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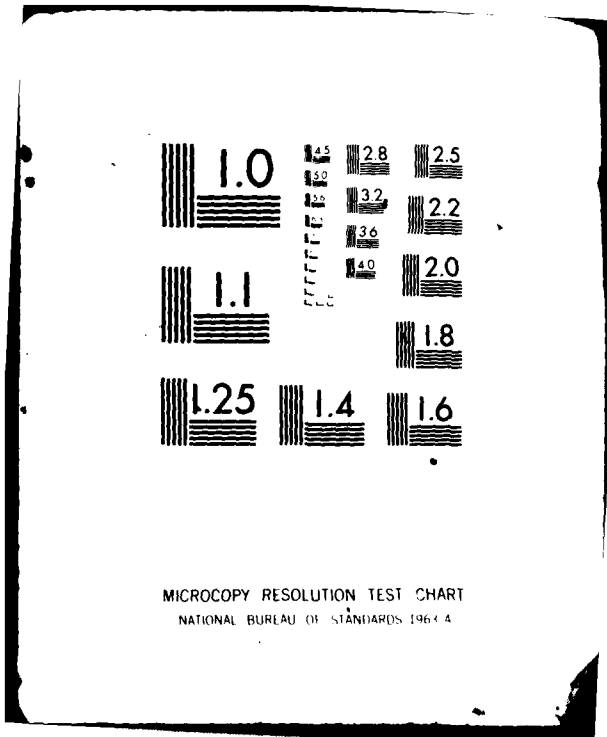
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by

Herve J. P. Raynaud

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HOW RESTRICTIVE ACTUALLY ARE THE VALUE RESTRICTION CONDITIONS*

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Herve J.P. Raynaud**

1. Preliminary Remarks and Notation

It has already been shown that the "value restriction conditions" are in a certain sense the best possible conditions ensuring the transitivity of the majority method of decision in Sen [to appear]; nevertheless, the actual restriction demanded by these conditions have not been yet clearly measured.

In a previous paper on the subject in Raynaud [1982], we have shown that, if $n \geq 4$ is the number of alternatives and M_n the maximum number of different votes in a profile following the value restriction condition, then $M_4 = 9$, $M_5 \geq 21$ and in general $M_n > 2^{n-1}$. As $4! = 24$ and $5! = 120$, it is reasonable to conjecture that the ratio $M_n/n!$ (which can be considered as a measure of the amount of individual freedom which remains to an individual in a culture following a value restriction condition) tends to zero as n gets large.

Section II recalls the previous materials on the subject which may aid in the clarification of the consequences of the result. The proof of this result is quickly obtained in Section III. The following notation is used throughout:

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- X denotes a finite set of alternatives;
- E (or E(X)) denotes a profile on X, i.e. an indexed set $\{\theta_1, \dots, \theta_N\}$ of N total orders on X called individual orders or votes;
- $\forall Y \subseteq X$, $\theta_i(Y)$ is the restriction of θ_i to only the alternatives in Y and E(Y) is the restriction of the profile E to the alternatives of Y, i.e. the set of the $\theta_i(Y)$'s.

2. The Framework of the Problem

This section is a "distillation" of well-known material.

Definition 1: A profile E follows a value restriction condition if and only if for any unordered triple T of alternatives, there exists an alternative x and a rank $k \in \{1, 2, 3\}$ such that, in E(T), x is never in the k-th rank.

Definition 2: A profile follows Ward's condition if and only if there is no couple (Y, θ) where Y is a subset $\{y_1, \dots, y_p\}$ of X, $p \geq 3$, and θ is a sequence $\theta_{y_1}, \dots, \theta_{y_p}$ of individual orders of E such that

$$\begin{aligned}\theta_{y_1}(Y) &= y_1 y_2 \dots y_p \\ \theta_{y_2}(Y) &= y_2 y_3 \dots y_p y_1 \\ &\vdots \\ \theta_{y_p}(Y) &= y_p y_1 \dots y_{p-1}\end{aligned}$$

Proposition 1: A profile follows Ward's condition if and only if it contains no Condorcet triple, i.e. no triple of alternatives, a,b,c , and no triple of individual orders, $\{\theta_a, \theta_b, \theta_c\}$, such that

$$\begin{aligned} \theta_a(\{a,b,c\}) &= a,b,c \\ \theta_b(\{a,b,c\}) &= b,c,a \\ \theta_c(\{a,b,c\}) &= c,a,b . \end{aligned}$$

Proof: Let E follow Ward's condition. It implies clearly the non-existence of a Condorcet triple. Conversely, suppose that E has no Condorcet triple but that there exists a $Y \subseteq X$, $Y = \{y_1, \dots, y_p\}$ with $p > 3$, and a sequence $\theta_{y_1}, \dots, \theta_{y_p}$ of individual orders such that

$$\begin{aligned} \theta_{y_1}(Y) &= y_1, y_2 \dots y_p \\ \theta_{y_2}(Y) &= y_2, y_3 \dots y_p y_1 \\ &\vdots \\ \theta_{y_p}(Y) &= y_p y_1 \dots y_{p-1} . \end{aligned}$$

$T = \{y_1, y_2, y_3\}$ would clearly make a Condorcet triple as

$$\begin{aligned} \theta_{y_1}(T) &= y_1 y_2 y_3 \\ \theta_{y_2}(T) &= y_2 y_3 y_1 \\ \theta_{y_3}(T) &= y_3 y_1 y_2 . \end{aligned}$$

Hence the hypothesis is contradictory.



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Proposition 2: A profile E follows Ward's condition if and only if it follows the value restriction condition.

Proof: Let us consider any unordered triple of alternatives $T = \{a, b, c\}$. If it was a Condorcet triple, there would exist three individual orders $\theta_a, \theta_b, \theta_c$ such that $\theta_a(T) = a, b, c$; $\theta_b(T) = b, c, a$; $\theta_c(T) = c, a, b$. Any of the three alternatives would be at least once in any of the three ranks and E would not follow the value restriction condition. Hence, if E follows the value restriction condition, it follows Ward's condition.

Let us now suppose that E follows Ward's condition and consider any triple $\{a, b, c\}$. Six rankings of these three objects are a priori possible: $(a, b, c), (b, c, a), (c, a, b), (a, c, b), (c, b, a), (b, a, c)$. As E follows Ward's condition, one ranking at least among the three first ones, and another one among the three last ones are prohibited--and it is trivial to check that each of the nine possible cases corresponds to a particular case of the value restriction conditions on T .

Proposition 3 (Sen [to appear]): The value restriction condition is the best possible ensuring the transitivity of the majority method of decision for any subset of alternatives and any subset of individual orders from the considered profile (the number of voters, of course, is odd).

The proof is left to the reader.

3. The Upper Bound for M_n

Theorem: If E_n denotes a profile on n alternatives following the value restriction condition, M_n , the number of different individual orders in E_n is necessarily smaller than $2(n-1)!$

Proof: The set of the $n!$ possible individual orders can be described by means of $(n-1)!$ circulant $(n \times n)$ matrices; each row of these matrices is deduced from the previous one by circular permutation, the set of all the rows being the set of the $n!$ possible permutation.

Consider now one of these matrices, say, if $X = \{x_1, \dots, x_n\}$,

$$\begin{array}{cccccc} x_1 & x_2 & \dots & & x_n & \\ x_2 & x_3 & \dots & x_n & x_1 & \\ \vdots & & & & & \\ x_n & x_1 & \dots & & x_{n-1} & \end{array}$$

In E_n , two at most among the rows of this matrix can be individual orders. If three of them were individual orders, say row p , row q , row r , with $p < q < r$, it is clear that $\{x_p, x_q, x_r\}$ would make a Condorcet triple and E_n would not follow the value restriction condition. Hence E cannot be made of more than $2(n-1)!$ different orders.

Proposition 4: Let n be a number of alternatives. Let ν be the maximum number of different individual orders in a profile following the value restriction conditions. Let $\mu = n!$ be the maximum number of different individual orders in a profile following no condition.

Then v/μ tends to zero faster than $2/n$ as n increases to infinity.

4. Conclusion

Let us consider a culture--or a system of procedures--imposing on an assembly a value restriction condition. Let us recall that such a condition is the most generous (in terms of permitted votes) if one wants to ensure the transitivity of the majority method of decision for any subset of individual orders and of alternatives.

If we regard v/μ as a measure of individual freedom in this culture, Proposition 4 implies that, on issues involving a large number of alternatives, the individual freedom will be dramatically reduced.

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