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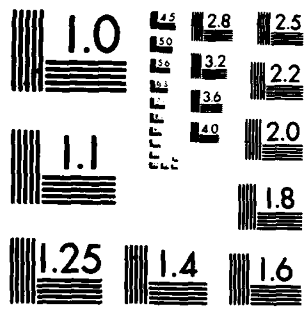

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Mean gravity anomalies are predicted to provide the average value of gravity within 1°x1° surface areas or surface areas of other sizes. Prediction methods include conventional, statistical, deterministic, and geophysical. The statistical methods appear to give the best results when adequate computer resources are available. The deterministic methods give good results with minimum time and computer requirements. The geophysical methods give usable predictions when little or no measured data is available. The most accurate conventional method is graphical interpolation, but this method is too time-consuming to be recommended for general use.

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MEAN GRAVITY ANOMALY PREDICTION TECHNIQUES WITH A COMPARATIVE ANALYSIS OF THE ACCURACY AND ECONOMY OF SELECTED METHODS

Luman E. Wilcox, Ph.D.
Sandral Jones, MSc

Defense Mapping Agency Aerospace Center
St. Louis AFS, MO 63118

BIOGRAPHICAL SKETCHES

Luman E. Wilcox received his BA degree in Geology from Wesleyan University, his M.S. degree in geodetic science and Ph.D. degree in Geology and Geophysics from the University of Hawaii. He has been employed by the Defense Mapping Agency Aerospace Center since 1954 in various supervisory and staff positions. For the past five years, he has also held the position of Affiliate Professor of Civil Engineering at Washington University. His research interests include various aspects of gravity and geodesy. He is a past Secretary of the Geodesy Section of the American Geophysical Union, and is a member of the American Congress on Surveying and Mapping and the American Society of Photogrammetry. Currently he is President of an International Association of Geodesy (IAG) Special Study Group on gravity predictions, a member of several IAG Working Groups on gravity, and Secretary of the US Interagency Gravity Standards Committee.

Sandral D. Jones received her BS degree in Mathematics from Lincoln University and her MS degree in Civil Engineering from Washington University. She has been employed by the Defense Mapping Agency Aerospace Center since 1962. She is a member of American Society of Photogrammetry.

ABSTRACT

Mean gravity anomalies are predicted to provide the average value of gravity within 1° x 1° surface areas or surface areas of other sizes. Prediction methods include conventional, statistical, deterministic, and geophysical. The statistical methods appear to give the best results when adequate computer resources are available. The deterministic methods give good results with minimum time and computer requirements. The geophysical methods give usable predictions when little or no measured data is available. The most accurate conventional method is graphical interpolation, but this method is too time-consuming to be recommended for general use.

DEFINITION AND SCOPE

Gravity prediction is defined as any process that enables the estimation of a gravity anomaly value (1) for a site where gravity has not been measured or (2) that represents the average value of gravity within a given surface area.

At the Defense Mapping Agency Aerospace Center (DMAAC), gravity prediction is used almost exclusively for estimation of mean gravity anomalies, that is, for estimation of gravity anomaly values that represent the average values of gravity within a given surface area.

Mean gravity anomalies are predicted for 1' x 1', 5' x 5', 15' x 15', 30' x 30', 1° x 1°, and 5° x 5° surface areas. In order to limit the scope of this paper, we will confine the discussion to 1° x 1° mean gravity anomaly prediction. However, most of the methods discussed are also suitable for prediction of values that represent surface areas of other sizes, and many can be used for prediction of gravity at specific sites.

The discussions of gravity prediction in this paper are limited to land areas. Different results may be obtained when these methods are applied in oceanic areas.

VALUE OF 1° x 1° MEAN GRAVITY ANOMALIES

A continuous gravity anomaly field covering the earth's surface is essential for solving many of the problems of physical geodesy. For example, with a continuous gravity anomaly field, geoid undulations and deflections of the vertical can be determined at the earth's surface, and gravity disturbance components can be determined in space above the earth's surface.

The classical integral formulas of physical geodesy, such as those of Stokes and Vening Meinesz, must be evaluated by summation. In these formulas, small finite compartments replace infinitesimal surface elements in the summation. These compartments are obtained by subdividing the surface of the earth by concentric circles and their radii, or by grid lines along the parallels and meridians to form "square" blocks, for example, 1° x 1° blocks. The "square" blocks are summed for a major portion of the earth's surface. The 1° x 1° blocks are a most important element in these summations.

In addition, a worldwide set of 1° x 1° mean gravity anomalies can be used to represent the potential of the earth. Alternatively, the 1° x 1° mean gravity anomalies can be summed in a suitable algorithm to obtain a set of geopotential harmonic coefficients that represent the earth's potential.

The 1° x 1° mean gravity anomaly values needed for the above applications all must be obtained by some suitable gravity prediction methods.

GRAVITY PREDICTION METHODS-INTRODUCTION

In general, a 1° x 1° mean gravity anomaly is predicted as a function of gravity that is measured at discrete points on the surface of the earth. If all of the measured gravity data lies within the 1° x 1° area for which the prediction is required, the prediction process is essentially interpolation or, in some cases, merely data averaging. If most of the measured data lies outside of the 1° x 1° area in question, the prediction becomes essentially an extrapolation process. Both interpolation and extrapolation, as well as data averaging, are included in many gravity prediction methods.

There is really only one way to obtain a rigorously correct 1° x1° mean gravity anomaly. To do this, it is necessary to apply a data averaging integral of the form (Heiskanen and Moritz, 1967):

$$\overline{\Delta g} = \frac{1}{ab} \int_{x=0}^{a} \int_{y=0}^{b} \Delta g(x,y)\, dx\, dy \qquad (1)$$

where

$\overline{\Delta g}$ = 1° x 1° mean gravity anomaly

a,b = rectangular dimensions of the 1° x 1° surface are for which $\overline{\Delta g}$ is required.

Δg = gravity anomaly values computed from measured gravity at discrete points (x,y) within the 1° x 1° area.

If the Δg are Bouguer gravity anomalies, the $\overline{\Delta g}$ is a 1° x 1° mean Bouguer anomaly. If the Δg are free-air gravity anomalies, the $\overline{\Delta g}$ is a 1° x 1° mean free-air gravity anomaly. Either anomaly form can be used successfully in (1).

In order to apply the integral (1), the measured gravity values must be available for all points (x,y), differentially small distances apart, throughout the 1° x 1° area. Of course, this condition is never fulfilled in practice, and the integral (1) can never be applied as written.

The automated gravity data holdings of the DoD Gravity Library (DoDGL) exceed 9.5 million measurements. If these measurements were to be divided equally among the 64,800 1° x 1° areas of the world, there would be slightly more than 145 measurements per 1° x 1° area. However, the 9.5 million measurements actually are very unequally divided among the 1° x 1° areas. The number of stations per 1° x 1° area varies from a maximum of 19,400 to a minimum of zero. Nevertheless, a 1° x 1° land area containing more than 100 measurements must be considered unusual (except within the U.S. and parts of Western Europe). In fact, most land areas contain a few tens of stations or less. Moreover, the measurements within a 1° x 1° area often are poorly distributed. Thus, the problem of gravity prediction usually is to obtain the best estimate of the mean anomaly based upon a limited amount of unequally distributed measured data.

GRAVITY PREDICTION METHODS-DESCRIPTION

Gravity prediction methods that have been used generally fall into four categories: conventional, statistical, geophysical, and deterministic. The conventional and geophysical methods are useful only for mean anomaly predictions. The statistical and deterministic methods can be used either for mean anomaly prediction or for prediction of gravity at sites where gravity has not been measured.

Graphical Interpolation

The graphical interpolation method, one of the oldest of the conventional methods, requires that a gravity anomaly map be constructed from the measured data. The gravity anomaly map may be constructed either by automated plotter or by hand.

A gravity anomaly map drawn by automated plotter is considerably more economical than such a map drawn by hand. The machine contouring algorithm can be based either on linear interpolation between data points or on least squares prediction. In areas containing ample amounts of well distributed measured data, machine contouring certainly is just as reliable as hand contouring. However, machine contours may be drawn inaccurately over areas containing relatively small amounts of measured data and/or poorly distributed data.

Gravity anomaly contour maps drawn by hand often provide a good representation -for data averaging purposes- of the gravity field even when measured data is relatively sparse and/or poorly distributed. In addition, such maps frequently can be improved by using data from topographic and geologic maps. When such supplementary information is used, the placement of gravity contours is influenced by considering topographic and geologic structure effects on gravity anomaly variations. Similar techniques may be used to enhance gravity anomaly contour maps drawn by automatic plotters. Hand contouring, although effective, is a very time-consuming and manpower intensive process, and the reliability of the resulting map depends upon the experience of the compiler. The contour interval selected and the amount of smoothing used in drawing the contours also influence the quality of the results.

Once the gravity anomaly map has been drawn, the integration (1) can be performed graphically with reference to the map. One method of graphical integration is to average contours by eye within, say, 5' x 5' blocks. Then the 144 5' x 5' averages within the 1° x 1° block are themselves averaged to obtain the final 1° x 1° mean gravity anomaly prediction. An alternate method is to read center values for each 5' x 5' area from the contour map, and then average the 144 center readings. Tests (undocumented) at DMAAC have shown essentially no difference in results between the two averaging methods. Consequently, the latter method is preferable because of its simplicity and because it requires less subjective judgment.

Predictions in continental areas always are made using Bouguer gravity anomalies because this anomaly form is nearly independent of correlation with short wavelength elevation variations and, therefore, well suited for the purposes of interpolation and extrapolation. The mean Bouguer gravity anomaly prediction must be converted to a mean free-air gravity anomaly which is the appropriate anomaly form for geodetic applications. The mean free-air gravity anomaly is computed using

$$\overline{\Delta g_F} = \overline{\Delta g_B} + 0.1119\bar{h} \tag{2}$$

where

$\overline{\Delta g_F}$ = 1° x 1° mean free-air gravity anomaly

$\overline{\Delta g_B}$ = 1° x 1° mean Bouguer gravity anomaly

$\bar{h}$ = mean elevation of the 1° x 1° area in meters

<u>Least Squares Prediction</u>

There are a limited number of gravity measurements scattered through a typical 1° x 1° area, rather than the infinite number required by the integral (1). A reasonable approach to this situation is to approximate the true mean gravity anomaly by some linear combination of the measured values using

$$\overline{\Delta g} = \sum_{i=1}^{n} \alpha_i \Delta g_i \tag{3}$$

where

$\overline{\Delta g}$ = 1° x 1° mean gravity anomaly

Δg_i = gravity anomaly value computed from measured data for the ith point

α_i = arbitrary coefficient to be determined.

The coefficients, α_i, which depend upon the relative positions of the gravity measurements and mean anomaly value, can be obtained in a number of different ways. Perhaps one of the best choices is to determine the coefficients in such a way that the standard error of prediction is minimized. Such a determination leads to the least squares prediction (or least squares estimation) method.

Least squares prediction has been the most widely used statistical prediction method. The formulation for least squares prediction was developed by Moritz and later modified by Rapp for practical application on digital computers. Details can be found in Heiskanen and Moritz (1967), and Rapp (1964).

Although least squares prediction can be used to predict mean anomalies directly, most have used a form that predicts the value of gravity at a discrete point. In the latter case, Moritz shows that, for least standard error of prediction, the coefficients α_i of (3) assume the form

$$\alpha_{P_k} = \sum_{i=1}^{n} C_{ik}^{(-1)} \, C_{P_i} \qquad (4)$$

Insertion of (4) into (3) yields the prediction equation for the point, P

$$\Delta g_P = \sum_{k=1}^{n} \alpha_{P_k} \, \Delta g_k = \sum_{i=1}^{n} \sum_{k=1}^{n} C_{ik}^{(-1)} \, C_{Pi} \, \Delta g_k \qquad (5)$$

where

Δg_P = predicted gravity anomaly at a point, P

$C_{ik}^{(-1)}$ = inverse of a matrix that expresses the autocovariance between all gravity anomalies in the vicinity of the point, P.

C_{Pi} = row matrix that expresses the autocovariance between the gravity anomaly at the point P and the gravity anomalies used in the prediction.

Δg_k = column matrix of the measured gravity anomalies used in the prediction.

Both interpolation and extrapolation may be involved in the application of (5).

The standard error of the prediction is given by

$$M_p^2 = C_o - \sum_{i=1}^{n} \sum_{k=1}^{n} C_{ik}^{(-1)} C_{p_i} C_{p_k} \tag{6}$$

where

M_p^2 = square of the standard error of prediction at the point P

C_o = variance of the gravity anomalies in the vicinity of the point P^o

It can be seen from equation (5) that least squares prediction does estimate the value of the gravity anomaly at a point. To obtain a 1° x 1° mean gravity anomaly, equation (5) is used to predict gravity anomaly values at the center of 5' x 5' blocks. Then the 144 5' x 5' predictions within the 1° x 1° block are averaged to obtain the final 1° x 1° prediction. It should be noted that the averaging of predictions at 5' x 5' block centers is identical to the procedure generally used to obtain the final 1° x 1° prediction in the graphical interpolation method.

Least squares prediction depends upon the availability of a reasonably representative gravity anomaly autocovariance function. The autocovariance function represents the average rate of change of the gravity anomaly field as a function of distance within a limited region, say, 5° x 5°. It is important that a local rather than a global covariance function be used for gravity anomaly prediction purposes. This will insure compatibility between the gravity anomaly covariance coefficients used for prediction and the gravity field characteristics of the prediction area.

The local covariance function for an area often can be represented by an analytical expression, first suggested by Hirvonen (1962), of the form

$$C(s) = \frac{C_o}{1+(s/d)^2} \tag{7}$$

where

C(s) = local autocovariance of the gravity anomalies for the distance, s

C_o = local variance of the gravity anomalies

d = arbitrary fixed distance that enables equation (7) to fit the local covariance function.

Alternatively, the local covariance function can be expressed by a polynomial fit to the data, as Rapp (1964) mentions, using a formula of the form

$$C(s) = C_o + C_1 s + C_2 s^2 + \ldots + C_n s^n \tag{8}$$

where

$C_1 \ldots C_n$ = coefficients that enable (8) to represent the local covariance function

In order to be used in the automated program (Rapp, 1964) generally applied for least squares prediction, the point gravity anomalies used for the prediction must be separated by a minimum of 0.4 minutes of arc.

This is done to prevent singularity in matrix inversion. Known systematic influences are eliminated by deletion or scaling.

Least squares prediction is among the most widely used of the gravity anomaly prediction methods because it is completely automated and yields results which are theoretically "best." However, when predicting gravity anomalies in areas containing large data sets, a large amount of computer memory is required.

Least Squares Collocation

Least squares collocation is a natural extension of least squares prediction. Like least squares prediction, least squares collocation depends upon the statistical properties of the gravity anomaly signal. In its purest form, however, least squares collocation combines linear prediction with a consideration of the geodetic boundary value problem.

Least squares collocation is of particular interest in prediction problems because it enables inclusion of various types of data, not just gravity anomalies, in the prediction algorithm. In theory, at least, a prediction can be based on not only measured gravity data but also topographic data, geologic and geophysical data, and the variances (accuracies) of these data.

However, because of the difficulty in specifying firm functional relationships between gravity anomalies and other types of data, or even obtaining enough numerical geological and geophysical data for use in a prediction, most investigators have been content to add only the variances of gravity anomaly data to the least squares prediction algorithm. Thus, in (5), the covariance matrix C_{ik} becomes $C_{ik} + D_{ik}$, where D_{ik} is a diagonal matrix of gravity anomaly error variances. Then the gravity anomaly prediction procedure follows exactly that described in the previous section for least squares prediction.

Least squares collocation is definitely the prediction method of the future, and several researchers now are active in development of the method for practical application.

Simple Average

The simplest way of applying the basic prediction equation (3) is to set all of the coefficients α_i equal to 1/n. This gives the equation for the conventional simple average method

$$\overline{\Delta g} = \frac{1}{n} \sum_{i=1}^{n} \Delta g_i \qquad (9)$$

Equation (9) must be applied with respect to Bouguer gravity anomalies. If free-air anomalies are used, a small correction term must be added to account for the dependence of free-air anomalies upon local elevation changes.

Sparse data and/or uneven distribution of the measured data causes the mean gravity anomaly predicted by the simple average method to be strongly biased in areas of high frequency gravity variations. With a dense, even distribution of data, strongly negative or positive areas within the 1° x 1° blocks should not unduly affect the result.

Modified Average

The modified average method has been used at DMAAC for rapid computer estimation of 1° x 1° mean gravity anomalies on a world-wide basis. In this method, a simple average mean gravity anomaly is computed using (9) for each 10' x 10' area that contains measured gravity data within a 1° x 1° block. These 10' x 10' mean anomalies are then averaged in two steps to obtain the final 1° x 1° mean gravity anomaly prediction.

The appropriate equations are

$$\overline{\Delta g}_{10} = \frac{1}{n} \sum_{i=1}^{n} \Delta g_i \tag{10}$$

$$\overline{\Delta g}_{30} = \frac{1}{L} \sum_{i=1}^{L} \Delta g_{10} \tag{11}$$

$$\overline{\Delta g} = \frac{1}{J} \sum_{i=1}^{J} \Delta g_{30} \tag{12}$$

where

$\overline{\Delta g}$ = predicted 1° x 1° mean gravity anomaly
Δg_{30} = 30' x 30' mean gravity anomaly
Δg_{10} = 10' x 10' mean gravity anomaly
n = number of measured gravity anomalies in the 10' x 10' block
L = number of non-empty 10' x 10' blocks in the 30' x 30' block
J = number of non-empty 30' x 30' blocks in the 1° x 1° block

Again, the prediction must be done in terms of Bouguer gravity anomalies for the equations to be applicable.

The modified average method is a significant improvement over the simple average. For example, if a 1° x 1° block has dense data in one corner, but little data elsewhere, the simple average may be seriously biased. The modified average method, on the other hand, gives equal weight to the contribution of each 10' x 10' block within the 1° x 1° area, thereby reducing biasing due to poorly distributed data.

Satisfactory results are often obtained from the modified average method provided that there are not too many empty 10' x 10' blocks. Indeed unsatisfactory results can be obtained when data is too sparse. The chief value of the method is its ease and simplicity of formulation which enables a rapid first approximation of 1° x 1° mean values.

Deterministic Methods

There are a large number of deterministic methods that might be used for gravity prediction. In all such methods, a smooth bivariate function is constructed that takes on the prescribed values at the data points. Most of these methods amount to fitting a smooth polynomial surface to the gravity anomaly data. The prediction is given by the value of the surface at, say, the center of a 5' x 5' block.

The two deterministic methods discussed here, the bicubic spline and multi-quadratic analysis, are the ones that appear to be best suited for gravity anomaly prediction.

Bicubic Spline

The bicubic spline approximation is basically a finite element method used to evaluate geodetic integral formulas. Functions to be integrated, such as gravity anomalies, are represented by a piecewise cubic two-dimensional polynomial. The polynomial is fitted together mathematically to form a continuous transition from one element to the next. This method combines the analytic handling of polynomials with a smooth representation by the resulting function.

For the purpose of digital computation, the earth's surface is divided into blocks, say 5° x 5°, bounded by latitudes and longitudes. Gravity anomaly data within the area of interest are used to compute the polynomial coefficients which determine the bicubic spline.

One good program (Michlik, 1977) uses a 5' integration grid and a minimum curvature routine for selection of data points having a significant change. The integration grid data is merged with the cubic spline (in both x and y) to predict gravity anomalies at the center of each 5' x 5' block. Then the 144 5' x 5' blocks within each 1° x 1° area are averaged to obtain the final 1° x 1° mean gravity anomaly prediction.

The spline is represented by (Davis and Kontis, 1970)

$$S(x,y) = \sum_{m=0}^{3} \sum_{n=0}^{3} M_{mn}(x-x_{j-1})^{m} (y-y_{j-1})^{n} \qquad (13)$$

where

M_{mn} = unknown for the two-dimensional bicubic surface.

$y_j = x_j$ = subintervals

The bicubic spline handles data distribution irregularities well since the interpolation surface passes through all the data points. The results obtained from this method usually are very good unless the data is very sparse, and the method is quite economical from the standpoint of computer resources required. The major drawback is that no prediction error function is available.

Multiquadric Analysis

Multiquadric Analysis is a least squares prediction method which applies the hypothesis that smooth or arbitrary mathematical surfaces can be approximated to any desired degree of exactness by the summation of quadric forms. However, the multiquadric functions are based upon geometric and physical consideration rather than stochastic processes (Hardy, 1978).

There are two forms of multiquadric functions that can be used for gravity anomaly prediction, non-harmonic and harmonic. The non-harmonic form fits a hyperboloid to the data points. The mathematics of the harmonic form depend upon the harmonic function reciprocal distance. The harmonic form can represent physical properties such as mass, and may be interpreted in terms of the point mass summation concept.

The general equations are:

Non-harmonic form

$$Z_i = \sum_{j=1}^{n} \left[(x_i - x_j)^2 + (y_i - y_j)^2 \right]^{-y} [C_j] \qquad (14)$$

$i=1, 2 \ldots m, m > n$

$$[C_i] = \sum_{j=1}^{n} \left[(x_i - x_j)^2 + (y_i - y_j)^2 \right]^{-1/2} Z_j \qquad (15)$$

Harmonic form

$$\Delta g_i = k \sum_{j=1}^{n} \left[\frac{R - r \cos \phi_{ij}}{\ell^3_{ij}} - \frac{2}{\ell_{ij} R} \right] [A_j] \qquad (16)$$

$$[A_i] = \frac{1}{k} \sum_{j=1}^{n} \left[\frac{R - r \cos \phi_{ij}}{\ell^3_{ij}} - \frac{2}{\ell_{ij} R} \right] \Delta g_j \qquad (17)$$

$$\phi_{ij} = \cos^{-1} [\cos \theta_i \cos \theta_j + \sin \theta_i \sin \theta_j \cos (\lambda_i - \lambda_j)] \qquad (18)$$

$$\ell_{ij} = \left(R^2 + r^2 - 2Rr \cos \phi_{ij} \right)^{1/2} \qquad (19)$$

where

k = universal gravitational constant
R = radius of the outer sphere
r = radius of the inner sphere
x_j, y_j = coordinates of the nodal points
x_i, y_i = coordinates of the data points
C_j, A_j = coefficients to be determined
θ = co-latitude
λ = longitude

The coefficients are computed from all known gravity anomaly values within an interpolation area so that one set of coefficients applies for the whole area. The derived coefficients then are used to predict gravity anomaly values at 5' x 5' intervals that can be averaged to obtain the final 1° x 1° mean gravity anomaly prediction.

A constant radius (r) for a so-called inner sphere on which the point masses lie is required for computation of the coefficients as well as the predicted point values. A table of recommended r values for n point mass anomalies is given by Hardy (1978).

Multiquadric analysis has been recommended as the most economical and accurate prediction method by Hein and Lenze (1979) and Franke (1979). However, it should be noted that these investigators were predicting elevations rather than gravity anomalies.

Geophysical Prediction

In areas containing very little measured gravity data, extrapolation based upon correlations between gravity anomaly variations and the corresponding variations in geological, geophysical, and topographical parameters can be developed. Thus,

$$\delta \overline{\Delta g} = f(\delta \overline{h}, \delta \overline{S}) \tag{20}$$

where

$\delta \overline{\Delta g}$ = variations in the 1° x 1° gravity anomaly.

$f(\delta \overline{h}, \delta \overline{S})$ = some function of topographic and structural variation, respectively.

If, for example, the changes in the regional portion of the 1° x 1° mean gravity anomalies are constant with respect to the corresponding changes in regional topography, which is the case within many regions of homogenious geological structure, then

$$\frac{\delta \overline{\Delta g}}{\delta \overline{h}} = \beta \tag{21}$$

where β is a constant.

Integration of (21) gives the function

$$\overline{\Delta g} = \beta \overline{h} + \alpha \tag{22}$$

where α is also a constant.

Equation (22) is a form of the so-called basic prediction that is used in one variety of geophysical prediction. A number of corrections are added to (22) to obtain the complete prediction formula.

$$\overline{\Delta g} = (\beta \overline{h} + \alpha) + f_1(S) + f_2(S) + f_3(\overline{h}) \tag{23}$$

In (23), $f_1(S)$ corrects for long period gravity variation that violate the constancy assumption inherent in (21). The corrections $f_2(S)$ and $f_3(\overline{h})$ add the gravitational effects of short period structural and topographic changes, respectively, that affect each 1° x 1° area in a different way, and, therefore, cannot be modeled by the basic predictor (22).

Details of this and other types of geophysical prediction methods may be found in Wilcox (1974).

The geophysical prediction method provides useable estimates of 1° x 1° mean gravity anomalies in areas where other types of prediction methods fail due to inadequate amounts of measured data being available. However, the principles of geophysical prediction potentially are valuable to improve predictions made by other methods even when adequate data is available. The most promising method for incorporation of geophysical, geological, and topographic parameters in gravity prediction is least squares collocation.

COMPARATIVE ANALYSIS OF SELECTED PREDICTION METHODS

A 5° x 5° area was selected in the United States to test five different prediction methods. The criteria used for selecting the area were (1) a good sample of elevation ranges and (2) an abundance of well distributed, dense measured gravity anomaly data. The area found that met these criteria is bounded by 35°N-40°N and 115°W-120°W.

The five methods selected for testing include (1) the modified average (MA), (2) graphical interpolation (GI), (3) cubic spline (SP), (4) simple average (SA), and (5) least squares prediction using the algorithms developed by Rapp (RP). We had originally intended to test multiquadric analysis fully, but due to problems arising in refining part of the basic formulation coupled with time constraints it was necessary to omit this method from the test.

The test was done in two parts. In part 1, some 33,000 measured gravity anomalies having a relatively even distribution were used in each prediction method to predict 1° x 1° mean Bouguer gravity anomalies. Where required by the method (e.g. cubic spline), Bouguer point anomalies were first predicted at the centers of a 5' x 5' integration grid within each 1° x 1° area, and these were averaged to obtain the final 1° x 1° mean Bouguer anomaly prediction.

In part 2, the data was thinned at random to simulate a sparse (moderate to extreme), poorly distributed data field. A total of 6,600 measured gravity anomalies were used. The mean Bouguer gravity anomalies were predicted as in part 1.

Using the predicted 1° x 1° mean Bouguer gravity anomalies and 1° x 1° mean elevation values, 25 1° x 1° mean free-air gravity anomalies were obtained using equation (2) for each data set: full (part 1) and sparse (part 2).

Least squares prediction for the full data set (part 1) was chosen as the standard for statistical comparison. The 1° x 1° mean free-air gravity anomalies, which were predicted by the various methods in both part 1 and part 2, were compared with those predicted by least squares prediction in part 1.

Mean free-air gravity anomalies predicted by each method for all 1° x 1° areas within the selected 5° x 5° area are tabulated in Table 1 for part 1 (full density) and Table 2 for part 2 (sparse density). The mean difference ($\bar{x}$), standard deviation (σ), and maximum difference between predictions made by each method and Rapp full density predictions (used as the standard) are given in Table 3.

From Table 3, it can be seen that least squares prediction (Rapp), graphical interpolation, and the cubic spline agree satisfactorily in both the full and sparse areas. There is no appreciable difference in the mean free-air gravity anomalies predicted by these three methods in part 1 (full distribution), and the differences in part 2 are relatively small. Thus any of these three methods might have been selected as the standard for comparison. It should be noted that, in both parts 1 and 2, graphical interpolation compares most favorably with least squares prediction.

The modified average method compares closely with least squares prediction in part 1 but not in the sparse case (part 2). Worst results in part 2 were for two 1° x 1° areas having exceptionally poor distribution and extremely sparse data. The simple average does not give good results in either case, full or sparse.

Since $\bar{x}$ (Table 3) is uniformly small, there does not appear to be a serious systematic error component in prediction for any method.

All prediction computations were made in a UNIVAC 1100 series computing system. A time study of the prediction methods tested were made (Table 4) based upon the following criteria: (1) preparation time (data tape preparation and computer deck set up), (2) total computer time (CPU + I/O) necessary for obtaining results, (3) other (time necessary for obtaining results other than by computer) - graphical interpolation was the only method having time requirements in this category, and (4) total time - sum of (1), (2) and (3).

The prediction method requiring the least amount of total time was judged to be the most economical. The bicubic spline proved to be the most economical in both the full and sparse areas. The other prediction methods ranked in the following order, (1) modified average/simple average, (2) least squares prediction (Rapp), and (3) graphical interpolation.

It is unfortunate that multiquadric analysis could not be included in this test. Other investigators (Hein and Lenze, 1979 and Franke, 1979) report that multiquadric analysis is both very accurate and very economical.

Based on our own test and those of others, we recommend the following:

(1) Where adequate computer resources and time are available, the most advantageous and accurate prediction method is most probably least squares prediction or its close cousin, least squares collocation.

(2) Where computer capacity or time is a constraint, either of the two deterministic methods - bicubic spline or multiquadric analysis, should give good results.

(3) Graphical interpolation gives good results - and is certainly one of the most accurate methods even when data is sparse or poorly distributed - but cannot be recommended for general use because it is the most time consuming, by far, of all methods tested.

(4) The modified average method should be used with caution. It is an acceptable method to obtain a rapid first approximation, but larger prediction errors may occur in some instances.

(5) The simple average method should never be used.

Table 1

Predicted Mean Free-Air Anomalies
Part 1 (Full Density)

40°N

MA	4.2805*	-12.2089	4.4102	13.2902	12.2522
GI	4.4597	-12.9616	4.3776	13.8333	12.3807
SP	3.7975	-12.5089	5.4142	13.4632	12.6642
SA	5.4805	-13.1089	4.2102	12.5902	11.5528
RP	3.9200	-13.0600	3.9300	13.5800	12.8100
MA	33.9871	- .7109	8.1029	10.7806	- 5.3134
GI	32.9580	- 1.1269	8.2730	10.8584	- 6.2669
SP	32.8531	- .8549	8.1469	10.8048	- 5.8544
SA	27.6871	- 2.2109	9.6029	12.7806	- 4.9134
RP	33.0300	- .9800	8.2500	10.9500	- 6.0900
MA	22.2060	33.8316	- 7.1673	- 1.9487	- 7.2170
GI	20.7261	34.2795	- 6.7499	- 1.6327	- 6.2753
SP	20.9750	34.1456	- 6.9753	- 1.1927	- 6.3070
SA	32.4060	31.0316	- 8.5673	- .4487	- 7.4170
RP	20.7600	34.3000	- 6.5200	- 1.6400	- 6.4200
MA	-35.1653	47.4328	-12.7870	-21.2781	13.0389
GI	-35.5764	46.8307	-13.4363	-22.4559	13.4485
SP	-35.7333	46.6088	-13.0860	-22.0861	13.4829
SA	-39.2653	42.4328	-17.3870	-14.3781	15.1389
RP	-35.5100	46.9200	-13.1300	-21.4100	13.4400
MA	-42.7607	10.4637	-17.3472	-22.5675	2.0923
GI	-42.4919	10.1630	-16.7819	-23.4418	2.1972
SP	-42.7970	9.9907	-16.9043	-23.1914	1.9853
SA	-46.1607	21.3637	-23.3472	-19.8675	- .5077
RP	-42.4300	10.1300	-16.9100	-23.2300	2.1100

35°N

120°W 115°W

MA = Modified Average
GI = Graphical Interpolation
SP = Spline
SA = Simple Average
RP = Rapp

*MGAL

Table 2

Predicted Mean Free-Air Anomalies
Part 2 (Sparse Density)

40°N

MA	3.8756*	-10.8977	9.7163	17.8529	13.9890
GI	4.0395	-11.8158	7.1011	16.7485	12.0591
SP	4.0425	-11.7919	7.3122	20.6552	14.6592
SA	4.9692	-15.2477	8.2114	17.9613	14.1267
RP	3.9500	-11.9500	7.3100	16.6600	14.1500
MA	34.8429	- 1.2623	7.2511	7.8838	- 4.0134
GI	36.3538	3.1648	6.1723	12.4487	- .1870
SP	34.2551	.2720	5.9579	12.0946	- 1.1784
SA	32.1871	2.6715	11.7380	5.0632	- 6.8134
RP	34.9600	- .0100	6.0700	11.1400	1.3600
MA	21.6139	33.0059	-10.4673	1.0513	- 4.8118
GI	22.1643	36.0593	- 9.5799	3.5916	- 5.6321
SP	21.2360	34.2326	- 9.8313	5.1893	- 4.6010
SA	23.6145	29.5273	- 8.3673	14.1013	- 5.8434
RP	20.8400	34.8000	- 8.7900	2.8200	- 5.3200
MA	-36.1111	47.6720	-33.1160	-17.6604	16.0125
GI	-35.5056	47.0307	-19.6301	-19.8031	14.6663
SP	-33.9279	46.0418	-16.3920	-16.8341	14.3569
SA	-32.5754	39.9168	-45.0033	-18.9781	19.0337
RP	-34.9500	47.1800	-19.1600	-15.1300	14.0000
MA	-41.4611	14.6848	-28.7605	-17.7605	2.8341
GI	-41.7183	11.7366	-22.4715	-22.5474	5.1867
SP	-44.6513	12.5447	-25.8952	-22.7094	3.2273
SA	-36.1213	28.2094	-36.1972	-15.8875	7.3038
RP	-41.2200	11.2600	-22.5000	-21.9600	4.1200

35°N

120°W — 115°W

MA = Modified Average
GI = Graphical Interpolation
SP = Spline
SA = Simple Average
RP = Rapp

*MGAL

Table 3

Statistical Analysis

Prediction Method	Density of Data	$\bar{x}$ (mgal)	σ (mgal)	Maximum Difference
Modified Average	Full	.1788	.5528	1.45
	Sparse	-.0340	5.4674	19.99
Graphical	Full	-.0489	.2971	1.05
	Sparse	.9133	2.914	6.5
Spline	Full	.0016	.4062	1.48
	Sparse	.8598	3.3652	8.985
Simple Average	Full	.2753	4.4937	11.65
	Sparse	.4320	9.8200	31.87
Rapp*	Full	*	*	
	Sparse	.9152	2.967	6.28

*Used as the standard

Table 4

Prediction Methods Time Comparison

Method	Density of Data	Preparation Time (hr, min)	Total Computer Time (CPU + I/O) (hr, min,sec)	Other	Total Time
MA/SA	Full	0 hr 25'	0 hr 5'26.0"		0 hr 30'26.0"
	Sparse	0 hr 15'	0 hr 2'10.0"		0 hr 17'10.0"
GI	Full	6 hr 0'	0 h 2' 5.0"	64 hr 2'*	70 hr 4' 5.0"
	Sparse	3 hr 0'	0 hr 1' 2.0"	24 hr 59'*	28 hr 1' 2.0"
SP	Full	0 hr 35'	0 hr 2'10.780"		0 hr 37'10.78"
	Sparse	0 hr 35'	0 hr 1'56.6"		0 hr 36'56.5"
RP	Full	1 hr 35'	8 hr 20'11.987"		9 hr 55'11.978"
	Sparse	1 hr 20'	3 hr 13'38.002"		4 hr 33'38.002"

*Time used to predict 5' x 5' center values per 1° square.

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