

AD-A110 451

WISCONSIN UNIV-MADISON MATHEMATICS RESEARCH CENTER
IMPROVED SUDAKOV-TYPE BOUNDS FOR OPTIMAL CONFIDENCE LIMITS ON T--ETC(U)
OCT 81 B HARRIS, A P SOMS

F/6 12/1

N00014-79-C-0321

NL

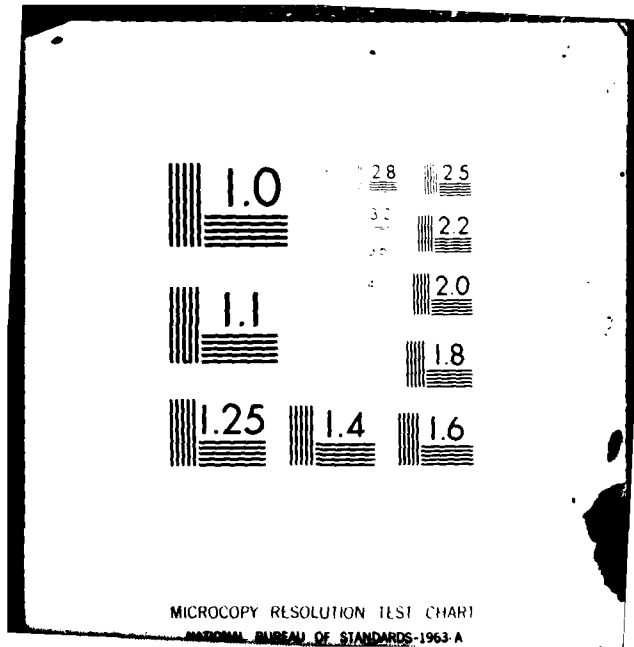
UNCLASSIFIED

MRC-TSR-2294

1 OF 1
AD-A
100000



END
DATE
FILMED
03-82
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

LEVEL

(2)

AD A110451

MRC Technical Summary Report # 2294

IMPROVED SUDAKOV-TYPE BOUNDS FOR
OPTIMAL CONFIDENCE LIMITS ON THE
RELIABILITY OF SERIES SYSTEMS

Bernard Harris and Andrew P. Soms

Mathematics Research Center
University of Wisconsin-Madison
610 Walnut Street
Madison, Wisconsin 53706

DTIC
SELECTED
FEB 4 1982
H

October 1981

(Received August 28, 1981)

Approved for public release
Distribution unlimited

Sponsored by

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

and

Office of Naval Research
Department of the Navy
Arlington, VA 22217

82 02 03 083

MRC FILE COPY

UNIVERSITY OF WISCONSIN - MADISON
MATHEMATICS RESEARCH CENTER

IMPROVED SUDAKOV-TYPE BOUNDS FOR
OPTIMAL CONFIDENCE LIMITS ON THE RELIABILITY
OF SERIES SYSTEMS

Bernard Harris* and Andrew P. Soms**

Technical Summary Report # 2294

October 1981

ABSTRACT

↳ A sharper Sudakov-type lower bound for the lower confidence limit on the reliability of a series system than the one given in Harris and Soms (1980) is obtained. Numerical examples, coverage probabilities and the listings of the short FORTRAN programs used are also provided. ↗

AMS (MOS) Subject Classifications: 62N05, 90B25.

Key Words: Optimal confidence bounds; Reliability; Series system.

Work Unit Number 4 - Statistics and Probability

Accession For	<input checked="" type="checkbox"/>
NTIS GRA&I	<input type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	<input type="checkbox"/>
By	
Distribution	
Availability Codes	
Dist	Special
A	

* University of Wisconsin-Madison

** University of Wisconsin-Milwaukee

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041 and the Office of Naval Research under Contract No. N00014-79-C-0321.



SIGNIFICANCE AND EXPLANATION

Serial Systems arise naturally in practice in engineering and physics. Therefore it is of substantial significance to be able to efficiently utilize data obtained on individual components for the purpose of obtaining an overall assessment of the reliability of the system. The methods of this paper may be employed for this purpose. As an example, suppose there are five subsystems, the sample sizes are (10, 15, 20, 25, 30) and the observed failures (1, 2, 1, 1, 2). Then the methods of this paper give that the optimum 95% lower confidence bound on the system reliability lies between .3871 and .3934.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

IMPROVED SUDAKOV-TYPE BOUNDS FOR
OPTIMAL CONFIDENCE LIMITS ON THE RELIABILITY
OF SERIES SYSTEMS

Bernard Harris* and Andrew P. Soms**

1. The Improved Sudakov-Type Bound

We will adhere to the notation of Harris and Soms (1980) and we refer the reader to this report for background material and additional references. Let V_i , $1 \leq i \leq k$, be independent binomial random variables with parameters n_i and p_i (the success probability) and let $X_i = n_i - V_i$, $q_i = 1 - p_i$, $1 \leq i \leq k$, with $n_1 \leq n_2 \leq \dots \leq n_k$ here and throughout. It is desired to obtain a "good" lower confidence limit on $\prod_{i=1}^k p_i$, the reliability of a series system with independent components. Let $g(\vec{x}) = g(x_1, x_2, \dots, x_k)$ be a monotonically decreasing or increasing (in each component) ordering function for real (not necessarily integral) x_i , $0 \leq x_i \leq n_i$, with large or small values, respectively, of $g(\vec{x})$ being best. Let $r_1 > r_2 > \dots > r_s$ be the ordered values of $g(\vec{x})$ in the decreasing case and $r_1 < r_2 < \dots < r_s$ in the increasing and let $A_i = \{\vec{x} | g(\vec{x}) = r_i\}$, $i = 1, 2, \dots, s$. Then (A_1, A_2, \dots, A_s) is a monotonic partition, i.e., $(0, 0, \dots, 0) \in A_1$, $(n_1, n_2, \dots, n_k) \in A_s$ and if $\vec{x}_1 = (x_{11}, \dots, x_{1k})$, $\vec{x}_2 = (x_{21}, \dots, x_{2k})$ with $x_{1i} \leq x_{2i}$, $i = 1, 2, \dots, k$, then $\vec{x}_1 \in A_i$ implies $\vec{x}_2 \in A_j$, $j \geq i$.

*University of Wisconsin-Madison.

**University of Wisconsin-Milwaukee.

Let

$$f(\tilde{x}; \tilde{p}) = P_{\tilde{p}}(\tilde{X}=\tilde{x}) = \prod_{i=1}^k \binom{n_i}{x_i} p_i^{n_i-x_i} q_i^{x_i} = \prod_{i=1}^k \binom{n_i}{v_i} p_i^{v_i} q_i^{n_i-v_i} \quad (2.1)$$

and for $1 \leq n \leq s-1$, let

$$a_n = \inf \left\{ \prod_{i=1}^k p_i \mid \sum_{i \leq n} \tilde{x}_j : \tilde{x}_j \in A_i, f(\tilde{x}_j; \tilde{p}) \geq \alpha \right\} \quad (2.2)$$

and $a_s = 0$. Each such partition may be identified with a function defined on the set of sample outcomes by defining the ordering function $g(\tilde{x})$, where

$$g(\tilde{x}) = n \quad \text{if} \quad \tilde{x} \in A_n, \quad 1 \leq n \leq s; \quad (2.3)$$

obviously $g(\tilde{x})$ inherits the monotonicity properties of the partition.

For defining Buehler's (1957) method of optimal lower confidence intervals on $\prod_{i=1}^k p_i$ we assume that $g(\tilde{x})$ has been redefined, if necessary, as in (2.3). Then we have, from Harris and Soms (1980),

Theorem 1. Let \tilde{X} be distributed by (2.1). Then $a_{g(\tilde{X})}$ is a $(1-\alpha)$ lower confidence bound for $\prod_{i=1}^k p_i$. If $b_{g(\tilde{X})}$ is also a $(1-\alpha)$ lower confidence bound for $\prod_{i=1}^k p_i$ which is monotonically decreasing in $g(\tilde{x})$, then $b_i \leq a_i, 1 \leq i \leq s$.

We now let $g(\tilde{x})$ denote the original ordering function, since this is necessary for the applications below. In order to obtain bounds for $a_{g(\tilde{x}_0)}$ we must assume that \tilde{x}_0 is such that for each $t = 1, 2, \dots, k$, the equation

$$g(i_1, i_2, \dots, i_{t-1}, y_t, 0, \dots, 0) = g(\tilde{x}_0) \quad (2.4)$$

has a unique solution $y_t, y_t < n_t$, where $i_r, r = 1, 2, \dots, t-1$, are integers, $0 \leq i_r \leq y_r$. Define y_i^* by $g(0, 0, \dots, 0, y_i^*, 0, \dots, 0) = g(\tilde{x}_0)$.

Theorem 2. Under the same assumptions as for (2.7),

$$u'(n_1, n_2, y_1, y_2, \alpha) \leq a_{g(\bar{x}_0)} \leq \min_{1 \leq i \leq k} u(n_1, [y_i^*], \alpha),$$

where $u'(n_1, n_2, y_1, y_2, \alpha)$ is the solution in a of the equation

$$\sum_{i=1}^{k \sup} \prod_{f=1}^i p_f \sum_{i_1=0}^{[y_1]} \binom{n_1}{n_1 - i_1} p_1^{n_1 - i_1} q_1^{i_1} \prod_{i=2}^k (n_2 - y_2 + y_2 + 1) = \alpha.$$

It follows from the proof in Harris and Soms (1980) that

$$u(n_1, y_1, \alpha) \leq u'(n_1, n_2, y_1, y_2, \alpha).$$

Thus the only question is how great the improvement will be in using u' . The numerical examples in 2. show that this can be substantial.

Remarks. u' can be calculated quickly and efficiently by using a short FORTRAN program. The listing is given in the Appendix along with the listing of the program used to calculate coverage probabilities. Two ordering functions that satisfy (2.5) and (2.6) are $g(\bar{x}) = \prod_{i=1}^k ((n_i - x_i)/n_i)$ if $g(\bar{x}_0) > 0$ and $g(\bar{x}) = \sum_{i=1}^k x_i/n_i$ for sufficiently large n_i , $1 \leq i \leq k$ (see Harris and Soms, 1980, for details).

2. Coverage Probabilities and Numerical Examples

The ordering function used here throughout is

$g(\bar{x}) = \prod_{i=1}^k ((n_i - x_i)/n_i)$. Table 1 gives the coverage probabilities for $k = 3$, $\bar{n} = (5, 7, 10)$, $\alpha = .10$ and selected $\bar{p} = (p_1, p_2, p_3)$, for both the lower LB and upper UB bounds of (2.7). While the optimality property of Theorem 1 implies that there are \bar{p} for which the coverage probability is less than .9, it does not seem that there

are very many such \tilde{p} .

1. Coverage Probabilities for $\tilde{n} = (5,7,10)$ and $\alpha = .10$

\tilde{p}	Coverages	
	UB	LB
(.95,.95,.95)	.9999	.9999
(.95,.95,.90)	1.0000	1.0000
(.95,.95,.85)	1.0000	1.0000
(.95,.95,.80)	.9999	.9999
(.95,.95,.75)	.9999	.9999
(.95,.95,.70)	1.0000	1.0000
(.95,.95,.65)	.9535	.9927
(.95,.95,.60)	.9737	.9967
(.95,.95,.55)	.9869	.9874
(.95,.95,.50)	.9940	.9940

Table 2 gives some comparisons of LB, UB, the improved lower bound LBI and the true value TV for $k = 2$, $\alpha = .05$ and selected $\tilde{n} = (n_1, n_2)$ and $\tilde{x}_0 = (x_{01}, x_{02})$. LBI gives substantial improvement when n_1 is small compared to n_2 and little or none if n_1 is approximately the same as n_2 . The TV for $\tilde{n} = (5,5)$ and $\tilde{x}_0 = (1,1)$ agrees with the value in Lipow and Riley (1959).

2. Comparison of Bounds and True Value for $k = 2$ and $\alpha = .05$

\tilde{n}	\tilde{x}_0	LB	LBI	TV	UB
(5,5)	(1,1)	.2166	.2166	.2776	.3426
(5,10)	(1,1)	.2761	.3317	.3317	.3426
(5,10)	(2,3)	.0859	.1521	.1529	.1893
(10,20)	(1,2)	.5037	.5675	.5691	.6058
(10,20)	(4,5)	.1851	.2137	.2137	.2224

Table 3 gives LB, UB and LBI for $k = 5$, $\alpha = .05$ and selected \tilde{n} and \tilde{x} .

3. Comparison of Bounds for $k = 5$ and $\alpha = .05$

\tilde{n}	\tilde{x}_0	LB	LBI	UB
(10,10,10,10,10)	(1,2,0,1,1)	.2893	.2893	.3035
(10,15,20,25,30)	(1,2,1,1,2)	.3599	.3871	.3934
(10,15,20,25,30)	(1,1,2,3,4)	.2838	.3017	.3035
(10,15,20,25,30)	(2,3,4,4,4)	.1319	.1478	.1500

References

- Buehler, Robert J. (1957), "Confidence Limits for the Product of Two Binomial Parameters", Journal of the American Statistical Association, 52, 482-93.
- Harris, Bernard and Soms, Andrew P. (1980), "Bounds for Optimal Confidence Limits for Series Systems", Mathematics Research Center technical summary report #2093, University of Wisconsin-Madison.
- Lipow, M. and Riley, J. (1959), "Tables of Upper Confidence Bounds on Failure Probability of 1, 2, and 3 Component Serial Systems", Vols. I and II, Space Technology Laboratories.
- Mann, Nancy R., Schafer, Ray E., and Singpurwalla, Nozer D. (1974), Methods for Statistical Analysis of Reliability and Life Data, New York: John Wiley and Sons.

Appendix

The listings of the coverage and improved bound FORTRAN programs are given below.

```
N'S SAMPLE SIZES FROM SMALLEST TO LARGEST, P'S SUCCESS PROBABILITIES
K NUMBER OF SUBSYSTEMS, 1-ALPHA CONFIDENCE LEVEL
  DIMENSION P(10),N(20),NY(10), PLIMU(10),PI(10,41),LL(10),II(10)
100 FORMAT ( )
  EQUIVALENCE (I1,II(1)),(I2,II(2)),(I3,II(3)),(I4,II(4)),
  I(I5,II(5)),(I6,II(6)),(I7,II(7)),(I8,II(8)),(I9,II(9)),(I10,II(10)
  2)
  1 READ 100,K,ALPHA
  IF (K.EQ.0) GO TO 21
  DO 221 I=1,10
  LL(I)=0
221 N(I)=0
  READ 100,(N(I),P(I),I=1,K)
  DO 343 I1=1,K
  DO 343 I2=0,N(I1)
  CALL MDBIN(I2,N(I1),P(I1),PT,PI(I1,I2+1),IER)
343 IF (PT.LE..0001) LL(I1)=LL(I1)+1
  PARAM=1.
  IPROD2=1
  DO 11 I=1,K
  IPROD2=IPROD2*N(I)
  11 PARAM=PARAM*P(I)
  PRL=0.
  PRU=0.
  DO 122 I1=LL(1),N(1)
  DO 122 I2=LL(2),N(2)
  DO 122 I3=LL(3),N(3)
  DO 122 I4=LL(4),N(4)
  DO 122 I5=LL(5),N(5)
  DO 122 I6=LL(6),N(6)
```

```

DO 122 I7=LL(7),N(7)
DO 122 I8=LL(8),N(8)
DO 122 I9=LL(9),N(9)
DO 122 I10=LL(10),N(10)
PROBT =1.
DO 511 IT=1,K
INT=II(IT)
511 PROBT=PROBT*PI(IT,INT+1)
IPROD1=1
DO 4 IJ=1,K
IN=II(IJ)
4 IPROD1=IPROD1*IN
IF (IPROD1.EQ.0) GO TO 15
ITEM1=IPROD2-IPROD1
Y1=N(1)*ITEM1/FLOAT(IPROD2)
DO 5 IT=1,K
NY(IT)=N(IT)*ITEM1/IPROD2
A=FLOAT(N(IT)-NY(IT))
R=FLOAT(NY(IT)+1)
5 CALL MDBETA(PARAM,A,B,PLIMU(IT),IER)
CALL MDBETA(PARAM,N(1)-Y1,Y1+1.,PLIML,IER)
IF (PLIML.GE.ALPHA) PRL=PRL+PROBT
IFLAG=0
DO 6 IT=1,K
6 IF (PLIMU(IT).GE.ALPHA) IFLAG=1
IF (IFLAG.EQ.1.) PRU=PRU+PROBT
GO TO 122
15 PRL=PRL+PROBT
PRU=PRU+PROBT
122 CONTINUE
PRINT 100,K,ALPHA,(N(I),P(I),I=1,K)
PRINT 100,PRU,PRL
GO TO 1
21 STOP
END

```

C SERIES WITH UNLIKE COMPONENTS

C MDBIN, MDBETA, MDBETI IMSL BINOMIAL, BETA, INVERSE BETA ROUTINES,
C N'S SAMPLE SIZES FROM SMALLEST TO LARGEST, X'S FAILURES, NINT NUMBER
C OF P VALUES CONSIDERED FOR SUP, EPS MAXIMUM ERROR IN SOLUTION FOR
C REFINED BOUND, NO X'S EQUAL TO N'S

REAL ITEM1,ITEM2,ITEM3,ITEMT
DIMENSION N(50),X(50)

100 FORMAT ()

DATA EPS/.001/

1 READ 100,K,ALPHA,NINT

IF (K.EQ.0) GO TO 99

READ 100,(N(I),X(I),I=1,K)

52 PRINT 100,'DATA',(N(I),X(I),I=1,K)

PRINT 100,'ALPHA=',ALPHA

ITEM1=1.

ITEM2=1.

DO 2 I=1,K

ITEM1=ITEM1*N(I)

2 ITEM2=ITEM2*(N(I)-X(I))

ITEM3=N(1)*(ITEM1-ITEM2)

Y1=ITEM3/ITEM1

NY1=Y1

CALL MDBETI(ALPHA,N(1)-Y1,Y1+1.,A,IER)

C COMPUTATION OF UPPER BOUND

BU=1.

DO 3 I=1,K

ITEMT=N(I)*(ITEM1-ITEM2)

NYT=ITEMT/ITEM1

CALL MDBETI(ALPHA,FLOAT(N(I)-NYT),NYT+1.,BUT,IER)

3 IF (BUT.LT.BU) BU=BUT

PRINT 100,'L-M LOWER BOUND =',A

PRINT 100,'UPPER BOUND =',BU

- C COMPUTATION OF IMPROVED LOWER BOUND

TEML=A

TEMU=RU

9 TEM=(TEML+TEMU)/2.

DELTA=(1-TEM)/NINT

```

SUP=0.
DO 42 I=1,NINT
P1=TEM+(I-1)*DELTA
P2=TEM/P1
Q1=1-P1
PROB=0.
DO 33 K1=0,NY1
Y2=N(2)-(ITEM2/ITEM1)*(N(1)*N(2)/(N(1)-K1)
CALL MDBIN(K1,N(1),Q1,PS1,PK1,IER)
CALL MDBETA(P2,N(2)-Y2,Y2+1.,PP2,IER)
33 PROB=PROB+PK1*PP2
IF (PROB.GT.SUP) SUP=PROB
42 CONTINUE
IF (SUP.LE.ALPHA) TEMPL=TEM
IF (SUP.GT.ALPHA) TEMU=TEM
IF (ABS(TEMU-TEMPL).LE.EPS) GO TO 10
GO TO 9
10 A=TEMPL
PRINT 100,'IMPROVED LOWER BOUND =',A
GO TO 1
99 STOP
END

```

CONFIDENTIAL - T - 1111

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER #2294	2. GOVT ACCESSION NO. A-4440	3. RECIPIENT'S CATALOG NUMBER 454
4. TITLE (and Subtitle) IMPROVED SUDAKOV-TYPE BOUNDS FOR OPTIMAL CONFIDENCE LIMITS ON THE RELIABILITY OF SERIES SYSTEMS		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Bernard Harris and Andrew P. Soms		8. CONTRACT OR GRANT NUMBER(s) N00014-79-C-0321 DAAG29-80-C-0041
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of 610 Walnut Street Madison, Wisconsin 53706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 4 - Statistics and Probability
11. CONTROLLING OFFICE NAME AND ADDRESS (see Item 18 below)		12. REPORT DATE October 1981
		13. NUMBER OF PAGES 11
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES U.S. Army Research Office and Office of Naval Research P. O. Box 12211 Department of the Navy Research Triangle Park Arlington, VA 22217 North Carolina 27709		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Optimal confidence bounds; Reliability; Series system		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A sharper Sudakov-type lower bound for the lower confidence limit on the reliability of a series system than the one given in Harris and Soms (1980) is obtained. Numerical examples, coverage probabilities and the listings of the short FORTRAN programs used are also provided.		

DATE
FILMED
8-8