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NPS55-81-025

NAVAL POSTGRADUATE SCHOOL

Monterey, California

DETERMINATION OF OPTIMAL AIRCRAFT MIX IN AN AIR FORCE

by

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December 1981

Prepared for:
Naval Postgraduate School
Monterey, California 93940

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REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS55-81-025 | 2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Determination of Optimal Aircraft Mix in an Air Force	5. TYPE OF REPORT & PERIOD COVERED Technical 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Ran Goren Dan O. Bausch Gerald G. Brown	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940	12. REPORT DATE December 1981 13. NUMBER OF PAGES 82
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Force Mix Planning, Life Cycle Costs, Strategic Planning, Optimal Force Mix, Mixed-Integer Optimization

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

(20. ABSTRACT Continued)

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TABLE OF CONTENTS

0. ABSTRACT -- i

I. INTRODUCTION -- 1

II. THE MODEL'S BASIC CONCEPT -- 4

III. RATIONALE FOR THE MODEL'S STRUCTURE -- 7

IV. SOLUTION STRATEGIES -- 18

V. AN EXAMPLE -- 20

VI. COMPUTATIONAL APPROACH AND RESULTS -- 39

VII. CONCLUSION -- 53

APPENDIX A: COST CONSIDERATIONS -- 54

APPENDIX B: SAMPLE MODEL INPUT -- 64

APPENDIX C: SAMPLE MODEL OUTPUT -- 70

BIBLIOGRAPHY -- 80

DETERMINATION OF OPTIMAL AIRCRAFT MIX IN AN AIR FORCE

I. INTRODUCTION

A. GENERAL

One of the major problems an air force (AF) faces is the *determination of its fighter aircraft mix*.

The fighter aircraft (a/c) component of the AF can be considered as a fairly homogeneous group with specific missions. This allows many trade-offs within the group but less with other weapon systems.[1] Most modern air forces compose their fighter force on the basis of a "high-low" mix concept. This prescribes a mix of highly capable and more expensive aircraft with less capable and less expensive ones. Of course, some intermediate a/c are possible as well. Some of the aircraft are old, some are new, and in future planning cases, some can also be at any stage of development.

Many factors affect the aircraft mix, of which the predominant ones are:

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[1] In fact, fighter aircraft missions are by no means exclusive to them. For example, fighters can be substituted for missiles in a strategic or even tactical strike mission; they can be used for assault helicopters in air-to-ground support; they can replace SAMs in air-defense missions, etc. But for the current context the basic assumption is that the missions of the total fighter force are determined after the trade-offs between different weapon systems have been resolved. Thus, the trade-offs in question are between the various a/c types within the fighter force. On the other hand, the analysis basically allows handling <u>any</u> mix of weapon systems, but that requires component (constraint) analysis beyond the scope of this report.

1) the assessment of the threat, and the operational mission requirements to face it;
2) the costs of the aircraft mix in the broadest sense of this term;
3) aircraft availability and performance, including a/c which are in development;
4) various "real-life" constraints such as manpower availability (quantity and quality), aerial training space, maintenance facilities, etc.

Usually the a/c mix is planned for several years in advance, since the acquisition cycle required to build the future force levels takes a long time, especially when Research and Development is involved. This increases uncertainty for the planner.

B. PROBLEM ISSUES

Several problems arise dealing with mix determination, such as:

1) how to convert the threat assessment into terms of aircraft types and quantities;
2) how to calculate cost and what cost components should be considered;
3) how to handle uncertainties;
4) how to consider <u>all</u> the significant factors and not only, say, cost and effectiveness.

The suggested analysis is aimed mainly at solving the last problem. The other issues are addressed in the analysis in an indirect fashion; a suggested model provides a solution approach for the first and third problems, while cost calculations are regarded in this report as given.

C. PURPOSE

The main purpose of this analysis is to determine the mix of aircraft that meets specified mission needs (i.e., specified requirements of effectiveness), with a minimal cost, taking into account some "real-life" constraints. In addition to this main purpose, some important sub-decisions can be made, such as:

1) whether or not it is worthwhile to begin development of a new aircraft for the mix;
2) what should the minimal relative performance and cost of a new aircraft be to justify its inclusion in the mix;
3) what is the maximum cost of additional airbase facilities or manpower that still warrants this addition, and more.

D. AN EXAMPLE

In Chapter V of this report a detailed example is presented. The example, though hypothetical, may be considered as a genuine representative of real-life cases. The example demonstrates the model's rationale and structure. It may be used as an illustration of the theoretical explanations of Chapter III, as well as a data base for solution trials.

II. THE MODEL'S BASIC CONCEPT

A. BASIC ASSUMPTIONS

1. General

Our model assumes a small or middle-sized AF, operating in a specific theater against a known enemy. On the other hand, the mathematical resolution is more general and applies to any force size. Thus, the model can be used for any case for which the basic assumptions are found to be applicable.

2. Detailed Basic Assumptions

1) Threat assessment and mission needs analysis can be done on relatively firm grounds 5-10 years into the future.
2) Several main war scenarios can be predicted and defined in relatively concrete terms.
3) There is good information about existing a/c in the future frame of time the model refers to. This includes knowledge about the following:
    a) a/c capabilities in the future battlefield;
    b) a/c availability for additional purchase;
    c) in case of obsolescence--whether a/c will be resold or discarded.
4) There is a relatively clear idea about future a/c. This includes:
    a) a/c availability;

b) a/c expected performance;

c) a/c predicted costs.

B. THE MODEL'S APPROACH TO THE SOLUTION

This is a cost-effectiveness model, with fixed effectiveness and variable costs. This approach has been selected under the assumption that meeting minimal mission needs, i.e., surviving the battle with reasonable outcomes, is the dominant factor. Cost, though driven to its minimal level, is subject to mission need constraints. But, since in real life "minimal" mission needs are not an absolute minimum, the model may have a provision for violating mission constraints in order to reach a reasonable solution.

The computational approach of the model is non-linear, integer programming. The objective function is concave, discontinuous and piecewise non-linear. (The behavior of the cost function is described in more detail in Appendix A.) Mixed-integer features are used because of the piecewise nature of the objective function, and because of the nature of the variables, namely, aircraft quantities. Furthermore, a/c usually are not purchased or discarded individually, rather, in groups of five, ten or twenty. Solution techniques may consider this fact. The programming approach allows taking into account all significant factors, in the form of constraints. The constraints are divided into three categories. These are:

(1) Mission needs constraints--which represent the minimal level of operational mission needs.

(2) Physical constraints--which represent the maximum levels of some "real life" limitations such as a/c availability, manpower, etc.

(3) Notational constraints--which represent the relationships between variables used within the model construct.

III. RATIONALE FOR THE MODEL'S STRUCTURE

A. THE OBJECTIVE FUNCTION

1. General

The objective of our model is to minimize cost. The considerations underlying the objective function development are:

(1) The costs recognized are those which are required to develop, produce, maintain and operate the a/c, and not the costs directly resulting from combat operations (e.g., a/c attrition costs, armament consumed, etc.).

(2) The model uses economic cost.

(3) Only relevant costs, i.e., costs that are related to or pursuant to the mix decision are evaluated.

(4) The cost considered is life cycle cost (LCC) (excluding irrelevant components), expressed in present value terms.

2. The Nature of the Cost Function

Total LCC as a function of quantity of a/c is composed from the three categories:

(1) Research and development costs (R&D).

(2) Investment costs which are divided into two main sub-categories:

   (a) Procurement cost of the major equipment (which usually corresponds to production costs); and

   (b) Other initial investments.

(3) Operating and Support Costs (O&S).

The resulting LCC function is generally of a concave, stepwise nature, as illustrated schematically in Figure III-1. (For more detail about the cost function see Appendix A.)

Figure III-1. Life Cycle Cost (LCC)

The initial cost step in Figure III-1 is attributed mainly to the R&D costs which are purely fixed costs. The other steps along the graph are attributed to some of the "other initial investments" which are semi-variable (e.g., technical facilities, initial inventories). The concavity is attributed mostly to the "learning curve" effect on the production costs.

In order to accommodate the LCC function with nonlinear, integer programming the concave stepwise function is separated into several disjoint component functions, each approximating a portion of the original overall function. The upper and the lower limits of each sub-function are set by either one of two criteria:

(1) In each place where the original LCC function has a discontinuity.

(2) Wherever necessary in order not to divert beyond a reasonable extent from the original function.

Figure III-2 demonstrates schematically how linear approximations are posed for the original function.

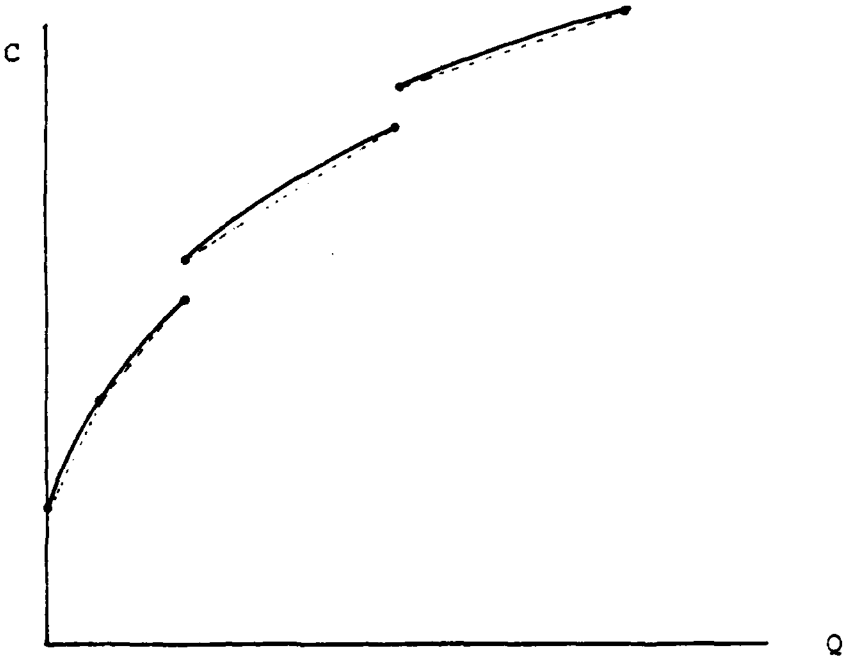

Figure III-2. Piecewise Linear Approximation of LCC

The result of this linear approximation is a set of linear functions, each applying to a specified disjoint range of a/c quantity. Since each a/c type has a different LCC function, such linear function sets are generated for each type separately. The overall range of a/c quantity upon which the above approximation process is implemented is unique to each type of a/c in the mix, and derived from its specific availability constraints.

The stepwise function can be constructed somewhat beyond the expected availability limits, to allow constraint violation (usually associated with some penalty).

The objective function is the sum of the individual cost functions of each a/c type of the mix. Consequently, it is a sum of the various <u>sets</u> of the approximated (linear) functions.

B. MISSION NEED CONSTRAINTS RATIONALE

1. General Assumptions

The threat assessment and the resulting mission needs to meet the threat are estimated on the grounds of several <u>wartime</u> <u>scenarios</u>. Each scenario is exclusive and exhausts a specific time frame in the war (i.e., two or more scenarios cannot occur concurrently), and each scenario relates to the <u>total activities</u> of the AF under discussion. On the other hand, several scenarios may appear alternately in the same war phase (e.g., several scenarios may alternately represent a possible initiation phase of war). Also, several scenarios may occur additively in the same war in different points of time. In case of a scenario posed in the midst of a war, the attrition rate up to this point in time should be taken into account in order to calculate the optimal mix, which is always regarded in this model as the quantities of a/c (by types) required at the <u>initiation</u> of the hostile activities.

2. Mission Constraints Rationale and Associated Definitions

Each scenario is defined in terms of specific combination of tactical missions (e.g., air defense, air-to-ground support, deep penetration strike, etc.). The specific combination is distinguished by the <u>kinds</u> of component missions and the <u>amount</u> of "mission units" (this term is explained below) in each of the individual missions.[1]

A "mission unit" is defined as the capability, or the combat effectiveness, of the <u>least capable</u> type of a/c used for the specified mission. It is defined separately for each type of a/c, in each mission, within each scenario.

Each type of a/c is assigned several "mission coefficients" (one for each mission in each scenario), which denotes the number of mission units it has, or, in other words, its effectiveness exchange ratio with the least capable a/c in the specific mission.

The total "mission units" for a specific mission within a specified scenario is the summation of the products of a/c amounts (by types) times their individual "mission coefficients." Thus,

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[1] In this context the authors refrain from an explanation of how these missions--by kinds and amounts--are derived from the threat assessment. There are several ways to do this. Just to mention the simplest: A "war game", or any tentative operational planning may provide such data.

$$(\text{III-1}) \quad TMU_{ij} = x_{ij}\cdot a_{xij} + y_{ij}\cdot a_{yij} + z_{ij}\cdot a_{zij} + u_{ij}\cdot a_{uij}$$

where:

TMU_{ij} = Total mission units of mission j in scenario i.

$x_{ij}, y_{ij}, z_{ij}, u_{ij}$ = Amount of a/c of types X,Y,Z and U, respectively, which participate in mission j at scenario i.

$a_{xij}, a_{yij}, a_{zij}, a_{uij}$ = The specific mission coefficients of a/c types X,Y,Z and U, respectively, for mission j in scenario i.

This linear summation assumes that the total amount of mission units required can be achieved by <u>any</u> combination of a/c that results in this amount. Equivalently, it suggests that the relative effectiveness among different a/c types is always the same, regardless of the operational force structure combination. This is, of course, untrue in real life. To enhance reality, the total mission unit amounts are complemented in the model by additional "minimal requirements", which are intended to meet some "qualitative needs". Only after these minimal requirements are met can we assume that the mission coefficients are valid and use an exchange ratio among a/c types. For example, assume that for the air-defense mission in a specific scenario, 300 mission units are required. There are two eligible a/c types for the mix--type X and type Y. A/c X has mission coefficient 1 for air defense. A/c Y, much more sophisticated and capable, has mission coefficient 3 for air defense. A simple calculation might suggest that the minimal mission need of 300 mission units can be met by either 300 a/c of type X or 100 a/c of type Y, or some combination of them. But in fact this is not true, since type X has no all-weather capability. To meet the all-weather threat, at least 50 type Y a/c are required in the mix. This provides 150 mission units. The remaining 150 units can be filled by either type. Thus the mission coefficients can be regarded as the relative marginal contribution of effectiveness, after some minimal requirements have been met. The contribution of each a/c is considered linear. Of course, many such "qualitative needs" constraints, which represent various operational planning considerations, can be stated to support and complement the total mission unit requirements.[1]

C. PHYSICAL CONSTRAINTS

1. General

In determining the optimal a/c mix, several "real life" constraints should be taken into account. These limit

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[1] Again, there is no intention in this context to demonstrate how such relative effectiveness expressed by the mission coefficients is reached. Just to mention the simplest ways:
--A survey among people involved may reveal the relative effectiveness according to their perception.
--Operations analysis concerning configuration, range and survivability may determine relative capability in a strike mission.
--Analysis of controlled exercises (e.g., "Red Flag" operations conducted at Nellis AF Base, Nevada) may result in some comparative data, etc.

significantly the range of possible alternative mixes, and may cause infeasible solutions in case of conflict with the mission need constraints. The physical constraints can be divided into three categories:

(1) availability constraints;

(2) capacity constraints;

(3) resource constraints.

2. Availability Constraints

The Availability Constraints are derived from production limitations (e.g., a production line is closed--no more a/c are available, or production capacity is limited--no more than a certain amount is available. These constraints also include minimal amounts required for production to be economically justifiable (usually considered as somewhere between 300 to 400 units). Also considered is the minimal amount to be purchased of a completely new type of a/c in the arsenal (say, 20-25 fighters for a small AF, and more for a larger one).

3. Capacity Constraints

The capacity constraints stem from the physical capacity limits (e.g., limits of parking space, runways, maintenance facilities, etc., in existing air bases. Limits on aerial training space and facilities are included as well in this category.) These constraints are not necessarily defined in terms of absolute a/c quantities. For example, the ground space and maintenance facilities required to maintain specified amounts of large and highly sophisticated aircraft can be used to maintain much larger amounts of smaller and simpler a/c. Thus, in the same sense as for the notion of mission units, these constraints can be stated in terms of "capacity units". While each a/c type has its individual coefficient, the total unit amount is constructed by the linear summation of the a/c type amounts multiplied by their respective capacity coefficients.

4. Resource Constraints

The Resource Constraints derive from other substantive limitations (e.g., manpower--quantity and quality). They are expressed in terms of maximum available amounts and in order to allow flexible interchangeability between a/c which have different resource requirements, they are also stated in "units".

There are no cost constraints in this context since cost is wholly represented in the objective function. In this sense the model can be regarded only as a first step in a decision process: first it is desired to know which alternate mix is the optimal one and what is the minimal cost required to obtain this mix. From this starting point one may proceed to other analysis approaches which impose cost constraints. The latter are beyond the scope of this report.

D. NOTATIONAL CONSTRAINTS

The notational constraints represent relationships among various variables used in the model. They allow more concise programming statements, and provide a convenient way to use some general factors such as technical availability or attrition rate. For example: Let X be the amount of a/c type X in the mix. Let x_{11}, x_{12}, x_{13}, x_{14} be the amounts of a/c type X participating in scenario 1 in missions 1, 2, 3 and 4 respectively. Obviously, the sum of the amounts of a/c type X participating in the various missions of scenario 1 cannot be larger than the total amount of a/c type X on hand. That implies:

$$X \geq x_{11} + x_{12} + x_{13} + x_{14}. \tag{III-2}$$

The same is true for scenarios 2 and 3. Thus:

$$X \geq x_{21} + x_{22} + x_{23} + x_{24}; \tag{III-3}$$

$$X \geq x_{31} + x_{32} + x_{33} + x_{34}. \tag{III-4}$$

Assume now that from the total amount of X a/c only 80% are technically ready for actual flight. Then instead of X in the above statements, .8X, or for the general case $X \cdot T_x$, should be written, where T_x is the technical availability coefficient of a/c type X. Statement (III-2) would look now as follows:

$$X \cdot T_x \geq x_{11} + x_{12} + x_{13} + x_{14}. \tag{III-5}$$

Assume further that X is the total a/c amount at the beginning of the war, but scenario 1 occurs at a later stage, by which time some of the a/c are lost. Then this rate of attrition should be taken into account. Statement (III-5) would look now as follows:

$$X \cdot T_x(1-A_{x1}) \geq x_{11} + x_{12} + x_{13} + x_{14}, \tag{III-6}$$

where A_{x1} is the attrition percentage of a/c type X from the war initiation up to the point in time in which scenario 1 is to take place.

All the above applies, of course, to each type of a/c individually.

The notational constraints allow the usage of the total amount of each a/c type, i.e., X, Y, Z, etc., wherever total amount is required (e.g., in the objective function or in the physical constraints), and the components of a/c participating in each mission within each scenario, i.e., x_{11}, x_{12}, x_{13}, etc.--in the mission needs constraints.

IV. SOLUTION STRATEGIES

A. SIMULTANEOUS SOLUTION

The non-linear integer programming model can be solved while satisfying all the scenario constraint sets simultaneously. Since the optimal mix should meet the constraints imposed by all scenarios, there is sense in this approach.

The weakness of this approach derives from the fact that the solution is determined by the severest constraints "accumulated" from all the scenarios. But in real life there are scenarios less critical than others, or missions within scenarios which are less significant, whose "minimal" requirements (i.e., constraints) are still subject to some compromises.

B. SOLUTION BY SEPARATE SCENARIOS

Here each scenario is solved separately and the optimal a/c mix for each scenario is determined. The consolidation of the different mixes can be done by several approaches:

1) The Severest Scenario--the mix of the severest scenario is selected as a basic reference. The other scenarios are evaluated using this mix. If the mission needs of the other scenarios are not met reasonably, slight changes in the mix are made, until satisfactory results are achieved.

2) The Most Expensive Mix--naturally, such a mix is a relatively highly capable one. This mix is evaluated for all scenarios and changes in the mix are made until the individual scenario requirements are satisfactorily fulfilled (although not necessarily completely fulfilled).

3) The Weighted Average--In cases with relatively few differences among the scenarios mixes, a weighted average mix is determined, where the mix of the severest scenario is assigned the largest weight. This surrogate mix is evaluated.

4) Goal Programming--Goals of total cost and a/c amounts are set. The mix that provides the smallest deviation from the goal is selected. The initial mixes are again assigned weights, where the severest scenario mix has the largest weight.

V. AN EXAMPLE

A. GENERAL

The example is fictitious, but assumptions and data represent real life as much as possible.

B. TYPES OF AIRCRAFT

There are four types of a/c which are included or eligible to be included in the example. These are:

(1) Type X

Unsophisticated a/c. It is limited to air-to-ground missions, and is almost incapable of air-to-air missions in the future time frame.

(2) Type Y

Air superiority a/c. Excellent in air-to-air missions, it is very limited in air-to-ground missions.

(3) Type Z

Light a/c. Good for air-to-air and air-to-ground missions, but it does not carry sophisticated weapons.

(4) Type U

An a/c still under development. Mainly for air-to-ground missions, it has good capability for simple air-to-air missions.

C. NOTATION AND NOTATIONAL CONSTRAINTS

1. Basic Notation

X,Y,Z,U = the number of a/c types X,Y,Z and U respectively. These are the a/c population in the initial mix (before the war starts).

$x_{ij}, y_{ij}, z_{ij}, u_{ij}$ = the number of each type that actually participate in mission j of scenario i. Not all a/c actually participate because of technical availability, attrition, etc.

T_x, T_y, T_z, T_u = Technical availability rate for a/c types X,Y,Z,U, respectively.

$A_{xi}, A_{yi}, A_{zi}, A_{ui}$ = Cumulative attrition of types X,Y, Z, and U, respectively, prior to scenario i.

2. Notational Constraints

(V-1) $X \cdot T_x(1-A_{x1}) - [x_{11} + x_{12} + x_{13} + x_{14}] \geq 0$

(V-2) $X \cdot T_x(1-A_{x2}) - [x_{21} + x_{22} + x_{23} + x_{24}] \geq 0$

(V-3) $X \cdot T_x(1-A_{x3}) - [x_{31} + x_{32} + x_{33} + x_{34}] \geq 0$

(V-4) $Y \cdot T_y(1-A_{y1}) - [y_{11} + y_{12} + y_{13} + y_{14}] \geq 0$

(V-5) $Y \cdot T_y(1-A_{y2}) - [y_{21} + y_{22} + y_{23} + y_{24}] \geq 0$

(V-6) $Y \cdot T_y(1-A_{y3}) - [y_{31} + y_{32} + y_{33} + y_{34}] \geq 0$

(V-7) $Z \cdot T_z(1-A_{z1}) - [z_{11} + z_{12} + z_{13} + z_{14}] \geq 0$

(V-8) $Z \cdot T_z(1-A_{z2}) - [z_{21} + z_{22} + z_{23} + z_{24}] \geq 0$

(V-9) $Z \cdot T_z(1-A_{z3}) - [z_{31} + z_{32} + z_{33} + z_{34}] \geq 0$

(V-10) $U \cdot T_u(1-A_{u1}) - [u_{11} + u_{12} + u_{13} + u_{14}] \geq 0$

(V-11) $U \cdot T_u(1-A_{u2}) - [u_{21} + u_{22} + u_{23} + u_{24}] \geq 0$

(V-12) $U \cdot T_u(1-A_{u3}) - [u_{31} + u_{32} + u_{33} + u_{34}] \geq 0$

Arbitrarily, all technical availability rates T_x, T_y, T_z, T_u are set to .8. The attrition rate for scenarios 1 and 3 is set to 0, as these are assumed to be initial conflict scenarios. A_{x2}, A_{y2}, A_{z2} and A_{u2} are set to .1 (i.e., the rate of attrition is equal for all types, and for each, 1/10 of the force is out of order at least for this conflict).

D. AVAILABILITY CONSTRAINTS

1. General

The availability constraints are common to all scenarios, and apply to the force mix at the beginning of the war.

2. Constraints--Rationale and Statements

a. Type X

1) Rationale

- There are 320 a/c on hand.
- The production line is closed, and no additional a/c are available.
- 25 a/c is the minimal amount for employing this a/c.

2) Constraint

$$X = 0, \quad \text{or} \quad 25 \leq X \leq 320 \tag{V-13}$$

b. Type Y

1) Rationale

- There are 100 a/c on hand. Reduction of this amount is out of the question.
- More a/c are available up to 200.
- Minimal incremental quantity, if any, is 10 a/c.

2) Constraint

(V-14) $Y = 100,\ \underline{\text{or}}\ 110 \leq Y \leq 200$

c) Type Z

1) Rationale

- There are 150 a/c on hand. Reduction of this amount is out of the question.

- Additional a/c are available up to 300.

- The minimal incremental quantity, if any, is 10 a/c.

2) Constraint

(V-15) $Z = 150\ \underline{\text{or}}\ 160 \leq Z \leq 300$

d. Type U

1) Rationale

- No a/c are on hand.

- The minimal amount to be purchased, if any, is 200 (to justify the production of completely new a/c).

- The maximum amount available in the time frame under study is 400.

2) Constraint

(V-16) $U = 0\ \underline{\text{or}}\ 200 \leq U \leq 400$

E. RESOURCE AND CAPACITY CONSTRAINTS

1. General

a. Only representative constraints are presented.

b. The constraints apply to all scenarios.

c. The coefficients represent the relative requirements of the different a/c types.

d. The right-hand side constant (in each constraint) represents a specified maximum amount of "units". (The units are unique to each resource.)

Table V-1. Resource and Capacity Coefficients

Type	Manpower	Ground Facilities	Training Space
X	10	1	1.5
Y	20	2	1
Z	15	1.5	2
U	12	1.5	2

e. The Constraints:

1) Manpower Constraint:

(V-17) $$10X + 20Y + 15Z + 12U \leq 11000;$$

2) Ground Facilities Constraint:

(V-18) $$X + 2Y + 1.5Z + 1.5U \leq 1000;$$

3) Training Space Constraint:

(V-19) $$1.5X + Y + 2Z + 2U \leq 1500 .$$

F. MISSION NEEDS CONSTRAINTS

1. General

a. These constraints are stated for each of three sample scenarios. All three scenarios consist of the following basic missions:

1) Air defense;
2) Air/Ground (A/G) support;
3) Deep penetration strike;
4) Miscellaneous.

b. Descriptive introduction of the scenarios is deliberately omitted. However, the main effort is apparent from the emphasis in meeting the threat, as reflected by the mission units allocation within each of the scenarios.

c. Besides the general mission needs constraint (i.e., the total amount of mission units required for each mission), there is an additional constraint for minimal "qualitative operational need" that compliments the general mission constraint (see explanation III.B.2.). This somewhat schematic constraint represents a wide spectrum of possible operational considerations expressed in terms of constraints.

d. Basically, a/c and crews are versatile. But there are missions which certain a/c types are not capable of accomplishing. In those cases their mission coefficient is set to 0.

2. Mission Coefficient Tables

Table V-2. Mission Coefficients--Scenario 1

Type	Air Defense	A/G Support	Deep penetration Strike	Miscellaneous
X	0	1	1	1
Y	3	0	1.5	1
Z	2	2	2.5	1
U	1.5	2	2	1

Table V-3. Mission Coefficients--Scenarios 2 and 3

Type	Air Defense	A/G Support	Deep penetration Strike	Miscellaneous
X	0	1	1.2	1
Y	3	0	1.5	1
Z	2	2	2.5	1
U	2	2	2	1

3. Scenario 1--Mission Needs Constraints

a. Air Defense:

(V-20) $$0x_{11} + 3y_{11} + 2z_{11} + 1.5u_{11} \geq 850,$$

of which "all-weather" capable a/c required are ("qualitative need constraint"):

(V-21) $$y_{11} \geq 80$$

b. A/G Support:

(V-22) $$x_{12} + 0y_{12} + 2z_{12} + 2u_{12} \geq 200$$

of which "smart bomb" carriers required are:

(V-23) $$z_{12} + u_{12} \geq 50$$

c. Deep Penetration Strike:

(V-24) $$x_{13} + 1.5y_{13} + 2.5z_{13} + 2u_{13} \geq 150$$

of which "very long range" capable a/c required are:

(V-25) $$y_{13} + z_{13} \geq 50$$

d. Miscellaneous:

(V-26) $$x_{14} + y_{14} + z_{14} + u_{14} \geq 100$$

4. Scenario 2--Mission Needs Constraints

a. Air Defense:

(V-27) $$0x_{21} + 3y_{21} + 2z_{21} + 2u_{21} \geq 280$$

of which "all-weather" required are:

(V-28) $$y_{21} \geq 50$$

b. A/G Support:

(V-29) $$x_{22} + 0y_{22} + 2z_{22} + 2u_{22} \geq 600$$

of which "smart bomb" carriers required are:

(V-30) $$z_{22} + u_{22} \geq 150$$

c. Deep Penetration Strike:

(V-31) $$1.2x_{23} + 1.5y_{23} + 2.5z_{23} + 2u_{23} \geq 200$$

of which "very long range" a/c required are:

(V-32) $$y_{23} + z_{23} \geq 70$$

d. Miscellaneous:

(V-33) $$x_{24} + y_{24} + z_{24} + u_{24} \geq 100$$

5. Scenario 3--Mission Needs Constraints

a. Air Defense:

(V-34) $$0x_{31} + 3y_{31} + 2z_{31} + 2u_{31} \geq 320$$

of which "all-weather" required are:

(V-35) $$y_{31} \geq 50$$

b. A/G Support:

(V-36) $$x_{32} + 0y_{32} + 2z_{32} + 2u_{32} \geq 300$$

or which "smart bomb" carriers are:

(V-37) $$z_{32} + u_{32} \geq 100$$

c. Deep Penetration Strike:

(V-38) $$1.2x_{33} + 1.5y_{33} + 2.5z_{33} + 2u_{33} \geq 550$$

of which "very long range" a/c required are:

(V-39) $$y_{33} + z_{33} \geq 100$$

d. Miscellaneous:

(V-40) $$x_{34} + y_{34} + z_{34} + u_{34} \geq 100$$

G. THE OBJECTIVE FUNCTION

1. General

a. The basic objective function is:

(V-41) $$\text{Minimize} \quad TC = TC_x + TC_y + TC_z + TC_u$$

where:

TC_x, TC_y, TC_z, and TC_u = Total relevant LCC for the types X,Y,Z, and U, respectively.

b. TC_x, TC_y, TC_z, and TC_u each represent a separable set of piecewise linear approximations of the original LCC function.

c. Although these cost functions are set arbitrarily for this example, special attention has been paid to compose them as realistically as possible. Furthermore, specific source data and distinctive underlying assumptions have been used in order to assure that solutions will be eligible for further analysis on the basis of some hypothetical, but readily validated case.

d. The piecewise linear approximations are provided over a reasonable range of a/c quantities. The upper range of the last function in each set is artificially high to allow for availability constraint violation.

e. The full development of the cost function is not presented here.

2. Cost Functions

a. Type X

(Comment: The reason for the simple function here is that the only relevant cost for a/c type X is O&S, which is assumed here to be purely linear.)

(V-42) $$TC_x = 4X$$

b. Type Y

$$\text{(V-43)} \quad TC_y = \begin{cases} 0 , & \text{if } 0 < Y \leq 100 \\ 45.33Y - 4430.3, & \text{if } 100 < Y \leq 125 \\ 44.37Y - 4216.4, & \text{if } 125 < Y \leq 150 \\ 43.55Y - 3955.8, & \text{if } 150 < Y \leq 175 \\ 42.74Y - 3732.9, & \text{if } 175 < Y \leq 200 \end{cases}$$

c. Type Z

$$\text{(V-44)} \quad TC_z = \begin{cases} 0 , & \text{if } 0 < Z \leq 150 \\ 23.2Z - 3430, & \text{if } 150 < Z \leq 175 \\ 23.1Z - 3365, & \text{if } 175 < Z \leq 200 \\ 23Z - 3275, & \text{if } 200 < Z \leq 225 \\ 22.9Z - 3210, & \text{if } 225 < Z \leq 250 \\ 22.8Z - 3120, & \text{if } 250 < Z \leq 275 \\ 22.7Z - 3055, & \text{if } 275 < Z \leq 300 \end{cases}$$

d. Type U

$$\text{(V-45)} \quad TC_u = \begin{cases} 12.83U + 1084.4, & \text{if } 200 \leq U \\ 12.73U + 1114.4, & \text{if } 200 < U \leq 225 \\ 12.5U + 1174.1, & \text{if } 225 < U \leq 250 \\ 12.29U + 1231.1, & \text{if } 250 < U \leq 275 \\ 12.09U + 1288.6, & \text{if } 275 < U \leq 300 \\ 11.9U + 1345 , & \text{if } 300 < U \leq 325 \\ 11.72U + 1399.7, & \text{if } 325 < U \leq 350 \\ 11.55U + 1454.9, & \text{if } 350 < U \leq 375 \\ 11.4U + 1509 , & \text{if } 375 < U \leq 400 \end{cases}$$

H. MODEL SUMMARY

1. The Objective Function

$$\text{Minimize} \quad TC = TC_x + TC_y + TC_z + TC_u$$

where:

a. Total (relevant) cost of type X:

$$TC_x = 4x \qquad \text{(V-46)}$$

b. Total cost of type Y:

$$TC_y = \begin{cases} 0 & \text{if } 0 < Y \leq 100 \\ 45.33Y - 4430.3, & \text{if } 100 < Y \leq 125 \\ 44.37Y - 4216.4, & \text{if } 125 < Y \leq 150 \\ 43.55Y - 3955.8, & \text{if } 150 < Y \leq 175 \\ 42.74Y - 3732.9, & \text{if } 175 < Y \leq 200 \end{cases} \qquad \text{(V-47)}$$

c. Total cost of type Z:

$$TC_z = \begin{cases} 0 \quad , & \text{if } 0 < Z \leq 150 \\ 23.2Z - 3430, & \text{if } 150 < Z \leq 175 \\ 23.1Z - 3365, & \text{if } 175 < Z \leq 200 \\ 23 \;\; Z - 3275, & \text{if } 200 < Z \leq 225 \\ 22.9Z - 3210, & \text{if } 225 < Z \leq 250 \\ 22.8Z - 3120, & \text{if } 250 < Z \leq 275 \\ 22.7Z - 3055, & \text{if } 275 < Z \leq 300 \end{cases} \qquad \text{(V-48)}$$

d. Total Cost of type U:

$$TC_u = \begin{cases} 12.83U + 1084.4, & \text{if } 200 = U \\ 12.73U + 1114.4, & \text{if } 200 < U \leq 225 \\ 12.5\ U + 1174.1, & \text{if } 225 < U \leq 250 \\ 12.29U + 1231.1, & \text{if } 250 < U \leq 275 \\ 12.09U + 1288.6, & \text{if } 275 < U \leq 300 \\ 11.9\ U + 1345\ , & \text{if } 300 < U \leq 325 \\ 11.72U + 1399.7, & \text{if } 325 < U \leq 350 \\ 11.55U + 1454.9, & \text{if } 350 < U \leq 375 \\ 11.4\ U + 1509\ , & \text{if } 375 < U \leq 400 \end{cases} \tag{V-49}$$

(X,Y,Z,U optionally integer)

2. Subject To (The Constraints):

a. Notational Constraints

1) Associated with Scenario 1

$$X \cdot T_x(1-A_{x1}) - [x_{11} + x_{12} + x_{13} + x_{14}] \geq 0 \tag{V-50}$$

$$Y \cdot T_y(1-A_{y1}) - [y_{11} + y_{12} + y_{13} + y_{14}] \geq 0 \tag{V-51}$$

$$Z \cdot T_z(1-A_{z1}) - [z_{11} + z_{12} + z_{13} + z_{14}] \geq 0 \tag{V-52}$$

$$U \cdot T_u(1-A_{u1}) - [u_{11} + u_{12} + u_{13} + u_{14}] \geq 0 \tag{V-53}$$

2) Associated with Scenario 2:

(V-54) $X \cdot T_x(1-A_{x2}) - [x_{21} + x_{22} + x_{23} + x_{24}] \geq 0$

(V-55) $Y \cdot T_y(1-Z_{y2}) - [y_{21} + y_{22} + y_{23} + y_{24}] \geq 0$

(V-56) $Z \cdot T_z(1-A_{z2}) - [z_{21} + z_{22} + z_{23} + z_{24}] \geq 0$

(V-57) $U \cdot T_u(1-A_{u2}) - [u_{21} + u_{22} + u_{23} + u_{24}] \geq 0$

3) Associated with Scenario 3:

(V-58) $X \cdot T_x(1-A_{x3}) - [x_{31} + x_{32} + x_{33} + x_{34}] \geq 0$

(V-59) $Y \cdot T_y(1-A_{y3}) - [y_{31} + y_{32} + y_{33} + y_{34}] \geq 0$

(V-60) $Z \cdot T_z(1-A_{z3}) - [z_{31} + z_{32} + z_{33} + z_{34}] \geq 0$

(V-61) $U \cdot T_u(1-A_{u3}) - [u_{31} + u_{32} + u_{33} + u_{34}] \geq 0$

b. Physical Constraints

1) Availability Constraints:

(V-62) $X = 0$ <u>or</u> $25 \leq X \leq 320$

(V-63) $Y = 100$, <u>or</u> $110 \leq Y \leq 200$

(V-64) $$Z = 150 \ \underline{\text{or}} \ 160 \le Z \le 300$$

(V-65) $$U = 0 \ \underline{\text{or}} \ 200 \le U \le 400$$

2) Resources Constraints:

a) Manpower Constraint:

(V-66) $$10X + 20Y + 15Z + 12U \le 11000$$

3) Capacity Constraints:

a) Ground Facilities Constraint:

(V-67) $$X + 2Y + 1.5Z + 1.5U \le 1000$$

b) Training Space Constraint:

(V-68) $$1.5X + Y + 2Z + 2U \le 1500$$

c. Scenario 1: Mission Needs Constraints

1) Air Defense:

(V-69) $$0x_{11} + 3y_{11} + 2z_{11} + 1.5u_{11} \ge 850$$

of which "all-weather" capable a/c required are:

(V-70) $$y_{11} \ge 80$$

2) A/G Support:

(V-71) $$x_{12} + 0y_{12} + 2z_{11} + 2u_{12} \geq 200$$

of which "smart bomb" carriers required are:

(V-72) $$z_{12} + u_{12} \geq 50$$

3) Deep Penetration Strike:

(V-73) $$x_{13} + 1.5y_{13} + 2.5z_{13} + 2u_{13} \geq 150$$

of which "very long range" capable a/c required are:

(V-74) $$y_{13} + z_{13} \geq 50$$

4) Miscellaneous:

(V-75) $$x_{14} + y_{14} + z_{14} + u_{14} \geq 100$$

d. Scenario 2: Missions Needs Constraints

1) Air Defense:

(V-76) $$0x_{21} + 3y_{21} + 2z_{21} + 2u_{21} \geq 280$$

of which "all-weather" required are:

(V-77) $y_{21} \geq 50$

2) A/G Support:

(V-78) $x_{22} + 0y_{22} + 2z_{22} + 2u_{22} \geq 600$

of which "smart bomb" carriers required are:

(V-79) $z_{22} + u_{22} \geq 150$

3) Deep Penetration Strike:

(V-80) $1.2x_{23} + 1.5y_{23} + 2.5z_{23} + 2u_{23} \geq 200$

of which "very long range" a/c required are:

(V-81) $y_{23} + z_{23} \geq 70$

4) Miscellaneous

(V-82) $x_{24} + y_{24} + z_{24} + u_{24} \geq 100$

e. Scenario 3: Mission Needs Constraints

1) Air Defense:

(V-83) $0x_{31} + 3y_{31} + 2z_{31} + 2u_{31} \geq 320$

of which "all-weather" required are:

(V-84) $$y_{31} \geq 50$$

2) A/G Support:

(V-85) $$x_{32} + 0y_{32} + 2z_{32} + 2u_{32} \geq 300$$

of which "smart bomb" carriers are:

(V-86) $$z_{32} + u_{32} \geq 100$$

3) Deep Penetration Strike:

(V-87) $$1.2x_{33} + 1.5y_{33} + 2.5z_{33} + 2u_{33} \geq 550$$

of which "very long range" a/c required are:

V(-88) $$y_{33} + z_{33} \geq 100$$

Miscellaneous:

(V-89) $$x_{34} + y_{34} + z_{34} + u_{34} \geq 100$$

VI. COMPUTATIONAL APPROACH AND RESULTS

A. COMPUTATIONAL APPROACH

The model presented in the preceding sections can be solved with existing optimization software packages if it is modified as follows.

Each segment of the discontinuous cost function is represented by a constraint of the form:

$$\text{MIN } \tau \leq s \leq \text{MAX } \tau \qquad \text{(each segment)},$$

where MIN and MAX are the lower and upper ranges of the variable s for the segment. τ is a binary variable which selects the segment. Note that $\tau = 0$ implies that the segment is void, and $\tau = 1$ gives

$$\text{MIN} \leq s \leq \text{MAX} \qquad \text{(segment range)}.$$

In this (linear) example, the objective function includes the variable and fixed segment costs as coefficients of s and τ, respectively.

For each discontinuous cost function f, the segments s(f) are coordinated with a mutual exclusion constraint:

$$\sum_{j \in s(f)} \tau_j = 1 \qquad \text{(each function)}.$$

A summation constraint for each function yields the composed argument value:

$$s = \sum_{j \in S(f)} s_j \qquad \text{(each function).}$$

In the example given, segments of each function have disjoint ranges, and each segment is linear. These restrictions are not necessary for the optimization system employed, but do produce a model which is somewhat easier to solve.

Appendix B displays a combined scenario model expressed in the international standard MPS format (e.g., [6]). Constraint and variable naming conventions are also given.

The model shown has no feasible solution in the classical sense. This is an artifact of the problem posed, and not a model oversight. In particular, no aircraft mix exists which completely satisfies the mission needs constraints and the physical constraints simultaneously.

In fact, the model shown includes a bounded logical ("slack", or "surplus") variable for each physical and for each mission need constraint. These variables allow limited violation of each constraint at a specified penalty cost. For mission needs, constraint violation is permitted up to a specified percentage at a penalty cost slightly less than that of an additional a/c. Physical constraints are violated within achievable limits with penalties reflecting the incremental costs for constructing additional bases, etc.

This parameterization of constraints is analogous to linear goal programming and frequently produces acceptable solutions for scenarios which would be intractable otherwise. An additional advantage of this approach is that the original requirements remain explicit in the model statement, along with the degree and penalty cost of admissible violations.

Unfortunately, many examples derived from that shown in Appendix B have no feasible solution in spite of limited violation of mission requirements. Analysis of these cases reveals that further relaxation is required. Accordingly, all model constraints have been stated with individual penalties for each unit of violation by any solution. This elastic model formulation guarantees that an optimal solution will be admissible in that:

1) it is an integer solution, and
2) it is the least cost solution in the complete sense of explicit objective function costs and penalty costs for constraint violations.

The integrality property is required in order to enfore the required model composition of the discontinuous functions. The least-cost property lends face validity to infeasible solutions.

The resulting models have been solved with the X-System (XS), an experimental optimization system of advanced design [3]. Key advantages of XS for this application include:

1) Sheer speed, permitting interactive solution and extensive experimentation,

2) Enhanced mixed integer enumeration, reliably yielding excellent quality integer solutions,

3) Logical incorporation of elastic penalties, significantly easing model preparation and solution efforts, and

4) Capability to include additional non-linear, integer, or other features (e.g., [4]).

XS output for the typical sample problem in Appendix B is shown in Appendix C. This small model exhibits 69 constraints and 124 variables (24 binary with fixed charges). An optimal integer solution was produced by XS (from a cold start) in 2.2 IBM 3033/VM seconds, and 695 pivots.

B. AN EXAMPLE HISTORY OF MODEL RUNS

1. General

To illustrate policy planning with the example at hand, a history of model runs, insights and model modifications follows. The analysis is intuitive, rather than elaborate. The specific results in Appendices B and C represent one of the later examples in this development.

2. First Attempts

Two solution approaches were tested:

a. Only mission constraints eligible for violation,

b. Only physical constraints (except availability) eligible for violation.

The former approach (Run a-mission constraints violation) resulted in unacceptable violations of the mission constraints (see Table VI-1a). The most significant violation of the second approach was 22.9% violation of the ground facilities (GNDFAC) constraint, which was considered slightly higher than an acceptable level of violation (see Table VI-1b).

From the results of both runs it was clear that the dominant mixes are those accepted for Scenario 1 (Scn. 1) and for the "Combined Scenario" (Comb. Scn.). These represent two concepts of mix:

a. The mix of Scn. 1, relative to the Comb. Scn. mix consists of a smaller total a/c quantity, but a larger portion of the "high capability" a/c. Scn. 1 costs more (ignoring penalty costs) but violates to a lesser extent the physical constraints.

b. The Comb. Scn. mix consists of larger total a/c quantity, with a greater proportion of the "low capability" a/c. It costs less than the Scn. 1 mix, but violates to a greater extent the physical constraints. Among the physical constraints, the ground facilities (GNDFAC) was found to be the most binding one. These observations remained valid in all subsequent runs.

3. <u>2nd Attempt</u>

As a result of the first attempt, a conclusion was drawn that in order to obtain "reasonable" violations, both mission and physical constraints should be eligible for violation simultaneously. Since it was known that even in such cases the model may select to violate the GNDFAC constraint above an acceptable level, an "absolute" upper range of 1200 units was specified for this constraint.

All other constraints (except availability) were left free for any penalized violation selected by the model. The results again reveal Scn. 1 and the Comb. Scn. mixes as the principal contenders (see Table VI-2a). In order to make sure that the Scn. 1 mix is an eligible contender, Run b was made. In this run the mix of Scn. 1 was evaluated for Scn. 2 and for Scn. 3. The results reveal acceptable violations for Scn. 2 (most significant--38.0% in Miscellaneous), and no mission constraints violations for Scn. 3 (see Table VI-2b). Thus, the mix of Scn. 1 was found to be adequate for Scn. 2 and Scn. 3, and an eligible contender.

Between the results of the 2nd attempt, Run a (the Comb. Scn. mix) was more attractive: it cost $2 billion less than the Scn. 1 mix, though it required 40 additional GNDFAC units. *Under the assumption* that each GNDFAC unit costs $5 million, the total cost of this mix is much lower. Therefore, further analysis concentrated on the Comb. Scn. mix only. The fact that Miscellaneous was violated by 59.1% in the Comb. Scn. (see Table VI-2a) required some modifications in the next attempt, although without affecting the basic preference for this mix.

4. <u>3rd Attempt</u>

In order to prevent mission constraints violations beyond an acceptable level (e.g., miscellaneous in the 2nd attempt) bounds were put to violations of each of the constraints, according to the percentages presented as upper bounds used in Table VI-3. Two runs took place in this attempt:

a. GNDFAC limited to 1100.

b. GNDFAC limited to 1200.

Ranges and violation bounds for all other constraints were the same in both runs.

The two runs resulted in significantly different mixes (see Table VI-3). The mission constraints violations were about the same in both cases. The mix cost (without penalties) of Run a (GNDFAC 1100) was $2 billion more than in Run b. On the other hand, in Run b an additional cost of the incremental 100 GNDFAC units relative to Run a should be added. But since each GNDFAC unit costs $5 million (which is $0.5 billion for 100 units), the total cost of the Run b mix was still $1.5 billion less than that of the Run a mix. Thus, the final mix selected was the one of Run b of the 3rd attempt.

Legend for Tables VI-1a--VI-3

WOPT - cost without penalty

X total, Y total, Z total, U total - Total quantities of a/c types X, Y, Z and U, respectively.

AIRDEF 1 - Air-defense mission constraint in scenario 1 (the associated digit may change to 2 or 3 for scenarios 2 and 3 respectively).

AGSUPP 2 - Air-to-ground support mission constraint for scenario 2.

DEEPPEN 3 - Deep penetration strike mission constraint for scenario 3.

MISCELL 3 - Miscellaneous mission constraint for scenario 3.

GNDFAC - Ground facilities constraint.

MANPOWER - Manpower constraint.

TRAINSPA - Training airspace constraint.

RHS - Right-hand side, giving constraint range.

Table VI-1a. First Attempt, Run a (Mission Violations Permitted)

* Run a

Scenario 1 (Mission Violation)

WOPT = 9720.50

X Total = 0

Y Total = 200.0

Z Total = 200.0

U Total = 200.0

AIRDEF 1: 771.25 vice 850 (9.2%)

MISCELL 1: 0 vice 100 (100%)

Scenario 2 (Mission Violation)

WOPT = 7330.6667

X Total = 0.0

Y Total = 129.6

Z Total = 150.0

U Total = 343.8

AGSUPP 2: 551.1 vice 600 (8.1%)

MISCELL 2: 0 vice 100 (100%)

Scenario 3 (Mission Violation)

WOPT = 8186.35

X Total = 37.5

Y Total = 125.0

Z Total = 275.0

U Total = 200.0

MISCELL 3: 30 vice 100 (70%)

Combined Scenarios (Mission Violation)

WOPT = 5420.30

X Total = 123.5

Y Total = 100.0

Z Total = 150.0

U Total = 301.0

Table VI-1a (CONTINUED)

AIRDEF 1: 572 vice 850 (32.7%)

MISCELL 1: 98.8 vice 100 (1.2%)

AGSUPP 2: 522.4 vice 600 (12.9%)

AIRDEF 2: 272 vice 280 (2.8%)

DEEPPEN 3: 520.2 vice 550 (5.4%)

MISCELL 2: 0 vice 100 (100%)

MISCELL 3: 0 vice 100 (100%)

*Mission: Weighted Equally

Table VI-1b. First Attempt, Run b (Physical violations allowed)

Run b

Scenario 1 (Physical Violation)

WOPT = 11307.0
X Total = 86.2
Y Total = 200.0
Z Total = 249.2
U Total = 200.0
GNDFAC: 1160 vice 1000 (16%)

Scenario 2 (Physical Violation)

WOPT = 7906.5
X Total = 138.9
Y Total = 129.6
Z Total = 150.0
U Total = 377.8
GNDFAC: 1189.8 vice 1000 (19%)

Scenario 3 (Mission Violation)

WOPT = 8536.4
X Total = 125.0
Y Total = 125.0
Z Total = 275.0
U Total = 200.0
GNDFAC: 1087.5 vice 1000 (8.8%)

Combined Scenario (Physical Violation)

WOPT = 9241.2
X Total = 63.8
Y Total = 132.5
Z Total = 200.0
U Total = 400.0
MANPOWER: 11087.8 vice 11000 (.7%)
GNDFAC: 1228.8 vice 1000 (22.9%)

Table VI-2a. Second Attempt, Run a (Relax the Binding Physical Constraint by 20% → GNDFAC ≤ 1200 vice 1000)

Scenario 1

WOPT = 11307.3 + Cost of 160 additional GNDFAC units

X Total = 86.2

Y Total = 200.0

Z Total = 249.2

U Total = 200.0

MISCELL 1: 68.9 vice 100 (31.1%)

Scenario 2

WOPT = 7105.7 + Cost of 200 additional GNDFAC units

X Total = 150.0

Y Total = 100.0

Z Total = 166.7

U Total = 400.0

No mission violations

Scenario 3

WOPT = 6413 + Cost of 143.6 additional GNDFAC units

X Total = 154.6

Y Total = 100.0

Z Total = 150.0

U Total = 376.0

No mission violations

Combined Scenarios

WOPT = 9371.6 + Cost of 200 additional GNDFAC units

X Total = 51.2

Y Total = 100.0

Z Total = 299.2

U Total = 333.3

MISCELL 1: 40.9 vice 100 (59.1%)*

MISCELL 2: 80.3 vice 100 (19.7%)

Comments: The RHS for GNDFAC was increased to 1200. Mission and physical constraints were assigned equal penalties with the exception of AIRDEF 1. AIRDEF 1 was found to be binding in both the Scenario 1 run and the combined Scenario runs. Since this was the constraint least desired to be violated, a high penalty was assigned to it. With the exception of AIRDEF 1, the optimizer was allowed to choose which mission constraints to violate.

Table VI-2b. Second Attempt, Run b (Use Scenario 1 Aircraft Mix in other Scenarios)

Values Used: (GNDFAC relaxed to 1200 from 1000)

WOPT = 11307.3

X Total = 86.2

Y Total = 200.0

Z Total = 249.2

U Total = 200.0

Combined Scenarios

MISCELL 1: 31% violation

MISCELL 2: 38% violation

AGSUPP 2: 8.7% violation

MANPOWER: At Upper Bound

GNDFAC: At Upper Bound

Scenario 2

AGSUPP 2: 8.7% violation

MISCELL 2: 38.0% violation

MANPOWER: At Upper Bound

GNDFAC: At 1160. Within 40 of Upper Bound.

Scenario 3

All constraints satisfied.

MANPOWER: At Upper Bound

GNDFAC: At 1160. Within 40 of Upper Bound.

Comment: Since the aircraft mix for Scenario 1 is significantly different from that for the other scenarios, a set of runs was made to determine how well the other scenarios could be satisfied with the Scenario 1 mix.

Table VI-3. Third Attempt: Combined Scenarios with Relaxations spread over all constraints

Upper Bounds Used:

GNDFAC: 10% and 20%

MANPOWER: 10%

TRAINSPA: 30%

AIRDEF 1,2,3: 0.0%

AGSUPP 1,2,3: 20.0%

DEEPPEN 1,2,3: 15.0%

All other: 10%

Run a

WOPT = 10203.3 (Cost of artificials "slacks" removed)

Relaxations Selected:

GFAC:	100.0	All available
AGS 1:	40.0	
DPN 1:	23.0	
VLR 1:	5.0	
MSC 1:	50.0	
AGS 2:	40.0	(120 available)
MSC 2:	50.0	All available
MSC 3:	14.4	(50 available)

Constraint Violated: MISCELL 1: 96.8 vice 100 (3.2%)

X Total = 58.6

Y Total = 150.0

Z Total = 294.3

U Total = 200.0

Run b

WOPT = 8012.7 (Cost of artificials "slacks" removed)

Relaxations Selected:

GFAC:	200.0	(20%)
AGS 1:	40.0	All Available
DPN 1:	23.0	
MSC 1:	50.0	
MSC 2:	12.6	(50 available)

No constraints violated

X Total = 80.7

Y Total = 100.0

Z Total = 212.9

U Total = 400.0

Comment: Surplus (slack) variables were added to each mission (physical) constraint. The upper bound on each surplus (slack) was set equal to a percentage of the RHS. Two runs were made, with the only difference being the percentage relaxations allowed for GNDFAC: 10% for Run #1, and 20% for Run #2. Each artificial (slack) was given a cost of 5.0 in the objective function.

VII. CONCLUSION

The approach to force mix planning presented here appears to have much to recommend it. Strategic planning is a complex task, with subtle relationships among the principal components of the problem. Use of a model in decision making can materially assist the planner by providing objective expression of the complex trade-offs and relationships, and by objectively evaluating and comparing alternatives.

The particular model proposed has proved to yield force mix decisions with excellent face validity--decisions that successfully address all of the simultaneous planning constraints and complicated relationships, that are available interactively to the planners, and that can be easily modified and retested as the perception of the nature of the problem at hand is sharpened.

Further work remains to be done. It is not clear whether the sensitivity of the solutions to changes in problem statement is due to the artificial simplicity of the examples used, or to some intrinsic properties of the mixed-integer model. Additional analysis of the construction and interpretation of penalties for infeasible, or nearly infeasible planning scenarios is also a topic of continuing interest.

APPENDIX A

COST CONSIDERATIONS

A. GENERAL

The objective function of the model is to minimize cost. Consequently, cost is a crucial ingredient in the model and warrants close scrutiny. Appendix A provides insight into the components of the cost function and describes the methodology by which the cost functions of the example model were generated.

B. LIFE CYCLE COST (LCC) CATEGORIES

LCC consists of three main categories. These are:

1) Research and Development Costs (R&D)--these are the resources required to develop a new capability to the point where it can be introduced into operational inventory at some desired level of reliability. In most cases R&D costs account for about 10%-12% of the total LCC.

2) Investment Costs--the one-time outlays required to introduce the capability into the operational inventory. This category is divided into two sub-categories:

a) The procurement costs of major equipment. These costs are basically derived from production costs, and they mimic behavior of a production "learning curve" function.

b) Other investment costs--which are incurred in various areas, such as building maintenance facilities, purchase of maintenance equipment, purchase of initial inventories, creating and training initial manpower, etc. In most cases investment costs (both sub-categories together) account for about 30%-40% of the total LCC.

3) Operating and Support Costs (O&S)--these are the recurring outlays required year-by-year to operate and maintain the capability in service over a specified period. In most cases O&S costs account for about 50%-60% of the total LCC ([5], p. 66-67).

C. CHARACTERISTICS OF THE LCC FUNCTION

1. General

LCC is by no means linear; in other words, LCC per unit is not a constant as the number of units change. This statement is based on the behavior of the LCC's main component functions. Thus, in order to explore the characteristics of the LCC function, the individual functions of each of LCC major category are examined.

2. R&D Cost Function

Essentially, R&D costs are fixed, and not mitigated by the amount of units produced. R&D cost allocated per unit changes in a simple algebraic ratio as the number of units increases. The R&D cost relationships are presented in Figures A-1 and A-2. The basic functions of R&D costs (simplified) are:

1) Total R&D costs:

$$C_{RD} = a$$

2) R&D cost per unit

$$c_{rd} = \frac{a}{Q}$$

where:

C_{RD} = total R&D costs

c_{rd} = R&D costs per unit

Q = Quantity of units

a = Specified total amount of R&D costs.

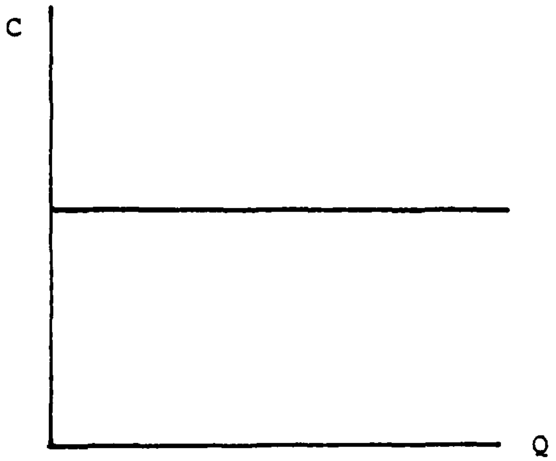

Figure A-1.
Total R&D Cost

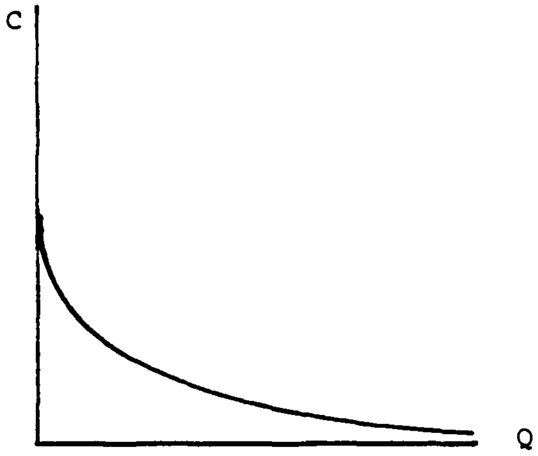

Figure A-2.
R&D Cost Per Unit

3. Investment Costs

a. Production Costs

Production costs are the main ingredient in procurement costs (the latter is a major subcategory of investment costs). Production costs as a function of units produced are not linear because of the so-called "learning curve" effect. Production costs exhibiting the "learning curve" effect, are illustrated in Figures A-3 and A-4.

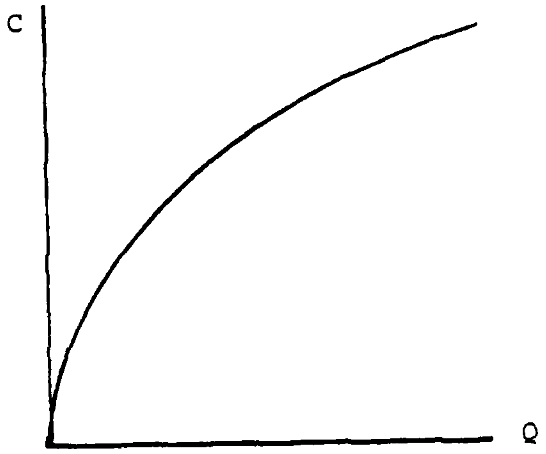

Figure A-3

Total Production Cost

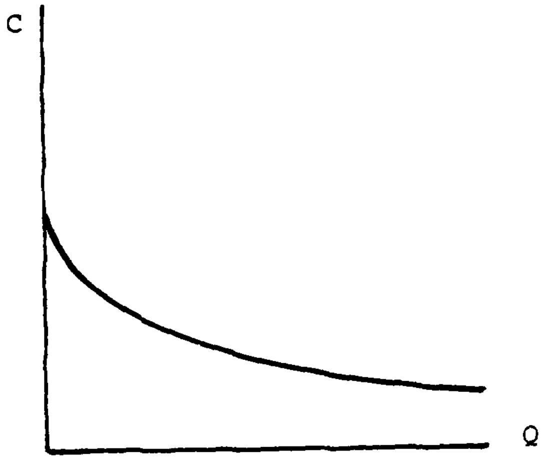

Figure A-4

Production Cost Per Unit

The basic functions for the "learning curve" effect are:

1) Total production costs:[1]

$$C_p = aQ^{b+1}$$

2) Average unit production cost:

$$c_p = aQ^b$$

---

[1] Both equations are valid for the case in which average unit cost is log-linear.

where:

a = the cost of the first unit produced

b = $\ln_2 S$

S = the fraction to which cost is reduced as the quantity is doubled.

The following example may provide further understanding of the meaning of the notation S: Assume that the 1st unit production cost is 100 and S = .8, then the average cost of the 2nd cost is 80. Or, if the average cost of the 100th unit is 20, the average cost of the 200th unit is 16, etc., ([2], pp. 95-100].

b. Other Investment Costs

Other investment costs are not linear. For example, initial inventories can behave in a very distinctive manner. In the case of aircraft procurement an initial increment is required as minimal amounts of inventory initiation. Inventories then increase while the number of aircraft increase, but the rate of growth may decrease as the number of aircraft increases because of greater interchangeability of the relatively large inventory. Then we can pose a complete change of the inventory system as size crosses a certain threshold, or a need to open a new storage depot if the amount of a/c requires stationing of squadrons at additional bases, and so forth. Maintenance facility costs, such as for hangars or repair labs, grow in increments with the number of squadrons or some similar criterion, but surely not as a discrete function of each additional a/c. The trajectory of investment costs (excluding major equipment procurement costs) appear as shown in Figure A-5.

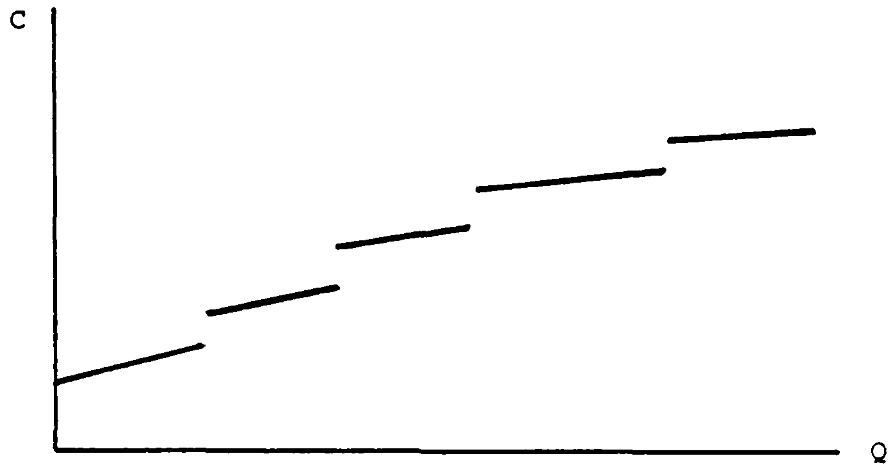

Figure A-5. "Other" Investment Costs (besides procurement costs of major equipment)

4. O&S Costs

Roughly, these costs might be considered as a linear function of the number of a/c. The general shape of O&S cost functions is illustrated in Figures A-6 and A-7.

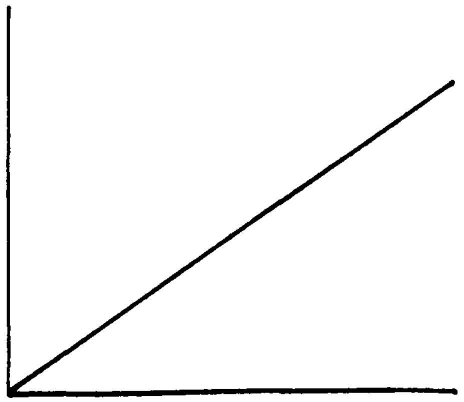

Figure A-6
Total O&S Cost

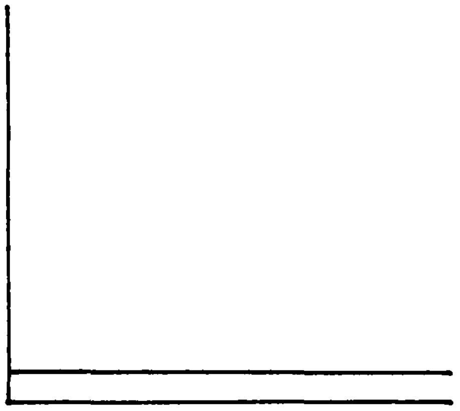

Figure A-7
O&S Cost Per Unit

5. Total LCC Function

Total LCC as a function of the number of a/c units is a summation of the individual LCC component functions. The shape of total LCC function is as illustrated in Figure A-8.

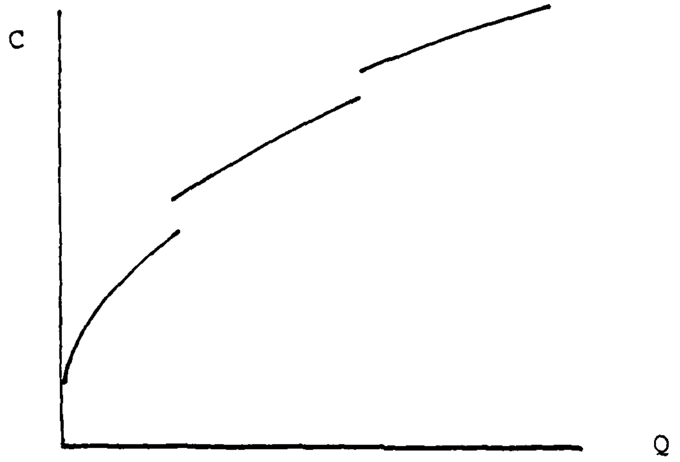

Figure A-8. LCC Function

D. COST FUNCTION CONSTRUCTION FOR THE EXAMPLE

1. Basic Considerations for Cost Calculation

a. Cost is calculated in 4 categories:

1) R&D.

2) Procurement (production for indigenous production, "fly away" price for foreign source).

3) Other investments (including facilities, initial ground equipment, spares inventories, initial manpower, initial training, etc.).

4) Operation and Support (for 5 years only, to create common lifetimes for all types).

b. In order to ease the example's cost data generation:

1) In each category (if relevant) cost is calculated by a simplified function that represents a much more detailed and elaborate "real-life" function (or model). There is no branching into sub-category levels.

2) The function at each category is supposed to:

a) Represent the actual general behavior of the "real life" cost category.

b) Result in reasonable numerical outcomes relative to the other cost categories for the same type of a/c, and relative to the parallel cost categories for the other a/c types.

3) For each type of a/c the cost categories approximately relate to each other according to the following ratio (arbitrarily set, though similar to real-life ratios);

a) R&D $\cong$ 10%

b) Procurement $\cong$ 30%

c) Other Investments $\cong$ 10%

d) O&S $\cong$ 50%.

c. In addition, the following principles are followed in cost function generation for the example:

1) The cost to be used is <u>economic cost</u>.

2) Only <u>relevant costs</u>, i.e., costs that are related to, or result by an alternate selection, are considered.

3) Basically, <u>life cycle cost</u> (LCC) figures are used, excluding the irrelevant components.

4) All costs are discounted to their present value.

2. The Cost Functions Construction Procedure

In the construction of the example cost functions the following procedure is followed (separately for each type of a/c):

a. Determine the functions for the four main LCC components, excluding the irrelevant components. Use simplified approximated functions.

b. Sum up the component functions, giving the total LCC function. Where the LCC function is available in the first place, it can be used directly.

c. In order to implement a particularly efficient non-linear programming method, approximate the LCC function with a set of linear functions. The break points, i.e., the levels of the independent variable (in our case--a/c quantities) that determine the upper and lower ranges for each linear sub-function, should be set by either of two main criteria:

1) In each place where the original LCC function exhibits a discontinuity (e.g., determined by the steps of the initial investment component).

2) In each case where the linear approximation would diverge unreasonably from the original function. Break points can be set at any level, and to not necessarily follow in equal steps.

Comment: In the example, steps in the initial investment cost function are set arbitrarily to occur at every 25 units. This also dictates the location of the break points of the linear approximation of the total LCC function.

d. The LCC function and the resulting set of linear approximations for each a/c type should be defined within a reasonable range. This reasonable range is different for each a/c type and derived from "real life" availability constraints or the like.

APPENDIX B

SAMPLE MODEL INPUT

```
NAME     COMBINED SCENARIOS: OPTIMAL AIRCRAFT MIX
ROWS
*  SELECTION CONSTRAINTS
   E PICK1PCX
   E PICK1PCY
   E PICK1PCZ
   E PICK1PCU
*  SUMMATION CONSTRAINTS
   G SUM    X
   G SUM    Y
   G SUM    Z
   G SUM    U
*  RANGE CONSTRAINTS
   L RANGE 1X
   L RANGE 2X
   L RANGE 1Y
   L RANGE 2Y
   L RANGE 3Y
   L RANGE 4Y
   L RANGE 5Y
   L RANGE 1Z
   L RANGE 2Z
   L RANGE 3Z
   L RANGE 4Z
   L RANGE 5Z
   L RANGE 6Z
   L RANGE 7Z
   L RANGE 1U
   L RANGE 2U
   L RANGE 3U
   L RANGE 4U
   L RANGE 5U
   L RANGE 6U
   L RANGE 7U
   L RANGE 8U
   L RANGE 9U
   L RANGE AU
*  PHYSICAL CONSTRAINTS
   L MANPOWER
   L GNDFAC
   L TRAINSPA
*  NOTATION CONSTRAINTS
   G SCNBALX1
   G SCNBALY1
   G SCNBALZ1
   G SCNBALU1
   G SCNBALX2
   G SCNBALY2
   G SCNBALZ2
   G SCNBALU2
   G SCNBALX3
   G SCNBALY3
   G SCNBALZ3
   G SCNBALU3
*  SCENARIO 1 MISSION CONSTRAINTS
   G AIRDEF1
   G ALWEATH1
   G AGSUPP1
   G SMRTBMB1
   G DEEPPEN1
   G VLONGRN1
   G MISCELL1
*  SCENARIO 2 MISSION CONSTRAINTS
   G AIRDEF2
   G ALWEATH2
   G AGSUPP2
   G SMRTBMB2
   G DEEPPEN2
   G VLONGRN2
   G MISCELL2
*  SCENARIO 3 MISSION CONSTRAINTS
   G AIRDEF3
```

```
  G ALWEATH3
  G AGSUPP3
  G SMRTBMB3
  G DEEPPEN3
  G VLONGRN3
  G MISCELL3
* OBJECTIVE FUNCTION
  N COSTS
COLUMNS
              'MARKER'                 'INTORG'
* BINARY VARIABLES USED TO SELECT INDICATED RANGE
    Y1X       PICK1FCX  1.0
    Y2X       PICK1FCX  1.0            RANGE 2X  -320.0
    Y1Y       PICK1FCY  1.0            RANGE 1Y  -100.0
    Y2Y       PICK1FCY  1.0            RANGE 2Y  -125.0
    Y2Y       COSTS     -4430.3
    Y3Y       PICK1FCY  1.0            RANGE 3Y  -150.0
    Y3Y       COSTS     -4216.4
    Y4Y       PICK1FCY  1.0            RANGE 4Y  -175.0
    Y4Y       COSTS     -3955.8
    Y5Y       PICK1FCY  1.0            RANGE 5Y  -200.0
    Y5Y       COSTS     -3732.9
    Y1Z       PICK1FCZ  1.0            RANGE 1Z  -150.0
    Y2Z       PICK1FCZ  1.0            RANGE 2Z  -175.0
    Y2Z       COSTS     -3430.0
    Y3Z       PICK1FCZ  1.0            RANGE 3Z  -200.0
    Y3Z       COSTS     -3365.0
    Y4Z       PICK1FCZ  1.0            RANGE 4Z  -225.0
    Y4Z       COSTS     -3275.0
    Y5Z       PICK1FCZ  1.0            RANGE 5Z  -250.0
    Y5Z       COSTS     -3210.0
    Y6Z       PICK1FCZ  1.0            RANGE 6Z  -275.0
    Y6Z       COSTS     -3120.0
    Y7Z       PICK1FCZ  1.0            RANGE 7Z  -300.0
    Y7Z       COSTS     -3055.0
    Y1U       PICK1FCU  1.0
    Y2U       PICK1FCU  1.0            RANGE 2U  -200.0
    Y2U       COSTS     1084.4
    Y3U       PICK1FCU  1.0            RANGE 3U  -225.0
    Y3U       COSTS     1114.1
    Y4U       PICK1FCU  1.0            RANGE 4U  -250.0
    Y4U       COSTS     1174.1
    Y5U       PICK1FCU  1.0            RANGE 5U  -275.0
    Y5U       COSTS     1231.1
    Y6U       PICK1FCU  1.0            RANGE 6U  -300.0
    Y6U       COSTS     1288.6
    Y7U       PICK1FCU  1.0            RANGE 7U  -325.0
    Y7U       COSTS     1345.0
    Y8U       PICK1FCU  1.0            RANGE 8U  -350.0
    Y8U       COSTS     1399.7
    Y9U       PICK1FCU  1.0            RANGE 9U  -375.0
    Y9U       COSTS     1454.9
    YAU       PICK1FCU  1.0            RANGE AU  -400.0
    YAU       COSTS     1509.0
              'MARKER'                 'INTEND'
* CONTINUOUS VARIABLES FOR INDICATED RANGE
    X1X       SUM    X  1.0            RANGE 1X  1.0
    X2X       SUM    X  1.0            RANGE 2X  1.0
    X2X       COSTS     4.0
    X1Y       SUM    Y  1.0            RANGE 1Y  1.0
    X2Y       SUM    Y  1.0            RANGE 2Y  1.0
    X2Y       COSTS     45.33
    X3Y       SUM    Y  1.0            RANGE 3Y  1.0
    X3Y       COSTS     44.37
    X4Y       SUM    Y  1.0            RANGE 4Y  1.0
    X4Y       COSTS     43.55
    X5Y       SUM    Y  1.0            RANGE 5Y  1.0
    X5Y       COSTS     42.74
    X1Z       SUM    Z  1.0            RANGE 1Z  1.0
    X2Z       SUM    Z  1.0            RANGE 2Z  1.0
    X2Z       COSTS     23.2
    X3Z       SUM    Z  1.0            RANGE 3Z  1.0
```

```
    X3Z        COSTS       23.1
    X4Z        SUM      Z  1.0                 RANGE 4Z   1.0
    X4Z        COSTS       23.0
    X5Z        SUM      Z  1.0                 RANGE 5Z   1.0
    X5Z        COSTS       22.9
    X6Z        SUM      Z  1.0                 RANGE 6Z   1.0
    X6Z        COSTS       22.8
    X7Z        SUM      Z  1.0                 RANGE 7Z   1.0
    X7Z        COSTS       22.7
    X1U        SUM      U  1.0                 RANGE 1U   1.0
    X2U        SUM      U  1.0                 RANGE 2U   1.0
    X2U        COSTS       12.83
    X3U        SUM      U  1.0                 RANGE 3U   1.0
    X3U        COSTS       12.73
    X4U        SUM      U  1.0                 RANGE 4U   1.0
    X4U        COSTS       12.50
    X5U        SUM      U  1.0                 RANGE 5U   1.0
    X5U        COSTS       12.29
    X6U        SUM      U  1.0                 RANGE 6U   1.0
    X6U        COSTS       12.09
    X7U        SUM      U  1.0                 RANGE 7U   1.0
    X7U        COSTS       11.90
    X8U        SUM      U  1.0                 RANGE 8U   1.0
    X8U        COSTS       11.72
    X9U        SUM      U  1.0                 RANGE 9U   1.0
    X9U        COSTS       11.55
    XAU        SUM      U  1.0                 RANGE AU   1.0
    XAU        COSTS       11.40
* CONTINUOUS VARIABLES INDICATING TOTAL NUMBER OF A/C X,Y,Z,OR U.
    XTOTAL     SUM      X  -1.0                MANPOWER   10.0
    XTOTAL     GNDFAC      1.0                 TRAINSPA   1.5
    XTOTAL     SCNBALX1      0.80              SCNBALX2   0.72
    XTOTAL     SCNBALX3      0.80
    YTOTAL     SUM      Y  -1.0                MANPOWER   20.0
    YTOTAL     GNDFAC      2.0                 TRAINSPA   1.0
    YTOTAL     SCNBALY1      0.80              SCNBALY2   0.72
    YTOTAL     SCNBALY3      0.80
    ZTOTAL     SUM      Z  -1.0                MANPOWER   15.0
    ZTOTAL     GNDFAC      1.5                 TRAINSPA   2.0
    ZTOTAL     SCNBALZ1      0.80              SCNBALZ2   0.72
    ZTOTAL     SCNBALZ3      0.80
    UTOTAL     SUM      U  -1.0                MANPOWER   12.0
    UTOTAL     GNDFAC      1.5                 TRAINSPA   2.0
    UTOTAL     SCNBALU1      0.80              SCNBALU2   0.72
    UTOTAL     SCNBALU3      0.80
* CONTINUOUS VARIABLES INDICATING NUM OF A/C  USED ON INDICATED MISSI
    X11        SCNBALX1     -1.0
    X12        SCNBALX1     -1.0               AGSUPP1    1.0
    X13        SCNBALX1     -1.0               DEEPPEN1   1.0
    X14        SCNBALX1     -1.0               MISCELL1   1.0
    X21        SCNBALX2     -1.0
    X22        SCNBALX2     -1.0               AGSUPP2    1.0
    X23        SCNBALX2     -1.0               DEEPPEN2   1.2
    X24        SCNBALX2     -1.0               MISCELL2   1.0
    X31        SCNBALX3     -1.0
    X32        SCNBALX3     -1.0               AGSUPP3    1.0
    X33        SCNBALX3     -1.0               DEEPPEN3   1.2
    X34        SCNBALX3     -1.0               MISCELL3   1.0
    Y11        SCNBALY1     -1.0               AIRDEF1    3.0
    Y11        ALWEATH1    1.0
    Y12        SCNBALY1     -1.0
    Y13        SCNBALY1     -1.0               DEEPPEN1   1.5
    Y13        VLONGRN1    1.0
    Y14        SCNBALY1     -1.0               MISCELL1   1.0
    Y21        SCNBALY2     -1.0               AIRDEF2    3.0
    Y21        ALWEATH2    1.0
    Y22        SCNBALY2     -1.0
    Y23        SCNBALY2     -1.0               DEEPPEN2   1.5
    Y23        VLONGRN2    1.0
    Y24        SCNBALY2     -1.0               MISCELL2   1.0
    Y31        SCNBALY3     -1.0               AIRDEF3    3.0
    Y31        ALWEATH3    1.0
```

```
   Y32     SCNBALY3   -1.0
   Y33     SCNBALY3   -1.0     DEEPPEN3 1.5
   Y33     VLONGRN3 1.0
   Y34     SCNBALY3   -1.0     MISCELL3 1.0
   Z11     SCNBALZ1   -1.0     AIRDEF1  2.0
   Z12     SCNBALZ1   -1.0     AGSUPP1  2.0
   Z12     SMRTBMB1 1.0
   Z13     SCNBALZ1   -1.0     DEEPPEN1 2.5
   Z13     VLONGRN1 1.0
   Z14     SCNBALZ1   -1.0     MISCELL1 1.0
   Z21     SCNBALZ2   -1.0     AIRDEF2  2.0
   Z22     SCNBALZ2   -1.0     AGSUPP2  2.0
   Z22     SMRTBMB2 1.0
   Z23     SCNBALZ2   -1.0     DEEPPEN2 2.5
   Z23     VLONGRN2 1.0
   Z24     SCNBALZ2   -1.0     MISCELL2 1.0
   Z31     SCNBALZ3   -1.0     AIRDEF3  2.0
   Z32     SCNBALZ3   -1.0     AGSUPP3  2.0
   Z32     SMRTBMB3 1.0
   Z33     SCNBALZ3   -1.0     DEEPPEN3 2.5
   Z33     VLONGRN3 1.0
   Z34     SCNBALZ3   -1.0     MISCELL3 1.0
   U11     SCNBALU1   -1.0     AIRDEF1  1.5
   U12     SCNBALU1   -1.0     AGSUPP1  2.0
   U12     SMRTBMB1 1.0
   U13     SCNBALU1   -1.0     DEEPPEN1 2.0
   U14     SCNBALU1   -1.0     MISCELL1 1.0
   U21     SCNBALU2   -1.0     AIRDEF2  2.0
   U22     SCNBALU2   -1.0     AGSUPP2  2.0
   U22     SMRTBMB2 1.0
   U23     SCNBALU2   -1.0     DEEPPEN2 2.0
   U24     SCNBALU2   -1.0     MISCELL2 1.0
   U31     SCNBALU3   -1.0     AIRDEF3  2.0
   U32     SCNBALU3   -1.0     AGSUPP3  2.0
   U32     SMRTBMB3 1.0
   U33     SCNBALU3   -1.0     DEEPPEN3 2.0
   U34     SCNBALU3   -1.0     MISCELL3 1.0
* SLACK VARIABLES USED TO RELAX PHYSICAL CONSTRAINTS
   MPWR    MANPOWER -1.0       COSTS    5.0
   GFAC    GNDFAC   -1.0       COSTS    5.0
   TSPA    TRAINSPA -1.0       COSTS    5.0
* ARTIFICIAL VARIABLES USED TO RELAX MISSION CONSTRAINTS
   ARD1    AIRDEF1  1.0        COSTS    5.0
   ALW1    ALWEATH1 1.0        COSTS    5.0
   AGS1    AGSUPP1  1.0        COSTS    5.0
   SMT1    SMRTBMB1 1.0        COSTS    5.0
   DPN1    DEEPPEN1 1.0        COSTS    5.0
   VLR1    VLONGRN1 1.0        COSTS    5.0
   MSC1    MISCELL1 1.0        COSTS    5.0
   ARD2    AIRDEF2  1.0        COSTS    5.0
   ALW2    ALWEATH2 1.0        COSTS    5.0
   AGS2    AGSUPP2  1.0        COSTS    5.0
   SMT2    SMRTBMB2 1.0        COSTS    5.0
   DPN2    DEEPPEN2 1.0        COSTS    5.0
   VLR2    VLONGRN2 1.0        COSTS    5.0
   MSC2    MISCELL2 1.0        COSTS    5.0
   ARD3    AIRDEF3  1.0        COSTS    5.0
   ALW3    ALWEATH3 1.0        COSTS    5.0
   AGS3    AGSUPP3  1.0        COSTS    5.0
   SMT3    SMRTBMB3 1.0        COSTS    5.0
   DPN3    DEEPPEN3 1.0        COSTS    5.0
   VLR3    VLONGRN3 1.0        COSTS    5.0
   MSC3    MISCELL3 1.0        COSTS    5.0
RHS
* LOGICAL RESTRICTION FORCING SELECTION OF ONLY ONE BINARY VARIABLE
   SCENALL PICK1FCX 1.0
   SCENALL PICK1FCY 1.0
   SCENALL PICK1FCZ 1.0
   SCENALL PICK1FCU 1.0
* PHYSICAL LIMITATIONS EXPRESSED IN PHYSICAL UNITS
   SCENALL MANPOWER 11000.0
   SCENALL GNDFAC   1000.0
```

```
    SCENALL   TRAINSPA  1500.0
* MISSION MINIMUMS EXPRESSED IN MISSION UNITS
    SCENALL   AIRDEF1   850.0
    SCENALL   ALWEATH1  80.0
    SCENALL   AGSUPP1   200.0
    SCENALL   SMRTBMB1  50.0
    SCENALL   DEEPPEN1  150.0
    SCENALL   VLONGRN1  50.0
    SCENALL   MISCELL1  100.
    SCENALL   AIRDEF2   280.0
    SCENALL   ALWEATH2  50.0
    SCENALL   AGSUPP2   600.0
    SCENALL   SMRTBMB2  150.0
    SCENALL   DEEPPEN2  200.0
    SCENALL   VLONGRN2  70.0
    SCENALL   MISCELL2  100.0
    SCENALL   AIRDEF3   320.0
    SCENALL   ALWEATH3  50.0
    SCENALL   AGSUPP3   300.0
    SCENALL   SMRTBMB3  100.0
    SCENALL   DEEPPEN3  550.0
    SCENALL   VLONGRN3  100.0
    SCENALL   MISCELL3  100.0
 RANGES
    R         RANGE 1X  0.0
    R         RANGE 2X  295.0
    R         RANGE 1Y  100.0
    R         RANGE 2Y  24.0
    R         RANGE 3Y  24.0
    R         RANGE 4Y  24.0
    R         RANGE 5Y  24.0
    R         RANGE 1Z  150.0
    R         RANGE 2Z  24.0
    R         RANGE 3Z  24.0
    R         RANGE 4Z  24.0
    R         RANGE 5Z  24.0
    R         RANGE 6Z  24.0
    R         RANGE 7Z  24.0
    R         RANGE 1U  0.0
    R         RANGE 2U  24.0
    R         RANGE 3U  24.0
    R         RANGE 4U  24.0
    R         RANGE 5U  24.0
    R         RANGE 6U  24.0
    R         RANGE 7U  24.0
    R         RANGE 8U  24.0
    R         RANGE 9U  24.0
    R         RANGE AU  24.0
 BOUNDS
  UP BOUND       XTOTAL     320.0
  LO BOUND       YTOTAL     100.0
  UP BOUND       YTOTAL     200.0
  LO BOUND       ZTOTAL     150.0
  LO BOUND       UTOTAL     200.0
  UP BOUND       X1Y        100.0
  UP BOUND       X1X        25.0
  UP BOUND       X1Z        150.0
  UP BOUND       X1U        200.0
* BOUNDS INDICATE MAXIMUM RELAXATION ALLOWED IN UNITS FOR EACH CONSTR
  UP BOUND       MPWR       1100.0
  UP BOUND       GFAC       200.0
  UP BOUND       TSPA       450.0
  UP BOUND       ARD1       0.0
  UP BOUND       ALW1       8.0
  UP BOUND       AGS1       40.0
  UP BOUND       SMT1       5.0
  UP BOUND       DPN1       23.00
  UP BOUND       VLR1       5.0
  UP BOUND       MSC1       50.0
  UP BOUND       ARD2       0.0
  UP BOUND       ALW2       5.0
  UP BOUND       AGS2       120.0
```

```
 UP BOUND      SMT2      30.00
 UP BOUND      DPN2      30.00
 UP BOUND      VLR2      7.00
 UP BOUND      MSC2      50.0
 UP BOUND      ARD3      0.0
 UP BOUND      ALW3      5.0
 UP BOUND      AGS3      60.00
 UP BOUND      SMT3      10.0
 UP BOUND      DPN3      83.00
 UP BOUND      VLR3      10.0
 UP BOUND      MSC3      50.0
ENDATA
```

APPENDIX C

SAMPLE MODEL OUTPUT

```
XS MATHEMATICAL PROGRAMMING SYSTEM (10/81)

MAX DATA REGION=  200000
MPN= 1369  MGP= 1370  NE= 16428  NPV=  4114
MPS DATA REGION=  199974

                              (MPS INPUT EDIT FROM UNIT  4)
                               PRIM              0    0
                               EDIT WARNING
                               MIP              13    0    0      0.1000000+01

MPS INPUT FROM UNIT  3

----HEADER   CARDS----
NAME     COMBINED SCENARIOS: OPTIMAL AIRCRAFT MIX
----ROW    SECTION----
* SELECTION CONSTRAINTS
* SUMMATION CONSTRAINTS
* RANGE CONSTRAINTS
* PHYSICAL CONSTRAINTS
* NOTATION CONSTRAINTS
* SCENARIO 1 MISSION CONSTRAINTS
* SCENARIO 2 MISSION CONSTRAINTS
* SCENARIO 3 MISSION CONSTRAINTS
* OBJECTIVE FUNCTION
                               OBJECTIVE ROW:   69   N COSTS

                               ROWS: TOTAL=    69   ACCEPTED=   69
                               PEN        1    32    1  0.1000000+06 0.1000000+06
                               PEN       36    48    1  0.1000000+06 0.1000000+06
                               PEN       33    35    1  0.1000000+05 0.1000000+05
                               PEN       49    68    1  0.1000000+05 0.1000000+05
                               SFAC       5    68    1  0.1000000-02
                               SFAC      69    69    1  0.1000000+00
                               SFAC      94   190    1  0.1000000+02
                              (END OF EDIT FILE)
----COLUMN SECTION----
* BINARY VARIABLES USED TO SELECT INDICATED RANGE
* CONTINUOUS VARIABLES FOR INDICATED RANGE
* CONTINUOUS VARIABLES INDICATING TOTAL NUMBER OF A/C X,Y,Z,OR U.
* CONTINUOUS VARIABLES INDICATING NUM OF A/C  USED IN INDICATED MISSION
* SLACK VARIABLES USED TO RELAX PHYSICAL CONSTRAINTS
* ARTIFICIAL VARIABLES USED TO RELAX MISSION CONSTRAINTS
                               COLUMNS=   124   ACCEPTED=   124
                               EXPLICIT ELS=   251   SUB ELS=     0
                               POOL ELS=   100
----RHS    SECTION----
* LOGICAL RESTRICTION FORCING SELECTION OF ONLY ONE BINARY VARIABLE
SCENALL
                               SELECTED: SCENALL
* PHYSICAL LIMITATIONS EXPRESSED IN PHYSICAL UNITS
* MISSION MINIMUMS EXPRESSED IN MISSION UNITS
                               POOL ELS=   132
----RANGES SECTION----
R
                               SELECTED: R
                               POOL ELS=   142
----BOUNDS SECTION----
BOUND
                               SELECTED: BOUND
* BOUNDS INDICATE MAXIMUM RELAXATION ALLOWED IN UNITS FOR EACH CONSTRAINT
                               POOL ELS=   190

                               ROWS: EXPLICIT=   69  SUB=     0
                                     EQUATIONS=   5          0

I/O UNITS: IUR=  1   IUT=  2

                               POOL ELS DELETED=     3

    0 MINOR ERRORS       0 MAJOR ERRORS


   24 BINARY VARIABLES FOUND
   24 BINARY VARIABLES ACCEPTED
MAX FIXED ORDER ENUMERATION=   13
MAX PIVOTS=32767
MAX SOLUTION VALUE TOLERANCE=    0.1000000+01

PRIMAL PROC INVOKED WITH IRD= 1  MSCN=   64  LOT=   8

MPS TIME=  0.36
```

```
ROW SUMMARY: (CONVERTED PROBLEM)

EXT     INT LABEL     T/  ELEMENTS     INT LABEL     T/  ELEMENTS     INT LABEL     T/  ELEMENTS     INT LABEL     T/  ELEMENTS
NO.     NO.           /S CNT RNG MX    NO.           /S CNT RNG MX    NO.           /S CNT RNG MX    NO.           /S CNT RNG MX
  1       1 PICKIFCX  E    2   0  0      2 PICKIFCY  E    5   0  0      3 PICKIFCZ  E    7   0  0      4 PICKIFCU  E   10   0  0
  5       5 SUM     X G    3   0 -1      6 SUM     Y G    6   0 -1      7 SUM     Z G    8   0 -1      8 SUM     U G   11   0 -1
  9       9 RANGE  1X L    1   0 -1     10 RANGE  2X R    2   2  0     11 RANGE  1Y R    2   1  0     12 RANGE  2Y R    2   2  0
 13      13 RANGE  3Y R    2   2  0     14 RANGE  4Y R    2   2  0     15 RANGE  5Y R    2   2  0     16 RANGE  1Z R    2   2  0
 17      17 RANGE  2Z R    2   2  0     18 RANGE  3Z R    2   2  0     19 RANGE  4Z R    2   2  0     20 RANGE  5Z R    2   2  0
 21      21 RANGE  6Z R    2   2  0     22 RANGE  7Z R    2   2  0     23 RANGE  1U L    1   0 -1     24 RANGE  2U R    2   2  0
 25      25 RANGE  3U R    2   2  0     26 RANGE  4U R    2   2  0     27 RANGE  5U R    2   2  0     28 RANGE  6U R    2   2  0
 29      29 RANGE  7U R    2   2  0     30 RANGE  8U R    2   2  0     31 RANGE  9U R    2   2  0     32 RANGE  AU R    2   2  0
 33      33 MANPOWER  L    5   2  0     34 GNDFAC    L    5   1  0     35 TRAINSPA  L    5   1  0     36 SCNBAL X1 G    5   1 -1
 37      37 SCNBAL Y1 G    5   1 -1     38 SCNBAL Z1 G    5   1 -1     39 SCNBAL U1 G    5   1 -1     40 SCNBAL X2 G    5   1 -1
 41      41 SCNBAL Y2 G    5   1 -1     42 SCNBAL Z2 G    5   1 -1     43 SCNBAL U2 G    5   1 -1     44 SCNBAL X3 G    5   1 -1
 45      45 SCNBAL Y3 G    5   1 -1     46 SCNBAL Z3 G    5   1 -1     47 SCNBAL U3 G    5   1 -1     48 AIRDEF1   G    4   1  0
 49      49 ALWEATH1  G    2   0 -1     50 AGSUPP1   G    4   1  0     51 SMRTBMB1  G    3   0 -1     52 DEEPPEN1  G    5   1  0
 53      53 VLONGRN1  G    3   0 -1     54 MISCELL1  G    5   0 -1     55 AIRDEF2   G    4   1  0     56 ALWEATH2  G    2   0 -1
 57      57 AGSUPP2   G    4   1  0     58 SMRTBMB2  G    3   0 -1     59 DEEPPEN2  G    5   1  0     60 VLONGRN2  G    3   0 -1
 61      61 MISCELL2  G    5   0 -1     62 AIRDEF3   G    4   1  0     63 ALWEATH3  G    2   0 -1     64 AGSUPP3   G    4   1  0
 65      65 SMRTBMB3  G    3   0 -1     66 DEEPPEN3  G    5   2  0     67 VLONGRN3  G    3   1 -1     68 MISCELL3  G    5   1 -1
 69      69 COSTS     N   63   3  3
            'RANGES'  LO  63   0  0
                      UP  31   0  2
            'PENALT'  LO  68   1  5
                      UP  68   1  5

          INITIAL COST=    0.0

LEGEND:  T/S ... TYPE/STATUS

   TYPE/     N   OBJECTIVE FUNCTION
             L   INF LOWER RANGE
             G   INF UPPER RANGE
             R   BOTH RANGES FINITE
             E   RANGES EQUAL
             +   EDIT DROP

   STATUS    G   GUB ROW

ELEMENTS:

CNT          NONZERO COEFFICIENTS
   RNG       ORDERS OF MAGNITUDE
       MX    LARGEST ELEMENT
```

```
COLUMN SUMMARY: (CONVERTED PROBLEM)

EXT     INT LABEL     T/  ELEMENTS     INT LABEL     T/  ELEMENTS     INT LABEL     T/  ELEMENTS     INT LABEL     T/  ELEMENTS
NO.     NO.           /S CNT RNG MX    NO.           /S CNT RNG MX    NO.           /S CNT RNG MX    NO.           /S CNT RNG MX
 70      70 Y1X       BI   1   0  0     71 Y2X       BI   2   1  0     72 Y1Y       BI   2   1  0     73 Y2Y       BI   2   1  0
 74      74 Y3Y       BI   2   1  0     75 Y4Y       BI   2   1  0     76 Y5Y       BI   2   1  0     77 Y1Z       BI   2   1  0
 78      78 Y2Z       BI   2   1  0     79 Y3Z       BI   2   1  0     80 Y4Z       BI   2   1  0     81 Y5Z       BI   2   1  0
 82      82 Y6Z       BI   2   1  0     83 Y7Z       BI   2   1  0     84 Y1U       BI   1   0  0     85 Y2U       BI   2   1  0
 86      86 Y3U       BI   2   1  0     87 Y4U       BI   2   1  0     88 Y5U       BI   2   1  0     89 Y6U       BI   2   1  0
 90      90 Y7U       BI   2   1  0     91 Y8U       BI   2   1  0     92 Y9U       BI   2   1  0     93 YAU       BI   2   1  0
 94      94 X1X       B    2   0 -1     95 X2X       L    2   0 -1     96 X1Y       B    2   0 -1     97 X2Y       L    2   0 -1
 98      98 X3Y       L    2   0 -1     99 X4Y       L    2   0 -1    100 X5Y       L    2   0 -1    101 X1Z       B    2   0 -1
102     102 X2Z       L    2   0 -1    103 X3Z       L    2   0 -1    104 X4Z       L    2   0 -1    105 X5Z       L    2   0 -1
106     106 X6Z       L    2   0 -1    107 X7Z       L    2   0 -1    108 X1U       B    2   0 -1    109 X2U       L    2   0 -1
110     110 X3U       L    2   0 -1    111 X4U       L    2   0 -1    112 X5U       L    2   0 -1    113 X6U       L    2   0 -1
114     114 X7U       L    2   0 -1    115 X8U       L    2   0 -1    116 X9U       L    2   0 -1    117 XAU       L    2   0 -1
118     118 XTOTAL    B    7   2  0    119 YTOTAL    B    7   2  0    120 ZTOTAL    L    7   2  0    121 UTOTAL    L    7   2  0
122     122 X11       L    1   0 -1    123 X12       L    2   0 -1    124 X13       L    2   0 -1    125 X14       L    2   0 -1
126     126 X21       L    1   0 -1    127 X22       L    2   0 -1    128 X23       L    2   1  0    129 X24       L    2   0 -1
130     130 X31       L    1   0 -1    131 X32       L    2   0 -1    132 X33       L    2   1  0    133 X34       L    2   0 -1
134     134 Y11       L    3   1  0    135 Y12       L    1   0 -1    136 Y13       L    3   1  0    137 Y14       L    2   0 -1
138     138 Y21       L    3   1  0    139 Y22       L    1   0 -1    140 Y23       L    3   1  0    141 Y24       L    2   0 -1
142     142 Y31       L    3   1  0    143 Y32       L    1   0 -1    144 Y33       L    3   1  0    145 Y34       L    2   0 -1
146     146 Z11       L    2   1  0    147 Z12       L    3   1  0    148 Z13       L    3   1  0    149 Z14       L    2   0 -1
150     150 Z21       L    2   1  0    151 Z22       L    3   1  0    152 Z23       L    3   1  0    153 Z24       L    2   0 -1
154     154 Z31       L    2   1  0    155 Z32       L    3   1  0    156 Z33       L    3   1  0    157 Z34       L    2   0 -1
158     158 U11       L    2   1  0    159 J12       L    3   1  0    160 U13       L    2   1  0    161 U14       L    2   0 -1
162     162 U21       L    2   1  0    163 U22       L    3   1  0    164 U23       L    2   1  0    165 U24       L    2   0 -1
166     166 U31       L    2   1  0    167 U32       L    3   1  0    168 U33       L    2   1  0    169 U34       L    2   0 -1
170     170 MPWR      B    1   0 -1    171 GFAC      B    1   0 -1    172 TSPA      B    1   0 -1    173 ARD1      X    1   0 -1
174     174 ALW1      B    1   0 -1    175 AGS1      B    1   0 -1    176 SMT1      B    1   0 -1    177 OPN1      B    1   0 -1
178     178 VLF1      B    1   0 -1    179 MSC1      B    1   0 -1    180 ARD2      X    1   0 -1    181 ALW2      B    1   0 -1
182     182 AGS2      B    1   0 -1    183 SMT2      B    1   0 -1    184 OPN2      B    1   0 -1    185 VLR2      B    1   0 -1
186     186 MSC2      B    1   0 -1    187 ARD3      X    1   0 -1    188 ALW3      B    1   0 -1    189 AGS3      B    1   0 -1
190     190 SMT3      B    1   0 -1    191 OPN3      B    1   0 -2    192 VLR3      B    1   0 -2    193 MSC3      B    1   0 -2
            'BOUNDS'  LO 124   0  2
                      UP  54   0  3

LEGEND:  T/S ... TYPE/STATUS

   TYPE/     N     INFINITE BOUNDS
             U     INF LOWER BOUND
             L     INF UPPER BOUND
             B     BOTH BOUNDS FINITE
             X     FIXED
             +     EDIT DROP

   STATUS    I     BINARY

ELEMENTS:

CNT              NONZERO COEFFICIENTS
   RNG           ORDERS OF MAGNITUDE
       MX        LARGEST ELEMENT

SUMMARY TIME=  0.09
```

```
XS DIMENSION KEY SETTINGS
 M=   69 N=  124 MG=     0 KSD= 3000 KSSD=    20 LOT=    8 IDP=  13 ND=    502 NRV=    187 NEL=  8917 IRD= 1  NPVX= 32767
 MSCN=   64

XS DATA REGION=   199982

XS MATHEMATICAL PROGRAMMING SYSTEM (8/81)

 M=   69 N=  124 MG=     0 KSD= 3000 KSSD=    20 LOT=    8 IDP=  13 ND=    502 NRV=    187 NEL=  9084 IPD= 1  NPVX= 32767
STABILITY EPB= 0.0                    EPNB= 0.43500000000D+01 EDB= 0.0                    EDNB= 0.44303000000D+03 OPT= 0.0

NPVT=      0 NVL =      27 VI =  -0.10000000000000+01 VS =  -0.5000000000000D-01
NPVT=      0 NVL =       0 VI =   0.0                  VS =  -0.1000000000000D+21
NPVT=      0 NVL =     108 VI =  -0.1004430300000D+06 VS =  -0.9500000000000D+01
NPVT=     16 NVL =      85 VI =  -0.3100000000000D+04 VS =  -0.9500000000000D+01
NPVT=     75 NVL =      42 VI =  -0.1013260000000D+04 VS =  -0.1363636363630D+00
NPVT=    100 KSZ=     48 NK=      1 OPT=  0.9690932215240D+04 TIME=      0.083
NPVT=    140 NVL =      29 VI =  -0.9612942708330D+02 VS =  -0.1183571428580D+00
NPVT=    175 NVL =      19 VI =  -0.1717781250000D+03 VS =  -0.2000000000000D+00
NPVT=    187 NVL =      13 VI =  -0.1013515521660D+02 VS =  -0.6757760830050D-01
NPVT=    200 KSZ=     55 NK=      2 OPT=  0.9767496428570D+03 TIME=      0.463
NPVT=    211 NVL =       1 VI =  -0.1601190476190D+01 VS =  -0.1601190476190D+01
NPVT=    211 NVL =       0 VI =   0.0                  VS =  -0.1000000000000D+21
NPVT=    213 NVL =       0 VI =   0.0                  VS =  -0.1000000000000D+21
NPVT=    217 KSZD=   -1 KSZP=   -3

LP- WOPT=  0.9462690476190D+03 COPT=  0.9462690476190D+03 OPT=  0.9462690476190D+03 NPVT=  217 NPVX=32767 MAX EL=   943 TIME=  0.4826


IMPROVED INTEGER  WOPT=  0.8521285714290D+03 BEST=  0.2029022380950D+05 OPT=  0.9462690476190D+03 TL=  0.1000000000000D+01 NVC=   0

NPVT=    219 NVL =       1 VI =  -0.8419047619050D-02 VS =  -0.8419047619050D-02
NPVT=    220 NVL =       0 VI =   0.0                  VS =  -0.1000000000000D+21
NPVT=    220 KSZD=    0 KSZP=    0

IMPROVED INTEGER  WOPT=  0.1087371428570D+04 BEST=  0.1365880000000D+05 OPT=  0.9593523809520D+03 TL=  0.1000000000000D+01 NVC=   1

NPVT=    224 NVL =       2 VI =  -0.3666666666670D-01 VS =  -0.6200000000010D-02
NPVT=    227 NVL =       1 VI =  -0.3666666666670D-01 VS =  -0.3666666666670D-01
NPVT=    229 NVL =       0 VI =   0.0                  VS =  -0.1000000000000D+21
NPVT=    235 KSZD=   -2 KSZP=   -8

IMPROVED INTEGER  WOPT=  0.1120360000000D+04 BEST=  0.8453693333330D+04 OPT=  0.9834870000000D+03 TL=  0.1000000000000D+01 NVC=   2

NPVT=    259 NVL =       2 VI =  -0.4391666666660D-01 VS =  -0.2666666666670D-01
NPVT=    264 NVL =       1 VI =  -0.2000000000000D-01 VS =  -0.2000000000000D-01
NPVT=    265 NVL =       0 VI =   0.0                  VS =  -0.1000000000000D+21
NPVT=    271 KSZD=   -3 KSZP=   -3

IMPROVED INTEGER  WOPT=  0.8414000000000D+03 BEST=  0.1617650000000D+04 OPT=  0.1617650000000D+04 TL=  0.1000000000000D+01 NVC=   3

NPVT=    271 NVL =       2 VI =  -0.1250000000000D+00 VS =  -0.1000000000000D+00
NPVT=    289 NVL =       4 VI =  -0.9773436691350D+11 VS =  -0.2500000000000D-01
NPVT=    293 NVL =       5 VI =  -0.5148506828480D+11 VS =  -0.2500000000000D-01
NPVT=    294 NVL =       6 VI =  -0.1161111111110D+01 VS =  -0.2500000000000D-01
NPVT=    300 KSZ=     40 NK=      2 OPT= -0.3120369212970D+03 TIME=      0.083
NPVT=    316 NVL =       3 VI =  -0.9045925925940D-01 VS =  -0.5800000000000D-02
NPVT=    328 NVL =       3 VI =  -0.1277777777780D+00 VS =  -0.2194057346260D-01
NPVT=    334 NVL =       0 VI =   0.0                  VS =  -0.1000000000000D+21
NPVT=    335 KSZD=    0 KSZP=   -1
STABILITY FAILURE WOPT=  0.1129920300930D+04 COPT=  0.1072135779010D+05 CPT=  0.1117192129600D+04
STABILITY EPB= 0.0                    EPNB= 0.43500000000D+01 EDB= 0.0                    EDNB= 0.8500000000000D+05 OPT= 0.3244240000000D+05

NPVT=    400 KSZ=     44 NK=      1 OPT=  0.9850476441800D+03 TIME=      0.100
NPVT=    403 NVL =       1 VI =  -0.9600000000000D-01 VS =  -0.9600000000000D-01
NPVT=    404 NVL =       0 VI =   0.0                  VS =  -0.1000000000000D+21
NPVT=    404 NVL =       0 VI =   0.0                  VS =  -0.1000000000000D+21
NPVT=    406 KSZD=   -2 KSZP=    0
NPVT=    414 NVL =       2 VI =  -0.6666666666670D-02 VS =  -0.3916666666670D-02
NPVT=    416 NVL =       0 VI =   0.0                  VS =  -0.1000000000000D+21
NPVT=    419 KSZD=   -2 KSZP=   -1

IMPROVED INTEGER  WOPT=  0.9394950000000D+03 BEST=  0.1117828333330D+04 OPT=  0.1117828333330D+04 TL=  0.1000000000000D+01 NVC=   4

NPVT=    430 NVL =       3 VI =  -0.2690104166670D+00 VS =  -0.5312500000000D-01
NPVT=    463 NVL =       3 VI =  -0.1050000000000D+00 VS =  -0.2400000000000D-01
NPVT=    468 NVL =       0 VI =   0.0                  VS =  -0.1000000000000D+21
NPVT=    468 KSZD=    0 KSZP=    0
```

```
IMPROVED INTEGER  WOPT=  0.9916833333330+03 BEST=  0.9916833333330+03 OPT=  0.9916833333333D+03 TL=  0.1000000000000D+01 NVC=   4

NPVT=   469 NVL =       3 VI =  -0.1000000000000D+01 VS =  -0.1000000000000D+00
NPVT=   472 NVL =       2 VI =  -0.3000000000000D+00 VS =  -0.3666666666667D-01
NPVT=   475 NVL =       1 VI =  -0.3000000000000D+00 VS =  -0.3000000000000D+00
NPVT=   477 NVL =       1 VI =  -0.1375000000000D+00 VS =  -0.1375000000000D+00
NPVT=   478 NVL =       1 VI =  -0.1250000000000D+00 VS =  -0.1250000000000D+00
NPVT=   479 NVL =       1 VI =  -0.1125000000000D+00 VS =  -0.1125000000000D+00
NPVT=   480 NVL =       1 VI =  -0.1000000000000D+00 VS =  -0.1000000000000D+00
NPVT=   481 NVL =       1 VI =  -0.8750000000000D-01 VS =  -0.8750000000000D-01
NPVT=   486 NVL =       1 VI =  -0.2666666666667D-01 VS =  -0.2666666666667D-01
NPVT=   488 NVL =       0 VI =   0.0                 VS =  -0.1000000000000D+21
NPVT=   491 KSZD=   -1 KSZP=   -2
NPVT=   492 NVL =       1 VI =  -0.7500000000000D-02 VS =  -0.7500000000000D-02
NPVT=   493 NVL =       1 VI =  -0.7500000000000D-02 VS =  -0.7500000000000D-02
NPVT=   494 NVL =       0 VI =   0.0                 VS =  -0.1000000000000D+21
NPVT=   494 KSZD=    0 KSZP=    0
NPVT=   503 KSZD=    0 KSZP=   -5
NPVT=   506 NVL =       2 VI =  -0.1000000000000D+01 VS =  -0.1000000000000D+00
NPVT=   508 NVL =       1 VI =  -0.2000000000000D+00 VS =  -0.2000000000000D+00
NPVT=   509 NVL =       1 VI =  -0.1000000000000D+00 VS =  -0.1000000000000D+00
NPVT=   510 NVL =       1 VI =  -0.6250000000000D-01 VS =  -0.6250000000000D-01
NPVT=   521 NVL =       3 VI =  -0.7555555555555D+00 VS =  -0.2500000000000D-01
NPVT=   533 NVL =       4 VI =  -0.1569178240740D+00 VS =  -0.1016666666667D-02
NPVT=   546 NVL =       2 VI =  -0.3228125000000D+00 VS =  -0.7912500000000D-01
NPVT=   594 NVL =       3 VI =  -0.1284000000000D+00 VS =  -0.3750000000000D-02
NPVT=   600 KSZ=     51 NK=        1 OPT=  0.9670107961907+03 TIME=     0.319
NPVT=   605 NVL =       1 VI =  -0.3916666666667D-01 VS =  -0.3916666666667D-01
NPVT=   613 NVL =       5 VI =  -0.5000000000000D-01 VS =  -0.8749999999999D-03
NPVT=   620 NVL =       1 VI =  -0.2250000000000D-01 VS =  -0.2250000000000D-01
NPVT=   621 NVL =       1 VI =  -0.2250000000000D-01 VS =  -0.2250000000000D-01
NPVT=   624 NVL =       0 VI =   0.0                 VS =  -0.1000000000000D+21
NPVT=   627 KSZD=   -1 KSZP=   -2

IMPROVED INTEGER  WOPT=  0.9725175000000D+03 BEST=  0.9725175000000D+03 OPT=  0.9725175000000D+03 TL=  0.1000000000000D+01 NVC=   4

NPVT=   629 NVL =       2 VI =  -0.1000000000000D+00 VS =  -0.1000000000000D+00
NPVT=   630 NVL =       0 VI =   0.0                 VS =  -0.1000000000000D+21
NPVT=   631 KSZD=    0 KSZP=  -1
NPVT=   632 NVL =       1 VI =  -0.2208333333333D-01 VS =  -0.2208333333333D-01
NPVT=   633 KSZD=    0 KSZP=   0
NPVT=   648 NVL =       0 VI =   0.0                 VS =  -0.1000000000000D+21
NPVT=   649 KSZD=    0 KSZP=  -1
NPVT=   651 NVL =       0 VI =   0.0                 VS =  -0.1000000000000D+21
NPVT=   652 KSZD=    0 KSZP=  -1

IMPROVED INTEGER  WOPT=  0.9640714285710D+03 BEST=  0.9640714285710D+03 OPT=  0.9640714285710D+03 TL=  0.1000000000000D+01 NVC=   5

NPVT=   653 NVL =       3 VI =  -0.1000000000000D+01 VS =  -0.1416666666667D-01
NPVT=   658 KSZD=    0 KSZP=  -1
NPVT=   674 NVL =       3 VI =  -0.4391666666667D+00 VS =  -0.2687500000000D-01
NPVT=   678 NVL =       2 VI =  -0.4391666666667D+00 VS =  -0.8750000000000D-01
NPVT=   680 NVL =       2 VI =  -0.4391666666667D+00 VS =  -0.7500000000000D-01
NPVT=   695 KSZD=    0 KSZP=  -9

ILP- BEST= 0.9640714286D+03 GOPT= 0.9640714286D+03 PEN= 0.0            NPVT=   695 NPVX=32767 MAX EL=   927 TIME=   2.27
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CON   NAME       G-LOWER RANGE      G-VALUE          G-UPPER RANGE      SCALE FACTOR     LOWER PENALTY              PENALTY     UPPER PENALTY
  1   PICK1FCX   0.10000000D+01   0.10000000D+01   0.10000000D+01   0.0              0.10000000D+06   0.0               0.10000000D+06
  2   PICK1FCY   0.10000000D+01   0.10000000D+01   0.10000000D+01   0.0              0.10000000D+06   0.0               0.10000000D+06
  3   PICK1FCZ   0.10000000D+01   0.10000000D+01   0.10000000D+01   0.0              0.10000000D+06   0.0               0.10000000D+06
  4   PICK1FCU   0.10000000D+01   0.10000000D+01   0.10000000D+01   0.0              0.10000000D+06   0.0               0.10000000D+06
  5   SIM     X  0.0             -0.13877788D-13   0.10000000D+24   0.10000000D-02   0.10000000D+06   0.13877788D-11    0.10000000D+06
  6   SIM     Y  0.0              0.0              0.10000000D+24   0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
  7   SIM     Z  0.0              0.38857806D-12   0.10000000D+24   0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
  8   SIM     U  0.0              0.29143354D-12   0.10000000D+24   0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
  9   RANGE  1X -0.10000000D+24   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 10   RANGE  2X -0.29500000D+03  -0.23928571D+03   0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 11   RANGE  1Y -0.10000000D+03   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 12   RANGE  2Y -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 13   RANGE  3Y -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 14   RANGE  4Y -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 15   RANGE  5Y -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 16   RANGE  1Z -0.15000000D+03   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 17   RANGE  2Z -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 18   RANGE  3Z -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 19   RANGE  4Z -0.24000000D+02  -0.12142857D+02   0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 20   RANGE  5Z -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 21   RANGE  6Z -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 22   RANGE  7Z -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 23   RANGE  1U -0.10000000D+24   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 24   RANGE  2U -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 25   RANGE  3U -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 26   RANGE  4U -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 27   RANGE  5U -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 28   RANGE  6U -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 29   RANGE  7U -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 30   RANGE  8U -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 31   RANGE  9U -0.24000000D+02   0.0              0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 32   RANGE  AU -0.24000000D+02  -0.24980018D-12   0.0              0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 33   MANPOWER  -0.10000000D+24   0.10000000D+05   0.11000000D+05   0.10000000D-02   0.10000000D+05   0.0               0.10000000D+05
 34   GNDFAC    -0.10000000D+24   0.10000000D+04   0.10000000D+04   0.10000000D-02   0.10000000D+05   0.71054274D-10    0.10000000D+05
 35   TRAINSPA  -0.10000000D+24   0.14467857D+04   0.15000000D+04   0.10000000D-02   0.10000000D+05   0.0               0.10000000D+05
 36   SCNBALX1   0.0              0.0              0.10000000D+24   0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 37   SCNBALY1   0.0              0.33306691D-12   0.10000000D+24   0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 38   SCNBALZ1   0.0              0.33219955D-12   0.10000000D+24   0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 39   SCNBALU1   0.0              0.33316449D-11   0.10000000D+24   0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 40   SCNBALX2   0.0             -0.53082538D-12   0.10000000D+24   0.10000000D-02   0.10000000D+06   0.53082538D-10    0.10000000D+06
 41   SCNBALY2   0.0              0.29143354D-12   0.10000000D+24   0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 42   SCNBALZ2   0.0             -0.40904780D-11   0.10000000D+24   0.10000000D-02   0.10000000D+06   0.40904780D-09    0.10000000D+06
 43   SCNBALU2   0.0             -0.18596236D-11   0.10000000D+24   0.10000000D-02   0.10000000D+06   0.18596236D-09    0.10000000D+06
 44   SCNBALX3   0.0              0.22428571D+02   0.10000000D+24   0.10000000D-02   0.10000000D+06   0.0               0.10000000D+06
 45   SCNBALY3   0.0             -0.13461454D-11   0.10000000D+24   0.10000000D-02   0.10000000D+06   0.13461454D-09    0.10000000D+06
 46   SCNBALZ3   0.0             -0.16514567D-11   0.10000000D+24   0.10000000D-02   0.10000000D+06   0.16514567D-09    0.10000000D+06
 47   SCNBALU3   0.0             -0.17031342D-09   0.10000000D+24   0.10000000D-02   0.10000000D+06   0.17031342D-07    0.10000000D+06
 48   AIRDEF1    0.85000000D+03   0.85000000D+03   0.10000000D+24   0.10000000D-02   0.10000000D+06   0.52319260D-09    0.10000000D+06
 49   ALWFATH1   0.80000000D+02   0.80000000D+02   0.10000000D+24   0.10000000D-02   0.10000000D+05   0.34694470D-11    0.10000000D+05
 50   AGSUPP1    0.20000000D+03   0.20000000D+03   0.10000000D+24   0.10000000D-02   0.10000000D+05   0.0               0.10000000D+05
```

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CON    NAME      G-LOWER RANGE      G-VALUE           G-UPPER RANGE     SCALE FACTOR      LOWER PENALTY           PENALTY       UPPER PENALTY
 51   SMRTAMB1   0.500000000D+02    0.727142870D+02   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.0               0.100000000D+05
 52   DEEPPEN1   0.150000000D+03    0.150000000D+03   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.0               0.100000000D+05
 53   VLONGRN1   0.500000000D+02    0.500000000D+02   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.667868540D-12   0.100000000D+05
 54   MISCELL1   0.100000000D+03    0.100000000D+03   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.138777880D-12   0.100000000D+05
 55   AIRDEF2    0.280000000D+03    0.280000000D+03   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.0               0.100000000D+05
 56   ALWEATH2   0.500000000D+02    0.720000000D+02   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.0               0.100000000D+05
 57   AGSUPP2    0.600000000D+03    0.600000000D+03   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.0               0.100000000D+05
 58   SMRTAMB2   0.150000000D+03    0.300000000D+03   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.0               0.100000000D+05
 59   DEEPPEN2   0.200000000D+03    0.200000000D+03   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.0               0.100000000D+05
 60   VLONGRN2   0.700000000D+02    0.800000000D+02   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.0               0.100000000D+05
 61   MISCELL2   0.100000000D+03    0.100000000D+03   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.192901250D-10   0.100000000D+05
 62   AIRDEF3    0.320000000D+03    0.320000000D+03   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.0               0.100000000D+05
 63   ALWEATH3   0.500000000D+02    0.800000000D+02   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.0               0.100000000D+05
 64   AGSUPP3    0.300000000D+03    0.300000000D+03   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.194289030D-11   0.100000000D+05
 65   SMRTAMB3   0.100000000D+03    0.150000000D+03   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.0               0.100000000D+05
 66   DEEPPFN3   0.550000000D+03    0.550000000D+03   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.0               0.100000000D+05
 67   VLONGFN3   0.100000000D+03    0.130285710D+03   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.0               0.100000000D+05
 68   MISCELL3   0.100000000D+03    0.100000000D+03   0.100000000D+24   0.100000000D-02   0.100000000D+05   0.0               0.100000000D+05
 69   COSTS      0.0               -0.990202910D+06   0.0               0.100000000D+00   0.0              -0.999843620D+05   0.0
```

```
VAR   NAME    Y-LOWER BOUND   Y-VALUE         Y-UPPER BOUND   SCALE FACTOR    D-LOWER BOUND   D-VALUE         D-UPPER BOUND
 70   Y1X     0.0             0.0             0.10000000D+01  0.0             0.0             0.0             0.10000000D+01
 71   Y2X     0.0             0.10000000D+01  0.10000000D+01  0.0             0.0             0.0             0.10000000D+01
 72   Y1Y     0.0             0.10000000D+01  0.10000000D+01  0.0             0.0             0.0             0.10000000D+01
 73   Y2Y     0.0             0.0             0.10000000D+01  0.0             0.0             0.80000000D+01  0.10000000D+01
 74   Y3Y     0.0             0.0             0.10000000D+01  0.0             0.0             0.0             0.10000000D+01
 75   Y4Y     0.0             0.0             0.10000000D+01  0.0             0.0             0.0             0.10000000D+01
 76   Y5Y     0.0             0.0             0.10000000D+01  0.0             0.0             0.0             0.10000000D+01
 77   Y1Z     0.0             0.0             0.10000000D+01  0.0             0.0             0.12028571D+02  0.10000000D+01
 78   Y2Z     0.0             0.0             0.10000000D+01  0.0             0.0             0.0             0.10000000D+01
 79   Y3Z     0.0             0.0             0.10000000D+01  0.0             0.0             0.50000000D+01  0.10000000D+01
 80   Y4Z     0.0             0.10000000D+01  0.10000000D+01  0.0             0.0             0.0             0.10000000D+01
 81   Y5Z     0.0             0.0             0.10000000D+01  0.0             0.0             0.32000000D+01  0.10000000D+01
 82   Y6Z     0.0             0.0             0.10000000D+01  0.0             0.0             0.12000000D+01  0.10000000D+01
 83   Y7Z     0.0             0.0             0.10000000D+01  0.0             0.0             0.80000000D+01  0.10000000D+01
 84   Y1U     0.0             0.0             0.10000000D+01  0.0             0.0             0.29257143D+01  0.10000000D+01
 85   Y2U     0.0             0.0             0.10000000D+01  0.0             0.0             0.40000000D+01  0.10000000D+01
 86   Y3U     0.0             0.0             0.10000000D+01  0.0             0.0             0.0             0.10000000D+01
 87   Y4U     0.0             0.0             0.10000000D+01  0.0             0.0             0.13028571D+02  0.10000000D+01
 88   Y5U     0.0             0.0             0.10000000D+01  0.0             0.0             0.0             0.10000000D+01
 89   Y6U     0.0             0.0             0.10000000D+01  0.0             0.0             0.24628571D+02  0.10000000D+01
 90   Y7U     0.0             0.0             0.10000000D+01  0.0             0.0             0.72714286D+01  0.10000000D+01
 91   Y8U     0.0             0.0             0.10000000D+01  0.0             0.0             0.10000000D+00  0.10000000D+01
 92   Y9U     0.0             0.0             0.10000000D+01  0.0             0.0             0.0             0.10000000D+01
 93   YAU     0.0             0.10000000D+01  0.10000000D+01  0.0             0.0             0.0             0.10000000D+01
 94   X1X     0.0             0.0             0.25000000D+02  0.10000000D+02  0.0             0.28800000D+02  0.25000000D+01
 95   X2X     0.0             0.80714286D+02  0.10000000D+22  0.10000000D+02  0.0             0.0             0.10000000D+21
 96   X1Y     0.0             0.10000000D+03  0.10000000D+03  0.10000000D+02  0.0             0.0             0.10000000D+02
 97   X2Y     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.0             0.10000000D+21
 98   X3Y     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.15000000D+02  0.10000000D+21
 99   X4Y     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.11214286D+02  0.10000000D+21
100   X5Y     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.57857143D+01  0.10000000D+21
101   X1Z     0.0             0.0             0.15000000D+03  0.10000000D+02  0.0             0.0             0.15000000D+02
102   X2Z     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.20000000D+02  0.10000000D+21
103   X3Z     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.0             0.10000000D+21
104   X4Z     0.0             0.21285714D+03  0.10000000D+22  0.10000000D+02  0.0             0.0             0.10000000D+21
105   X5Z     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.0             0.10000000D+21
106   X6Z     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.40000000D+01  0.10000000D+21
107   X7Z     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.0             0.10000000D+21
108   X1U     0.0             0.0             0.20000000D+03  0.10000000D+02  0.0             0.23000000D+01  0.20000000D+02
109   X2U     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.0             0.10000000D+21
110   X3U     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.50000000D+01  0.10000000D+21
111   X4U     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.0             0.10000000D+21
112   X5U     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.0             0.10000000D+21
113   X6U     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.0             0.10000000D+21
114   X7U     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.0             0.10000000D+21
115   X8U     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.0             0.10000000D+21
116   X9U     0.0             0.0             0.10000000D+22  0.10000000D+02  0.0             0.0             0.10000000D+21
117   XAU     0.0             0.40000000D+03  0.10000000D+22  0.10000000D+02  0.0             0.12628571D+01  0.10000000D+21
118   XTOTAL  0.0             0.80714286D+02  0.32000000D+03  0.10000000D+02  0.0             0.0             0.32000000D+02
119   YTOTAL  0.10000000D+03  0.10000000D+03  0.20000000D+03  0.10000000D+02  0.10000000D+02  0.0             0.20000000D+02
```

```
VAR    NAME     X-LOWER BOUND      X-VALUE           X-UPPER BOUND     SCALE FACTOR      D-LOWER BOUND              D-VALUE      D-UPPER BOUND
 120   ZTOTAL   0.15000000D+03    0.21285714D+03    0.10000000D+22    0.10000000D+02    0.15000000D+02    0.0                    0.10000000D+21
 121   UTOTAL   0.20000000D+03    0.40000000D+03    0.10000000D+22    0.10000000D+02    0.20000000D+02    0.0                    0.10000000D+21
 122   X11      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 123   X12      0.0               0.14571429D+02    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 124   X13      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 125   X14      0.0               0.50000000D+02    0.10000000D+22    0.10000000D+02    0.0               0.03377776D-78         0.10000000D+21
 126   X21      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.23915958D-77         0.10000000D+21
 127   X22      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.15856197D-77         0.10000000D+21
 128   X23      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.52964744D-77         0.10000000D+21
 129   X24      0.0               0.58114286D+02    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 130   X31      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 131   X32      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 132   X33      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 133   X34      0.0               0.42142857D+02    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 134   Y11      0.0               0.80000000D+02    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 135   Y12      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 136   Y13      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 137   Y14      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 138   Y21      0.0               0.72000000D+02    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 139   Y22      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 140   Y23      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 141   Y24      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 142   Y31      0.0               0.80000000D+02    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 143   Y32      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 144   Y33      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 145   Y34      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 146   Z11      0.0               0.12028571D+03    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 147   Z12      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 148   Z13      0.0               0.50000000D+02    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 149   Z14      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 150   Z21      0.0               0.32000000D+02    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 151   Z22      0.0               0.12000000D+02    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 152   Z23      0.0               0.80000000D+02    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 153   Z24      0.0               0.29257143D+02    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 154   Z31      0.0               0.40000000D+02    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 155   Z32      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 156   Z33      0.0               0.13028571D+03    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 157   Z34      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 158   U11      0.0               0.24628571D+03    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 159   U12      0.0               0.72714286D+02    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 160   U13      0.0               0.10000000D+01    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 161   U14      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 162   U21      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 163   U22      0.0               0.28800000D+03    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 164   U23      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 165   U24      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 166   U31      0.0               0.0               0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 167   U32      0.0               0.15000000D+03    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 168   U33      0.0               0.11214286D+03    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
 169   U34      0.0               0.57857143D+02    0.10000000D+22    0.10000000D+02    0.0               0.0                    0.10000000D+21
```

```
VAR   NAME    Y-LOWER BOUND       Y-VALUE          Y-UPPER BOUND     SCALE FACTOR     D-LOWER BOUND          D-VALUE      D-UPPER BOUND
 170  MPWR    0.0              0.0                 0.110000000D+04   0.100000000D+02  0.0               0.0               0.110000000D+03
 171  GFAC    0.0              0.200000000D+03     0.200000000D+03   0.100000000D+02  0.0               0.0               0.200000000D+02
 172  TSPA    0.0              0.0                 0.450000000D+03   0.100000000D+02  0.0               0.0               0.450000000D+02
 173  ARD1    0.0              0.0                 0.0               0.100000000D+02  0.0               0.0               0.0
 174  ALW1    0.0              0.0                 0.800000000D+01   0.100000000D+02  0.0               0.0               0.800000000D+00
 175  AGS1    0.0              0.400000000D+02     0.400000000D+02   0.100000000D+02  0.0               0.0               0.400000000D+01
 176  SMT1    0.0              0.0                 0.500000000D+01   0.100000000D+02  0.0               0.0               0.500000000D+00
 177  DPN1    0.0              0.230000000D+02     0.230000000D+02   0.100000000D+02  0.0               0.0               0.230000000D+01
 178  VLR1    0.0              0.0                 0.500000000D+01   0.100000000D+02  0.0               0.0               0.500000000D+00
 179  MSC1    0.0              0.500000000D+02     0.500000000D+02   0.100000000D+02  0.0               0.0               0.500000000D+01
 180  AFD2    0.0              0.0                 0.0               0.100000000D+02  0.0               0.0               0.0
 181  ALW2    0.0              0.0                 0.500000000D+01   0.100000000D+02  0.0               0.0               0.500000000D+00
 182  AGS2    0.0              0.0                 0.120000000D+03   0.100000000D+02  0.0               0.0               0.120000000D+02
 183  SMT2    0.0              0.0                 0.300000000D+02   0.100000000D+02  0.0               0.0               0.300000000D+01
 184  DPN2    0.0              0.0                 0.300000000D+02   0.100000000D+02  0.0               0.0               0.300000000D+01
 185  VLR2    0.0              0.0                 0.700000000D+01   0.100000000D+02  0.0               0.0               0.700000000D+00
 186  MSC2    0.0              0.126285710D+02     0.500000000D+02   0.100000000D+02  0.0               0.0               0.500000000D+01
 187  AFD3    0.0              0.0                 0.0               0.100000000D+02  0.0               0.0               0.0
 188  ALW3    0.0              0.0                 0.500000000D+01   0.100000000D+02  0.0               0.0               0.500000000D+00
 189  AGS3    0.0              0.0                 0.600000000D+02   0.100000000D+02  0.0               0.0               0.600000000D+01
 190  SMT3    0.0              0.0                 0.100000000D+02   0.100000000D+02  0.0               0.0               0.100000000D+01
 19   DPN3    0.0              0.0                 0.830000000D+02   0.0              0.0               0.0               0.830000000D+02
 192  VLR3    0.0              0.0                 0.100000000D+02   0.0              0.0               0.0               0.100000000D+02
 193  MSC3    0.0              0.0                 0.500000000D+02   0.0              0.0               0.0               0.500000000D+02

ITERATIONS=    0    CPU SECONDS=      2.87

FCN CALLS:        FUNCTION                    DIFFERENCING
              LINEAR      NON-LINEAR      LINEAR      NON-LINEAR    PIVOTS

                0            0              0            0           695
              C.0          0.0            0.0          0.0           0.0  (PER ITERATION)
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