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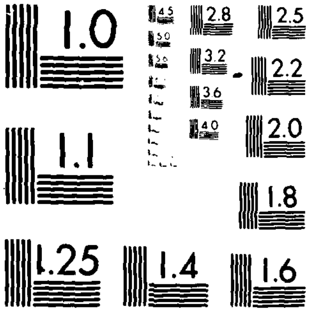

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INFINITE EXCESSIVE AND INVARIANT MEASURES

by

Michael I. Taksar

Technical Report No. 334
May 1981

A REPORT OF THE
CENTER FOR RESEARCH ON ORGANIZATIONAL EFFICIENCY
STANFORD UNIVERSITY

Contract ONR-N00014-79-C-0685, United States Office of Naval Research

THE ECONOMICS SERIES

INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES
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1. Formulation of Results

1.1. In the paper [9] the following problem was considered. Given a contraction semigroup T_t on a Borel space D and an excessive measure ν, when is it possible to find another contraction semigroup $\tilde{T}_t$ such that $\tilde{T}_t \geq T_t$ and ν is invariant with respect to $\tilde{T}_t$? The most restrictive condition under which this problem was solved is the finiteness of the excessive measure ν. This condition excludes such an interesting case as the semigroup T_t generated by the transition function of Wiener's process and the Lebesque measure ν. In the present paper we extend the results of [9] to all quasi-finite null-excessive measures ν.

Definition. Let T_t be a semigroup. A measure ν is called null-excessive with respect to T_t if for each $\Gamma \subset D$, subject to $\nu(\Gamma) < \infty$

$$\nu T_t(\Gamma) \downarrow 0 \quad \text{as} \quad t \to \infty \quad .$$

An excessive measure ν is called quasi-finite with respect to T_t if for some $s > 0$ the difference between ν and νT_s is a finite measure.

The principal part of the proof of the main res[illegible] the same as that of [9]. We consider the transition function p which generate T_t, then we construct a stationary Markov process $(w(s),P)$ with the transition function p and the one-dimensional distribution ν. (Actually the process $w(\cdot)$ has random

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*This research was supported by the Office of Naval Research Grant ONR-N0014-7-C-0685 at the Center for Research on Organizational Efficiency, Stanford University.

birth and death times and the measure P is infinite.) We add a single point V to the space D and we look for a stationary Markov process $(x_t, \bar{P})$ with the state space $E = D \cup V$ such that

1.1.α. The birth time of x_t is equal to $-\infty$ and the death time of x_t is equal to $+\infty$.

1.1.β. The one-dimensional distribution of $\bar{P}$ is equal to ν.

1.1.γ. $p(t,x,\Gamma) = \bar{P}_x\{x_t \in \Gamma;\ x_s \bar{\in} V \text{ for all } s < t\}$.

A process $(x_t, \bar{P})$ satisfying 1.1.α-1.1.γ is called a covering process for $(w(s), P)$ (see [8] for a more detailed discussion). If the measure ν is infinite then so has to be $\bar{P}$, and we cannot apply the results of [8] for the construction of $(x_t, \bar{P})$. In order to extend the results of [9] to infinite measures ν we have to develop the whole theory anew. Accordingly, all the definitions and notations of [7] and [8] will be used without explicit mentioning.

In the second part of this section we give precise formulations of the main results and give the conditions under which they are proved. In Section 2 we prove the existence of $(0,\Pi)$-generated random set M for any measure Π which is the Levi measure of an increasing process with independent increments. (In [7] such sets were constructed only for Π having the first moment.) Here the most important tool is the theorem of B. Maisonneuve in [6], which enables us to find an invariant distribution for the "jump process" of the process with independent increments. Using this result, we prove the existence of a covering process for any stationary Markov process with a quasi-finite one-dimensional distribution

(Section 3). Section 4 is devoted to the construction of a semigroup $\tilde{T}_t$ with respect to which ν is invariant.

In the case when the proof is similar to the one given in [7], [8], or [9], we shall only outline it, without going into details.

As always the same letter is used for a measure and the integral with respect to this measure. The word "function" stands for a nonnegative bounded measureable function.

1.2. Let D be a Borel space and T_t, $t \geq 0$, be a semigroup in the Banach space of bounded measurable functions on D (we say for brevity that T_t is a semigroup on D). The semigroup T_t is called a positivity preserving normal contraction semigroup if

1.2.A. For any $t \geq 0$ and each function $g \geq 0$

$$T_t g \geq 0 \quad .$$

1.2.B. For each $x \in D$

$$T_t 1(x) \leq 1 \quad , \quad \text{and} \quad \lim_{t \downarrow 0} T_t 1(x) = 1 \quad .$$

1.2.C. If $f(x_0) = 0$ then $T_0 f(x_0) = 0$.

A semigroup T_t is called continuous if

1.2.D. For each $x \in D$

$$T_t 1(x) \text{ is a continuous function of } t > 0 \quad .$$

A positivity preserving normal contraction semigroup is denoted S-semigroup. If S-semigroup T_t satisfies 1.2.E below, then T_t is called dying or SD-semigroup; if T_t satisfies 1.2.E', then T_t is called conservative or SC-semigroup.

1.2.E. For each $x \in D$

$$\lim_{t\to\infty} T_t 1(x) = 0 \quad .$$

1.2.E'. $T_t 1 \equiv 1$ for each $t > 0$.

(Note that 1.2.E' implies 1.2.D).

If T_t and $\tilde{T}_t$ are two semigroups on D and for each function g

$$T_t g \le \tilde{T}_t g \quad , \tag{1.2.1}$$

then we say that $\tilde{T}_t$ is larger than T_t, or $\tilde{T}_t$ is an enhancing of T_t.

We write $T_t = \tilde{T}_t$ a.e.μ if for any function g for μ-almost all x $T_t g(x) = \tilde{T}_t g(x)$.

In this paper we are going to prove the following theorems.

Theorem 1. Given a continuous SD-semigroup T_t and a quasi-finite null-excessive measure ν, one can find a SC-semigroup $\tilde{T}_t$ which is larger than T_t and for which ν is invariant.

Theorem 2. If T_t and ν satisfy the conditions of Theorem 1 and if in addition ν is an extreme excessive measure then $\tilde{T}_t$ is unique up to the measure ν.

2. Regenerative Sets with Infinite Underlying Measures

2.1. Let (Ω, F) be a measurable space and Q be a measure on F (not necessarily finite). A subset $M \subset T \times \Omega$ is called a random set (r.s.) if it is $\mathcal{B} \times \mathcal{F}$-measurable and $M(\omega)$ is nonempty for a.e. ω. (Here T is the real line $]-\infty,+\infty[$ and $\mathcal{B}$ is its Borel σ-field.) A r.s. M is called closed (closed from the right, perfect, discrete, etc.) if for a.e. ω, $M(\omega)$ is closed (closed from the right, perfect, discrete, etc.). Only closed random sets will be considered in the sequeal. We refer the reader to [7] for the definitions of the associated random process z_t, sets M_t, M^t, $\tilde{M}_t$, $\tilde{M}^t$, etc.; the definitions of regenerativity, translation invariancy, as well as the definitions of (α,Π)-processes, (α,Π)-generated set.

A r.s. M is said to have a σ-finite distribution (or M is a σf-set) if

2.1.A The process z_t has σ-finite one-dimensional distributions.

For example, consider any increasing process with independent increments with the Lebesque initial distribution (i.e. initial distribution uniform on T). The range of this process is a r.s. whose distribution is not σ-finite. Let us take now any σ-finite measure ν with support on $[0,1]$ and let Π be a unit measure concentrated in the point 1. If we consider the range of the $(0,\Pi)$-process with initial distribution ν, then this r.s. has a σ-finite distribution.

Any measure Π on $]0,\infty[$ subject to

$$(2.1.1) \qquad \int_0^\infty x \wedge 1 \Pi(dx) < \infty$$

may be considered as the Levi measure of an increasing process with independent increments (subordinator), and any subordinator has the Levi measure satisfying (2.1.1). The range of any subordinator is a right regenerative set; all translation invariant sets of such type with finite underlying distributions are described in [7]. These sets are in one-to-one correspondence with the ranges of all (α,Π)-processes with Π having the first moment. It is possible to perform a similar analysis for all r.r.t.i. σf-sets, but we restrict our attention only to the theorem of existence.

Theorem 2.1.1. For any $\alpha > 0$ and any measure Π on $]0,\infty[$ subject to (2.1.1) there exists a t.i. (α,Π)-generated σf-set M. The set M is left regenerative and moreover, $-M$ has the same distribution as M.

Let the complement of M be the union of disjoint open intervals $]\gamma,\delta[$. Then for any function f on $T \times T$

$$Q\{\sum_{\gamma} f(\gamma,\delta)\} = \int_{-\infty}^{\infty}\{\int_{0}^{\infty} f(s,s+y)\Pi(dy)\}ds \quad . \tag{2.1.2}$$

2.2 For simplicity of calculations we shall consider only the case of $\alpha = 0$. The modification of the proof for $\alpha > 0$ is trivial. Let y_t be a $(0,\Pi)$-process and Q_y be its transition probabilities. We denote by σ_ℓ the first hitting time of $]\ell,\infty[$ by y_t; and by $Y_\ell = (U_\ell,V_\ell) \equiv (y_{\sigma_\ell -},y_{\sigma_\ell})$ we denote the "jump" process of y_t (see [7] Section 2). V_t as well as Y_t is a Markov process. Let

$$q(s,x;\, t,\Gamma) = \begin{cases} 1_\Gamma(x) \ , & \text{if } x \geq t \\ Q_x\{V_t \in \Gamma\} \ , & \text{if } x < t \ , \quad \Gamma \subset T \end{cases}$$

be the transition function of the process V_t. Let $\Pi(x;-)$, $\Pi_x(-)$, λ_b, etc. be the measures and the kernels defined in Section 2 of [7]. Denote

$$R_t =]-\infty,t[\quad , \quad R^t =]t,\infty[$$

$$\mu_t(\Gamma) = \int_{-\infty}^{t} \Pi(x;\Gamma)dx \quad , \quad \Gamma \subset R^t$$

By the theorem of Maisonneuve (see [6], Th. (3.2)) the family μ_t is an entrance law with respect to q. Note that $\mu_t(\Gamma) = \mu_0(\Gamma - t)$. Consider the Markov process (v_t,Q) with the one-dimensional distributions ν_t and with the transition function q. (The measure Q is finite iff μ_o is a finite measure.) The existence of such a Markov process is proved in [5]. The same way as in Lemma 6.2 of [7], we can show that v_t is a stochastically continuous increasing process; hence there exists a right-continuous version of it. Consider the random set M which is the range of v_t (i.e. the closure of the set of values of v_t). We are going to prove that M is the set we are looking for.

Lemma 2.2.1. The set M is a translation invariant right-regenerative $(0,\Pi)$-generated set with the associated process z_t having the one-dimensional distributions

$$\nu_t(\Gamma) = \int_{-\infty}^{t} \Pi_x(\Gamma)dx \quad , \quad \Gamma \subset R_t \times R^t \quad . \tag{2.2.1}$$

Proof: Fix $s \in T$. Consider a $(0,\Pi)$-process $y_\cdot$ with initial distribution μ_s. Let $V_u = y_{\sigma_u}$. By the constraction of (v_t,Q) the process v_t, $t \geq s$ has the same finite-dimensional distributions as V_t, $t \geq s$. Both processes are right-continuous, therefore their ranges have equal distributions. But the range of $V_\cdot$ is equal to that of $y_\cdot$, and that proves that M is $(0,\Pi)$-generated (right-regenerativity is a consequence of this fact).

By the construction, the process $v_t - t$ is Markov with stationary transition function and stationary one-dimensional distributions (equal to μ_0). Hence M is a t.i. set. Any $(0,\Pi)$-generated set is thin; as a result, for the t.i. set M we have

$$(2.2.2) \qquad Q\{t \in M\} = 0 \quad .$$

Since M is $(0,\Pi)$-generated

$$Q\{z_t \in \Gamma | z_s^+\} = Q_{z_s^+}\{Y_t \in \Gamma\} \quad ,$$

$$\text{a.e. } Q \text{ on the set } \{z_s^+ < t\} \quad , \quad \Gamma \subset R_t \times R^t \quad .$$

To prove (2.2.1) we can consider only bounded sets Γ of the form $\Delta_1 \times \Delta_2$. By virtue of (2.2.2), $Q\{z_t = (t,t)\} = 0$; consequently we may take $\Delta_1 < t$ and $\Delta_2 > t$. Since Δ_1 is bounded there exists s such that $\Delta_1 > s$. The distribution of z_s^+ is equal to that of v_s, and we can write

$$(2.2.3) \qquad Q\{z_t \in \Gamma\} = \int_{R^s} \mu_s(dx) Q\{z_t \in \Gamma | z_s^+ = x\}$$

$$= \int_{R^s} \mu_s(dx) Q_x\{Y_t \in \Gamma\} \quad .$$

The last equality in (2.2.3) due to the fact that $\{z_s^+ < t\} = \{z_t^- > s\}$. By virtue of Lemma 2.1 of [7] the right hand side of (2.2.3) is equal to

$$(2.2.4) \qquad \int_{-\infty}^{s} dy \int_{s}^{t} \Pi(y; dx) \int_{\Delta_1} \lambda_x(dz)\Pi(z; \Delta_2) \quad .$$

Let $y_t^* = -y_t$ and let Q_b^*, λ_b^*, $\Pi^*(x; \Gamma)$, ν_t^*, etc. be defined as in Section 6 of [7]. Performing the same transformations as in Lemma 6.6 of [7], we get that (2.2.4) equals

$$(2.2.5) \qquad \int_{\Delta_2} dx \int_{\Delta_1} \Pi^*(x; dz) \int_{s}^{z} \lambda_z^*(dy)\Pi^*(y; R_s) \quad .$$

By virtue of Lemma 2.1 of [7]

$$\int_{s}^{z} \lambda_z^*(dy)\Pi^*(y; R_s) = Q_z^*\{y^*_{\sigma_s^*} < s\} = 1 \quad ,$$

here σ_s^* is the first hitting time of $]-\infty, s]$. Hence (2.2.5) is equal to (we use (6.11) of [7])

$$(2.2.6) \qquad \int_{\Delta_2} dx\Pi^*(x; \Delta_1) = \int_{\Delta_1} dx\Pi(x; \Delta_2) = \int_{-\infty}^{t} \Pi_x(\Delta_1 \times \Delta_2)dx \quad .$$

Corollary: The distribution of $-M$ is equal to that of M.

Proof: Since M is a $(0,\Pi)$-generated set, z_t is a Markov process with the transition function p given by (5.2) of [7]. As a result, ν_t is an entrance law with respect to p. Let ν_t^* be defined as in Lemma 6.5 of [7]. Formula (2.2.6) shows that $\nu_t^* = \nu_{-t}$. Repeating the proof of Lemma 6.6 in our case, we get that z_t has backward transition function p^*. Then we must argue in the same way as in Lemma 6.7 of [7].

Lemma 2.2.2. The set M satisfies (2.1.2).

Proof: In (2.1.2) we may consider only the functions f such that

$$f(x,y) = 0 \quad \text{if} \quad x \geq y \quad . \tag{2.2.7}$$

Put $R_{st} = \{(x,y)\colon\ x < s,\ y > t\}$, $f_{st} = f1_{R_{st}}$. For $\Lambda = r_1, r_2, \dots r_k$ set $R_\Lambda = R_{r_1 r_1} \cup R_{r_2 r_2} \cup \dots \cup R_{r_k r_k}$, $f_\Lambda = f1_{R_\Lambda}$. If $r_1, r_2, \dots r_k \dots$ is a sequence of all rational numbers and $\Lambda(n) = \{r_1, \dots r_n\}$, then $f_{\Lambda(n)} \uparrow f$ for any function f subject to (2.2.7). Trivial computations show that the function $f_{\Lambda(n)}$ is a linear combination of the functions f_{st}, $s < t$. Since both sides of (2.1.2) are stable under linear operations and monotone passage to the limit, we have to verify (2.1.2) only for the functions f_{st}, $s < t$.

$$\begin{aligned}
Q\Big\{\sum_\gamma f_{st}(\gamma,\delta)\Big\} &= Q\Big\{\sum_\gamma 1_{\gamma<s} f(\gamma,\delta) 1_{\delta>t}\Big\} \\
&= Q\{f_{st}(z_t)\} \\
&= \nu_t(f_{st}) \\
&= \int_{-\infty}^{t} \Pi_x(f_{st})\,dx \\
&= \int_{-\infty}^{\infty} \Pi_x(f_{st})\,dx \\
&= \int_{-\infty}^{\infty} dx \int_0^{\infty} f_{st}(x, x+y)\,\Pi(dy) \quad .
\end{aligned}$$

3. Stationary Markov Processes with Infinite Underlying Distributions and their Subprocesses.

3.1. Consider a (generalized) stationary Markov process $(x_t,\bar{P})$, that is a process satisfying the definition given in Section 1.2 of [8]. Assume that x_t is conservative, i.e. $\bar{P}\{\alpha \neq -\infty\} = \bar{P}\{\beta \neq +\infty\} = 0$. Suppose that the state space E of this process is divided into two sets D and V in such a way that

$$(3.1.1) \qquad M = \{t:\ x_t \in V\} \text{ is closed a.e. } \bar{P} .$$

We denote by $]\gamma,\delta[$ an element of the set of all open intervals contiguous to M. For each path $x_\cdot$ and each $]\gamma,\delta[$ we associate a trajectory w_δ^γ in D by the formula $w_\delta^\gamma(s) = x_s$, $\gamma < s < \delta$. The set of all trajectories in D with random birth time α and death time β is denoted by W. If M satisfies 1.2.α of [8] then it is possible to define a measure P on W in the following way (W is endowed with the Kolmogorov σ-field G).

$$(3.1.2) \qquad P\{A\} = \bar{P}\{\sum_\gamma 1_A(w_\delta^\gamma)\} .$$

The process $(w(s),P)$ is called a subprocess in D of the process $(x_t,\bar{P})$. Let ν, $\bar{p}$, $\bar{P}_x$ be respectively the one-dimensional distribution, the transition function and the transition probabilities of the process $(x_t,\bar{P})$. The formula (1.2.2) of [8] shows that P is a Markov measure with the transition function p defined by 1.1.γ. If the measure ν is σ-finite, then so is P, and if for each t

$$(3.1.3) \qquad \bar{P}\{x_t \in V\} = 0 ,$$

then the one-dimensional distribution of P is equal to that of $\bar{P}$ (namely to ν). In the sequel we shall consider only processes $(x_t, \bar{P})$ subject to (3.1.3). Put $\tau_s = \inf\{t > s: x_t \in V\}$; $\tau = \tau_0$. If

3.1.A. For each $x \in D$

$$\bar{P}_x\{\tau > t\} \to 0 \quad \text{as} \quad t \to \infty \quad ,$$

then for each $x \in D$

$$p(t,x;\, D) \to 0 \quad \text{as} \quad t \to \infty \quad . \tag{3.1.4}$$

If

3.1.B. For any set $\Gamma \subset D$ such that $\nu(\Gamma) < \infty$

$$\bar{P}\{x_s \in \Gamma,\ \tau > s\} \to 0 \quad \text{as} \quad s \to \infty \quad ,$$

then for any set Γ such that $P\{w(0) \in \Gamma\} < \infty$

$$P\{w(s) \in \Gamma,\ \alpha \leq 0,\ \beta > s\} \to 0 \quad \text{as} \quad s \to \infty \quad . \tag{3.1.5}$$

If

3.1.C. For some $s > 0$

$$\bar{P}\{\tau < s\} < \infty$$

then

$$P\{\alpha \leq 0,\ 0 < \beta < s\} < \infty \quad . \tag{3.1.6}$$

Let T_t be the semigroup generated by the transition function p. Note that (3.1.4) is true iff T_t is a SD-semigroup. The condition (3.1.5) holds iff ν is null-excessive measure; (3.1.6) is true iff ν is quasi-finite excessive with respect to T_t measure. If both (3.1.5) and (3.1.6) are satisfied then we say that the process (w(s),P) has a null-quasi-finite one-dimensional distribution.

Let Ω be the sample space of the process $(x_t,\overline{P})$ and $\mathcal{F}$ be the basic σ-field in Ω on which the measure $\overline{P}$ is defined, and which is supposed to contain all sets of $\overline{P}$-measure zero. Denote by $\mathcal{F}_s$ the completion with respect to $\overline{P}$ of $\sigma(x_u, u < s)$ and by $\mathcal{C}_s$ the completion with respect to $\overline{P}$ of the σ-field generated by the sets

$$\{\tau_u < r\} \quad , \quad u,r < s$$

(If the process x_t is regular, then $\mathcal{C}_s \subset \mathcal{F}_s$.) We say that the set D is regular for $(x_t,\overline{P})$ if for $t > s$, $\mathcal{C}_s \vee \mathcal{F}_s$ and x_t are conditionally independent given x_s. (This definition certainly assumes $\mathcal{C}_s \subset \mathcal{F}$).

A Markov process (x_t^1,Q_1) with the state space $E_1 = D \cup V_1$ and a Markov process (x_t^2,Q_2) with the state space $E_2 = D \cup V_2$ are said to be equivalent, if the one-dimensional distributions of both processes are concentrated on D and their finite-dimensional distributions coincide.

The following theorems are similar to Theorems 1 and 2 in [8].

<u>Theorem 3.1.1</u>. Let (w(s),P) be a stationary Markov process in the state space D with the transition function p, subject to (3.1.4). If the one-dimensional distribution of P is null-quasi-finite, then this process is a subprocess of a concervative stationary Markov process $(x_t,\overline{P})$ satisfying 3.1.A - 3.1.C for which D is a regular set.

Just as in [8] the set of all stationary Markov measures with transition function p is denoted by $S(p)$.

Theorem 3.1.2. If $(w(s),P)$ satisfies the conditions of Theorem 3.1.1 and if in addition P is a minimal element of $S(p)$, then the process $(x_t,\overline{P})$ is unique up to equivalence.

3.2. In this section we prove Theorem 3.1.1. Consider the one-dimensional distribution ν of $(w(s),\overline{P})$. It was proved in [3] that

$$\nu = \int_0^\infty \nu^s ds \quad , \tag{3.2.1}$$

where ν^s is an entrance law for p. We denote by P^* a Markov measure on G with the transition function p and the one-dimensional distributions ν^s. Put

$$\Pi(\Gamma) = P^*\{\beta \in \Gamma\} \quad . \tag{3.2.2}$$

Suppose that the process $(x_t,\overline{P})$ is constructed and M is defined by (3.1.1). The same heuristic arguments as in Section 3.1 of [8] show that the set M must be translation invariant $(0,\Pi)$-generated and all the cuts w_δ^γ must be conditionally independent, when M is fixed.

The next three lemmas prove that Π, defined by (3.2.2), satisfies (2.1.1).

Lemma 3.2.1. For any $u > 0$ the measure $\nu - \nu T_u$ is finite.

Proof: By our assumptions $\mu = \nu - \nu T_s$ is a finite measure for some $s > 0$. For each $r > 0$ $\mu T_r < \infty$, and for $u = ks$ we have

$$(3.2.3) \qquad \nu - \nu T_{ks} = \nu - \nu T_s + \nu T_s - \nu T_{2s} + \dots + \nu T_{(k-1)s} - \nu T_{ks}$$

$$= \mu + \mu T_s + \dots + \mu T_{(k-1)s} \quad .$$

Each summond in the right side of (3.2.3) is a finite measure; and so is $\nu - \nu T_{ks}$.

By virtue of (3.2.1)

$$(3.2.4) \qquad \nu - \nu T_u = \int_0^u \nu^t dt \quad .$$

Hence if $u \leq ks$, then $\nu - \nu T_u \leq \nu - \nu T_{ks}$, and the lemma is proved.

Lemma 3.2.1 shows that $\nu^s(D)$ is finite for m-almost all $s > 0$ (m is the Lebesque measure). On the other hand for $t > s$

$$\nu^t(D) = \nu^t(1) = \nu^s T_{t-s} 1 \quad \nu^s(1) = \nu^s(D) \quad .$$

Therefore $\nu^s(D)$ is finite for all $s > 0$ and is a decreasing function of s. Consequently

$$(3.2.5) \qquad P^*\{\beta > s\} = P^*\{w(s) \in D\} = \nu^s(D) < \infty \quad , \quad s > 0 \quad .$$

Formula (3.2.5) shows that the restriction on any interval $]s,\infty]$ of the measure Π, defined by (3.2.2), is a finite measure; as a result, Π is σ-finite.

<u>Lemma 3.2.2</u>. The measure Π defined by (3.2.2) satisfies (2.1.1).

<u>Proof</u>: Put $f(s) = \nu^s(D)$.

$$(3.2.6) \qquad \int_0^\infty x \wedge 1\Pi(dx) = \int_0^1 x\Pi(dx) + f(1)$$

$$= \iint_C dx\Pi(dy) + f(1) \quad ,$$

where $C = \{(x,y): \; x \geq 0, \; y \geq 0, \; x + y \leq 1\}$. By Fubini's Theorem (3.2.6) equals

$$\int_0^1 \{\int_y^1 \Pi(dy)\}dx + f(1) = \int_0^1 (\Pi(R^x) - \Pi(R^1))dx + f(1)$$

$$= \int_0^1 (f(x) - f(1))dx + f(1)$$

$$= \int_0^1 f(x)dx = \int_0^1 \nu^x(D)dx \quad .$$

By virtue of (3.2.4) the right side of the above formula is equal to $(\nu - \nu T_1)(D)$. Lemma 3.2.1 implies that this expression is finite.

By virtue of Theorem 2.1.1 and Lemma 3.2.2 we are able to construct a $(0,\Pi)$-generated translation invariant set M, subject to (2.1.2). Let $\tilde{\Omega}$ be the corresponding sample space and Q be the corresponding measure.

Lemma 3.2.3. For any function f on $T \times T$

$$(3.2.7) \quad P\{f(\alpha,\beta)\} = Q\{\sum_\gamma f(\gamma,\delta)\} \quad .$$

Proof: Let P_t^* be defined by (3.2.1) of [8]. By formula (3.2.2) of [8]

$$(3.2.8)\quad P\{f(\alpha,\beta)\} = \int_{-\infty}^{\infty} P_t^*\{f(\alpha,\beta)\}dt$$

$$= \int_{-\infty}^{\infty} P_t^*\{f(t,\beta)\}dt$$

$$= \int_{-\infty}^{\infty} P^*\{f(t,\beta + t)\}dt$$

$$= \int_{-\infty}^{\infty}\{\int_{0}^{\infty} f(t,y + t)\Pi(dy)\}dt \quad .$$

By virtue of (2.1.2), the right side of (3.2.8) is equal to the right side of (3.2.7).

Consider a measure $\overline{N}$ on $T \times T \times W$ defined below.

$$\overline{N}(\Gamma \times \Delta \times A) = P\{\alpha \in \Gamma, \beta \in \Delta, w \in A\} \quad , \quad \Gamma,\Delta \subset T, A \in G \quad .$$

Put

$$(3.2.9)\ N(B) = \overline{N}(B \times W) \quad , \quad B \subset T \times T \quad .$$

Lemma 3.2.4. The measure N, defined by (3.2.9) is σ-finite.

Proof: If Π satisfies (2.1.1), then for $t > 0$

$$\Pi\ (R^t) < \infty \quad .$$

The support of measure N is the set $C = \{(x,y):\ y > x\}$. The set C may be represented as a countable union of rectangles $R =]u,v[\times]r,q[$, where $u < v < r < q$. For such rectangle R

$$
\begin{aligned}
N(R) &= P\{u < \alpha < v,\ r < \beta < q\} \\
&= \int_u^v P_t^*\{r < \beta < q\}dt \\
&\leq \int_u^v P_t^*\{\beta > r\}dt \\
&\leq \int_u^v P^*\{\beta > r - v\}dt \\
&= (v - u)\Pi(R^{r-v}) \\
&< \infty \quad .
\end{aligned}
$$

Further steps in the construction of $(x_t, \bar{P})$ do not differ from the analogous ones in [8]. We take the stochastic N-quasi kernel $m(x,y; A)$ which is a Radon-Nikodym derivative of $\bar{N}(dx \times dy \times A)$ with respect to $N(dx \times dy)$. Then we define a sequence of stochastic Q-quasi kernels $n_k(\tilde{\omega}; A)$ in the same way as it was done in Lemma 3.3.2 of [8]. We put $\Omega = \tilde{\Omega} \times W^\infty$ and define $\bar{P}$ on Ω by the formula (3.3.3) of [8] (it is necessary only to replace $\tilde{P}$ in the right side of (3.3.3) by Q). To justify the existence of such a measure $\bar{P}$, we use Theorem 3.3.1 of [8], which is true for σ-finite measure Q as well. We take $E = D \cup V$, where V is a singleton, and put $x_t(\omega) = x_t(\omega, w_1, w_2, \ldots) = V$ if $t \in M(\tilde{\omega})$ and we put $x_t(\omega) = w_{k(t)}(t)$ otherwise (see the end of Section 3 of [8] for details).

<u>Lemma 3.2.5</u>. The process $(x_t, \bar{P})$ is a conservative stationary Markov process. The subprocess in D of $(x_t, \bar{P})$ is equal to $(w(s), P)$.

To prove Lemma 3.2.5 we have to repeat without variations all the arguments of Section 4 of [8].

<u>Lemma 3.2.6</u>. The set D is a regular set for $(x_t, \bar{P})$.

<u>Proof</u>: Let $u_1, u_2, \dots, u_k$, $v_1, v_2, \dots v_k$, $s_1, s_2, \dots s_n < s < t$. We need to show that for each $\Gamma, \Gamma_1, \dots, \Gamma_n, \Delta \subset E$ there exists a function g on E such that

$$(3.2.10) \qquad \bar{P}\{x_s \in \Gamma,\ x_{s_1} \in \Gamma_1, \dots, x_{s_n} \in \Gamma_n,\ x_t \in \Delta,\ \tau_{u_1} < v_1, \dots, \tau_{u_n} < v_n\}$$

$$= \bar{P}\{g(x_s);\ x_s \in \Gamma,\ x_{s_1} \in \Gamma_1, \dots, x_{s_n} \in \Gamma_n,\ \tau_{u_1} < v_1, \dots, \tau_{u_n} < v_n\} \quad .$$

For simplicity of calculations we consider only the case of $n = k = 1$, $u < v < s_1$. Since the one-dimensional distributions of $\bar{P}$ are concentrated on D we may consider only the case in which Γ, Γ_1 and Δ are subsets of D. Put

$$D(s,t) = \{w \in W:\ \alpha(w) < s < t < \beta(w)\} \quad ,$$

$$E(s,t) = \{w \in W:\ \alpha(w) < s < \beta(w) < t\} \quad ,$$

$$A = \{w \in W:\ w(s_1) \in \Gamma_1\} \quad , \quad B = \{w \in W:\ w(s) \in \Gamma\} \quad ,$$

$$C = \{w \in W:\ w(t) \in \Delta\} \quad .$$

Denote by $\lambda_i(s,t)$ the indicator of the set $\{\gamma_i < s < t < \delta_i\}$, by $\delta_i(s,t)$ the indicator of the set $\{\gamma_i < s < \delta_i < t\}$, and by w_i the cut off $w_{\delta_i}^{\gamma_i}$, $i = 1,2,\dots$.

$$(3.2.11) \quad \bar{P}\{x_{s_1} \in \Gamma_1, x_s \in \Gamma, x_t \in \Delta, \tau_u < v\}$$

$$= \bar{P}\{ \sum_{\gamma_1<\gamma_2} \delta_1(u,v)\lambda_2(s_1,t)1_{ABC}(w_2)$$

$$+ \sum_{\gamma_1<\gamma_2<\gamma_3} \delta_1(u,v)\lambda_2(s_1,s_1)1_A(w_2)\lambda_3(s,t)1_{BC}(w_3)$$

$$+ \sum_{\gamma_1<\gamma_2<\gamma_3} \delta_1(u,v)\lambda_2(s_1,s)1_{AB}(w_2)\lambda_3(t,t)1_C(w_3)$$

$$\sum_{\gamma_1<\gamma_2<\gamma_3<\gamma_4} \delta_1(u,v)\lambda_2(s_1,s_1)1_A(w_2)\lambda_3(s,s)1_B(w_3)\lambda_4(t,t)1_C(w_4)$$

The first term in the right hand side of (3.2.11) is equal to

$$(3.2.12) \quad Q\{ \sum_{\gamma_1<\gamma_2} m(\gamma_1,\delta_1;\ E(u,v))m(\gamma_2,\delta_2;\ D(s_1,t)AB)\}$$

$$= Q\{\sum_{\gamma} m(\gamma,\delta;\ D(s_1,t)ABC)\phi(\gamma)\}$$

$$= P\{\phi(\alpha);\ ABCD(s_1,t)\}$$

$$= P\{\phi(\alpha)p(t-s;\ w(s),\Delta);\ AB, \alpha < s_1 < s < \beta\} \quad ,$$

where

$$\phi(x) = Q_x^* \sum_{t \in J} m(y_t, y_{t-};\ E(u,v))\} \quad .$$

(The first equality in (3.2.12) is due to Lemma 6.8 in [7], the second to Lemma 4.1.1 in [8], the last equality is due to the Markov property of $(w(s),P)$.) Similarly, we get that the sum of the second, the third and the fourth term in (3.2.11) equals

$$(3.2.13)\qquad P\{\phi(\alpha)\psi(w(s));\ AB, \alpha < s_1 < s < \beta\}$$
$$+\ P\{\phi_1(\alpha)p(t - s;\ w(s), \Delta);\ B, \alpha < s < \beta\}$$
$$+\ P\{\phi_1(\alpha)\psi(w(s));\ B, \alpha < s < \beta\}\quad ,$$

where

$$\phi_1(x) = Q^*_x\{\sum_{r,z\in J, r<z} m(y_r, y_{r-}, AD(s_1, s_1))m(y_z, y_{z-};\ E(u,v))\}\quad ,\quad x \in T$$

$$(3.2.14)\qquad \psi(x) = \int_0^\infty P_x\{\beta \in dy\}Q_y\{\sum_{r\in J} m(y_{r-}, y_r;\ w(t - s) \in \Delta)\}\quad ,\quad x \in D\quad .$$

(Compare to (4.1.5)-(4.1.8) in [8]). Adding (3.2.13) to (3.2.12), we get (3.2.10) with

$$(3.2.15)\qquad g(x) = p(t - s, x;\ \Delta) + \psi(x)\quad .$$

<u>Lemma 3.2.7.</u> The transition function of $(x_t, \bar{P})$ is

$$(3.2.16)\qquad \bar{p}(u,x;\ \Delta) = p(u,x;\ \Delta) + \int_0^\infty P_x\{\beta \in dy\}Q_y\{\int_0^\sigma {}^u P^*_{y_t}\{w(u) \in \Delta\}du\}\quad ,\quad \Delta \subset D\quad .$$

<u>Proof</u>: The Kolmogorov-Chapman equation for $\bar{p}$ was verified in Section 2 of [8].

Putting $t - w = u$ in (3.2.15), one can see that for ν-a.e. x $p(u,x,;\ \Delta) = g(x)$, and for the proof of (3.2.16), it is enough to verify equality between $\psi(x)$, given by (3.2.14), and the second term in the right hand side of (3.2.16) for ν-almost all x. Put

$$\theta(y) = Q_y\{\sum_{r\in J} m(y_{r-}, y_r;\ w(u) \in \Delta\}\quad ,\quad y \in T\quad .$$

Applying successively the Markov property of $(w(s),P)$, Lemma 4.1.1 of [8] and Lemma 6.8 in [7], we get

$$\begin{aligned} \int_{\Gamma}\psi(x)\nu(dx) &= P\{1_{\Gamma}(w(s))\psi(w(s))\} \\ &= P\{1_{\Gamma}(w(s)),\theta(\beta)\} \\ &= Q\{\sum_{\gamma} m(\gamma,\delta;\ w(s) \in \Gamma)\theta(\delta)\} \\ &= Q\{\sum_{\gamma} m(\gamma,\delta;\ w(u) \in \Delta)\theta'(\gamma)\} \quad , \end{aligned} \tag{3.2.17}$$

where

$$\theta'(y) = Q^{*}_{y}\{\sum_{r\in J} m(y_{r},y_{r-};\ w(s) \in \Gamma)\} \quad .$$

In view of Lemma 4.1.1 in [8], (3.2.17) equals

$$P\{\theta'(\alpha)1_{\Delta}(w(u))\} = P\{\theta'(\alpha)\xi(\alpha);\ \beta > u\} \quad , \tag{3.2.18}$$

where

$$\xi(y) = 1_{y<u}P^{*}_{y}\{w(u) \in \Delta\}|P^{*}_{y}\{\beta > u\} \quad .$$

Applying Lemma 4.1.1 of [8], Lemma 6.8 of [7] and again Lemma 4.1.1 of [8], we get that (3.2.18) equals

$$\begin{aligned} Q\{\sum_{\gamma<u<\delta} \theta'(\gamma)\xi(\gamma)\} &= Q\{\sum_{\gamma_1<\gamma_2} m(\gamma_1,\delta_1;\ w(s) \in \Gamma)\xi(\gamma_2)\lambda_2(u,u)\} \\ &= Q\{\sum_{\gamma} m(\gamma,\delta;\ w(s) \in \Gamma)\xi'(\delta)\} \\ &= P\{1_{\Gamma}(w(s))\xi'(\beta)\} \quad , \end{aligned} \tag{3.2.19}$$

where

$$\xi'(y) = Q_y\{\sum_{\gamma<u<\delta} \xi(\gamma)\} \quad .$$

By virtue of Lemma 2.1 in [7]

$$\xi'(y) = Q_y\{\int_0^{\infty} 1_{y_t<u} \xi_{y_t} \Pi(y_t;\ R^u)dt\}$$

$$= Q_y\{\int_0^{\sigma_u} \xi_{y_t} P^*_{y_t}\{\beta > u\}dt\}$$

$$= Q_y\{\int_0^{\sigma_u} P^*_{y_t}\{w(u) \in \Delta\}dt\} \quad .$$

Substituting the expression for $\xi'(y)$ in (3.2.19), we see that for any set Γ

$$(3.2.20) \qquad \int_{\Gamma} \psi(x)\nu(dx) = \int_{\Gamma} \psi_1(x)\nu(dx) \quad ,$$

where $\psi_1(x)$ is the second term in the right hand side of (3.2.16). Formula (3.2.20) implies

$$\psi(x) = \psi_1(x) \quad \text{a.e.} \quad \nu \quad .$$

3.3. The proof of Theorem 3.1.2 does not differ from the proof of Theorem 2 in [8]. Lemma 5.2.1 in [8] is true in our case as well. If $P^*\{W\} < \infty$, then it is necessary to repeat the proofs of Lemmas 5.3.1-5.3.4 in order to arrive to the expression (5.3.10) in [8] for the two-dimensional distributions of the process $(x_t,\bar{P})$.

If $P^*\{W\} = \infty$, then we must consider the local time ξ_t of $(x_t, \bar{P})$ at V. We have to introduce a filtration $A_t = C_t \vee F_t$, with respect to which the local time ξ_t is adapted. Repeating Lemmas 5.4.1-5.4.3 and 5.5.1-5.5.2 in our case, we obtain the expression (5.5.3), which is true in the case of infinite underlying distribution $\bar{P}$ as well. (The proofs of the above lemmas were based on the lemmas and theorems of Chapters 4 and 5 in [7]. The whole theory in [7] was developed under the assumption that the underlying measure is a probability one. Nevertheless everything remains the same in the case of infinite underlying distribution.) Then we have to consider the process y_s, which is the right-continuous inverse of the local time $\xi_\bullet$. Repeating the proofs of Lemmas 5.6.1-5.6.3, we get that $y_t - y_s$ and A_{y_s} are independent; therefore $(y_s, \bar{P})$ is the process with independent increments, whose Levi's measure can be obtained through P^*. Lemmas 5.7.1 and 5.7.2 of [8] are also true in the case of infinite underlying distribution, and they show that the two-dimensional distributions of $(x_t, \bar{P})$ are uniquely determined by the measure P.

4. Enhancing of Semigroups

4.1. Now we consider the semigroup T_t and the measure ν described in Theorem 1.

If T_t is a S-semigroup then there exists a transition function $p(t,x;\Gamma)$ such that

$$T_t f(x) = p(t,x;f)$$

(see [4], Theorem 2.1). If T_t is a dying semigroup then

$$p(t,x;D) \to 0 \quad \text{as} \quad t \to \infty \ .$$

By the theorem of Kuznecov (see [5]) there exists a stationary Markov process $(w(s),P)$ with random birth and death times whose one-dimensional distribution is equal to ν and transition function is equal to p. (To construct such a process $(w(s),P)$ we need also to specify an excessive function h, but in our case $h(x) \equiv 1$.) The conditions of Theorem 1 imply that the one-dimensional distribution of P is null-quasi-finite. By Theorem 3.1.1 there exists a covering process $(x_t,\bar{P})$ with the state space $E = D \cup V$.

Let $\bar{p}(t,x;\Gamma)$ be the transition function of $\bar{P}$. The same way as in [9] we can show that the condition 1.2.D implies

$$(4.1.1) \qquad \bar{p}(t,x;D) = 1 \quad \text{for all} \quad t \quad \text{and} \quad x \in D \ .$$

Therefore $\bar{p}(t,x;V) \equiv 0$ and $\bar{p}(t,x;-)$, considered as a kernel from D into D, is a transition function. By Theorem 3.1.1 $(x_t,\bar{P})$ is a stationary conservative process with the one-dimensional distribution ν; consequently, ν is invariant with respect to $\bar{p}$. The semigroup $\tilde{T}_t$ generated by $\bar{p}$ is the semigroup we are looking for. The properties 1.2.A-1.2.C are automatically satisfied by any semigroup generated by a transition function. The property 1.2.E' is a consequence of (4.1.1); and (1.2.1) follows from the fact that $(x_t,\bar{P})$ is a covering process for $(w(s),P)$, for which 1.1.γ holds. Lemma 3.2.7 gives us the explicit expressiong for $\tilde{T}_tf(x)$ in terms of "internal" characteristics of ν and T_t (we make trivial transformations in (3.2.16) to obtain the formula below).

$$(4.1.2) \qquad \tilde{T}_tf(x) = T_tf(x) + \int_0^\infty Q_0\{\int_0^{\sigma_{t-y}} \nu^{ys}(f)ds\}\mu_x(dy) \quad ,$$

where μ_x is a measure on $]0,\infty[$ such that $\mu]r,u] = T_r1(x) - T_u1(x)$, the family ν^s is an entrance law with respect to T_t for which (3.2.1) holds, and (y_s,Q_0) is an increasing process with independent increments with translation constant 0 and the Levi measure Π such that

$$\Pi]r,u] = \nu^r(D) - \nu^u(D) \quad .$$

It is interesting to compare the formula (4.1.2) with the result of Getoor (see [4], Theorem (8.1)). He solves the inverse problem, namely, he finds an invariant distribution ν for the transition function $\bar{p}$ given by the formula analogous to (3.2.16). The expression for ν he obtains is similar to (3.2.1).

4.2. The proof of Theorem 2 does not differ from the proof of Theorem 2 in [9]. We have to consider a conservative stationary Markov process $(x_t,\bar{P})$ with the one-dimensional distribution ν and the transition function $\bar{p}$ which generates the semigroup $\tilde{T}_t$ (but in contrast to the situation in [9], now $\bar{P}$ may be an infinite measure). A multiplicative functional α_t is constructed in such a way that

$$p(s,x;\Gamma) = \bar{P}_x\{1_\Gamma(x_s)\alpha_s\} \quad .$$

Let Ω be the sample space of the process x_t. We put $\tilde{\Omega} = \Omega \times (T)^\infty$ and construct a measure Q on $\tilde{\Omega}$ and a family of random variables $\tau_s(\tilde{\omega})$ in such a way that

4.2.A. The marginal distribution of Q on Ω is equal to $\bar{P}$.

4.2.B. For $t > s$ the conditional probability of τ_s to be greater than t, given ω is equal to $\alpha_{t-s}(\theta_s\omega)$.

4.2.C. For $t > s$ $\tau_s = \tau_t$ on the set $\{\tau_s > t\}$.

4.2.D. The σ-field $\mathcal{C}_s \vee \mathcal{F}_s$ and the pair (x_t, τ_t) are conditionally independent, given x_s, where $\mathcal{C}_s$ is the minimal σ-field in $\bar{\Omega}$ generated by the sets $\{\tau_r < u; r,u \leq s$.

The family τ_s has the same properties as the family of the first hitting times of a set in the state space. We put $M(\tilde{\omega})$ to be the closure of the set of values of the function $\tau_{\cdot}(\tilde{\omega})$. We put $x_t^*(\tilde{\omega})$ to be equal to $x_t(\omega)$ if $t \bar{\in} M(\tilde{\omega})$ and $x_t^*(\tilde{\omega}) \in V$ otherwise. In the same way as in [9] one can show that (x_t^*, Q) and $(x_t, \bar{P})$ has the same finite-dimensional distributions and the subprocess in D of (x_t^*, Q) is equal to $(w(s), P)$ where P is a Markov measure with the one-dimensional distribution ν and the transition function p. Now we need only to apply Theorem 3.2.1 to obtain the final result.

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