] to obtain the single curve ABCDEF and then deleting all matrix elements associated with the connecting line CD. Line 17 sets $k\rho = 1$ at the midpoint of the connecting line CD. Here, k is the propagation constant and ρ is the distance from the z axis. If line 17 were absent, this $k\rho$ would be zero and divisions by zero would occur. The value 1 assigned to $k\rho$ in line 17 is not critical because, as stated earlier, all matrix elements associated with the connecting line CD are deleted. Line 17 also enables YZA to calculate the moment matrix for

an H-field solution for the two conducting bodies in [1, Fig. 13].

However, this H-field solution, being a solution to [2, Eq. (1)] without the $\underline{n} \times$, would fail according to the discussion in the third from the last paragraph in [1, Section III]. The H-field solution shown in [1, Fig. 15] was obtained by using the subroutine YZ of [2] modified by setting $k\rho = 1$ at the midpoint of the connecting line CD in [1, Fig. 13].

The major difference between YZ of [2] and YZA lies in the fact that there is an $\underline{n} \times$ in (6) for Y_{nij}^{rs} calculated by YZ of [2] but not in (3) for Y_{nij}^{rs} calculated by YZA. Because $\underline{W}_{ni}^r$ in (3) is a tangential vector, (3) can be written as

$$Y_{nij}^{rs} = - \langle \underline{n} \times \underline{W}_{ni}^r, \underline{n} \times \underline{H}(\underline{J}_{nj}^s, 0) \rangle \quad \begin{cases} r = t, \phi \\ s = t, \phi \end{cases} \quad (9)$$

In view of (7) and the assumptions that $\underline{W}_{ni}^t$ has only a t component and $\underline{W}_{ni}^\phi$ has only a ϕ component, (9) becomes

$$Y_{nij}^{ts} = - \hat{Y}_{nij}^{\phi s} \quad (10a)$$

$$Y_{nij}^{\phi s} = \hat{Y}_{nij}^{ts} \quad (10b)$$

where

$$\hat{Y}_{nij}^{rs} = - \langle \underline{\hat{W}}_{ni}^r, \underline{n} \times \underline{H}(\underline{J}_{nj}^s, 0) \rangle \quad \begin{cases} r = t, \phi \\ s = t, \phi \end{cases} \quad (11)$$

where

$$\underline{\hat{W}}_{ni}^t = \underline{u}_t (\underline{W}_{ni}^\phi \cdot \underline{u}_\phi) \quad (12a)$$

$$\underline{\hat{W}}_{ni}^\phi = \underline{u}_\phi (\underline{W}_{ni}^t \cdot \underline{u}_t) \quad (12b)$$

According to (10)-(12), Y_{nij}^{ts} is the negative of $Y_{nij}^{\phi s}$ modified from (6) and $Y_{nij}^{\phi s}$ is Y_{nij}^{ts} modified from (6). The expression "modified from (6)" means (6) with the t and ϕ components of $\underline{W}_{ni}^r$ interchanged.

Now, the matrix elements (6) are given by [2, Eq. (12)] and it is evident from [2, Eq. (9)] that

$$\underline{W}_{ni}^t \cdot \underline{u}_t = \frac{T_i(t)}{\rho} e^{-jn\phi} \quad (13a)$$

$$\underline{W}_{ni}^\phi \cdot \underline{u}_\phi = \frac{P_i(t)}{\rho_i} e^{-jn\phi} \quad (13b)$$

Hence, (10) becomes

$$Y_{nij}^{tt} = -\hat{Y}_{nij}^{\phi t} \quad (14a)$$

$$Y_{nij}^{\phi t} = \hat{Y}_{nij}^{tt} \quad (14b)$$

$$Y_{nij}^{t\phi} = -\hat{Y}_{nij}^{\phi\phi} \quad (14c)$$

$$Y_{nij}^{\phi\phi} = \hat{Y}_{nij}^{t\phi} \quad (14d)$$

where

$$\hat{Y}_{nij}^{\phi t} = Y_{nij}^{\phi t} \text{ of [2, Eq. (12b)] with } \frac{P_i(t)}{\rho_i} \text{ replaced by } \frac{T_i(t)}{\rho} \quad (15a)$$

$$\hat{Y}_{nij}^{tt} = Y_{nij}^{tt} \text{ of [2, Eq. (12a)] with } \frac{T_i(t)}{\rho} \text{ replaced by } \frac{P_i(t)}{\rho_i} \quad (15b)$$

$$\hat{Y}_{nij}^{\phi\phi} = Y_{nij}^{\phi\phi} \text{ of [2, Eq. (12d)] with } \frac{P_i(t)}{\rho_i} \text{ replaced by } \frac{T_i(t)}{\rho} \quad (15c)$$

$$\hat{Y}_{nij}^{t\phi} = Y_{nij}^{t\phi} \text{ of [2, Eq. (12c)] with } \frac{T_i(t)}{\rho} \text{ replaced by } \frac{P_i(t)}{\rho_i} \quad (15d)$$

Obviously, the interchange of functions in (15) must be accompanied by

appropriate changes in the limits of integration with respect to t in [2, Eq. (12)]. Also, because the magnetic fields in [2, Eq. (12)] as it stands are evaluated on the side of the electric current sheet where the tail of $\underline{n}$ is, the Ampere's law contributions to [2, Eq. (12)] should be multiplied by the input argument IN of YZA. These contributions are the single integrals with respect to t in [2, Eq. (12)].

Consequently, the Ampere's law contributions to (14) are given by

$$\bar{Y}_{nij}^{\phi t} = \frac{\pi(IN)}{\rho_i} \int_{t_i}^{t_{i+1}^-} P_i(t) T_j(t) dt \quad (16a)$$

$$\bar{Y}_{nij}^{t\phi} = - \frac{\pi(IN)}{\rho_j} \int_{t_j}^{t_{j+1}^-} T_i(t) P_j(t) dt \quad (16b)$$

where the horizontal bar on the left-hand sides of (16) denotes Ampere's law contribution. If the q th interval is defined to be (t_q^-, t_{q+1}^-) , then the contributions to (16) due to the integrations over the q th interval are given by

$$\dot{Y}_{nqj}^{\phi t} = \frac{\pi(IN)}{\rho_q} \int_{t_q^-}^{t_{q+1}^-} P_q(t) T_j(t) dt \quad \begin{array}{l} j = q-1, q \\ j \neq 0 \\ j \neq NP - 1 \end{array} \quad (17a)$$

$$\dot{Y}_{niq}^{t\phi} = - \frac{\pi(IN)}{\rho_q} \int_{t_q^-}^{t_{q+1}^-} T_i(t) P_q(t) dt \quad \begin{array}{l} i = q-1, q \\ i \neq 0 \\ i \neq NP - 1 \end{array} \quad (17b)$$

where NP is one of the input arguments of YZA. The dot on the left-hand sides of (17) denotes Ampere's law contribution due to integration over the q th interval. Thanks to the definitions [2, Eqs. (5) and (6)], (17) reduces to

$$\dot{Y}_{nqj}^{\phi t} = \frac{\Delta_q \pi(IN)}{2\rho_q} \quad \begin{cases} j = q-1, q \\ j \neq 0 \\ j \neq NP - 1 \end{cases} \quad (18a)$$

$$\dot{Y}_{niq}^{t\phi} = - \frac{\Delta_q \pi(IN)}{2\rho_q} \quad \begin{cases} i = q-1, q \\ i \neq 0 \\ i \neq NP - 1 \end{cases} \quad (18b)$$

where

$$\Delta_q = t_{q+1}^- - t_q^- \quad (19)$$

Expressions (18) are the Ampere's law contributions to the matrix elements Y_{nij}^{rs} of (14) due to the integrations over the q th interval.

The contributions to (14) referred to in this paragraph and the next paragraph are exclusive of the Ampere's law contributions. If the p qth region is the region for which

$$t_p^- \leq t \leq t_{p+1}^-$$

$$t_q^- \leq t' \leq t_{q+1}^-$$

then the contributions to (14) due to the integrations over the p qth region are given by expressions similar to [2, Eq. (18)]. If the generating curve of the surface of revolution is assumed to be a series of straight line segments connecting the points t_1^- , t_2^- , ..., t_{NP}^- and if the integrations with respect to t are approximated by sampling the integrands at $t = t_p$ and multiplying by Δ_p , then the contributions to (14) due to the integrations over the p qth region reduce to

$$\dot{Y}_{nij}^{*,tt} = -\frac{1}{2} \text{ (right-hand side of [2, Eq. (22b)])} \quad (20a)$$

$$\dot{Y}_{npj}^{*,\phi t} = 2 \text{ (right-hand side of [2, Eq. (22a)])} \quad (20b)$$

$$\dot{Y}_{niq}^{*,t\phi} = -\frac{1}{2} \text{ (right-hand side of [2, Eq. (22d)] without the } \delta_{pq} \text{ term)} \quad (20c)$$

$$\dot{Y}_{npq}^{*,\phi\phi} = 2 \text{ (right-hand side of [2, Eq. (22c)])} \quad (20d)$$

The asterisk on the left-hand sides of (20) denotes contribution due to integration over the pq th region. The ranges of values of i and j in (20) are given by

$$\left. \begin{array}{l} i = p-1, p \\ i \neq 0 \\ i = NP-1 \end{array} \right\} \quad (21a)$$

$$\left. \begin{array}{l} j = q-1, q \\ j \neq 0 \\ j \neq NP-1 \end{array} \right\} \quad (21b)$$

The factors $\frac{1}{2}$ and 2 in (20) are due to sampling the integrands at $t = t_p$. When $t = t_p$, the functions $\frac{P_i(t)}{\rho_i}$ and $\frac{T_i(t)}{\rho}$ which are being interchanged in (15) reduce to

$$\frac{P_i(t_p)}{\rho_i} = \begin{cases} \frac{1}{\rho_p}, & i = p \\ 0 & i \neq p \end{cases} \quad (22)$$

$$\frac{T_i(t_p)}{\rho} = \begin{cases} \frac{1}{2\rho_p}, & i = p-1, p \\ 0, & \text{otherwise} \end{cases} \quad (23)$$

Since [2, Eq. (22)] is equal to [2, Eq. (24)], (20) can be re-written as

$$\bar{Y}_{n,p-1,q-1}^{*,tt} = -\frac{1}{2} (UC - UD) , p \neq 0, q \neq 0 \quad (24a)$$

$$\bar{Y}_{np,q-1}^{*,tt} = -\frac{1}{2} (UC - UD) , p \neq NP-1, q \neq 0 \quad (24b)$$

$$\bar{Y}_{n,p-1,q}^{*,tt} = -\frac{1}{2} (UC + UD) , p \neq 0, q \neq NP-1 \quad (24c)$$

$$\bar{Y}_{npq}^{*,tt} = -\frac{1}{2} (UC + UD) , p \neq NP-1, q \neq NP-1 \quad (24d)$$

$$\bar{Y}_{np,q-1}^{*,\phi t} = 2 (UA) , q \neq 0 \quad (24e)$$

$$\bar{Y}_{npq}^{*,\phi t} = 2 (UB) , q \neq NP-1 \quad (24f)$$

$$\bar{Y}_{n,p-1,q}^{*,t\phi} = -\frac{1}{2} (UF) , p \neq 0 \quad (24g)$$

$$\bar{Y}_{npq}^{*,t\phi} = -\frac{1}{2} (UF) , p \neq NP-1 \quad (24h)$$

$$\bar{Y}_{npq}^{*,\phi\phi} = 2 (UE) \quad (24i)$$

where

$$UA = \text{right-hand side of [2, Eq. (24a)] for } j = q-1 \quad (25a)$$

$$UB = \text{right-hand side of [2, Eq. (24a)] for } j = q \quad (25b)$$

$$UC-UD = \text{right-hand side of [2, Eq. (24b)] for } j = q-1 \quad (25c)$$

$$UC+UD = \text{right-hand side of [2, Eq. (24b)] for } j = q \quad (25d)$$

$$UE = \text{right-hand side of [2, Eq. (24c)]} \quad (25e)$$

$$UF = \text{right-hand side of [2, Eq. (24d)] without the } \delta_{pq} \text{ term} \quad (25f)$$

The notation UA, UB,...UF on the left-hand sides of (25) is the same as the notation used in YZ of [2]. Expressions (24) are the contributions to the matrix elements Y_{nij}^{rs} of (14) due to the integrations over the pqth region. No Ampere's law contributions are included in (24). The Ampere's law contributions are given by (18).

The Ampere's law contributions in (18) are different from those in [2] which consist of [2, Eq. (23)] and the δ_{pq} term in [2, Eq. (24d)]. Hence, the statements which realize the Ampere's law contributions in YZA are different from those in YZ of [2]. Lines 50-51, 74-75, 86-88, 93-99, and 424-429 of YZ of [2] obtain the Ampere's law contributions. Line 50 of YZA puts $\pi(IN)$ of (18) in PN1. Line 73 puts the right-hand side of (18a) in P1.

The index JQ of DO loop 15 obtains the subscript q which appears in (18a) and (24). The index IP of DO loop 16 obtains the subscript p which appears in (24). The purpose of the statement IF(IP.NE.JQ) in lines 121, 270, and 424 of YZ of [2] is to check if the IPth interval coincides with the JQth interval. If the IPth and JQth intervals coincide, then the numerical evaluation of the integrals $G_{m\alpha}$ in [2, Section III] is affected and Ampere's law contributions are taken into account. However, YZ of [2] was not designed to accommodate a generating curve which closes upon itself.

The subroutine YZA allows for a generating curve which closes upon itself. Such a curve is treated in the following manner. A triangle function whose peak is at the first data point is needed and is obtained by overlapping the last interval of the generating curve with the first

interval as in [1, Fig. 5]. This triangle function is called $T_{NP-2}(t)$. In addition to $T_{NP-2}(t)$, the process of overlapping obtains the pulse function $P_{NP-1}(t)$. $P_{NP-1}(t)$ is not wanted because it is identical to $P_1(t)$. The effect of $P_{NP-1}(t)$ can be eliminated by discarding all matrix elements (3) and (4) for which either $\frac{W}{n_i}$ or $\frac{J}{n_j}$ contain $P_{NP-1}(t)$. These elements must be discarded after exit from YZA, because YZA does not contain any logic for discarding matrix elements.

Unfortunately, (18) was derived for a generating curve which does not overlap on itself. If the last interval of the generating curve overlaps the first interval, the quantities on the right-hand side of (18) are still correct but more values of i and j are needed in (18) when $q=1$ and $q=NP-1$ to allow for the overlapping so as to account for all Ampere's law contributions to the matrix elements (3). It will be shown that all Ampere's law contributions can be accounted for by defining ZIP to be the electrical distance from the center of the JQth interval to the center of the IPth interval and using the statement IF(ZIP.NE.O.) instead of the statement IF(IP.NE.JQ). The statement IF(ZIP.NE.O.) appears in lines 110, 259, and 408. If the generating curve does not overlap on itself, the action of the statement IF(ZIP.NE.O.) is the same as the action of the statement IF(IP.NE.JQ) and all Ampere's law contributions are accounted for. However, if the last interval of the generating curve overlaps the first interval, then it remains to be shown that (ZIP = 0.) obtains all Ampere's law contributions.

If the last interval of the generating curve overlaps the first interval, then ZIP is zero not only for

$$IP = JQ = m, \quad m=1,2,\dots, NP-1 \quad \text{Case 1}$$

but also for

$$\left. \begin{array}{l} IP = 1 \\ JQ = NP-1 \end{array} \right\} \quad \text{Case 2}$$

and for

$$\left. \begin{array}{l} IP = NP-1 \\ JQ = 1 \end{array} \right\} \quad \text{Case 3}$$

In what is to follow, the matrix elements Y_{nij}^{rs} of (3) are viewed as interactions between testing functions and expansion functions. The combination of Cases 1, 2, and 3 will give all Ampere's law contributions if it covers all possible interactions between parts of testing functions on the m th interval and parts of expansion functions on the m th interval for $m = 1, 2, \dots, NP-2$.

With regard to the Ampere's law contributions (18), the last statement is more general than necessary because not all interactions between testing functions and expansion functions are involved in (18). For example, the interactions Y_{nij}^{tt} between t directed testing functions and t directed expansion functions are not involved in (18) and neither are the interactions $Y_{nij}^{\phi\phi}$ between ϕ directed testing functions and ϕ directed expansion functions. However, the matrix elements Y_{nij}^{tt} and $Y_{nij}^{\phi\phi}$ of (6) calculated by YZ of [2] do have Ampere's law contributions. It would be useful to know that YZ of [2] can be modified to allow for a closed generating curve by using the same technique as in YZA. This technique consists of overlapping the last interval of the generating curve with the first, replacing the statement $IF(IP.NE.JQ)$ by the statement $IF(ZIP.NE.O.)$,

and deleting the matrix elements associated with $P_{NP-1}(t)$ after exit from the subroutine.

It is evident that the portion of Case 1 for which

$$IP = JQ = m, m = 2, \dots, NP-2$$

covers all possible interactions between parts of testing functions on the m th interval and expansion functions on the m th interval for $m = 2, 3, \dots, NP-2$. However, clarification is needed with regard to the first interval because, due to the overlapping, the first interval is sometimes disguised as the $(NP-1)$ th interval.

All possible interactions between parts of testing functions on the first interval and parts of expansion functions on the first interval are listed as

- 1) $T_{NP-2}^b(t)$ with $T_{NP-2}^b(t)$
- 2) $T_1^a(t)$ with $T_{NP-2}^b(t)$
- 3) $P_1(t)$ with $T_{NP-2}^b(t)$
- 4) $T_{NP-2}^b(t)$ with $T_1^a(t)$
- 5) $T_1^a(t)$ with $T_1^a(t)$
- 6) $P_1(t)$ with $T_1^a(t)$
- 7) $T_{NP-2}^b(t)$ with $P_1(t)$
- 8) $T_1^a(t)$ with $P_1(t)$
- 9) $P_1(t)$ with $P_1(t)$

In each of the foregoing 9 interactions, the first function is the part of the testing function and the second function is the part of the expansion function. $T_{NP-2}^b(t)$ is the second part of the $(NP-2)$ th triangle

function, the downward sloping part. $T_1^a(t)$ is the first part of the first triangle function, the upward sloping part.

The interactions covered by Cases 2 and 3 and the parts of Case 1 for which $IP = JQ = 1$ and $IP = JQ = NP-1$ are listed as

- | | | |
|---|--|----------------|
| 2) $T_1^a(t)$ with $T_{NP-2}^b(t)$ | $\left. \begin{array}{l} IP = 1 \\ JQ = NP-1 \end{array} \right\}$ | Case 2 |
| 3) $P_1(t)$ with $T_{NP-2}^b(t)$ | | |
| 10) $T_1^a(t)$ with $P_{NP-1}(t)$ | | |
| 11) $P_1(t)$ with $P_{NP-1}(t)$ | | |
| 4) $T_{NP-2}^b(t)$ with $T_1^a(t)$ | $\left. \begin{array}{l} IP = NP-1 \\ JQ = 1 \end{array} \right\}$ | Case 3 |
| 12) $P_{NP-1}(t)$ with $T_1^a(t)$ | | |
| 7) $T_{NP-2}^b(t)$ with $P_1(t)$ | | |
| 13) $P_{NP-1}(t)$ with $P_1(t)$ | | |
| 5) $T_1^a(t)$ with $T_1^a(t)$ | $\left. \begin{array}{l} IP = JQ = 1 \end{array} \right\}$ | Part of Case 1 |
| 6) $P_1(t)$ with $T_1^a(t)$ | | |
| 8) $T_1^a(t)$ with $P_1(t)$ | | |
| 9) $P_1(t)$ with $P_1(t)$ | | |
| 1) $T_{NP-2}^b(t)$ with $T_{NP-2}^b(t)$ | $\left. \begin{array}{l} IP = JQ = NP-1 \end{array} \right\}$ | Part of Case 1 |
| 14) $P_{NP-1}(t)$ with $T_{NP-2}^b(t)$ | | |
| 15) $T_{NP-2}^b(t)$ with $P_{NP-1}(t)$ | | |
| 16) $P_{NP-1}(t)$ with $P_{NP-1}(t)$ | | |

The preceding 16 interactions were numbered so as to facilitate comparison with the 9 interactions in the last paragraph.

Of the 16 interactions in the last paragraph, numbers 1 to 9 are the interactions in the second from the last paragraph and numbers 10 to 16 are to be discarded because they contain $P_{NP-1}(t)$. Drawing an equivalence between the interactions in the last paragraph and the interactions in the second from the last paragraph, we conclude that the combination of Cases 1, 2, and 3 covers all possible interactions between parts of testing functions on the m th interval and parts of expansion functions on the m th interval for $m = 1, 2, \dots, NP-2$. Hence, $(ZIP = 0.)$ obtains all Ampere's law contributions when the last interval on the generating curve overlaps the first interval. If such overlapping is intended, then the input arguments RH and ZH of YZA must satisfy

$$\left. \begin{aligned} RH(NP-1) &= RH(1) \\ RH(NP) &= RH(2) \\ ZH(NP-1) &= ZH(1) \\ ZH(NP) &= ZH(2) \end{aligned} \right\} \quad (26)$$

exactly. Otherwise, the computed values of ZIP will not be zero in Cases 2 and 3.

With the intention of showing that YZA implements (24), we list in Table 1 some variables in YZA whose values are different from those in YZ of [2].

Table 1. Comparison of some variables in YZA with those in YZ of [2].

Line in YZA	Variable in YZA	Expression in terms of variables in YZ of [2]
379	W1	2.*W1
380	W2	2.*W2
385	H1C	-.5*H1C
387	H3C	2.*H3C
388	H2C	2.*H2C
389	H3C	2.*H3C
390	W3	-.5*W3
391	W4	-.5*W4
392	W5	-.5*W5
405	UC	2.*UC
406	UB	2.*UB
407	UF	-.5*UF
411	UA	2.*UA
412	UB	2.*UB
415	UC	-.5*(UC-UD)
416	UD	-.5*(UC+UD)
417	UE	2.*UE

In Table 1, the variable in the second column is defined by the statement in YZA whose line number is given in the first column. The expression in the third column is the value of the variable in the second column. The values of the variables appearing in the third column are the values which exist in YZ just after the counterpart to the variable in the second column has been defined in YZ. For example, the value of W1 in the first line of the third column is defined in line 391 of YZ of [2]. As an exception, the

value of UD intended in the third from the last line of the third column is that defined by line 433 of YZ of [2]. Table 1 was constructed under the assumption that no Ampere's law contributions come into play. This amounts to assuming that lines 425-429 of YZ of [2] are not executed and that lines 409-410 of YZA are not executed. The Ampere's law contributions (18) will be considered later.

The last 6 variables in the second column of Table 1 are used to construct the matrix elements Y_{nij}^{rs} of (3). The statements in YZA which perform this task are listed in Table 2.

Table 2. Construction of matrix elements Y_{nij}^{rs}

Line in YZA	Statement in YZA	Matrix element
461,476	$Y(K1) = Y(K1) + UC$	$Y_{n,p-1,q-1}^{tt}$
466,483	$Y(K2) = Y(K2) + UC$	$Y_{n,p,q-1}^{tt}$
448,477	$Y(K3) = Y(K3) + UD$	$Y_{n,p-1,q}^{tt}$
453,484	$Y(K4) = UD$	Y_{npq}^{tt}
458,471	$Y(K5) = Y(K5) + UA$	$Y_{np,q-1}^{\phi t}$
445,472	$Y(K6) = UB$	$Y_{npq}^{\phi t}$
449,462,478	$Y(K7) = Y(K7) + UF$	$Y_{n,p-1,q}^{t\phi}$
454,467,485	$Y(K8) = UF$	$Y_{npq}^{t\phi}$
489	$Y(K9) = UE$	$Y_{npq}^{\phi\phi}$

In Table 2, the variable being modified by the statement in the second column represents the matrix element in the third column. For example,

$Y(K1)$ which appears in the first line of the second column of Table 2 is the storage location of the matrix element $Y_{n,p-1,q-1}^{tt}$. The subscripts n, p , and q appearing in the third column of Table 2 are given in terms of variables in YZA by

$$n = M + M1 - 1$$

$$p = IP$$

$$q = JQ$$

Here, M is the index of DO loop 31, $M1$ is one of the input arguments of YZA, IP is the index of DO loop 16, and JQ is the index of DO loop 15. In view of the fact that the variables UA, UB, UC, UD, UE , and UF appearing in the second column of Table 2 are given by the last 6 entries of the third column of Table 1, it is now evident that YZA implements (24).

From (18), the correct values of the Ampere's law contributions are $\pm(\Delta_q \pi(IN))/(2\rho_q)$. In the paragraph which follows (25) we established that line 73 stores $(\Delta_q \pi(IN))/(2\rho_q)$ in $P1$. In the paragraph prior to the introduction of Table 1, we concluded that the Ampere's law contributions come into play whenever $ZIP = 0$. Accordingly, lines 409-410 are executed when $ZIP = 0$. The effect of line 409 is to add $P1$ to the values of UA and UB calculated by lines 411 and 412. Line 410 subtracts $P1$ from the value of UF calculated by line 407. The previously mentioned variables UA, UB , and UF appear in the second column of Table 1 and were transferred to the second column of Table 2. It is now evident from Table 2 that execution of lines 409-410 adds $P1$ to $Y_{np,q-1}^{\phi t}$ and $Y_{npq}^{\phi t}$ and subtracts $P1$ from $Y_{n,p-1,q}^{t\phi}$ and $Y_{npq}^{t\phi}$. Hence, YZ takes proper account of the Ampere's law contributions.

```

001C LISTING OF THE SUBROUTINE YZA
002C THE SUBROUTINE YZA CALLS THE FUNCTION BLOG
003 SUBROUTINE YZA(M1,M2,NP,NPHI,NT,IN,RH,ZH,X,A,XT,AT,Y,Z)
004 COMPLEX Y(2209),Z(2209),U,U1,U2,U3,U4,U5,U6,H1A,H2A,H3A,GA(48)
005 COMPLEX GB(48),GC(48),GD(48),GE(48),H1B,H2B,H3B,H1C,H2C,H3C,H4A
006 COMPLEX H5A,H6A,H4B,H5B,H6B,UA,UB,UC,UD,UE,UF,G1A(10),G2A(10)
007 COMPLEX G3A(10),G1B(10),G2B(10),G3B(10),G1C(10),G2C(10),G3C(10)
008 COMPLEX G4A(10),G5A(10),G6A(10),G4B(10),G5B(10),G6B(10),CMPLX
009 DIMENSION RH(43),ZH(43),X(48),A(48),XT(10),AT(10),RS(42),ZS(42)
010 DIMENSION D(42),DR(42),DZ(42),DN(42),C1(48),C2(48),C3(48),C4(200)
011 DIMENSION C5(200),C6(200),R2(10),Z2(10),R7(10),Z7(10)
012 CT=2.
013 CP=.1
014 DO 10 I=2,NP
015 I2=I-1
016 RS(I2)=.5*(RH(I)+RH(I2))
017 IF(RS(I2).EQ.0.) RS(I2)=1.
018 ZS(I2)=.5*(ZH(I)+ZH(I2))
019 D1=.5*(RH(I)-RH(I2))
020 D2=.5*(ZH(I)-ZH(I2))
021 D(I2)=SQRT(D1*D1+D2*D2)
022 DR(I2)=D1
023 DZ(I2)=D2
024 DN(I2)=D(I2)/RS(I2)
025 10 CONTINUE
026 M3=M2-M1+1
027 M4=M1-1
028 P12=1.570796
029 PP=9.869604
030 DO 11 K=1,NPHI
031 PH=PI2*(X(K)+1.)
032 C1(K)=PH
033 C2(K)=PH*PH
034 SN= SIN(.5*PH)
035 C3(K)=4.*SN*SN
036 A1=P12*A(K)
037 D4=.5*A1*C3(K)
038 D5=A1*COS(PH)
039 D6=A1*SIN(PH)
040 M5=K
041 DO 29 M=M1,M3
042 PHM=(M4+M)*PH
043 A2=COS(PHM)
044 C4(M5)=D4*A2
045 C5(M5)=D5*A2
046 C6(M5)=D6*SIN(PHM)
047 M5=M5+NPHI
048 29 CONTINUE
049 11 CONTINUE
050 PNI=3.141593*IN
051 U=(0.,1.)
052 U1=.5*U
053 U2=2.*U
054 NP=NP-1
055 NT=NP-1
056 N=NT+NP
057 N2N=NT*N
058 N2=N*N
059 JN=-1-N
060 DO 15 JQ=1,NP

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061      KQ=2
062      IF(JQ.EQ.1) KQ=1
063      IF(JQ.EQ.MP) KQ=3
064      R1=RS(JQ)
065      Z1=ZS(JQ)
066      D1=D(JQ)
067      D2=DR(JQ)
068      D3=DZ(JQ)
069      D4=D2/R1
070      D5=D1/R1
071      SV=D2/D1
072      CV=D3/D1
073      P1=PN1*D5
074      P3=2.*D1
075      P4=2.*D4
076      P5=D4*D4
077      P6=D1*D1
078      P7=P6*D1
079      T6=CT*D1
080      T62=T6+D1
081      T62=T62*T62
082      R6=CP*R1
083      R62=R6*R6
084      DO 12 L=1,NT
085      D6=XT(L)
086      R2(L)=R1+D2*D6
087      Z2(L)=Z1+D3*D6
088 12 CONTINUE
089      DO 16 IP=1,MP
090      R3=RS(IP)
091      Z3=ZS(IP)
092      R4=R1-R3
093      Z4=Z1-Z3
094      U3=D2*U1
095      U4=D3*U1
096      DO 40 L=1,NT
097      D7=R2(L)-R3
098      D8=Z2(L)-Z3
099      R7(L)=R3+R2(L)
100      Z7(L)=D7*D7+D8*D8
101 40 CONTINUE
102      PH=R4*SV+Z4*CV
103      A1=ABS(PH)
104      A2=ABS(R4*CV-Z4*SV)
105      D6=A2
106      IF(A1.LE.D1) GO TO 26
107      D6=A1-D1
108      D6=SQRT(D6*D6+A2*A2)
109 26 Z(P=R4+R4+Z4*Z4
110      IF(ZIP.NE.0..AND.(R6.GT.D6.OR.T6.LE.D6)) GO TO 41
111      Z5=ZIP
112      R5=R3+R1
113      PHM=.5*R3*SV
114      DO 33 K=1,NPHI
115      A1=C3(K)
116      RR=Z5+R5*A1
117      H1A=0.
118      H2A=0.
119      H3A=0.
120      H4A=0.

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121      H5A=0.
122      IF(RR.LT.T62) GO TO 34
123      DO 35 L=1,NT
124      W=Z7(L)+R7(L)*A1
125      R=SQRT(W)
126      SN=-SIN(R)
127      CS=COS(R)
128      D6=AT(L)/R
129      H1B=D6/W*CMPLX(CS-R*SN,SN+R*CS)
130      H1A=H1B+H1A
131      H2B=XT(L)*H1B
132      H2A=H2B+H2A
133      H3A=XT(L)*H2B+H3A
134      H4B=D6*CMPLX(CS,SN)
135      H4A=H4B+H4A
136      H5A=XT(L)*H4B+H5A
137 35 CONTINUE
138      GO TO 36
139 34 DO 37 L=1,NT
140      W=Z7(L)+R7(L)*A1
141      R=SQRT(W)
142      IF(R.GT..5) GO TO 14
143      CS=R*(W*(.6944444E-2-W*.1736111E-3)-.125)
144      SN=W*(.3333333E-1-W*.1190476E-2)-.3333333
145      H1B=AT(L)*CMPLX(CS,SN)
146      CS=R*(W*(.4166667E-1-.1388889E-2*W)-.5)
147      SN=W*(W*(.1984126E-3*W-.8333333E-2)+.1666667)-1.
148      H4B=AT(L)*CMPLX(CS,SN)
149      GO TO 43
150 14 SN=-SIN(R)
151      CS=COS(R)
152      D6=AT(L)/R
153      H1B=D6*((CMPLX(CS-R*SN,SN+R*CS)-1.)/W-.5)
154      H4B=D6*CMPLX(CS-1.,SN)
155 43 H1A=H1B+H1A
156      H2B=XT(L)*H1B
157      H2A=H2B+H2A
158      H3A=XT(L)*H2B+H3A
159      H4A=H4B+H4A
160      H5A=XT(L)*H4B+H5A
161 37 CONTINUE
162      A1=PH+PHM*A1
163      A2=ABS(A1)
164      R=RR-A2+A2
165      D6=A2-D1
166      D7=A2+D1
167      D62=D6*D6
168      D72=D7*D7
169      D8=SQRT(D62+R)
170      D9=SQRT(D72+R)
171      IF(R-(RR*1.E-5)) 52,52,53
172 52 IF(D6.LT.0.) STOP
173      W4=.5/D62-.5/D72
174      GO TO 54
175 53 W4=(D7/D9-D6/D8)/R
176 54 IF(D6.GE.0.) GO TO 38
177      W=ALOG((D7+D9)*(-D6+D8)/R)
178      GO TO 39
179 38 W=ALOG((D7+D9)/(D6+D8))
180 39 W1=(W4+.5*W)/D1

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181      W5=A2/D1
182      W2=(.5*(D9-D8)-1./D9+1./D8)/P6-W5*W1
183      W3=(.25*(D7+D9-D6+D8)+W-R*(W4+.25*W1))/P7-W5*(2.*W2+W5*W1)
184      W4=W/D1
185      W5=(D9-D8-A2*W)/P6
186      IF(A1.GE.0.) GO TO 27
187      W2=-W2
188      W5=-W5
189      27 H1A=W1+H1A
190      H2A=W2+H2A
191      H3A=W3+H3A
192      H4A=W4+H4A
193      H5A=W5+H5A
194      36 GA(K)=H1A
195      GB(K)=H2A
196      GC(K)=H3A
197      GD(K)=H4A
198      GE(K)=H5A
199      33 CONTINUE
200      K1=0
201      DO 45 M=1,M3
202      H1A=0.
203      H2A=0.
204      H3A=0.
205      H1B=0.
206      H2B=0.
207      H3B=0.
208      H1C=0.
209      H2C=0.
210      H3C=0.
211      H4A=0.
212      H5A=0.
213      H6A=0.
214      H4B=0.
215      H5B=0.
216      H6B=0.
217      DO 46 K=1,NPHE
218      K1=K1+1
219      D6=C4(K1)
220      D7=C5(K1)
221      D8=C6(K1)
222      UA=GA(K)
223      UB=GB(K)
224      UC=GC(K)
225      UD=GD(K)
226      UE=GE(K)
227      H1A=D6*UA+H1A
228      H2A=D7*UA+H2A
229      H3A=D8*UA+H3A
230      H1B=D6*UB+H1B
231      H2B=D7*UB+H2B
232      H3B=D8*UB+H3B
233      H1C=D6*UC+H1C
234      H2C=D7*UC+H2C
235      H3C=D8*UC+H3C
236      H4A=D6*UD+H4A
237      H5A=D7*UD+H5A
238      H6A=D8*UD+H6A
239      H4B=D6*UE+H4B
240      H5B=D7*UE+H5B

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241      H6B=D8*UE+H6B
242  46 CONTINUE
243      G1A(M)=H1A
244      G2A(M)=H2A
245      G3A(M)=H3A
246      G1B(M)=H1B
247      G2B(M)=H2B
248      G3B(M)=H3B
249      G1C(M)=H1C
250      G2C(M)=H2C
251      G3C(M)=H3C
252      G4A(M)=H4A
253      G5A(M)=H5A
254      G6A(M)=H6A
255      G4B(M)=H4B
256      G5B(M)=H5B
257      G6B(M)=H6B
258  45 CONTINUE
259      IF(ZIP-NE.0.) GO TO 47
260      A1=D5*D5
261      D8=0.
262      D9=0.
263      DO 63 K=1,NPHI
264      D8=D8+A(K)/SQRT(C2(K)+A1)
265      D9=D9+A(K)*BLOG(D5/C1(K))
266  63 CONTINUE
267      A2=3.141593/D5
268      D8=(BLOG(A2)-PI2*D8)/(R5+R1)
269      D9=2./R1*(BLOG(A2)+A2*BLOG(1./A2))-3.141593/D1*D9
270      DO 67 M=1,M3
271      G1A(M)=D8+G1A(M)
272      G2A(M)=0.
273      G2B(M)=0.
274      G2C(M)=0.
275      G3A(M)=0.
276      G5A(M)=D9+G5A(M)
277  67 CONTINUE
278      GO TO 47
279  41 DO 25 M=1,M3
280      G1A(M)=0.
281      G2A(M)=0.
282      G3A(M)=0.
283      G1B(M)=0.
284      G2B(M)=0.
285      G3B(M)=0.
286      G1C(M)=0.
287      G2C(M)=0.
288      G3C(M)=0.
289      G4A(M)=0.
290      G5A(M)=0.
291      G6A(M)=0.
292      G4B(M)=0.
293      G5B(M)=0.
294      G6B(M)=0.
295  25 CONTINUE
296      DO 13 L=1,NT
297      R5=R7(L)
298      Z5=Z7(L)
299      DO 17 K=1,NPHI
300      W=Z5+R5+C3(K)

```

```

301      R=SQRT(W)
302      SN=-SIN(R)
303      CS=COS(R)
304      GA(K)=CMPLX(CS-R*SN,SN+R*CS)/(W*R)
305      GD(K)=CMPLX(CS,SN)/R
306  17 CONTINUE
307      IF(R62.LE.Z5) GO TO 51
308      D6=0.
309      D7=0.
310      D9=0.
311      DO 62 K=1,NPHI
312      W2=C2(K)
313      W=1./(Z5+R5*W2)
314      W1=A(K)+SQRT(W)
315      D6=D6+W1+W2*W
316      D7=D7+W1*(.5+W*(1.+0.125*W*R5+W2*W2))
317      D9=D9+W1
318  62 CONTINUE
319      W1=R5/Z5
320      W2=PP*W1
321      W=SQRT(W2)
322      W3=1.+W2
323      R=SQRT(W3)
324      W4=SQRT(R5)
325      W5=ALOG(W+R)
326      D8=-PI2*D6-(W/R-W5)/(R5*W4)
327      D6=.5*D8
328      D7=((W/R*(W1-(0.125+.1666667*W2)/W3)+.125*W5)/R5+.5*W5)/W4-PI2*D7
329      D9=W5/W4-PI2*D9
330  51 A1=AT(L)
331      A2=XT(L)*A1
332      A3=XT(L)*A2
333      K1=0
334      DO 30 M=1,M3
335      W=M+M4
336      H1A=0.
337      H2A=0.
338      H3A=0.
339      H4A=0.
340      H5A=0.
341      H6A=0.
342      DO 32 K=1,NPHI
343      K1=K1+1
344      H1B=GA(K)
345      W4=C4(K1)
346      W5=C5(K1)
347      W6=C6(K1)
348      H1A=W4*H1B+H1A
349      H2A=W5*H1B+H2A
350      H3A=W6*H1B+H3A
351      H1B=GD(K)
352      H4A=W4*H1B+H4A
353      H5A=W5*H1B+H5A
354      H6A=W6*H1B+H6A
355  32 CONTINUE
356      IF(R62.LE.Z5) GO TO 44
357      H1A=D6+H1A
358      H2A=D7-(W*W+1.)*D6+H2A
359      H3A=W*D8+H3A
360      H5A=D9+H5A

```

```

361 44 G1A(M)=A1*H1A+G1A(M)
362 G2A(M)=A1*H2A+G2A(M)
363 G3A(M)=A1*H3A+G3A(M)
364 G1B(M)=A2*H1A+G1B(M)
365 G2B(M)=A2*H2A+G2B(M)
366 G3B(M)=A2*H3A+G3B(M)
367 G1C(M)=A3*H1A+G1C(M)
368 G2C(M)=A3*H2A+G2C(M)
369 G3C(M)=A3*H3A+G3C(M)
370 G4A(M)=A1*H4A+G4A(M)
371 G5A(M)=A1*H5A+G5A(M)
372 G6A(M)=A1*H6A+G6A(M)
373 G4B(M)=A2*H4A+G4B(M)
374 G5B(M)=A2*H5A+G5B(M)
375 G6B(M)=A2*H6A+G6B(M)
376 30 CONTINUE
377 13 CONTINUE
378 47 A2=D(IP)
379 W1=A2*(R4*D3-Z4*D2)
380 W2=-A2*R3*D3
381 A3=DZ(IP)
382 D6=DR(IP)
383 D7=Z4*D6
384 D9=D3*D6
385 H1C=(D2*(R3*A3+D7)-R1*D9)*U1
386 D8=A2*D1
387 H3C=D8*U2
388 H2C=Z4*H3C
389 H3C=D3*H3C
390 W3=D1*(D7-R4*A3)
391 W4=D1*(D9-D2*A3)
392 W5=-D1*R1*A3
393 A1=DR(IP)
394 U5=A1*U3
395 U6=A3*U4
396 D6=-D2*A2
397 D7=D1*A1
398 A3=DM(IP)
399 JM=JN
400 DO 31 N=1,M3
401 H2A=G2A(M)
402 H1A=G1A(M)
403 H2B=G2B(M)
404 H1B=G1B(M)
405 UC=W1*H2A+W2*H1A
406 UB=W1*H2B+W2*H1B
407 UF=W3*(H2A+D4*H2B)+W4*(H2B+D4*G2C(M))+W5*(H1A+P4*H1B+P5*G1C(M))
408 IF(ZIP-NE-0.) GO TO 48
409 UC=UC+P1
410 UF=UF-P1
411 48 UA=UC-UB
412 UB=UC+UB
413 H3A=G3A(M)
414 H3B=G3B(M)
415 UC=H1C*(H3A-H3B)
416 UD=H1C*(H3A+H3B)
417 UE=H2C*(H3A+D4*H3B)+H3C*(H3B+D4*G3C(M))
418 H5A=G5A(M)
419 H5B=G5B(M)
420 H4A=G4A(M)+H5A

```

```

421      H4B=G4B(M)+H5B
422      H6A=G6A(M)
423      H6B=G6B(M)
424      H3A=U5*H5A+U6*H4A
425      H1B=U5*H5B+U6*H4B
426      H1A=H3A-H1B
427      H2A=H3A+H1B
428      H3A=-U1*H4A
429      H1B=D6*H6A
430      B=M+H4
431      A1=W*H3
432      H2B=D6*H6B-A1*H4A
433      H3B=D7*(H6A+D4*H6B)
434      H4A=W*H5*H4A
435      K1=IP+JM
436      K2=K1+1
437      K3=K1+N
438      K4=K2+N
439      K5=K2+MT
440      K6=K4+MT
441      K7=K3+N2N
442      K8=K4+N2N
443      K9=K8+MT
444      GO TO (18,20,19),K0
445 18 Y(K6)=UB
446      Z(K6)=H1B+H2B
447      IF(IP.EQ.1) GO TO 21
448      Y(K3)=Y(K3)+UD
449      Y(K7)=Y(K7)+UF
450      Z(K3)=Z(K3)+H2A-H3A
451      Z(K7)=Z(K7)+H3B-H4A
452      IF(IP.EQ.MP) GO TO 22
453 21 Y(K4)=UD
454      Y(K8)=UF
455      Z(K4)=H2A+H3A
456      Z(K8)=H3B+H4A
457      GO TO 22
458 19 Y(K5)=Y(K5)+UA
459      Z(K5)=Z(K5)+H1B-H2B
460      IF(IP.EQ.1) GO TO 23
461      Y(K1)=Y(K1)+UC
462      Y(K7)=Y(K7)+UF
463      Z(K1)=Z(K1)+H1A+H3A
464      Z(K7)=Z(K7)+H3B-H4A
465      IF(IP.EQ.MP) GO TO 22
466 23 Y(K2)=Y(K2)+UC
467      Y(K8)=UF
468      Z(K2)=Z(K2)+H1A-H3A
469      Z(K8)=H3B+H4A
470      GO TO 22
471 20 Y(K5)=Y(K5)+UA
472      Y(K6)=UB
473      Z(K5)=Z(K5)+H1B-H2B
474      Z(K6)=H1B+H2B
475      IF(IP.EQ.1) GO TO 24
476      Y(K1)=Y(K1)+UC
477      Y(K3)=Y(K3)+UD
478      Y(K7)=Y(K7)+UF
479      Z(K1)=Z(K1)+H1A+H3A
480      Z(K3)=Z(K3)+H2A-H3A
481      Z(K7)=Z(K7)+H3B-H4A
482      IF(IP.EQ.MP) GO TO 22
483 24 Y(K2)=Y(K2)+UC
484      Y(K4)=UD
485      Y(K8)=UF
486      Z(K2)=Z(K2)+H1A-H3A
487      Z(K4)=H2A+H3A
488      Z(K8)=H3B+H4A
489 22 Y(K9)=UE
490      Z(K9)=U2*(D8*(H5A+D4*H5B)-A1*H4A)
491      JM=JM+N2
492 31 CONTINUE
493 16 CONTINUE
494      JN=JN+N
495 15 CONTINUE
496      RETURN
497      END

```

III. THE FUNCTION BLOG AND THE SUBROUTINES PLANE, DECOMP, AND SOLVE

The function BLOG is exactly the same as in [3, page 56]. The only difference between the subroutine PLANE (of the present report) and the subroutine PLANE of [3, pages 57-62] is that the statement

IF(R1.EQ.0.) R1 = .5

in line 39 of PLANE has no counterpart in the subroutine PLANE of [3, pages 57-62]. The action of this statement is similar to that of the statement in line 17 of the subroutine YZA. Except for a difference in the actual space allocated to the variables UL, SCL, and IPS, the subroutine DECOMP is the same as the subroutine DECOMP of [3, pages 63-64]. Minimum allocations in DECOMP are the same as those in the subroutine DECOMP of [3, pages 63-64]. The subroutine SOLVE differs from the subroutine SOLVE of [3, pages 63-64] only in the actual space allocated to the variables UL, B, X, and IPS. Minimum allocations in SOLVE are the same as those in the subroutine SOLVE of [3, pages 63-64].

```

001C    LISTING OF THE FUNCTION BLOG
002    FUNCTION BLOG(X)
003    IF(X.GT..1) GO TO 1
004    X2=X*X
005    BLOG=((-.075*X2-1.666667)*X2+1.)*X
006    RETURN
007    1 BLOG=ALOG(X+SQRT(1.+X*X))
008    RETURN
009    END
010    LISTING OF THE SUBROUTINE PLANE
011    SUBROUTINE PLANE(M1,M2,NF,NP,NT,RH,ZH,XT,AT,THR,R)
012    COMPLEX R(240),U,U1,UA,UB,FA(10),FB(10),F2A,F2B,F1A,F1B,U2,U3,U4
013    COMPLEX US,CNPLX
014    DIMENSION RH(43),ZH(43),XT(10),AT(10),THR(3),CS(3),SN(3),R2(10)
015    DIMENSION Z2(10),BJ(50)
016    NP=NP-1
017    NT=NP-1
018    N=NT+NP
019    N2=2*N
020    DO 11 K=1,NF
021    X=THR(K)
022    CS(K)=COS(X)
023    SN(K)=SIN(X)
024    11 CONTINUE
025    U=(0.,1.)
026    U1=3.141593*U**M1
027    M3=M1+1
028    M4=M2+3
029    IF(M1.EQ.0) M3=2
030    M5=M1+2
031    M6=M2+2
032    DO 12 IP=1,NP
033    K2=IP
034    I=IP+1
035    DR=.5*(RH(I)-RH(IP))
036    DZ=.5*(ZH(I)-ZH(IP))
037    D1=SQRT(DR*DR+DZ*DZ)
038    R1=.25*(RH(I)+RH(IP))
039    IF(R1.EQ.0.) R1=.5
040    Z1=.5*(ZH(I)+ZH(IP))
041    DR=.5*DR
042    D2=DR/R1
043    DO 13 L=1,NT
044    R2(L)=R1+DR*XT(L)
045    Z2(L)=Z1+DZ*XT(L)
046    13 CONTINUE
047    DO 14 K=1,NF
048    CC=CS(K)
049    SS=SN(K)
050    D3=DR*CC
051    D4=-DZ*SS
052    D5=D1*CC
053    DO 23 M=M3,M4
054    FA(M)=0.
055    FB(M)=0.
056    23 CONTINUE
057    DO 15 L=1,NT
058    X=SS*R2(L)
059    IF(X.GT..5E-7) GO TO 19
060    DO 20 M=M3,M4

```

```

061      HJ(M)=0.
062  20 CONTINUE
063      BJ(2)=1.
064      S=1.
065      GO TO 18
066  19 M=2.8*X+14.-2./X
067      IF(X.LT..5) M=11.8+ALOG10(X)
068      IF(M.LT.M4) M=M4
069      BJ(M)=0.
070      JM=M-1
071      BJ(JM)=1.
072      DO 16 J=4.M
073      J2=JM
074      JM=JM-1
075      J1=JM-1
076      BJ(JM)=J1/X*BJ(J2)-BJ(JM+2)
077  16 CONTINUE
078      S=0.
079      IF(M.LE.4) GO TO 24
080      DO 17 J=4.M.2
081      S=S+BJ(J)
082  17 CONTINUE
083  24 S=BJ(2)+2.*S
084  18 ARG=Z2(L)*CC
085      UA=AT(L)/S*CMPLX(COS(ARG),SIN(ARG))
086      UB=XT(L)*UA
087      DO 25 M=M3.M4
088      FA(M)=BJ(M)*UA+FA(M)
089      FB(M)=BJ(M)*UB+FB(M)
090  25 CONTINUE
091  15 CONTINUE
092      IF(M1.NE.0) GO TO 26
093      FA(1)=-FA(3)
094      FB(1)=-FB(3)
095  26 UA=U1
096      DO 27 M=M5.M6
097      M7=M-1
098      M8=M+1
099      F2A=UA*(FA(M8)+FA(M7))
100      F2B=UA*(FB(M8)+FB(M7))
101      UB=U*UA
102      F1A=UB*(FA(M8)-FA(M7))
103      F1B=UB*(FB(M8)-FB(M7))
104      U4=D4*UA
105      U2=D3*F1A+U4*FA(M)
106      U3=D3*F1B+U4*FB(M)
107      U4=DR*F2A
108      U5=DR*F2B
109      K1=K2-1
110      K4=K1+N
111      K5=K2+N
112      R(K2+M7)=-D5*(F2A+D2*F2B)
113      R(K5+M7)=D1*(F1A+D2*F1B)
114      IF(IP.EQ.1) GO TO 21
115      R(K1)=R(K1)+U2-U3
116      R(K4)=R(K4)+U4-U5
117      IF(IP.EQ.MP) GO TO 22
118  21 R(K2)=U2+U3
119      R(K5)=U4+U5
120  22 K2=K2+N2

```

```

121      UA=UB
122      27 CONTINUE
123      14 CONTINUE
124      12 CONTINUE
125      RETURN
126      END
127C     LISTING OF THE SUBROUTINE DECOMP
128      SUBROUTINE DECOMP(N,IPS,UL)
129      COMPLEX UL(6241),PIVOT,EM
130      DIMENSION SCL(79),IPS(79)
131      DO 5 I=1,N
132      IPS(I)=I
133      RN=0.
134      J1=I
135      DO 2 J=1,N
136      ULM=ABS(REAL(UL(J1)))+ABS(AIMAG(UL(J1)))
137      J1=J1+N
138      IF(RN-ULM) 1,2,2
139      1 RN=ULM
140      2 CONTINUE
141      SCL(I)=1./RN
142      5 CONTINUE
143      NM1=N-1
144      K2=0
145      DO 17 K=1,NM1
146      BIG=0.
147      DO 11 I=K,N
148      IP=IPS(I)
149      IPK=IP+K2
150      SIZE=(ABS(REAL(UL(IPK)))+ABS(AIMAG(UL(IPK))))*SCL(IP)
151      IF(SIZE-BIG) 11,11,10
152      10 BIG=SIZE
153      IPV=I
154      11 CONTINUE
155      IF(IPV-K) 14,15,14
156      14 J=IPS(K)
157      IPS(K)=IPS(IPV)
158      IPS(IPV)=J
159      15 KPP=IPS(K)+K2
160      PIVOT=UL(KPP)
161      KP1=K+1
162      DO 16 I=KP1,N
163      KP=KPP
164      IP=IPS(I)+K2
165      EM=-UL(IP)/PIVOT
166      16 UL(IP)=-EM
167      DO 16 J=KP1,N
168      IP=IP+N
169      KP=KP+N
170      UL(IP)=UL(IP)+EM*UL(KP)
171      16 CONTINUE
172      K2=K2+N
173      17 CONTINUE
174      RETURN
175      END
176      LISTING OF THE SUBROUTINE SOLVE
177      SUBROUTINE SOLVE(N,IPS,UL,B,X)
178      COMPLEX UL(6241),B(79),X(79),SUM
179      DIMENSION IPS(79)
180      NPI=N+1
181      IP=IPS(1)
182      X(1)=B(IP)
183      DO 2 I=2,N
184      IP=IPS(I)
185      IPB=IP
186      IM1=I-1
187      SUM=0.
188      DO 1 J=1,IM1
189      SUM=SUM+UL(IP)*X(J)
190      1 IP=IP+N
191      2 X(1)=B(IPB)-SUM
192      K2=N*(N-1)
193      IP=IPS(N)+K2
194      X(N)=X(N)/UL(IP)
195      DO 4 IBACK=2,N
196      I=NPI-IBACK
197      K2=K2-N
198      IP1=IPS(I)+K2
199      IP1=IP1
200      SUM=0.
201      IP=IP1
202      DO 3 J=IP1,N
203      IP=IP+N
204      3 SUM=SUM+UL(IP)*X(J)
205      4 X(1)=(X(1)-SUM)/UL(IP1)
206      RETURN
207      END

```

IV. THE SUBROUTINE PRNT

The subroutine PRNT uses knowledge of the superscripted I_{nj} 's or V_{nj} 's and the $k^+ \bar{\rho}^+$'s in a portion of [1, Eqs. (87a), (88a), and (89a)] and in the corresponding portion of [1, Eqs. (87b), (88b), and (89b)] in order to calculate and print out these portions of [1, Eqs. (87)-(89)]. By calling the subroutine PRNT repeatedly, it is possible to print out all the quantities on the right-hand sides of [1, Eqs. (87)-(89)] except those for the cases in which $j=1$ and $j=N^+$ in [1, Eq. (87a)] and $j=M^++1$ and $j=M^++M$ in [1, Eq. (89a)]. PRNT is not designed to treat these cases. They are included in [1, Eqs. (87a) and (89a)] merely for convenience.

Except for the cases in which $j=1$ and $j=N^+$ in [1, Eq. (87a)] and $j=M^++1$ and $j=M^++M$ in [1, Eq. (89a)], any portion of [1, Eqs. (87a), (88a), and (89a)] can be written as

$$C1 * XX(J + JX)/RA(J + J3), J = 1, 2, \dots, J1 \quad (27)$$

and the corresponding portion of [1, Eqs. (87b), (88b), and (89b)] can be written as

$$C2 * XX (J+J1+JX)/(RA(J+J4-1) + RA(J+J4)), J = 1, 2, \dots, J2 \quad (28)$$

In (27) and (28), XX represents the I_{nj} 's or V_{nj} 's of [1], and RA represents the $k^+ \bar{\rho}^+$'s of [1]. In (27), C1 is either ϵ_n or $2j$. In (28), C2 is either $4j$ or $2\epsilon_n$. The sum of the two RA's in the denominator of (28) is due to [1, Eq. (12)]. The complex numbers (27) are t directed currents, either electric or magnetic. The complex numbers (28) are ϕ directed currents. The set of complex numbers (28) is linked to the set

(27) by the fact that the XX's used in (28) occur immediately after those used in (27).

First, the subroutine PRNT (J1, J2, J3, J4, RA) prints out the real part, the imaginary part, and the magnitude of each of the complex numbers (27) under the heading which appears in statement 10. Then, PRNT prints out the real part, the imaginary part, and the magnitude of each of the complex numbers (28) under the heading which appears in statement 13. The variables in (27) and (28) enter PRNT by means of the arguments of PRNT and the statement

```
COMMON C1, C2, XX, JX
```

in line 6. If $J1 = 0$, nothing from (27) is printed out. If $J2 = 0$, nothing from (28) is printed out. All the arguments of PRNT are input arguments. The common variables C1, C2, and XX are input variables. However, the common variable JX functions as both an input variable and an output variable. PRNT adds $J1 + J2$ to the original value of JX.

Minimum allocations in PRNT are given by

```
COMPLEX XX(J2 + J1 + JX)
```

```
DIMENSION RA(Max(J1 + J3, J2 + J4))
```

where Max denotes the larger of the two values in the parentheses. Of course, the space allocated to XX in the calling program must be exactly the same as the space allocated to XX in line 4 of PRNT.

DO loop 11 prints out the complex numbers (27). Line 14 obtains (27). DO loop 14 prints out the complex numbers (28). Line 27 obtains (28).

```

001 C      LISTING OF THE SUBROUTINE PRNT
002      SUBROUTINE PRNT(J1,J2,J3,J4,RA)
003      COMPLEX C1,C2,U
004      COMPLEX XX(79)
005      DIMENSION RA(43)
006      COMMON C1,C2,XX,JX
007      IF(J1.EQ.0) GO TO 15
008      WRITE(3,10)
009 10 FORMAT('0 REAL JT      IMAG JT      MAG JT*')
010      K3=J3
011      DO 11 J=1,J1
012      K3=K3+1
013      JX=JX+1
014      U=C1/RA(K3)*XX(JX)
015      W=CABS(U)
016      WRITE(3,12) U,W
017 12 FORMAT('1X,3E11.4')
018      11 CONTINUE
019 15 IF(J2.EQ.0) RETURN
020      WRITE(3,13)
021 13 FORMAT('0 REAL JP      IMAG JP      MAG JP*')
022      K4=J4
023      DO 14 J=1,J2
024      JX=JX+1
025      K3=K4
026      K4=K4+1
027      U=C2/(RA(K3)+RA(K4))*XX(JX)
028      W=CABS(U)
029      WRITE(3,12) U,W
030 14 CONTINUE
031      RETURN
032      END

```

V. THE MAIN PROGRAM

The main program accepts input data and calls the subroutines YZA, PLANE, DECOMP, SOLVE, and PRNT in order to calculate and print out [1, Eqs. (87)-(89)]. The input data are read from punched cards according to

```

      READ(1,15) NT, NPHI
15    FORMAT(2I3)
      READ(1,10)(XT(K), K=1, NT)
      READ(1,10)(AT(K), K=1, NT)
10    FORMAT(5E14.7)
      READ(1,10)(X(K), K=1, NPHI)
      READ(1,10)(A(K), K=1, NPHI)
      READ(1,16) NA, NB, MA, MB, MC, LA, LB, LC,
           M1, M2, BK, UR, ER, THR(1)
16    FORMAT(10I3/4E14.7)
      READ(1,18)(RA(I), I=1, NA)
      READ(1,18)(ZA(I), I=1, NA)
18    FORMAT(10F8.4)
      READ(1,18)(RB(I), I=1, NB)
      READ(1,18)(ZB(I), I=1, NB)

```

Most of the input variables in the main program represent variables in [1] and [2]. Table 3 relates input variables in the main program to variables in [1] and [2]. The input variable NT in Table 3 represents both n_t and n_T . It is assumed that $n_t = n_T$ in the main program. The Gaussian quadrature data $x_{\ell}^{(n_t)}$, $A_{\ell}^{(n_t)}$, $x_{\ell}^{(n_{\phi})}$, and $A_{\ell}^{(n_{\phi})}$ are given in [4, Appendix A]. It is

Table 3. Input data for the main program.

Variable in main program	Variable in [1] or [2]	Equation or figure number in [1] or [2]
NT	$n_t = n_T$	[2, Eqs. (35) and (79)]
NPHI	n_ϕ	[2, Eq. (36)]
XT	$x_{\ell'}^{(n_t)}, \ell' = 1, 2, \dots, n_t$	[2, Eq. (35)]
AT	$A_{\ell'}^{(n_t)}, \ell' = 1, 2, \dots, n_t$	[2, Eq. (35)]
X	$x_{\ell}^{(n_\phi)}, \ell = 1, 2, \dots, n_\phi$	[2, Eq. (37)]
A	$A_{\ell}^{(n_\phi)}, \ell = 1, 2, \dots, n_\phi$	[2, Eq. (36)]
NA	N^+	[1, Fig. 4]
NB	N^-	[1, Fig. 5]
MA	M^+	[1, Fig. 4]
MB	M^-	[1, Fig. 5]
MC	M	[1, Figs. 4 and 5]
BK	k^+	[1, Eqs. (73)-(84)]
UR	μ^-/μ^+	[1, Fig. 1]
ER	ϵ^-/ϵ^+	[1, Fig. 1]
THR(1)	θ_t	[1, Eq. (92)]
RA	$\bar{\rho}_j^+, j = 1, 2, \dots, N^+$	[1, Eq. (11)]
ZA	$\bar{z}_j^+, j = 1, 2, \dots, N^+$	[1, Eq. (11)]
RB	$\bar{\rho}_j^-, j = 1, 2, \dots, N^-$	[1, Eq. (11)]
ZB	$\bar{z}_j^-, j = 1, 2, \dots, N^-$	[1, Eq. (11)]

assumed that $N^+ \geq 3$, $N^- \geq 3$, and $M \geq 1$. It is also assumed that either

$$\begin{cases} M^+ = 0 \\ M^- = 0 \end{cases} \text{ or } \begin{cases} M^+ > 0 \\ M^- > 0 \end{cases} \text{ and that either } \begin{cases} N^+ - M^+ - M = 0 \\ N^- - M^- - M = 0 \end{cases} \text{ or}$$

$$\begin{cases} N^+ - M^+ - M > 0 \\ N^- - M^- - M > 0 \end{cases} . \quad \text{If } \begin{cases} M^+ = 0 \\ M^- = 0 \end{cases} , \text{ then there is no conductor below}$$

the aperture as in [1, Fig. 18]. If $\begin{cases} N^+ - M^+ - M = 0 \\ N^- - M^- - M = 0 \end{cases}$, then there is

no conductor above the aperture as in [1, Fig. 10]. In Table 3, $\text{THR}(1)$ is in radians.

The input variables LA, LB, LC, M1, and M2 are not listed in Table 3.

The input variables LA, LB, and LC allow for generating curves different from those in [1, Figs. 4 and 5]. LA, LB, and LC are either 0 or 1.

LA = 0 if the generating curve of the surface $(S^+ + A)$ does not close upon itself.

LA = 1 if the generating curve of the surface $(S^+ + A)$ closes upon itself.

LB = 0 if the generating curve of the surface $(S^- + A)$ does not close upon itself.

LB = 1 if the generating curve of the surface $(S^- + A)$ closes upon itself.

LC = 0 if the generating curve of the aperture does not close upon itself.

LC = 1 if the generating curve of the aperture closes upon itself.

For example, LA = 0, LB = 1, and LC = 0 for the generating curves in [1,

Figs. 4 and 5]. If LA = 1, it is assumed that $M^+ \geq 1$ and $N^+ - M^+ - M \geq 2$.

If $LB = 1$, it is assumed that $M^- \geq 1$ and that $N^- - M^- - M \geq 2$. $LC = 1$ implies that the object under consideration reduces to a homogeneous material obstacle with parameters (μ^-, ϵ^-) in contrast with the parameters of the external environment. As for $M1$ and $M2$, [1, Eqs. (87)-(89)] are printed out for $n = M1, M1 + 1, \dots, M2$.

Minimum allocations in the main program are given by

```
COMPLEX XX(N), YP(M*JA*JA), ZP(M*JA*JA),
      YM(M*JB*JB), ZM(M*JB*JB), R(2*M*JA),
      T(N*N), Y(N)
DIMENSION XT(NT), AT(NT), X(NPHI), A(NPHI), RA(NA),
      ZA(NA), RB(NB), ZB(NB), IPS(N)
```

where

$$N = \sum_{I=1}^4 L1A(I) + \sum_{I=1}^4 L1B(I) + L2(1) + L2(2) + L3(1) + L3(2) \quad (29)$$

$$M = M2 - M1 + 1 \quad (30)$$

$$JA = 2 * NA - 3 \quad (31)$$

$$JB = 2 * NB - 3 \quad (32)$$

The L 's on the right-hand side of (29) are calculated in the main program. At any rate, N is the order of the moment matrix T_n given by [1, Eq. (33)].

At the end of this section, the main program is listed along with sample input and output data. The sample data are for the object of [1, Fig. 13] with $k^+a = 2.5$ and $\epsilon_r = 2$ as in [1, Fig. 17]. The output data printed under the heading "Electric current on first part of outside conductor" are, with reference to [1, Eq. (87)],

$$J_n^{t+\theta}(\bar{t}_j^+), \quad j = 2, 3, \dots, M^+ \quad (33)$$

and

$$J_n^{\phi+\theta}(t_j^+), \quad j = 1, 2, 3, \dots, M^+ \quad (34)$$

The real and imaginary parts and magnitudes of (33) are printed under the heading "Real JT Imag JT Mag JT." The real and imaginary parts and magnitudes of (34) are printed under the heading "Real JP Imag JP Mag JP." The output data printed under the heading "Electric current on second part of outside conductor" are

$$J_n^{t+\theta}(\bar{t}_j^+), \quad j = M^+ + M + 1, M^+ + M + 2, \dots, N^+ - 1 \quad (35)$$

and

$$J_n^{\phi+\theta}(\bar{t}_j^+), \quad j = M^+ + M, M^+ + M + 1, \dots, N^+ - 1 - LA \quad (36)$$

The output data printed under the heading "Electric current on first part of inside conductor" are, with reference to [1, Eq. (88)],

$$J_n^{t-\theta}(\bar{t}_j^-), \quad j = 2, 3, \dots, M^- \quad (37)$$

and

$$J_n^{\phi-\theta}(\bar{t}_j^-), \quad j = 1, 2, 3, \dots, M^- \quad (38)$$

The output data printed under the heading "Electric current on second part of inside conductor" are

$$J_n^{t-\theta}(\bar{t}_j^-), \quad j = M^- + M + 1, M^- + M + 2, \dots, N^- - 1 \quad (39)$$

and

$$J_n^{\phi-\theta}(\bar{t}_j^-), \quad j = M^- + M, M^- + M + 1, \dots, N^- - 1 - LB \quad (40)$$

The output data printed under the heading "Electric current in aperture" are, with reference to [1, Eq. (87)],

$$J_n^{t+\theta}(\bar{t}_j^+), \quad j = M^+ + 1, M^+ + 2, \dots, M^+ + M \quad (41)$$

and

$$J_n^{\phi+\theta}(\bar{t}_j^+), \quad j = M^+ + 1, M^+ + 2, \dots, M^+ + M - 1 - LC \quad (42)$$

If, as in [1, Fig. 18], there is no conducting surface below the aperture, then $j = M^+ + 1$ is deleted in (41). If, as in [1, Fig. 10], there is no conducting surface above the aperture, then $j = M^+ + M$ is deleted in (41). The output data printed under the heading "Magnetic current in aperture" are, with reference to [1, Eq. (89)],

$$M_n^{t\theta}(\bar{t}_j^+), \quad j = M^+ + 2, M^+ + 3, \dots, M^+ + M - 1 \quad (43)$$

and

$$M_n^{\phi\theta}(\bar{t}_j^+), \quad j = M^+ + 1, M^+ + 2, \dots, M^+ + M - 1 - LC \quad (44)$$

If, in any of (33)-(44), the upper limit on j is less than the starting value of j , then no values of j are to be taken. For example, if $M^+ = 1$ in (33), then no values of j are to be taken in (33).

Since [1, Eq. (12)] is used for $\rho_j^\pm$, the factors $2, 4j, 2j$, and 4 are needed in [1, Eqs. (87)-(89)] when $n \geq 1$. In the main program, lines 48-51 store these factors in C3, C4, C5, and C6. Line 53 stores in BB the propagation constant of the medium characterized by (μ^-, ϵ^-) in [1, Fig. 1]. This propagation constant is called k^- . With regard to [1, Eq. (33)], line 54 stores η_r in ET and line 55 stores $1/\eta_r$ in ET1. DO loop 23 puts $k^{\pm\pm}\rho_I^+$ in RA(I) and $k^{\pm\pm}z_I^+$ in ZA(I). DO loop 24 puts $k^{\pm\pm}\rho_I^-$ in RB(I) and $k^{\pm\pm}z_I^-$ in ZB(I).

The elements $Y_{nij}^{pq\pm}$ and $Z_{nij}^{pq\pm}$ of the submatrices on the right-hand side of [1, Eq. (33)] are given by [1, Eqs. (36) and (35)]. The testing functions $w_{ni}^{p\pm}$ and the expansion functions $J_{nj}^{q\pm}$ appear in [1, Eqs. (36) and (35)]. However, the submatrices supplied by the subroutine YZA are $Y_n^{rs\pm}$ and $Z_n^{rs\pm}$ whose elements are defined by

$$\left. \begin{aligned} Y_{ni'j'}^{rs\pm} &= - \langle W_{ni'}^{r\pm}, H^{\pm}(J_{nj'}^{s\pm}, 0) \rangle \\ Z_{ni'j'}^{rs\pm} &= - \langle W_{ni'}^{r\pm}, \frac{1}{\eta^{\pm}} E^{\pm}(J_{nj'}^{s\pm}, 0) \rangle \end{aligned} \right\} \begin{aligned} r &= t, \phi \\ s &= t, \phi \end{aligned} \quad (45)$$

$$(46)$$

Equations (45) and (46) are (3) and (4) with the superscript $\pm$ appended, with i replaced by i' , and with j replaced by j' . In (45) and (46), the testing functions are $W_{ni'}^{r\pm}$, and the expansion functions are $J_{nj'}^{s\pm}$. To obtain $Y_{nij}^{pq\pm}$ from $Y_{ni'j'}^{rs\pm}$, and $Z_{nij}^{pq\pm}$ from $Z_{ni'j'}^{rs\pm}$, we must express $W_{ni}^{p\pm}$ in terms of $W_{ni'}^{r\pm}$, and $J_{nj}^{q\pm}$ in terms of $J_{nj'}^{s\pm}$. From [1, Eqs. (23), (24), (27), (28), (30), and (37)], we obtain

$$W_{ni}^{p\pm} = (J_{ni}^{p\pm})^* \quad (47)$$

where $*$ denotes complex conjugate. [1, Eq. (48)] and (47) state that the testing functions are the complex conjugates of the expansion functions. Hence, the expressions for $W_{ni}^{p\pm}$ in terms of $W_{ni'}^{r\pm}$, will be similar to the expressions for $J_{nj}^{q\pm}$ in terms of $J_{nj'}^{s\pm}$. Consequently, it suffices to express $J_{nj}^{q\pm}$ in terms of $J_{nj'}^{s\pm}$.

[1, Eq. (16)] with the choice of superscript $+$ is rewritten as

$$J_{nj}^{1+} = \begin{cases} J_{nj'}^{+}, j' = 1+M1A(1), 2+M1A(1), \dots, L1A(1) + M1A(1) & (48a) \\ J_{nj'}^{+}, j' = 1+M1A(2), 2+M1A(2), \dots, L1A(2) + M1A(2) & (48b) \\ J_{nj'}^{+}, j' = 1+M1A(3), 2+M1A(3), \dots, L1A(3) + M1A(3) & (48c) \\ J_{nj'}^{+}, j' = 1+M1A(4), 2+M1A(4), \dots, L1A(4) + M1A(4) & (48d) \end{cases}$$

where

$$\underline{J}_{nj}^+ = \begin{cases} \underline{J}_{nj}^{t+}, & j' = 1, 2, \dots, N^+ - 2 \\ \underline{J}_{n,j'-N^++2}^{\phi+}, & j' = N^+ - 1, N^+, \dots, 2N^+ - 3 \end{cases} \quad (49a)$$

$$(49b)$$

The vector functions (49a) are the t directed expansion functions on the surface $(S^+ + A)$ in [1, Fig. 4]. The vector functions (49b) are the ϕ directed expansion functions on the surface $(S^+ + A)$ in [1, Fig. 4].

Instead of expressing each j' in (48) in terms of j , we state that $\underline{J}_{nj}^{1+}$ is the j th vector function listed on the right-hand side of (48). For example, if $j = L1A(1)+3$ and if $L1A(2) \geq 3$, then $\underline{J}_{nj}^{1+} = \underline{J}_{n,3+M1A(2)}^+$.

In (48),

$$L1A(1) = \text{Max}(0, M^+ - 1) \quad (50a)$$

$$L1A(2) = M^+ \quad (50b)$$

$$L1A(3) = \text{Max}(0, N^+ - M^+ - M - 1) \quad (50c)$$

$$L1A(4) = N^+ - M^+ - M - LA \quad (50d)$$

$$M1A(1) = 0 \quad (51a)$$

$$M1A(2) = N^+ - 2 \quad (51b)$$

$$M1A(3) = M^+ + M - 1 \quad (51c)$$

$$M1A(4) = N^+ + M^+ + M - 3 \quad (51d)$$

where Max denotes maximum value. The parameters $(L1A(I), I=1,2,3,4)$ and $(M1A(I), I=1,2,3,4)$ are calculated by lines 64-71 of the main program.

The vector functions (48a) are the t directed expansion functions on the first part of the outside conductor. The first part of the

outside conductor is the part of S^+ below the aperture in [1, Fig. 4]. The vector functions (48b) are the ϕ directed expansion functions on the first part of the outside conductor. The vector functions (48c) are the t directed expansion functions on the second part of the outside conductor. The second part of the outside conductor is the part of S^+ above the aperture in [1, Fig. 4]. The vector functions (48d) are the ϕ directed expansion functions on the second part of the outside conductor.

$$J_{nj}^{1-} = \begin{cases} J_{nj}^-, j' = 1+M1B(1), 2+M1B(1), \dots L1B(1) + M1B(1) & (52a) \\ J_{nj}^-, j' = 1+M1B(2), 2+M1B(2), \dots L1B(2) + M1B(2) & (52b) \\ J_{nj}^-, j' = 1+M1B(3), 2+M1B(3), \dots L1B(3) + M1B(3) & (52c) \\ J_{nj}^-, j' = 1+M1B(4), 2+M1B(4), \dots L1B(4) + M1B(4) & (52d) \end{cases}$$

where

$$J_{nj}^- = \begin{cases} J_{nj}^{t-}, & j' = 1, 2, \dots N^- - 2 & (53a) \\ J_{n, j' - N^- + 2}^{\phi-}, & j' = N^- - 1, N^-, \dots 2N^- - 3 & (53b) \end{cases}$$

The vector functions (53a) are the t directed expansion functions on the surface $(S^- + A)$ in [1, Fig. 5]. The vector functions (53b) are the ϕ directed expansion functions on the surface $(S^- + A)$ in [1, Fig. 5]. Equation (52) means that J_{nj}^{1-} is the j th vector function listed on the right-hand side of (52). In (52),

$$L1B(1) = \text{Max}(0, M^- - 1) \quad (54a)$$

$$L1B(2) = M^- \quad (54b)$$

$$L1B(3) = \text{Max}(0, N^- - M^- - M - 1) \quad (54c)$$

$$L1B(4) = N^- - M^- - M - LB \quad (54d)$$

$$M1B(1) = 0 \quad (55a)$$

$$M1B(2) = N^- - 2 \quad (55b)$$

$$M1B(3) = M^- + M - 1 \quad (55c)$$

$$M1B(4) = N^- + M^- + M - 3 \quad (55d)$$

The parameters (L1B(I), I = 1,2,3,4) and (M1B(I), I = 1,2,3,4) are calculated by lines 72-79 of the main program.

The vector functions (52a) are the t directed expansion functions on the first part of the inside conductor and (52b) are the ϕ directed expansion functions there. The first part of the inside conductor is the part of $(S^- + A)$ below the aperture in [1, Fig. 5]. The vector functions (52c) are the t directed expansion functions on the second part of the inside conductor and (52d) are the ϕ directed expansion functions there. The second part of the inside conductor is the part of $(S^- + A)$ above the aperture in [1, Fig. 5].

[1, Eq. (17)] with the choice of superscript $+$ is rewritten as

$$\underline{J}_{nj}^{2+} = \begin{cases} \underline{J}_{nj}^+, & j' = 1+M2A(1), 2+M2A(1), \dots, L2(1) + M2A(1) \\ \underline{J}_{nj}^+, & j' = 1+M2A(2), 2+M2A(2), \dots, L2(2) + M2A(2) \end{cases} \quad (56a)$$

$$\underline{J}_{nj}^+, \quad j' = 1+M2A(2), 2+M2A(2), \dots, L2(2) + M2A(2) \quad (56b)$$

where $\underline{J}_{nj}^+$ is given by (49). Equation (56) means that $\underline{J}_{nj}^{2+}$ is the j th vector function listed on the right-hand side of (56). In (56),

$$L2(1) = \begin{cases} M & M^+ \neq 0, \quad L1A(4) \neq 0 \\ M-1 & M^+ = 0, \quad L1A(4) \neq 0 \\ M-1 & M^+ \neq 0, \quad L1A(4) = 0 \\ M-2 & M^+ = 0, \quad L1A(4) = 0 \end{cases} \quad (57a)$$

$$L2(2) = M - 1 - LC \quad (57b)$$

$$M2A(1) = \text{Max}(0, M^+ - 1) \quad (58a)$$

$$M2A(2) = N^+ - 2 + M^+ \quad (58b)$$

where $L1A(4)$ is given by (50d). The parameters $L2(1)$, $L2(2)$, $M2A(1)$, and $M2A(2)$ are calculated by lines 80-85 of the main program.

The vector functions (56a) consist of the t directed expansion function which straddles the first part of the outside conductor and the aperture, the t directed expansion functions in the aperture, and the t directed expansion function which straddles the aperture and the second part of the outside conductor. The vector functions (56b) are the ϕ directed expansion functions in the aperture. Four different cases are necessary in (57a) because there is no t directed expansion function which straddles the first part of the outside conductor and the aperture if the first part of the outside conductor is absent and there is no t directed expansion function which straddles the aperture and the second part of the outside conductor if the second part of the outside conductor is absent.

[1, Eq. (17)] with the choice of superscript - is rewritten as

$$J_{nj}^{2-} = \begin{cases} J_{nj}^-, & j' = 1+M2B(1), 2+M2B(1), \dots, L2(1) + M2B(1) \quad (59a) \\ J_{nj}^-, & j' = 1+M2B(2), 2+M2B(2), \dots, L2(2) + M2B(2) \quad (59b) \end{cases}$$

where J_{nj}^- is given by (53). Equation (59) means that J_{nj}^{2-} is the j th vector function listed on the right-hand side of (59). In (59), $L2(1)$

and $L2(2)$ are given by (57). $M2B(1)$ and $M2B(2)$ are given by

$$M2B(1) = \text{Max}(0, M^- - 1) \quad (60a)$$

$$M2B(2) = N^- - 2 + M^- \quad (60b)$$

$M2B(1)$ and $M2B(2)$ are calculated by lines 86-87 of the main program. The vector functions (59a) consist of the t directed expansion function which straddles the first part of the inside conductor and the aperture, the t directed expansion functions in the aperture, and the t directed expansion function which straddles the aperture and the second part of the inside conductor. The vector functions (59b) are the ϕ directed expansion functions in the aperture. Because of [1, Eqs. (15b) and (15d)], the vector functions in the aperture in (59) are the same as the vector functions in the aperture in (56).

[1, Eq. (22)] is rewritten as

$$J_{nj}^{3+} = \begin{cases} J_{nj}^+, & j' = 1+M3A(1), 2+M3A(1), \dots L3(1) + M3A(1) \\ J_{nj}^+, & j' = 1+M3A(2), 2+M3A(2), \dots L3(2) + M3A(2) \end{cases} \quad (61a)$$

$$J_{nj}^+, \quad j' = 1+M3A(2), 2+M3A(2), \dots L3(2) + M3A(2) \quad (61b)$$

where J_{nj}^+ is given by (49). Equation (61) means that J_{nj}^{3+} is the j th vector function listed on the right-hand side of (61). In (61),

$$L3(1) = \text{Max}(0, M - 2) \quad (62a)$$

$$L3(2) = L2(2) \quad (62b)$$

$$M3A(1) = M^+ \quad (63a)$$

$$M3A(2) = N^+ - 2 + M^+ \quad (63b)$$

where $L2(2)$ is given by (57b). $L3(1)$, $L3(2)$, $M3A(1)$, and $M3A(2)$ are

calculated by lines 88-91 of the main program. The vector functions (61a) are the t directed expansion functions in the aperture. The vector functions (61b) are the ϕ directed expansion functions in the aperture.

Replacement of the superscript $+$ in [1, Eq. (22)] by the superscript $-$ gives

$$\underline{J}_{nj}^{3-} = \begin{cases} \underline{J}_{n,j+M^-}^{t-} & , j = 1, 2, \dots M-2 \\ \underline{J}_{n,j+M^- - M+2}^{\phi-} & , j = M-1, M, \dots 2M-3 \end{cases} \quad (64)$$

Equation (64) is consistent with [1, Eqs. (22), (37), (15b), and (15d)].

Equation (64) can be rewritten as

$$\underline{J}_{nj}^{3-} = \begin{cases} \underline{J}_{nj}^-, & , j' = 1+M3B(1), 2+M3B(1), \dots L3(1) + M3B(1) \end{cases} \quad (65a)$$

$$\underline{J}_{nj}^-, & , j' = 1+M3B(2), 2+M3B(2), \dots L3(2) + M3B(2) \end{cases} \quad (65b)$$

where $\underline{J}_{nj}^-$ is given by (53). Equation (65) means that $\underline{J}_{nj}^{3-}$ is the j th vector function listed on the right-hand side of (65). In (65), $L3(1)$ and $L3(2)$ are given by (62a) and (62b). $M3B(1)$ and $M3B(2)$ are given by

$$M3B(1) = M^- \quad (66a)$$

$$M3B(2) = N^- - 2 + M^- \quad (66b)$$

$M3B(1)$ and $M3B(2)$ are calculated by lines 92-93 of the main program. The vector functions on the right-hand side of (65) are the same as those on the right-hand side of (61).

To accompany the expansion functions $\underline{J}_{nj}^+$, of (49) and (53), testing functions $\underline{W}_{ni}^+$, are defined by

$$\underline{W}_{ni}^+ = (\underline{J}_{ni}^+)^* \quad (67)$$

Equations (49), (53), and (67) allow (45) and (46) to be recast as

$$Y_{ni,j}^{\pm} = - \langle \underline{W}_{ni}^+ , H^{\pm}(\underline{J}_{nj}^+ , 0) \rangle \quad (68)$$

$$Z_{ni,j}^{\pm} = - \langle \underline{W}_{ni}^+ , \frac{1}{\eta^{\pm}} E^{\pm}(\underline{J}_{nj}^+ , 0) \rangle \quad (69)$$

Expressions (68) and (69) are the matrix elements supplied by the sub-routine YZA.

Line 94 stores in JA the order of the matrices Y_n^+ and Z_n^+ whose $i'j'$ 'th elements are given by (68) and (69). Line 95 stores in JB the order of the matrices Y_n^- and Z_n^- whose $i'j'$ 'th elements are given by (68) and (69). Line 96 stores in IA the total number of vector functions listed on the right-hand side of (48). IA is the order of the submatrix Z_n^{11+} in [1, Eq. (33)]. Line 97 stores in IB the total number of vector functions listed on the right-hand side of (52). IB is the order of the submatrix Z_n^{11-} in [1, Eq. (33)]. Line 98 stores in N the order of the matrix T_n of [1, Eq. (33)].

Line 99 puts Y_n^+ of (68) in YP $((n-M1)*JA*JA+1)$ to YP $((n-M1+1)*JA*JA)$ and Z_n^+ of (69) in the corresponding region of ZP for $n = M1, M1+1, \dots M2$. Storage of Y_n^+ and Z_n^+ in YP and ZP is by columns. Line 100 puts Y_n^- of (68) in YM $((n-M1)*JB*JB+1)$ to YM $((n-M1+1)*JB*JB)$ and Z_n^- of (69) in the corresponding region of ZM for $n = M1, M1+1, \dots M2$. Storage of Y_n^- and Z_n^- in YM and ZM is by columns. With reference to [1, Eq. (103)], line 101 puts

$$\left. \begin{aligned}
 V_{ni}^{t\theta} & \text{ in } R(i' + J1) & , & \quad i' = 1, 2, \dots, NA-2 \\
 -V_{ni}^{\phi\theta} & \text{ in } R(i' + J1 + NA - 2) & , & \quad i' = 1, 2, \dots, NA-1 \\
 -V_{ni}^{t\phi} & \text{ in } R(i' + J1 + JA) & , & \quad i' = 1, 2, \dots, NA-2 \\
 V_{ni}^{\phi\phi} & \text{ in } R(i' + J1 + JA + NA-2) & , & \quad i' = 1, 2, \dots, NA-1
 \end{aligned} \right\} (70)$$

where

$$J1 = (n - M1) * 2 * JA \quad (71)$$

and

$$n = M1, M1 + 1, \dots, M2 \quad (72)$$

DO loop 82 puts $k_{\rho_j}^{+-}$ in RB(J). $k_{\rho_j}^{+-}$ is needed in order to calculate [1, Eq. (88a)].

DO loop 86 obtains n according to $n = M-1$. If $n \neq M1$, DO loop 89 moves Y_n^{+} down into YP(1) to YP(JA*JA) and Z_n^{+} down into ZP(1) to ZP(JA*JA), DO loop 90 moves Y_n^{-} down into YM(1) to YM(JB*JB) and Z_n^{-} down into ZM(1) to ZM(JB*JB), and DO loop 91 moves the V's of (70) down into R(1) to R(2*JA).

The only difference between $Y_{nij}^{pq\pm}$ of [1, Eq. (36)] and $Y_{ni,j}^{\pm}$ of (68) lies in the testing functions and expansion functions. Likewise, the only difference between $Z_{nij}^{pq\pm}$ of [1, Eq. (35)] and $Z_{ni,j}^{\pm}$ of (69) lies in the testing functions and expansion functions. The expansion functions in [1, Eqs. (36) and (35)] are related to the expansion functions in (68) and (69) by (48), (52), (56), (59), (61), and (65). According to (67) and (47), the testing functions are the complex conjugates of the expansion functions. Hence, the complex conjugates of (48), (52), (56), (59), (61), and (65) relate the testing functions in [1, Eqs. (36) and (35)] to the testing functions in (68) and (69). As a result,

$$Y_{nij}^{pq\pm} = Y_{ni'j'}^{\pm} \quad (73)$$

$$Z_{nij}^{pq\pm} = Z_{ni'j'}^{\pm} \quad (74)$$

where j' is the subscript of the j th vector function listed on the right-hand side of either (48), (52), (56), (59), (61), or (65), whichever is appropriate. Similarly, i' is the subscript of the i th vector function listed on the right-hand side of either (48), (52), (56), (59), (61), or (65), whichever is appropriate. For example, if $p=2$ and $q=1$ and if the superscript $+$ is chosen in (73), then (73) becomes

$$Y_{nij}^{21+} = Y_{ni'j'}^{+} \quad (75)$$

where j' is the subscript of the j th vector function listed on the right-hand side of (48) and i' is the subscript of the i th vector function listed on the right-hand side of (56).

With regard to [1, Eq. (33)], DO loop 25 uses (73) and (74) to store

$$\begin{bmatrix} Z_n^{11+} \\ 0 \\ Z_n^{21+} \\ Y_n^{31+} \end{bmatrix} \quad (76)$$

by columns in T . The elements of the submatrices in (76) depend on the expansion functions (48). Hence, j' in (73) and (74) is the subscript of the j th vector function listed on the right-hand side of (48). The index of DO loop 25 is JJ . If $JJ = 1$, inner DO loop 26 obtains the values of j for which j' is given by (48a). If $JJ = 2$, DO loop 26 obtains the values

of j for which j' is given by (48b). Similarly, if $JJ = 3$, DO loop 26 obtains the values of j covered by (48c). Finally, if $JJ = 4$, DO loop 26 obtains the values of j covered by (48d).

The elements of Z_n^{11+} in (76) depend on the testing functions W_{ni}^{1+} . These testing functions are related to W_{ni}^+ , of (67) by the testing function version of (48). The testing function version of (48) is (48) with J_{nj}^{1+} replaced by W_{ni}^{1+} , J_{nj}^+ replaced by W_{ni}^+ , and j' replaced by i' . It is now apparent that i' in (74) is the subscript of the i th vector function listed on the right-hand side of the testing function version of (48). Inside nested DO loops 27 and 28, line 141 stores the appropriate element of Z_n^{11+} of (76) in T . The index of DO loop 27 is II . If $II = 1$, DO loop 28 obtains the values of i for which i' is given by the testing function version of (48a). If $II = 2$, DO loop 28 obtains the values of i for which i' is given by the testing function version of (48b). Similarly, if $II = 3$, DO loop 28 obtains the values of i covered by the testing function version of (48c). Finally, if $II = 4$, DO loop 28 obtains the values of i covered by the testing function version of (48d).

DO loop 29 takes care of the 0 in (76).

The elements of Z_n^{21+} in (76) depend on the testing functions W_{ni}^{2+} . These testing functions are related to W_{ni}^+ , of (67) by the testing function version of (56). Hence, i' in (74) is the subscript of the i th vector function listed on the right-hand side of the testing function version of (56). Inside nested DO loops 30 and 31, line 157 stores the appropriate element of Z_n^{21+} of (76) in T . The index of DO loop 30 is II . If $II = 1$, DO loop 31 obtains the values of i for which i' is

given by the testing function version of (56a). If $II = 2$, DO loop 31 obtains the values of i for which i' is given by the testing function version of (56b).

The elements of Y_n^{31+} in (76) depend on the testing functions W_{ni}^{3+} . These testing functions are related to W_{ni}^+ , of (67) by the testing function version of (61). Hence i' in (73) is the subscript of the i th vector function listed on the right-hand side of the testing function version of (61). Inside nested DO loops 32 and 33, line 168 stores the appropriate element of Y_n^{31+} of (76) in T . The index of DO loop 32 is II . If $II = 1$, DO loop 33 obtains the values of i for which i' is given by the testing function version of (61a). If $II = 2$, DO loop 33 obtains the values of i for which i' is given by the testing function version of (61b).

With regard to [1, Eq. (33)], DO loop 34 uses (73) and (74) to store

$$\begin{bmatrix} 0 \\ \eta_r z_n^{11-} \\ \eta_r z_n^{21-} \\ y_n^{31-} \end{bmatrix} \quad (77)$$

by columns in T . Nested DO loops 34 and 35 obtain the values of j in (73) and (74). In (73) and (74), j' is the subscript of the j th vector function listed on the right-hand side of (52). DO loop 36 obtains the 0 in (77). Nested DO loops 37 and 38 obtain the values of i for z_{nij}^{11-} of (74) in which i' is the subscript of the i th vector function listed on the right-hand side of the testing function version of (52). Nested DO loops 39 and 40 obtain the values of i for z_{nij}^{21-} of (74) in which i' is determined by the testing function version of (59). Nested DO loops 41 and 42 obtain the values of i for

Y_{nij}^{31-} of (73) in which i' is determined by the testing function version of (65).

With regard to [1, Eq. (33)], DO loop 43 uses (73) and (74) to store

$$\begin{bmatrix} Z_n^{12+} \\ \eta_r Z_n^{12-} \\ Z_n^{22+} + \eta_r Z_n^{22-} \\ Y_n^{32+} + Y_n^{32-} \end{bmatrix} \quad (78)$$

by columns in T. Nested DO loops 43 and 44 obtain the values of j in (73) and (74) in which j' is determined by (56) for the submatrices with superscript $+$ in (78) and by (59) for the submatrices with superscript $-$ in (78). Nested DO loops 45 and 46 obtain the values of i for Z_{nij}^{12+} of (74) with i' determined by the testing function version of (48). Nested DO loops 47 and 48 obtain the values of i for Z_{nij}^{12-} of (74) with i' determined by the testing function version of (52). Nested DO loops 49 and 50 obtain the values of i for $Z_{nij}^{22+} + \eta_r Z_{nij}^{22-}$ of (78). Z_{nij}^{22+} is given by (74) with i' determined by the testing function version of (56). Z_{nij}^{22-} is given by (74) with i' determined by the testing function version of (59). Nested DO loops 51 and 52 obtain the values of i for $Y_{nij}^{32+} + Y_{nij}^{32-}$ of (78). Y_{nij}^{32+} is given by (73) with i' determined by the testing function version of (61). Y_{nij}^{32-} is given by (73) with i' determined by the testing function version of (65).

With regard to [1, Eq. (33)], DO loop 53 uses (73) and (74) to store

$$\begin{bmatrix} -Y_n^{13+} \\ -Y_n^{13-} \\ -Y_n^{23+} - Y_n^{23-} \\ Z_n^{33+} + \left(\frac{1}{\eta_r}\right) Z_n^{33-} \end{bmatrix} \quad (79)$$

by columns in T. Nested DO loops 53 and 54 obtain the values of j in (73) and (74) with j' determined by (61) for the submatrices with superscript + in (79) and by (65) for the submatrices with superscript - in (79). Nested DO loops 55 and 56 obtain the values of i for Y_{nij}^{13+} of (73) with i' determined by the testing function version of (48). Nested DO loops 57 and 58 obtain the values of i for Y_{nij}^{13-} of (73) with i' determined by the testing function version of (52). Nested DO loops 59 and 60 obtain the values of i for $-Y_{nij}^{23+} - Y_{nij}^{23-}$ of (79). Y_{nij}^{23+} is given by (73) with i' determined by the testing function version of (56). Y_{nij}^{23-} is given by (73) with i' determined by the testing function version of (59). Nested DO loops 61 and 62 obtain the values of i for $Z_{nij}^{33+} + \left(\frac{1}{\eta_r}\right) Z_{nij}^{33-}$ of (79). Z_{nij}^{33+} is given by (74) with i' determined by the testing function version of (61). Z_{nij}^{33-} is given by (74) with i' determined by the testing function version of (65).

Now that the moment matrix T_n has been stored in T, we turn to the excitation vector $\tilde{B}_n^\theta$ of [1, Eq. (101)]. The V's on the right-hand side of [1, Eq. (103)] reside in R. Because DO loop 91 has been executed, storage in R is not according to (70) as it stands but according to (70) without the offset J1. The auxiliary equation involving the W's in

[1, Eq. (103a)] is the testing function version of (48). Likewise, the auxiliary equation in [1, Eq. (103b)] is the testing function version of (56), and the auxiliary equation in [1, Eq. (103c)] is the testing function version of (61).

Based on the considerations in the preceding paragraph, lines 336-378 store $\tilde{B}_n^\theta$ of [1, Eq. (101)] in Y. Nested DO loops 63 and 64 store $\tilde{V}_n^{11+\theta}$ of [1, Eq. (101)] in Y. The sign factor S in line 346 compensates for the minus sign in front of $V_{ni}^{\phi\theta}$, in (70). The number of elements in the row vector 0 in [1, Eq. (101)] is IB because this row vector can be traced back to [1, Eq. (39b)]. DO loop 65 stores this row vector in Y. Nested DO loops 76 and 77 store $\tilde{V}_n^{1\theta}$ of [1, Eq. (101)] in Y. Line 364 compensates for the minus sign in front of $V_{ni}^{\phi\theta}$, in (70). Nested DO loops 66 and 67 store $\tilde{I}_n^{1\theta}$ of [1, Eq. (101)] in Y. The offset JA in line 370 is mandated by the superscript ϕ in $V_{ni}^{r\phi}$, in [1, Eq. (103c)]. Line 376 accounts for the net effect of the minus sign in front of $V_{ni}^{r\phi}$, in [1, Eq. (103c)] and the minus sign in front of $V_{ni}^{t\phi}$, in (70).

Line 379 decomposes T_n into the product of a lower triangular matrix with an upper triangular matrix. Line 380 stores the solution $\tilde{X}_n^\theta$ to [1, Eq. (65a)] in XX. The elements [1, Eq. (68a)] of $\tilde{X}_n^\theta$ are the I's and V's in [1, Eqs. (87)-(89)]. Lines 384-385 store ϵ_n in C1. Line 386 stores 4j in C2. If $L1A(2) = 0$, then it is evident (50a) and (50b) that $L1A(1) = 0$ so that no values of j are to be taken in (33) and (34). If $L1A(2) > 0$, line 392 prints (33) and (34). If $L1A(4) = 0$, then it is evident from (50c) and (50d) that $L1A(3) = 0$ so that no values of j are to be taken in (35) and (36). If $L1A(4) > 0$, line 398 prints (35) and (36). Line 403 prints (37) and (38). Line 409 prints (39) and (40).

If $L2(1) = L2(2) = 0$, then it is evident from (57a), (57b), (62a), and (62b) that $L3(1) = L3(2) = 0$ so that no values of j are to be taken in (41) - (44). If either $L2(1) > 0$ or $L2(2) > 0$, then line 417 prints (41)-(42). If $L2(2) = 0$, then it is evident from (57b), (62a) and (62b) that $L3(1) = L3(2) = 0$ so that no values of j are to be taken in (43) and (44). If $L2(2) > 0$, line 421 puts $2j$ in $C1$, lines 422 and 423 put $2\epsilon_n$ in $C2$, and line 424 prints (43) and (44).

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001 C      LISTING OF THE MAIN PROGRAM
002 C      THE SUBPROGRAMS YZA,BLOG,PLANE,DECOMP,SOLVE, AND PRNT ARE NEEDED
003 //PGM JOB (XXXX,XXXX,2,2),*MAUTZ,JOE*,REGION=275K
004 // EXEC WATFIV
005 //GO.SYSIN DD *
006 $JOB      MAUTZ,TIME=5,PAGES=60
007      COMPLEX C1,C2,C3,C4,C5,C6,XX(79),YP(2209),ZP(2209),YM(2209)
008      COMPLEX ZN(2209),R(240),T(6241),Y(79)
009      DIMENSION XT(10),AT(10),X(48),A(48),THR(3),RA(43),ZA(43),RB(43)
010      DIMENSION ZB(43),LIA(4),MIA(4),LIB(4),MIB(4),L2(2),M2A(2),M2B(2)
011      DIMENSION L3(2),M3A(2),M3B(2),IPS(79)
012      COMMON C1,C2,XX,JX
013      READ(1,15) NT,NPHI
014      15 FORMAT(2I3)
015      WRITE(3,9) NT,NPHI
016      9 FORMAT(' NT NPHI'/(1X,I3,I5))
017      READ(1,10)(XT(K),K=1,NT)
018      READ(1,10)(AT(K),K=1,NT)
019      10 FORMAT(5E14.7)
020      WRITE(3,11)(XT(K),K=1,NT)
021      WRITE(3,12)(AT(K),K=1,NT)
022      11 FORMAT(' XT'/(1X,5E14.7))
023      12 FORMAT(' AT'/(1X,5E14.7))
024      READ(1,10)(X(K),K=1,NPHI)
025      READ(1,10)(A(K),K=1,NPHI)
026      WRITE(3,13)(X(K),K=1,NPHI)
027      WRITE(3,14)(A(K),K=1,NPHI)
028      13 FORMAT(' X'/(1X,5E14.7))
029      14 FORMAT(' A'/(1X,5E14.7))
030      READ(1,16) NA,NB,MA,MB,MC,LA,LB,LC,M1,M2,BK,UR,ER,THR(1)
031      16 FORMAT(10I3/4E14.7)
032      WRITE(3,17) NA,NB,MA,MB,MC,LA,LB,LC,M1,M2,BK,UR,ER,THR(1)
033      17 FORMAT(' NA NB MA MB MC LA LB LC M1 M2'/(1X,10I3/7X,'BK',12X,'UR',
034      112X,'ER',10X,'THR(1)'/(1X,4E14.7))
035      READ(1,18)(RA(I),I=1,NA)
036      READ(1,18)(ZA(I),I=1,NA)
037      18 FORMAT(10F8.4)
038      WRITE(3,19)(RA(I),I=1,NA)
039      WRITE(3,20)(ZA(I),I=1,NA)
040      19 FORMAT(' RA'/(1X,10F8.4))
041      20 FORMAT(' ZA'/(1X,10F8.4))
042      READ(1,18)(RB(I),I=1,NB)
043      READ(1,18)(ZB(I),I=1,NB)
044      WRITE(3,21)(RB(I),I=1,NB)
045      21 FORMAT(' RB'/(1X,10F8.4))
046      WRITE(3,22)(ZB(I),I=1,NB)
047      22 FORMAT(' ZB'/(1X,10F8.4))
048      C3=2.
049      C4=4.*(0..1.)
050      C5=2.*(0..1.)
051      C6=4.
052      UE=SQRT(UR*ER)
053      BB=BK*UE
054      ET=SQRT(UR/ER)
055      ET1=1./ET
056      DO 23 I=1,NA
057      RA(I)=BK*RA(I)
058      ZA(I)=BK*ZA(I)
059      23 CONTINUE
060      DO 24 I=1,NB

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061      RB(I)=BB*RB(I)
062      ZB(I)=BB*ZB(I)
063  24 CONTINUE
064      L1A(1)=MAX0(0,MA-1)
065      L1A(2)=MA
066      L1A(3)=MAX0(0,NA-MA-MC-1)
067      L1A(4)=NA-MA-MC-LA
068      M1A(1)=0
069      M1A(2)=NA-2
070      M1A(3)=MA+MC-1
071      M1A(4)=NA+MA+MC-3
072      L1B(1)=MAX0(0,MB-1)
073      L1B(2)=MB
074      L1B(3)=MAX0(0,NB-MB-MC-1)
075      L1B(4)=NB-MB-MC-LB
076      M1B(1)=0
077      M1B(2)=NB-2
078      M1B(3)=MB+MC-1
079      M1B(4)=NB+MB+MC-3
080      L2(1)=MC
081      IF(MA.EQ.0) L2(1)=MC-1
082      IF(L1A(4).EQ.0) L2(1)=L2(1)-1
083      L2(2)=MC-1-LC
084      M2A(1)=MAX0(0,MA-1)
085      M2A(2)=NA-2+MA
086      M2B(1)=MAX0(0,MB-1)
087      M2B(2)=NB-2+MB
088      L3(1)=MAX0(0,MC-2)
089      L3(2)=L2(2)
090      M3A(1)=MA
091      M3A(2)=NA-2+MA
092      M3B(1)=MB
093      M3B(2)=NB-2+MB
094      JA=2*NA-3
095      JB=2*NB-3
096      IA=L1A(1)+L1A(2)+L1A(3)+L1A(4)
097      IB=L1B(1)+L1B(2)+L1B(3)+L1B(4)
098      N=IA+IB+L2(1)+L2(2)+L3(1)+L3(2)
099      CALL YZA(M1,M2,NA,NPHI,NT,0,RA,ZA,X,A,XT,AT,YP,ZP)
100      CALL YZA(M1,M2,NB,NPHI,NT,0,RB,ZB,X,A,XT,AT,YM,ZM)
101      CALL PLANE(M1,M2,1,NA,NT,RA,ZA,XT,AT,THR,R)
102      DO 82 J=1,NB
103      RB(J)=RB(J)/UE
104  82 CONTINUE
105      JA2=JA+JA
106      JB2=JB+JB
107      M3=M1+1
108      M4=M2+1
109      DO 86 M=M3,M4
110      IF(M.EQ.M3) GO TO 88
111      J2=M-M3
112      J1=J2+JA2
113      DO 89 J=1,JA2
114      YP(J)=YP(J+J1)
115      ZP(J)=ZP(J+J1)
116  89 CONTINUE
117      J1=J2+JB2
118      DO 90 J=1,JB2
119      YM(J)=YM(J+J1)
120      ZM(J)=ZM(J+J1)

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121 90 CONTINUE
122 J3=2*JA
123 J1=J2+J3
124 DO 91 J=1,J3
125 R(J)=R(J+J1)
126 91 CONTINUE
127 88 JT=0
128 DO 25 JJ=1,4
129 J2=L1A(JJ)
130 IF(J2.EQ.0) GO TO 25
131 J1=M1A(JJ)*JA
132 DO 26 J=1,J2
133 DO 27 I=1,4
134 I3=L1A(I)
135 IF(I3.EQ.0) GO TO 27
136 J3=J1+M1A(I)
137 I1=J3+1
138 I2=J3+I3
139 DO 28 I=I1,I2
140 JT=JT+1
141 T(JT)=ZP(I)
142 28 CONTINUE
143 27 CONTINUE
144 IF(I8.EQ.0) GO TO 79
145 DO 29 I=1,I8
146 JT=JT+1
147 T(JT)=0.
148 29 CONTINUE
149 79 DO 30 I=1,2
150 I3=L2(I)
151 IF(I3.EQ.0) GO TO 30
152 J3=J1+M2A(I)
153 I1=J3+1
154 I2=J3+I3
155 DO 31 I=I1,I2
156 JT=JT+1
157 T(JT)=ZP(I)
158 31 CONTINUE
159 30 CONTINUE
160 DO 32 I=1,2
161 I3=L3(I)
162 IF(I3.EQ.0) GO TO 32
163 J3=J1+M3A(I)
164 I1=J3+1
165 I2=J3+I3
166 DO 33 I=I1,I2
167 JT=JT+1
168 T(JT)=YP(I)
169 33 CONTINUE
170 32 CONTINUE
171 J1=J1+JA
172 26 CONTINUE
173 25 CONTINUE
174 DO 34 JJ=1,4
175 J2=L18(JJ)
176 IF(J2.EQ.0) GO TO 34
177 J1=M18(JJ)*JB
178 DO 35 J=1,J2
179 IF(I1A.EQ.0) GO TO 80
180 DO 36 I=1,I1A

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181      JT=JT+1
182      T(JT)=0.
183      36 CONTINUE
184      80 DO 37 I=1.4
185          I3=L1B(I)
186          IF(I3.EQ.0) GO TO 37
187          J3=J1+M1B(I)
188          I1=J3+1
189          I2=J3+I3
190          DO 38 I=I1.I2
191              JT=JT+1
192              T(JT)=ET*ZN(I)
193      38 CONTINUE
194      37 CONTINUE
195      DO 39 I=1.2
196          I3=L2(I)
197          IF(I3.EQ.0) GO TO 39
198          J3=J1+M2B(I)
199          I1=J3+1
200          I2=J3+I3
201          DO 40 I=I1.I2
202              JT=JT+1
203              T(JT)=ET*ZN(I)
204      40 CONTINUE
205      39 CONTINUE
206      DO 41 I=1.2
207          I3=L3(I)
208          IF(I3.EQ.0) GO TO 41
209          J3=J1+M3B(I)
210          I1=J3+1
211          I2=J3+I3
212          DO 42 I=I1.I2
213              JT=JT+1
214              T(JT)=YM(I)
215      42 CONTINUE
216      41 CONTINUE
217          J1=J1+J8
218      35 CONTINUE
219      34 CONTINUE
220      DO 43 JJ=1.2
221          J2=L2(JJ)
222          IF(J2.EQ.0) GO TO 43
223          J1A=M2A(JJ)*JA
224          J1B=M2B(JJ)*JB
225          DO 44 J=1.J2
226              DO 45 I=1.4
227                  I3=L1A(I)
228                  IF(I3.EQ.0) GO TO 45
229                  J3=J1A+M1A(I)
230                  I1=J3+1
231                  I2=J3+I3
232                  DO 46 I=I1.I2
233                      JT=JT+1
234                      T(JT)=ZP(I)
235      46 CONTINUE
236      45 CONTINUE
237          DO 47 I=1.4
238              I3=L1B(I)
239              IF(I3.EQ.0) GO TO 47
240              J3=J1B+M1B(I)

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241      I1=J3+1
242      I2=J3+I3
243      DO 48 I=1,12
244      JT=JT+1
245      T(JT)=ET+ZN(I)
246  48 CONTINUE
247  47 CONTINUE
248      DO 49 I1=1,2
249      I3=L2(I1)
250      IF(I3.EQ.0) GO TO 49
251      J3=J1A+M2A(I1)
252      I1=J3+1
253      I2=J3+I3
254      J4=J1B+M2B(I1)
255      DO 50 I=1,12
256      J4=J4+1
257      JT=JT+1
258      T(JT)=ZP(I)+ET+ZN(J4)
259  50 CONTINUE
260  49 CONTINUE
261      DO 51 I1=1,2
262      I3=L3(I1)
263      IF(I3.EQ.0) GO TO 51
264      J3=J1A+M3A(I1)
265      I1=J3+1
266      I2=J3+I3
267      J4=J1B+M3B(I1)
268      DO 52 I=1,12
269      J4=J4+1
270      JT=JT+1
271      T(JT)=YP(I)+YM(J4)
272  52 CONTINUE
273  51 CONTINUE
274      J1A=J1A+JA
275      J1B=J1B+JB
276  44 CONTINUE
277  43 CONTINUE
278      DO 53 JJ=1,2
279      J2=L3(JJ)
280      IF(J2.EQ.0) GO TO 53
281      J1A=M3A(JJ)*JA
282      J1B=M3B(JJ)*JB
283      DO 54 J=1,J2
284      DO 55 I1=1,4
285      I3=L1A(I1)
286      IF(I3.EQ.0) GO TO 55
287      J3=J1A+M1A(I1)
288      I1=J3+1
289      I2=J3+I3
290      DO 56 I=1,12
291      JT=JT+1
292      T(JT)=-YP(I)
293  56 CONTINUE
294  55 CONTINUE
295      DO 57 I1=1,4
296      I3=L1B(I1)
297      IF(I3.EQ.0) GO TO 57
298      J3=J1B+M1B(I1)
299      I1=J3+1
300      I2=J3+I3

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301      DO 58 I=1,12
302      JT=JT+1
303      T(JT)=YM(I)
304      58 CONTINUE
305      57 CONTINUE
306      DO 59 II=1,2
307      I3=L2(II)
308      IF(I3.EQ.0) GO TO 59
309      J3=J1A+M2A(II)
310      I1=J3+1
311      I2=J3+I3
312      J4=J1B+M2B(II)
313      DO 60 I=1,12
314      J4=J4+1
315      JT=JT+1
316      T(JT)=YP(I)-YM(J4)
317      60 CONTINUE
318      59 CONTINUE
319      DO 61 II=1,2
320      I3=L3(II)
321      IF(I3.EQ.0) GO TO 61
322      J3=J1A+M3A(II)
323      I1=J3+1
324      I2=J3+I3
325      J4=J1B+M3B(II)
326      DO 62 I=1,12
327      J4=J4+1
328      JT=JT+1
329      T(JT)=ZP(I)+ET1*ZM(J4)
330      62 CONTINUE
331      61 CONTINUE
332      J1A=J1A+JA
333      J1B=J1B+JB
334      54 CONTINUE
335      53 CONTINUE
336      JY=0
337      S=1.
338      DO 63 II=1,4
339      I3=L1A(II)
340      IF(I3.EQ.0) GO TO 83
341      J3=M1A(II)
342      I1=J3+1
343      I2=J3+I3
344      DO 64 I=1,12
345      JY=JY+1
346      Y(JY)=S*R(I)
347      64 CONTINUE
348      83 S=-S
349      63 CONTINUE
350      IF(I8.EQ.0) GO TO 81
351      DO 65 I=1,18
352      JY=JY+1
353      Y(JY)=0.
354      65 CONTINUE
355      81 DO 76 II=1,2
356      I3=L2(II)
357      IF(I3.EQ.0) GO TO 76
358      J3=M2A(II)
359      I1=J3+1
360      I2=J3+I3

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361      DO 77 I=1,12
362      JY=JY+1
363      Y(JY)=R(I)
364      IF(I1.EQ.2) Y(JY)=-Y(JY)
365      77 CONTINUE
366      76 CONTINUE
367      DO 66 I1=1,2
368      I3=L3(I1)
369      IF(I3.EQ.0) GO TO 66
370      J3=M3A(I1)+JA
371      I1=J3+1
372      I2=J3+I3
373      DO 67 I=1,12
374      JY=JY+1
375      Y(JY)=R(I)
376      IF(I1.EQ.2) Y(JY)=-Y(JY)
377      67 CONTINUE
378      66 CONTINUE
379      CALL DECOMP(N,IPS,T)
380      CALL SOLVE(N,IPS,T,Y,XX)
381      M5=M-1
382      WRITE(3,96) M5
383      96 FORMAT('0',I3,'TH MODE ELECTRIC AND MAGNETIC CURRENTS')
384      C1=C3
385      IF(M5.EQ.0) C1=1.
386      C2=C4
387      JX=0
388      J2=L1A(2)
389      IF(J2.EQ.0) GO TO 84
390      WRITE(3,68)
391      68 FORMAT('0ELECTRIC CURRENT ON FIRST PART OF OUTSIDE CONDUCTOR')
392      CALL PRNT(L1A(1),J2,1,1,RA)
393      84 J2=L1A(4)
394      IF(J2.EQ.0) GO TO 74
395      WRITE(3,69)
396      69 FORMAT('0ELECTRIC CURRENT ON SECOND PART OF OUTSIDE CONDUCTOR')
397      J3=MA+MC
398      CALL PRNT(L1A(3),J2,J3,J3,RA)
399      74 J2=L1B(2)
400      IF(J2.EQ.0) GO TO 85
401      WRITE(3,70)
402      70 FORMAT('0ELECTRIC CURRENT ON FIRST PART OF INSIDE CONDUCTOR')
403      CALL PRNT(L1B(1),J2,1,1,RB)
404      85 J2=L1B(4)
405      IF(J2.EQ.0) GO TO 75
406      WRITE(3,71)
407      71 FORMAT('0ELECTRIC CURRENT ON SECOND PART OF INSIDE CONDUCTOR')
408      J3=MB+MC
409      CALL PRNT(L1B(3),J2,J3,J3,RB)
410      75 J1=L2(1)
411      J2=L2(2)
412      IF((J1+J2).EQ.0) GO TO 86
413      WRITE(3,72)
414      72 FORMAT('0ELECTRIC CURRENT IN APERTURE')
415      J3=M2A(1)+1
416      J4=MA+1
417      CALL PRNT(J1,J2,J3,J4,RA)
418      IF(J2.EQ.0) GO TO 86
419      WRITE(3,73)
420      73 FORMAT('0MAGNETIC CURRENT IN APERTURE')

```

```

421      C1=C5
422      C2=C6
423      IF(M5.EQ.0) C2=2.
424      CALL PRNT(L3(1),J2,J4,J4,RA)
425      86 CONTINUE
426      STOP
427      END

```

\$DATA

2 20

```

-0.5773503E+00 0.5773503E+00
0.1000000E+01 0.1000000E+01
-0.9931286E+00-0.9639719E+00-0.9122344E+00-0.8391170E+00-0.7463319E+00
-0.6360537E+00-0.5108670E+00-0.3737061E+00-0.2277859E+00-0.7652652E-01
0.7652652E-01 0.2277859E+00 0.3737061E+00 0.5108670E+00 0.6360537E+00
0.7463319E+00 0.8391170E+00 0.9122344E+00 0.9639719E+00 0.9931286E+00
0.1761401E-01 0.4060143E-01 0.6267205E-01 0.8327674E-01 0.1019301E+00
0.1181945E+00 0.1316886E+00 0.1420961E+00 0.1491730E+00 0.1527534E+00
0.1527534E+00 0.1491730E+00 0.1420961E+00 0.1316886E+00 0.1181945E+00
0.1019301E+00 0.8327674E-01 0.6267205E-01 0.4060143E-01 0.1761401E-01
15 13 5 4 5 0 0 0 1 1
0.2500000E+01 0.1000000E+01 0.2000000E+01 0.0000000E+00
0.0000 0.2588 0.5000 0.7071 0.8660 0.9659 0.9914 1.0000 0.9914 0.9659
0.8660 0.7071 0.5000 0.2588 0.0000
-1.0000 -0.9659 -0.8660 -0.7071 -0.5000 -0.2588 -0.1305 0.0000 0.1305 0.2588
0.5000 0.7071 0.8660 0.9659 1.0000
0.0000 0.2415 0.4830 0.7244 0.9659 0.9914 1.0000 0.9914 0.9659 0.7244
0.4830 0.2415 0.0000
-0.2588 -0.2588 -0.2588 -0.2588 -0.2588 -0.1305 0.0000 0.1305 0.2588 0.2588
0.2588 0.2588 0.2588

```

\$STOP

/*

//

PRINTED OUTPUT

NT NPHI

2 20

XT

-0.5773503E+00 0.5773503E+00

AT

0.1000000E+01 0.1000000E+01

X

```

-0.9931286E+00-0.9639719E+00-0.9122344E+00-0.8391170E+00-0.7463319E+00
-0.6360537E+00-0.5108670E+00-0.3737061E+00-0.2277859E+00-0.7652652E-01
0.7652652E-01 0.2277859E+00 0.3737061E+00 0.5108670E+00 0.6360537E+00
0.7463319E+00 0.8391170E+00 0.9122344E+00 0.9639719E+00 0.9931286E+00

```

A

```

0.1761401E-01 0.4060143E-01 0.6267208E-01 0.8327675E-01 0.1019301E+00
0.1181945E+00 0.1316886E+00 0.1420961E+00 0.1491730E+00 0.1527534E+00
0.1527534E+00 0.1491730E+00 0.1420961E+00 0.1316886E+00 0.1181945E+00
0.1019301E+00 0.8327675E-01 0.6267208E-01 0.4060143E-01 0.1761401E-01

```

NA NB MA MB MC LA LB LC MI M2

15 13 5 4 5 0 0 0 1 1

```

BK UR ER THR(1)
0.2500000E+01 0.1000000E+01 0.2000000E+01 0.0000000E+00

```

RA

```

0.0000 0.2588 0.5000 0.7071 0.8660 0.9659 0.9914 1.0000 0.9914 0.9659
0.8660 0.7071 0.5000 0.2588 0.0000

```

ZA

```

-1.0000 -0.9659 -0.8660 -0.7071 -0.5000 -0.2588 -0.1305 0.0000 0.1305 0.2588
0.5000 0.7071 0.8660 0.9659 1.0000

```

RB

06

0.0000 0.2115 0.4830 0.7244 0.9659 0.9914 1.0000 0.9914 0.9659 0.7244
0.4830 0.2115 0.0000

ZB

-0.2588 -0.2588 -0.2588 -0.2588 -0.2588 -0.1305 0.0000 0.1305 0.2588 0.2588
0.2588 0.2588 0.2588

1TH MODE ELECTRIC AND MAGNETIC CURRENTS

ELECTRIC CURRENT ON FIRST PART OF OUTSIDE CONDUCTOR

REAL JT	IMAG JT	MAG JT
0.6125E+00	-0.8715E+00	0.1077E+01
0.5889E+00	-0.8223E-01	0.5946E+00
0.2590E+00	0.7627E+00	0.8055E+00
-0.4205E+00	0.1188E+01	0.1261E+01

REAL JP	IMAG JP	MAG JP
-0.6092E+00	0.1065E+01	0.1227E+01
-0.5386E+00	0.6909E+00	0.8760E+00
-0.3348E+00	0.4160E+00	0.5339E+00
-0.8079E-01	0.2966E+00	0.3074E+00
-0.5854E-01	0.5308E+00	0.5340E+00

ELECTRIC CURRENT ON SECOND PART OF OUTSIDE CONDUCTOR

REAL JT	IMAG JT	MAG JT
-0.4426E+00	-0.1791E+01	0.1844E+01
0.4508E+00	-0.1904E+01	0.1957E+01
0.1105E+01	-0.1607E+01	0.1950E+01
0.1478E+01	-0.1261E+01	0.1943E+01

REAL JP	IMAG JP	MAG JP
-0.8943E+00	-0.5344E+00	0.1042E+01
-0.2190E+00	-0.1271E+01	0.1290E+01
0.5212E+00	-0.1483E+01	0.1572E+01
0.1118E+01	-0.1365E+01	0.1765E+01
0.1525E+01	-0.1183E+01	0.1930E+01

ELECTRIC CURRENT ON FIRST PART OF INSIDE CONDUCTOR

REAL JT	IMAG JT	MAG JT
0.1267E+01	0.3871E-01	0.1268E+01
0.2608E+00	0.5877E-01	0.2673E+00
-0.7262E+00	0.1498E+00	0.7415E+00

REAL JP	IMAG JP	MAG JP
-0.1558E+01	-0.3328E-01	0.1558E+01
-0.1116E+01	-0.6796E-01	0.1118E+01
-0.7062E+00	-0.7621E-01	0.7103E+00
-0.1707E+00	-0.3173E+00	0.3603E+00

ELECTRIC CURRENT ON SECOND PART OF INSIDE CONDUCTOR

REAL JT	IMAG JT	MAG JT
-0.7549E+00	-0.2561E+00	0.8005E+00
0.2513E+00	-0.4657E-01	0.2556E+00
0.1280E+01	0.8836E-01	0.1283E+01

REAL JP	IMAG JP	MAG JP
0.6677E+00	-0.1862E+00	0.6932E+00
0.7989E+00	0.7428E-02	0.7989E+00

0.1129E+01 0.6751E-01 0.1131E+01
 0.1584E+01 0.1214E+00 0.1589E+01

ELECTRIC CURRENT IN APERTURE

REAL JT	IMAG JT	MAG JT
-0.1102E+01	0.6770E+00	0.1294E+01
-0.1261E+01	0.3286E+00	0.1303E+01
-0.1339E+01	-0.4550E-01	0.1339E+01
-0.1305E+01	-0.4478E+00	0.1380E+01
-0.1196E+01	-0.9023E+00	0.1498E+01

REAL JP	IMAG JP	MAG JP
-0.1580E+00	0.2658E+00	0.3092E+00
-0.2728E+00	0.1534E+00	0.3130E+00
-0.3750E+00	0.5005E-01	0.3783E+00
-0.5571E+00	-0.5482E-01	0.5598E+00

MAGNETIC CURRENT IN APERTURE

REAL JT	IMAG JT	MAG JT
0.1429E+00	0.7546E-01	0.1616E+00
0.1589E+00	0.1393E+00	0.2113E+00
0.1049E+00	0.1579E+00	0.1896E+00

REAL JP	IMAG JP	MAG JP
0.6528E+00	0.1541E+00	0.6708E+00
0.1559E+00	0.3855E+00	0.4158E+00
-0.1000E+00	0.4523E+00	0.4633E+00
-0.8395E+00	0.4811E+00	0.9675E+00

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- [4] V. I. Krylov, Approximate Calculation of Integrals, translated by A. H. Stroud, Macmillan Co., New York, 1962.

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