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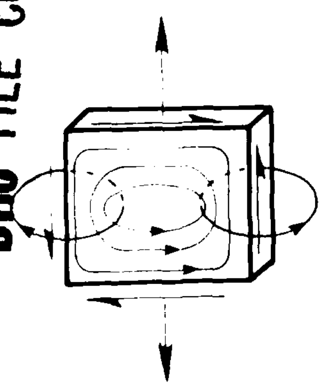

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Topical Report

THE VIRIAL THEOREM AND SCALING LAWS FOR

SUPERCONDUCTING MAGNET SYSTEMS[(1)]

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The structural masses of over thirty superconducting magnet designs are shown to exceed that required by the virial theorem by factors of from two to ten or more. Designs for MHD and toroidal field (TF) fusion magnets from many different magnet groups around the world obey a stored energy-mass scaling law of the form $E = CM^{4/3}$. This scaling law appears to be valid over three to four orders of magnitude of stored magnetic energy. The 4/3 power law differs from the linear scaling derived from the virial theorem.

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20. Abstract (continued)

and appears to result from the thermal stability requirements of the superconductor. However the masses of large scale TF magnets are approaching the regime where the structural mass required by the virial theorem will place greater constraints on the design than the thermal stability.

INTRODUCTION

Mass in superconducting magnet systems is required for three primary purposes; to carry current, to dissipate heat, and to withstand stresses due to magnetic, gravitational and earthquake related loads. In normal magnets, the first two needs are paramount whereas in superconducting systems the structural loads assume a greater role in determining the total mass, especially as the magnets increase in size.

Two countervailing ideas have intrigued many magnet designers regarding structural mass; one is the concept of a force-free magnet and the other is the principle embodied in the so called "virial theorem". This theorem relates the stored magnetic energy in the system to the integral of the trace of the stress tensor over the magnet volume. The first idea promises a mass reduced design based on thermal and current density constraints alone while the second says that there is a minimum structural mass required of every magnetic energy storage system.

One of the earliest references to force free fields is that of Chandrasekhar [1]. For nonferromagnetic materials one seeks a current distribution $\underset{\sim}{J}(\underset{\sim}{r})$ for which the resulting magnetic field $\underset{\sim}{B}$ is parallel to $\underset{\sim}{J}$, i.e.,

$$\underset{\sim}{f} = \underset{\sim}{J} \times \underset{\sim}{B} = 0 \tag{1}$$

For steady currents this condition is equivalent to finding a current distribution such that,

$$\nabla \times \underset{\sim}{B} = \mu_o \alpha \underset{\sim}{B} \tag{2}$$

where α may be a scalar function of position.

Solutions to this equation can be found but require unbounded current systems. This concept has been discussed by Furth et al. [2] and Wakefield [3]. Wells and Mills [4] experimented with force reduced magnets in the shape of toroids. Mawardi [5] has suggested that such force reduced magnets can be applied to the design of tokamak fusion magnets and magnets for energy storage.

The idea of force-free magnets has been challenged by many on the basis of the virial theorem. For a bounded nonferromagnetic system in equilibrium under Lorentz body forces, this theorem relates the stress in the body to the stored magnetic energy, i.e.

$$\int \mathrm{Tr}(\underset{\approx}{t})dv = \int_{\mathrm{space}} \frac{B^2}{2\mu_o}\, dv \equiv E \tag{3}$$

where $\mathrm{Tr}(\underset{\approx}{t})$ is the trace of the stress tensor $\underset{\approx}{t}$. Parker [6] discussed the implications of this theorem for laboratory magnets and Levy [7] used this theorem to argue that a lower bound on structural mass was required for superconducting magnets for space application. Wakefield [3] proposed that for fusion reactors the magnets near the plasma region be made force-free, while the force bearing structure required by virial theorem be located far from the plasma region.

Recently Moses [8] and Eyssa [9] of the University of Wisconsin discussed the limitation the virial theorem places on the design of superconducting magnetic energy storage systems.

As the scale of superconducting magnet designs approaches the $\$10^8$ to $\$10^9$ cost range, it becomes important to minimize the mass of material used. With this motivation the author sought to find out how far present designs of superconducting structures are from the minimum mass required by the virial theorem. Results of this study are presented for MHD magnets, toroidal field coils (TF) for fusion, and energy storage solenoids. These studies indicate that present designs may have twice to ten times the mass required by the virial theorem. Thus while force-free systems may be impossible, there appears to be room for great mass reduction in designs of large superconducting structures.

THE VIRIAL THEOREM FOR MAGNETO-SOLIDS

One definition of a virial is given by Maxwell [10]; "half the product of the stress [between two points] into the distance between two points is called the Virial of the stress". Clausius used this idea to derive a theorem for gases relating the mean kinetic energy of a gas to the virial of the pressure and internal forces between the molecules. It appears however that Maxwell was the first to extend this theorem to a solid continuum [11]. In modern notation the theorem for continua in equilibrium is derived from the identity

$$\frac{1}{2}\int \underset{\sim}{r}\cdot\nabla\cdot\underset{\approx}{t}\,dv + \frac{1}{2}\int \underset{\sim}{r}\cdot\underset{\sim}{f}\,dv = 0 \tag{4}$$

where $\underset{\sim}{r}$ is a position vector, $\underset{\approx}{t}$ the stress tensor in the body, and $\underset{\sim}{f}$ is the body force on the body. Maxwell's theorem in this notation relates the mean stress in a structure under gravitational body forces alone

$$\int \mathrm{Tr}(\underset{\approx}{t})\,dv + Mgz_c = 0 \tag{5}$$

where z_c is the height of the center of mass. The only surface stresses are those on $z = 0$ required to equilibrate the total gravitational force on the body.

A general discussion of the virial theorem and a number of generalizations is given by Truesdell and Toupin [12]. The extension of the virial theorem to magnetohydrodynamics was claimed by Chandrasekhar and Fermi [13] and a generalization using the tensor product $\underset{\sim}{r} \otimes \underset{\sim}{f}$ rather than the inner product in (4) was given by Chandrasekhar [14]. Since the theorem is independent of the stress-strain constitutive relations, its extension to solids and in particular magnets is trivial although apparently independently derived by Parker [6] and Levy [7]. For magnets, the body forces are both gravitational and magnetic, i.e.

$$\underset{\sim}{f} = -\rho g \underset{\sim}{e}_z + \underset{\sim}{J} \times \underset{\sim}{B} \tag{6}$$

For non ferromagnetic solids we may replace $\underset{\sim}{J} \times \underset{\sim}{B}$ by

$$\nabla \cdot \underset{\approx}{T} = \underset{\sim}{J} \times \underset{\sim}{B} \tag{7}$$

where $\underset{\approx}{T} = (\underset{\sim}{B}\underset{\sim}{B} - \underset{\approx}{\delta} B^2/2)/\mu_o$ is the Maxwell stress tensor, and $\underset{\approx}{\delta}$ is the identity tensor. Integration of the virial of $\underset{\sim}{J} \times \underset{\sim}{B}$ over the entire volume occupied by the magnetic fields leads to the theorem

$$\int \mathrm{Tr}(\underset{\approx}{t})dv = \mathcal{E} - mgz_c \tag{8}$$

where the only surface stresses are those on $z = 0$ required to support the gravitational loads. $\mathcal{E}$ is the magnetic energy given by

$$\mathcal{E} = \frac{1}{2}\mu_o \int_{space} B^2 dv \tag{9}$$

where the integral is carried out over all space.

This theorem states that the mean principal stress under magnetic fields alone is tensile. Gravity can increase the average tension if the structure is hung $(z_c < 0)$ or can decrease the average tension if the center of mass of the structure is above the plane $z = 0$. An important consequence of (8) is that in a structure with pure bending $\int \mathrm{Tr}(\underset{\approx}{t})dv = 0$. Thus <u>bending is a waste of structural material</u>. The ideal magneto-mechanical design is a truss, or bending free structure (hence the importance of the "D" shaped coils in toroidal field, fusion magnets; see File, Mills and Sheffield [15]).

For truss structures, we assume each element has only one principal stress and no bending. For design stresses in tension and compression respectively of S_T and S_c, the virial theorem becomes

$$S_T V_T - S_c V_c = E - mgz_c \tag{10}$$

($E = 0$ is Maxwell's theorem). A more familiar form of this theorem when $S_T = S_c \equiv S_o$ uses the average density ρ and the ratio of compressive volume to total volume, $\beta = V_c/(V_c+V_T)$,

$$M = \frac{\rho E}{[S_o(1-2\beta)+\rho g z_c]} \tag{11}$$

This implies that gravity loads will become more important as the devices get larger. However for $z_c \sim 10$ m, the gravitational term is less than 10^{-2} of the internal stress term for conventional structural materials and one can write

$$M = \frac{\rho E}{S_o(1-2\beta)} \tag{12}$$

This form of the virial theorem has been used by Moses [8], and Eyssa [9] to analyze minimum mass structures for energy storage.

However the truss or cable structure is an exception. For most structures there is more than one principal stress at a point. And what is perhaps more important, the structure reaches its maximum stress at only a few locations. Thus most of the structure is under utilized vis a vis stress.

For the long thin solenoid, the circumferential stress $t_{\theta\theta}$, and the axial stress t_{zz} are related to the magnetic pressure $P_m = B_o^2/2\mu_o$, (Fig. 1)

$$t_{\theta\theta} = P_m R/\Delta \quad , \quad t_{zz} = -t_{\theta\theta}/2 \tag{13}$$

where R is the radius of the solenoid and Δ the wall thickness. The maximum radial stress t_{rr} is of order P_m, which is small compared to $t_{\theta\theta}$, t_{zz}. Thus,

$$\mathrm{Tr}(\underset{\approx}{t}) = \frac{1}{2}\, t_{\theta\theta} \tag{14}$$

If the structural material fails by yielding,then the maximum shear stress τ is given by $S_Y/2$ where S_Y is the yield stress in tension. For a biaxial state of stress in the solenoid, $t_{\theta\theta} < 4\tau/3 = 2S_Y/3$ and

$$\mathrm{Tr}(\underset{\approx}{t}) < S_Y/3 \tag{15}$$

The virial theorem for thin solenoids then becomes

$$M = 3\,\frac{\rho E}{S_Y} \tag{16}$$

This formula was derived by Sviatoslavsky and Young [16] using the concept of separate axial and radial load structures. The analysis here shows this is not necessary.

The thick wall cylinder was also analyzed by Sviatoslavsky and Young neglecting axial stresses. Their conclusion is that thin wall structures are more efficient as regards structural mass usage.

Dynamic Effects

The extension of the theorems of Clausius and Maxwell to steady state dynamic problems with zero stress vector on the boundary of the structure, i.e. $\underset{\approx}{t}\cdot\underset{\sim}{n} = 0$, is

$$\int \mathrm{Tr}(\underset{\approx}{t})dv = E + 2T \tag{17}$$

where T is the total kinetic energy of the body. This form of the theorem has implications for superconducting rotating machinery. Defining a mean principal stress as $\bar{S}_o$ and a radius of gyration of R, the mass-magnetic energy relation becomes

$$M = \frac{\rho E}{(\bar{S}_o - \rho\omega^2 R^2)} \tag{18}$$

where ω is the rotational speed of the device.

In the discussion below we were not able to find sufficient data in the literature to check (18) out for rotating superconducting devices.

STORED ENERGY-MASS SCALING IN SUPERCONDUCTING DESIGNS

The virial theorem (3) or (12) implies that the minimum structural mass is linearly proportional to the stored magnetic energy. Given this minimum, one can ask how close do contempory designs of superconducting structures come to the virial mass limit? Another question is whether there is another constraint on mass in superconducting structures that places an even higher limit on mass in actual designs. To answer these questions the author has compiled three tables and four graphs comparing the stored magnetic energy and cold mass of over thirty superconducting magnet designs. Many of these designs are just preliminary or scoping exercises while others have actually been built. The magnets are grouped into MHD magnets, toroidal field magnets for fusion, and solenoids.

MHD Magnets

The stored energy-mass comparisons for MHD magnets are listed in Table I and Figure 3. These magnets are essentially a pair of dipole magnets, (Fig. 2) each of which is wound in a nonplanar configuration. This suggests that bending forces will be the rule in these designs and hence they will be the least efficient as regards structural mass. In Table I we have tried to include only the conductor and structural mass, excluding the dewar mass which ideally carries no load. However in all of the tables it was not always possible to ascertain exactly what mass was included in the given reference. Some designs were not included because either the energy or mass was not given in the literature. In a few cases, phone calls to U.S. magnet design centers provided the missing data.

The theoretical virial law plotted in Figure 3 is the ideal case with no compression members, bending or multidimensional states of stress. The average density chosen was that of stainless steel and a working stress of 34,500 N/cm^2 (50,000 psi). The materials in the designs however included stainless steel, copper, niobium titanium, and fiber glass epoxy composite. Also working stresses far below 34,000 N/cm^2 were cited in the references.

As shown in Figure 3 the Log E - Log M graph shows a remarkable linear relationship, with little scatter over several orders of magnitude of stored energy. The empirical scaling law that emerges does not follow the virial theorem although all designs exceed the minimum mass by a factor ranging from 10 to 50. It is clear that a pure tension MHD magnet is not possible and thus the ideal virial mass is probably higher than the theoretical curve shown in Figure 3. The scaling law that seems to emerge has the form

$$E = cM^{4/3} \tag{19}$$

This is remarkable considering the variety of materials and design stresses employed. It suggests that the primary constraint on these designs is not stress and that perhaps current density, magnetic field, or thermal stability may be the controlling element in all these designs. This subjuct is discussed in a later section. The trend does seem to indicate that the design mass approaches the virial limit for larger devices.

Also included in Figure 3 is a point for the Mirror Fusion Test Facility (MFTF) fusion magnet built by the Lawrence Livermore Laboratory at California. This magnet pair is not planar and must carry bending loads. As can be seen it seems to fit right on the MHD scaling law.

Not included in Table I is a small design recently published by Magnet Corporation of America in Massachusetts. The cited stored energy and mass was 3.1 MJ and 2.7 Tonne. This design does not fit the scaling law. However the reported mass may not include all the structural mass.

Toroidal Field Fusion Magnets

The energy-mass relationship for toroidal field (TF) magnets for fusion magnets are given in Table II and Figures 4, 5. The number and diversity of the designs is greater than for the MHD group, with designs from the

USA, Japan, Europe and the USSR, and a stored energy range of over four decades. However in almost all of the designs the conductor was niobium-titanium in a copper matrix and the structural material was stainless steel. Also all of the designers used a "D" shape pure tension shape to avoid bending. (Fig. 3) However out of plane bending induced by poloidal magnetic field forces accounted for a significant part of the structural mass which is not accounted for in the theoretical curve in Figure 4.

The points LCP (Large Coil Project - Oak Ridge National Laboratory) actually represents six different TF coils from manufacturers in both the US, Japan and Europe.

The Log E - Log M graph in Figure 4 again shows a 4/3 scaling law, i.e.

$$E = C_2 M^{4/3} \tag{20}$$

The exception is the point UW-III. However this design calls for an aluminum structure. When the mass is scaled to stainless steel, it falls right on the scaling curve. Again there are perhaps a half dozen designs not included because of missing data in the literature.

Also shown on Figure 4 is a point for the Joint European Torus (JET) which has normal copper coils. Remarkably it also falls on the scaling curve. (Data for the TFTR at Princeton was not available.)

The scaling law for TF magnet sets seems to be identical to that for MHD magnets except for the constants C_1, C_2.

Although the virial theorem applies to isolated magnet systems, it was found that the individual TF coils seemed to obey an energy-mass scaling law even though the number of coils in each torus varied significantly (see Table II). The data in Figure 5 shows a power law of the form

$$E = C_3 M^n \quad , \quad n = 1.43 \tag{21}$$

This differs from the law for the total torus. This result again suggests a design constraint not solely related to stress.

It should be noted that Table II does not include the mass of the bucking cylinder or intercoil structure. Since these structures carry stress due to magnetic forces, their mass should be added to the values in Table II, but this data was not available for all the magnets. Thus the points should properly be moved further away from the ideal virial mass limit.

Solenoid Magnets

The data for solenoid magnets is shown in Table III and Figure 6. Although the solenoid or cylindrically shaped magnet or coil has the most design experience, (see e.g. Montgomery [17] or Brechna [18]) the energy-mass data seem to have greater scatter than either the MHD or TF magnets. Also solenoids have little bending, although they most often are in a two or three dimensional stress state. The theoretical virial limit shown in Figure 6 is for the case of the long thin solenoid (Eqn. 16) and thus has three times the mass for a given energy compared to the ideal limits shown in Figures 3, 4, 5.

A large number of energy storage designs by the University of Wisconsin were not included since they require underground geological support for the magnet forces in addition to the cold magnet mass itself. Also some designs listed in Brechna [18] were not included since it appeared that only the conductor mass was given.

Some applications call for ferromagnetic material to shape the flux path. Since this mass will experience magnetic forces its mass should be added to that of the superconducting structure. However this data was not available for most of the designs in Table III and might account for some of the scatter in Figure 6.

While all designs exceed the minimum theoretical virial mass, an unequivocal scaling law does not emerge. Smaller devices seem to have a linear $\mathcal{E}$-$\mathcal{M}$ relationship, while the larger designs seem to approach the virial limit in the same way that the MHD, and TF coils behave.

Discussion

The data in Figures 3, 4 for MHD and TF magnets suggests that some scaling law other than that based on stress alone is implicitly being employed by different magnet design groups around the world. The simplest fit to the data appears to be a power law of the form

$$\mathcal{E} = cM^{4/3} \tag{22}$$

Potential candidates for scaling principles besides stress are constant current density, constant magnetic field and thermal stability.

I) Constant current density scaling.

Superconducting materials have a limitation on the current density for a given magnetic field and temperature. Suppose we assume that all designs of a certain class of magnets will have the same current density. The stored energy of the system can be written in the form

$$\mathcal{E} = \frac{1}{2} LI^2 \tag{23}$$

for a single current input magnet. If R represents a global dimension of the magnet and r is representative of the cross-section through which I flows, then L may be written in the form

$$L = \mu_o Rf(R/r) \tag{24}$$

where $\mu_o = 4\pi 10^{-7}$ in MKS units. For a certain class of magnet designs we assume R/r, and I/r^2 to be constants. Then it is easy to show that the mass $\mathcal{M}$ will scale as R^3 and

$$E = \alpha M^{5/3} \tag{25}$$

This law cannot fit the data in Figures 3, 4 over three and four decades of stored energy.

II. Constant magnetic field scaling.

Another candidate scaling law is that based on constant magnetic field. In this case we assume that B is proportional to I/R. However for $B =$ constant, this leads to a linear energy mass relation similar to the Virial law.

$$E = \gamma M \tag{26}$$

III. Thermal stability scaling.

To avoid quenching or the magnets becoming normal, the heat transfer must be sufficient to take out energy input due to Joule heating. This condition may be expressed in the form (see e.g. Montgomery [17]).

$$I^2 \rho_e / A_o \leq h_o A_1 \Delta T \tag{27}$$

where h_o is a heat transfer coefficient; A_1 is a surface area per unit length and is proportional to r; ΔT is a temperature difference; A_o is the cross sectional area through which I flows and is proportional to r^2; and ρ_e is an equivalent electrical resistivity of the superconductor composite conductor. Assuming that h_o, ρ_e, ΔT, R/r all remain constant for a certain class of magnets, then one can see that $I^2 \sim R^3$. Upon substitution of I^2 from (27) into the expression for the stored energy (23) it follows that E has the form

$$E \leq \eta M^{4/3} \tag{28}$$

where η depends on the following physical constants

$$\eta \propto \frac{\mu_o h_o \Delta T}{\rho_e}$$

Thus thermal stability seems to be the most likely candidate for a practical scaling law in the lower stored energy regime. However it is clear that for energy levels greater than 10^5 MJ the virial law must hold. In this limit there will be more mass for thermal stability than required by the scaling law (28). One is tempted to conclude that larger devices will be more thermally stable than smaller ones since the virial minimum mass for stress resistance will provide more mass than is required for thermal stability. However this analysis neglects the dynamics of the quenching process and further study might be required to establish this design principal.

The above analysis however does not explain the scatter for the solenoid cases. One possible explanation however is that the solenoid cases have too much diversity in terms of aspect ratio and application, e.g. bubble chamber versus utility power peak shaving.

Conclusion

A summary of the major results of this analysis is shown in Figure 7. First it demonstrates that toroidal energy storage is more efficient as regards mass requirements. Second it is clear that all designs meet the virial limit imposed by stress limitations. In present designs, it appears that improvements in structural design could achieve great reductions in mass barring other constraints. However the data suggest that the impediment to further mass reduction is the thermal stability since conventional magnet designs in the MHD and TF classes appear to be governed by a thermal stability scaling law.

Finally the scale of advanced MHD and TF magnet designs is rapidly approaching the virial mass limit. In this regime, the need to use mass efficiently will require more optimally designed superconducting structures than have been developed to date.

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TABLE I

MHD MAGNET DESIGNS

CODE NAME	DESIGN GROUP	STORED ENERGY MJ	MASS: CONDUCTOR + STRUCTURE 10^3 kgm	REFERENCE
U-25	Argonne Nat. Lab.	20	28	IEEE Trans. Mag-13, No. 1, p. 632 (1977)
Stanford	MIT, General Dynamics-Convair	93	84*	IEEE Trans. Mag-17, No. 1, p. 344 (1981)
CFFF	Argonne Nat. Lab.	168	145	IEEE Trans. Mag-17, No. 1, p. 529 (1981)
CDIF	MIT	240	144	A. Dawson, F. Bitter Nat. Magnet Lab., MIT, (personal comm.)
ETF	MCA Corp.	483	290	IEEE Trans. Mag-13, No. 1, p. 636 (1977)
Base Load ('76)	MCA Corp.	4480	1560	IEEE Trans. Mag-13, No. 1, p. 636 (1977)
Base Load ('81)	MIT	5300	2150	A. Dawson, F. Bitter Nat. Magnet Lab., MIT, (personal comm.)
Base Load ('78)	MCA Corp.	6700	1880	IEEE Trans. Mag-15, No. 1, p. 306 (1979)
MFTF (Fusion)	Livermore Lab.	409	300	IEEE Trans. Mag-15, No. 1, p. 534 (1979)

* May include Dewar mass.

TABLE II

TOROIDAL FIELD MAGNET DESIGNS

FOR MAGNETIC FUSION REACTORS

CODE NAME	DESIGN GROUP	TOTAL STORED ENERGY MJ	TOTAL MASS OF COILS: CONDUCTOR + STRUCTURE 10^3 kgm	NO. OF COILS	REFERENCE
T-7	Kurchatov Atomic Energy Institute,	20	12	--	IEEE Trans. Mag-15, No. 1, p. 550 (1979)
Jaeri-Cluster	Japan Atomic Energy Research Institute	20.7	13.2	2	IEEE Trans. Mag-17, No. 1, p. 494 (1981)
Torus II-Supra	Euratom	440	144	24	IEEE Trans. Mag-15, No. 1, p. 542 (1979)
LCP	GA, GD, GE, WESTH. JAERI, Eur. *	894	240	6	W.H. Gray, Oak Ridge Nat. Lab. (personal comm.)
TNS	General Atomic Corp.	10 10^3	1500	12	GA TNS Project, General Atomic Report, GA-A15100, Vol. V, UC-2nd, Oct. 1978, p. 5.3-144, 5.3-179
ANL-EPR	Argonne Nat. Lab.	15.6 10^3	2800	16	Tokamak Exp. Power Reactor Studies, Argonne Nat. Lab. Report ANL/CTR-75-2, June 1975, p. II-12, II-31
GA-EPR	General Atomic Co.	16.7 10^3	1700	16	Exp. Fusion Power Reactor Conceptual Design Study Report No. EPRI ER-289, Vol. II, Dec. 1976, p. 5-70, Table 5.3-1
ANL-10T	Argonne Nat. Lab.	30 10^3	3328	16	IEEE Trans. Mag-13, No. 1 p. 605 (1977)
ETF	MIT/FBNML-GE	38 10^3	3260	10	ETF Interim Design Description Document, Oak Ridge Nat. Lab., July 1980
ETF	MIT/FBNML-GE	45 10^3	3580	10	ETF Interim Design Description Document, Oak Ridge Nat. Lab., July 1980
HFCTR/MIT	FBNML/MIT	40 10^3	6800	16	7th Symp. Engr'g. Prob. of Fusion Research, IEEE Publ. No. 77CH1267-4-NPS, Vol. 1, p. 629

TABLE II

(continued)

Fintor	LNF del CNEN, Italy	$60\ 10^3$	5808	24	IEEE Trans. Mag-13, No. 1 p. 617 (1977)
UW-III	Univ. of Wisconsin	$108\ 10^3$	3268	18	UWMAK-III Design Report Univ. Wisconsin, Madison, Wisc., USA, Report No. UWFDM-150, July 1976, Table IV-A-1
UW-1	Univ. of Wisconsin	$158\ 10^3$	9960	12	UWMAK-III Design Report Univ. Wisconsin, Madison, Wisc., USA, Report No. UWFDM-150, July 1976, Table IV-A-1
UW-II	Univ. of Wisconsin	$223\ 10^3$	$16.1\ 10^3$	24	UWMAK-III Design Report Univ. Wisconsin, Madison, Wisc., USA, Report No. UWFDM-150, July 1976, Table IV-A-1
JET (Normal coils)	Joint European Torus Design Group, Culham Lab. England	$1.45\ 10^3$	384	32	7th Symp. Engr'g. Prob. of Fusion Research, IEEE Publ. No. 77CH1267-4-NPS, Vol. 1, p. 28

* General Atomic (GA), General Dynamics (GD), General Electric (GE), Westinghouse (WESTH), Japanese Atomic Energy Research Institute (JAERI).

Table III

SOLENOID MAGNET DESIGNS

Code Name	Design Group	Stored Energy MJ	Mass: Conductor + Structure 10^3 kgm	Reference
LBL	Lawrence Laboratory Berkeley, Calif.	0.55	0.22	IEEE Trans. Mag-13, No. 1, p. 78 (1977)
MIT	MIT/FBNML	2.0	0.68	Superconducting Magnet Systems, Brechna, p. 558
Japan	National Lab. for High Energy Physics, Japan	3.0	0.67	IEEE Trans. Mag-15, No. 1, p. 318 (1979)
PCTF	Argonne National Laboratory	3.5	1.2	IEEE Trans. Mag-17, No. 1, p. 502 (1981)
SLAC	Stanford	5.5	1.6	Superconducting Magnet Systems, Brechna, p. 571
BPA	General Atomic Co.	30	17.2*	IEEE Trans. Mag-17, No. 1, p. 521 (1981)
ANL	Argonne National Laboratory	80	47.5	Superconducting Magnet Systems, Brechna, p. 548
CERN (BEBC)	European Organization for Nuclear Research	800	166	Superconducting Magnet Systems, Brechna, p. 551
UW-II	Univ. of Wisconsin	3600	510	Sixth Symp. Engr'g. Problems of Fusion Research, Proc. IEEE Publ. 75CH 1097-5-NPS, p. 291 (1976)
UW-I	Univ. of Wisconsin	54 10^3	5400	Sixth Symp. Engr'g. Problems of Fusion Research, Proc. IEEE Publ. 75CH 1097-5-NPS, p. 291 (1976)

* May include Dewar.

Figure 1

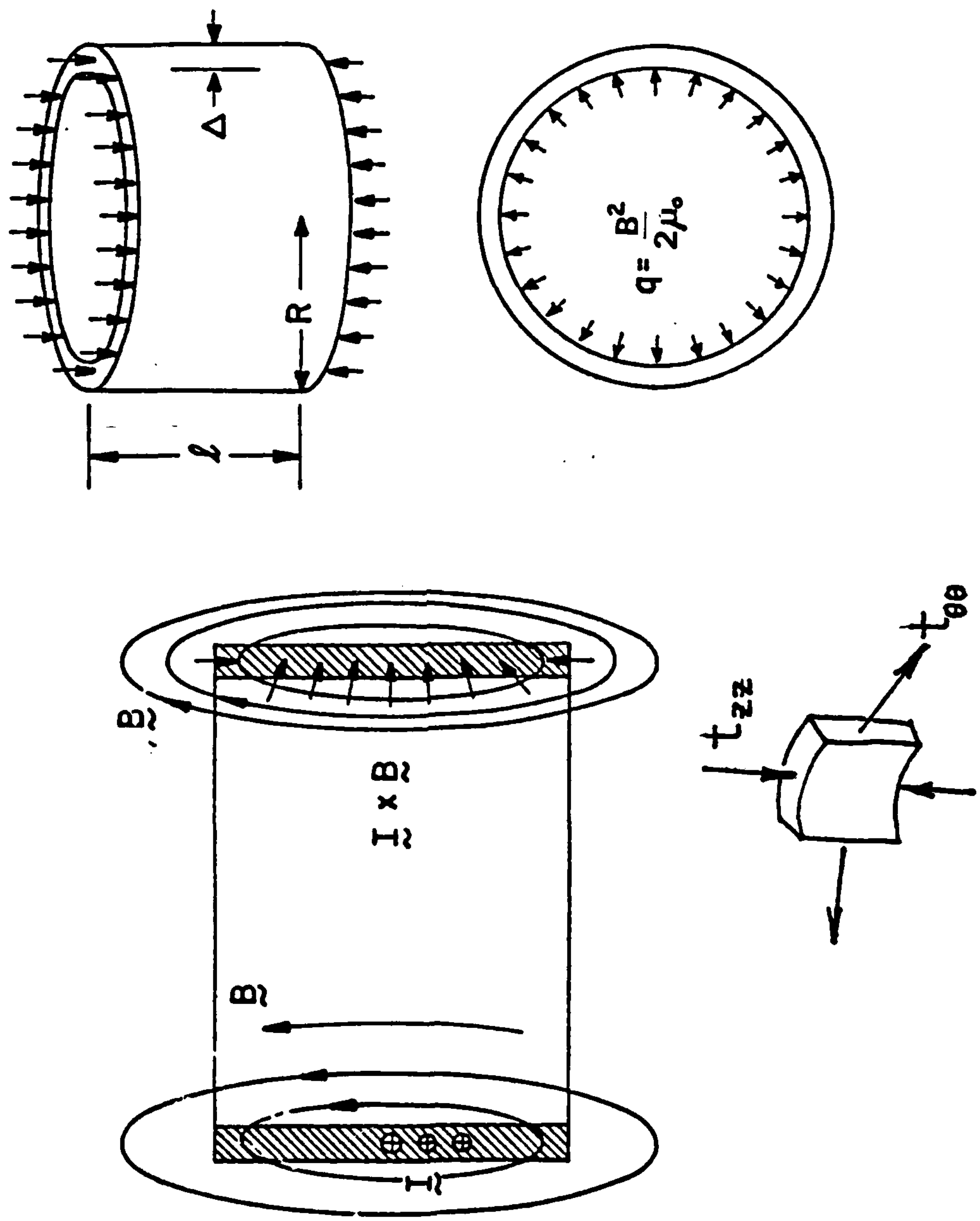
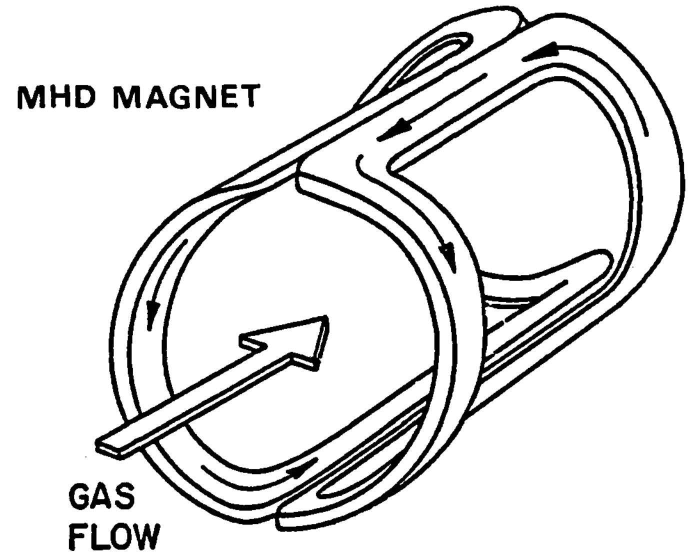


Figure 2


MHD MAGNET
GAS
FLOW

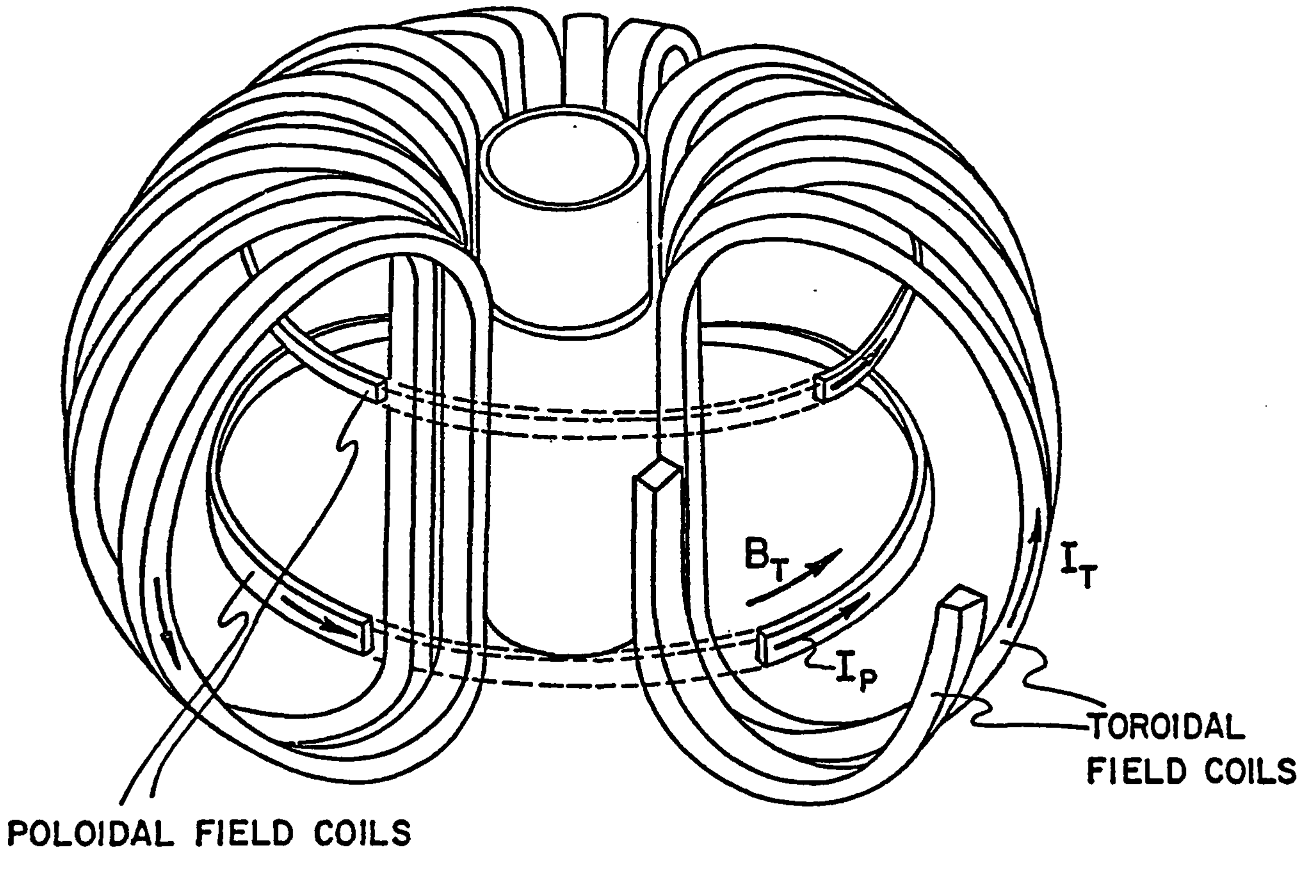

B_T
I_T
I_P
TOROIDAL
FIELD COILS
POLOIDAL FIELD COILS

Figure 3

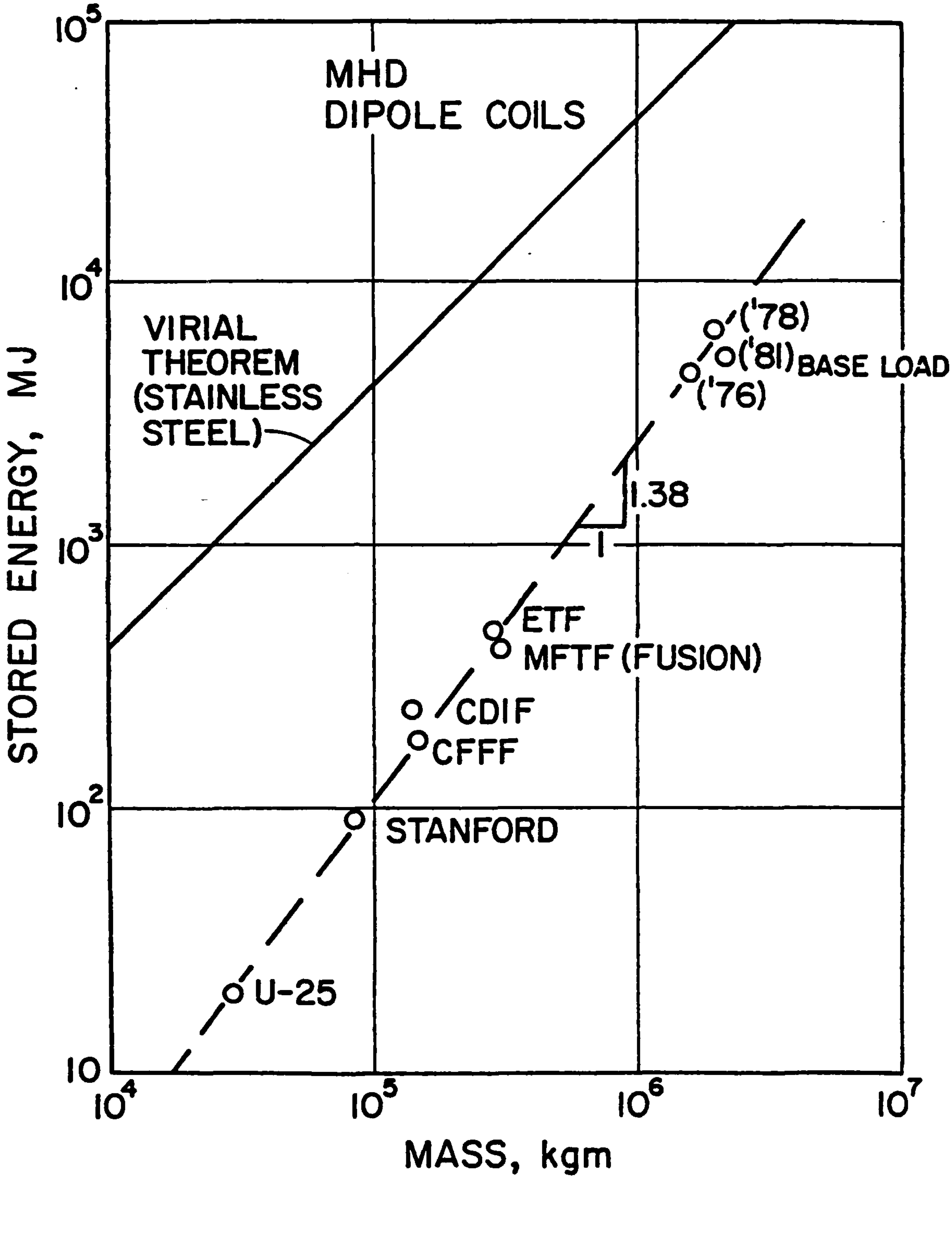

MHD
DIPOLE COILS
VIRIAL
THEOREM
(STAINLESS
STEEL)
1.38
1
('78)
('81) BASE LOAD
('76)
ETF
MFTF (FUSION)
CDIF
CFFF
STANFORD
U-25
STORED ENERGY, MJ
MASS, kgm
10
10^2
10^3
10^4
10^5
10^6
10^7

Figure 4

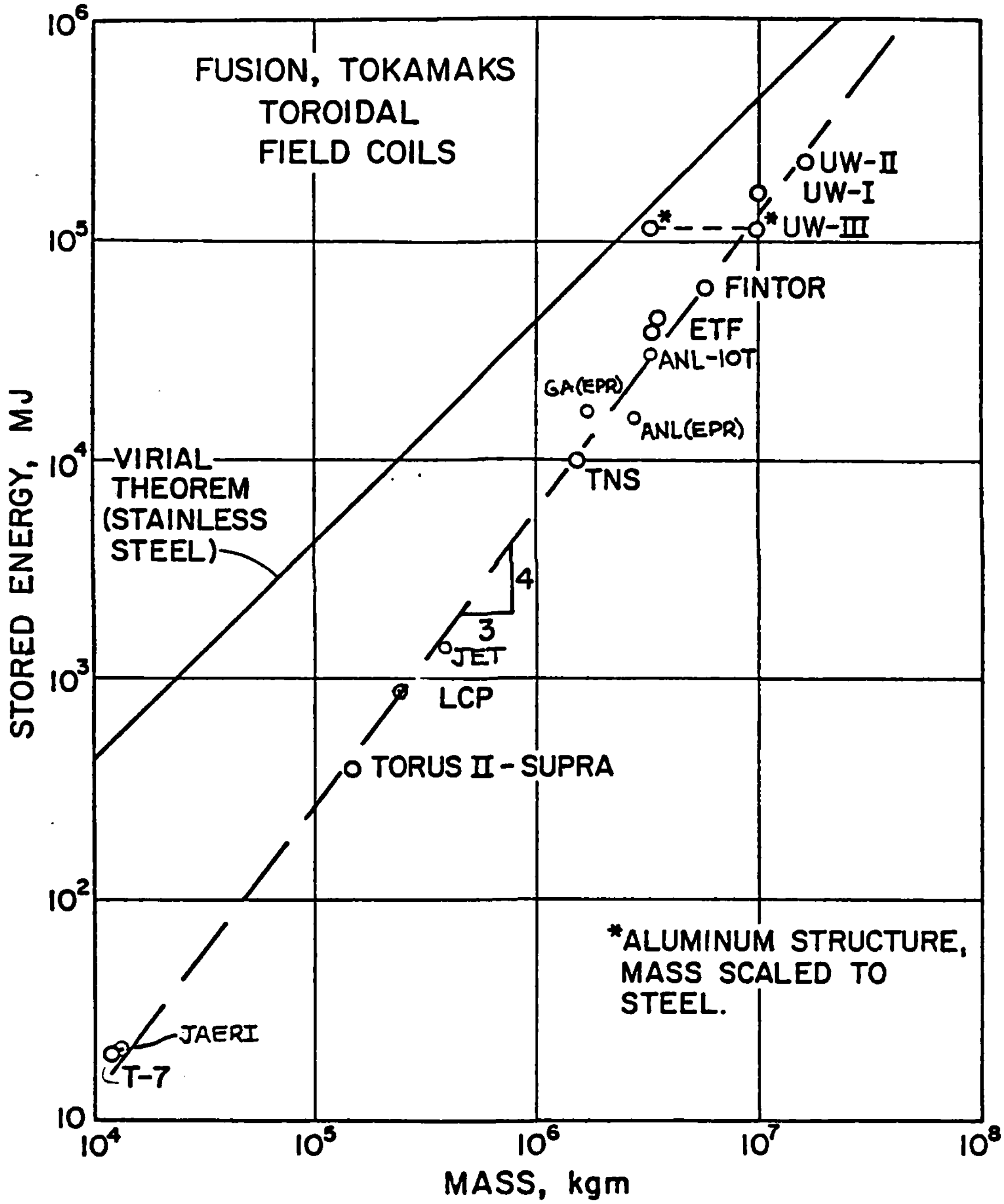

FUSION, TOKAMAKS
TOROIDAL
FIELD COILS
VIRIAL THEOREM (STAINLESS STEEL)
UW-II
UW-I
*UW-III
FINTOR
ETF
ANL-IOT
GA(EPR)
ANL(EPR)
TNS
4
3
JET
LCP
TORUS II - SUPRA
JAERI
T-7
*ALUMINUM STRUCTURE, MASS SCALED TO STEEL.
STORED ENERGY, MJ
MASS, kgm
10
10^2
10^3
10^4
10^5
10^6
10^7
10^8

Figure 5

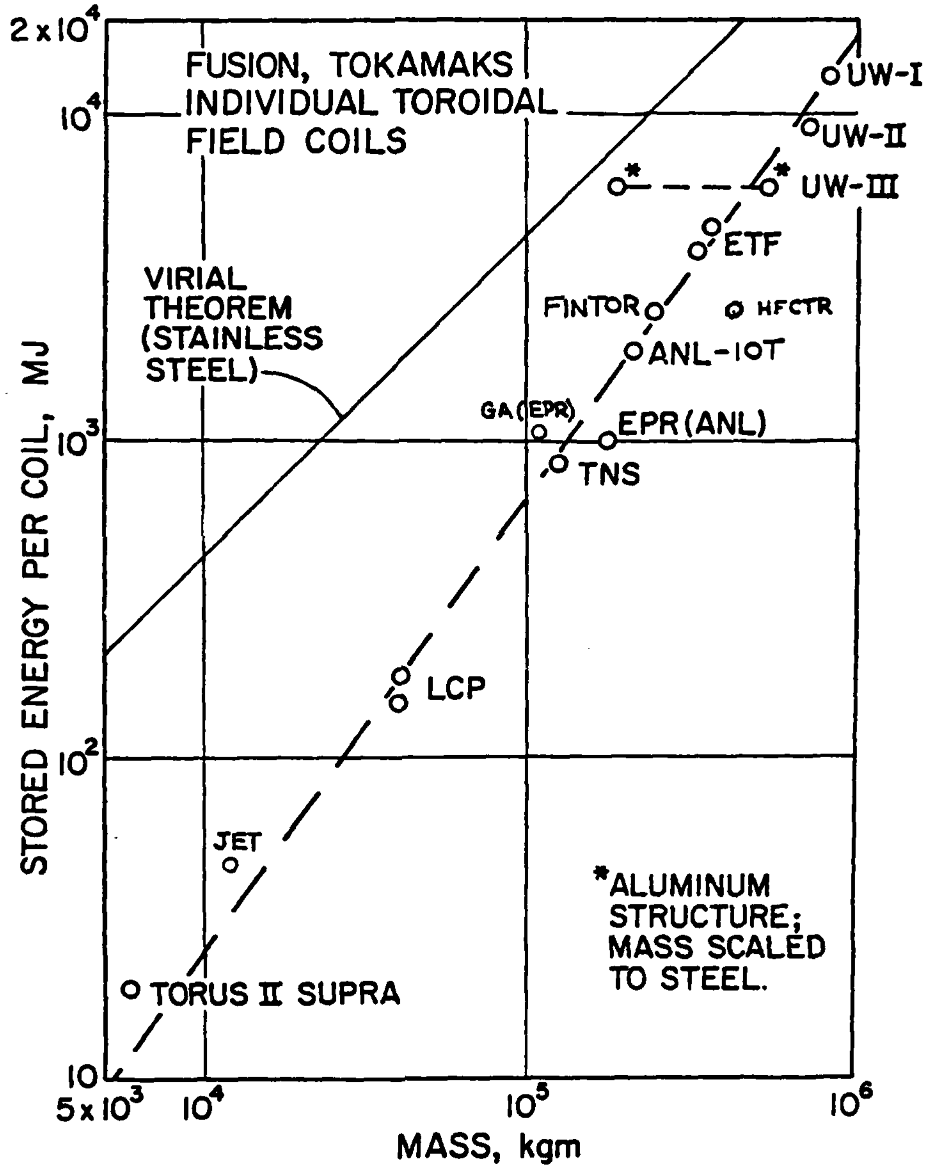

FUSION, TOKAMAKS
INDIVIDUAL TOROIDAL
FIELD COILS
VIRIAL
THEOREM
(STAINLESS
STEEL)
UW-I
UW-II
UW-III
ETF
FINTOR
HFCTR
ANL-10T
GA (EPR)
EPR (ANL)
TNS
LCP
JET
TORUS II SUPRA
*ALUMINUM
STRUCTURE;
MASS SCALED
TO STEEL.
STORED ENERGY PER COIL, MJ
MASS, kgm
2 x 10^4
10^4
10^3
10^2
10
5 x 10^3
10^4
10^5
10^6

Figure 6

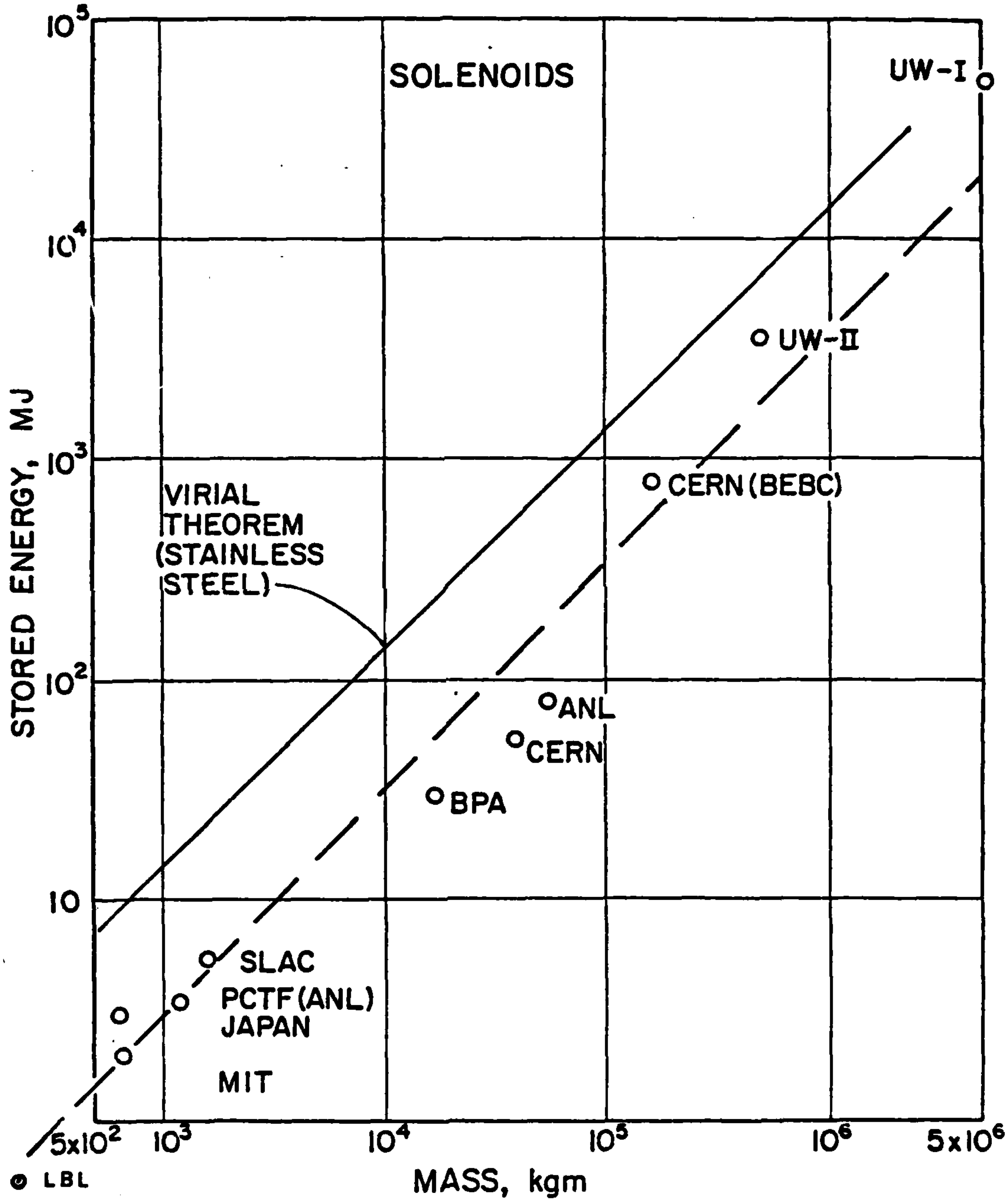
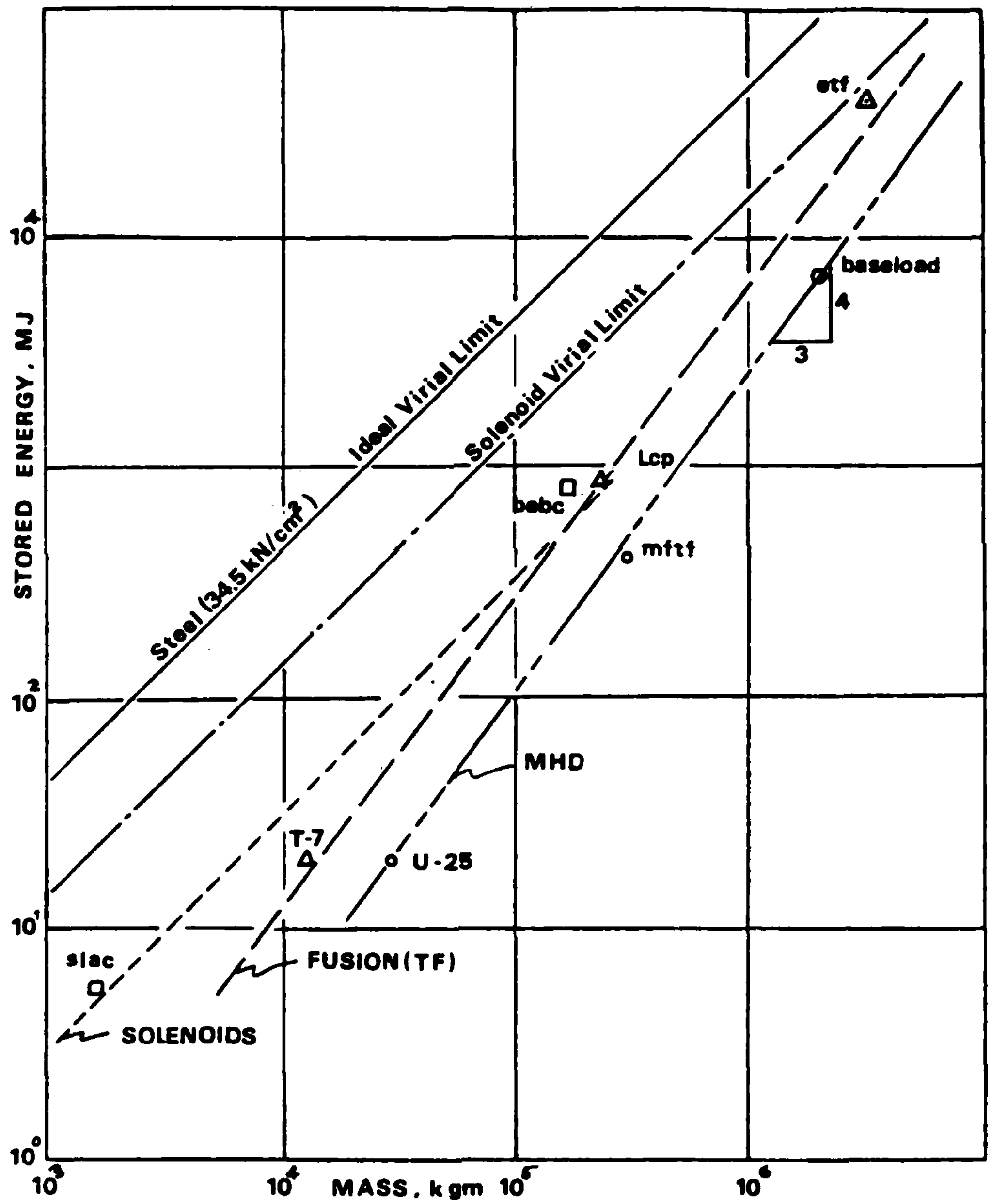

SOLENOIDS
UW-I
UW-II
CERN (BEBC)
VIRIAL THEOREM (STAINLESS STEEL)
ANL
CERN
BPA
SLAC
PCTF (ANL)
JAPAN
MIT
LBL
STORED ENERGY, MJ
MASS, kgm
10^5
10^4
10^3
10^2
10
5×10^2
10^3
10^4
10^5
10^6
5×10^6

Figure 7


STORED ENERGY, MJ
MASS, kgm
10^0
10^1
10^2
10^4
10^3
10^4
10^5
10^6
Steel (34.5 kN/cm^2)
Ideal Virial Limit
Solenoid Virial Limit
etf
baseload
4
3
Lcp
bebc
mftf
MHD
T-7
U-25
slac
FUSION (TF)
SOLENOIDS

COMPOSITE LIST OF TECHNICAL REPORTS
TO THE
OFFICE OF NAVAL RESEARCH

NUMERICAL SOLUTIONS FOR COUPLED MAGNETOTHERMOMECHANICS

Task Number NR 064-621

Departments of Structural Engineering and
Theoretical and Applied Mechanics,
Cornell University,
Ithaca, New York 14853

1. K.Y. Yuan, F.C. Moon, and J.F. Abel, "Numerical Solutions for Coupled Magnetomechanics", Department of Structural Engineering Report Number 80-5, February 1980.

2. F.C. Moon and K. Hara, "Detection of Vibrations in Metallic Structures Using Small Passive Magnetic Fields", January 1981.

3. S. Mukherjee, M.A. Morjaria, and F.C. Moon, "Eddy Current Flows Around Cracks in Thin Plates for Nondestructive Testing", March 1981.

4. K.Y. Yuan, F.C. Moon, and J.F. Abel, "Finite Element Analysis of Coupled Magnetomechanical Problems of Conducting Plates", Department of Structural Engineering Report Number 81-10, May 1981.

5. F.C. Moon, "The Virial Theorem and Scaling Laws for Superconducting Magnet Systems", May 1981.

6. K.Y. Yuan, "Finite Element Analysis of Magnetoelastic Plate Problems", Department of Structural Engineering Report Number 81-14, August 1981.

7. K.Y. Yuan <u>et al.</u>, "Two Papers on Eddy Current Calculations in Thin Plates", September 1981.

DATE
FILMED

3-8