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The technique of linear regression has been applied as a tool for predicting the cost of an item based on its most important characteristics. Often these characteristics (variables) tend to be highly intercorrelated (the data are said to exhibit multicollinearity) causing least squares estimates of the regression coefficients to be unstable and possibly leading to erroneous predictions.

Ridge regression, a possible remedy for the problems caused by multicollinearity proposed by Hoerl and Kennard, is a biased estimation technique which reduces the variance of estimators and provides more precision (as measured by mean square error of the coefficients) than ordinary least squares (OLS) estimators.

A comparison was made between these techniques to determine when ridge regression provides better cost equation coefficient estimates than OLS as a function of the degree of multicollinearity in the data, the number of predictor variables in the model, the degree of model fit (R^2), and the amount of bias (k) of the estimate.

Monte Carlo simulation was used to generate data for linear and log-linear model forms. A regression analysis of both sets showed that the degree of multicollinearity and amount of bias interact in explaining the major part of the improvement (degradation) in the mean square coefficient error.

Estimates of $k \le 0.04$ limit the degradation and allow slight improvements in the MSE for low levels of multicollinearity and enable large improvements to be made for higher levels of multicollinearity.

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AN EVALUATION OF RIDGE REGRESSION IN COST ESTIMATION

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

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Abstract

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AN EVALUATION OF RIDGE REGRESSION IN COST ESTIMATION

I. Introduction

Background

Cost estimation of military and civilian hardware systems based on a limited amount of information has been an integral part of development and acquisition processes for many years. The technique of linear regression has been applied as a tool for predicting the overall cost of an item based on characteristics the system possesses.

During the early stages in the life-cycle of a system, only a limited knowledge of the system characteristics is available. This is especially true for new systems which have no existing counterpart. Here, there is no past experience or data to draw upon to aid in the analysis. Later, as the system takes shape, predictions must be made using only essential data to insure that the predictions are timely and are made at the minimum cost. Thus, it is necessary to identify and collect data for the most important characteristics of the system, identify the appropriate model for making the cost prediction, determine the parameters of the model and, finally, make a prediction (point or interval) of the cost of each item.

In cost estimation, only one (or in unusual circumstances, a few) prediction(s) of the cost of a system will be made. Obviously, it is desired that the prediction be as close to the actual cost incurred as possible. Often the most important characteristics used to predict the cost (predictor/independent variables) tend to be intercorrelated. When this intercorrelation is very high, the data are said to exhibit multicollinearity.

When multicollinearity is present, a tradeoff must be made between models such as ordinary least squares (OLS), which produce unbiased estimates of the cost, and biased techniques such as ridge regression which introduce some bias but reduce the variance of the estimate. The least squares estimates of the individual regression coefficients tend to be unstable. This can lead to erroneous predictions, especially if the degree of multicollinearity is very high.

A possible remedy to the estimation problems caused by multicollinearity was introduced by Hoerl and Kennard (Ref 11). Although biased, the estimates of the regression coefficients tend to have more precision (as measured by mean square error) than the ordinary least squares estimators (Kendall (Ref 18:37-43) and McCallum (Ref 23: 110-113)). The ridge technique is directly applicable to cost estimation because it has the potential of significantly reducing the chance of making a truly bad estimate for a single or small number of trials.

The regression model is $Y=X\beta+u$ where Y is an $n \times 1$ vector of observations on the response variable (cost), X is an $n \times p$ matrix of observations on p explanatory variables (characteristics of the system to be cost estimated), β is a $p \times 1$ vector of regression coefficients and u is an $n \times 1$ vector of residuals such that $E(u)=0$ and $E(u\,u')=\sigma^2 I$. Ordinary least squares estimates of the regression coefficients are $\hat{\beta}=(X'X)^{-1} X'Y$. On the other hand, for the class of ridge regression estimators indexed by the parameter $k>0$, the estimates of the regression coefficients (for a given value of k) are $\hat{\beta}^*(k)=(X'X+kI)^{-1} X'Y$. As k increases from zero, bias of the estimates increases. As k continues to increase without bound, the regression estimates all tend toward zero.

The total variance, the sum of the variances of the parameter estimates, is a decreasing function of k. The idea of ridge regression, as suggested by Hoerl and Kennard (Ref 12:58-63), is to pick a value of k for which the reduction in total variance is not exceeded by the increase in bias. Ultimately, forecasts of the response variable corresponding to values of the explanatory variables which were not included in the set of data used to estimate the regression coefficients tend to be more accurate.

Statement of the Problem

A comparison has not been made to determine when ridge regression provides a better prediction of the cost of a system than the ordinary least squares estimates (k=0 for the ridge regression model) as a function of the degree of multicollinearity in the data, the size of the model (number of predictor variables in the model), the degree of model fit (as measured by coefficient of determination, R^2, of the regression model), and the amount of bias (controlled by selecting values for k) of the estimate. Mean square error of the coefficients is used as the criterion for evaluating the alternative modeling procedures.

Objective

The objective of this thesis is to determine when ridge regression provides a better prediction of the cost of a system compared to ordinary least squares for data simulated by varying the amounts of multicollinearity in the data, the number of variables in the regression model (2-4), the degree of model fit (R^2), and the amount of bias controlled by the ridge regression parameter, k. The investigation considers both linear and log-linear model forms.

Scope and Limitations

The analysis is limited to the comparison of ridge regression and ordinary least squares estimates within the constructs of cost estimating relationships where linear or log-linear regression models are appropriate.

It is assumed that the correct model form is being used to analyze the data throughout the investigation; that is to say, the investigation will not consider alternative model forms.

Assumptions

It is assumed that the error term of the regression model is normally distributed with mean zero and common variance σ^2.

II. Theoretical Background

Least Squares Estimation

The solution to the general linear model $Y=X\beta+u$, where the elements are defined as in Chapter I and $E(u)=0$ and $E(u'u)=\sigma^2 I$ is $\hat{\beta}=(X'X)^{-1}X'Y$. It is well known that the vector $\hat{\beta}$ is an unbiased estimate of the coefficient vector β. The variance of $\hat{\beta}_j$ is given by the formula

$$V(\hat{\beta}_j) = c_{jj}\sigma^2 \tag{2.1}$$

where the c_{jj}'s are the diagonal elements of $(X'X)^{-1}$. According to the Gauss-Markov theorem (Theil (Ref 35: 119-120)) no linear unbiased estimator has a smaller sampling variance than the least squares estimator. If the X's have been standardized, so that the $X'X$ matrix is in correlation form, the $(X'X)^{-1}$ matrix (for a two variable model) is

$$C = (X'X)^{-1} = \begin{bmatrix} \dfrac{1}{(1-r_{12}^2)} & \dfrac{-r_{12}}{(1-r_{12}^2)} \\ \dfrac{-r_{12}}{(1-r_{12}^2)} & \dfrac{1}{(1-r_{12}^2)} \end{bmatrix} \tag{2.2}$$

and the estimators of the parameters are

$$\hat{\beta}_1 = \frac{X_1'Y - r_{12}X_2'Y}{1 - r_{12}^2} \tag{2.3a}$$

$$\hat{\beta}_2 = \frac{X_2'Y - r_{12}X_1'Y}{1 - r_{12}^2} \qquad (2.3b)$$

where r_{12} is the simple correlation between X_1 and X_2 and $X_1'Y$ and $X_2'Y$ are elements of the $X'Y$ vector.

If multicollinearity is present, X_1 and X_2 are highly correlated and $|r_{12}| \to 1$. It can be seen that the variances and covariances of the regression coefficients become very large, since $V(\hat{\beta}_j) = c_{jj}\sigma^2 \to \infty$ as $|r_{12}| \to 1$ and $\mathrm{Cov}\,(\hat{\beta}_1, \hat{\beta}_2) = c_{12}\sigma^2 \to \pm\infty$ depending on whether $r_{12} \to \pm 1$ (Ref 10: 428-429). The large variances for $\hat{\beta}$ imply that the regression coefficients are very poorly estimated; they are very likely to change significantly for small changes in the data.

The constants of proportionality along the diagonal of the inverse of the correlation matrix (c_{jj}'s) are referred to as variance inflation factors, VIF's. In general, $\mathrm{VIF}(\hat{\beta}_j) = \frac{1}{1-R_j^2}$ where R_j^2 is the coefficient of multiple determination resulting from regressing X_j on the other k-1 regressor variables. As R_j^2 tends toward 1 indicating the presence of a linear relationship in the X's, the VIF for the estimated coefficient of X_j tends to infinity. On the other hand, if the explanatory variables are orthogonal, the VIF's will all equal 1 (Ref 10:429-430).

The average of the variance inflation factors for a given set of data is denoted as R_L where

$$R_L = \frac{\sum_{i=1}^{p} VIF_i}{p} . \tag{2.4}$$

This ratio measures the squared error in the OLS estimators relative to the size of that error if the data were orthogonal; it is called an "index of multicollinearity."

<u>Ridge Regression Estimation</u>

In ridge regression, the parameter estimates are obtained by solving $\hat{\beta}^*(k)=(X'X+kI)^{-1}X'Y$ where k is a non-negative constant. One approach for selecting the value of k for the problem, as suggested by Hoerl and Kennard (Ref 12:64-65), is examination of the ridge trace, a plot of the estimated values of the parameters as a function of k. The value of k is selected as soon as the coefficients stabilize in magnitude. Other methods of selecting the value of k are discussed in the literature review in Chapter III.

<u>Properties of the Ridge Regression Estimator</u>

Ridge estimation produces biased estimates since the expected value of $\hat{\beta}^*(k)$ is

$$E[\hat{\beta}^*(k)] = (X'X+kI)^{-1}X'X\beta . \tag{2.5}$$

The variance-covariance matrix is

$$VAR[\hat{\beta}^*(k)] = (X'X+kI)^{-1}X'X(X'X+kI)^{-1}\sigma^2 . \tag{2.6}$$

The ridge solution requires some increase in the residual sum of squares above the least squares sum of squares as is shown in

$$[Y-X\hat{\beta}^*(k)]'[Y-X\hat{\beta}^*(k)] = (Y-X\hat{\beta})'(Y-X\hat{\beta})$$

$$+ (\hat{\beta}^*(k)-\hat{\beta})'X'X(\hat{\beta}^*(k)-\hat{\beta}) \qquad (2.7)$$

where $(Y-X\hat{\beta})'(Y-X\hat{\beta})$ is the OLS residual sum of squares. The mean square error function of $\hat{\beta}^*$ is

$$E[L^2(k)] = E[(\hat{\beta}^*-\beta)'(\hat{\beta}^*-\beta)]$$

$$= \sigma^2 \sum_{i=1}^{p} \lambda_i/(\lambda_i+k)^2 + k^2\beta'(X'X+kI)^{-2}\beta$$

$$= \sum_{i=1}^{p} \mathrm{Var}(\hat{\beta}_i^*) + \mathrm{Bias}^2(\hat{\beta}^*) . \qquad (2.8)$$

The first element is the sum of the variances (total variance) of the parameter estimates while the second is the square of the bias introduced when $\hat{\beta}^*$ is used instead of $\hat{\beta}$.

The total variance is a continuous, monotonically decreasing function of k and the squared bias is a continuous, monotonically increasing function of k.

Figure 1 shows the qualitative form of the relationships between the variances, the squared bias, and the parameter k. As is indicated by the dotted line, the sum of the variance and squared bias, the possibility exists that there are values of k (admissible values) for which the mean square error is less for $\hat{\beta}^*$ than it is for $\hat{\beta}$.

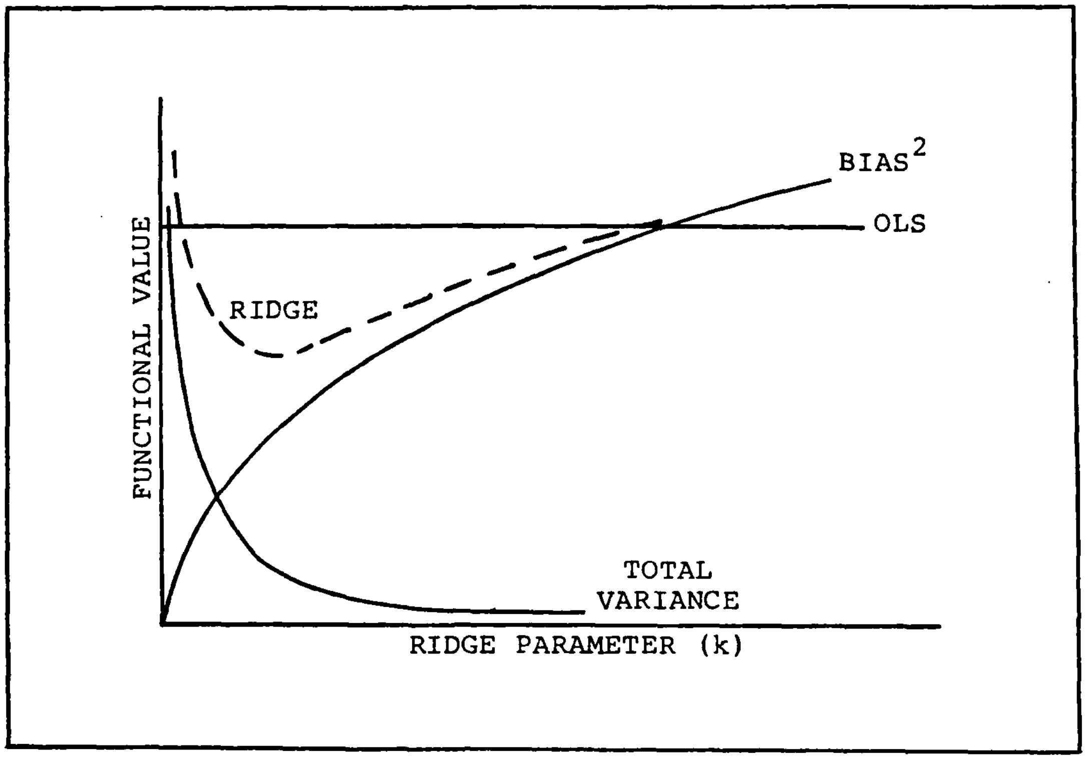

Fig. 1. Mean Square Error Functions

Mathematically, it can be shown that $E[L_1^2(k)]$ will go through a minimum and that the squared bias approaches $\beta'\beta$ as an upper limit. As the magnitude of $\beta'\beta$ increases, the minimum will move toward k=0. Because $\beta'\beta$ is not boundless in practice, there should be a value (or values) of k that will put $\hat{\beta}^*$ closer to β than $\hat{\beta}$. See Hoerl and Kennard (Ref 12) for proofs and complete development of the theory.

<u>The Ridge Trace</u>

The ridge trace is a two-dimensional plot of the ridge coefficient estimates, $\hat{\beta}^*(k)$, against k. It helps provide an insight into the structure of the problem and the sensitivity of the results to the particular set of data at hand. For nearly orthogonal data, the ridge trace provides little additional information. However, as multicollinearity in the data increases, the ridge trace stabilizes more rapidly (the rate of change of the standardized coefficients gets rapidly smaller) providing a significant contribution in the analysis of the problem (for example, see Hoerl and Kennard (Ref 13)).

The ridge trace is a characterization of a constrained optimization problem. The residual sum of squares of an estimator B of the vector β is

$$\begin{aligned} \Phi &= (Y-XB)'(Y-XB) \\ &= (Y-X\hat{\beta})'(Y-X\hat{\beta}) + (B-\hat{\beta})'X'X(B-\hat{\beta}) \\ &= \Phi_{LS} + \Phi(B) \end{aligned} \tag{2.9}$$

which is the minimum found by the least squares solution plus the value of the quadratic form. A move from the minimum sum of squares point (Φ_{LS}) might be reasonable if the length of regression vector (B) could be shortened.

The ridge trace follows a path through the sum of squares surface so that for a fixed value of Φ, a minimum value of B is chosen. Stated mathematically, the optimization problem is

$$\begin{aligned} &\text{minimize} \quad B'B \\ &\text{subject to} \quad (B-\hat{\beta})'X'X(B-\hat{\beta}) = \Phi_0 \ . \end{aligned} \tag{2.10}$$

Using the Lagrangian approach, the solution to the problem is

$$B = \hat{\beta}* = (X'X+kI)^{-1} X'Y ,$$

the ridge estimator where k is chosen to satisfy the constraint (equation 2.10). In practice, it is easier to choose $k>0$. This results in an increase in the residual sum of squares and reduction in the R^2, however, it allows for minimization of the regression vector.

Monte Carlo Simulation of the Data

The comparison of ridge and least squares estimators requires a large number of trials. Consequently, Monte Carlo simulation was used to generate 1,000 data sets for each of the predictor variable combinations. Several subroutines from the International Mathematical and Statistical Library, IMSL (Ref 15), were used in FORTRAN programs DATA and DATAL to generate the predictor and criterion variable values for the linear and log-linear model forms. The FORTRAN statements in these programs are contained in Appendices A (program DATA) and B (program DATAL).

For the linear model, subroutine GGNSM was used to produce the predictor variable deviate vectors with known correlation matrix from a multivariate normal distribution. Corresponding criterion variable values were generated by adding a mean corrected random error deviate from a univariate normal distribution (subroutine GGNML), adjusted to variance σ^2, to a linear combination of the predictor variable deviates and known coefficient parameters (β's).

For the log-linear model, the predictor variable vectors were produced by mean correcting the natural logarithms of the multivariate normal deviates generated by subroutine GGNSM. The criterion variable values (ln(Y)'s) were generated by adding a random error similar to that described for the linear model to a linear combination of the known coefficient parameters and the predictor variable values (ln(X)'s).

The random error terms of both models were mean corrected so that the X's, Y's, ln(X)'s, and ln(Y)'s would have zero means and the resulting regression models would have no Y-intercept term. The coefficient parameters were chosen as 1's for all trials so that all variables would have equal weighting in the analysis.

The relative magnitude of the random error term, with respect to the value of the linear combination of the known coefficient parameters and predictor variables, was used to control the model fit (R^2). The correlation matrix for the simulated predictor deviates was used to control the size of the variance inflation factors. Separate data was generated for each model size (2, 3, or 4 variable models); however, the same random number seed was used for all data sets.

In keeping within the usual limitations of cost estimating data sets, a sample set of 20 random vectors was used to perform each least squares and ridge regression analysis. Similar analyses of the 1,000 random sample sets provided the data for comparison of the two estimators.

<u>Predictor Variables</u>

The predictor variables for the model used to compare OLS and ridge estimator performance include the number of variables (NVAR), the value of k (K), the index of multicollinearity (RL), and the model fit under least squares (RSQ). Each variable will be explained in detail in succeeding paragraphs.

To determine the impact of model size on comparison of estimators, the analysis was performed for 2, 3, and 4 variable models. Data for each model size was generated using different correlation matrices. The correlation matrices were chosen so as to have reasonably similar determinants for the three levels of multicollinearity investigated.

The value of k was chosen by analyzing the ridge trace and the variance inflation factors for the first sample set. Once a value of k was selected, it was used for all 1,000 analyses.

The index of multicollinearity was chosen as

$$R_L = \frac{\sigma^2 \sum_{i=1}^{p} VIF_i}{\sigma^2 p} = \frac{\sum_{i=1}^{p} VIF_i}{p} . \qquad (2.11)$$

It measures the squared error in the OLS estimators relative to the size of that error if the data were orthogonal (Ref 1:183). It is interesting to note that this index can also be thought of as the average variance inflation factor of the $(X'X)^{-1}$ matrix since p is the number of predictor variables. The mean of the average variance inflation factors for the 1,000 trials was treated as the predictor variable RL.

The model fit is measured by coefficient of determination (R^2), the ratio of SSR (sum of squares regression) and SST (sum of squares total) in the regression model. The mean of the least squares R^2 (RSQLS) for the 1,000 trials was treated as the predictor variable RSQ.

Measure of Effectiveness

The criterion for evaluation of the two estimators is the mean square error (MSE) of the regression coefficients. The mean square error for the vector $(\hat{\beta})_i$, least squares solution for trial i, is defined as

$$\mathrm{MSE}(\hat{\beta})_i = E[\sum_{j=1}^{p} (\hat{\beta}_j - \beta_j)^2]_i \quad . \qquad (2.12)$$

For the 1,000 trials, the average MSE $(\hat{\beta})$ becomes

$$\overline{\mathrm{MSE}}(\hat{\beta}) = \frac{\sum_{i=1}^{1000} \mathrm{MSE}(\hat{\beta})_i}{1000} \quad . \qquad (2.13)$$

Similarly, the mean square error for the vector $(\hat{\beta}^*)_i$, ridge regression solution for trial i, is defined as

$$\mathrm{MSE}(\hat{\beta}^*)_i = E[\sum_{j=1}^{p} (\hat{\beta}_j^* - \beta_j)^2]_i \ . \qquad (2.14)$$

For the 1,000 trials, the average MSE $(\hat{\beta}^*)$ becomes

$$\overline{\mathrm{MSE}}(\hat{\beta}^*) = \frac{\sum_{i=1}^{1000} \mathrm{MSE}(\hat{\beta}^*)_i}{1000} \ . \qquad (2.15)$$

The ratio of these averages,

$$\frac{\overline{\mathrm{MSE}}(\hat{\beta})}{\overline{\mathrm{MSE}}(\hat{\beta}^*)} \ , \qquad (2.16)$$

indicates the relative improvement or deterioration of the ridge regression model with respect to the least squares model in a mean square error sense. A ratio greater than one indicates an improvement by the ridge regression; the greater the ratio, the greater the relative improvement. A ratio less than one indicates that the ridge regression model provides a worse model in the mean square error sense; the smaller the ratio, the worse the model.

III. Literature Review

Introduction

A considerable amount of research has been done based on the results of Hoerl and Kennard (Ref 12) who proposed and developed a comprehensive theory supporting the argument that it is helpful to augment the diagonal of the normal equations matrix by a small positive quantity in order to prevent "inflation" of the elements of the regression coefficient vector. The most significant portion of the research has been focused in the following areas:

1. Estimation methods for the value of k, the ridge bias parameter.

2. (a) Development of other ridge estimators and the comparison of these estimators with the standard ridge estimator, $\hat{\beta}^* = (X'X+kI)^{-1}X'Y$, and the ordinary least squares estimator.

(b) Development of alternative solutions to the ridge trace and generalized ridge regression procedures presented in Hoerl and Kennard (Ref 12:63-66).

3. (a) Development of the theoretical properties of the ridge regression family of estimators.

(b) Attempts to identify the probability distribution of ridge estimators.

4. Alternative methods to MSE of the coefficients for evaluating the regression estimates.

5. Simulation studies comparing ridge and other biased estimators with unbiased estimators (principally OLS) using Monte Carlo techniques.

6. Studies dealing with the practical application of ridge regression.

Estimation Methods for k

In their original papers (Refs 12; 13), Hoerl and Kennard discussed the use of the ridge trace as the "best method for achieving a better estimate $\hat{\beta}*$" with respect to mean square error. This method involves selecting a single value of k (all $k_i=k$) once the system has stabilized and has the general characteristics of an orthogonal system. Although reasonably simple, this method has been criticized by Smith and Campbell (Ref 33), Thisted (Ref 36), Van Nostrand (Ref 37), and others who oppose restricting the parameters of the model and mechanically manipulating the data without knowledge of the phenomena being modeled (using a priori information about the coefficients).

Another approach presented in the original papers to achieve a better estimate $\hat{\beta}*$ was generalized ridge regression (GRR). The general linear regression model $Y=X\beta+u$ is reduced to canonical form by transformations so that the $X'X$ matrix is diagonal. Iteration is used to find optimal k_i's which achieve stability in estimates in the canonical form. The GRR estimates are obtained from the canonical model estimates through an inverse transformation.

As an alternative to the iterative approach of Hoerl and Kennard, Hemmerle (Ref 9) proposed a non-iterative, closed form solution. This solution was shown to depend on certain convergence/divergence conditions which related to the ordinary least squares estimator. When the proper conditions are met, an explicit solution for the optimum canonical model estimates is obtained leading directly to estimates of the k_i's.

Directed ridge regression, a modification of the procedure of Hoerl and Kennard, was proposed by Guilkey and Murphy (Ref 4). This method alters only diagonal elements corresponding to low eigenvalues in an attempt to produce less bias in the coefficient estimates than methods that alter all the diagonal elements.

<u>Other Ridge Related Estimators and Solution Techniques</u>

To overcome the objections to using a subjective estimate of k (or k_i's), a number of estimators for k have been suggested. Hoerl, Kennard, and Baldwin (Ref 14) proposed the estimator $k=p\hat{\sigma}^2/\hat{\beta}'\hat{\beta}$ where $\hat{\beta}$ and p have been defined previously and $\hat{\sigma}^2$ is an unbiased estimate of σ^2. McDonald and Galarneau (Ref 24) suggested "ridge-type" estimators formed by first estimating the squared length of the unknown coefficient vector and then choosing the value of k so that the ridge estimator squared length was equal to this estimated quantity.

Since

$$E\,(\hat{\beta}'\hat{\beta}) = \beta'\beta + \sigma^2 \sum_{i=1}^{p} (1/\lambda_i) \qquad (3.1)$$

the quantity

$$Q = \hat{\beta}'\hat{\beta} - \hat{\sigma}^2 \sum_{i=1}^{p} (1/\lambda_i) \qquad (3.2)$$

is an unbiased estimator of $\beta'\beta$. Therefore, for $Q>0$ McDonald and Galarneau suggested the estimator $\hat{\beta}_k$, such that $\hat{\beta}_k{}'\hat{\beta}_k=Q$. For $Q<0$, they suggested selecting k equal to zero (OLS estimator) or infinity (zero vector estimate). Simulations by both sets of authors have shown that the estimators did well for some selections of the parameters but worse for others as compared to the OLS estimator. McDonald and Galarneau also concluded that the performance of the ridge estimators depended on "the variance of the random error, the correlations among the explanatory variables, and the unknown coefficient vector."

A computer iteration technique for finding the k value associated with the minimum mean square error of estimation was proposed by Kasarda and Shih (Ref 17). This technique is based on the monotonic properties of the total variance and squared bias terms as shown by Hoerl and Kennard (Ref 12:60-63). The MSE is estimated by a variable ($\widehat{MSE}$) which, through computer iteration, converges on the minimum point, yielding the optimum k value. This technique is also applicable to the directed ridge regression approach discussed earlier.

Another alternative to subjectively selecting a k value, the minimum mean square error estimator, was proposed by Farebrother (Ref 3). This estimator was extended and simulated in Vinod (Ref 39) where it was found to be inferior (in MSE) to a Stein-Rule estimator (see Judge, Bock and Yancey (Ref 16) for a detailed study of Stein-Rule estimators).

Several nonstochastic estimators of k have been proposed by Gunst and Hua (Ref 6) and Vinod (Ref 38). The two methods proposed by Gunst and Hua include one where k is chosen so that $|X'X+kI|=1$ (forcing the ridge system to behave orthogonally) and a second where k is chosen so that the largest variance inflation equals 4. The method proposed by Vinod involves choosing k so that the "multicollinearity allowance"

$$m = p - \sum_{j=1}^{p} \lambda_j(\lambda_j+k)^{-1} \quad . \tag{3.3}$$

Here, m has the interpretation as the assigned deficiency in the rank of $(X'X)$. The value of k is found iteratively once the rank deficiency has been assigned. Vinod also proposed using a "ridge trace" as a function of the multicollinearity allowance (m) instead of the standard ridge trace (function of k). He proposed the Index of Stability of Relative Magnitudes (ISRM) to quantify the stable region of m values.

Gunst and Hua (Ref 6:8-21) found that use of the minimum ISRM proposed by Vinod performed erratically and often indicated several local minima, one of which had to be subjectively selected. They also found fault with the nonstochastic rule requiring $|X'X+kI|=1$. In some cases, the determinant of $X'X$ matrix was so small that a large value of k caused the bias of the ridge estimator to overcome the reduction in variance negating the advantage of using ridge regression.

Theoretical Properties of Ridge Estimators

Several theoretical studies were conducted to provide additional information about the properties of ridge estimators. Hawkes and Alam (Ref 8) discussed the theoretical properties of ridge estimators using both classical and Bayesian statistics. They showed that for certain choices of k, depending on Y, the ridge estimator had uniformly smaller mean square error than the least squares estimator, provided that a number of the characteristic roots of the $X'X$ matrix were sufficiently small.

An investigation of the probability distributions of ridge estimators was conducted by Lewis (Ref 20) so that hypothesis tests and computation of confidence bounds could be made for $\hat{\beta}^*$. It was found that the distribution of $\hat{\beta}^*$ depended on the objective rule used to select k.

The objective rules selected by the author did not lead to useful probability distributions.

Alternative Evaluation Methods

The majority of the studies comparing estimators used the concept of mean square error of the coefficients as the criteria for evaluation. Gunst and Mason (Ref 7), on the other hand, used integrated mean square error (IMSE) in evaluating ridge, principal component, and least squares estimators. This method was found to introduce the problem of choosing a weighting function to determine the IMSE in addition to the other estimation problems. Also, the results were not considered conclusive since the determination of the effects of variable selection on the technique and the impact of restrictions on the estimators used in the analysis required more research.

Simulation Studies of Ridge and Other Estimators

Many comparisons have been made between ridge regression and other estimators using the Monte Carlo simulation technique. Newhouse and Oman (Ref 29) performed a study which was restricted to the case of two predictors having two different values of r, the correlation between predictors, and a number of methods for choosing k. They concluded that for the two variable case the ridge estimators did worse than the OLS estimators for at least some of the models. The failures were "by a sufficient margin and in a 'sufficient' number of cases" that they recommended against use of ridge regression. As was pointed out in Eskew (Ref 2:18), however, Sclove (Ref 32) showed in 1967 that no estimator is better in the total mean square error sense than the least squares estimator when only two parameters are estimated. Therefore, the experimental results of Newhouse and Oman only confirm the theoretical work of Sclove.

Lawless and Wang (Ref 19) found results contrary to Newhouse and Oman in their evaluation of ridge estimators. Further, they concluded that it may not be worthwhile to consider generalized ridge estimators since they were found to have inferior mean square error properties than the ordinary ridge estimators.

Simulation results by Newman (Ref 30) supported the first conclusion of Lawless and Wang. He showed that the ridge estimator $\hat{\beta}^*(k)$, found by selecting the value of k from the ridge trace, outperformed other estimators including least squares.

Mitra and Ling (Ref 27) found several ridge estimators, those proposed by Hoerl, Kennard and Baldwin (Ref 14), Farebrother (Ref 3:128), McDonald and Galarneau (Ref 24:409, rule 2), Hoerl and Kennard (Ref 12:63), Guilkey and Murphy (Ref 4:770), and several others, superior to the OLS estimator (in mean square error) and provided a ranking of these estimators based on parameters of the basic regression model.

Su and Chai (Ref 34) performed a comparison of the ridge estimator proposed by Hoerl, Kennard and Baldwin and the least squares estimator using squared error of estimation ($L^2=(B-\beta)'(B-\beta)$), squared error of prediction ($SSE=(Y-\hat{Y})'(Y-\hat{Y})$), and cross validity (the Pearson's product moment correlation between the observed values in the second sample and the predictions made for the second sample using the ridge estimator estimated from the first sample). The results showed that the ridge estimator was better for nonorthogonal data and the least squares estimator better for orthogonal data (except in one case).

A study by Lindell (Ref 21) considered ridge (Hoerl, Kennard and Baldwin (Ref 14)), ordinary least squares, and jackknife estimators (Mosteller and Tukey (Ref 28)) evaluated using the criteria of mean square error of the coefficients and the size of the t-statistics associated with these coefficients. The design considered two factors, the sample size to number of predictors ratio (N/p) and the metric quality of the data (dichotomous and polychotomous data were used to assess the sensitivity of the estimators to different levels of violation of the regression assumptions). The results of the study showed that the ridge estimator performed better for smaller N/p ratios and worse for higher levels.

Eskew (Ref 2) and other authors, some of whom include Lindley and Smith (Ref 22), Smith and Campbell (Ref 33), Thisted (Ref 36), and Van Nostrand (Ref 37), have proposed the use of "a priori" information along with ridge regression in the estimation of the regression coefficients (Bayesian approach). In this approach, if the prior estimate of the coefficients is closer than the origin (zero-prior of the classical ridge estimator) to the true model parameters, then the squared bias of the ridge estimators will be reduced without an increase in variance. This results in an even greater improvement over the OLS estimator (in mean square error). Eskew showed that with "good" or even "fair" prior information that the ridge method was superior to OLS for estimation of the model parameters.

Practical Application Studies

Some practical aspects of ridge regression have been addressed by Marquardt and Snee (Ref 26) and Gunst (Ref 5). Three practical application examples were presented in Marquardt and Snee. They noted that models with no constant term required a smaller value of k (often $\leq .01$) than models with a constant term. Also, they claimed that models with lower R^2 statistics required larger values of k than better fitting models. The study further showed that the ridge regression coefficients performed better for prediction and extrapolation than least squares and were useful for selecting variables. Hocking (Ref 11:11, 23,28-31,37-44) also supported the use of ridge regression for variable selection.

Gunst applied ridge regression and two other biased estimators to a data set of automobile emissions. A number of selection rules for determining k were tried yielding similar estimates from the data. The resulting ridge model generated coefficients with magnitudes and signs inconsistent with a priori beliefs. However, it was judged superior to the OLS model.

Conclusions

The ridge regression technique has been shown to possess valuable theoretical and empirical properties which appear advantageous when the predictor variables are collinear. Many methods have been suggested for determining the amount of bias which is "optimal." The body of research has shown that no one method of choosing k is clearly superior to the others.

In terms of mean square error of the coefficients, improvements can be made for prediction and extrapolation by using ridge regression. The criticisms of several authors have pointed out that ridge regression must be used carefully in order to fulfill two important requirements: the model produced must make sense from the physical nature of the problem, and it must provide predictions close to reality.

IV. Research Methodology

Overall Approach

The overall approach of the investigation involved generating mean square error data for various levels of the independent variables (RSQ, RL, NVAR, and K) and analyzing this data using a linear regression model to determine which factor(s) produced a significant impact on the mean square error improvement (degradation) for the ridge versus OLS models. The regression approach was chosen because the independent variable levels could not be specified as is required for the treatments in an Analysis of Variance (ANOVA). In particular, the variables RL and RSQ were outputs of programs DATA and DATAL controlled by a correlation matrix with varying intercorrelations and dimensions (for RL) and the variance of the random error of the known model (for RSQ).

Model Development

A FORTRAN program, RIDGE, developed by McNichols (Ref 25) from theory presented in Chatterjee and Price (Ref 1:181-187), was used to portray the ridge trace and provide other outputs for the first data set (20 random vectors). The computer code for this program is contained in Appendix C.

The outputs of program RIDGE consisted of the following:

1. Sample means and standard deviations of both dependent and independent variables.

2. Sample covariance matrix of all variables.

3. Values of the standardized coefficient estimates, $\hat{b}^*$, for each value of k.

4. Values of the unnormalized coefficient estimates ($\hat{\beta}$ for k=0 and $\hat{\beta}^*$ for k>0) for each value of k computed from

$$\hat{\beta}_j^* = \hat{b}^*(s_j/s_y) \tag{4.1}$$

where s_j is the sample standard deviation of the jth variable and s_y is the sample standard deviation of the dependent variable.

5. Values of the Variance Inflation Factors (VIF's) for each coefficient at each value of k. The VIF's are the diagonal elements of the matrix

$$(X'X+kI)^{-1} X'X (X'X+kI)^{-1} \tag{4.2}$$

which when multiplied by σ^2 is the variance/covariance matrix of $\hat{b}^*$.

The values of k were selected at three or more levels including k=0 (OLS solution), the k value corresponding to all VIF's less than or equal to 10 (where appropriate), and others selected where the ridge trace (standardized coefficient estimates) and VIF's appeared to stabilize.

A modified RIDGE program was used to perform 1,000 Monte Carlo trials using the k value selected by reviewing the output from RIDGE. A listing of the program is contained in Appendix D. The program generated the following data for each trial (the variables listed are SPSS variable names used in the CONDESCRIPTIVE analysis discussed below):

1. Unstandardized coefficient estimates (variables BLS1 to BLS4).

2. Ridge coefficient estimates (variables BR1 to BR4).

3. Variance Inflation Factors for the OLS solution (k=0) and selected k value (variables VIFLS and VIFR, respectively).

4. Model fit (R^2) for the OLS solution (variable RSQLS) and selected k value (variable RSQR).

5. Index of multicollinearity (variable RL).

6. Mean square error for the OLS solution and selected k value (variables MSELS and MSER, respectively).

Several statistics were computed using subprogram CONDESCRIPTIVE of SPSS (Statistical Package for the Social Sciences) (Ref 31) for the data produced by the Monte Carlo RIDGE program.

The mean, standard deviation, variance, range, and minimum and maximum values for each variable (dependent and independent) were computed from the 1,000 cases at both the k=0 and selected k>0 levels.

The independent variables for the regression analysis (RSQ, RL, NVAR, and K) were taken from both the CONDESCRIPTIVE output (RSQ and RL) and the Monte Carlo inputs (NVAR and K). The variable RSQ was the mean of the R^2 values for the 1,000 trials; RL was the mean of the RL's for the trials.

The dependent variable for the regression analysis was the mean square error ratio (MSERATIO) as defined and interpreted in Chapter II. The numerator, MSELS, was computed in the Monte Carlo analysis for k=0; the denominator, MSER, for the selected k>0 value. The ratio (MSELS/MSER) was computed in the regression analysis through use of a COMPUTE statement (Ref 31:96-97).

Generation of the Data

The levels of the independent variables were chosen to generate data consistent with the usual limitations of cost estimating data sets. Each variable will be discussed separately in the succeeding paragraphs.

Three levels of RL were considered corresponding to low (RL (mean) approximately equal to 1.5 - 2), medium (approximately 10), and high (approximately 100) degrees of multicollinearity. RL values below 1.5 were considered too orthogonal and above 120 excessively collinear. The correlation matrices (dimensions 2-4) used to generate the independent variable deviates (X's) were chosen from actual data sets by evaluating the determinant of each matrix and grouping matrices with similar determinants together (one from each dimension). Correlation matrices with determinants very close to zero produced data with high multicollinearity; matrices with large determinants produced near-orthogonal data.

Only models with RSQ values between 52 and 99.8 percent were used in the analysis since estimates involving cost would not frequently be made with models of poorer fit.

Separate data were generated for models of each size (NVAR); however, the same random number seed was used to generate each set of data. Three SPSS programs were necessary to analyze the data output from the Monte Carlo RIDGE program due to the three model sizes considered. The computer code for these programs is contained in Appendix E.

Analysis of the Data

The REGRESSION subprogram of SPSS was used as the descriptive tool to identify the structural nature of the relationship between mean square error improvement (degradation) of ridge versus OLS models as a function of linear combinations of the predictor variables. For this problem it was appropriate to isolate the smallest subset of predictor variables that yielded the greatest impact on the model. Therefore, the stepwise solution procedure was selected. This procedure combined forward inclusion, the entering of independent variables that met pre-established statistical criteria, with deletion of variables that met specified exit criteria at each successive step.

The variables considered by the model included the four predictor variables, all first order cross products (six interaction terms), and squared predictor variable terms.

The principal criteria for evaluating the terms of the model included the following:

1. Comparing the coefficient of determination (R^2) for each model step.

2. Comparing the relative size of the partial F-statistics for all variables within each model step.

3. Comparing the relative size of the partial F-statistic for the variables entered during each model step.

The size of the coefficient of determination was interpreted as the percent of total variation explained by the variables in the regression model. The change in the R^2 for a step indicated the additional percentage of variation explained by the variable entering the model, given the variables already in the model. For this analysis, the R^2 statistic was clearly the most important measure for determining the key variables (terms).

The size of the partial F-statistic for each variable within a model step indicated the significance (relative importance) of that variable with respect to the model formed for that step. The most significant variable contained the highest partial F-statistic and so on.

The partial F-statistic for each variable entering the model was compared between model steps as a further measure of the relative importance of each additional variable to the model.

The SPSS programs used to perform the regression analysis are contained in Appendix F. The output from these programs will be presented and discussed, along with the results, in Chapter V.

V. Results and Conclusions

Linear Model Data Analysis

The regression analysis of the linear model data produced a seven variable model. This model was based on 115 cases which are presented in Appendix G. A summary of the linear model regression results is contained in Table 1.

Based on the criteria used to evaluate the terms of the model discussed in Chapter IV, the key term in the model was determined to be CROSS4, the cross product of K and RL. This term explained 82.6 percent of the total variation in the data and was much more significant than the other terms entering the model as is shown by the partial F-statistics under the "F to enter or remove" column of Table 1. Also, throughout all of the regression steps, the CROSS4 term remained much more significant than the other variables. This is shown in columns "Partial F of CROSS4" and "Partial F of the Next Most Significant Term."

Because of the high amount of variation explained by the CROSS4 term, the small contributions made in explaining the remaining variation by the other terms, and the relative sizes of the partial F-statistics discussed above, CROSS4 was selected as the key variable explaining nature of the relationship between mean square error improvement (degradation) of the ridge versus OLS models.

TABLE 1

SUMMARY OF REGRESSION ANALYSIS RESULTS--LINEAR MODEL DATA

Step	Variable Entered (Removed)	R^2	R^2 Change	F to Enter or Remove	Partial F of CROSS4 Term	Partial F of Next Most Significant Term (Variable)
1	CROSS4	.826	.826	537.65	537.65	14.29 (CROSS3)
2	CROSS3	.846	.020	14.29	612.05	14.29 (CROSS3)
3	CROSS6	.851	.005	3.74	625.25	10.42 (CROSS3)
4	K	.856	.005	3.72	590.78	4.90 (CROSS6)
5	RL	.860	.004	3.25	267.71	6.45 (CROSS6)
6	CROSS5	.885	.025	22.95	343.01	26.75 (RL)
7	CROSS2	.886	.001	1.72	313.56	28.31 (RL)

<u>NOTES</u>:

[a]Variables defined: CROSS2 - RSQ x K; CROSS3 - RSQ x NVAR; CROSS4 - RL x K; CROSS5 - RL x NVAR; CROSS6 - K x NVAR.

[b]All variables (terms) with F-statistics less than 1 are omitted from the table.

Graphical Analysis of the Linear Model Data

The interrelationship of the variables in the CROSS4 term is shown in Figure 2. The graph is a plot of MSERATIO (improvement/degradation of the ridge versus OLS model) versus K at fixed levels of RL. For two of the lower values of RL (1.536 and 1.830), the results were mixed. Slight improvements (MSERATIO between 1.0 and 1.60); were shown for 16 cases; however, slight degradations (MSERATIO between 0.19 and 1.0) were shown for 4 cases. For RL levels 2.196 to 13.424, greater improvements (MSERATIO between 1.0 and 4.16) were made while there were no degradations. The size of the improvements increased consistently with k for a given level of RL and with RL for a given level of k. For high levels of RL (50.841 to 119.772), large improvements (MSERATIO between 1.0 and 86.94) were realized, especially for models of poorer fit (R^2 values of 90 percent or lower).

Results of the Linear Model Analysis

For the linear model, ridge regression provided the greatest MSE improvement in situations with high multicollinearity, especially for models with an R^2 value of 90 percent or lower. Only in a few situations did the technique show a degradation in the mean square error. These were due to overestimating the value of k, causing the bias to overcome the reduction in variance of the ridge

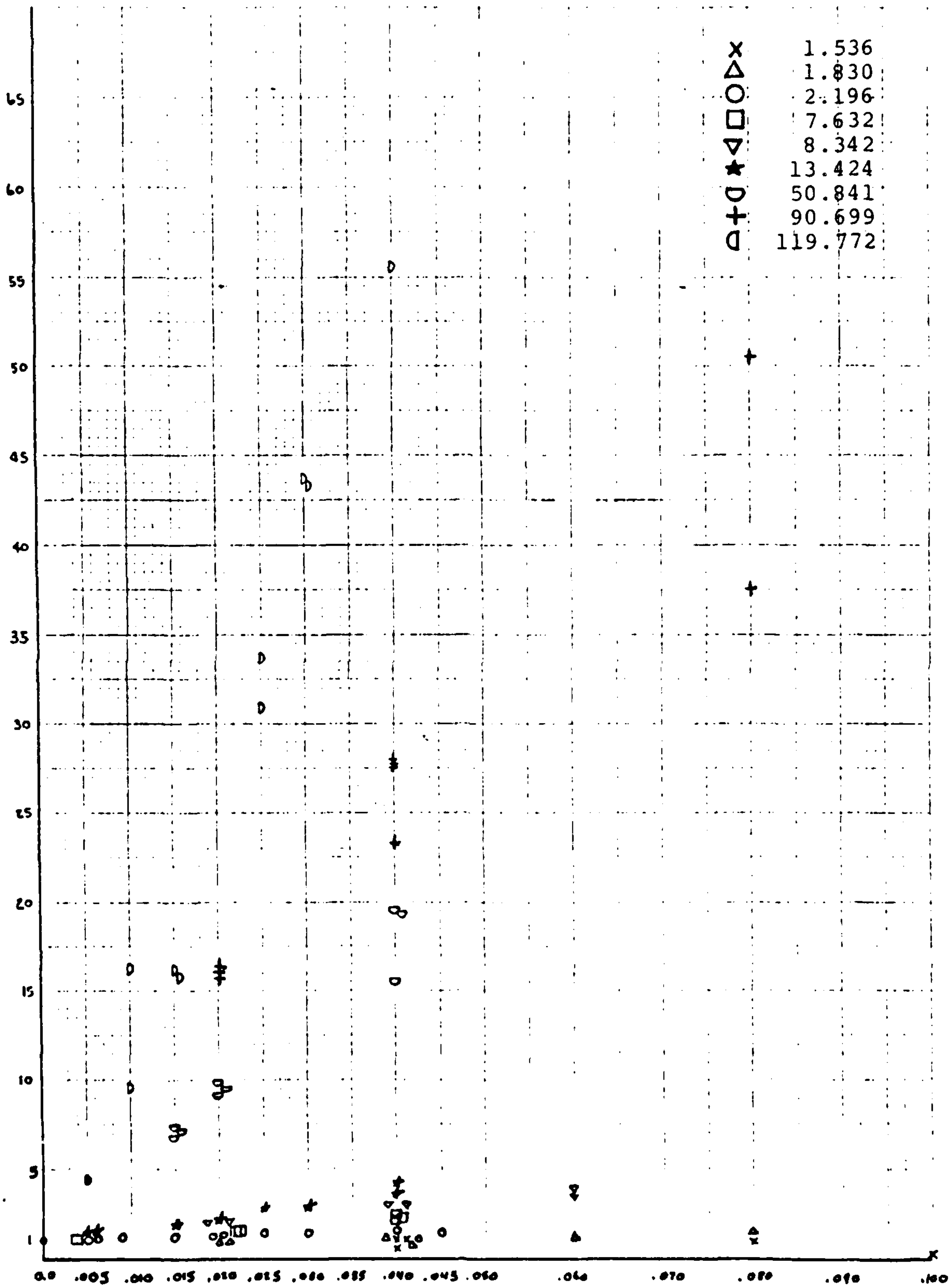

Fig. 2. Graphical Analysis--Linear Model Data

estimator. Consequently, conservative estimates of k, less than or equal to 0.04, limit worsening the mean square error for data with low RL values (RL less than 2) and enable larger improvements to be made for more collinear data (RL greater than 2).

Log-Linear Model Data Analysis

The regression analysis for a log-linear model data produced a nine variable model. The model was based on 108 cases which are presented in Appendix H.

Using the same evaluation criteria as the linear model, CROSS4 (cross product of K and RL) was again selected as the key term explaining the improvement (degradation) in the MSE. A summary of the log-linear regression results is contained in Table 2.

For the log-linear model, the CROSS4 term explained 92.4 percent of the total variation in the data; each of the remaining eight terms explained less than 0.9 percent of the variation (contributed less than .009 to the R^2 statistic) as they were added to the model.

As in the linear model, CROSS4 was much more significant (partial F-statistic to enter equal to 1280.23) than any other variable entering the model. The next most significant term was CROSS6 with an F to enter of 15.28. Throughout the iterations of the stepwise regression, CROSS4 remained the most significant term by a wide margin

TABLE 2

SUMMARY OF REGRESSION ANALYSIS RESULTS--LOG-LINEAR MODEL DATA

Step	Variable Entered (Removed)	R^2	R^2 Change	F to Enter or Remove	Partial F of CROSS4 Term	Partial F of Next Most Significant Term (Variable)
1	CROSS4	.924	.924	1280.23	1280.23	15.28 (CROSS6)
2	CROSS6	.933	.009	15.28	1320.80	15.28 (CROSS6)
3	CROSS5	.941	.008	14.33	976.55	22.58 (CROSS6)
4	RL	.946	.005	10.24	638.00	22.87 (CROSS5)
5	CROSS1	.950	.004	6.74	677.23	28.84 (CROSS5)
6	NVAR	.951	.001	2.77	690.04	28.70 (CROSS5)
7	K	.953	.002	4.02	616.59	27.99 (CROSS5)
8	CROSS2	.954	.001	1.98	615.59	27.57 (CROSS5)
9	CROSS3	.955	.001	1.39	615.86	27.69 (CROSS5)
10	(NVAR)	.955	.000	.056	622.47	28.60 (CROSS5)

NOTES:

[a]Variables defined: CROSS1 - RSQ x RL; CROSS2 - RSQ x K; CROSS3 - RSQ x NVAR; CROSS4 - RL x K; CROSS5 - RL x NVAR; CROSS6 - K x NVAR.

[b]All variables (terms) with F-statistics less than 1 are omitted from the table.

as is shown in the comparison of partial F-statistics of CROSS4 and the next most significant term in Table 2.

Graphical Analysis of the Log-Linear Model Data

The interrelationship for the variables in the CROSS4 term of the log-linear model is shown in Figure 3. The graph shows results very similar to the linear model. Mixed results were obtained for RL values 1.528 and 1.836. As with the linear model, the degradations were due to overestimates of k. Slight improvements (MSERATIO between 1.0 and 1.18) were shown for 21 cases while 3 cases showed a slight degradation (MSERATIO between .25 and 1.0) in the mean square error. Similar improvements (MSERATIO between 1 and 3.66) were shown for RL levels between 3.174 to 12.127 as the 2.196 to 13.424 levels in the linear model. Again, greater improvements (MSERATIO between 1.0 and 64.29) were made for RL levels 50.926 to 120.029 with the largest corresponding to models with poorer fit (R^2 values of 92 percent or lower).

Results of the Log-Linear Model Analysis

The overall results of ridge regression for the log-linear model are the same as those for the linear model. Conservative estimates of k, less than or equal to 0.04, limit the worsening effects of bias in the ridge regression estimates (RL values less than 2) while enabling

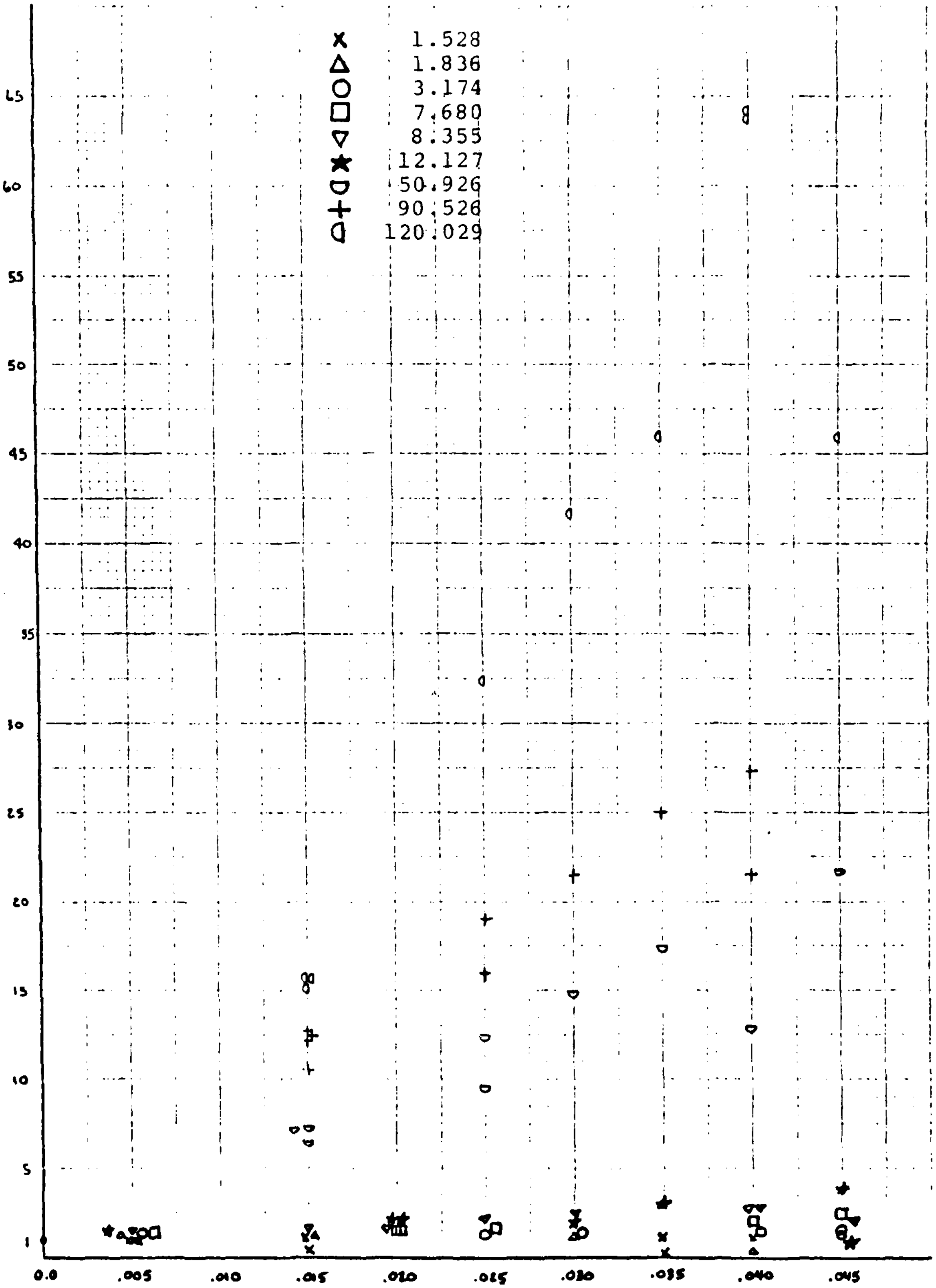

Fig. 3. Graphical Analysis--Log-Linear Model Data

more significant improvements to be made for data with higher degrees of multicollinearity (RL values greater than 3).

Conclusions

The regression and graphical analyses showed, within the limitations and assumptions of the investigation, when ridge regression provided a better estimate of the coefficients of a system cost model compared to ordinary least squares for simulated data. From the regression analysis, the interaction of the value of k and the degree of multicollinearity (RL) determined the amount of relative improvement in the mean square error. The graphical analysis showed in detail how the variables interacted and what degree of improvement (degradation) could be expected from the data.

In using the ridge trace to select the value of k, it was important to consider the tradeoff between reduction in variance and increase in bias on the mean square error of the coefficient vector. Conservative estimates ($k \leq 0.04$) allowed for reduction in variance desired in the biased estimate for RL values as low as 1.536 for the linear model and 1.528 for the log-linear model. Further, the ridge trace and variance inflation factors of the $(X'X)^{-1}$ matrix were valuable in choosing the value of k, particularly for RL levels greater than or equal to five.

Recommendations

Based on the results of this investigation, it is recommended that ridge regression be used as a tool to construct cost estimating models for data exhibiting multicollinearity. It is particularly valuable for high levels of multicollinearity (RL levels greater than 50) but also shows moderate improvements in MSE for lower RL levels (RL levels as low as 2).

The ridge trace and variance inflation factors can be used to select k which determines the amount of bias in the regression coefficient estimates. It is recommended that conservative estimates be made so that k is less than or equal to 0.04. However, for data producing one or more of the VIF's greater than or equal to 10, the estimate of k should be large enough to reduce the largest VIF to below 10. A priori information about the signs of the coefficients can also be used, within the boundaries recommended, in constructing the model. The value of k is selected after the standardized coefficients have stabilized to the "correct" sign and magnitude.

Suggested Follow-on Research

Further research could be directed at comparing predictions of the ridge and OLS models using actual data. These comparisons could be made using several existing statistics used by cost analysts.

Further Monte Carlo comparisons could be conducted examining prediction intervals of the two estimators. Selection of the appropriate technique might be based on the smallest prediction interval as an alternative to improvements in mean square error of the coefficient vector.

Finally, continued research could be conducted comparing the different methods for selecting k, using the basic variables considered in this study. Although the ridge trace and variance inflation factors provided valuable information for selecting a value of k, additional guidance concerning non-subjective methods for choosing k could simplify the modeling procedure and lead to expanded use of the ridge technique in the area of cost estimation.

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Appendix A

FORTRAN Code for Program DATA

```
      PROGRAM DATA(INPUT,TAPE4)
*
*
*
      REAL SIGMA,V(4,4),VVEC(10),XVEC(20,4),WKVEC(4),R(20),ET(20),B(4),
     1     C(20),Y(20),ETC(20),SUMX(4),MEANX(4),XVECMC(20,4)
      INTEGER M,N,NR,IER,K,I,J
      DOUBLE PRECISION DSEED,DSEED1
*
*****************************************************************************
*
*     PROGRAM GENERATES DATA FOR REGRESSION ANALYSIS IN PROGRAM RIDGE.
*     INDEPENDENT VARIABLE DATA GENERATED IN MEAN CORRECTED FORM USING IMSL
*        SUBROUTINE GGNSM(TRIANGULAR FACTORIZATION METHOD).
*
*     DEPENDENT VARIABLE DATA Y(I) GENERATED FROM MODEL Y=XB+E WHERE E IS A
*        NORMAL RANDOM ERROR TERM WITH MEAN ZERO AND VARIANCE SIGMA SQUARED.
*        IMSL SUBROUTINE GGNML IS USED TO GENERATE NORMAL(0,1) DEVIATES WHICH
*        ARE ADJUSTED TO VARIANCE SIGMA SQUARED.
*
*     MATRIX V( , ) IS IN CORRELATION FORM.
*     B( ) IS A VECTOR OF KNOWN COEFFICIENTS.
*     SIGMA IS THE VARIANCE OF THE RANDOM ERROR OF THE Y'S.
*     M IS THE ROW DIMENSION OF THE CORRELATION MATRIX V.
*     N IS THE COLUMN DIMENSION OF THE CORRELATION MATRIX V. ALSO, IT IS THE
*        NUMBER OF ELEMENTS(VARIABLES) IN THE X-VECTOR.
*     NR IS THE NUMBER OF N-ELEMENT X-VECTORS TO BE GENERATED IN EACH ITERATION.
*
*     DATA CARD FORMAT:
*     FIRST CARD--COL 1 NUMBER OF ROWS IN THE CORRELATION MATRIX(LIMIT IS 4).
*                 COL 3 NUMBER OF COLUMNS IN THE CORRELATION MATRIX(LIMIT IS 4).
*                       ALSO, IT IS THE NUMBER OF VARIABLES IN THE X-VECTOR.
*                 COLS 5-6 NUMBER OF MULTIVARIATE VECTORS(X) TO BE GENERATED.
*                 COLS 10-14 VARIANCE OF THE RANDOM ERROR OF THE Y
*                       OBSERVATIONS(3 DECIMAL PLACES).
*     SECOND CARD--COLS 1-5 COEFFICIENT B1.
*                  COLS 10-14 COEFFICIENT B2.
*                  COLS 20-24 COEFFICIENT B3.
*                  COLS 30-34 COEFFICIENT B4.
*     REMAINING CARDS-- ELEMENTS OF CORRELATION MATRIX V(I,J). EACH CARD
*                       CONTAINS ONE ROW OF THE MATRIX IN F FORMAT(4 DECIMAL
*                       PLACES).
*                       COLS 2-8 V(1,J)
*                       COLS 12-18 V(2,J)
*                       COLS 22-28 V(3,J)
*                       COLS 32-38 V(4,J)
*
*****************************************************************************
*
*
      DSEED=466364003.D0
      DSEED1=123457.D0
      READ'(I1,T3,I1,T5,I2,T10,F5.3)',M,N,NR,SIGMA
*INITIALIZE MATRICES AND VECTORS.
      DO 10 I=1,M
            B(I)=0.0
                  DO 20 J=1,N
                        V(I,J)=0.0
20                CONTINUE
10    CONTINUE
      DO 30 I=1,(N*(N+1))/2
            VVEC(I)=0.0
30    CONTINUE
      DO 40 I=1,NR
            DO 50 J=1,N
                  XVEC(I,J)=0.0
50          CONTINUE
40    CONTINUE
      DO 60 I=1,N
            WKVEC(I)=0.0
```

```
60      CONTINUE
        DO 70 I=1,NR
             ET(I)=0.0
             R(I)=0.0
             C(I)=0.0
             Y(I)=0.0
70      CONTINUE
        READ'(F5.2,T10,F5.2,T20,F5.2,T30,F5.2,)',(B(J),J=1,N)
*READ CORRELATION MATRIX
        DO 100 I=1,M
            READ'(T2,F7.4,T12,F7.4,T22,F7.4,T32,F7.4)',(V(I,J),J=1,N)
100     CONTINUE
*CONVERT MATRIX V TO SYMETRIC STORAGE MODE.
        CALL VCVTFS(V,N,4,VVEC)
*WRITE PARAMETER VECTOR TO PERMONENT FILE.
        WRITE(4,25) (B(J),J=1,N)
25      FORMAT(F5.2,T10,F5.2,T20,F5.2,T30,F5.2)
*LOOP FOR NUMBER OF DATA SETS WITH PARAMETERS SPECIFIED IN DATA INPUT.
        DO 110 K=1,1000

             IF(K.EQ.1) WKVEC(1)=0.0
             IF(K.GT.1) WKVEC(1)=1.0
*GENERATE X VALUES.  MATRIX XVEC IS DIMENSION NR X N.
             CALL GGNSM(DSEED,NR,N,VVEC,20,XVEC,WKVEC,IER)
*MEAN CORRECT XVEC MATRIX.
        DO 75 I=1,N
             SUMX(I)=0.0
75      CONTINUE
        DO 80 I=1,NR
             DO 90 J=1,N
                  SUMX(J)=SUMX(J)+XVEC(I,J)
90           CONTINUE
80      CONTINUE
        DO 95 I=1,N
             MEANX(I)=SUMX(I)/NR
95      CONTINUE
        DO 96 I=1,NR
             DO 97 J=1,N
                  XVECMC(I,J)=XVEC(I,J)-MEANX(J)
97           CONTINUE
96      CONTINUE
*GENERATE NR STANDARD NORMAL DEVIATES(VECTOR R).
             CALL GGNML(DSEED1,NR,R)
*ADJUST DEVIATES TO VARIANCE SIGMA SQUARED.
             DO 120 I=1,NR
                  ET(I)=R(I)*SIGMA
120          CONTINUE
*MEAN ADJUST ERROR DEVIATES.
        SUMET=0.0
        DO 125 I=1,NR
             SUMET=SUMET+ET(I)
125     CONTINUE
        ETMEAN=SUMET/NR
        DO 126 I=1,NR
             ETC(I)=ET(I)-ETMEAN
126     CONTINUE
*CREATE MATRIX PRODUCT XB.
             CALL VMULFF(XVECMC,B,NR,N,1,20,4,C,20,IER)
*ADD RANDOM ERROR TO MATRIX PRODUCT XB.
             DO 130 I=1,NR
                  Y(I)=C(I)+ETC(I)
130          CONTINUE
*WRITE DATA TO PERMANENT FILE.
             DO 140 I=1,NR
          WRITE(4,15) Y(I),(XVECMC(I,J),J=1,N)
15                FORMAT(T2,F8.4,T12,F8.4,T22,F8.4,T32,F8.4,T42,F8.4)
140          CONTINUE
110     CONTINUE
        END
```

Appendix B

FORTRAN Code for Program DATAL

```
      PROGRAM DATAL(INPUT,TAPE4)
*
*
*
      REAL SIGMA,V(4,4),VVEC(10),XVEC(20,4),WKVEC(4),R(20),ET(20),B(4),
     *     C(20),LOGY(20),ETC(20),SUMX(4),MEANX(4),XVECMC(20,4)
      INTEGER M,N,NR,IER,K,I,J
      DOUBLE PRECISION DSEED,DSEED1
*
**************************************************************************
*
*     PROGRAM GENERATES DATA FOR REGRESSION ANALYSIS IN PROGRAM RIDGE.
*     INDEPENDENT VARIABLE DATA GENERATED IN STANDARDIZED FORM USING IMSL
*        SUBROUTINE GGNSM(TRIANGULAR FACTORIZATION METHOD).
*
*        IMSL SUBROUTINE GGNML IS USED TO GENERATE NORMAL(0,1) DEVIATES WHICH
*        ARE ADJUSTED TO VARIANCE SIGMA SQUARED.
*
*     DEPENDENT VARIABLE DATA LOGY(I) GENERATED FROM MODEL Y=LOG(X)B+E WHERE E
*        IS A NORMAL RANDOM ERROR TERM WITH MEAN ZERO AND VARIANCE SIGMA
*        SQUARED.
*
*     MATRIX V( , ) IS IN CORRELATION FORM.
*     B( ) IS A VECTOR OF LOG LINEAR PARAMETERS.
*     SIGMA IS THE VARIANCE OF THE ERROR OF THE LOGY'S.
*     M IS THE ROW DIMENSION OF THE CORRELATION MATRIX V.
*     N IS THE COLUMN DIMENSION OF THE CORRELATION MATRIX V. ALSO, IT IS THE
*        NUMBER OF ELEMENTS(VARIABLES) IN THE X-VECTOR.
*     NR IS THE NUMBER OF N-ELEMENT X-VECTORS TO BE GENERATED IN EACH ITERATION.
*
*     DATA CARD FORMAT:
*     FIRST CARD--COL 1 NUMBER OF ROWS IN THE CORRELATION MATRIX(LIMIT IS 4).
*                 COL 3 NUMBER OF COLUMNS IN THE CORRELATION MATRIX(LIMIT IS 4).
*                       ALSO, IT IS THE NUMBER OF VARIABLES IN THE X-VECTOR.
*                 COLS 5-6 NUMBER OF MULTIVARIATE VECTORS(X) TO BE GENERATED.
*                 COLS 10-14 VARIANCE OF THE RANDOM ERROR OF THE LOG(Y)
*                       OBSERVATIONS(3 DECIMAL PLACES).
*     SECOND CARD--COLS 1-5 MULTIPLICATIVE CONSTANT PARAMETER A.
*                  COLS 10-14 EXPONENT PARAMETER B1.
*                  COLS 20-24 EXPONENT PARAMETER B2.
*                  COLS 30-34 EXPONENT PARAMETER B3.
*                  COLS 40-44 EXPONENT PARAMETER B4.
*     REMAINING CARDS-- ELEMENTS OF CORRELATION MATRIX V(I,J). EACH CARD
*                       CONTAINS ONE ROW OF THE MATRIX IN F FORMAT(4 DECIMAL
*                       PLACES).
*                       COLS 2-8 V(1,J)
*                       COLS 12-18 V(2,J)
*                       COLS 22-28 V(3,J)
*                       COLS 32-38 V(4,J)
*
**************************************************************************
*
*
      DSEED=466764003.D0
      DSEED1=123457.D0
      READ'(I1,T3,I1,T5,I2,T10,F5.3)',M,N,NR,SIGMA
*INITIALIZE MATRICES AND VECTORS.
      DO 10 I=1,M
          B(I)=0.0
              DO 20 J=1,N
                  V(I,J)=0.0
20            CONTINUE
10    CONTINUE
      DO 30 I=1,(N*(N+1))/2
          VVEC(I)=0.0
```

```
30      CONTINUE
        DO 40 I=1,NR
             DO 50 J=1,N
                  XVEC(I,J)=0.0
50           CONTINUE
40      CONTINUE
        DO 60 I=1,N
             WKVEC(I)=0.0
60      CONTINUE
        DO 70 I=1,NR
             ET(I)=0.0
             R(I)=0.0
             C(I)=0.0
             LOGY(I)=0.0
70      CONTINUE
        READ'(F5.2,T10,F5.2,T20,F5.2,T30,F5.2,T40,F5.2)',A,(B(J),J=1,N)
*READ CORRELATION MATRIX
        DO 100 I=1,M
            READ'(T2,F7.4,T12,F7.4,T22,F7.4,T32,F7.4)',(V(I,J),J=1,N)
100     CONTINUE
*CONVERT MATRIX V TO SYMETRIC STORAGE MODE.
        CALL VCVTFS(V,N,4,VVEC)
*WRITE PARAMETER VECTOR TO PERMANENT FILE.
        WRITE(4,25) (B(J),J=1,N)

25      FORMAT(F5.2,T10,F5.2,T20,F5.2,T30,F5.2)
*LOOP FOR NUMBER OF DATA SETS WITH PARAMETERS SPECIFIED IN DATA INPUT.
        DO 110 K=1,1000
            IF(K.EQ.1) WKVEC(1)=0.0
            IF(K.GT.1) WKVEC(1)=1.0
*GENERATE X VALUES.  MATRIX XVEC IS DIMENSION NR X N.
            CALL GGNSM(DSEED,NR,N,VVEC,20,XVEC,WKVEC,IER)
*TRANSLATE X'S 6 STANDARD DEVIATIONS INTO POSITIVE QUADRANT.
        DO 71 I=1,NR
             DO 72 J=1,N
                  XVEC(I,J)=XVEC(I,J)+6
                  IF(XVEC(I,J).LE.0.0) THEN
                       XVEC(I,J)=0.01
                  ENDIF
                  XVEC(I,J)=LOG(XVEC(I,J))
72           CONTINUE
71      CONTINUE
*MEAN CORRECT XVEC MATRIX.
        DO 75 I=1,N
             SUMX(I)=0.0
75      CONTINUE
        DO 80 I=1,NR
             DO 90 J=1,N
                  SUMX(J)=SUMX(J)+XVEC(I,J)
90           CONTINUE
80      CONTINUE
        DO 95 I=1,N
             MEANX(I)=SUMX(I)/NR
95      CONTINUE
        DO 96 I=1,NR
             DO 97 J=1,N
                  XVECMC(I,J)=XVEC(I,J)-MEANX(J)
97           CONTINUE
96      CONTINUE
*GENERATE NR STANDARD NORMAL DEVIATES(VECTOR R).
             CALL GGNML(DSEED1,NR,R)
*ADJUST DEVIATES TO VARIANCE SIGMA SQUARED.
             DO 120 I=1,NR
                  ET(I)=R(I)*SIGMA
120          CONTINUE
*MEAN ADJUST ERROR DEVIATES.
        SUMET=0.0
        DO 125 I=1,NR
             SUMET=SUMET+ET(I)
125     CONTINUE
        ETMEAN=SUMET/NR
```

```
      DO 126 I=1,NR
           ETC(I)=ET(I)-ETMEAN
126   CONTINUE
*CREATE MATRIX PRODUCT LN(X)B.
          CALL VMULFF(XVECMC,B,NR,N,1,20,4,C,20,IER)
*ADD RANDOM ERROR TO MATRIX PRODUCT XB.
          DO 130 I=1,NR
               LOGY(I)=C(I)+ETC(I)+LOG(A)
130       CONTINUE
*WRITE DATA TO PERMANENT FILE.
          DO 140 I=1,NR
       WRITE(4,15) LOGY(I),(XVECMC(I,J),J=1,N)
15             FORMAT(T2,F8.4,T12,F8.4,T22,F8.4,T32,F8.4,T42,F8.4)
140       CONTINUE
110   CONTINUE
      END
```

Appendix C

FORTRAN Code for Program RIDGE

```
      PROGRAM RIDGE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7)
*
*
******************************************************************************
*
*
C     PROGRAM TO PERFORM RIDGE REGRESSION ANALYSIS
C     C.MCNICHOLS -- MAY 1980 -- AIR FORCE INSTITUTE OF TECHNOLOGY
C     PROCEDURE AS DOCUMENTED IN CHATTERJEE AND PRICE
C     "REGRESSION ANALYSIS BY EXAMPLE" WILEY,1977
C     MULTIPLE REGRESSION PROCEDURE IS EFROYMSON'S ALGORITHM
C     IN "MATHEMATICAL METHODS FOR DIGITAL COMPUTERS" ED. BY
C     RALSTON AND WILF WILEY,1960
C     DATA BASE FORMAT:
C     FIRST CARD -- COLS 1-2 NUMBER OF VARIABLES (INCLUDING DEPENDENT)
C                        LIMIT IS 16. COLS 3-4 INDEX OF DEPENDENT VAR.
C                        COLS 5-6, ANY NON-ZERO VALUE GENERATES
C                        LOG-LINEAR MODEL. COLS 7-11 K-INCREMENT VALUE,
C                        MUST BE GREATER THAN ZERO AND LE .02.
C                        COLS 12-13 NUMBER OF OBSERVATION VECTORS IN
C                        EACH CASE.
C     SECOND CARD -- FORTRAN FORMAT STATEMENT FOR INPUT DATA. MUST BE
C                    F-TYPE SPECIFICATIONS AND ACCOUNT FOR NUMBER OF
C                    VARIABLES STATED ON FIRST CARD
C     REMAINING CARDS -- OBSERVATIONS IN FORMAT SPECIFIED BY SECOND CARD
C     CORRELATION MATRIX CONSTRUCTED IN A(,)
C     CORRELATION MATRIX COPIED TO R(,) FOR EACH ITERATION
C     M() IS MEAN VECTOR, S() IS STD.DEV. VECTOR
C     B(,) IS MATRIX OF STANDARDIZED COEFFICIENTS
C     PLT(,) IS PLOT BUFFER FOR RIDGE TRACE; PLT(I,52) IS R-SQUARE
C     PKMX() CONTAINS VALUES OF K FOR EACH ITERATION
C     VIF(,) CONTAINS VARIANCE INFLATION FACTORS FOR EACH ITERATION
*
*
******************************************************************************
*
*
      DIMENSION A(16,16),R(16,16),M(16),S(16),X(16),B(50,16),PLT(50,52),
     1        FMT(8),ALNUM(16),PKMX(51),VIF(50,16),BPAR(16)
      REAL M,INCK
      DATA ALNUM/"1","2","3","4","5","6","7","8","9",
     1  "A","B","C","D","E","F","G"/
      N=0
C     LOAD DATA BASE. FIRST READ NO. VARS:NV, INDEX OF DEPENDENT: IXD
C     FLAG FOR LOG-LINEAR: LOGF, INCREMENT FOR K-VALUE: INCK
      READ(5,10) NV,IXD,LOGF,INCK,NUMC
10    FORMAT(3I2,F5.3,I2)
      IF(INCK.LE.0.0) INCK=.005
      IF(INCK.GT..02) INCK=.005
      NVM1=NV-1
C     INITIALIZE VARIABLES,VECTORS,AND ARRAYS.
      DO 100 I=1,NV
       M(I)=0.0
       S(I)=0.0
       DO 100 J=1,NV
        A(I,J)=0.0
100   CONTINUE
      DO 150 I=1,50
       DO 150 J=1,16
        B(I,J)=0.0
150   CONTINUE
C     READ FORMAT STATEMENT DESCRIBING DATA BASE
      READ(5,15) FMT
15    FORMAT(8A10)
C     READ COEFFICIENT PARAMETERS OF MODEL AS SPECIFIED BY FORMAT
C         STATEMENT 25.
      READ(7,25) (BPAR(I),I=1,NVM1)
```

```
25      FORMAT(F5.2,T10,F5.2,T20,F5.2,T30,F5.2)
C       READ OBSERVATIONS ACCORDING TO USER INPUT FORMAT STATEMENT
        DO 300 I=1,NUMC
200     READ(7,FMT) (X(J),J=1,NV)
        IF(LOGF.EQ.0) GO TO 250
        DO 225 J=1,NV
         X(J)=ALOG(X(J))
225     CONTINUE
250     N=N+1
C       CONSTRUCT MEAN VECTOR AND COVARIANCE MATRIX
        DO 300 J=1,NV
         M(J)=M(J)+X(J)
         DO 300 J1=J,NV
          A(J,J1)=A(J,J1)+X(J)*X(J1)
300     CONTINUE
C       END OF INPUT DATA, CALCULATE MEANS,SIGMAS,CORRELATION MATRIX
400     DO 500 J=1,NV
         S(J)=SQRT((A(J,J)-M(J)*M(J)/N)/(N-1.0))
         M(J)=M(J)/N
500     CONTINUE
        DO 600 J=1,NV
         DO 600 J1=J,NV
*         A(J,J1)=(A(J,J1)-N*M(J)*M(J1))/((N-1.0)*S(J)*S(J1))
600     CONTINUE
        DO 700 J=1,NVM1
         JP1=J+1
         DO 700 J1=JP1,NV
          A(J1,J)=A(J,J1)
700     CONTINUE
C       PRINT MEANS,STD.DEVIATIONS,CORRELATION MATRIX
        WRITE(6,30) INCK,N
30      FORMAT(1H1,"RIDGE REGRESSION PROGRAM -- AIR FORCE INSTITUTE OF",
       1 " TECHNOLOGY"/1H0,"K-VALUE INCREMENT IS ",F6.4///
       2 1H0,I8," CASES READ FROM INPUT FILE"//
       3 1H0,"VARIABLE NUMBER       MEAN       STD.DEV."/)
        DO 800 J=1,NV
        WRITE(6,35) J,M(J),S(J)
35      FORMAT(1H ,7X,I2,6X,F12.5,F12.4)
800     CONTINUE
        IF(LOGF.EQ.0) GO TO 850
        WRITE(6,37)
37      FORMAT(1H0/1H0,"LOG-LINEAR OPTION, ALL VARIABLES TRANSFORMED"/)
850     WRITE(6,40) (NN,NN=1,NV)
40      FORMAT(1H0/1H0,"CORRELATION MATRIX"/1H0,"VARIABLE",
       1  16I7)
        DO 900 J=1,NV
        WRITE(6,45) J,(A(J,J1),J1=1,NV)
45      FORMAT(1H0,I6,4X,16F7.3)
900     CONTINUE
C       COPY CORRELATION MATRIX FROM A TO R FOR EACH ITERATION
C       FK IS VALUE OF K FOR RIDGE ESTIMATES
        FK=0.0
        FKMX(1)=0.0
        WRITE(6,50) (NN,NN=1,NV)
        WRITE(6,52)
50      FORMAT(1H1,"NORMALIZED (STANDARDIZED) REGRESSION COEFFICIENTS"/
       1 1H0,"  VARIABLE:",16I7)
52      FORMAT(1H ,"K-VALUE")
        DO 1650 IXK=1,50
         DO 1000 J=1,NV
          DO 1000 J1=1,NV
           R(J,J1)=A(J,J1)
1000     CONTINUE
C        ALTER DIAGONAL OF R MATRIX REPRESENTING X'X
         DO 1100 J=1,NV
          IF(J.EQ.IXD) GO TO 1100
          R(J,J)=R(J,J)+FK
1100     CONTINUE
```

```
C       MATRIX INVERSION -- SOLVES FOR REGRESSION COEFFICIENTS
        DO 1500 I=1,NV
         IF(I.EQ.IXD) GO TO 1500
         DO 1300 J=1,NV
          IF(J.EQ.I) GO TO 1300
          V=R(J,I)/R(I,I)
          DO 1200 K=1,NV
           IF(K.EQ.I) GO TO 1200
           R(J,K)=R(J,K)-V*R(I,K)
1200      CONTINUE
         R(J,I)=-V
1300     CONTINUE
         DO 1400 K=1,NV
          IF(K.EQ.I) GO TO 1400
         R(I,K)=R(I,K)/R(I,I)
1400     CONTINUE
         R(I,I)=1.0/R(I,I)
1500    CONTINUE
C       SAVE COEFFICIENTS FROM THIS ITERATION
C       CALCULATE VIF'S AND SAVE:
C       DIAGONAL ELS OF COEFFICIENT COVAR. MTX. DIVIDED BY SIGMA**2
        BSQ=0.0
        DO 1600 J=1,NV
         VIF(IXK,J)=0.0
         IF(J.EQ.IXD) GO TO 1600
         B(IXK,J)=R(J,IXD)
         BSQ=BSQ+B(IXK,J)*B(IXK,J)
         DO 1575 L=1,NV
          TVIF=0.0
          IF(L.EQ.IXD) GO TO 1575
          DO 1550 K=1,NV
           IF(K.EQ.IXD) GO TO 1550
            TVIF=TVIF+A(L,K)*R(K,J)
1550      CONTINUE
         VIF(IXK,J)=VIF(IXK,J)+R(J,L)*TVIF
1575     CONTINUE
1600    CONTINUE
C       SAVE R-SQUARE VALUE IN PLOT BUFFER: B'X'Y+K*B'B
        PLT(IXK,52)=1.0-R(IXD,IXD)+FK*BSQ
C       END OF LOOP OVER VALUES OF K
C       PRINT COEFFICIENTS FOR THIS ITERATION
        WRITE(6,55) FK,(B(IXK,J),J=1,NV)
55      FORMAT(1H ,F5.3,6X,16F7.3)
C       ALTER K VALUE
        FK=FK+INCK
        FKMX(IXK+1)=FK
1650    CONTINUE
C       CALCULATE UNNORMALIZED COEFFICIENTS
        WRITE(6,51) (NN,NN=1,NV)
51      FORMAT(1H1,"UNNORMALIZED COEFFICIENTS"/
       1 1H0," VARIABLE:INTERCEPT   ",I4,15I7)
        WRITE(6,52)
        DO 1800 I=1,50
        CNST=M(IXD)
        DO 1700 J=1,NV
         IF(J.EQ.IXD) GO TO 1700
         CNST=CNST-(R(I,J)*S(IXD)/S(J))*M(J)
         X(J)=R(I,J)*S(IXD)/S(J)
1700    CONTINUE
        X(IXD)=0.0
        IF(LOGF.NE.0) CNST=EXP(CNST)
        WRITE(6,56) FKMX(I),CNST,(X(J),J=1,NV)
56      FORMAT(1H ,F5.3,2X,G12.4,16F7.3)
1800    CONTINUE
        IF(LOGF.EQ.0) GO TO 2050
        WRITE(6,58)
58      FORMAT(1H0/1H0,"LOG-LINEAR MODEL: INTERCEPT CONVERTED TO ANTILOG")
```

```
C      GENERATE RIDGE TRACE
2050   DO 2100 I=1,50
        DO 2100 J=1,51
         PLT(I,J)=1H
2100   CONTINUE
C      FIND MIN AND MAX NORMALIZED COEFFICIENT VALUES
       SM=+1E99
       BG=-1E99
       DO 2200 I=1,50
        DO 2200 J=1,NV
         IF(J.EQ.IXD) GO TO 2200
         IF(B(I,J).LT.SM) SM=B(I,J)
         IF(B(I,J).GT.BG) BG=B(I,J)
2200   CONTINUE
C      LOAD PLOT BUFFER
       XI=(BG-SM)/50.0
       DO 2400 I=1,50
        J1=1.0-SM/XI
        IF(J1.GT.0.AND.J1.LE.51) PLT(I,J1)=1H.
        DO 2400 J=1,NV
         IF(J.EQ.IXD) GO TO 2400
         J1=1.0+(B(I,J)-SM)/XI
         PLT(I,J1)=ALNUM(J)
2400   CONTINUE
C      PRINT RIDGE TRACE
       WRITE(6,60)SM,BG
60     FORMAT(1H1,"RIDGE TRACE: NORMALIZED COEFFICIENTS"/
      1  1H0,"COEFFICIENT RANGE:",F12.4," TO",F12.4/
      2  1H0,"K-VALUE",1X,51(1H.)," R-SQUARE"/)
       DO 2500 I=1,50
        WRITE(6,65) FKMX(I),(PLT(I,J),J=1,51),PLT(I,52)
65     FORMAT(1H ,F5.3,3X,51A1,F7.4)
2500   CONTINUE
C      OUTPUT VARIANCE INFLATION FACTORS (VIF)
       WRITE(6,70)(NN,NN=1,NV)
70     FORMAT(1H1,"VARIANCE INFLATION FACTORS FOR REGRESSION"
      1  " COEFFICIENTS"/ 1H0,"  VARIABLE:",16I7)
       WRITE(6,57)
       DO 2600 I=1,50
        WRITE(6,75)FKMX(I),(VIF(I,J),J=1,NV)
75     FORMAT(1H ,F5.3,6X,16F7.1)
2600   CONTINUE
       WRITE(6,80)
80     FORMAT(1H1)
       STOP
       END
```

Appendix D

FORTRAN Code for Program RIDGE (Monte Carlo Modified Version)

```
      PROGRAM RIDGE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7,TAPE8)
*
C     MONTE CARLO ANALYSIS.
*
*
*****************************************************************************
*
*
C     PROGRAM TO PERFORM RIDGE REGRESSION ANALYSIS
C     J.MAKIN--JULY 1981--AIR FORCE INSTITUTE OF TECHNOLOGY
C     PROCEDURE AS DOCUMENTED IN CHATTERJEE AND PRICE
C     "REGRESSION ANALYSIS BY EXAMPLE" WILEY,1977
C     MULTIPLE REGRESSION PROCEDURE IS EFROYMSON'S ALGORITHM
C     IN "MATHEMATICAL METHODS FOR DIGITAL COMPUTERS" ED. BY
C     RALSTON AND WILF WILEY,1960
C     DATA BASE FORMAT:
C     FIRST CARD -- COLS 1-2 NUMBER OF VARIABLES (INCLUDING DEPENDENT)
C                        LIMIT IS 16. COLS 3-4 INDEX OF DEPENDENT VAR.
C                        COLS 5-6. ANY NON-ZERO VALUE GENERATES
C                        LOG-LINEAR MODEL. COLS 7-11 K-VALUE SELECTED
C                        FROM PROGRAM RIDGE FOR MONTE CARLO TRIALS.
C                        COLS 12-13 NUMBER OF OBSERVATION VECTORS IN
C                        EACH CASE.
C     SECOND CARD -- FORTRAN FORMAT STATEMENT FOR INPUT DATA. MUST BE
C                    F-TYPE SPECIFICATIONS AND ACCOUNT FOR NUMBER OF
C                    VARIABLES STATED ON FIRST CARD.
C     THIRD CARD-- FORTRAN FORMAT STATEMENT FOR OUTPUT DATA. MUST BE
C                  F-TYPE SPECIFICATIONS AND ACCOUNT FOR ALL VARIABLES
C                  IN WRITE STATEMENT 999.
C     REMAINING CARDS -- OBSERVATIONS IN FORMAT SPECIFIED BY SECOND CARD
*
*
C     CORRELATION MATRIX CONSTRUCTED IN A(,)
C     CORRELATION MATRIX COPIED TO R(,) FOR EACH ITERATION
C     M() IS MEAN VECTOR, S() IS STD.DEV. VECTOR
C     B(,) IS MATRIX OF STANDARDIZED COEFFICIENTS
C     VIF(,) CONTAINS VARIANCE INFLATION FACTORS FOR EACH ITERATION
*
*
*****************************************************************************
*
      DIMENSION A(16,16),R(16,16),M(16),S(16),X(16),B(2,16),FMT(8),
     1        VIF(2,16),SDE(2),RSQ(50),FMTOUT(8),BPAR(16),MSE(2),XX(2,16)
      REAL M,INCK,MSE
      DATA SDE/2*0.0/
      NC=0
C     LOAD DATA BASE. FIRST READ NO. VARS:NV. INDEX OF DEPENDENT: IXD
C     FLAG FOR LOG-LINEAR: LOGF. INCREMENT FOR K-VALUE: INCK
      READ(5,10) NV,IXD,LOGF,INCK,NUMC
10    FORMAT(3I2,F5.3,I2)
      NVM1=NV-1
C     READ FORMAT STATEMENTS DESCRIBING DATA BASE INPUT AND OUTPUT.
      READ(5,15) FMT
      READ(5,15) FMTOUT
15    FORMAT(8A10)
C     READ PARAMETER COEFFICIENTS OF MODEL.
      READ(7,25) (BPAR(I),I=1,NVM1)
25    FORMAT(F5.2,T10,F5.2,T20,F5.2,T30,F5.2)
C     INITIALIZE VARIABLES,VECTORS,AND ARRAYS.
200   CONTINUE
      N=0
      DO 100 I=1,NV
       M(I)=0.0
       S(I)=0.0
       DO 100 J=1,NV
        A(I,J)=0.0
```

```
100   CONTINUE
      DO 125 I=1,2
           SDE(I)=0.0
           MSE(I)=0.0
125   CONTINUE
      DO 150 I=1,2
       DO 150 J=1,16
        R(I,J)=0.0
150   CONTINUE
C     READ"OBSERVATIONS ACCORDING TO USER INPUT FORMAT STATEMENT
      DO 350 II=1,NUMC
      READ(7,FMT) (X(J),J=1,NV)
      IF(EOF(7).NE.0) GO TO 1900
      IF(LOGF.EQ.0) GO TO 250
      DO 275 J=1,NV
       X(J)=ALOG(X(J))
225   CONTINUE
250   N=N+1
C     CONSTRUCT MEAN VECTOR AND COVARIANCE MATRIX
      DO 300 J=1,NV
       M(J)=M(J)+X(J)
       DO 300 J1=J,NV
        A(J,J1)=A(J,J1)+X(J)*X(J1)
300   CONTINUE
350   CONTINUE
C     CALCULATE MEANS,SIGMAS,CORRELATION MATRIX FOR THIS SET OF DATA.
400   DO 500 J=1,NV
       S(J)=SQRT((A(J,J)-M(J)*M(J)/N)/(N-1.0))
       M(J)=M(J)/N
500   CONTINUE
      DO 600 J=1,NV
       DO 600 J1=J,NV
        A(J,J1)=(A(J,J1)-N*M(J)*M(J1))/((N-1.0)*S(J)*S(J1))
600   CONTINUE
      DO 700 J=1,NVM1
       JP1=J+1
       DO 700 J1=JP1,NV
        A(J1,J)=A(J,J1)
700   CONTINUE
C     COPY CORRELATION MATRIX FROM A TO R FOR EACH ITERATION
C     FK IS VALUE OF K FOR RIDGE ESTIMATES
      FK=0.0
      DO 1650 IXK=1,2
       DO 1000 J=1,NV
        DO 1000 J1=1,NV
         R(J,J1)=A(J,J1)
1000   CONTINUE
C     ALTER DIAGONAL OF R MATRIX REPRESENTING X'X
      DO 1100 J=1,NV
       IF(J.EQ.IXD) GO TO 1100
       R(J,J)=R(J,J)+FK
1100  CONTINUE
C     MATRIX INVERSION -- SOLVES FOR REGRESSION COEFFICIENTS
      DO 1500 I=1,NV
       IF(I.EQ.IXD) GO TO 1500
       DO 1300 J=1,NV
        IF(J.EQ.I) GO TO 1300
        V=R(J,I)/R(I,I)
        DO 1200 K=1,NV
         IF(K.EQ.I) GO TO 1200
         R(J,K)=R(J,K)-V*R(I,K)
1200    CONTINUE
       R(J,I)=-V
1300   CONTINUE
       DO 1400 K=1,NV
        IF(K.EQ.I) GO TO 1400
       R(I,K)=R(I,K)/R(I,I)
1400   CONTINUE
       R(I,I)=1.0/R(I,I)
```

```
1500  CONTINUE
C     SAVE COEFFICIENTS FROM THIS ITERATION
C     CALCULATE VIF'S AND SAVE:
C     DIAGONAL ELS OF COEFFICIENT COVAR. MTX. DIVIDED BY SIGMA**2
      BSQ=0.0
      DO 1600 J=1,NV
       VIF(IXK,J)=0.0
       IF(J.EQ.IXD) GO TO 1600
       B(IXK,J)=R(J,IXD)
       BSQ=BSQ+B(IXK,J)*B(IXK,J)
       DO 1575 L=1,NV
        TVIF=0.0
        IF(L.EQ.IXD) GO TO 1575
        DO 1550 K=1,NV
         IF(K.EQ.IXD) GO TO 1550
          TVIF=TVIF+A(L,K)*R(K,J)
1550    CONTINUE
       VIF(IXK,J)=VIF(IXK,J)+R(J,L)*TVIF
1575   CONTINUE
1600  CONTINUE
*CALCULATE THE UNNORMALIZED COEFFICIENTS.
      DO 1610 I=1,2
           DO 1620 J=1,NV
                IF(J.EQ.IXD) GO TO 1620
                XX(I,J)=B(I,J)*S(IXD)/S(J)
1620       CONTINUE
1610  CONTINUE
C     COMPUTE RL STATISTIC(INDEX OF MULTICOLLINEARITY)
      SVIF=0.0
      DO 1625 I=1,NV
           IF(I.EQ.IXD) GO TO 1625
           SVIF=VIF(1,I)+SVIF
1625  CONTINUE
      IF(IXK.EQ.1) RL=SVIF/(NV-1)
C     COMPUTE AND SAVE R-SQUARED.
      RSQ(IXK)=1.0-R(IXD,IXD)+FK*BSQ
      FK=FK+INCK
1650  CONTINUE
C     CALCULATE MEAN SQUARE ERROR.
      DO 1800 I=1,2
           DO 1750 J=2,NV
                SQE(I)=SQE(I)+(XX(I,J)-BPAR(J-1))**2
1750       CONTINUE
1800  CONTINUE
      DO 1850 I=1,2
           MSE(I)=SQE(I)/NVM1
1850  CONTINUE
999   WRITE(8,FMTOUT) ((XX(II,JJ),JJ=2,NV),II=1,2),((VIF(KK,LL),LL=2,NV)
     ;              ,KK=1,2),(RSQ(III),III=1,2),RL,(MSE(JJJ),JJJ=1,2)
      NC=NC+1
C     NEXT CASE
      GO TO 200
1900  CONTINUE
      PRINT(6,2000)BPAR(1),BPAR(2),BPAR(3),BPAR(4),INCK,NVM1
2000  FORMAT(1H1////"    MODEL PARAMETERS(B'S)"//"      B1= ",F5.2,"  B2= "
     ;        ,F5.2,"  B3= ",F5.2,"  B4= ",F5.2//"    K-VALUE:",F5.3,
     ;         "    NUMBER OF VARIABLES:    ",I2//"   RANDOM ERROR ADDED:"
     ;         /)
       STOP
       END
```

Appendix E

SPSS Programs Containing Subprogram CONDESCRIPTIVE (2, 3, and 4 Variable Models)

```
RUN NAME          RIDGE,OLS ANALYSIS
FILE NAME         RIDGE,ANAL OF MONTE CARLO SIM(OLS AND RIDGE REGRESSION)
VARIABLE LIST     BLS1,BLS2,BR1,BR2,VIFLS1,VIFLS2,VIFR1,VIFR2,RSOLS,RSOR,RL,MSELS,
                  MSER
INPUT FORMAT      FIXED(1X,4F8.3,1X,4F8.3,1X,2F7.4,1X,F7.2,1X,2F8.4)
N OF CASES        1000
INPUT MEDIUM      CARD
VAR LABELS        BLS1,B1 LEAST SQUARES/BLS2,B2 LEAST SQUARES/
                  BR1,B1 RIDGE/BR2,B2 RIDGE/
                  VIFLS1,VIF LS VAR 1/VIFLS2,VIF LS VAR 2/VIFR1,VIF RIDGE VAR 1/
                  VIFR2,VIF RIDGE VAR 2/RSOLS,R-SQUARED LEAST SQUARES/
                  RSOR,R-SQUARED RIDGE/RL,INDEX OF MULTICOLLINEARITY/MSELS,
                  MEAN SQUARE ERROR LS/MSER,MEAN SQUARE ERROR RIDGE/
PRINT FORMATS     BLS1,BLS2,BR1,BR2,(3)/VIFLS1,VIFLS2,VIFR1,VIFR2(2)/
                  RSOLS,RSOR,RL,MSELS,MSER(4)/
LIST CASES        CASES=20/VARIABLES=ALL/
CONDESCRIPTIVE    BLS1,BLS2,BR1,BR2,VIFLS1,VIFLS2,VIFR1,VIFR2,RSOLS,RSOR,RL,MSELS,
                  MSER
STATISTICS        1,5,6,9,10,11
READ INPUT DATA
FINISH
```

Fig. 4. Subprogram CONDESCRIPTIVE--Two Variable Model

```
RUN NAME          RIDGE,OLS ANALYSIS
FILE NAME         RIDGE,ANAL OF MONTE CARLO SIM(OLS AND RIDGE REGRESSION)
VARIABLE LIST     BLS1 TO BLS3,BR1 TO BR3,VIFLS1 TO VIFLS3,VIFR1 TO VIFR3,RSOLS,
                  RSOR,RL,MSELS,MSER
INPUT FORMAT      FIXED(1X,6F8.3,1X,6F8.3,1X,2F7.4,1X,F7.2,1X,2F8.4)
N OF CASES        1000
INPUT MEDIUM      CARD
VAR LABELS        BLS1,B1 LEAST SQUARES/BLS2,B2 LEAST SQUARES/BLS3,B3 LEAST
                  SQUARES/
                  BR1,B1 RIDGE/BR2,B2 RIDGE/BR3,B3 RIDGE/
                  VIFLS1,VIF LS VAR 1/VIFLS2,VIF LS VAR 2/VIFLS3,VIF LS VAR 3/
                  VIFR1,VIF RIDGE VAR1/VIFR2,VIF RIDGE VAR 2/VIFR3,VIF RIDGE VAR 3/
                  RSOR,R-SQUARED RIDGE/RL,INDEX OF MULTICOLLINEARITY/MSELS,
                  MEAN SQUARE ERROR LS/MSER,MEAN SQUARE ERROR RIDGE/
PRINT FORMATS     BLS1 TO BLS3,BR1 TO BR3(3)/VIFLS1 TO VIFLS3,VIFR1 TO VIFR3(2)/
                  RSOLS,RSOR,RL,MSELS,MSER(4)/
LIST CASES        CASES=20/VARIABLES=ALL/
CONDESCRIPTIVE    BLS1 TO BLS3,BR1 TO BR3,VIFLS1 TO VIFLS3,VIFR1 TO VIFR3,RSOLS,
                  RSOR,RL,MSELS,MSER
STATISTICS        1,5,6,9,10,11
READ INPUT DATA
FINISH
```

Fig. 5. Subprogram CONDESCRIPTIVE--Three Variable Model

```
RUN NAME          RIDGE,OLS ANALYSIS
FILE NAME         RIDGE,ANAL OF MONTE CARLO SIM(OLS AND RIDGE REGRESSION)
VARIABLE LIST     BLS1 TO BLS4,BR1 TO BR4,VIFLS1 TO VIFLS4,VIFR1 TO VIFR4,RSOLS,
                  RSQR,RL,MSELS,MSER
INPUT FORMAT      FIXED(1X,8F8.3,1X,8F8.3/1X,2F7.4,1X,F7.2,1X,2F8.4)
N OF CASES        1000
INPUT MEDIUM      CARD
VAR LABELS        BLS1,B1 LEAST SQUARES/BLS2,B2 LEAST SQUARES/BLS3,B3 LEAST SQUARES
                  /BLS4,B4 LEAST SQUARES/BR1,B1 RIDGE/BR2,B2 RIDGE/BR3,B3 RIDGE/
                  BR4,B4 RIDGE/VIFLS1,VIF LS VAR 1/VIFLS2,VIF LS VAR 2/
                  VIFLS3,VIF LS VAR 3/VIFLS4,VIF LS VAR 4/VIFR1,VIF RIDGE VAR 1/
                  VIFR2,VIF RIDGE VAR 2/VIFR3,VIF RIDGE VAR 3/VIFR4,VIF RIDGE VAR 4
                  /RSOLS,R-SQUARED LS/RSQR,R-SQUARED RIDGE/RL,INDEX OF MULTICOLLINE
                  ARITY/  MSELS,MEAN SQUARE ERROR LEAST SQUARES/MSER,MEAN
                  SQUARE ERROR RIDGE/
PRINT FORMATS     BLS1 TO BLS4,BR1 TO BR4(3)/VIFLS1 TO VIFLS4,VIFR1 TO VIFR4(2)/
                  RSOLS,RSQR,RL(4)/MSELS,MSER(4)/
LIST CASES        CASES=20/VARIABLES=ALL/
CONDESCRIPTIVE    BLS1 TO BLS4,BR1 TO BR4,VIFLS1 TO VIFLS4,VIFR1 TO VIFR4,RSOLS,
                  RSQR,RL,MSELS,MSER
STATISTICS        1,5,6,9,10,11
READ INPUT DATA
FINISH
```

Fig. 6. Subprogram CONDESCRIPTIVE--Four Variable Model

Appendix F

SPSS Program for Regression Analysis

```
RUN NAME          REGRESSION OF RIDGE REGRESSION DATA
FILE NAME         REGANAL,REGRESSION OF SIMULATED DATA
VARIABLE LIST     RSQ,RL,K,NVAR,MSE,MSELS
INPUT FORMAT      FIXED(F4.3,T10,F7.3,T20,F5.3,T30,F1.0,T32,F6.3,T40,F6.3)
N OF CASES        115
INPUT MEDIUM      CARD
VAR LABELS        RSQ,R-SQUARED<LS>/RL,INDEX OF MULTICOLLINEARITY/K,K-INCREMENT/
                  NVAR,NUMBER OF VARIABLES IN MODEL/MSE,MEAN SQUARE ERROR/
                  MSELS,MEAN SQUARE ERROR LEAST SQUARES/
COMPUTE           MSERATIO=MSELS/MSE
COMPUTE           CROSS1=RSQ*RL
COMPUTE           CROSS2=RSQ*K
COMPUTE           CROSS3=RSQ*NVAR
COMPUTE           CROSS4=RL*K
COMPUTE           CROSS5=RL*NVAR
COMPUTE           CROSS6=K*NVAR
COMPUTE           RLSQ=RL**2
COMPUTE            KSQ=K**2
COMPUTE           NVARSQ=NVAR**2
COMPUTE           RSQSQ=RSQ**2
PRINT FORMATS       MSE,MSELS,MSERATIO,RSQ,K,CROSS1 TO CROSS6,RLSQ,KSQ,RSQSQ,
                  RL(3)/
LIST CASES        CASES=115/VARIABLES=ALL/
REGRESSION        METHOD=STEPWISE/
                  VARIABLES=RSQ,RL,K,NVAR,MSERATIO,CROSS1 TO CROSS6,RLSQ,KSQ,RSQSQ,
                  NVARSQ/
                  REGRESSION=MSERATIO(10,1.0,.001,0.9) WITH RSQ,K,NVAR,RL,
                  CROSS1 TO CROSS6(1) RESID=0/
STATISTICS        ALL
READ INPUT DATA
FINISH
```

Fig. 7. Regression Program--Linear Model

```
RUN NAME        REGRESSION OF RIDGE REGRESSION DATA
FILE NAME       REGANAL,REGRESSION OF SIMULATED DATA
VARIABLE LIST   RSQ,RL,K,NVAR,MSE,MSELS
INPUT FORMAT    FIXED(F4.3,T10,F7.3,T20,F5.3,T30,F1.0,T32,F6.3,T40,F6.3)
N OF CASES      108
INPUT MEDIUM    CARD
VAR LABELS      RSQ,R-SQUARED<LS>/RL,INDEX OF MULTICOLLINEARITY/K,K-INCREMENT/
                NVAR,NUMBER OF VARIABLES IN MODEL/MSE,MEAN SQUARE ERROR/
                MSELS,MEAN SQUARE ERROR LEAST SQUARES/
COMPUTE         MSERATIO=MSELS/MSE
COMPUTE         CROSS1=RSQ*RL
COMPUTE         CROSS2=RSQ*K
COMPUTE         CROSS3=RSQ*NVAR
COMPUTE         CROSS4=RL*K
COMPUTE         CROSS5=RL*NVAR
COMPUTE         CROSS6=K*NVAR
COMPUTE         RLSQ=RL**2
COMPUTE          KSQ=K**2
COMPUTE         NVARSQ=NVAR**2
COMPUTE         RSQSQ=RSQ**2
PRINT FORMATS     MSE,MSELS,MSERATIO,RSQ,K,CROSS1 TO CROSS6,RLSQ,KSQ,RSQSQ,
                RL(3)/
LIST CASES      CASES=108/VARIABLES=ALL/
REGRESSION      METHOD=STEPWISE/
                VARIABLES=RSQ,RL,K,NVAR,MSERATIO,CROSS1 TO CROSS6,RLSQ,KSQ,RSQSQ,
                NVARSQ/
                REGRESSION=MSERATIO(10,1.0,.001,0.9) WITH RSQ,K,NVAR,RL,
                CROSS1 TO CROSS6(1) RESID=0/
STATISTICS      ALL
READ INPUT DATA
FINISH
```

Fig. 8. Regression Program--Log-Linear Model

Appendix G

Linear Model Data

```
CONTENTS OF CASE NUMBER      1
SEQNUM         1.      SUBFILE  REGANAL      CASWGT     1.0000     RSQ          .787     RL          90.699
K              0       NVAR          4.      MSE       22.591      MSELS      22.591     MSERATIO     1.000
CROSS1    71.380       CROSS2        0       CROSS3     3.148      CROSS4          0     CROSS5     362.796
CROSS6         0       RLSQ   8226.309       KSQ            0      NVARSQ        16.     RSQSQ         .619

CONTENTS OF CASE NUMBER      2
SEQNUM         2.      SUBFILE  REGANAL      CASWGT     1.0000     RSQ          .787     RL          90.699
K           .020       NVAR          4.      MSE        1.399      MSELS      22.591     MSERATIO    16.148
CROSS1    71.380       CROSS2      .016      CROSS3     3.148      CROSS4      1.814     CROSS5     362.796
CROSS6      .080       RLSQ   8226.309       KSQ         .000      NVARSQ        16.     RSQSQ         .619

CONTENTS OF CASE NUMBER      3
SEQNUM         3.      SUBFILE  REGANAL      CASWGT     1.0000     RSQ          .787     RL          90.699
K           .040       NVAR          4.      MSE         .801      MSELS      22.591     MSERATIO    28.203
CROSS1    71.380       CROSS2      .031      CROSS3     3.148      CROSS4      3.628     CROSS5     362.796
CROSS6      .160       RLSQ   8226.309       KSQ         .002      NVARSQ        16.     RSQSQ         .619

CONTENTS OF CASE NUMBER      4
SEQNUM         4.      SUBFILE  REGANAL      CASWGT     1.0000     RSQ          .910     RL          90.699
K              0       NVAR          4.      MSE        7.888      MSELS       7.888     MSERATIO     1.000
CROSS1    82.536       CROSS2        0       CROSS3     3.640      CROSS4          0     CROSS5     362.796
CROSS6         0       RLSQ   8226.309       KSQ            0      NVARSQ        16.     RSQSQ         .828

CONTENTS OF CASE NUMBER      5
SEQNUM         5.      SUBFILE  REGANAL      CASWGT     1.0000     RSQ          .910     RL          90.699
K           .020       NVAR          4.      MSE         .488      MSELS       7.888     MSERATIO    16.164
CROSS1    82.536       CROSS2      .018      CROSS3     3.640      CROSS4      1.814     CROSS5     362.796
CROSS6      .080       RLSQ   8226.309       KSQ         .000      NVARSQ        16.     RSQSQ         .828

CONTENTS OF CASE NUMBER      6
SEQNUM         6.      SUBFILE  REGANAL      CASWGT     1.0000     RSQ          .910     RL          90.699
K           .040       NVAR          4.      MSE         .280      MSELS       7.888     MSERATIO    28.171
CROSS1    82.536       CROSS2      .036      CROSS3     3.640      CROSS4      3.628     CROSS5     362.796
CROSS6      .160       RLSQ   8226.309       KSQ         .002      NVARSQ        16.     RSQSQ         .828

CONTENTS OF CASE NUMBER      7
SEQNUM         7.      SUBFILE  REGANAL      CASWGT     1.0000     RSQ          .944     RL          90.699
K              0       NVAR          4.      MSE        4.688      MSELS       4.688     MSERATIO     1.000
CROSS1    85.620       CROSS2        0       CROSS3     3.776      CROSS4          0     CROSS5     362.796
CROSS6         0       RLSQ   8226.309       KSQ            0      NVARSQ        16.     RSQSQ         .891

CONTENTS OF CASE NUMBER      8
SEQNUM         8.      SUBFILE  REGANAL      CASWGT     1.0000     RSQ          .944     RL          90.699
K           .040       NVAR          4.      MSE         .166      MSELS       4.688     MSERATIO    28.241
CROSS1    85.620       CROSS2      .038      CROSS3     3.776      CROSS4      3.628     CROSS5     362.796
CROSS6      .160       RLSQ   8226.309       KSQ         .002      NVARSQ        16.     RSQSQ         .891

CONTENTS OF CASE NUMBER      9
SEQNUM         9.      SUBFILE  REGANAL      CASWGT     1.0000     RSQ          .944     RL          90.699
K           .080       NVAR          4.      MSE         .093      MSELS       4.688     MSERATIO    50.409
CROSS1    85.620       CROSS2      .076      CROSS3     3.776      CROSS4      7.256     CROSS5     362.796
CROSS6      .320       RLSQ   8226.309       KSQ         .006      NVARSQ        16.     RSQSQ         .891

CONTENTS OF CASE NUMBER      10
SEQNUM        10.      SUBFILE  REGANAL      CASWGT     1.0000     RSQ          .998     RL          90.699
K              0       NVAR          4.      MSE         .187      MSELS        .187     MSERATIO     1.000
CROSS1    90.578       CROSS2        0       CROSS3     3.992      CROSS4          0     CROSS5     362.796
CROSS6         0       RLSQ   8226.309       KSQ            0      NVARSQ        16.     RSQSQ         .996
```

CONTENTS OF CASE NUMBER 11

SEQNUM	11.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	90.659
K	.020	NVAR	4.	MSE	.012	MSELS	.187	MSERATIO	15.583
CROSS1	90.518	CROSS2	.020	CROSS3	3.992	CROSS4	1.814	CROSS5	362.796
CROSS6	.000	RLSQ	8226.309	KSQ	.000	NVARSQ	16.	RSQSQ	.996

CONTENTS OF CASE NUMBER 12

SEQNUM	12.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	90.699
K	.040	NVAR	4.	MSE	.008	MSELS	.187	MSERATIO	23.375
CROSS1	90.518	CROSS2	.040	CROSS3	3.992	CROSS4	3.628	CROSS5	362.796
CROSS6	.160	RLSQ	8226.309	KSQ	.002	NVARSQ	16.	RSQSQ	.996

CONTENTS OF CASE NUMBER 13

SEQNUM	13.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	90.699
K	.080	NVAR	4.	MSE	.005	MSELS	.187	MSERATIO	37.400
CROSS1	90.518	CROSS2	.080	CROSS3	3.992	CROSS4	7.256	CROSS5	362.796
CROSS6	.320	RLSQ	8226.309	KSQ	.006	NVARSQ	16.	RSQSQ	.996

CONTENTS OF CASE NUMBER 14

SEQNUM	14.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.708	RL	50.841
K	0	NVAR	3.	MSE	10.941	MSELS	10.941	MSERATIO	1.000
CROSS1	35.995	CROSS2	0	CROSS3	2.124	CROSS4	0	CROSS5	152.523
CROSS6	0	RLSQ	2584.807	KSQ	0	NVARSQ	9.	RSQSQ	.501

CONTENTS OF CASE NUMBER 15

SEQNUM	15.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.708	RL	50.841
K	.015	NVAR	3.	MSE	1.497	MSELS	10.941	MSERATIO	7.309
CROSS1	35.995	CROSS2	.011	CROSS3	2.124	CROSS4	.763	CROSS5	152.523
CROSS6	.045	RLSQ	2584.807	KSQ	.000	NVARSQ	9.	RSQSQ	.501

CONTENTS OF CASE NUMBER 16

SEQNUM	16.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.708	RL	50.841
K	.020	NVAR	3.	MSE	1.119	MSELS	10.941	MSERATIO	9.777
CROSS1	35.995	CROSS2	.014	CROSS3	2.124	CROSS4	1.017	CROSS5	152.523
CROSS6	.060	RLSQ	2584.807	KSQ	.000	NVARSQ	9.	RSQSQ	.501

CONTENTS OF CASE NUMBER 17

SEQNUM	17.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.708	RL	50.841
K	.040	NVAR	3.	MSE	.560	MSELS	10.941	MSERATIO	19.537
CROSS1	35.995	CROSS2	.028	CROSS3	2.124	CROSS4	2.034	CROSS5	152.523
CROSS6	.120	RLSQ	2584.807	KSQ	.002	NVARSQ	9.	RSQSQ	.501

CONTENTS OF CASE NUMBER 18

SEQNUM	18.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.883	RL	50.841
K	0	NVAR	3.	MSE	3.310	MSELS	3.310	MSERATIO	1.000
CROSS1	44.893	CROSS2	0	CROSS3	2.649	CROSS4	0	CROSS5	152.523
CROSS6	0	RLSQ	2584.807	KSQ	0	NVARSQ	9.	RSQSQ	.780

CONTENTS OF CASE NUMBER 19

SEQNUM	19.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.883	RL	50.841
K	.015	NVAR	3.	MSE	.454	MSELS	3.310	MSERATIO	7.291
CROSS1	44.673	CROSS2	.013	CROSS3	2.649	CROSS4	.763	CROSS5	152.523
CROSS6	.045	RLSQ	2584.807	KSQ	.000	NVARSQ	9.	RSQSQ	.780

CONTENTS OF CASE NUMBER 20

SEQNUM	20.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.883	RL	50.841
K	.020	NVAR	3.	MSE	.339	MSELS	3.310	MSERATIO	9.764
CROSS1	44.893	CROSS2	.010	CROSS3	2.649	CROSS4	1.017	CROSS5	157.523
CROSS6	.060	RLSQ	2584.007	KSQ	.000	NVARSQ	9.	RSQSQ	.780

CONTENTS OF CASE NUMBER 21

SEQNUM	21.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.883	RL	50.841
K	.040	NVAR	3.	MSE	.170	MSELS	3.310	MSERATIO	19.471
CROSS1	44.893	CROSS2	.035	CROSS3	2.649	CROSS4	2.034	CROSS5	152.523
CROSS6	.120	RLSQ	2584.807	KSQ	.002	NVARSQ	9.	RSQSQ	.780

CONTENTS OF CASE NUMBER 22

SEQNUM	22.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.996	RL	50.841
K	0	NVAR	3.	MSE	.109	MSELS	.109	MSERATIO	1.000
CROSS1	50.638	CROSS2	0	CROSS3	2.988	CROSS4	0	CROSS5	152.523
CROSS6	0	RLSQ	2584.807	KSQ	0	NVARSQ	9.	RSQSQ	.992

CONTENTS OF CASE NUMBER 23

SEQNUM	23.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.996	RL	50.841
K	.015	NVAR	3.	MSE	.016	MSELS	.109	MSERATIO	6.812
CROSS1	50.638	CROSS2	.015	CROSS3	2.988	CROSS4	.763	CROSS5	152.523
CROSS6	.045	RLSQ	2584.807	KSQ	.000	NVARSQ	9.	RSQSQ	.992

CONTENTS OF CASE NUMBER 24

SEQNUM	24.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.996	RL	50.841
K	.020	NVAR	3.	MSE	.012	MSELS	.109	MSERATIO	9.033
CROSS1	50.638	CROSS2	.020	CROSS3	2.988	CROSS4	1.017	CROSS5	152.523
CROSS6	.060	RLSQ	2584.807	KSQ	.000	NVARSQ	9.	RSQSQ	.992

CONTENTS OF CASE NUMBER 25

SEQNUM	25.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.996	RL	50.841
K	.040	NVAR	3.	MSE	.007	MSELS	.109	MSERATIO	15.571
CROSS1	50.638	CROSS2	.040	CROSS3	2.988	CROSS4	2.034	CROSS5	152.523
CROSS6	.120	RLSQ	2584.807	KSQ	.002	NVARSQ	9.	RSQSQ	.992

CONTENTS OF CASE NUMBER 26

SEQNUM	26.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.798	RL	8.342
K	0	NVAR	4.	MSE	1.617	MSELS	1.617	MSERATIO	1.000
CROSS1	6.657	CROSS2	0	CROSS3	3.192	CROSS4	0	CROSS5	33.368
CROSS6	0	RLSQ	69.589	KSQ	0	NVARSQ	16.	RSQSQ	.637

CONTENTS OF CASE NUMBER 27

SEQNUM	27.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.798	RL	8.342
K	.020	NVAR	4.	MSE	.854	MSELS	1.617	MSERATIO	1.893
CROSS1	6.657	CROSS2	.016	CROSS3	3.192	CROSS4	.167	CROSS5	33.368
CROSS6	.080	RLSQ	69.589	KSQ	.000	NVARSQ	16.	RSQSQ	.637

CONTENTS OF CASE NUMBER 28

SEQNUM	28.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.798	RL	8.342
K	.040	NVAR	4.	MSE	.588	MSELS	1.617	MSERATIO	2.750
CROSS1	6.657	CROSS2	.032	CROSS3	3.192	CROSS4	.334	CROSS5	33.368
CROSS6	.160	RLSQ	69.589	KSQ	.002	NVARSQ	16.	RSQSQ	.637

CONTENTS OF CASE NUMBER 29

SEQNUM	29.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.829	RL	8.342
K	0	NVAR	4.	MSE	1.294	MSELS	1.294	MSERATIO	1.000
CROSS1	6.916	CROSS2	0	CROSS3	3.316	CROSS4	0	CROSS5	33.368
CROSS6	0	RLSQ	69.589	KSQ	0	NVARSQ	16.	RSQSQ	.687

CONTENTS OF CASE NUMBER 30

SEQNUM	30.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.829	RL	8.342
K	.020	NVAR	4.	MSE	.684	MSELS	1.294	MSERATIO	1.892
CROSS1	6.916	CROSS2	.017	CROSS3	3.316	CROSS4	.167	CROSS5	33.368
CROSS6	.080	RLSQ	69.589	KSQ	.000	NVARSQ	16.	RSQSQ	.687

CONTENTS OF CASE NUMBER 31

SEQNUM	31.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.829	RL	8.342
K	.060	NVAR	4.	MSE	.361	MSELS	1.294	MSERATIO	3.584
CROSS1	6.916	CROSS2	.059	CROSS3	3.316	CROSS4	.501	CROSS5	33.368
CROSS6	.240	RLSQ	69.589	KSQ	.004	NVARSQ	16.	RSQSQ	.687

CONTENTS OF CASE NUMBER 32

SEQNUM	32.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.931	RL	8.342
K	0	NVAR	4.	MSE	.448	MSELS	.448	MSERATIO	1.000
CROSS1	7.766	CROSS2	0	CROSS3	3.724	CROSS4	0	CROSS5	33.368
CROSS6	0	RLSQ	69.589	KSQ	0	NVARSQ	16.	RSQSQ	.867

CONTENTS OF CASE NUMBER 33

SEQNUM	33.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.931	RL	8.342
K	.020	NVAR	4.	MSE	.237	MSELS	.448	MSERATIO	1.890
CROSS1	7.766	CROSS2	.019	CROSS3	3.724	CROSS4	.167	CROSS5	33.368
CROSS6	.080	RLSQ	69.589	KSQ	.000	NVARSQ	16.	RSQSQ	.867

CONTENTS OF CASE NUMBER 34

SEQNUM	34.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.931	RL	8.342
K	.060	NVAR	4.	MSE	.126	MSELS	.448	MSERATIO	3.556
CROSS1	7.766	CROSS2	.056	CROSS3	3.724	CROSS4	.501	CROSS5	33.368
CROSS6	.240	RLSQ	69.589	KSQ	.004	NVARSQ	16.	RSQSQ	.867

CONTENTS OF CASE NUMBER 35

SEQNUM	35.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.997	RL	8.342
K	0	NVAR	4.	MSE	.018	MSELS	.018	MSERATIO	1.000
CROSS1	8.317	CROSS2	0	CROSS3	3.988	CROSS4	0	CROSS5	33.368
CROSS6	0	RLSQ	69.589	KSQ	0	NVARSQ	16.	RSQSQ	.994

CONTENTS OF CASE NUMBER 36

SEQNUM	36.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.997	RL	8.342
K	.020	NVAR	4.	MSE	.010	MSELS	.018	MSERATIO	1.800
CROSS1	8.317	CROSS2	.020	CROSS3	3.988	CROSS4	.167	CROSS5	33.368
CROSS6	.080	RLSQ	69.589	KSQ	.000	NVARSQ	16.	RSQSQ	.994

CONTENTS OF CASE NUMBER 37

SEQNUM	37.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.997	RL	8.342
K	.040	NVAR	4.	MSE	.007	MSELS	.018	MSERATIO	2.571
CROSS1	8.317	CROSS2	.040	CROSS3	3.988	CROSS4	.334	CROSS5	33.368
CROSS6	.160	RLSQ	69.589	KSQ	.002	NVARSQ	16.	RSQSQ	.994

CONTENTS OF CASE NUMBER 38

SEQNUM	38.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.704	RL	7.632
K	0	NVAR	3.	MSE	1.690	MSELS	1.690	MSERATIO	1.000
CROSS1	5.373	CROSS2	0	CROSS3	2.112	CROSS4	0	CROSS5	22.896
CROSS6	0	RLSQ	58.247	KSQ	0	NVARSQ	9.	RSQSQ	.496

CONTENTS OF CASE NUMBER 39

SEQNUM	39.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.704	RL	7.632
K	.005	NVAR	3.	MSE	1.445	MSELS	1.690	MSERATIO	1.170
CROSS1	5.373	CROSS2	.004	CROSS3	2.112	CROSS4	.038	CROSS5	22.896
CROSS6	.015	RLSQ	58.247	KSQ	.000	NVARSQ	9.	RSQSQ	.496

CONTENTS OF CASE NUMBER 40

SEQNUM	40.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.704	RL	7.632
K	.020	NVAR	3.	MSE	1.008	MSELS	1.690	MSERATIO	1.677
CROSS1	5.373	CROSS2	.014	CROSS3	2.112	CROSS4	.153	CROSS5	22.896
CROSS6	.060	RLSQ	58.247	KSQ	.000	NVARSQ	9.	RSQSQ	.496

CONTENTS OF CASE NUMBER 41

SEQNUM	41.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.704	RL	7.632
K	.200	NVAR	3.	MSE	.200	MSELS	1.690	MSERATIO	8.450
CROSS1	5.373	CROSS2	.141	CROSS3	2.112	CROSS4	1.526	CROSS5	22.896
CROSS6	.600	RLSQ	58.247	KSQ	.040	NVARSQ	9.	RSQSQ	.496

CONTENTS OF CASE NUMBER 42

SEQNUM	42.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.804	RL	7.632
K	0	NVAR	3.	MSE	.951	MSELS	.951	MSERATIO	1.000
CROSS1	6.136	CROSS2	0	CROSS3	2.412	CROSS4	0	CROSS5	22.896
CROSS6	0	RLSQ	58.247	KSQ	0	NVARSQ	9.	RSQSQ	.646

CONTENTS OF CASE NUMBER 43

SEQNUM	43.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.804	RL	7.632
K	.005	NVAR	3.	MSE	.813	MSELS	.951	MSERATIO	1.170
CROSS1	6.136	CROSS2	.004	CROSS3	2.412	CROSS4	.038	CROSS5	22.896
CROSS6	.015	RLSQ	58.247	KSQ	.000	NVARSQ	9.	RSQSQ	.646

CONTENTS OF CASE NUMBER 44

SEQNUM	44.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.804	RL	7.632
K	.020	NVAR	3.	MSE	.567	MSELS	.951	MSERATIO	1.677
CROSS1	6.136	CROSS2	.016	CROSS3	2.412	CROSS4	.153	CROSS5	22.096
CROSS6	.060	RLSQ	58.247	KSQ	.000	NVARSQ	9.	RSQSQ	.645

CONTENTS OF CASE NUMBER 45

SEQNUM	45.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.804	RL	7.632
K	.120	NVAR	3.	MSE	.180	MSELS	.951	MSERATIO	5.283
CROSS1	6.136	CROSS2	.096	CROSS3	2.412	CROSS4	.916	CROSS5	22.896
CROSS6	.360	RLSQ	58.247	KSQ	.014	NVARSQ	9.	RSQSQ	.646

CONTENTS OF CASE NUMBER 46

SEQNUM	46.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.900	RL	7.632
K	0	NVAR	3.	MSE	.423	MSELS	.423	MSERATIO	1.000
CROSS1	6.869	CROSS2	0	CROSS3	2.700	CROSS4	0	CROSS5	22.896
CROSS6	0	RLSQ	58.247	KSQ	0	NVARSQ	9.	RSQSQ	.810

CONTENTS OF CASE NUMBER 47

SEQNUM	47.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.900	RL	7.632
K	.020	NVAR	3.	MSE	.252	MSELS	.423	MSERATIO	1.679
CROSS1	6.869	CROSS2	.018	CROSS3	2.700	CROSS4	.153	CROSS5	22.896
CROSS6	.060	RLSQ	58.247	KSQ	.000	NVARSQ	9.	RSQSQ	.810

CONTENTS OF CASE NUMBER 48

SEQNUM	48.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.900	RL	7.632
K	.040	NVAR	3.	MSE	.179	MSELS	.423	MSERATIO	2.363
CROSS1	6.869	CROSS2	.035	CROSS3	2.700	CROSS4	.305	CROSS5	22.896
CROSS6	.120	RLSQ	58.247	KSQ	.002	NVARSQ	9.	RSQSQ	.810

CONTENTS OF CASE NUMBER 49

SEQNUM	49.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.975	RL	7.632
K	0	NVAR	3.	MSE	.017	MSELS	.017	MSERATIO	1.000
CROSS1	7.594	CROSS2	0	CROSS3	2.985	CROSS4	0	CROSS5	22.896
CROSS6	0	RLSQ	58.247	KSQ	0	NVARSQ	9.	RSQSQ	.990

CONTENTS OF CASE NUMBER 50

SEQNUM	50.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.975	RL	7.632
K	.020	NVAR	3.	MSE	.010	MSELS	.017	MSERATIO	1.700
CROSS1	7.594	CROSS2	.020	CROSS3	2.985	CROSS4	.153	CROSS5	22.896
CROSS6	.060	RLSQ	58.247	KSQ	.000	NVARSQ	9.	RSQSQ	.990

CONTENTS OF CASE NUMBER 51

SEQNUM	51.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.995	RL	7.632
K	.040	NVAR	3.	MSE	.008	MSELS	.017	MSERATIO	2.125
CROSS1	7.594	CROSS2	.040	CROSS3	2.985	CROSS4	.305	CROSS5	22.096
CROSS6	.120	RLSQ	58.247	KSQ	.002	NVARSQ	9.	RSQSQ	.990

CONTENTS OF CASE NUMBER 52

SEQNUM	52.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.832	RL	1.536
K	0	NVAR	4.	MSE	.090	MSELS	.090	MSERATIO	1.000
CROSS1	1.278	CROSS2	0	CROSS3	3.328	CROSS4	0	CROSS5	6.144
CROSS6	0	RLSQ	2.359	KSQ	0	NVARSQ	16.	RSQSQ	.692

CONTENTS OF CASE NUMBER 53

SEQNUM	53.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.832	RL	1.536
K	.040	NVAR	4.	MSE	.082	MSELS	.090	MSERATIO	1.098
CROSS1	1.278	CROSS2	.033	CROSS3	3.328	CROSS4	.061	CROSS5	6.144
CROSS6	.160	RLSQ	2.359	KSQ	.002	NVARSQ	16.	RSQSQ	.692

CONTENTS OF CASE NUMBER 54

SEQNUM	54.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.832	RL	1.536
K	.120	NVAR	4.	MSE	.086	MSELS	.090	MSERATIO	1.047
CROSS1	1.278	CROSS2	.100	CROSS3	3.328	CROSS4	.184	CROSS5	6.144
CROSS6	.480	RLSQ	2.359	KSQ	.014	NVARSQ	16.	RSQSQ	.692

CONTENTS OF CASE NUMBER 55

SEQNUM	55.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.832	RL	1.536
K	.380	NVAR	4.	MSE	.154	MSELS	.090	MSERATIO	.584
CROSS1	1.278	CROSS2	.316	CROSS3	3.328	CROSS4	.584	CROSS5	6.144
CROSS6	1.520	RLSQ	2.359	KSQ	.144	NVARSQ	16.	RSQSQ	.692

CONTENTS OF CASE NUMBER 56

SEQNUM	56.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.908	RL	1.536
K	0	NVAR	4.	MSE	.044	MSELS	.044	MSERATIO	1.000
CROSS1	1.395	CROSS2	0	CROSS3	3.632	CROSS4	0	CROSS5	6.144
CROSS6	0	RLSQ	2.359	KSQ	0	NVARSQ	16.	RSQSQ	.824

CONTENTS OF CASE NUMBER 57

SEQNUM	57.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.908	RL	1.536
K	.040	NVAR	4.	MSE	.042	MSELS	.044	MSERATIO	1.048
CROSS1	1.395	CROSS2	.036	CROSS3	3.632	CROSS4	.061	CROSS5	6.144
CROSS6	.160	RLSQ	2.359	KSQ	.002	NVARSQ	16.	RSQSQ	.824

CONTENTS OF CASE NUMBER 58

SEQNUM	58.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.908	RL	1.536
K	.080	NVAR	4.	MSE	.047	MSELS	.044	MSERATIO	.936
CROSS1	1.395	CROSS2	.073	CROSS3	3.632	CROSS4	.123	CROSS5	6.144
CROSS6	.320	RLSQ	2.359	KSQ	.006	NVARSQ	16.	RSQSQ	.824

CONTENTS OF CASE NUMBER 59

SEQNUM	59.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.992	RL	1.536
K	0	NVAR	4.	MSE	.004	MSELS	.004	MSERATIO	1.000
CROSS1	1.524	CROSS2	0	CROSS3	3.968	CROSS4	0	CROSS5	6.144
CROSS6	0	RLSQ	2.359	KSQ	0	NVARSQ	16.	RSQSQ	.984

CONTENTS OF CASE NUMBER 60

SEQNUM	60.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.992	RL	1.536
K	.040	NVAR	4.	MSE	.007	MSELS	.004	MSERATIO	.571
CROSS1	1.524	CROSS2	.040	CROSS3	3.968	CROSS4	.061	CROSS5	6.144
CROSS6	.160	RLSQ	2.359	KSQ	.002	NVARSQ	16.	RSQSQ	.984

CONTENTS OF CASE NUMBER 61

SEQNUM	61.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.992	RL	1.536
K	.100	NVAR	4.	MSE	.021	MSELS	.004	MSERATIO	.190
CROSS1	1.524	CROSS2	.099	CROSS3	3.968	CROSS4	.154	CROSS5	6.144
CROSS6	.400	RLSQ	2.359	KSQ	.010	NVARSQ	16.	RSQSQ	.984

CONTENTS OF CASE NUMBER 62

SEQNUM	62.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.681	RL	1.830
K	0	NVAR	3.	MSE	.348	MSELS	.348	MSERATIO	1.000
CROSS1	1.246	CROSS2	0	CROSS3	2.043	CROSS4	0	CROSS5	5.490
CROSS6	0	RLSQ	3.349	KSQ	0	NVARSQ	9.	RSQSQ	.464

CONTENTS OF CASE NUMBER 63

SEQNUM	63.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.681	RL	1.830
K	.040	NVAR	3.	MSE	.281	MSELS	.348	MSERATIO	1.238
CROSS1	1.246	CROSS2	.027	CROSS3	2.043	CROSS4	.073	CROSS5	5.490
CROSS6	.120	RLSQ	3.349	KSQ	.002	NVARSQ	9.	RSQSQ	.464

CONTENTS OF CASE NUMBER 64

SEQNUM	64.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.681	RL	1.830
K	.080	NVAR	3.	MSE	.237	MSELS	.348	MSERATIO	1.468
CROSS1	1.246	CROSS2	.054	CROSS3	2.043	CROSS4	.146	CROSS5	5.490
CROSS6	.240	RLSQ	3.349	KSQ	.006	NVARSQ	9.	RSQSQ	.464

CONTENTS OF CASE NUMBER 65

SEQNUM	65.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.631	RL	1.830
K	.120	NVAR	3.	MSE	.206	MSELS	.348	MSERATIO	1.689
CROSS1	1.246	CROSS2	.032	CROSS3	2.043	CROSS4	.220	CROSS5	5.490
CROSS6	.360	RLSQ	3.349	KSQ	.014	NVARSQ	9.	RSQSQ	.464

CONTENTS OF CASE NUMBER 66

SEQNUM	66.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.909	RL	1.830
K	0	NVAR	3.	MSE	.069	MSELS	.069	MSERATIO	1.000
CROSS1	1.663	CROSS2	0	CROSS3	2.727	CROSS4	0	CROSS5	5.490
CROSS6	0	RLSQ	3.349	KSQ	0	NVARSQ	9.	RSQSQ	.826

CONTENTS OF CASE NUMBER 67

SEQNUM	67.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.909	RL	1.830
K	.020	NVAR	3.	MSE	.061	MSELS	.069	MSERATIO	1.131
CROSS1	1.663	CROSS2	.018	CROSS3	2.727	CROSS4	.037	CROSS5	5.490
CROSS6	.060	RLSQ	3.349	KSQ	.000	NVARSQ	9.	RSQSQ	.826

CONTENTS OF CASE NUMBER 68

SEQNUM	68.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.909	RL	1.830
K	.060	NVAR	3.	MSE	.052	MSELS	.069	MSERATIO	1.327
CROSS1	1.663	CROSS2	.055	CROSS3	2.727	CROSS4	.110	CROSS5	5.490
CROSS6	.180	RLSQ	3.349	KSQ	.004	NVARSQ	9.	RSQSQ	.826

CONTENTS OF CASE NUMBER 69

SEQNUM	69.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.994	RL	1.830
K	0	NVAR	3.	MSE	.004	MSELS	.004	MSERATIO	1.000
CROSS1	1.819	CROSS2	0	CROSS3	2.982	CROSS4	0	CROSS5	5.490
CROSS6	0	RLSQ	3.349	KSQ	0	NVARSQ	9.	RSQSQ	.988

CONTENTS OF CASE NUMBER 70

SEQNUM	70.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.994	RL	1.830
K	.020	NVAR	3.	MSE	.004	MSELS	.004	MSERATIO	1.000
CROSS1	1.819	CROSS2	.020	CROSS3	2.982	CROSS4	.037	CROSS5	5.490
CROSS6	.060	RLSQ	3.349	KSQ	.000	NVARSQ	9.	RSQSQ	.988

CONTENTS OF CASE NUMBER 71

SEQNUM	71.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.994	RL	1.830
K	.040	NVAR	3.	MSE	.004	MSELS	.004	MSERATIO	1.000
CROSS1	1.819	CROSS2	.040	CROSS3	2.982	CROSS4	.073	CROSS5	5.490
CROSS6	.120	RLSQ	3.349	KSQ	.002	NVARSQ	9.	RSQSQ	.988

CONTENTS OF CASE NUMBER 72

SEQNUM	72.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.526	RL	119.772
K	0	NVAR	2.	MSE	27.772	MSELS	27.772	MSERATIO	1.000
CROSS1	63.000	CROSS2	0	CROSS3	1.052	CROSS4	0	CROSS5	239.544
CROSS6	0	RLSQ	14345.332	KSQ	0	NVARSQ	4.	RSQSQ	.277

CONTENTS OF CASE NUMBER 73

SEQNUM	73.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.526	RL	119.772
K	.005	NVAR	2.	MSE	6.221	MSELS	27.772	MSERATIO	4.464
CROSS1	63.000	CROSS2	.003	CROSS3	1.052	CROSS4	.599	CROSS5	239.544
CROSS6	.010	RLSQ	14345.332	KSQ	.000	NVARSQ	4.	RSQSQ	.277

CONTENTS OF CASE NUMBER 74

SEQNUM	74.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.526	RL	119.772
K	.010	NVAR	2.	MSE	2.891	MSELS	27.772	MSERATIO	9.606
CROSS1	63.000	CROSS2	.005	CROSS3	1.052	CROSS4	1.198	CROSS5	239.544
CROSS6	.020	RLSQ	14345.332	KSQ	.000	NVARSQ	4.	RSQSQ	.277

CONTENTS OF CASE NUMBER 75

SEQNUM	75.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.526	RL	119.772
K	.025	NVAR	2.	MSE	.825	MSELS	27.772	MSERATIO	33.663
CROSS1	63.000	CROSS2	.013	CROSS3	1.052	CROSS4	2.994	CROSS5	239.544
CROSS6	.050	RLSQ	14345.332	KSQ	.001	NVARSQ	4.	RSQSQ	.277

CONTENTS OF CASE NUMBER 76

SEQNUM	76.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.744	RL	119.772
K	0	NVAR	2.	MSE	9.998	MSELS	9.998	MSERATIO	1.000
CROSS1	89.110	CROSS2	0	CROSS3	1.488	CROSS4	0	CROSS5	239.544
CROSS6	0	RLSQ	14345.332	KSQ	0	NVARSQ	4.	RSQSQ	.554

CONTENTS OF CASE NUMBER 77

SEQNUM	77.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.744	RL	119.772
K	.015	NVAR	2.	MSE	.614	MSELS	9.998	MSERATIO	16.283
CROSS1	89.110	CROSS2	.011	CROSS3	1.488	CROSS4	1.797	CROSS5	239.544
CROSS6	.030	RLSQ	14345.332	KSQ	.000	NVARSQ	4.	RSQSQ	.554

CONTENTS OF CASE NUMBER 78

SEQNUM	78.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.744	RL	119.772
K	.030	NVAR	2.	MSE	.228	MSELS	9.998	MSERATIO	43.851
CROSS1	89.110	CROSS2	.022	CROSS3	1.488	CROSS4	3.593	CROSS5	239.544
CROSS6	.060	RLSQ	14345.332	KSQ	.001	NVARSQ	4.	RSQSQ	.554

CONTENTS OF CASE NUMBER 79

SEQNUM	79.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.744	RL	119.772
K	.040	NVAR	2.	MSE	.115	MSELS	9.998	MSERATIO	86.939
CROSS1	89.110	CROSS2	.030	CROSS3	1.488	CROSS4	4.791	CROSS5	239.544
CROSS6	.080	RLSQ	14345.332	KSQ	.002	NVARSQ	4.	RSQSQ	.554

CONTENTS OF CASE NUMBER 80

SEQNUM	80.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.893	RL	119.772
K	0	NVAR	2.	MSE	3.402	MSELS	3.402	MSERATIO	1.000
CROSS1	106.956	CROSS2	0	CROSS3	1.786	CROSS4	0	CROSS5	239.544
CROSS6	0	RLSQ	14345.332	KSQ	0	NVARSQ	4.	RSQSQ	.797

CONTENTS OF CASE NUMBER 81

SEQNUM	81.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.893	RL	119.772
K	.015	NVAR	2.	MSE	.209	MSELS	3.402	MSERATIO	16.278
CROSS1	106.956	CROSS2	.013	CROSS3	1.786	CROSS4	1.797	CROSS5	239.544
CROSS6	.030	RLSQ	14345.332	KSQ	.000	NVARSQ	4.	RSQSQ	.797

CONTENTS OF CASE NUMBER 82

SEQNUM	82.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.893	RL	119.772
K	.030	NVAR	2.	MSE	.078	MSELS	3.402	MSERATIO	43.615
CROSS1	106.956	CROSS2	.027	CROSS3	1.786	CROSS4	3.593	CROSS5	239.544
CROSS6	.060	RLSQ	14345.332	KSQ	.001	NVARSQ	4.	RSQSQ	.797

CONTENTS OF CASE NUMBER 83

SEQNUM	83.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.893	RL	119.772
K	.045	NVAR	2.	MSE	.044	MSELS	3.402	MSERATIO	77.318
CROSS1	106.956	CROSS2	.040	CROSS3	1.786	CROSS4	5.390	CROSS5	239.544
CROSS6	.090	RLSQ	14345.332	KSQ	.002	NVARSQ	4.	RSQSQ	.797

CONTENTS OF CASE NUMBER 84

SEQNUM	84.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.990	RL	119.772
K	0	NVAR	2.	MSE	.278	MSELS	.278	MSERATIO	1.000
CROSS1	118.574	CROSS2	0	CROSS3	1.980	CROSS4	0	CROSS5	239.544
CROSS6	0	RLSQ	14345.332	KSQ	0	NVARSQ	4.	RSQSQ	.980

CONTENTS OF CASE NUMBER 85

SEQNUM	85.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.990	RL	119.772
K	.010	NVAR	2.	MSE	.017	MSELS	.278	MSERATIO	16.353
CROSS1	118.574	CROSS2	.010	CROSS3	1.980	CROSS4	1.198	CROSS5	239.544
CROSS6	.020	RLSQ	14345.332	KSQ	.000	NVARSQ	4.	RSQSQ	.980

CONTENTS OF CASE NUMBER 86

SEQNUM	86.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.990	RL	119.772
K	.025	NVAR	2.	MSE	.009	MSELS	.278	MSERATIO	30.899
CROSS1	118.574	CROSS2	.025	CROSS3	1.980	CROSS4	2.994	CROSS5	239.544
CROSS6	.050	RLSQ	14345.332	KSQ	.001	NVARSQ	4.	RSQSQ	.980

CONTENTS OF CASE NUMBER 87

SEQNUM	87.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.990	RL	119.772
K	.040	NVAR	2.	MSE	.005	MSELS	.278	MSERATIO	55.600
CROSS1	118.574	CROSS2	.040	CROSS3	1.980	CROSS4	4.791	CROSS5	239.544
CROSS6	.080	RLSQ	14345.332	KSQ	.002	NVARSQ	4.	RSQSQ	.980

CONTENTS OF CASE NUMBER 88

SEQNUM	88.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.740	RL	13.424
K	0	NVAR	2.	MSE	1.136	MSELS	1.136	MSERATIO	1.000
CROSS1	9.934	CROSS2	0	CROSS3	1.480	CROSS4	0	CROSS5	26.848
CROSS6	0	RLSQ	180.204	KSQ	0	NVARSQ	4.	RSQSQ	.548

CONTENTS OF CASE NUMBER 89

SEQNUM	89.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.740	RL	13.424
K	.005	NVAR	2.	MSE	.880	MSELS	1.136	MSERATIO	1.291
CROSS1	9.934	CROSS2	.004	CROSS3	1.480	CROSS4	.067	CROSS5	26.848
CROSS6	.010	RLSQ	180.204	KSQ	.000	NVARSQ	4.	RSQSQ	.548

CONTENTS OF CASE NUMBER 90

SEQNUM	90.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.740	RL	13.424
K	.025	NVAR	2.	MSE	.435	MSELS	1.136	MSERATIO	2.611
CROSS1	9.934	CROSS2	.018	CROSS3	1.480	CROSS4	.336	CROSS5	26.848
CROSS6	.050	RLSQ	180.204	KSQ	.001	NVARSQ	4.	RSQSQ	.548

CONTENTS OF CASE NUMBER 91

SEQNUM	91.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.740	RL	13.424
K	.040	NVAR	2.	MSE	.273	MSELS	1.136	MSERATIO	4.161
CROSS1	9.934	CROSS2	.030	CROSS3	1.400	CROSS4	.537	CROSS5	26.848
CROSS6	.080	RLSQ	180.204	KSQ	.002	NVARSQ	4.	RSQSQ	.548

CONTENTS OF CASE NUMBER 92

SEQNUM	92.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.863	RL	13.424
K	0	NVAR	2.	MSE	.505	MSELS	.505	MSERATIO	1.000
CROSS1	11.585	CROSS2	0	CROSS3	1.726	CROSS4	0	CROSS5	26.848
CROSS6	0	RLSQ	180.204	KSQ	0	NVARSQ	4.	RSQSQ	.745

CONTENTS OF CASE NUMBER 93

SEQNUM	93.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.863	RL	13.424
K	.005	NVAR	2.	MSE	.391	MSELS	.505	MSERATIO	1.292
CROSS1	11.585	CROSS2	.004	CROSS3	1.726	CROSS4	.067	CROSS5	26.848
CROSS6	.010	RLSQ	180.204	KSQ	.000	NVARSQ	4.	RSQSQ	.745

CONTENTS OF CASE NUMBER 94

SEQNUM	94.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.863	RL	13.424
K	.020	NVAR	2.	MSE	.224	MSELS	.505	MSERATIO	2.254
CROSS1	11.585	CROSS2	.017	CROSS3	1.726	CROSS4	.268	CROSS5	26.848
CROSS6	.040	RLSQ	180.204	KSQ	.000	NVARSQ	4.	RSQSQ	.745

CONTENTS OF CASE NUMBER 95

SEQNUM	95.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.863	RL	13.424
K	.040	NVAR	2.	MSE	.135	MSELS	.505	MSERATIO	3.741
CROSS1	11.585	CROSS2	.035	CROSS3	1.726	CROSS4	.537	CROSS5	26.848
CROSS6	.080	RLSQ	180.204	KSQ	.002	NVARSQ	4.	RSQSQ	.745

CONTENTS OF CASE NUMBER 96

SEQNUM	96.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.990	RL	13.424
K	0	NVAR	2.	MSE	.032	MSELS	.032	MSERATIO	1.000
CROSS1	13.290	CROSS2	0	CROSS3	1.980	CROSS4	0	CROSS5	26.848
CROSS6	0	RLSQ	180.204	KSQ	0	NVARSQ	4.	RSQSQ	.980

CONTENTS OF CASE NUMBER 97

SEQNUM	97.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.990	RL	13.424
K	.005	NVAR	2.	MSE	.025	MSELS	.032	MSERATIO	1.280
CROSS1	13.290	CROSS2	.005	CROSS3	1.980	CROSS4	.067	CROSS5	26.848
CROSS6	.010	RLSQ	180.204	KSQ	.000	NVARSQ	4.	RSQSQ	.980

CONTENTS OF CASE NUMBER 98

SEQNUM	98.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.990	RL	13.424
K	.015	NVAR	2.	MSE	.017	MSELS	.032	MSERATIO	1.882
CROSS1	13.290	CROSS2	.015	CROSS3	1.980	CROSS4	.201	CROSS5	26.848
CROSS6	.030	RLSQ	180.204	KSQ	.000	NVARSQ	4.	RSQSQ	.980

CONTENTS OF CASE NUMBER 99

SEQNUM	99.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.990	RL	13.424
K	.030	NVAR	2.	MSE	.011	MSELS	.032	MSERATIO	2.909
CROSS1	13.290	CROSS2	.030	CROSS3	1.980	CROSS4	.403	CROSS5	26.848
CROSS6	.060	RLSQ	180.204	KSQ	.001	NVARSQ	4.	RSQSQ	.980

CONTENTS OF CASE NUMBER 100

SEQNUM	100.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.714	RL	2.196
K	0	NVAR	2.	MSE	.194	MSELS	.194	MSERATIO	1.000
CROSS1	1.568	CROSS2	0	CROSS3	1.428	CROSS4	0	CROSS5	4.392
CROSS6	0	RLSQ	4.822	KSQ	0	NVARSQ	4.	RSQSQ	.510

CONTENTS OF CASE NUMBER 101

SEQNUM	101.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.714	RL	2.196		
K	.005	NVAR	2.	MSE	.187	MSELS	.194	MSERATIO	1.037		
CROSS1	1.568	CROSS2	.004	CROSS3	1.428	CROSS4	.011	CROSS5	4.392		
CROSS6	.010	RLSQ	4.822	KSQ	.000	NVARSQ	4.	RSQSQ	.510		

CONTENTS OF CASE NUMBER 102

SEQNUM	102.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.714	RL	2.196
K	.020	NVAR	2.	MSE	.170	MSELS	.194	MSERATIO	1.141
CROSS1	1.568	CROSS2	.014	CROSS3	1.428	CROSS4	.044	CROSS5	4.392
CROSS6	.040	RLSQ	4.822	KSQ	.000	NVARSQ	4.	RSQSQ	.510

CONTENTS OF CASE NUMBER 103

SEQNUM	103.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.714	RL	2.196
K	.040	NVAR	2.	MSE	.151	MSELS	.194	MSERATIO	1.285
CROSS1	1.568	CROSS2	.029	CROSS3	1.428	CROSS4	.088	CROSS5	4.392
CROSS6	.080	RLSQ	4.822	KSQ	.002	NVARSQ	4.	RSQSQ	.510

CONTENTS OF CASE NUMBER 104

SEQNUM	104.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.780	RL	2.196
K	0	NVAR	2.	MSE	.135	MSELS	.135	MSERATIO	1.000
CROSS1	1.713	CROSS2	0	CROSS3	1.560	CROSS4	0	CROSS5	4.392
CROSS6	0	RLSQ	4.822	KSQ	0	NVARSQ	4.	RSQSQ	.608

CONTENTS OF CASE NUMBER 105

SEQNUM	105.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.780	RL	2.196
K	.005	NVAR	2.	MSE	.130	MSELS	.135	MSERATIO	1.038
CROSS1	1.713	CROSS2	.004	CROSS3	1.560	CROSS4	.011	CROSS5	4.392
CROSS6	.010	RLSQ	4.822	KSQ	.000	NVARSQ	4.	RSQSQ	.608

CONTENTS OF CASE NUMBER 106

SEQNUM	106.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.780	RL	2.196
K	.015	NVAR	2.	MSE	.122	MSELS	.135	MSERATIO	1.107
CROSS1	1.713	CROSS2	.012	CROSS3	1.560	CROSS4	.033	CROSS5	4.392
CROSS6	.030	RLSQ	4.822	KSQ	.000	NVARSQ	4.	RSQSQ	.608

CONTENTS OF CASE NUMBER 107

SEQNUM	107.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.780	RL	2.196
K	.035	NVAR	2.	MSE	.108	MSELS	.135	MSERATIO	1.250
CROSS1	1.713	CROSS2	.027	CROSS3	1.560	CROSS4	.077	CROSS5	4.392
CROSS6	.070	RLSQ	4.822	KSQ	.001	NVARSQ	4.	RSQSQ	.608

CONTENTS OF CASE NUMBER 108

SEQNUM	108.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.906	RL	2.196
K	0	NVAR	2.	MSE	.049	MSELS	.049	MSERATIO	1.000
CROSS1	1.990	CROSS2	0	CROSS3	1.812	CROSS4	0	CROSS5	4.392
CROSS6	0	RLSQ	4.822	KSQ	0	NVARSQ	4.	RSQSQ	.821

CONTENTS OF CASE NUMBER 109

SEQNUM	109.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.906	RL	2.196
K	.010	NVAR	2.	MSE	.045	MSELS	.049	MSERATIO	1.089
CROSS1	1.990	CROSS2	.009	CROSS3	1.812	CROSS4	.022	CROSS5	4.392
CROSS6	.020	RLSQ	4.822	KSQ	.000	NVARSQ	4.	RSQSQ	.821

CONTENTS OF CASE NUMBER 110

SEQNUM	110.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.906	RL	2.196
K	.025	NVAR	2.	MSE	.041	MSELS	.049	MSERATIO	1.195
CROSS1	1.990	CROSS2	.023	CROSS3	1.812	CROSS4	.055	CROSS5	4.392
CROSS6	.050	RLSQ	4.822	KSQ	.001	NVARSQ	4.	RSQSQ	.821

CONTENTS OF CASE NUMBER 111

SEQNUM	111.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.906	RL	2.196
K	.045	NVAR	2.	MSE	.037	MSELS	.049	MSERATIO	1.324
CROSS1	1.990	CROSS2	.041	CROSS3	1.812	CROSS4	.099	CROSS5	4.392
CROSS6	.090	RLSQ	4.822	KSQ	.002	NVARSQ	4.	RSQSQ	.821

CONTENTS OF CASE NUMBER 112

SEQNUM	112.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.989	RL	2.196
K	0	NVAR	2.	MSE	.005	MSELS	.005	MSERATIO	1.000
CROSS1	2.172	CROSS2	0	CROSS3	1.978	CROSS4	0	CROSS5	4.392
CROSS6	0	RLSQ	4.822	KSQ	0	NVARSQ	4.	RSQSQ	.978

CONTENTS OF CASE NUMBER 113

SEQNUM	113.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.989	RL	2.196
K	.005	NVAR	2.	MSE	.005	MSELS	.005	MSERATIO	1.000
CROSS1	2.172	CROSS2	.005	CROSS3	1.978	CROSS4	.011	CROSS5	4.392
CROSS6	.010	RLSQ	4.822	KSQ	.000	NVARSQ	4.	RSQSQ	.978

CONTENTS OF CASE NUMBER 114

SEQNUM	114.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.989	RL	2.196
K	.020	NVAR	2.	MSE	.005	MSELS	.005	MSERATIO	1.000
CROSS1	2.172	CROSS2	.020	CROSS3	1.978	CROSS4	.044	CROSS5	4.392
CROSS6	.040	RLSQ	4.822	KSQ	.000	NVARSQ	4.	RSQSQ	.978

CONTENTS OF CASE NUMBER 115

SEQNUM	115.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.989	RL	2.196
K	.040	NVAR	2.	MSE	.005	MSELS	.005	MSERATIO	1.000
CROSS1	2.172	CROSS2	.040	CROSS3	1.978	CROSS4	.088	CROSS5	4.392
CROSS6	.080	RLSQ	4.822	KSQ	.002	NVARSQ	4.	RSQSQ	.978

Appendix H

Log-Linear Model Data

CONTENTS OF CASE NUMBER 1

SEQNUM	1.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.735	RL	3.174
K	0	NVAR	2.	MSE	.259	MSELS	.259	MSERATIO	1.000
CROSS1	2.333	CROSS2	0	CROSS3	1.470	CROSS4	0	CROSS5	6.348
CROSS6	0	RLSQ	10.074	KSQ	0	NVARSQ	4.	RSQSQ	.540

CONTENTS OF CASE NUMBER 2

SEQNUM	2.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.735	RL	3.174
K	.005	NVAR	2.	MSE	.245	MSELS	.259	MSERATIO	1.057
CROSS1	2.333	CROSS2	.004	CROSS3	1.470	CROSS4	.016	CROSS5	6.348
CROSS6	.010	RLSQ	10.074	KSQ	.000	NVARSQ	4.	RSQSQ	.540

CONTENTS OF CASE NUMBER 3

SEQNUM	3.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.735	RL	3.174
K	.030	NVAR	2.	MSE	.193	MSELS	.259	MSERATIO	1.342
CROSS1	2.333	CROSS2	.022	CROSS3	1.470	CROSS4	.075	CROSS5	6.348
CROSS6	.060	RLSQ	10.074	KSQ	.001	NVARSQ	4.	RSQSQ	.540

CONTENTS OF CASE NUMBER 4

SEQNUM	4.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.735	RL	3.174
K	.045	NVAR	2.	MSE	.171	MSELS	.259	MSERATIO	1.515
CROSS1	2.333	CROSS2	.033	CROSS3	1.470	CROSS4	.143	CROSS5	6.348
CROSS6	.090	RLSQ	10.074	KSQ	.002	NVARSQ	4.	RSQSQ	.540

CONTENTS OF CASE NUMBER 5

SEQNUM	5.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.915	RL	3.174
K	0	NVAR	2.	MSE	.065	MSELS	.065	MSERATIO	1.000
CROSS1	2.904	CROSS2	0	CROSS3	1.830	CROSS4	0	CROSS5	6.348
CROSS6	0	RLSQ	10.074	KSQ	0	NVARSQ	4.	RSQSQ	.837

CONTENTS OF CASE NUMBER 6

SEQNUM	6.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.915	RL	3.174
K	.005	NVAR	2.	MSE	.061	MSELS	.065	MSERATIO	1.066
CROSS1	2.904	CROSS2	.005	CROSS3	1.830	CROSS4	.016	CROSS5	6.348
CROSS6	.010	RLSQ	10.074	KSQ	.000	NVARSQ	4.	RSQSQ	.837

CONTENTS OF CASE NUMBER 7

SEQNUM	7.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.915	RL	3.174
K	.025	NVAR	2.	MSE	.051	MSELS	.065	MSERATIO	1.275
CROSS1	2.904	CROSS2	.023	CROSS3	1.830	CROSS4	.079	CROSS5	6.348
CROSS6	.050	RLSQ	10.074	KSQ	.001	NVARSQ	4.	RSQSQ	.837

CONTENTS OF CASE NUMBER 8

SEQNUM	8.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.915	RL	3.174
K	.040	NVAR	2.	MSE	.045	MSELS	.065	MSERATIO	1.444
CROSS1	2.904	CROSS2	.037	CROSS3	1.830	CROSS4	.127	CROSS5	6.348
CROSS6	.080	RLSQ	10.074	KSQ	.002	NVARSQ	4.	RSQSQ	.837

CONTENTS OF CASE NUMBER 9

SEQNUM	9.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.996	RL	3.174
K	0	NVAR	2.	MSE	.003	MSELS	.003	MSERATIO	1.000
CROSS1	3.161	CROSS2	0	CROSS3	1.992	CROSS4	0	CROSS5	6.348
CROSS6	0	RLSQ	10.074	KSQ	0	NVARSQ	4.	RSQSQ	.992

CONTENTS OF CASE NUMBER 10

SEQNUM	10.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.996	RL	3.174
K	.005	NVAR	2.	MSE	.002	MSELS	.003	MSERATIO	1.500
CROSS1	3.161	CROSS2	.005	CROSS3	1.992	CROSS4	.016	CROSS5	6.348
CROSS6	.010	RLSQ	10.074	KSQ	.000	NVARSQ	4.	RSQSQ	.992

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CONTENTS OF CASE NUMBER   11
SEQNUM     11.     SUBFILE  REGANAL   CASWGT   1.0000   RSQ       .996   RL          3.174
K         .030     NVAR          2.   MSE        .002   MSELS     .003   MSERATIO    1.500
CROSS1   3.161     CROSS2      .030   CROSS3    1.992   CROSS4    .095   CROSS5      6.348
CROSS6    .060     RLSQ      10.074   KSQ        .001   NVARSQ      4.   RSQSQ        .992

CONTENTS OF CASE NUMBER   12
SEQNUM     12.     SUBFILE  REGANAL   CASWGT   1.0000   RSQ       .996   RL          3.174
K         .045     NVAR          2.   MSE        .003   MSELS     .003   MSERATIO    1.000
CROSS1   3.161     CROSS2      .045   CROSS3    1.992   CROSS4    .143   CROSS5      6.348
CROSS6    .090     RLSQ      10.074   KSQ        .002   NVARSQ      4.   RSQSQ        .992

CONTENTS OF CASE NUMBER   13
SEQNUM     13.     SUBFILE  REGANAL   CASWGT   1.0000   RSQ       .749   RL         12.127
K            0     NVAR          2.   MSE        .950   MSELS     .950   MSERATIO    1.000
CROSS1   9.083     CROSS2         0   CROSS3    1.498   CROSS4       0   CROSS5     24.254
CROSS6       0     RLSQ     147.064   KSQ           0   NVARSQ      4.   RSQSQ        .561

CONTENTS OF CASE NUMBER   14
SEQNUM     14.     SUBFILE  REGANAL   CASWGT   1.0000   RSQ       .749   RL         12.127
K         .005     NVAR          2.   MSE        .755   MSELS     .950   MSERATIO    1.258
CROSS1   9.083     CROSS2      .004   CROSS3    1.498   CROSS4    .061   CROSS5     24.254
CROSS6    .010     RLSQ     147.064   KSQ        .000   NVARSQ      4.   RSQSQ        .561

CONTENTS OF CASE NUMBER   15
SEQNUM     15.     SUBFILE  REGANAL   CASWGT   1.0000   RSQ       .749   RL         12.127
K         .020     NVAR          2.   MSE        .453   MSELS     .950   MSERATIO    2.097
CROSS1   9.083     CROSS2      .015   CROSS3    1.498   CROSS4    .243   CROSS5     24.254
CROSS6    .040     RLSQ     147.064   KSQ        .000   NVARSQ      4.   RSQSQ        .561

CONTENTS OF CASE NUMBER   16
SEQNUM     16.     SUBFILE  REGANAL   CASWGT   1.0000   RSQ       .749   RL         12.127
K         .035     NVAR          2.   MSE        .313   MSELS     .950   MSERATIO    3.035
CROSS1   9.083     CROSS2      .026   CROSS3    1.498   CROSS4    .424   CROSS5     24.254
CROSS6    .070     RLSQ     147.064   KSQ        .001   NVARSQ      4.   RSQSQ        .561

CONTENTS OF CASE NUMBER   17
SEQNUM     17.     SUBFILE  REGANAL   CASWGT   1.0000   RSQ       .920   RL         12.127
K            0     NVAR          2.   MSE        .238   MSELS     .238   MSERATIO    1.000
CROSS1  11.157     CROSS2         0   CROSS3    1.840   CROSS4       0   CROSS5     24.254
CROSS6       0     RLSQ     147.064   KSQ           0   NVARSQ      4.   RSQSQ        .846

CONTENTS OF CASE NUMBER   18
SEQNUM     18.     SUBFILE  REGANAL   CASWGT   1.0000   RSQ       .920   RL         12.127
K         .005     NVAR          2.   MSE        .189   MSELS     .238   MSERATIO    1.259
CROSS1  11.157     CROSS2      .005   CROSS3    1.840   CROSS4    .061   CROSS5     24.254
CROSS6    .010     RLSQ     147.064   KSQ        .000   NVARSQ      4.   RSQSQ        .846

CONTENTS OF CASE NUMBER   19
SEQNUM     19.     SUBFILE  REGANAL   CASWGT   1.0000   RSQ       .920   RL         12.127
K         .020     NVAR          2.   MSE        .114   MSELS     .238   MSERATIO    2.088
CROSS1  11.157     CROSS2      .018   CROSS3    1.840   CROSS4    .243   CROSS5     24.254
CROSS6    .040     RLSQ     147.064   KSQ        .000   NVARSQ      4.   RSQSQ        .846

CONTENTS OF CASE NUMBER   20
SEQNUM     20.     SUBFILE  REGANAL   CASWGT   1.0000   RSQ       .920   RL         12.127
K         .045     NVAR          2.   MSE        .065   MSELS     .238   MSERATIO    3.662
CROSS1  11.157     CROSS2      .041   CROSS3    1.840   CROSS4    .546   CROSS5     24.254
CROSS6    .090     RLSQ     147.064   KSQ        .002   NVARSQ      4.   RSQSQ        .846
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CONTENTS OF CASE NUMBER 21

SEQNUM	21.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.999	RL	12.127
K	0	NVAR	2.	MSE	.002	MSELS	.002	MSERATIO	1.000
CROSS1	12.115	CROSS2	0	CROSS3	1.998	CROSS4	0	CROSS5	24.254
CROSS6	0	RLSQ	147.064	KSQ	0	NVARSQ	4.	RSQSQ	.998

CONTENTS OF CASE NUMBER 22

SEQNUM	22.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.999	RL	12.127
K	.005	NVAR	2.	MSE	.002	MSELS	.002	MSERATIO	1.000
CROSS1	12.115	CROSS2	.005	CROSS3	1.998	CROSS4	.061	CROSS5	24.254
CROSS6	.010	RLSQ	147.064	KSQ	.000	NVARSQ	4.	RSQSQ	.998

CONTENTS OF CASE NUMBER 23

SEQNUM	23.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.999	RL	12.127
K	.030	NVAR	2.	MSE	.001	MSELS	.002	MSERATIO	2.000
CROSS1	12.115	CROSS2	.030	CROSS3	1.998	CROSS4	.364	CROSS5	24.254
CROSS6	.060	RLSQ	147.064	KSQ	.001	NVARSQ	4.	RSQSQ	.998

CONTENTS OF CASE NUMBER 24

SEQNUM	24.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.999	RL	12.127
K	.045	NVAR	2.	MSE	.002	MSELS	.002	MSERATIO	1.000
CROSS1	12.115	CROSS2	.045	CROSS3	1.998	CROSS4	.546	CROSS5	24.254
CROSS6	.090	RLSQ	147.064	KSQ	.002	NVARSQ	4.	RSQSQ	.998

CONTENTS OF CASE NUMBER 25

SEQNUM	25.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.752	RL	120.029
K	0	NVAR	2.	MSE	9.194	MSELS	9.194	MSERATIO	1.000
CROSS1	90.262	CROSS2	0	CROSS3	1.504	CROSS4	0	CROSS5	240.058
CROSS6	0	RLSQ	14406.961	KSQ	0	NVARSQ	4.	RSQSQ	.566

CONTENTS OF CASE NUMBER 26

SEQNUM	26.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.752	RL	120.029
K	.015	NVAR	2.	MSE	.582	MSELS	9.194	MSERATIO	15.797
CROSS1	90.262	CROSS2	.011	CROSS3	1.504	CROSS4	1.800	CROSS5	240.058
CROSS6	.030	RLSQ	14406.961	KSQ	.000	NVARSQ	4.	RSQSQ	.566

CONTENTS OF CASE NUMBER 27

SEQNUM	27.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.752	RL	120.029
K	.025	NVAR	2.	MSE	.283	MSELS	9.194	MSERATIO	32.468
CROSS1	90.262	CROSS2	.019	CROSS3	1.504	CROSS4	3.001	CROSS5	240.058
CROSS6	.050	RLSQ	14406.961	KSQ	.001	NVARSQ	4.	RSQSQ	.566

CONTENTS OF CASE NUMBER 28

SEQNUM	28.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.752	RL	120.029
K	.040	NVAR	2.	MSE	.143	MSELS	9.194	MSERATIO	64.294
CROSS1	90.262	CROSS2	.030	CROSS3	1.504	CROSS4	4.801	CROSS5	240.058
CROSS6	.080	RLSQ	14406.961	KSQ	.002	NVARSQ	4.	RSQSQ	.566

CONTENTS OF CASE NUMBER 29

SEQNUM	29.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.922	RL	120.029
K	0	NVAR	2.	MSE	2.298	MSELS	2.298	MSERATIO	1.000
CROSS1	110.667	CROSS2	0	CROSS3	1.844	CROSS4	0	CROSS5	240.058
CROSS6	0	RLSQ	14406.961	KSQ	0	NVARSQ	4.	RSQSQ	.850

CONTENTS OF CASE NUMBER 30

SEQNUM	30.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.922	RL	120.029
K	.015	NVAR	2.	MSE	.146	MSELS	2.298	MSERATIO	15.740
CROSS1	110.667	CROSS2	.014	CROSS3	1.844	CROSS4	1.800	CROSS5	240.058
CROSS6	.030	RLSQ	14406.961	KSQ	.000	NVARSQ	4.	RSQSQ	.850

CONTENTS OF CASE NUMBER 31

SEQNUM	31.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.922	RL	120.029
K	.030	NVAR	2.	MSE	.055	MSELS	2.298	MSERATIO	41.782
CROSS1	110.667	CROSS2	.028	CROSS3	1.844	CROSS4	3.601	CROSS5	240.058
CROSS6	.060	RLSQ	14406.961	KSQ	.001	NVARSQ	4.	RSQSQ	.850

CONTENTS OF CASE NUMBER 32

SEQNUM	32.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.922	RL	120.029
K	.040	NVAR	2.	MSE	.036	MSELS	2.298	MSERATIO	63.833
CROSS1	110.667	CROSS2	.037	CROSS3	1.844	CROSS4	4.801	CROSS5	240.058
CROSS6	.080	RLSQ	14406.961	KSQ	.002	NVARSQ	4.	RSQSQ	.850

CONTENTS OF CASE NUMBER 33

SEQNUM	33.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.997	RL	120.029
K	0	NVAR	2.	MSE	.092	MSELS	.092	MSERATIO	1.000
CROSS1	119.669	CROSS2	0	CROSS3	1.994	CROSS4	0	CROSS5	240.058
CROSS6	0	RLSQ	14406.961	KSQ	0	NVARSQ	4.	RSQSQ	.994

CONTENTS OF CASE NUMBER 34

SEQNUM	34.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.997	RL	120.029
K	.015	NVAR	2.	MSE	.006	MSELS	.092	MSERATIO	15.333
CROSS1	119.669	CROSS2	.015	CROSS3	1.994	CROSS4	1.800	CROSS5	240.058
CROSS6	.030	RLSQ	14406.961	KSQ	.000	NVARSQ	4.	RSQSQ	.994

CONTENTS OF CASE NUMBER 35

SEQNUM	35.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.997	RL	120.029
K	.035	NVAR	2.	MSE	.002	MSELS	.092	MSERATIO	46.000
CROSS1	119.669	CROSS2	.035	CROSS3	1.994	CROSS4	4.201	CROSS5	240.058
CROSS6	.070	RLSQ	14406.961	KSQ	.001	NVARSQ	4.	RSQSQ	.994

CONTENTS OF CASE NUMBER 36

SEQNUM	36.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.997	RL	120.029
K	.045	NVAR	2.	MSE	.002	MSELS	.092	MSERATIO	46.000
CROSS1	119.669	CROSS2	.045	CROSS3	1.994	CROSS4	5.401	CROSS5	240.058
CROSS6	.090	RLSQ	14406.961	KSQ	.002	NVARSQ	4.	RSQSQ	.994

CONTENTS OF CASE NUMBER 37

SEQNUM	37.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.828	RL	1.836
K	0	NVAR	3.	MSE	.147	MSELS	.147	MSERATIO	1.000
CROSS1	1.520	CROSS2	0	CROSS3	2.484	CROSS4	0	CROSS5	5.508
CROSS6	0	RLSQ	3.371	KSQ	0	NVARSQ	9.	RSQSQ	.686

CONTENTS OF CASE NUMBER 38

SEQNUM	38.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.828	RL	1.836
K	.005	NVAR	3.	MSE	.143	MSELS	.147	MSERATIO	1.028
CROSS1	1.520	CROSS2	.004	CROSS3	2.484	CROSS4	.009	CROSS5	5.508
CROSS6	.015	RLSQ	3.371	KSQ	.000	NVARSQ	9.	RSQSQ	.686

CONTENTS OF CASE NUMBER 39

SEQNUM	39.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.828	RL	1.836
K	.015	NVAR	3.	MSE	.135	MSELS	.147	MSERATIO	1.089
CROSS1	1.520	CROSS2	.012	CROSS3	2.484	CROSS4	.028	CROSS5	5.508
CROSS6	.045	RLSQ	3.371	KSQ	.000	NVARSQ	9.	RSQSQ	.686

CONTENTS OF CASE NUMBER 40

SEQNUM	40.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.828	RL	1.836
K	.030	NVAR	3.	MSE	.125	MSELS	.147	MSERATIO	1.176
CROSS1	1.520	CROSS2	.025	CROSS3	2.484	CROSS4	.055	CROSS5	5.508
CROSS6	.090	RLSQ	3.371	KSQ	.001	NVARSQ	9.	RSQSQ	.686

CONTENTS OF CASE NUMBER 41

SEQNUM	41.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.917	RL	1.836
K	0	NVAR	3.	MSE	.062	MSELS	.062	MSERATIO	1.000
CROSS1	1.684	CROSS2	0	CROSS3	2.751	CROSS4	0	CROSS5	5.508
CROSS6	0	RLSQ	3.371	KSQ	0	NVARSQ	9.	RSQSQ	.841

CONTENTS OF CASE NUMBER 42

SEQNUM	42.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.917	RL	1.836
K	.005	NVAR	3.	MSE	.060	MSELS	.062	MSERATIO	1.033
CROSS1	1.684	CROSS2	.005	CROSS3	2.751	CROSS4	.009	CROSS5	5.508
CROSS6	.015	RLSQ	3.371	KSQ	.000	NVARSQ	9.	RSQSQ	.841

CONTENTS OF CASE NUMBER 43

SEQNUM	43.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.917	RL	1.836
K	.015	NVAR	3.	MSE	.057	MSELS	.062	MSERATIO	1.088
CROSS1	1.684	CROSS2	.014	CROSS3	2.751	CROSS4	.028	CROSS5	5.508
CROSS6	.045	RLSQ	3.371	KSQ	.000	NVARSQ	9.	RSQSQ	.841

CONTENTS OF CASE NUMBER 44

SEQNUM	44.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.917	RL	1.836
K	.030	NVAR	3.	MSE	.053	MSELS	.062	MSERATIO	1.170
CROSS1	1.684	CROSS2	.028	CROSS3	2.751	CROSS4	.055	CROSS5	5.508
CROSS6	.090	RLSQ	3.371	KSQ	.001	NVARSQ	9.	RSQSQ	.841

CONTENTS OF CASE NUMBER 45

SEQNUM	45.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	1.836
K	0	NVAR	3.	MSE	.001	MSELS	.001	MSERATIO	1.000
CROSS1	1.832	CROSS2	0	CROSS3	2.994	CROSS4	0	CROSS5	5.508
CROSS6	0	RLSQ	3.371	KSQ	0	NVARSQ	9.	RSQSQ	.996

CONTENTS OF CASE NUMBER 46

SEQNUM	46.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	1.836
K	.005	NVAR	3.	MSE	.001	MSELS	.001	MSERATIO	1.000
CROSS1	1.832	CROSS2	.005	CROSS3	2.994	CROSS4	.009	CROSS5	5.508
CROSS6	.015	RLSQ	3.371	KSQ	.000	NVARSQ	9.	RSQSQ	.996

CONTENTS OF CASE NUMBER 47

SEQNUM	47.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	1.836
K	.025	NVAR	3.	MSE	.001	MSELS	.001	MSERATIO	1.000
CROSS1	1.832	CROSS2	.025	CROSS3	2.994	CROSS4	.046	CROSS5	5.508
CROSS6	.075	RLSQ	3.371	KSQ	.001	NVARSQ	9.	RSQSQ	.996

CONTENTS OF CASE NUMBER 48

SEQNUM	48.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	1.836
K	.040	NVAR	3.	MSE	.002	MSELS	.001	MSERATIO	.500
CROSS1	1.832	CROSS2	.040	CROSS3	2.994	CROSS4	.073	CROSS5	5.508
CROSS6	.120	RLSQ	3.371	KSQ	.002	NVARSQ	9.	RSQSQ	.996

CONTENTS OF CASE NUMBER 49

SEQNUM	49.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.753	RL	7.680
K	0	NVAR	3.	MSE	1.305	MSELS	1.305	MSERATIO	1.000
CROSS1	5.783	CROSS2	0	CROSS3	2.259	CROSS4	0	CROSS5	23.040
CROSS6	0	RLSQ	58.982	KSQ	0	NVARSQ	9.	RSQSQ	.567

CONTENTS OF CASE NUMBER 50

SEQNUM	50.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.753	RL	7.680
K	.005	NVAR	3.	MSE	1.115	MSELS	1.305	MSERATIO	1.170
CROSS1	5.783	CROSS2	.004	CROSS3	2.259	CROSS4	.038	CROSS5	23.040
CROSS6	.015	RLSQ	58.982	KSQ	.000	NVARSQ	9.	RSQSQ	.567

CONTENTS OF CASE NUMBER 51

SEQNUM	51.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.753	RL	7.680
K	.020	NVAR	3.	MSE	.777	MSELS	1.305	MSERATIO	1.680
CROSS1	5.783	CROSS2	.015	CROSS3	2.259	CROSS4	.154	CROSS5	23.040
CROSS6	.060	RLSQ	58.982	KSQ	.000	NVARSQ	9.	RSQSQ	.567

CONTENTS OF CASE NUMBER 52

SEQNUM	52.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.753	RL	7.680
K	.035	NVAR	3.	MSE	.595	MSELS	1.305	MSERATIO	2.193
CROSS1	5.783	CROSS2	.026	CROSS3	2.259	CROSS4	.269	CROSS5	23.040
CROSS6	.105	RLSQ	58.982	KSQ	.001	NVARSQ	9.	RSQSQ	.567

CONTENTS OF CASE NUMBER 53

SEQNUM	53.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.920	RL	7.680
K	0	NVAR	3.	MSE	.326	MSELS	.326	MSERATIO	1.000
CROSS1	7.066	CROSS2	0	CROSS3	2.760	CROSS4	0	CROSS5	23.040
CROSS6	0	RLSQ	58.982	KSQ	0	NVARSQ	9.	RSQSQ	.846

CONTENTS OF CASE NUMBER 54

SEQNUM	54.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.920	RL	7.680
K	.005	NVAR	3.	MSE	.279	MSELS	.326	MSERATIO	1.168
CROSS1	7.066	CROSS2	.005	CROSS3	2.760	CROSS4	.038	CROSS5	23.040
CROSS6	.015	RLSQ	58.982	KSQ	.000	NVARSQ	9.	RSQSQ	.846

CONTENTS OF CASE NUMBER 55

SEQNUM	55.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.920	RL	7.680
K	.025	NVAR	3.	MSE	.177	MSELS	.326	MSERATIO	1.842
CROSS1	7.066	CROSS2	.023	CROSS3	2.760	CROSS4	.192	CROSS5	23.040
CROSS6	.075	RLSQ	58.982	KSQ	.001	NVARSQ	9.	RSQSQ	.846

CONTENTS OF CASE NUMBER 56

SEQNUM	56.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.920	RL	7.680
K	.045	NVAR	3.	MSE	.129	MSELS	.326	MSERATIO	2.527
CROSS1	7.066	CROSS2	.041	CROSS3	2.760	CROSS4	.346	CROSS5	23.040
CROSS6	.135	RLSQ	58.982	KSQ	.002	NVARSQ	9.	RSQSQ	.846

CONTENTS OF CASE NUMBER 57

SEQNUM	57.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	7.680
K	0	NVAR	3.	MSE	.006	MSELS	.006	MSERATIO	1.000
CROSS1	7.665	CROSS2	0	CROSS3	2.994	CROSS4	0	CROSS5	23.040
CROSS6	0	RLSQ	58.982	KSQ	0	NVARSQ	9.	RSQSQ	.996

CONTENTS OF CASE NUMBER 58

SEQNUM	58.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	7.680
K	.005	NVAR	3.	MSE	.005	MSELS	.006	MSERATIO	1.200
CROSS1	7.665	CROSS2	.005	CROSS3	2.994	CROSS4	.038	CROSS5	23.040
CROSS6	.015	RLSQ	58.982	KSQ	.000	NVARSQ	9.	RSQSQ	.996

CONTENTS OF CASE NUMBER 59

SEQNUM	59.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	7.680
K	.020	NVAR	3.	MSE	.004	MSELS	.006	MSERATIO	1.500
CROSS1	7.665	CROSS2	.020	CROSS3	2.994	CROSS4	.154	CROSS5	23.040
CROSS6	.060	RLSQ	58.982	KSQ	.000	NVARSQ	9.	RSQSQ	.996

CONTENTS OF CASE NUMBER 60

SEQNUM	60.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	7.680
K	.040	NVAR	3.	MSE	.003	MSELS	.006	MSERATIO	2.000
CROSS1	7.665	CROSS2	.040	CROSS3	2.994	CROSS4	.307	CROSS5	23.040
CROSS6	.120	RLSQ	58.982	KSQ	.002	NVARSQ	9.	RSQSQ	.996

CONTENTS OF CASE NUMBER 61

SEQNUM	61.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.756	RL	50.926
K	0	NVAR	3.	MSE	8.512	MSELS	8.512	MSERATIO	1.000
CROSS1	38.500	CROSS2	0	CROSS3	2.268	CROSS4	0	CROSS5	152.778
CROSS6	0	RLSQ	2593.457	KSQ	0	NVARSQ	9.	RSQSQ	.572

CONTENTS OF CASE NUMBER 62

SEQNUM	62.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.756	RL	50.926
K	.015	NVAR	3.	MSE	1.169	MSELS	8.512	MSERATIO	7.281
CROSS1	38.500	CROSS2	.011	CROSS3	2.268	CROSS4	.764	CROSS5	152.778
CROSS6	.045	RLSQ	2593.457	KSQ	.000	NVARSQ	9.	RSQSQ	.572

CONTENTS OF CASE NUMBER 63

SEQNUM	63.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.756	RL	50.926
K	.025	NVAR	3.	MSE	.695	MSELS	8.512	MSERATIO	12.247
CROSS1	38.500	CROSS2	.019	CROSS3	2.268	CROSS4	1.273	CROSS5	152.778
CROSS6	.075	RLSQ	2593.457	KSQ	.001	NVARSQ	9.	RSQSQ	.572

CONTENTS OF CASE NUMBER 64

SEQNUM	64.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.756	RL	50.926
K	.035	NVAR	3.	MSE	.496	MSELS	8.512	MSERATIO	17.161
CROSS1	38.500	CROSS2	.026	CROSS3	2.268	CROSS4	1.782	CROSS5	152.778
CROSS6	.105	RLSQ	2593.457	KSQ	.001	NVARSQ	9.	RSQSQ	.572

CONTENTS OF CASE NUMBER 65

SEQNUM	65.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.921	RL	50.926
K	0	NVAR	3.	MSE	2.128	MSELS	2.128	MSERATIO	1.000
CROSS1	46.903	CROSS2	0	CROSS3	2.763	CROSS4	0	CROSS5	152.778
CROSS6	0	RLSQ	2593.457	KSQ	0	NVARSQ	9.	RSQSQ	.848

CONTENTS OF CASE NUMBER 66

SEQNUM	66.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.921	RL	50.926
K	.015	NVAR	3.	MSE	.273	MSELS	2.128	MSERATIO	7.763
CROSS1	46.903	CROSS2	.014	CROSS3	2.763	CROSS4	.764	CROSS5	152.778
CROSS6	.045	RLSQ	2593.457	KSQ	.000	NVARSQ	9.	RSQSQ	.848

CONTENTS OF CASE NUMBER 67

SEQNUM	67.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.921	RL	50.926
K	.030	NVAR	3.	MSE	.145	MSELS	2.128	MSERATIO	14.676
CROSS1	46.903	CROSS2	.028	CROSS3	2.763	CROSS4	1.528	CROSS5	152.778
CROSS6	.090	RLSQ	2593.457	KSQ	.001	NVARSQ	9.	RSQSQ	.848

CONTENTS OF CASE NUMBER 68

SEQNUM	68.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.921	RL	50.926
K	.045	NVAR	3.	MSE	.098	MSELS	2.128	MSERATIO	21.714
CROSS1	46.903	CROSS2	.041	CROSS3	2.763	CROSS4	2.292	CROSS5	152.778
CROSS6	.135	RLSQ	2593.457	KSQ	.002	NVARSQ	9.	RSQSQ	.848

CONTENTS OF CASE NUMBER 69

SEQNUM	69.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	50.926
K	0	NVAR	3.	MSE	.038	MSELS	.038	MSERATIO	1.000
CROSS1	50.824	CROSS2	0	CROSS3	2.994	CROSS4	0	CROSS5	152.778
CROSS6	0	RLSQ	2593.457	KSQ	0	NVARSQ	9.	RSQSQ	.996

CONTENTS OF CASE NUMBER 70

SEQNUM	70.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	50.926
K	.015	NVAR	3.	MSE	.006	MSELS	.038	MSERATIO	6.333
CROSS1	50.824	CROSS2	.015	CROSS3	2.994	CROSS4	.764	CROSS5	152.778
CROSS6	.045	RLSQ	2593.457	KSQ	.000	NVARSQ	9.	RSQSQ	.996

CONTENTS OF CASE NUMBER 71

SEQNUM	71.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	50.926
K	.025	NVAR	3.	MSE	.004	MSELS	.038	MSERATIO	9.500
CROSS1	50.824	CROSS2	.025	CROSS3	2.994	CROSS4	1.273	CROSS5	152.778
CROSS6	.075	RLSQ	2593.457	KSQ	.001	NVARSQ	9.	RSQSQ	.996

CONTENTS OF CASE NUMBER 72

SEQNUM	72.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	50.926
K	.040	NVAR	3.	MSE	.003	MSELS	.038	MSERATIO	12.667
CROSS1	50.824	CROSS2	.040	CROSS3	2.994	CROSS4	2.037	CROSS5	152.778
CROSS6	.120	RLSQ	2593.457	KSQ	.002	NVARSQ	9.	RSQSQ	.996

CONTENTS OF CASE NUMBER 73

SEQNUM	73.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.759	RL	1.528
K	0	NVAR	4.	MSE	.146	MSELS	.146	MSERATIO	1.000
CROSS1	1.160	CROSS2	0	CROSS3	3.036	CROSS4	0	CROSS5	6.112
CROSS6	0	RLSQ	2.335	KSQ	0	NVARSQ	16.	RSQSQ	.576

CONTENTS OF CASE NUMBER 74

SEQNUM	74.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.759	RL	1.528
K	.005	NVAR	4.	MSE	.144	MSELS	.146	MSERATIO	1.014
CROSS1	1.160	CROSS2	.004	CROSS3	3.036	CROSS4	.008	CROSS5	6.112
CROSS6	.020	RLSQ	2.335	KSQ	.000	NVARSQ	16.	RSQSQ	.576

CONTENTS OF CASE NUMBER 75

SEQNUM	75.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.759	RL	1.528
K	.015	NVAR	4.	MSE	.139	MSELS	.146	MSERATIO	1.050
CROSS1	1.160	CROSS2	.011	CROSS3	3.036	CROSS4	.023	CROSS5	6.112
CROSS6	.060	RLSQ	2.335	KSQ	.000	NVARSQ	16.	RSQSQ	.576

CONTENTS OF CASE NUMBER 76

SEQNUM	76.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.759	RL	1.528
K	.040	NVAR	4.	MSE	.130	MSELS	.146	MSERATIO	1.123
CROSS1	1.160	CROSS2	.030	CROSS3	3.036	CROSS4	.061	CROSS5	6.112
CROSS6	.160	RLSQ	2.335	KSQ	.002	NVARSQ	16.	RSQSQ	.576

CONTENTS OF CASE NUMBER 77

SEQNUM	77.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.922	RL	1.528
K	0	NVAR	4.	MSE	.037	MSELS	.037	MSERATIO	1.000
CROSS1	1.409	CROSS2	0	CROSS3	3.688	CROSS4	0	CROSS5	6.112
CROSS6	0	RLSQ	2.335	KSQ	0	NVARSQ	16.	RSQSQ	.850

CONTENTS OF CASE NUMBER 78

SEQNUM	78.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.922	RL	1.528
K	.005	NVAR	4.	MSE	.036	MSELS	.037	MSERATIO	1.028
CROSS1	1.409	CROSS2	.005	CROSS3	3.688	CROSS4	.008	CROSS5	6.112
CROSS6	.020	RLSQ	2.335	KSQ	.000	NVARSQ	16.	RSQSQ	.850

CONTENTS OF CASE NUMBER 79

SEQNUM	79.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.922	RL	1.528
K	.015	NVAR	4.	MSE	.035	MSELS	.037	MSERATIO	1.057
CROSS1	1.409	CROSS2	.014	CROSS3	3.688	CROSS4	.023	CROSS5	6.112
CROSS6	.060	RLSQ	2.335	KSQ	.000	NVARSQ	16.	RSQSQ	.850

CONTENTS OF CASE NUMBER 80

SEQNUM	80.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.922	RL	1.528
K	.035	NVAR	4.	MSE	.036	MSELS	.037	MSERATIO	1.028
CROSS1	1.409	CROSS2	.032	CROSS3	3.688	CROSS4	.053	CROSS5	6.112
CROSS6	.140	RLSQ	2.335	KSQ	.001	NVARSQ	16.	RSQSQ	.850

CONTENTS OF CASE NUMBER 81

SEQNUM	81.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.997	RL	1.528
K	0	NVAR	4.	MSE	.001	MSELS	.001	MSERATIO	1.000
CROSS1	1.523	CROSS2	0	CROSS3	3.908	CROSS4	0	CROSS5	6.112
CROSS6	0	RLSQ	2.335	KSQ	0	NVARSQ	16.	RSQSQ	.994

CONTENTS OF CASE NUMBER 82

SEQNUM	82.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.997	RL	1.528
K	.005	NVAR	4.	MSE	.001	MSELS	.001	MSERATIO	1.000
CROSS1	1.523	CROSS2	.005	CROSS3	3.988	CROSS4	.008	CROSS5	6.112
CROSS6	.020	RLSQ	2.335	KSQ	.000	NVARSQ	16.	RSQSQ	.994

CONTENTS OF CASE NUMBER 83

SEQNUM	83.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.997	RL	1.528
K	.015	NVAR	4.	MSE	.002	MSELS	.001	MSERATIO	.500
CROSS1	1.523	CROSS2	.015	CROSS3	3.988	CROSS4	.023	CROSS5	6.112
CROSS6	.060	RLSQ	2.335	KSQ	.000	NVARSQ	16.	RSQSQ	.994

CONTENTS OF CASE NUMBER 84

SEQNUM	84.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.997	RL	1.528
K	.035	NVAR	4.	MSE	.004	MSELS	.001	MSERATIO	.250
CROSS1	1.523	CROSS2	.035	CROSS3	3.988	CROSS4	.053	CROSS5	6.112
CROSS6	.140	RLSQ	2.335	KSQ	.001	NVARSQ	16.	RSQSQ	.994

CONTENTS OF CASE NUMBER 85

SEQNUM	85.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.619	RL	90.523
K	0	NVAR	4.	MSE	56.262	MSELS	56.262	MSERATIO	1.000
CROSS1	56.034	CROSS2	0	CROSS3	2.476	CROSS4	0	CROSS5	362.092
CROSS6	0	RLSQ	8194.414	KSQ	0	NVARSQ	16.	RSQSQ	.383

CONTENTS OF CASE NUMBER 86

SEQNUM	86.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.619	RL	90.523
K	.015	NVAR	4.	MSE	4.570	MSELS	56.262	MSERATIO	12.311
CROSS1	56.034	CROSS2	.009	CROSS3	2.476	CROSS4	1.358	CROSS5	362.092
CROSS6	.060	RLSQ	8194.414	KSQ	.000	NVARSQ	16.	RSQSQ	.383

CONTENTS OF CASE NUMBER 87

SEQNUM	87.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.619	RL	90.523
K	.030	NVAR	4.	MSE	2.594	MSELS	56.262	MSERATIO	21.689
CROSS1	56.034	CROSS2	.019	CROSS3	2.476	CROSS4	2.716	CROSS5	362.092
CROSS6	.120	RLSQ	8194.414	KSQ	.001	NVARSQ	16.	RSQSQ	.383

CONTENTS OF CASE NUMBER 88

SEQNUM	88.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.619	RL	90.523
K	.040	NVAR	4.	MSE	2.057	MSELS	56.262	MSERATIO	27.351
CROSS1	56.034	CROSS2	.025	CROSS3	2.476	CROSS4	3.621	CROSS5	362.092
CROSS6	.160	RLSQ	8194.414	KSQ	.002	NVARSQ	16.	RSQSQ	.383

CONTENTS OF CASE NUMBER 89

SEQNUM	89.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.850	RL	90.523
K	0	NVAR	4.	MSE	14.325	MSELS	14.325	MSERATIO	1.000
CROSS1	76.945	CROSS2	0	CROSS3	3.400	CROSS4	0	CROSS5	362.092
CROSS6	0	RLSQ	8194.414	KSQ	0	NVARSQ	16.	RSQSQ	.723

CONTENTS OF CASE NUMBER 90

SEQNUM	90.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.850	RL	90.523
K	.015	NVAR	4.	MSE	1.141	MSELS	14.325	MSERATIO	12.555
CROSS1	76.945	CROSS2	.013	CROSS3	3.400	CROSS4	1.358	CROSS5	362.092
CROSS6	.060	RLSQ	8194.414	KSQ	.000	NVARSQ	16.	RSQSQ	.723

CONTENTS OF CASE NUMBER 91

SEQNUM	91.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.850	RL	90.523
K	.025	NVAR	4.	MSE	.751	MSELS	14.325	MSERATIO	19.075
CROSS1	76.945	CROSS2	.021	CROSS3	3.400	CROSS4	2.263	CROSS5	362.092
CROSS6	.100	RLSQ	8194.414	KSQ	.001	NVARSQ	16.	RSQSQ	.723

CONTENTS OF CASE NUMBER 92

SEQNUM	92.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.850	RL	90.523
K	.035	NVAR	4.	MSE	.573	MSELS	14.325	MSERATIO	25.000
CROSS1	76.945	CROSS2	.030	CROSS3	3.400	CROSS4	3.168	CROSS5	362.092
CROSS6	.140	RLSQ	8194.414	KSQ	.001	NVARSQ	16.	RSQSQ	.723

CONTENTS OF CASE NUMBER 93

SEQNUM	93.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.999	RL	90.523
K	0	NVAR	4.	MSE	.064	MSELS	.064	MSERATIO	1.000
CROSS1	90.432	CROSS2	0	CROSS3	3.996	CROSS4	0	CROSS5	362.092
CROSS6	0	RLSQ	8194.414	KSQ	0	NVARSQ	16.	RSQSQ	.998

CONTENTS OF CASE NUMBER 94

SEQNUM	94.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.999	RL	90.523
K	.015	NVAR	4.	MSE	.006	MSELS	.064	MSERATIO	10.667
CROSS1	90.432	CROSS2	.015	CROSS3	3.996	CROSS4	1.358	CROSS5	362.092
CROSS6	.060	RLSQ	8194.414	KSQ	.000	NVARSQ	16.	RSQSQ	.998

CONTENTS OF CASE NUMBER 95

SEQNUM	95.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.999	RL	90.523
K	.025	NVAR	4.	MSE	.004	MSELS	.064	MSERATIO	16.000
CROSS1	90.432	CROSS2	.025	CROSS3	3.996	CROSS4	2.263	CROSS5	362.092
CROSS6	.100	RLSQ	8194.414	KSQ	.001	NVARSQ	16.	RSQSQ	.998

CONTENTS OF CASE NUMBER 96

SEQNUM	96.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.999	RL	90.523
K	.040	NVAR	4.	MSE	.003	MSELS	.064	MSERATIO	21.333
CROSS1	90.432	CROSS2	.040	CROSS3	3.996	CROSS4	3.621	CROSS5	362.092
CROSS6	.160	RLSQ	8194.414	KSQ	.002	NVARSQ	16.	RSQSQ	.998

CONTENTS OF CASE NUMBER 97

SEQNUM	97.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.687	RL	8.355
K	0	NVAR	4.	MSE	3.047	MSELS	3.047	MSERATIO	1.000
CROSS1	5.740	CROSS2	0	CROSS3	2.748	CROSS4	0	CROSS5	33.420
CROSS6	0	RLSQ	69.806	KSQ	0	NVARSQ	16.	RSQSQ	.472

CONTENTS OF CASE NUMBER 98

SEQNUM	98.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.687	RL	8.355
K	.015	NVAR	4.	MSE	1.838	MSELS	3.047	MSERATIO	1.658
CROSS1	5.740	CROSS2	.010	CROSS3	2.748	CROSS4	.125	CROSS5	33.420
CROSS6	.060	RLSQ	69.806	KSQ	.000	NVARSQ	16.	RSQSQ	.472

CONTENTS OF CASE NUMBER 99

SEQNUM	99.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.687	RL	8.355
K	.030	NVAR	4.	MSE	1.330	MSELS	3.047	MSERATIO	2.291
CROSS1	5.740	CROSS2	.021	CROSS3	2.748	CROSS4	.251	CROSS5	33.420
CROSS6	.120	RLSQ	69.806	KSQ	.001	NVARSQ	16.	RSQSQ	.472

CONTENTS OF CASE NUMBER 100

SEQNUM	100.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.687	RL	8.355
K	.040	NVAR	4.	MSE	1.176	MSELS	3.047	MSERATIO	2.706
CROSS1	5.740	CROSS2	.027	CROSS3	2.748	CROSS4	.334	CROSS5	33.420
CROSS6	.160	RLSQ	69.806	KSQ	.002	NVARSQ	16.	RSQSQ	.472

CONTENTS OF CASE NUMBER 101

SEQNUM	101.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.822	RL	8.355
K	0	NVAR	4.	MSE	1.354	MSELS	1.354	MSERATIO	1.000
CROSS1	6.868	CROSS2	0	CROSS3	3.288	CROSS4	0	CROSS5	33.420
CROSS6	0	RLSQ	69.806	KSQ	0	NVARSQ	16.	RSQSQ	.676

CONTENTS OF CASE NUMBER 102

SEQNUM	102.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.822	RL	8.355
K	.005	NVAR	4.	MSE	1.105	MSELS	1.354	MSERATIO	1.225
CROSS1	6.868	CROSS2	.004	CROSS3	3.288	CROSS4	.042	CROSS5	33.420
CROSS6	.020	RLSQ	69.806	KSQ	.000	NVARSQ	16.	RSQSQ	.676

CONTENTS OF CASE NUMBER 103

SEQNUM	103.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.822	RL	8.355
K	.020	NVAR	4.	MSE	.725	MSELS	1.354	MSERATIO	1.868
CROSS1	6.868	CROSS2	.016	CROSS3	3.288	CROSS4	.167	CROSS5	33.420
CROSS6	.080	RLSQ	69.806	KSQ	.000	NVARSQ	16.	RSQSQ	.676

CONTENTS OF CASE NUMBER 104

SEQNUM	104.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.822	RL	8.355
K	.040	NVAR	4.	MSE	.501	MSELS	1.354	MSERATIO	2.703
CROSS1	6.868	CROSS2	.033	CROSS3	3.288	CROSS4	.334	CROSS5	33.420
CROSS6	.160	RLSQ	69.806	KSQ	.002	NVARSQ	16.	RSQSQ	.676

CONTENTS OF CASE NUMBER 105

SEQNUM	105.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	8.355
K	0	NVAR	4.	MSE	.014	MSELS	.014	MSERATIO	1.000
CROSS1	8.338	CROSS2	0	CROSS3	3.992	CROSS4	0	CROSS5	33.420
CROSS6	0	RLSQ	69.806	KSQ	0	NVARSQ	16.	RSQSQ	.996

CONTENTS OF CASE NUMBER 106

SEQNUM	106.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	8.355
K	.005	NVAR	4.	MSE	.011	MSELS	.014	MSERATIO	1.273
CROSS1	8.338	CROSS2	.005	CROSS3	3.992	CROSS4	.042	CROSS5	33.420
CROSS6	.020	RLSQ	69.806	KSQ	.000	NVARSQ	16.	RSQSQ	.996

CONTENTS OF CASE NUMBER 107

SEQNUM	107.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	8.355
K	.025	NVAR	4.	MSE	.007	MSELS	.014	MSERATIO	2.000
CROSS1	8.338	CROSS2	.025	CROSS3	3.992	CROSS4	.209	CROSS5	33.420
CROSS6	.100	RLSQ	69.806	KSQ	.001	NVARSQ	16.	RSQSQ	.996

CONTENTS OF CASE NUMBER 108

SEQNUM	108.	SUBFILE	REGANAL	CASWGT	1.0000	RSQ	.998	RL	8.355
K	.045	NVAR	4.	MSE	.006	MSELS	.014	MSERATIO	2.333
CROSS1	8.338	CROSS2	.045	CROSS3	3.992	CROSS4	.376	CROSS5	33.420
CROSS6	.180	RLSQ	69.806	KSQ	.002	NVARSQ	16.	RSQSQ	.996

VITA

James Richard Makin was born 30 May 1950 in Washington, D.C. He graduated from high school in Bel Air, Maryland in 1968 and attended Drexel University from which he received the degree of Bachelor of Science in Commerce and Engineering in June 1973. Upon graduation, he received a commission in the United States Army through the ROTC program and was called to active duty in July 1973. He completed the Infantry Officer Basic Course and airborne training prior to being assigned to the 9th Infantry Division, Ft. Lewis, Washington in February 1974. During this assignment he performed duties as platoon leader, executive officer, company commander, maintenance officer and others. He completed the Ordnance Officer Advanced Course in December 1979 and subsequently served as tank/automotive maintenance officer of the 2d Infantry Division, Camp Casey, Korea until entering the School of Engineering, Air Force Institute of Technology, in June 1980.

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