END
DATE
FILMED
3-82
DTIC

1.0

1.1

1.25 1.4 1.6

4.5 2.8 2.5
3.2 2.2
3.6
4.0 2.0

1.8

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
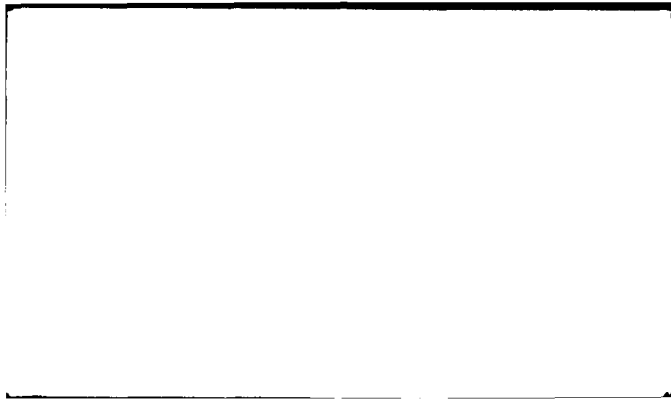
DESMATICS, INC.

Applied Research in Statistics - Mathematics - Operations Research

VALIDATION OF
COST ALLOCATION METHODOLOGIES

by

Dennis E. Smith
and
Robert L. Gardner

TECHNICAL REPORT NO. 115-1

February 1982
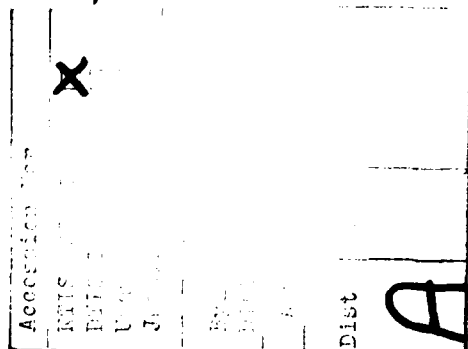
Introduction

This report is essentially a transcription of a presentation given at the Resource Analysis and Management Working Group of the 48th Military Operations Research Symposium. Because the topic discussed, the validation of cost allocation methodologies, may be of interest to a wider audience, we have prepared this report for dissemination.

Slide #1

In this paper we present a general discussion of a difficult validation problem--the validation of the algorithms used to allocate operating and support (O&S) costs in a military cost reporting system. It is based on a study currently being conducted by Desmatics, Inc. for the Office of VAMOSC II, within the Air Force Logistics Command (AFLC), USAF.

Cost allocation is an imprecise art at best; cost accountants have wrestled for years with the problems involved in allocating factory overhead among production departments. The military counterpart to that problem is much more complex than that usually encountered in the typical industrial model.

Slide #2

Although this paper is intended to have broad applicability in the area of cost allocation validation, it will be discussed in terms of a specific system currently under development by the U.S. Air Force. This system is called the Weapon Systems Support Cost (WSSC) system. It is a subsystem of the Visibility and Management of Support Costs (VAMOSC) system. Actually

the Air Force refers to its VAMOSC system as VAMOSC II, since the system
now under development is a replacement for, and considerable enhancement of,
an existing system which has been in operation since 1976. WSSC is concerned
with aircraft weapon systems as distinguished from large ground-based radar/
communication systems or system "black box" components. These two categories
are handled by the Communication-Electronics System (C-E) and the Component
Support Cost System (CSCS).

It should be pointed out that the WSSC system deals with actual, incurred
costs for the operation and support of aircraft weapon systems. The costs
portrayed in WSSC do not include any R&D or production costs. Furthermore,
WSSC is a cost collection system, not an accounting system or a cost estimat-
ing system.

Slide #3

WSSC's primary product is an annual report which provides a cost break-
down for each fleet of aircraft weapon systems, identified by MDS (mission-
design-series), such as B52H, C5A, F15A. Slide 3 illustrates in highly sim-
plified format the type of operating and support cost data presented in the
primary WSSC system report product, the WSSC AF Detail Report. Shown here are
just four of the major cost categories into which O&S costs are divided. Each
of these is further broken down into two or more subcategories. For example,
unit operations include aircrew, command staff, security, fuel, munitions
training and other unit costs. Installation Support includes real property
maintenance (RPM), base operating support (BOS) and communications (COM).

A factor common to most of the cost categories is that the cost information
required to fill in this matrix is usually not available directly, but must be
allocated to the MDS level. Within these cost categories are several unique

-2-

allocation problems. For instance fuel cost, medical cost, aircrew cost, maintenance cost and installation support cost data are each available from different sources and at different levels of identification, thus necessitating different treatments to get the required costs at the MDS level. This means that several different allocation algorithms must be used by WSSC.

Slide #4

This slide illustrates the flow of data into the WSSC system. Most of the O&S costs are incurred at airbases within the several commands of the U.S. Air Force. Data flows up through commands into several data systems which existed before VAMOSC and WSSC were designed.

An important constraint on WSSC operation is that WSSC must utilize data from existing systems; separate data collection facilities may not be constructed for WSSC. Much of the cost information required by WSSC is available through the Air Force's Accounting and Budget Distribution System (ABDS). However, fuel costs and depot maintenance costs must be obtained from other sources, as shown in the illustration.

The cost data in ABDS is reported by base and is identified by program element codes (PEC), responsibility center/cost center codes (RC/CC) and element of expense/investiment codes (EEIC), but not by MDS. Thus, to get the required fleet costs for each MDS, it is necessary to classify costs by type, apply appropriated allocation algorithms at base level and roll these costs up to fleet level.

Slide #5

Personnel strength, flying operations and maintenance manhour data are

-3-

utilized by WSSC as the basis for allocating costs to aircraft. Slide 5
summarizes the types of data used by WSSC for cost allocations. Personnel
strengths come from the E300Z Advanced Personnel System. Flying hours and
possessed hours (from which aircraft counts are derived) come from the GO33B
AVISURS system. Maintenance manhours are obtained from the E506 AMMIS System.

In this paper we will not describe all of the categories used in WSSC,
nor will we discuss all the allocation algorithms employed by WSSC. We will
concentrate instead on just one of these cost categories---installation
support costs---since they resemble the typical overhead allocation problem
encountered in industrial cost accounting.

Slide #6

This slide shows the task which WSSC must perform with respect to the
installation support costs incurred at base level. ABDS reports the annual
costs by base as a number of individual line items. Some of these items are
clearly fixed costs which, according to DOD guidelines, are not to be allocated
to aircraft. Other costs, such as BOS, RPM, and COM, have both fixed a : vari-
able components. The fixed components cannot be identified directly, so a
portion should be factored out of the total. The variable costs (the portion
remaining after the fixed component has been removed) constitutes the total
burden to be allocated among all tenants, including both the aircraft weapon
systems at that base and any non-aircraft tenants. This requires application
of another allocation algorithm.

The remaining BOS, RPM and COM costs constitute the aircraft variable
cost burden which is to be allocated among the base aircraft. The question
is, "What is the most reasonable basis for allocating these costs?". In this

-4-

example there are 72 TAC fighters, 15 SAC bombers and 15 SAC tankers at the same base, closely resembling the configuration on at least one AF CONUS base. The flying hours, crew sizes and total supported strengths represent typical figures taken from an Air Force planning document[1]. They are representative of the type of information available for use in allocation of overhead costs. In this example the fighters have more flying hours, crew and total strenth and probably should bear the majority of the overhead burden. Between the bombers and tankers, however, the picture is mixed. The tankers fly more, but the bombers require more personnel for whom support costs must be incurred.

That essentially is the type of allocation problem which a system such as the WSSC system must address in order to provide the type of cost breakdowns required to achieve the necessary visibility of operating and support costs for aircraft weapon systems. The main features of the problem are: (1) a requirement to provide costs at the weapon system level, (2) availablility of cost data from several sources, but lack of available data at the required level, thus necessitating the use of allocation algorithms, and (3) availability of some (possibly conflicting) types of data for use as the basis of allocation.

The task in validating the specific choices made in the design of a military cost allocation system is to identify the reasonable alternatives and assess them in a framework which is as objective as the situation permits. The remainder of this paper addresses a general methodology for completing this task, using WSSC as an illustrative case.

---

[1]AFP 173-13 "USAF Cost and Planning Factors," 1 February 1980.

Slide #7

In a general context, there are a number of cost categories (N) to be considered. For each cost category, we have the total expenditures over all MDS's. These total expenditures must be apportioned to each MDS, often by means of an allocation algorithm.

Slide #8

The purpose of the type of validation effort discussed in this paper is to examine critically the underlying algorithms for allocating the costs to each MDS. It is not concerned with the related task of determining whether the WSSC computer programs are correct, i.e., whether the algorithms have been coded correctly and thoroughly debugged.

If we consider C_{ij} , the costs in category j allocated by WSSC to MDS i, the validation task we are discussing has as its objective the comparison of C_{ij} with T_{ij} , the corresponding "true" costs. Of course, because we are concerned with each of a number (K) of MDS's and each of N cost categories, we can consider the overall task as comparing the NxK matrix $\underline{C}$ with the NxK matrix $\underline{T}$.

Notice, however, that the term "true" costs was put in quotations. This indicates that, in many cases, the actual costs that should be assigned to a given MDS are unknown, except perhaps by an omniscient being.

Slide #9

Thus, direct validation by examination of the differences between the WSSC allocation cost matrix $\underline{C}$ and the true cost matrix $\underline{T}$ is not possible, in general. Instead, any validation effort must take an indirect route. Such indirect validation is concerned with two general types of validity. One,

face validity, involves a qualitative assessment, while the other, mathe-
matical validity, involves a quantitative assessment.

Slide #10

Face validity, as we will use the term, refers to the examination of the
algorithms on a subjective, common-sense basis. In examining face validity,
we must ask whether the allocation algorithms appear reasonable, particularly
when compared to possible alternative allocation schemes. We must also con-
sider whether the algorithms appear to provide equitable results, e.g., whether
the choice of an allocation basis is fair, and does not make one group of MDS's
bear an inordinate amount of the costs.

We must also ask ourselves if the data base can be improved. Perhaps the
data base currently used in cost allocation can be replaced by a different data
base which permits allocation at a lower, and thus more accurate, level. Al-
though the data base really does not form a part of any allocation algorithm,
a good algorithm can, of course, be made useless if it is based on inaccurate
data. Thus, some assessment must be made of data base accuracy.

In conjunction with each of these steps, a major check on face validity
is to consult with potential users throughout the development of a system
of cost allocation algorithms. Their suggestions and/or criticisms can often
result in major improvements which yield increased face validity. As an aside,
we should note that the Air Force Office of VAMOSC has worked closely with
potential users of WSSC during its development.

Slide #11

Parallel with the consideration of face validity, we must also look at the

algorithms from a quantitative standpoint. This involves the concept of mathematical validity, which refers to the quantitative evaluation of the mathematical framework underlying the cost allocation algorithms. We must, as part of this type of validation, examine whether the algorithms are consistent.

In general, an allocation algorithm involves (implicitly or explicitly), one or more parameters to which values must be assigned. A major question, therefore, is how sensitive the cost allocations are to the assigned parameter values. If, for example, we find that cost allocations are very sensitive to a certain parameter, more study could be focused on that parameter and the associated algorithm.

We should also examine whether the parameter values used in the algorithms appear reasonable. For example, in a particular algorithm involving a parameter r, does a value of $r=100$ appear more reasonable from a mathematical viewpoint than does a value of $r=5$? Also we should ask whether the parameter values are stable. That is, can the same value of r be used year after year, or must it be revised each year?

Slide #12

Slide #12 presents an example of a consistency check. Let us assume that two cost allocation algorithms are being examined, and that each algorithm uses two items of data, X_1 and X_2 , as the basis for allocation.

Consider situation #1. Here, each algorithm provides the same allocation of costs to the four MDS's. Now, let us look at situation #2, where the data being used for allocation (X_1 and X_2) remains identical to that in situation #1 except for MDS #3 for which the values of both X_1 and X_2 have increased.

-8-

One of the first things we see in this situation is that both of the algorithms result in the same allocation of costs ($300K) to MDS #1. We might, therefore, venture that both algorithms are inconsistent because they do not allocate $240K as in situation #1. We might reason that since $X_1 = 200$ and $X_2 = 1000$ for MDS #1 in both situations, the identical cost amount should be allocated to this MDS in both situations. However, such a conclusion would be incorrect because other variables do not remain the same from the first to the second situation. At the very least, the situations must be separated in time. Thus, the assumption of "ceteris paribus" cannot be made. Hence, the allocation of $300K instead of $240K is not an indication of inconsistency.

However, since X_1 and X_2 are the basis for allocation, the _relative_ costs allocated to MDS #1, MDS #2 and #4 should be the same in both situations. Thus, since

$$240:420:460$$

are in the same ratio as

$$300:525:575 ,$$

allocation algorithm #1 is consistent in this respect. However, algorithm #2 is not consistent because

$$240:420:460$$

are not in the same ratio as

$$300:480:500 .$$

Slide #13

As far as sensitivity is concerned, we can examine the relative sensitivity

within a cost category (i.e., the proportional change) based only on the allocation data. However, to get an idea of absolute sensitivity (i.e., the actual change in dollars), we would need the total dollar amounts included in each category.

In any event, to evaluate the sensitivity of cost allocation algorithms to a particular parameter r, we can examine the partial derivative, taken with respect to r, of the relative (or absolute) allocated costs or we can investigate the extremes, i.e., the allocated costs if r is at its lowest possible value compared with the allocated costs if r is at its highest possible value.

Slide #14

In this slide, we present a hypothetical situation involving what is essentially one of the WSSC cost allocation procedures, the use of a weighted average of flying hours (FH) and possessed hours (PH). This weighted average is of the form

$$pFH + (1-p)PH$$

where $0 \leq p \leq 1$. This weighted average involves one parameter, p, which can range in value from p=0 (allocation based completely on possessed hours) to p=1 (allocation based completely on flying hours).

The simple illustration presented here provides an example involving only four MDS's. (WSSC deals with approximately one hundred MDS's.) Here we have considered sensitivity of the cost allocations based on the minimum and maximum values of p. As can be seen, MDS #3 is most sensitive as p varies from 0 to 1, with the corresponding change of -23% on a relative scale or -$460K on an absolute scale assuming a total of $2000K to allocate.

-10-

If we examine the distribution of the sensitivity values by plotting them, we may gain a great deal of information. Of course, in our simple example of four MDS's, plotting is not worthwhile as it would be when used with actual WSSC data.

Slide #15

It should be noted that because of the way the sensitivity values are defined, their mean is always zero. Therefore, we know that to some degree the sensitivity values will cluster around zero. What we need to examine is how they cluster. Specifically, we must pay attention to the spread in the data, to the existence of outliers, and to the existence of any particular patterns.

For example, consider the three sensitivity value distributions in the lower half of the slide. We certainly would have more concern about the second and the third than we would about the first. Our concern about the second distribution would be focused on its wide spread which tells us that the cost allocations are very sensitive to the value selected for the parameter. Thus, particular attention should be paid to the specification of this value. For the third distribution, there are indications that a few MDS's are extremely sensitive to the assignment of the parameter value. In this case, attention must be paid to these particular MDS's to determine whether perhaps a revised allocation algorithm might be required.

Slide #16

As previously noted, we cannot directly verify the costs allocated to each MDS. However, if data from a number of years is available, we can ex-

amine stability/reasonableness by looking at the relationship between total costs for a given cost category and the total allocation basis, which involves a corresponding parameter vector r. In other words, for a given cost category such as installation support, we can check the relationship between total costs and the total allocation basis when different allocation parameters are used.

In general, we would hope that for the best choice of the parameter vector r there would be a relatively strong relationship, at least when total costs are adjusted for the effects of other variables (for example, by means of the analysis of covariance). In exploring such adjustments, examination of existing cost estimating relationships may prove of value.

If, for a given value of the parameter vector r, we obtain a good relationship, it would probably be best to use that value of r in the allocations, assuming that there is no evidence to the contrary. For example, in this slide, the use of the parameter vector value r_0 provides a better relationship than does r_1. Thus, r_0 is probably a better value to use. Of course, if we obtain negative results, this would reflect on either unreasonableness or on lack of stability.

Summary

In summary we must point out that, to many people, the term validation has the connotation of proving that a cost allocation methodology is either correct or incorrect. Actually, the most a validation effort can usually do is to provide increased confidence in the procedure if no defects are revealed. We hope that this paper has provided some general guidelines that may prove of value in validation studies concerned with systems such as WSSC.

VALIDATION OF
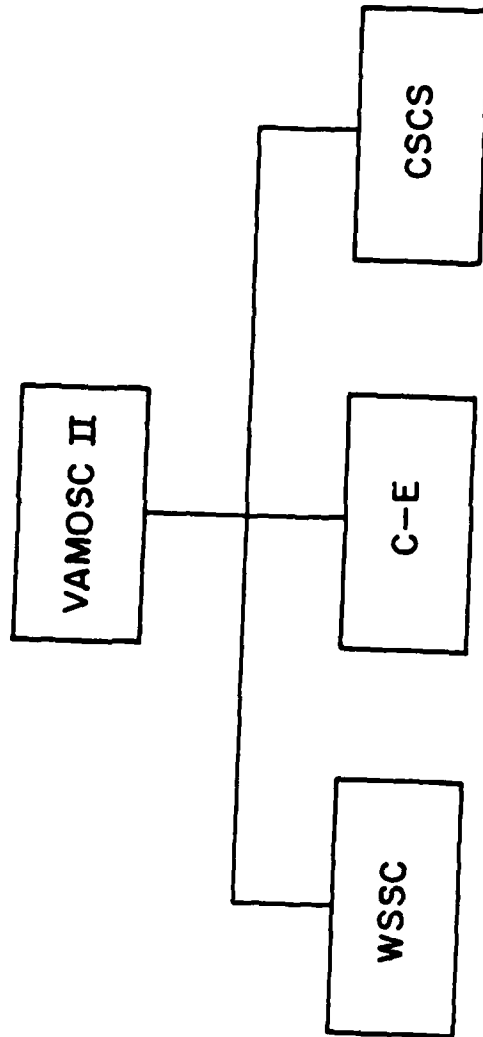
COST ALLOCATION METHODOLOGIES

BY

DENNIS E. SMITH

ROBERT L. GARDNER

DESMATICS, INC.

BASED ON A RESEARCH STUDY FOR

THE AIR FORCE OFFICE OF VAMOSC

Slide #1

-13-

USAF VAMOSC II SYSTEM ELEMENTS

VAMOSC II

CSCS

C—E

WSSC

Slide # 2

-14-

WSSC ANNUAL REPORT FORMAT

AIRCRAFT O & S COSTS

MDS B52G   FY 81

COST ($MILLIONS)

COST CATEGORY	TOTAL	MATERIEL	CONTRACT	PAY	OTHER
UNIT OPERATIONS					
INSTALL. SUPPORT					
DEPOT MAINT.					
BELOW DEPOT MAINT.					

Slide #3
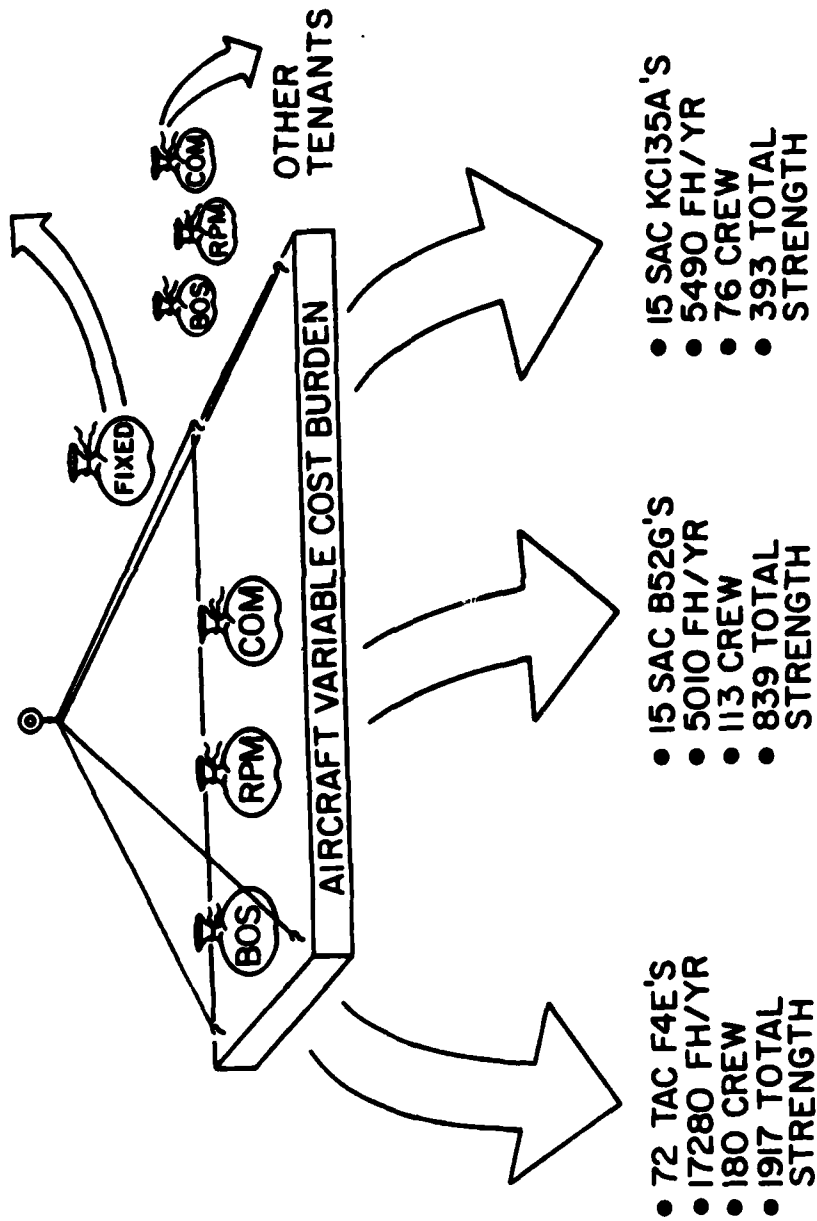
-15-

FLOW OF DATA INTO THE WSSC SYSTEM

Slide #4

-16-
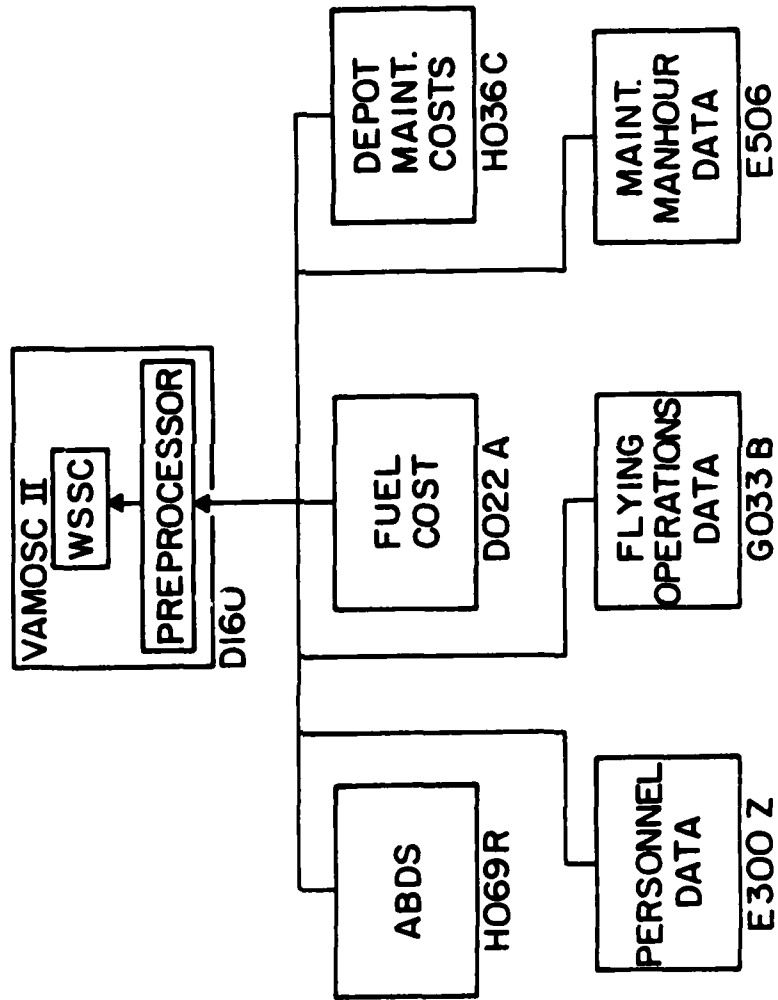
TYPES OF DATA USED FOR WSSC COST ALLOCATIONS

- PERSONNEL STRENGTHS

- NUMBER OF AIRCRAFT (POSSESSED HOURS)

- FLYING HOURS

- MAINTENANCE MANHOURS

Slide #5

-17-

ALLOCATION OF INSTALLATION SUPPORT COSTS

Slide #6

-18-

WSSC COST ALLOCATION

COST CATEGORIES

	#1	#2		#N
TOTAL EXPENDITURES:				
ALL MDS	C_1	C_2	. . .	C_N

WSSC

ALLOCATED EXPENDITURES:
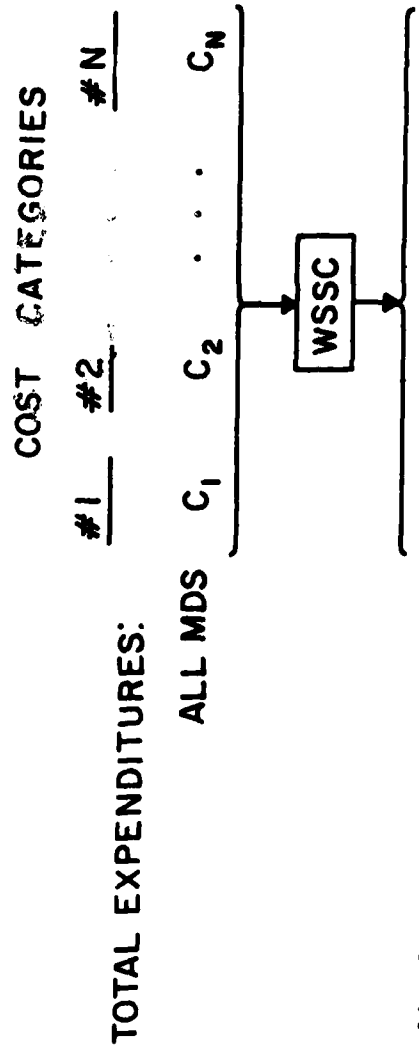
MDS #1	C_{11}	C_{21}	. . .	C_{N1}
MDS #2	C_{12}	C_{22}	. . .	C_{N2}
. . .	. . .	. . .	. . .	. . .
MDS #K	C_{1K}	C_{2K}	. . .	C_{NK}

Slide #7

VALIDATION OF WSSC METHODOLOGY

• FOCUS IS ON THE UNDERLYING ALGORITHMS

• TWO MATRICES:

$$\underline{C} = \begin{bmatrix} C_{11} \cdots C_{1K} \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ C_{N1} \cdots C_{NK} \end{bmatrix} \qquad \underline{T} = \begin{bmatrix} T_{11} \cdots T_{1K} \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ T_{N1} \cdots T_{NK} \end{bmatrix}$$

C_{ij} — COSTS IN CATEGORY i ALLOCATED BY WSSC TO MDS j

T_{ij} — "TRUE" COSTS OF MDS j WITHIN CATEGORY i

• OBJECTIVE IS TO COMPARE $\underline{C}$ AGAINST $\underline{T}$

Slide #8

-20-

VALIDATION APPROACHES

- DIRECT VALIDATION NOT POSSIBLE, IN GENERAL

- INDIRECT VALIDATION:

  - FACE VALIDITY (QUALITATIVE)

  - MATHEMATICAL VALIDITY (QUANTITATIVE)

Slide #9

-21-

FACE VALIDITY

- ARE THE ALLOCATION ALGORITHMS REASONABLE ?

- ARE THE ALLOCATION ALGORITHMS EQUITABLE ?

- CAN THE DATA BASE BE IMPROVED ?

- DO POTENTIAL USERS HAVE ANY SUGGESTIONS ?

Slide #10

-22-

MATHEMATICAL VALIDITY

- ARE THE ALLOCATION ALGORITHMS CONSISTENT ?

- HOW SENSITIVE ARE THE ALGORITHMS TO ASSIGNED PARAMETER VALUES ?

- DO PARAMETER VALUES APPEAR MATHEMATICALLY REASONABLE ?

- ARE PARAMETER VALUES STABLE ?

Slide #11

-23-

AN EXAMPLE OF A CONSISTENCY CHECK

SITUATION #1

	ALLOCATION DATA		COST ALLOCATION (IN $1000)	
	X_1	X_2	#1	#2
MDS #1	200	1000	240	240
MDS #2	100	2000	420	420
MDS #3	(400	5000)	880	880
MDS #4	300	2000	460	460

SITUATION #2

	ALLOCATION DATA		COST ALLOCATION (IN $1000)	
	X_1	X_2	#1	#2
MDS #1	200	1000	300	300
MDS #2	100	2000	525	480
MDS #3	(500	8000)	1250	1300
MDS #4	300	2000	575	500

ALLOCATION #1 IS CONSISTENT (420/240 = 525/300, ETC.)

ALLOCATION #2 IS INCONSISTENT (420/240 ≠ 480/300, ETC.)

Slide #12

-24-

SENSITIVITY

- CAN CONSIDER:

  - PROPORTIONAL CHANGE IN ALLOCATED COSTS
    (RELATIVE SENSITIVITY)

  - TOTAL CHANGE IN ALLOCATED COSTS
    (ABSOLUTE SENSITIVITY)

- CAN EXAMINE SENSITIVITY BY MEANS OF:

  - PARTIAL DERIVATIVES [$\partial C(r) / \partial r$]

 - EXTREMES [$C(r_L)$, $C(r_U)$]

Slide #13

-25-

SENSITIVITY EXAMPLE

- CONSIDER WEIGHTED AVERAGE $pFH + (1-p)PH$

- ASSUME COSTS OF $2000K TO ALLOCATE

- AN ILLUSTRATION:

	FH	PH	RELATIVE ALLOCATIONS		SENSITIVITY VALUES	
			p=0	p=1	RELATIVE	ABSOLUTE
MDS#1	150	2,800	.14	.15	+.01	+$ 20K
MDS#2	280	4,200	.21	.28	+.07	+$140K
MDS#3	200	8,600	.43	.20	-.23	-$460K
MDS#4	370	4,400	.22	.37	+.15	+$300K
TOTALS	1,000	20,000	1.00	1.00	0	0

Slide #14

-26-

SENSITIVITY EXAMINATION

● EXAMINE SENSITIVITY VALUES FOR :

    – SPREAD
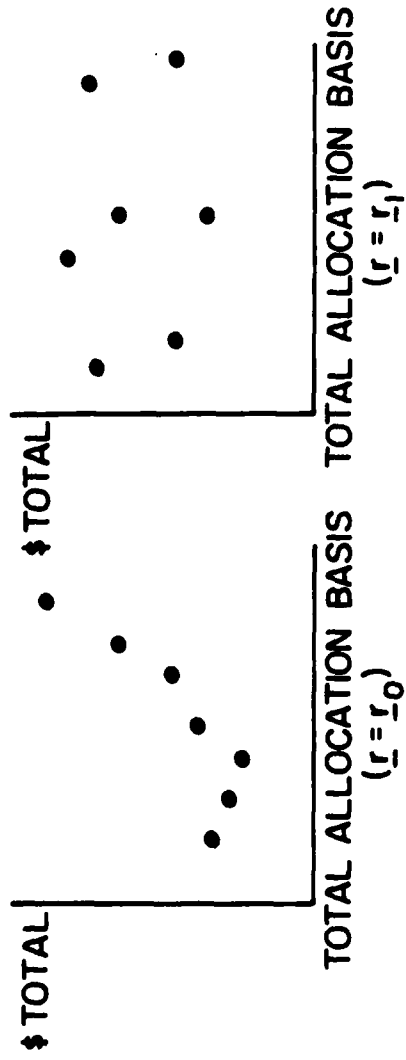
    – OUTLIERS

    – PATTERNS

● EXAMPLES :

Slide #15

-27-
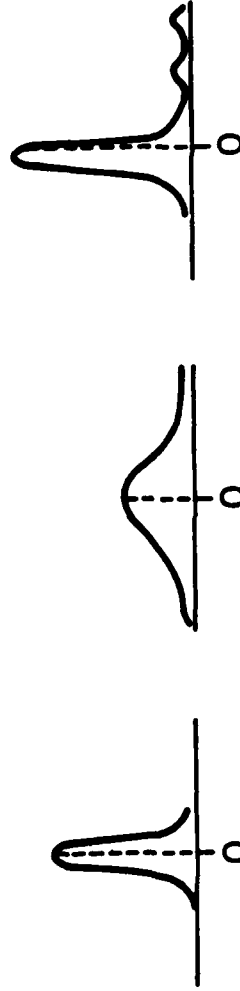
STABILITY/REASONABLENESS

- USE DATA FROM A NUMBER OF YEARS

- CHECK TOTAL CATEGORY COSTS AGAINST TOTAL ALLOCATION BASIS USING DIFFERENT PARAMETER VALUES r

- EXAMPLES:

$TOTAL

TOTAL ALLOCATION BASIS
$(r = r_0)$

$TOTAL

TOTAL ALLOCATION BASIS
$(r = r_1)$

REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2 GOVT ACCESSION NO	3. RECIPIENT'S CATALOG NUMBER
115-1	A110 771	

4. TITLE (and Subtitle)	5. TYPE OF REPORT & PERIOD COVERED
VALIDATION OF COST ALLOCATION METHODOLOGIES	Technical Report
	6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)	8. CONTRACT OR GRANT NUMBER(s)
Dennis E. Smith and Robert L. Gardner	F33600-80-C-0554

9. PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT PROJECT, TASK AREA & WORK UNIT NUMBERS
Desmatics, Inc. P.O. Box 618 State College, PA 16801	

11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
HQ AFLC/LOC (VAMOSC) Wright-Patterson AFB, OH 45433	February 1982
	13. NUMBER OF PAGES 28

14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report)
	Unclassified
	15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution of this report is unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Presented at the Resource Analysis and Management Working Group
of the 48th Military Operations Research Symposium
(Monterey, CA, 1-3 December 1981)

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Cost Allocation Validation
O&S Costs
VAMOSC

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report presents a discussion of the validation of the algorithms used
to allocate operating and support (O&S) costs in a military cost reporting
system. It also provides some general guidelines that may prove of value in
validation studies concerned with similar systems.

DD FORM 1473 EDITION OF 1 NOV 65 IS OBSOLETE