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TRANSVERSE BEAM DYNAMICS IN THE MODIFIED BETATRON.(U)
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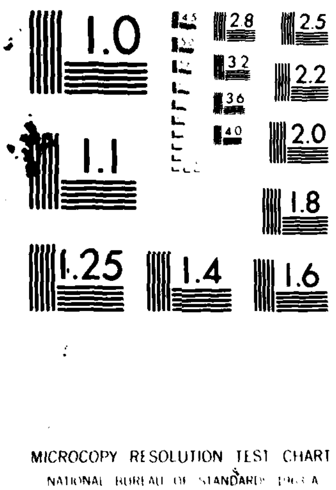
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The linearized equations governing the motion of the center of a beam about its equilibrium position in a modified betatron, as well as equations governing the motion of an individual particle about the beam center, are presented and solved. Self field effects, including toroidal hoop stresses and wall image forces, are included in the analysis. All fields, both self and applied, are assumed to be azimuthally symmetric but are allowed to have arbitrary time dependences. The solutions to the equations of motion are analyzed for stability and conditions for stability are obtained. Further study of the solutions illustrates two phenomena of experimental interest: (1) the unavoidable

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20. ABSTRACT (Continued)

traversal of a finite "instability gap" in parameter space during acceleration and (2) the adiabatic increase in the amplitude of the betatron oscillations during removal of the toroidal magnetic field, prior to beam ejection. By careful design, the effects of these phenomena can be reduced to insignificant levels in an actual accelerator.

CONTENTS

TRANSVERSE BEAM DYNAMICS IN THE MODIFIED BETATRON

I. INTRODUCTION

It has been suggested[1,2] that the current carrying capacity of a conventional betatron accelerator might be improved dramatically by the addition of a strong toroidal magnetic field. Such a field acts to confine the beam during injection and early stages of acceleration when γ, the usual relativistic factor, is small and space charge effects which tend to expand the beam are large. After acceleration is complete, γ is large, space charge effects are small, and the usual weak focussing betatron fields are sufficient to confine the beam; the toroidal field may then be removed to facilitate beam ejection. In general both vertical and toroidal magnetic fields may be changing simultaneously during beam injection and ejection. It is the purpose of this paper to examine the behavior of the beam in such time-varying fields.

We shall derive and solve equations governing the motion of the center of an electron beam confined in a modified betatron as well as equations governing the motion of an individual particle within the beam. Whole beam and single particle stability criteria will be presented; the stabilizing effect of the toroidal field for both beam and single particle motions, noted earlier,[1,2] will be apparent.

When the fields are allowed to vary in time two interesting phenomena occur. The first phenomenon, which occurs during acceleration, has no analogue in a conventional betatron: As the beam accelerates (γ increases) the betatron makes a transition from a region in parameter space in which the toroidal field is *essential* to stability (modified betatron regime) to a region in which the toroidal field is *superfluous* to stability (conventional betatron regime). It turns out that, except under extraordinary circumstances, the system must pass through an "instability gap"—a region of parameter space, separating the modified and conventional betatron regimes, in which single particle motion is

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unstable, though beam center motion may not be, *irrespective* of the magnitude of the toroidal magnetic field. However, though the size of the instability gap is independent of the toroidal field, the instability growth rate within the gap is inversely proportional to this field. We find below that by judicious magnet design and sufficiently rapid acceleration, this gap may be successfully traversed with minimal beam disturbance.

The second phenomenon occuring in time varying fields does have an analogue in a conventional betatron; this is the adiabatic change in the amplitude of the betatron oscillations.[3] Since the frequency of these oscillations depends now on both the vertical and toroidal fields a slow change in either is expected to alter the amplitude of the betatron oscillations. During acceleration we find, as in a conventional accelerator,[3] that the oscillation amplitude decreases as the vertical field increases. If one now considers removal of the toroidal field prior to beam ejection, we find that, as long as the toroidal magnetic field is much larger than the vertical field, the beam motion will describe orbits of increasing amplitude as the toroidal field is decreased. Once the toroidal field becomes comparable to the vertical field, however, the motion becomes more complicated and the betatron oscillations no longer continue to increase in amplitude. We find that, by careful choice of field strengths, the ratio of the betatron oscillation amplitude before acceleration to the amplitude of oscillation following complete removal of the toroidal field can be adjusted to be near one.

In the following analysis we assume "perfect," i.e. azimuthally symmetric fields. By neglecting the possibility of azimuthal variation in the self fields (due to beam bunching or kinking) we omit here consideration of a variety of beam instabilities that may occur;[4] by neglecting similar azimuthal variation in the applied fields ("field errors") we neglect the effects of orbital resonances. These will be addressed in a separate report.[5]

II. EQUILIBRIUM RADIAL FORCE BALANCE

The geometry of the modified betatron is shown in Fig. 1. The field configuration is that of an ordinary betatron with the addition of a toroidal magnetic field, $B_{\theta 0}$, here taken to be positive and constant across the minor cross section of the torus. We consider an electron beam of circular cross section, as shown in Fig. 2, with center located at $(r_c, z_c) = (r_0 + \Delta r, \Delta z)$ where r_0 is the equilibrium radius for the center of the beam at which the electric, magnetic, and centrifugal forces on a particle at the center of the beam are in balance. We shall take r_0 to be the major radius of the accelerator chamber. In the absence of self field effects radial force balance requires the electron circulation frequency at $r = r_0$, $z = 0$ to be given by

$$\dot{\theta}_0 = \Omega_{z0} \equiv eB_{z0}/m\gamma_0 c \quad \text{(no self field effects)} \tag{1}$$

where B_{z0} is the value of the applied vertical betatron field at the location of the orbit, γ_0 is the usual relativistic factor, $e(>0)$ is the magnitude of the electron charge, m is the electron rest mass, and c is the speed of light.

Self field effects will modify Eq. (1) however. A nonneutral current ring produces both a zero order vertical magnetic field and a radial electric field. In general, for a reference particle at $r = r_0$, $z = 0$, radial force balance requires

$$-\gamma_0 r_0 \dot{\theta}_0^2 = \left[E_r^{(0)} + \frac{1}{c} r_0 \dot{\theta}_0 B_z^{(0)}\right] \tag{2}$$

where $E_r^{(0)}$ and $B_z^{(0)}$ are the zero order fields at $r = r_0$, $z = 0$. From Appendix A, Eqs. (A-25c, 26c, 26d)

$$B_z^{(0)} = B_{z0} - \pi n_0 e \beta_0 \frac{r_b^2}{r_0} l_B \tag{3}$$

$$E_r^{(0)} = -\pi n_0 e \frac{r_b^2}{r_0} l_E \tag{4}$$

where the notation is defined in Appendix A.

The terms proportional to l_B in Eq. (3) and l_E in Eq. (4) are toroidal corrections to the self fields of a cylindrical beam. They represent "hoop stresses"— self forces on a nonneutral ring of current which act to expand the ring. Since we do not attempt here to construct a consistent equilibrium for the beam we leave l_B and l_E arbitrary in the analysis below since their precise values depend upon the particular distributions of charge and current in the beam. Still, one expects the leading order logarithms in the expressions for l_B and l_E, Eqs. (A-27,28), to be correct.

Using now the zero order fields, Eqs. (3,4), in Eq. (2) we may write the condition for radial force balance as

$$\left[1 + \frac{\nu}{\gamma_0} l_B\right]\dot{\theta}_0^2 - \Omega_{z0}\dot{\theta}_0 + \frac{\nu}{\gamma_0}\frac{c^2}{r_0^2} l_E = 0 \tag{5}$$

where

$$\nu/\gamma_0 = \frac{1}{\gamma_0}\left[\pi r_b^2 n_0 \frac{e^2}{mc^2}\right] = \frac{1}{4}\frac{\omega_b^2 r_b^2}{c^2} \tag{6}$$

and where ω_b is the beam plasma frequency, $(4\pi n_0 e^2/m\gamma_0)^{1/2}$. Here and below Ω_{z0} retains the definition assigned to it in Eq. (1).

Equation (5) is a quadratic equation for the circulation frequency, $\dot{\theta}_0$. The solution which approaches Ω_{z0} as $\nu/\gamma_0 \rightarrow 0$ is, to first order in ν/γ_0:

$$\dot{\theta}_0 \approx \Omega_{z0}\left[1 - \frac{\nu}{\gamma_0}\left(\frac{1}{\alpha^2} l_E + l_B\right)\right] \tag{7}$$

where $\alpha = \Omega_{z0} r_0/c$. Self field effects, represented by the ν/γ_0 term, are seen to *reduce* the single particle circulation frequency below that expected for a zero density; the correction term can be significant (20-30%) in presently contemplated devices. The general result, Eq. (7), will be needed below in the derivation of the first order equations of motion.

III. FIRST ORDER EQUATIONS OF MOTION

In this section the equations governing the motion of a beam and motion of an electron within the beam are obtained and discussed. We shall consider in detail only motion transverse to the toroidal magnetic field, assuming that all fields, both self and applied, are independent of θ.

The equations of motion for a particle in the fields of (A-25, 26) to first order in the displacements from the reference orbit $(r_0, 0)$, are derived in Appendix B. They are

$$\ddot{r}_1 + \frac{\dot{\gamma}_0}{\gamma_0}\dot{r}_1 + \Omega_{z0}^2\left[1 - n^* - \frac{\nu}{\gamma_0}\left(\frac{3}{\alpha^2}l_E + 2l_B\right)\right]r_1 - n_s\Omega_{z0}^2\left(\delta r + \frac{r_b^2}{a^2}\Delta r\right)$$
$$= \frac{e\dot{B}_{\theta 0}}{2m\gamma_0 c}z_1 + \Omega_{\theta 0}\dot{z}_1 + \Omega_{z0}\frac{P_{\theta 1}}{\gamma_0 m r_0}\left[1 - \frac{\nu}{\gamma_0}\left(\frac{1+\gamma_0^{-2}}{\alpha^2}l_E + l_B\right)\right] \tag{8a}$$

$$\ddot{z}_1 + \frac{\dot{\gamma}_0}{\gamma_0}\dot{z}_1 + \Omega_{z0}^2 n^* z_1 - n_s\Omega_{z0}^2\left(\delta z + \frac{r_b^2}{a^2}\Delta z\right)$$

$$= -\frac{e\dot{B}_{\theta 0}}{2m\gamma_0 c}r_1 - \Omega_{\theta 0}\dot{r}_1 \tag{8b}$$

where

$$r_1 = r - r_0 = \Delta r + \delta r$$
$$z_1 = z = \Delta z + \delta z$$
$$n^* = n\left[1 - \frac{\nu}{\gamma_0}\left(\frac{1}{\alpha^2}l_E + l_B\right)\right]$$
$$n_s = \omega_b^2/\left(2\gamma_0^3\Omega_{z0}^2\right)$$
$$\Omega_{\theta o} = eB_{\theta 0}/m\gamma_0 c$$

and where $P_{\theta 1}$ is equal to the canonical angular momentum of the particle at (r, z) minus the canonical angular momentum of the reference particle at $(r_0, 0)$, to first order in small quantities. It may be shown, using the definition of P_θ, $P_\theta \equiv r\left(m\gamma V_\theta - \frac{e}{c}A_\theta\right)$, that

$$P_{\theta 1} = m\gamma_0 r_0\left[V_1\gamma_0^2 - \frac{\nu}{\gamma_0}\frac{1}{\alpha^2}\Omega_{z0}l_E r_1\right] \tag{9}$$

where $V_1 = V_\theta - V_{\theta 0}$.

As they stand Eqs. (8a) and (8b) are not easily solved since, before they can be solved for the coordinates of a particle (r_1, z_1) the beam position $(\Delta r, \Delta z)$ must somehow be known as a function of time. However, a set of consistent equations for beam and particle motion may be obtained by performing an ensemble average of Eqs. (8a,b) over initial particle coordinates and velocities. Denoting such an average by brackets it may be shown that, as long as the beam is assumed not to kink (Δr, Δz independent of θ), we will have

$$<r_1> = \Delta r, \ <\delta r> = <\dot{\delta} r> = <\ddot{\delta} r> = 0 \tag{10a}$$

$$<z_1> = \Delta z, \ <\delta z> = <\dot{\delta} z> = <\ddot{\delta} z> = 0. \tag{10b}$$

Upon performing this averaging procedure on Eqs. (8a,b) we will obtain equations governing the motion of the center of the beam. These may subsequently be subtracted from the original, unaveraged Eqs. (8a,b) to obtain equations governing the motion of a single particle within the beam. Both resulting sets of equations may be summarized by the following single set:

$$\ddot{x} + \omega_x^2 x = \Omega_{\theta 0} \dot{y} + \frac{1}{2} \dot{\Omega}_{\theta 0} y + F \tag{11a}$$

$$\ddot{y} + \omega_y^2 y = -\Omega_{\theta 0} \dot{x} - \frac{1}{2} \dot{\Omega}_{\theta 0} x \tag{11b}$$

where the various quantities appearing in Eqs. (11a,b) are defined in Table I.

Table I — Definition of Quantities Appearing in Equations of Motion, Eqs. (11a,b).

	Beam Equations	Particle Equations
(x,y)	$\gamma_0^{1/2}(\Delta r, \Delta z)$	$\gamma_0^{1/2}(\delta r, \delta z)$
ω_x^2	$\Omega_{z0}^2\left[1 - n^* - \frac{r_b^2}{a^2} n_s - \frac{\nu}{\gamma_0}\left(\frac{3}{\alpha^2} l_E + 2 l_B\right)\right] - \frac{1}{2}\frac{\ddot{\gamma}_0}{\gamma_0} + \frac{1}{4}\left(\frac{\dot{\gamma}_0}{\gamma_0}\right)^2$	$\Omega_{z0}^2\left[1 - n^* - n_s - \frac{\nu}{\gamma_0}\left(\frac{3}{\alpha^2} l_E + 2 l_B\right)\right] - \frac{1}{2}\frac{\ddot{\gamma}_0}{\gamma_0} + \frac{1}{4}\left(\frac{\dot{\gamma}_0}{\gamma_0}\right)^2$
ω_y^2	$\Omega_{z0}^2\left[n^* - \frac{r_b^2}{a^2} n_s\right] - \frac{1}{2}\frac{\ddot{\gamma}_0}{\gamma_0} + \frac{1}{4}\left(\frac{\dot{\gamma}_0}{\gamma_0}\right)^2$	$\Omega_{z0}^2[n^* - n_s] - \frac{1}{2}\frac{\ddot{\gamma}_0}{\gamma_0} + \frac{1}{4}\left(\frac{\dot{\gamma}_0}{\gamma_0}\right)^2$
F	$\Omega_{z0}\frac{<P_{\theta 1}>}{m\gamma_0 r_0} \times \left[1 - \frac{\nu}{\gamma_0}\left(\frac{1+\gamma_0^{-2}}{\alpha^2} l_E + l_B\right)\right]$	$\Omega_{z0}[P_{\theta 1} - <P_{\theta 1}>]\frac{1}{m\gamma_0 r_0} \times \left[1 - \frac{\nu}{\gamma_0}\left(\frac{1+\gamma_0^{-2}}{\alpha^2} l_E + l_B\right)\right]$

Equations (11a,b) are our basic starting points for the analysis to be presented below. In the following sections we will derive and study the WKB solutions to Eqs. (11a,b). First we make a few remarks on the equations themselves.

The term proportional to x on the left hand side of Eq. (11a) and the term proportional to y on the left hand side of Eq. (11b) represent radial and vertical focussing forces respectively. In general the coefficients of x and y in these terms are not equal which suggests that an initially circular beam may not remain circular. The value of n which makes these terms equal (the value required to maintain a circular beam cross section) is

$$n_{\mathrm{cir}} = \frac{1}{2}\left[1 - \frac{\nu}{\gamma_0}\left(\frac{2}{\alpha^2} l_E + l_B\right)\right] \tag{12}$$

which depends on γ_0 and therefore on time. In what follows we will leave n arbitrary, though we shall assume implicitly that its value is close to n_{cir}. This is necessary for self consistency since we obtained the beam self fields Eqs. (A-26) assuming a circular beam cross section.

In the case of constant fields Eqs. (11a,b) are elementary. For this case we have

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} F/\omega_x^2 \\ 0 \end{pmatrix} + \sum_{j=1}^{4} C_j \begin{pmatrix} \sqrt{\omega_y^2 - \omega_j^2} \\ \sqrt{\omega_j^2 - \omega_x^2} \end{pmatrix} e^{i\omega_j t} \tag{13}$$

where the eigenfrequencies (frequencies of betatron oscillations) are given by

$$\omega_j = \pm\left[\frac{\omega_x^2 + \omega_y^2 + \Omega_{\theta 0}^2 \pm [(\omega_x^2 + \omega_y^2 + \Omega_{\theta 0}^2)^2 - 4\omega_x^2\omega_y^2]^{1/2}}{2}\right]^{1/2} \tag{14}$$

and where the C_j, $j = 1, 2, 3, 4$ are constants.

Stability conditions result in the usual way by requiring $\omega^2 > 0$. We postpone examination of these conditions, however, until the following section. We note here only that for values of γ above a value dependent on geometry (r_b, a, r_0, n) but *not* on beam density, the self field contributions to ω_x^2 and ω_y^2 fall off as γ_0^{-1}, rather than γ_0^{-3}. For whole beam motion the value of γ at which the ν/γ_0 terms become comparable to the $r_b^2\, n_s/a^2$ term can be modest ($\gamma \sim 10$) for typical laboratory parameters ($r_b = 1$ cm, $a = 10$ cm, $r_0 = 100$ cm, $n = 0.5$).

The particular solution in Eq. (13) represents physically for a particle motion a first order radial shift of a particle which, while located initially at the reference orbit $(r_0, 0)$ does not have the correct energy to be maintained there by the local vertical magnetic field. It therefore moves in or out slightly depending on the sign of the energy mismatch. If, however, the radial focussing forces, represented by ω_x^2, happen to vanish the behavior becomes secular (no equilibrium radius exists).

The solution to the homogeneous part of Eqs. (11a,b) also becomes secular when $\omega_x^2 = 0$. In fact, when $\omega_x^2 = 0$ and $\omega_y^2 \neq \omega_x^2$ $(n \neq n^*)$, the point $\omega_x^2 = 0$ corresponds to a turning point (transition from stable to unstable behavior) in the WKB solution presented in the next section. Since ω_x^2 for particle motion *will pass through zero* during acceleration, it becomes important to examine the behavior of the solutions to Eqs. (11a,b) for time dependent fields. In general, for slowly time varying fields, a numerical solution to Eqs. (11a,b) over the entire acceleration cycle is prohibitive since the numerical integration time step must be small compared to $\Omega_{\theta 0}^{-1}$ which in turn is extremely small compared to typical acceleration times. An explicit solution for this case is therefore essential.

IV. MOTION OF BEAM IN SLOWLY VARYING EXTERNAL FIELDS

A. Stability Considerations

If the coefficients of the derivatives of x and y in Eqs. (11a,b) are slowly varying during a period of a betatron oscillation, the equations may be solved by the WKB method. (See Appendix C.) To leading order the solution is

$$\begin{pmatrix} x \\ y \end{pmatrix} \sim \frac{1}{\omega_\Delta} \sum_{j=1}^{4} \frac{A_j}{\omega_j^{1/2}} \begin{pmatrix} (\omega_y^2 - \omega_j^2)^{1/2} \\ (\omega_j^2 - \omega_x^2)^{1/2} \end{pmatrix} e^{i\int^t \omega_j dt'} + \int^t dt' \begin{bmatrix} K_x(t,t') \\ K_y(t,t') \end{bmatrix} F(t') \qquad (15)$$

where the eigenfrequencies are those given in (14) in which now all quantities may depend on time,

$$\omega_\Delta \equiv [(\omega_x^2 + \omega_y^2 + \Omega_{\theta 0}^2)^2 - 4\omega_x^2\omega_y^2]^{1/4}, \qquad (16)$$

and where the kernels $K_x(t,t')$ and $K_y(t,t')$ are given in Appendix C. The A_j, $j = 1, 2, 3, 4$ are constants in this approximation.

This solution, Eq. (16), is valid far from any turning point, i.e. where any ω_j vanishes. Turning points will occur if $\omega_x^2\,\omega_y^2 = 0$ and if $\omega_x^2 \neq \omega_y^2$. (See below.) Initially we shall confine attention to a cold beam (no longitudinal momentum spread) for which the particular solution in (15) vanishes identically. Later we shall comment on the effect of temperature.

The solution is unstable (exponentially growing) in time for such times that $\mathrm{Im}(\omega_j) < 0$ for any j. Unstable behavior will occur therefore whenever either of the following conditions is violated:

$$\omega_x^2\omega_y^2 > 0 \tag{17a}$$

$$\omega_x^2 + \omega_y^2 + \Omega_{\theta 0}^2 > 2(\omega_x^2\omega_y^2)^{1/2} \tag{17b}$$

For $n = n_{\mathrm{cir}}$ $(\omega_x^2 = \omega_y^2)$ inequality (17a) is trivial and (17b) gives the simplified stability condition:

$$\Omega_{\theta 0}^2 > \max(0, -4\omega_x^2) \tag{18}$$

If $n \neq n_{\mathrm{cir}}$ then both conditions (17a,b) must be simultaneously satisfied for stability. Condition (17a) in particular *cannot always be satisfied.* At injection n_s is typically quite large and both ω_x^2 and ω_y^2 for particle motion (and perhaps for beam motion) are negative. During acceleration, as γ_0 increases n_s decreases $(n_s \sim \gamma_0^{-3})$ and ω_x^2 and ω_y^2 change sign (for different values of γ_0, if $n \neq n_{\mathrm{cir}}$); an instability "gap" therefore exists while ω_x^2 and ω_y^2 have opposite signs.

It is important to point out that ω_x^2 and ω_y^2 for beam center motion (Re: Table I) may start out and remain positive throughout the injection-acceleration cycle while ω_x^2 and ω_y^2 for particle motion change sign. We recall from Table I that the small quantity $(r_b/a)^2$ multiplies n_s in the expressions for ω_x^2 and ω_y^2 for beam center motion but not for single particle motion. Therefore unless n_s is extremely large initially, beam center motion will remain stable.

The inequalities Eq. (17a-b) are illustrated graphically in Fig. 3. The stable regions of the $\left(\dfrac{\omega_y}{\Omega_{\theta 0}}\right)^2, \left(\dfrac{\omega_x}{\Omega_{\theta 0}}\right)^2$ plane are those shaded regions I and II in the figure. After injection but before acceleration both $\left(\dfrac{\omega_y}{\Omega_{\theta 0}}\right)^2$ and $\left(\dfrac{\omega_x}{\Omega_{\theta 0}}\right)^2$ for particle motion are negative and in region I. In this region the toroidal magnetic field is essential for stability (modified betatron regime). Following acceleration both $\left(\frac{\omega_y}{\Omega_{\theta 0}}\right)^2$ and $\left(\frac{\omega_x}{\Omega_{\theta 0}}\right)^2$ are positive, i.e., in region II in which the toroidal field is no longer required for stability (conventional betatron regime). Only by passing precisely through the origin (e.g., trajectory b in Fig. 3) can instability be avoided altogether. While the size of the instability gap does not depend on the magnitude of $B_{\theta 0}$ the value of $\mathrm{Im}(\omega_j)$ in the gap does and is inversely proportional to $B_{\theta 0}$. Therefore by choosing a sufficiently large toroidal field it should be possible to pass through the instability gap safely (within a few growth times, or less).

We may be quantitative for a case in which toroidal effects may be neglected: When Eq. (17a) is violated and if $\Omega_{\theta 0}^2 >> |\omega_x^2|, |\omega_y^2|$ then for the unstable mode, from Eq. (14),

$$\mathrm{Im}\,\omega_j \approx \frac{\sqrt{-\omega_x^2\omega_y^2}}{\Omega_{\theta 0}} \tag{19}$$

which has a peak value, assuming only γ_0 and not $B_{\theta 0}$ is changing in time, of

$$\Omega_{z0}\left(\frac{B_{z0}}{B_{\theta 0}}\right)|n - 1/2| \equiv \tau_g^{-1}. \tag{20}$$

If

$$\int_{t_1}^{t_2} dt\,\mathrm{Im}\,\omega_j \equiv \int_{\gamma_1}^{\gamma_2} \mathrm{Im}\,\omega_j \frac{d\gamma_0}{\dot{\gamma}_0} << 1 \tag{21}$$

where t_1 and t_2 are the times at which the instability gap is entered and exitted, respectively, then one expects that the transit through the gap will not significantly disrupt the beam; Eq. (21) translates into a constraint on $\dot{\gamma}_0$:

$$\frac{\dot{\gamma}_0}{\gamma_0} >> \frac{\pi}{3}\,\Omega_{z0}\frac{B_{z0}}{B_{\theta 0}}(n - 1/2)^2. \tag{22}$$

If the acceleration is fast enough to satisfy Eq. (22) particle motion will be essentially unaffected by passage through the gap. It should be possible to choose a machine design (i.e., a sufficiently large toroidal field and a field index close to 1/2) so that Eq. (22) is well satisfied.

The instability which occurs while $\omega_x^2\omega_y^2 < 0$ has an interesting dynamical origin. Let us consider the equations of motion, Eqs. (11a,b), taking $F = 0$, and taking the external fields to be constant in time:

$$\ddot{x} + \omega_x^2 x = \Omega_{\theta 0}\dot{y} \tag{23a}$$

$$\ddot{y} + \omega_y^2 y = -\Omega_{\theta 0}\dot{x} \tag{23b}$$

These equations are just those governing the motion of a particle in an effective electric field

$$E_x^{\text{eff}} = \frac{m}{e}\,\omega_x^2 x \tag{24a}$$

$$E_y^{\text{eff}} = \frac{m}{e}\,\omega_y^2 y \tag{24b}$$

and a magnetic field $B_{\theta 0}/\gamma_0$. Converting to polar coordinates ρ, ϕ we have

$$E_\rho^{\text{eff}} = \frac{m}{e}\,\rho\,[\omega_x^2 \cos^2\phi + \omega_y^2 \sin^2\phi] \tag{25a}$$

$$E_\phi^{\text{eff}} = \frac{m}{e}\,\rho\,[\omega_y^2 - \omega_x^2]\,\sin\phi\cos\phi. \tag{25b}$$

The particle behavior may be understood as follows. Let us assume that $n > 1/2$, from which it follows that $\omega_y^2 > \omega_x^2$ always, and let us consider first the modified betatron regime $(\omega_x^2 < 0,\ \omega_y^2 < 0)$. E_ρ^{eff} in this regime is everywhere negative thereby giving rise to a *clockwise* $\vec{E} \times \vec{B}$ drift, assuming $B_{\theta 0}$ is positive. E_ϕ^{eff}, which is much smaller in magnitude than E_ρ^{eff}, gives a radial drift of alternating sign as the particle moves from quadrant to quadrant, thereby producing an elliptical orbit. Stable motion is established by balancing the outward radial electrostatic + outward centrifugal forces against the $\vec{V} \times \vec{B}$ confining force.

In the conventional betatron regime $\omega_x^2 > 0$, $\omega_y^2 > 0$ and the sign of E_ρ^{eff} is reversed. Azimuthal particle drift is now *counter-clockwise* and the major axis of the elliptical orbit is rotated by 90°. Stable motion is achieved by balancing the inward radial electrostatic force against the centrifugal force; the toroidal field is no longer needed.

In the instability gap E_ρ^{eff} has zeroes at polar angles given by

$$\cos^2\phi_0 = \left(1 - \frac{\omega_x^2}{\omega_y^2}\right)^{-1} \tag{26}$$

at which points the azimuthal drift velocity vanishes. The radial drift velocity, $cE_\phi^{\text{eff}}/B_\theta$, cannot also vanish at the same point. Consequently the particle drifts radially, with increasing velocity, since

$E_\phi^{\rm eff} \sim \rho$, at the angle ϕ_0, as long as $\omega_x^2\, \omega_y^2 < 0$. Increasing the toroidal B field, thereby reducing the radial drift velocity, reduces the growth rate of this instability, a fact reflected in Eq. (19).

Typical orbits during transit of the instability gap are illustrated for a simple case in Figs. 4 and 5 in which results of a numerical integration of Eqs. (11a,b) are plotted. In Fig. 4 condition (21) is not well satisfied. The dramatic drift direction reversal and instability are evident. In Fig. 5 condition (21) is well satisfied (n is near 1/2); particle motion is virtually unaffected, except for the reversal of drift direction, by passage through the gap. The two graphs, in Figs. 4 and 5 differ only by the value of n used; all other external parameters and total integration time are identical.

So far no mention has been made of the effect of temperature, the inhomogeneous term in Eqs. (11a,b), on particle orbit behavior in or near the instability gap. Particles having an energy mismatch—either too little or too much energy to be maintained at the reference orbit by the local vertical field—will seek out their new equilibrium orbits about which they will execute betatron oscillators. Secular behavior is expected, as discussed earlier, when ω_x^2 vanishes.

The effect of energy mismatch on a particle orbit is illustrated in Fig. 6 where the particle of Fig. 5 has been given an energy mismatch of

$$\frac{P_{\theta 1} - \langle P_{\theta 1} \rangle}{m r_0 c} \approx \gamma_1 - \langle \gamma_1 \rangle = 0.10.$$

The effect is twofold. The orbit center shifts slightly outward and the amplitude of betatron oscillations following passage through the instability gap has increased by a factor of ~ 35 over the zero mismatch case. Such a large expansion of the particle orbits cannot, in fact, be reliably computed using the linearized Eqs. (11a,b) used here. One non-linear effect in particular, namely the reduction of beam density during the orbit expansion, will clearly speed the passage of a particle through the instability gap. (Recall that n_s is proportional to density.) Due to this density reduction the actual degree of orbit expansion to be anticipated in a real device is likely to be significantly less than that seen in Fig. 6. Still, these calculations suggest that a fairly cold beam will be required for successful acceleration through the instability gap. Poorly "matched" particles are likely to be lost as ω_x^2 goes through zero. It should be pointed out as well that a strong toroidal field greatly reduces the effects of energy mismatch.

The computer runs necessarily employ a very modest toroidal field (600 gauss in the case of Figs. 4-6) due to time step considerations. A stronger field, by further restricting radial motion, is expected to improve the confinement properties of a warm beam.

B. Adiabatic Behavior

Let us next briefly consider, using the solutions to the equations of motion, Eq. (15), the effects on the particle orbits of the removal of the toroidal magnetic field. The toroidal field may need to be removed in order to facilitate beam extraction though this may not be essential. Let us assume that Eq. (15) is valid throughout the acceleration cycle, i.e. that ω_x^2 and ω_y^2 pass through zero simultaneously and that the solution to the homogeneous equation (the sum in Eq. (15)) dominates the solution. This is certainly true for matched particles ($P_\theta = \langle P_\theta \rangle = 0$) when $n = 1/2$ and when toroidal effects may be neglected ($\nu/\gamma \ll 1$). One may show, using Eq. (15) for such a case, that for beam center motion in either the fast or slow oscillation mode

$$\frac{[(\Delta r)^2 + (\Delta z)^2]_f}{[(\Delta r)^2 + (\Delta z)^2]_i} = \left[\frac{\left[\left(\frac{1}{2} - \frac{r_b^2}{a^2} n_s\right) B_{z0}^2 + \frac{1}{4} B_{\theta 0}^2\right]_i}{\left[\left(\frac{1}{2} - \frac{r_b^2}{a^2} n_s\right) B_{z0}^2 + \frac{1}{4} B_{\theta 0}^2\right]_f}\right]^{1/2} \tag{27}$$

while for particle motion about the beam center[6]

$$\frac{[(\delta r)^2 + (\delta z)^2]_f}{[(\delta r)^2 + (\delta z)^2]_i} = \left[\frac{\left[\left(\frac{1}{2} - n_s\right) B_{z0}^2 + \frac{1}{4} B_{\theta 0}^2\right]_i}{\left[\left(\frac{1}{2} - n_s\right) B_{z0}^2 + \frac{1}{4} B_{\theta 0}^2\right]_f}\right]^{1/2} \tag{28}$$

where the subscripts i and f correspond to any initial and final states. The latter expression, Eq. (28), may be interpreted as the fractional change in beam cross sectional area. Note that for large B_θ the area of the orbits $\sim B_\theta^{-1}$, as expected.

Expressions for these ratios in the case that toroidal effects are not negligible and $n \neq 1/2$ may be obtained from Eq. (15). The expressions are complicated, however, and will not be cited here.

As a numerical example we consider a 1 kA beam of 1 cm initial radius in an initial state corresponding to $\gamma_i = 7$, $B_{z0,i} = 120$ g, $B_{\theta 0,i} = 1.5$ kg and a final state with $\gamma_f = 100$, $B_{z0,f} = 1.7$ kg, and $B_{\theta 0,f} = 0$. In such a case Eq. (27) gives for the orbital area ratio a value of 0.63 while Eq. (28) gives for the ratio of beam cross sectional areas a value of 0.60.

We conclude that it should be possible both to accelerate the beam and to remove the toroidal field to facilitate beam ejection without causing either the beam orbit or individual particle orbits to expand without limit.

V. CONCLUSIONS

The beam in a modified betatron can be stably confined both during the acceleration phase and during the subsequent gradual removal of the toroidal magnetic field prior to beam ejection. As the beam is accelerated, however, unless very special conditions are satisfied, a region of instability will be passed through; however if the time of transit through this instability gap is small compared to the time specified in Eq. (20) the net effect should be small.

As the toroidal field is removed to facilitate beam extraction following acceleration no further instability gaps occur but the magnitude of the beam betatron oscillations will change adiabatically. By arranging that the ratios, Eqs. (28,29), be near one, one expects the beam to be well behaved during the removal of the toroidal field.

It should be remarked however that changing the toroidal field changes the "tune" of the betatron which, in general, will necessitate the passage through orbital resonances as the toroidal field is removed. These resonances, due to the periodic encounter by a particle of a field error or "bump" are currently under investigation. It is anticipated that a condition governing the minimum speed with which B_θ must be removed, expressed as a function of the magnitude of the field error, will be obtained.

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Appendix A

FIELDS IN THE MODIFIED BETATRON

In this appendix we calculate the fields seen by a particle in a modified betatron. The particle is assumed to be close to the axis of the torus, that is, the coordinates of the particle are taken to be (refer to Figs. 1 and 2 in the text)

$$(r,z) = (r_0 + \Delta r + \delta r, \Delta z + \delta z)$$

and all fields will be calculated to first order in Δr, δr, Δz, and δz. Fields will be given in the (r, θ, z) coordinate system of Fig. 1 and all will be assumed to be independent of θ. Superscripts a and s will be used below to denote applied and self fields, respectively.

Part I (Applied Fields)

Magnetic Field

The usual weak focussing betatron field has r and z components. The z component is taken to behave near r_0 as

$$B_z^a \approx B_{z0}(r_0/r)^n \approx B_{z0}\left(1 - n\frac{\Delta r + \delta r}{r_0}\right) \tag{A-1}$$

where B_{z0} depends only on time and n, taken as a constant to this order, is the so-called vacuum field index. The radial field is obtained by requiring $(\vec{\nabla} \times \vec{B})_\theta = 0$ and $B_r(z = 0) = 0$ (making the $z = 0$ plane a plane of symmetry). The result is:

$$B_r^a \approx -nB_{z0}\left(\frac{\Delta z + \delta z}{r_0}\right). \tag{A-2}$$

The applied toroidal field generally falls off as r^{-1} across the minor cross section of the torus:

$$B_\theta^a \approx B_{\theta 0}\left(1 - \frac{\Delta r + \delta r}{r_0}\right)$$

where $B_{\theta 0}$ depends only on time. However, in the equations of motion B_θ multiplies only $\dot{r}$ and $\dot{z}$ terms which are already first order. Therefore the gradient of B_θ does not enter the linearized equations of motion and we take only the zero order value,

$$B_\theta^a \simeq B_{\theta 0} \tag{A-3}$$

Electric Field

All applied electric fields are inductive. The toroidal electric field is governed by the changing central flux and is taken to be a specified function of time

$$E_\theta^a = E_\theta(t) \tag{A-4}$$

E_θ is negative for electron acceleration with B_{z0} positive.

Changing the toroidal magnetic field, $B_{\theta 0}$, will induce a poloidal electric field, the r and z components of which are easily found:

$$E_r^a = -\frac{1}{2c}\dot{B}_{\theta 0}(\Delta z + \delta z) \tag{A-5}$$

$$E_z^a = \frac{1}{2c}\dot{B}_{\theta 0}(\Delta r + \delta r) \tag{A-6}$$

where a dot indicates a time derivative.

Part II (Self Fields)

Since we neglect beam diamagnetism and the possibility of a change in self flux due to time varying beam current we take $B_\theta^s = E_\theta^s = 0$. It remains to calculate the r and z components of the beam self electric and magnetic fields.

Consider a beam circulating inside a perfectly conducting toroidal chamber of circular cross section as shown in Fig. A-1. (The beam displacement is exaggerated for clarity; we will assume $\Delta << r_b$). The chamber major and minor radii are r_0 and a respectively. The beam major and minor radii are R_b and r_b respectively. We must calculate the fields *inside* the beam ($\rho < r_b$), assuming the chamber is a perfect conductor. To proceed we define a scalar potential $\Phi(\rho, \phi)$ and a magnetic flux or stream function $\Psi(\rho, \phi) \equiv rA_\theta$ where A_θ is the usual vector potential. The equations for Φ and Ψ are

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2}=4\pi e n_0(\rho,\phi)-\frac{\left[\cos\phi\dfrac{\partial\Phi}{\partial\rho}-\dfrac{1}{\rho}\sin\phi\dfrac{\partial\Phi}{\partial\phi}\right]}{R_b+\rho\cos\phi} \tag{A-7}$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Psi}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2\Psi}{\partial\phi^2}=-\frac{4\pi}{c}(R_b+\rho\cos\phi)J_\theta(\rho,\phi)+\frac{\left[\cos\phi\dfrac{\partial\Psi}{\partial\rho}-\dfrac{1}{\rho}\sin\phi\dfrac{\partial\Psi}{\partial\phi}\right]}{R_b+\rho\cos\phi} \tag{A-8}$$

where n_0, the beam number density and J_θ, the beam current density, are assumed to have been specified. Here we shall take both n_0 and J_θ constant, independent of ρ and ϕ.

The boundary conditions on Φ and Ψ are the same; they both must vanish at the surface of the chamber, specified by

$$\rho \approx a - \Delta\cos(\psi-\phi), \tag{A-9}$$

correct to first order in Δ/a.

Scalar Potential and Electric Field

The general solution for Φ, including the first toroidal correction, is

$$\Phi=\begin{cases}\Phi_0+q\left(1-\rho^2/r_b^2\right)+\dfrac{q\rho^3}{4R_b r_b^2}\cos\phi+A\dfrac{\rho}{r_b}\sin\phi+B\dfrac{\rho}{r_b}\cos\phi & \rho<r_b\\[2ex] \Phi_0-2q\ln\rho/r_b+\dfrac{q\rho}{R_b}\ln\rho/r_b\cos\phi+\left(A'\dfrac{\rho}{r_b}+C'\dfrac{r_b}{\rho}\right)\sin\phi & \\[2ex] \quad+\left(B'\dfrac{\rho}{r_b}+D'\dfrac{r_b}{\rho}\right)\cos\phi & \rho>r_b\end{cases} \tag{A-10}$$

where $q \equiv -en_0\pi r_b^2$ and Φ_0, A, B, A', B', C', and D' are constants.

Applying now the correct boundary conditions both at the beam surface and the wall determines all of the constants:

$$\Phi_0 = 2q\ln a/r_b \tag{A-11a}$$

$$A = A' = -2q\frac{\Delta}{a}\frac{r_b}{a}\sin\psi \tag{A-11b}$$

$$B = B' = -q\frac{r_b}{R_b}\ln a/r_b - \frac{qr_b^3}{4R_b a^2} - 2q\frac{\Delta}{a}\frac{r_b}{a}\cos\psi \tag{A-11c}$$

$$C' = 0 \tag{A-11d}$$

$$D' = \frac{q}{4}\,\frac{r_b}{R_b}. \tag{A-11e}$$

Using this result in Eq. (A-10) we may calculate the r and z components of $\overline{E}^s$ inside the beam, to first order:

$$\begin{aligned} E_r^s &= -\frac{\partial\Phi}{\partial\rho}\cos\phi + \frac{1}{\rho}\,\frac{\partial\Phi}{\partial\phi}\sin\phi \\ &= \frac{2q}{r_b^2}\left[\delta r + \frac{r_b^2}{a^2}\Delta r\right] + \frac{q}{R_b}\ln\frac{a}{r_b} \end{aligned} \tag{A-12}$$

$$\begin{aligned} E_z^s &= -\frac{\partial\Phi}{\partial\rho}\sin\phi - \frac{1}{\rho}\,\frac{\partial\Phi}{\partial\phi}\cos\phi \\ &= \frac{2q}{r_b^2}\left[\delta z + \frac{r_b^2}{a^2}\Delta z\right] \end{aligned} \tag{A-13}$$

where $(\delta r,\ \delta z) \equiv \rho\ (\cos\phi,\ \sin\phi)$ and $(\Delta r,\ \Delta z) \equiv \Delta\ (\cos\psi,\ \sin\psi)$.

Magnetic Flux (or Stream) Function and Magnetic Field

The general solution for Ψ, including the first toroidal correction, is

$$\Psi = \begin{cases} \Psi_0 + Q\left(1 - \rho^2/r_b^2\right) - \dfrac{3}{4}\,\dfrac{Q}{R_b}\,\dfrac{\rho^3}{r_b^2}\cos\phi + \overline{A}\,\dfrac{\rho}{r_b}\sin\phi + \overline{B}\,\dfrac{\rho}{r_b}\cos\phi & \rho < r_b \\ \Psi_0 - 2Q\ln\rho/r_b - \dfrac{Q\rho}{R_b}\ln\rho/r_b\cos\phi + \left(\overline{A}'\dfrac{\rho}{r_b} + \overline{C}'\dfrac{r_b}{\rho}\right)\sin\phi & \\ \quad + \left(\overline{B}'\dfrac{\rho}{r_b} + \overline{D}'\dfrac{r_b}{\rho}\right)\cos\phi & \rho > r_b \end{cases} \tag{A-14}$$

where $Q \equiv \pi r_b^2 J_\theta R_b/c = -\pi r_b^2 e n_0 \beta_0 R_b$, $\beta_0 = V_{\theta 0}/c$, and Ψ_0, $\overline{A}$, $\overline{B}$, $\overline{A}'$, $\overline{B}'$, $\overline{C}'$, and $\overline{D}'$ are constants.

Applying the boundary conditions gives

$$\Psi_0 = 2Q\ln\frac{a}{r_b} \tag{A-15a}$$

$$\overline{A} = \overline{A}' = -2Q\frac{\Delta}{a}\,\frac{r_b}{a}\sin\psi \tag{A-15b}$$

$$\bar{B} = \bar{B}' + Q\frac{r_b}{R_b}$$

$$= -2Q\frac{\Delta}{a}\frac{r_b}{a}\cos\psi + Q\frac{r_b}{R_b}\ln\frac{a}{r_b} + Q\frac{r_b}{R_b} - \frac{1}{4}\frac{Q}{R_b}\frac{r_b^3}{a^2} \quad \text{(A-15c)}$$

$$\bar{C}' = 0 \quad \text{(A-15d)}$$

$$\bar{D}' = \frac{1}{4}Q\frac{r_b}{R_b}. \quad \text{(A-15e)}$$

The resulting magnetic field, to first order, is

$$B_r^s = -\frac{1}{r}\left(\frac{\partial\Psi}{\partial\rho}\sin\phi + \frac{\partial\Psi}{\partial\phi}\frac{\cos\phi}{\rho}\right)$$

$$= -2\pi e n_0 \beta_0\left(\delta z + \frac{r_b^2}{a^2}\Delta z\right) \quad \text{(A-16)}$$

$$B_z^s = \frac{1}{r}\left(\frac{\partial\Psi}{\partial\rho}\cos\phi - \frac{\partial\Psi}{\partial\phi}\frac{\sin\phi}{\rho}\right)$$

$$= 2\pi e n_0 \beta_0\left(\delta r + \frac{r_b^2}{a^2}\Delta r - \frac{r_b^2}{2R_b}\left(\ln\frac{a}{r_b} + 1\right)\right). \quad \text{(A-17)}$$

If the circulation frequency, $\dot{\theta}_0$, rather than the current itself, is taken to be constant across the beam (current $\sim r$) then it is straightforward to show that $\ln\frac{a}{r_b} + 1$ in Eq. (A-17) is replaced by $\ln\frac{a}{r_b} + 2$.

If the magnetic field of the beam has diffused completely through the wall then the field surrounding the beam is most directly calculated using the free space Green's function:

$$\vec{A}(\vec{r}) = \frac{1}{c}\int d\vec{r}'\,\frac{\vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|}. \quad \text{(A-18)}$$

If $\vec{J} = J_\theta\hat{\theta}$ is constant across the beam and J_θ is independent of θ then $\vec{A} = A_\theta\hat{\theta}$ where

$$A_\theta(r, z) = \frac{J_\theta}{c}\int_{R_b - r_b}^{R_b + r_b} r'dr' \int_0^{2\pi} d\theta' \int_{-z_b(r')}^{z_b(r')} dz' \frac{\cos\theta'}{\left[r^2 + r'^2 - 2rr'\cos\theta' + (z - z')^2\right]^{1/2}}$$

$$z_b(r') = \left[r_b^2 - (r' - R_b)^2\right]^{1/2}. \quad \text{(A-19)}$$

The integral over θ' may be expressed in terms of the complete elliptic integrals[1]

$$A_\theta = \frac{4J_\theta}{c} \int_{R_b-r_b}^{R_b+r_b} r'dr' \int_{-z_b(r')}^{z_b(r')} dz' \frac{1}{\left[(r-r')^2+(z-z')^2\right]^{1/2}} \left[(1+\frac{2}{m})K(-m) - \frac{2}{m}E(-m)\right] \quad \text{(A-20)}$$

where

$$m = \frac{4rr'}{(r-r')^2+(z-z')^2}.$$

In the beam interior m is large. Using the asymptotic expansions for K and E one may show that

$$(1+\frac{2}{m})K(-m) - \frac{2}{m}E(-m) \sim \frac{1}{2}m^{-1/2}(\ln m + 4\ln 2 - 4). \quad \text{(A-21)}$$

Using Eq. (A-21) in Eq. (A-20) the resulting integrals are elementary. The result, for the vector potential inside the beam is

$$A_\theta^s = \frac{I}{c}\left[2\ln\left[\frac{8R_b}{e^2 r_b}\right] + 1 - \rho^2/r_b^2 + \frac{\rho}{R_b}\cos\phi\left[-\ln\frac{8R_b}{r_b} + 3\right]\right] \quad \text{(A-22)}$$

where $I = \pi r_b^2 J_\theta$, from which it follows that the fields inside the beam, to first order in ρ, are

$$B_r^s = \frac{2I}{c}\frac{\rho}{r_b^2}\sin\phi \quad \text{(A-23)}$$

$$B_z^s = \frac{I}{cr_b}\left[\frac{r_b}{R_b}\ln\frac{8R_b}{r_b} - 2\frac{\rho\cos\phi}{r_b}\right]. \quad \text{(A-24)}$$

We may summarize all of the foregoing results as follows:

The applied fields are,

$$B_r^a = -nB_{z0}\frac{\Delta z + \delta z}{r_0} \quad \text{(A-25a)}$$

$$B_\theta^a = B_{\theta 0} \quad \text{(A-25b)}$$

$$B_z^a = B_{z0}\left[1 - n\left(\frac{\Delta r + \delta r}{r_0}\right)\right] \quad \text{(A-25c)}$$

$$E_r^a = -\frac{1}{2c}\dot{B}_{\theta 0}(\Delta z + \delta z) \quad \text{(A-25d)}$$

$$E_\theta^a = E_{\theta 0} \quad \text{(A-25e)}$$

[1] Handbook of Mathematical Functions, M. Abramowitz and I. Stegun, eds. Dover Publications, ch 17.

$$E_z^a = \frac{1}{2c}\, \dot{B}_{\theta 0}\, (\Delta r + \delta r) \tag{A-25f}$$

where B_{z0}, $B_{\theta 0}$, and $E_{\theta 0}$ are taken to be prescribed functions of time.

The self fields are:

$$B_r^s = -2\pi n_0 e \beta_0 \left[\delta z + \frac{r_b^2}{a^2}\Delta z\right] \tag{A-26a}$$

$$B_\theta^s = 0 \tag{A-26b}$$

$$B_z^s = 2\pi n_0 e \beta_0 \left[\delta r + \frac{r_b^2}{a^2}\Delta r - \frac{r_b^2}{2r_0}\, l_B\right] \tag{A-26c}$$

$$E_r^s = -2\pi n_0 e \left[\delta r + \frac{r_b^2}{a^2}\Delta r + \frac{r_b^2}{2r_0}\, l_E\right] \tag{A-26d}$$

$$E_\theta^s = 0 \tag{A-26e}$$

$$E_z^s = -2\pi n_0 e \left[\delta z + \frac{r_b^2}{a^2}\Delta z\right] \tag{A-26f}$$

where

$$l_B = \begin{cases} \ln \dfrac{a}{r_b} + 2 \text{ if circulation frequency, } \dot{\theta}, \text{ is constant across the beam} \\ \ln \dfrac{a}{r_b} + 1 \text{ if current density is constant across the beam} \end{cases} \tag{A-27}$$

$$l_E = \ln \frac{a}{r_b} \text{ if density is constant across the beam.} \tag{A-28}$$

For times long compared to the time it takes the magnetic field to diffuse through the chamber wall the result (A-24) shows that one must replace a in the logarithm in the definition of l_B by $(8\, r_0/e) \approx 2.9 r_0$. This suggests that fields in an actual device may have to be programmed in time to compensate for this extra change (reduction) in B_z, in order to hold the beam in place.

Appendix B

LINEARIZED EQUATIONS OF MOTION FOR A PARTICLE IN THE MODIFIED BETATRON

In this appendix the equations of motion for a particle in the fields (A-25) and (A-26) are obtained, correct to first order in small quantities.

The complete equations of motion are

$$\frac{d}{dt}(\gamma\dot{r}) - \gamma r\dot{\theta}^2 = -\frac{e}{m}\left[E_r + \frac{1}{c}(r\dot{\theta}B_z - \dot{z}B_\theta)\right] \tag{B-1}$$

$$\frac{1}{r}\frac{d}{dt}(\gamma r^2\dot{\theta}) = -\frac{e}{m}\left[E_\theta + \frac{1}{c}(\dot{z}B_r - \dot{r}B_z)\right] \tag{B-2}$$

$$\frac{d}{dt}(\gamma\dot{z}) = -\frac{e}{m}\left[E_z + \frac{1}{c}(\dot{r}B_\theta - r\dot{\theta}B_r)\right]. \tag{B-3}$$

We consider first the linearization of Eq. (B-2). This equation has an exact first integral, assum ing the fields do not depend on θ; it is the canonical angular momentum

$$P_\theta \equiv r\left[m\gamma r\dot{\theta} - \frac{e}{c}A_\theta\right]. \tag{B-4}$$

We now write all quantities Q as $Q = Q_0 + Q_1$ where $Q_1 << Q_0$ and Q_0 refers to quantities evaluated at the reference orbit $(r, z) = (r_0, 0)$. Defining $V_\theta = r\dot{\theta}$ it is straightforward to show from Eq. (B-4) that

$$V_{\theta 1} = \frac{P_{\theta 1}}{m\gamma_0^3 r_0} - \frac{r_1}{\gamma_0^2}\left[\dot{\theta}_0 - \frac{eB_{z0}^{(0)}}{m\gamma_0 c}\right] \tag{B-5}$$

where

$$B_{z0}^{(0)} = B_{z0} - \pi n_0 e\beta_0\frac{r_b^2}{r_0} l_B$$

and where we have used $\gamma_1 = V_{\theta 0}\gamma_0^3 V_{\theta 1}/c^2$. Now, using the expression for $\dot{\theta}_0$ in Eq. (7), one obtains

$$V_{\theta 1} = \frac{P_{\theta 1}}{m\gamma_0^3 r_0} + \frac{r_1}{\gamma_0^2}\,\frac{\nu}{\gamma_0}\,\frac{1}{\alpha^2}\Omega_{z0}\, l_E \tag{B-6}$$

where

$$\frac{\nu}{\gamma_0} = \frac{1}{\gamma_0}\left[\pi r_b^2 n_0 \frac{e^2}{mc^2}\right] = \frac{1}{4}\,\frac{\omega_b^2 r_b^2}{c^2}$$

$$\omega_b^2 = 4\pi n_0 e^2/m\gamma_0$$

$$\alpha = \Omega_{z0} r_0/c.$$

The expression Eq. (B-6) will be needed next in the linearization of the radial equation, (B-1). Carrying out a straightforward linearization of Eq. (B-1), using the zero order fields from Eqs. (A-25b,c) and Eqs. (A-26c,d) gives

$$\ddot{r}_1 = -\frac{e}{m\gamma_0}\left[E_{r1} + \frac{V_{\theta 0}}{c}B_{z1}\right] + \Omega_{\theta 0}\dot{z}_1 - \frac{\dot{\gamma}_0}{\gamma_0}\dot{r}_1 - \dot{\theta}_0^2 r_1$$
$$+ V_1\Omega_{z0}\gamma_0^2\left[1 - \frac{\nu}{\gamma_0}\left(\frac{1+\gamma_0^{-2}}{\alpha^2}\, l_E + l_B\right)\right] \tag{B-7}$$

where

$$\Omega_{\theta 0} = eB_{\theta 0}/m\gamma_0 c.$$

Using now Eq. (B-6) and keeping terms only to first order in ν/γ_0 and using Eqs. (A-25c,d) and Eqs. (A-26c,d) to write

$$-\frac{e}{m\gamma_0}\left[E_{r1} + \frac{V_{\theta 0}}{c}B_{z1}\right] = \frac{e\dot{B}_{\theta 0}}{2m\gamma_0 c}z_1 + \frac{\omega_b^2}{2\gamma_0^2}\left(\delta r + \frac{r_b^2}{a^2}\Delta r\right) + n\dot{\theta}_0\Omega_{z0}r_1 \tag{B-8}$$

we obtain our final result for the radial equation:

$$\ddot{r}_1 + \frac{\dot{\gamma}_0}{\gamma_0}\dot{r}_1 + \Omega_{z0}^2\left[1 - n - \frac{\nu}{\gamma_0}\left((3-n)\frac{1}{\alpha^2}\, l_E + (2-n)\, l_B\right)\right] r_1$$
$$= \frac{e\dot{B}_{\theta 0}}{2m\gamma_0 c}z_1 + \Omega_{\theta 0}\dot{z}_1 + n_s\Omega_{z0}^2\left(\delta r + \frac{r_b^2}{a^2}\Delta r\right)$$
$$+ \Omega_{z0}\frac{P_{\theta 1}}{\gamma_0 m r_0}\left[1 - \frac{\nu}{\gamma_0}\left(\frac{1+\gamma_0^{-2}}{\alpha^2}\, l_E + l_B\right)\right] \tag{B-9}$$

where $n_s \equiv \omega_b^2/2\gamma_0^2\Omega_{z0}^2$ is the "self field index."

The analogous linearization of the z equation (Eq. (B-3)) is completely straightforward, using the fields (A-25a,b,f) and (A-26a,b,f). The result is

$$\ddot{z}_1 + \frac{\dot{\gamma}_0}{\gamma_0}\dot{z}_1 + n\Omega_{z0}^2\left[1 - \frac{\nu}{\gamma_0}\left(\frac{1}{\alpha^2} l_E + l_B\right)\right]z_1$$

$$= -\frac{e\dot{B}_{\theta 0}}{2m\gamma_0 c} r_1 - \Omega_{\theta 0}\dot{r}_1 + n_s\Omega_{z0}^2\left(\delta r + \frac{r_b^2}{a^2}\Delta r\right). \qquad \text{(B-10)}$$

Appendix C

WKB SOLUTION OF EQUATIONS OF MOTION

The linearized equations of motion are given in the text, Eqs. (11a,b). Below we shall obtain first, an approximate solution to the homogeneous version of Eqs. (11a,b), assuming that all coefficients are slowly varying. We will then give the solution to the full, inhomogeneous equations.

The homogeneous equations are

$$\ddot{x} + \omega_x^2 x = \Omega_{\theta 0}\dot{y} + \frac{1}{2}\dot{\Omega}_{\theta 0} y \qquad \text{(C-1a)}$$

$$\ddot{y} + \omega_y^2 y = -\Omega_{\theta 0}\dot{x} - \frac{1}{2}\dot{\Omega}_{\theta 0} x \qquad \text{(C-1b)}$$

All coefficients, ω_x^2, ω_y^2, and $\Omega_{\theta 0}$ will be assumed to vary significantly only over a slow time scale. To carry out a formal asymptotic expansion then we define

$$\tau = t/\lambda$$

where λ is a large dimensionless parameter. Denoting $\frac{d}{d\tau}$ by a prime ($'$), Eqs. (C-1a,b) become

$$x'' + \lambda^2\omega_x^2 x = \lambda\Omega_{\theta 0} y' + \frac{\lambda}{2}\Omega_{\theta 0}' y \qquad \text{(C-2a)}$$

$$y'' + \lambda^2\omega_y^2 y = -\lambda\Omega_{\theta 0} x' - \frac{\lambda}{2}\Omega_{\theta 0}' x. \qquad \text{(C-2b)}$$

Now writing

$$x = a_1(\tau;\lambda)e^{i\lambda\int\omega(\tau')d\tau'} \qquad \text{(C-3a)}$$

$$y = a_2(\tau;\lambda)e^{i\lambda\int\omega(\tau')d\tau'} \qquad \text{(C-3b)}$$

we proceed to express a_1 and a_2 in formal asymptotic series:

$$a_1(\tau;\lambda) \sim \sum_{n=0}^{\infty}\frac{a_{1n}(\tau)}{\lambda^n} \qquad \text{(C-4a)}$$

$$a_2(\tau;\lambda) \sim \sum_{n=0}^{\infty}\frac{a_{2n}(\tau)}{\lambda^n}. \qquad \text{(C-4b)}$$

We must now find the a_{1n}, a_{2n}, and ω.

Substituting Eqs. (C-4a,b) in Eqs. (C-2a,b) one finds the leading order (λ^2) result

$$(\omega_x^2 - \omega^2)a_{10} - i\omega\Omega_{\theta 0}a_{20} = 0 \tag{C-5a}$$

$$i\omega\Omega_{\theta 0}a_{10} + (\omega_y^2 - \omega^2)a_{20} = 0 \tag{C-5b}$$

from which it follows that ω must be one of the four quantities:

$$\omega = \pm\left[\frac{\omega_x^2 + \omega_y^2 + \Omega_{\theta 0}^2 \pm \left\{\left[\omega_x^2 + \omega_y^2 + \Omega_{\theta 0}^2\right]^2 - 4\omega_x^2\omega_y^2\right\}^{1/2}}{2}\right]^{1/2}. \tag{C-6}$$

The next order (λ^1) relation may be written, after some manipulation, as

$$i\left[2\omega - \frac{\omega^2 - \omega_y^2}{\omega}\right]a'_{20} + i\left[\omega' - \frac{1}{2}\frac{\omega^2 - \omega_y^2}{\omega}\frac{\Omega'_{\theta 0}}{\Omega_{\theta 0}}\right]a_{20}$$

$$= \left[-\Omega_{\theta 0} + \frac{2}{\Omega_{\theta 0}}(\omega^2 - \omega_y^2)\right]a'_{10} + \left[-\frac{1}{2}\Omega'_{\theta 0} + \frac{\omega'}{\omega}\frac{(\omega^2 - \omega_y^2)}{\Omega_{\theta 0}}\right]a_{10}. \tag{C-7}$$

Using Eq. (C-5a) or Eq. (C-5b) and Eq. (C-7) an equation for just a_{10} (or just a_{20}) may be obtained. The solutions are

$$a_{10} = A\omega^{-1/2}(\omega_y^2 - \omega^2)^{1/2}(\omega_x^2 + \omega_y^2 + \Omega_{\theta 0}^2 - 2\omega^2)^{-1/2} \tag{C-8a}$$

$$a_{20} = A\omega^{-1/2}(\omega^2 - \omega_x^2)^{1/2}(\omega_x^2 + \omega_y^2 + \Omega_{\theta 0}^2 - 2\omega^2)^{-1/2} \tag{C-8b}$$

where A is an arbitrary complex constant.

Using Eq. (C-8a,b) and the definition of ω we may write the leading order WKB solution to Eqs. (C-1a,b) as

$$\begin{pmatrix} x \\ y \end{pmatrix} \sim \left[\left[\omega_x^2 + \omega_y^2 + \Omega_{\theta 0}^2\right]^2 - 4\omega_x^2\omega_y^2\right]^{-1/4} \sum_{j=1}^{4} \frac{A_j}{\omega_j^{1/2}} \begin{pmatrix} (\omega_y^2 - \omega_j^2)^{1/2} \\ (\omega_j^2 - \omega_x^2)^{1/2} \end{pmatrix} e^{i\int \omega_j dt} \tag{C-9}$$

where the sum extends over the four values of ω in Eq. (C-6) and where the A_j are constants.

This solution is expected to be valid as long as ω_x, ω_y, and $\Omega_{\theta 0}$ are slowly varying compared to any ω_j, i.e.,

$$\left|\frac{d}{dt}\ln\tilde{\omega}\right| << |\omega_j|,\ j = 1, 2, 3, 4,$$

where $\bar{\omega}$ is ω_x, ω_y, or $\Omega_{\theta 0}$. The solution is therefore expected to fail when $\omega_j \approx 0$, that is, near a turning point. From Eq. (C-6) this can happen when

$$\omega_x^2 \omega_y^2 = 0. \tag{C-10}$$

Equation (C-9) is bounded, however, if in addition $\omega_x^2 = \omega_y^2 = 0$. Breakdown of Eq. (C-9) (and a transition to unstable behavior) occurs only if $\omega_x^2 \omega_y^2 = 0$ and $\omega_x^2 \neq \omega_y^2$.

Once the solution to the homogeneous equations have been found the solution to the inhomogeneous equations follows by the usual variation of parameters or some similar method. Writing four independent solutions to the homogeneous equations as

$$\begin{Bmatrix} x^{(j)} \\ y^{(j)} \end{Bmatrix} \quad j = 1,\ 2,\ 3,\ 4 \tag{C-11}$$

one finds in a straightforward way that a particular solution to Eqs. (11a,b) in the text is given by

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \int^t dt' \begin{Bmatrix} K_x(t,t') \\ K_y(t,t') \end{Bmatrix} F(t') \tag{C-12}$$

where

$$K_x(t,t') = -\frac{1}{W}\,\epsilon_{jklm}\, x^{(j)}(t)\, x^{(k)}(t')\, y^{(l)}(t')\, \dot{y}^{(m)}(t')$$

$$K_y(t,t') = -\frac{1}{W}\,\epsilon_{jklm}\, y^{(j)}(t)\, x^{(k)}(t')\, y^{(l)}(t')\, \dot{y}^{(m)}(t')$$

$$W = \epsilon_{jklm}\, x^{(j)}(t)\, \dot{x}^{(k)}(t)\, y^{(l)}(t)\, \dot{y}^{(m)}(t),$$

and where the summation convention is understood. The Wronskian W is a constant, independent of time; its value is determined once a choice is made for the $x^{(j)}$, $y^{(j)}$.

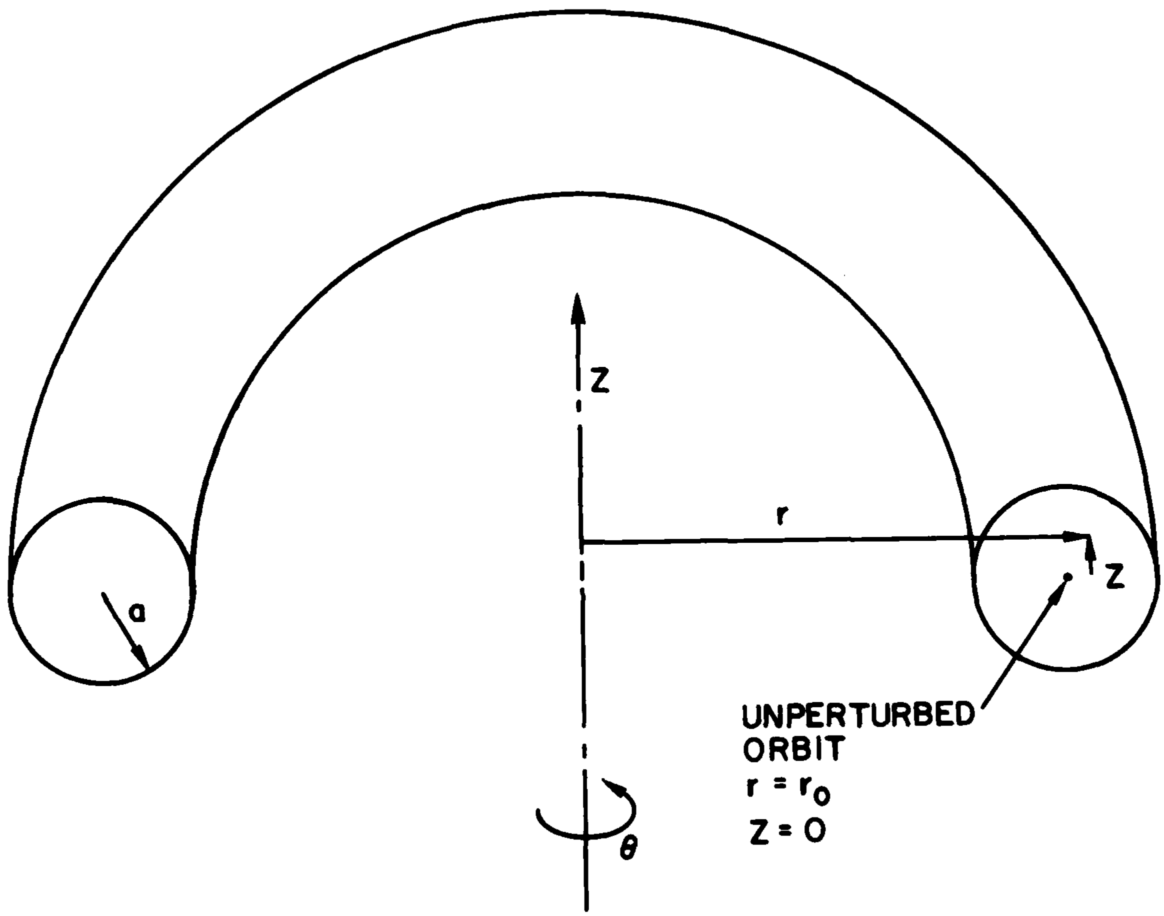

Fig. 1 — Cutaway view of modified betatron geometry

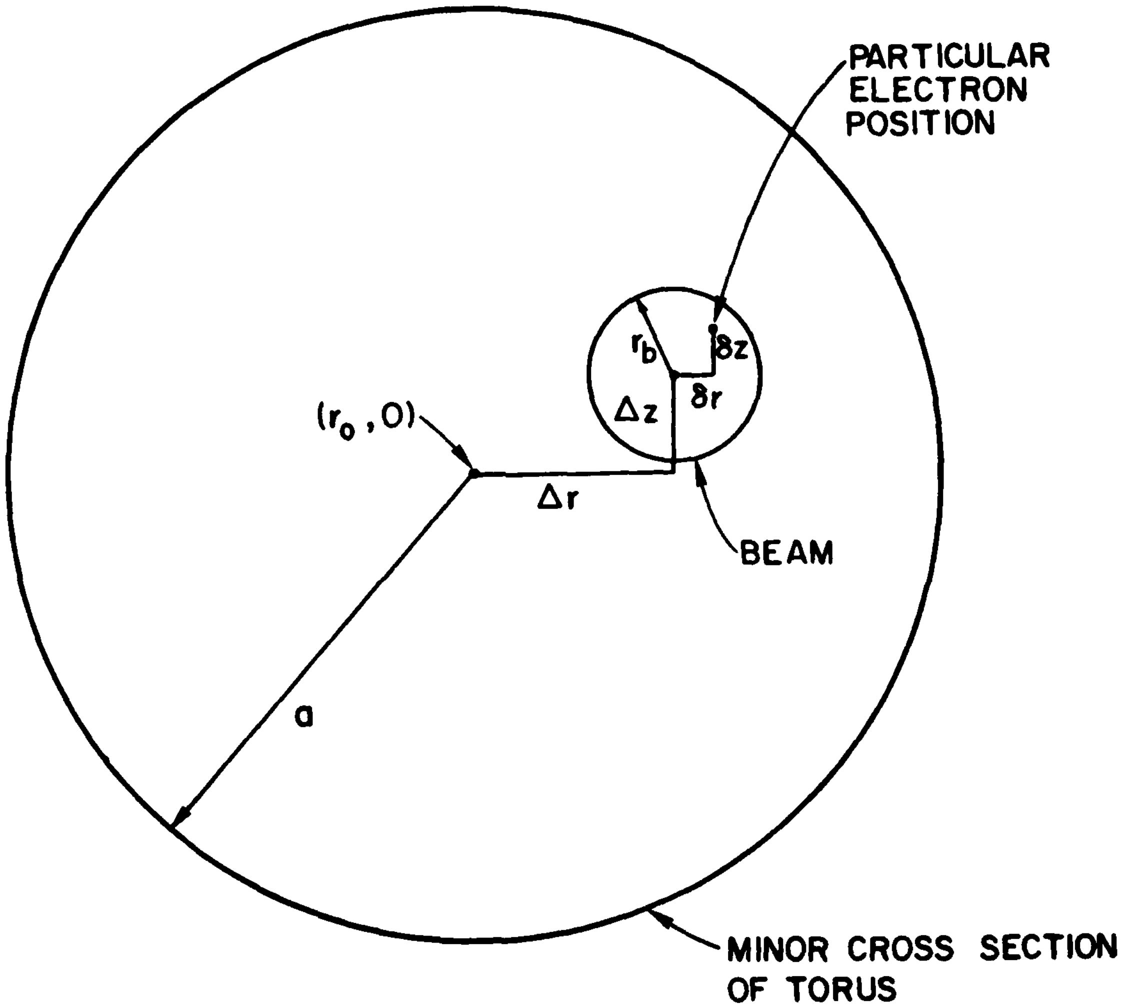

Fig. 2 — Coordinates of beam and particle in modified betatron. Center of beam is at $(r, z) = (r_0 + \Delta r, \Delta z)$. Electron is at $(r, z) = (r_0 + \Delta r + \delta r, \Delta z + \delta z)$.

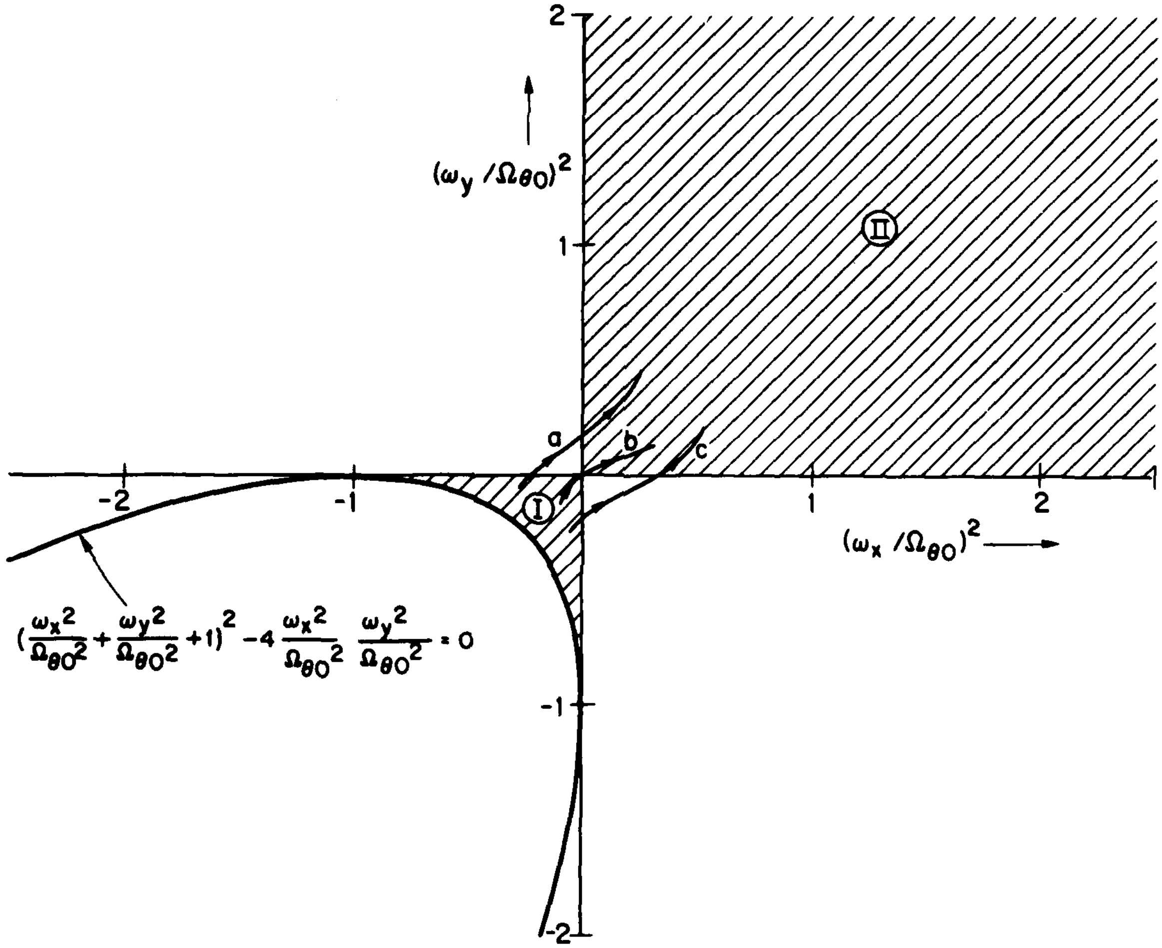

Fig. 3 — The $\left(\frac{\omega_y}{\Omega_{\theta 0}}\right)^2, \left(\frac{\omega_x}{\Omega_{\theta 0}}\right)^2$ plane. Shaded regions are stable. Trajectors *a* and *c* pass through unstable regions. Only trajectories, such as *b*, avoid all unstable behavior.

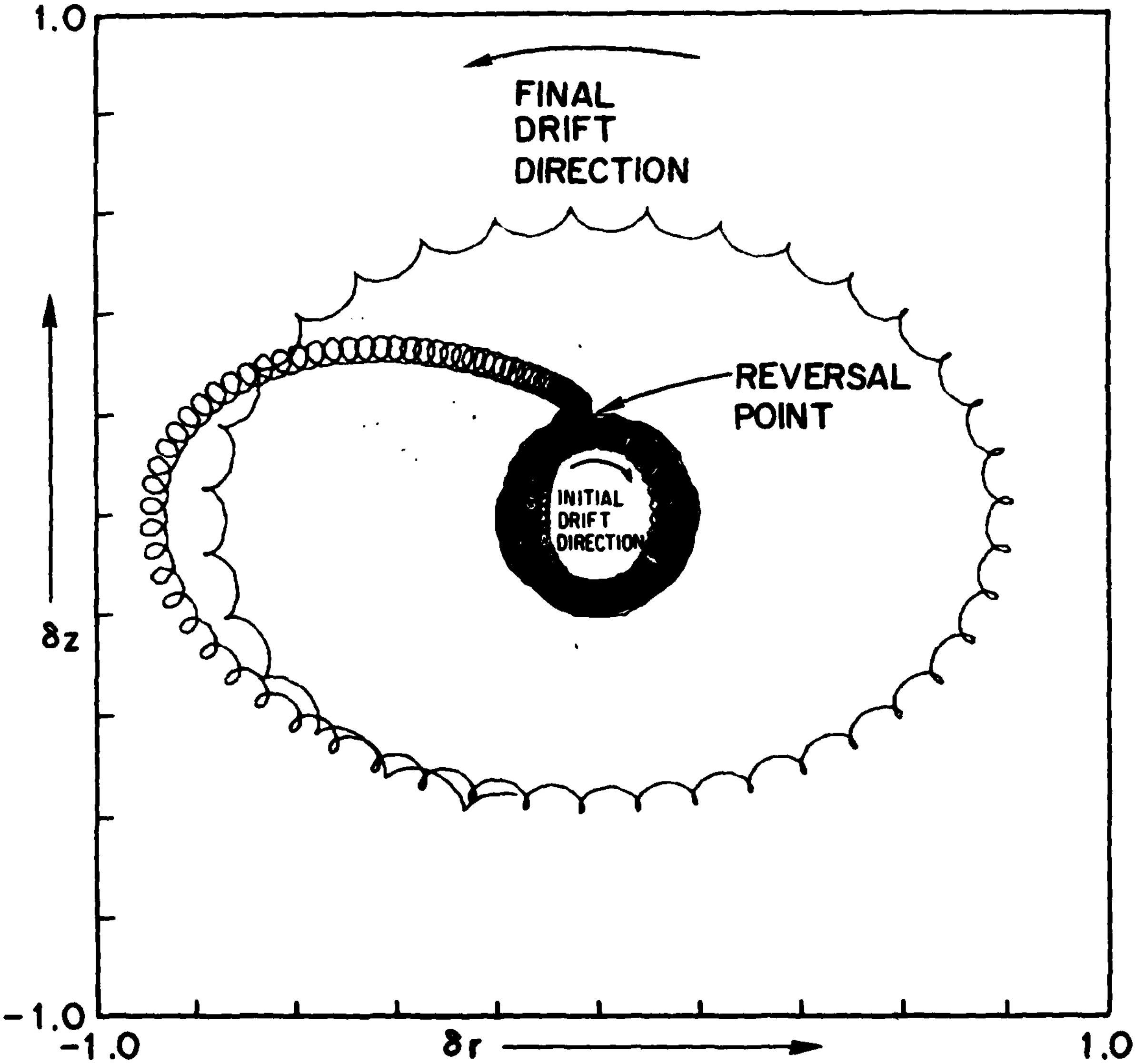

Fig. 4 — Particle trajectory (δz vs. δr) in the modified betatron during transit of the instability gap. γ varies linearly in time from 7.0 to 16.1 in 2.4 μs. B_θ = 600 gauss, r_0 = 100 cm, a = 10 cm, r_b = 1 cm. n = 0.53, ν/γ = 8.4×10^{-3} at t = 0.

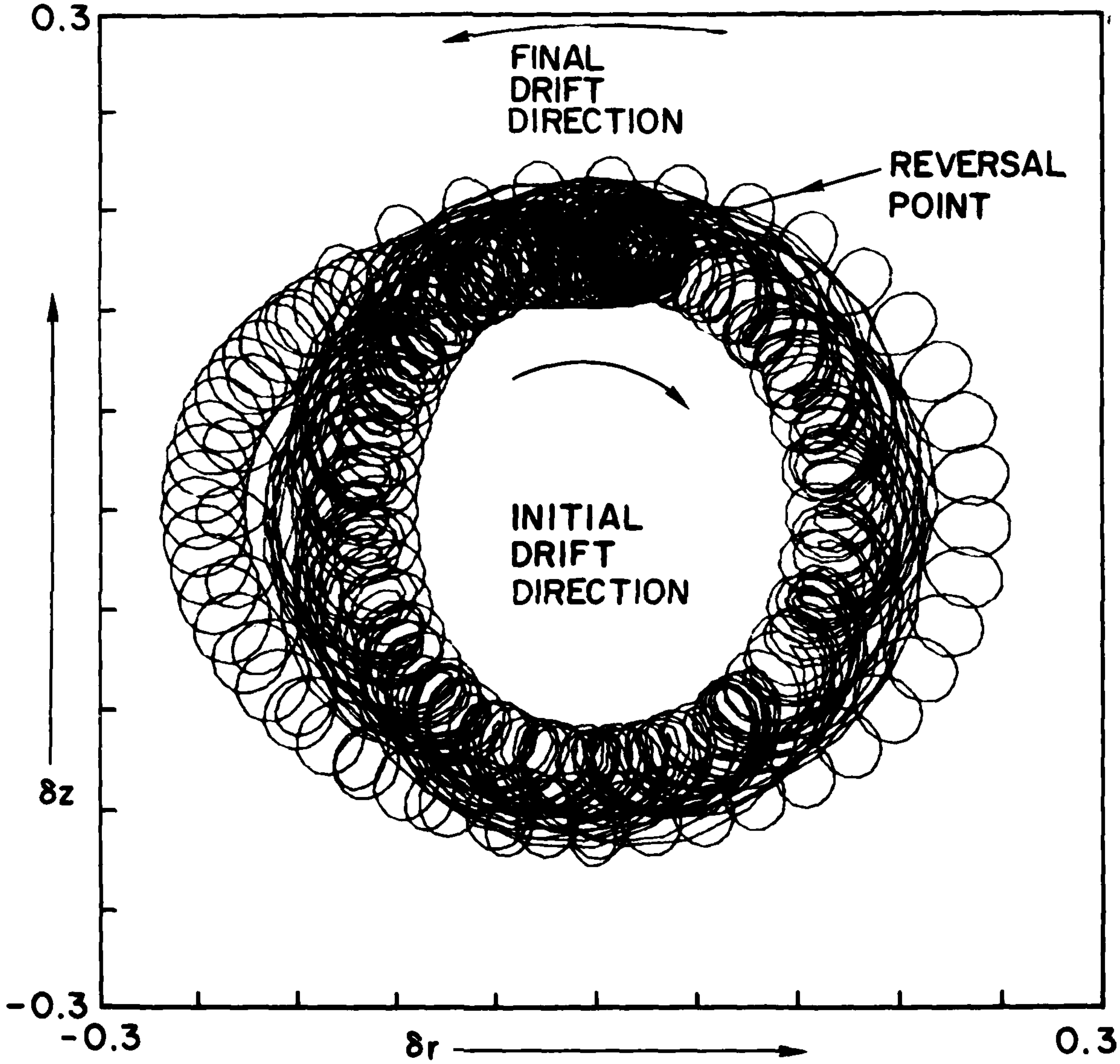

Fig. 5 — Particle trajectory (δz vs. δr) in the modified betatron during transit of the instability gap. All parameters are as in Fig. 4 except $n = 0.51$.

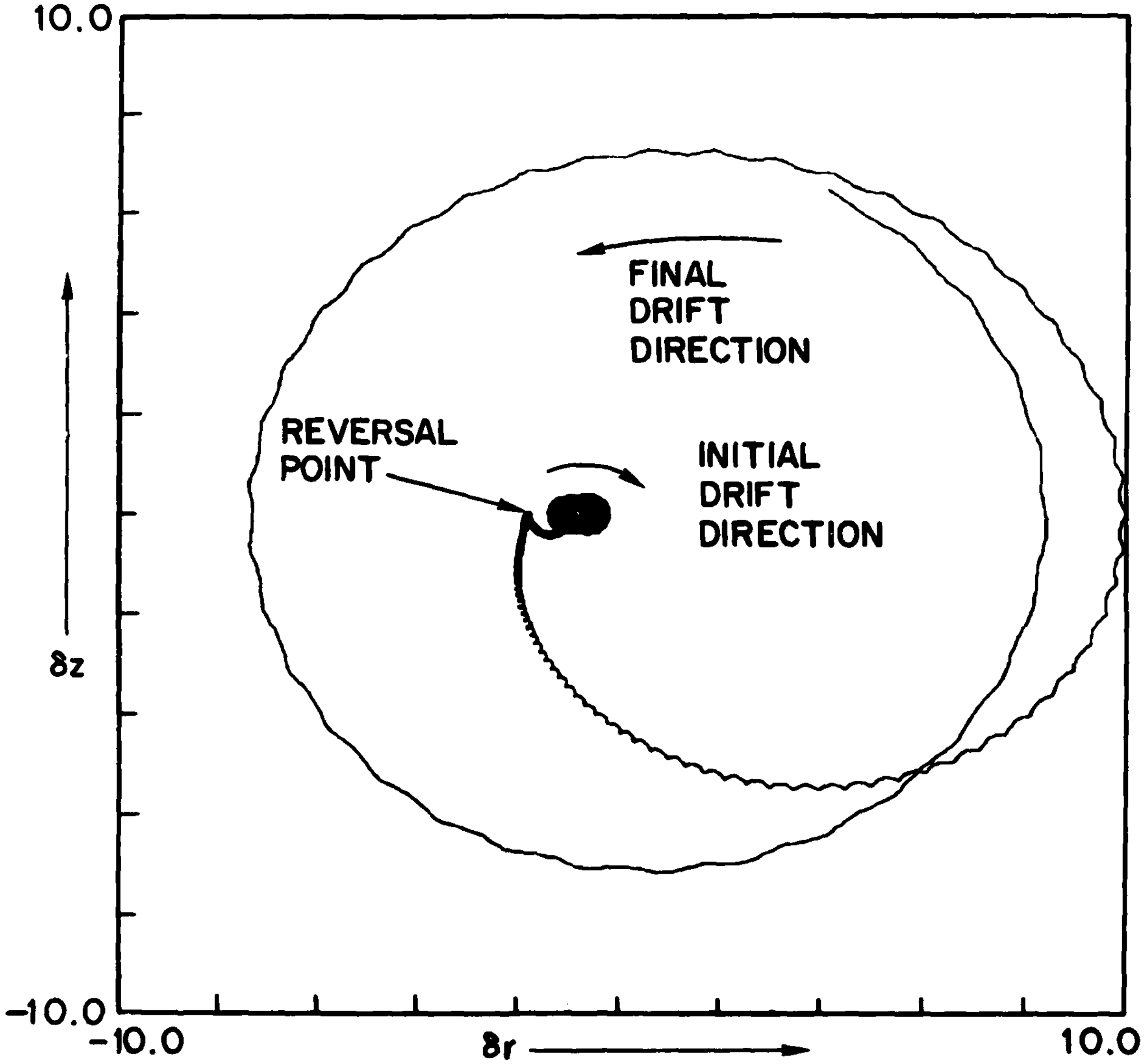

Fig. 6 — Particle trajectory (δz vs. δr) in the modified betatron during transit of the instability gap, including energy mismatch. All parameters are as in Fig. 5 except an energy mismatch of $(P_{\theta_1} - \langle P_{\theta 1} \rangle)/m r_0 c = 0.10$ has been introduced.

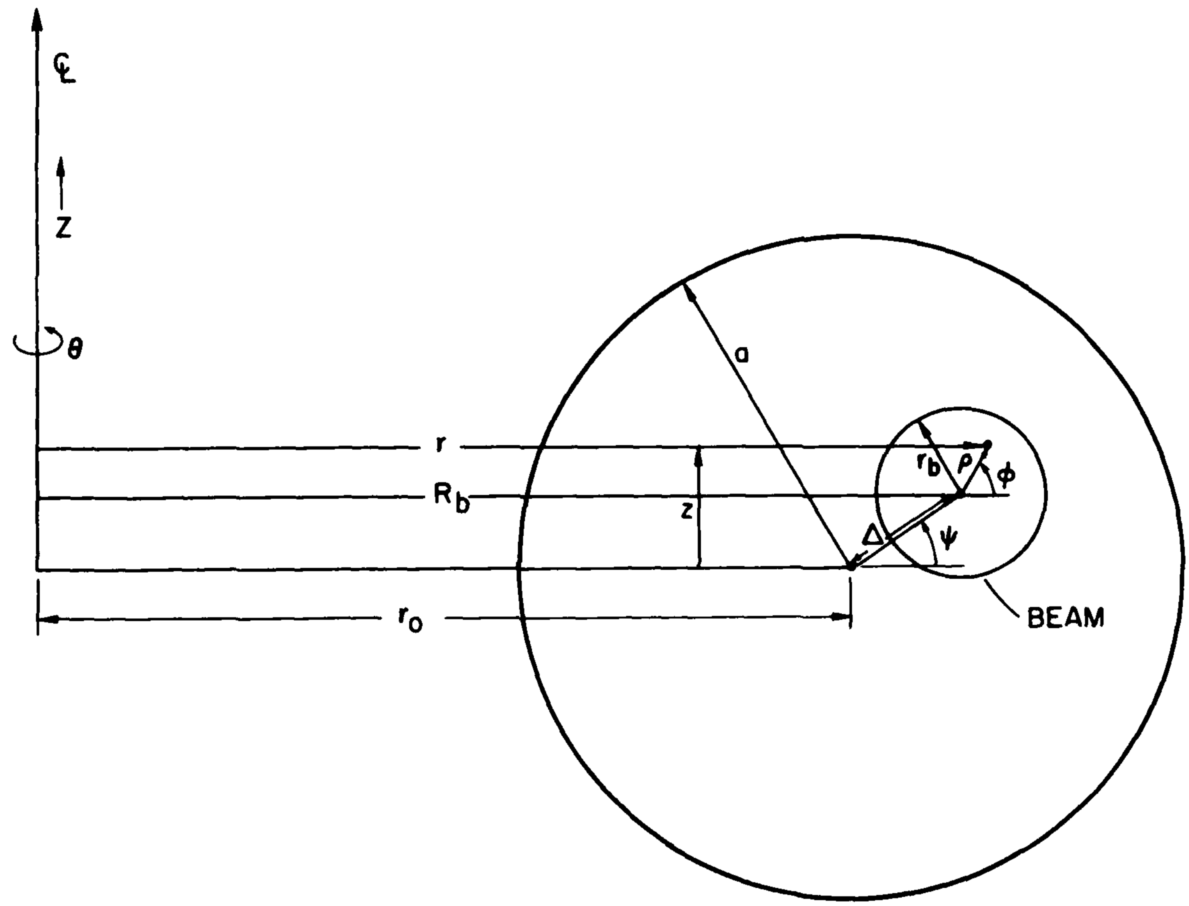

Fig. A-1 — Geometry for self field calculation

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