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STATISTICAL METHODS FOR THE
JOINT OIL ANALYSIS PROGRAM

by

Toke Jayachandran
and
H.J. Larson

January 1982

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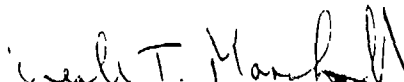


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STATISTICAL METHODS FOR THE JOINT OIL ANALYSIS PROGRAM

1. INTRODUCTION

This report contains the results of a continuing effort, sponsored by the Kelly AFB, to develop statistical procedures to improve the analyses of the spectrometric wearmetal readings in used oil samples; the results of the analyses are used as an aid in monitoring the wear condition of equipment with oil-wetted parts. For FY 81, we were tasked to address the development of (1) a statistical procedure for the daily standardization check of the Baird Atomic emission spectrometers (2) formulas and tables to convert wearmetal readings from an emission spectrometer to an equivalent set of readings from an absorption spectrometer (3) a moving regression methodology for monitoring the condition of a given piece of equipment based on the wearmetal buildup "signature" exhibited by the piece of equipment and (4) to assist the Joint Oil Analysis Program Data Coordinating Group in improving the program's data base and the periodically issued management and technical reports.

The new methodology for daily standardization check of the atomic emission spectrometers is presented in Section 1. Whereas the current procedure requires all laboratories stocking oil standards with 0, 3, 10, 30, 50, 100 and 300 ppm concentrations the new one requires stocking either 0, 50 and 100 ppm or 0, 50, and 300 ppm standards only. In addition, the current procedure only ensures that the accuracy of a spectrometer is within bounds; the one proposed examines both the accuracy and repeatability of a spectrometer.

Regression based formulas and tables for the conversion of measurements from an emission spectrometer into an equivalent set for an absorption spectrometer and vice versa are in Section 2. When an aircraft moves from its home base to a different base it may happen that the two bases have the two different types of spectrometers respectively. The conversion formulas will be useful in maintaining continuous oil analysis records. The formulas were derived using used oil sample records from the correlation test results for the period Mar 80 - Feb 81. The coefficient of determination R^2 measuring the goodness-of-fit of the regression was about .99 for all the elements tested except copper. This would indicate that the formulas should provide reasonably good conversions, in general. However, we have been informed by Mr. Richard E. Lee, the Army representative, JOAP-TSC, that care should be exercised in using the formulas; if the size of the wearmetal particles exceeds the capability of the emission spectrometer one can get misleading results.

Section 3 contains the new moving regression methodology for tracking the wear condition of a piece of equipment. The criteria presently in use for evaluating the oil analysis results consist of trend tables and decision guidelines that are derived from pooled historical data on all equipment of a given type. We believe that each piece of equipment will exhibit a unique "signature" of wearmetal buildup that can be exploited to improve the decision process. To this end, we developed a moving regression methodology that uses the most recent fixed number of observations to assess if a new observation should be considered normal, suspect or indicates abnormal wear needing attention. The procedures have been applied to several cases of unit failure histories that led to a JOAP hit using the

current criteria. In all cases, except one, the moving regression detected an abnormality at or before the time of the JOAP hit.

In addition to the foregoing, we met with the data base committee of the Joint Oil Analysis Program Coordinating Group, and provided suggestions for the structure of the data base and the types of managerial and technical reports needed for a successful oil analysis program.

2. DAILY STANDARDIZATION CHECK PROCEDURE

2.1 The Recommended Procedure

The current daily standardization check allows the laboratory operator to select three concentrations from the 0, 3, 10, 30, 50, 100, 300 ppm standards; it is possible that a laboratory then could choose 3, 10 and 30 ppm standards, say, for its daily check, totally avoiding the two operator-controlled standardization points at 0 and 100 ppm. The current procedure then compares the three single-burns at the selected levels with tabled limits, apparently the same for essentially all elements. If one or more of the single-burns does not fall within the specified limits, two more burns are made of the offending standard and the average of three burns then is compared with the same limits. If this average of three burns, for one or more standards (and presumably one or more elements) still lies outside the prescribed limits, the operator must then perform a complete standardization procedure.

The new recommended daily standardization check requires two daily burns at each of the 0 (offset), 50 and 100 ppm levels, (or 0 (offset), 50 and 300 ppm levels), which then are used to check both the accuracy and the precision of the instrument for all elements which the laboratory expects to monitor on that day. For the two burns at the same concentration (for each element of interest) the operator must then

(a) Average the two values and subtract the known true concentration (50 for the 0 offset value and for 50 ppm, 100 for the 100 ppm, 300 for the 300 ppm level). The magnitude d_i of this difference (ignoring whether it is positive or negative) then is compared with the appropriate entry of Table 1. This is done for each element the lab expects to analyze that day.

So long as each computed magnitude is no larger than the entry in Table 1, the lab passes the accuracy check and goes on to the precision check in (b). If one or more elements have a difference whose magnitude exceeds the tabled limit, the lab must make two more burns of the standard (at the same concentration level), again average these two new values, subtract the true value and compare the magnitudes (for the elements monitored) with the Table 1 limits. If again one or more elements produces a magnitude in excess of the value in Table 1, the operator must perform a complete standardization procedure.

(b) For the two burns which passed the accuracy check, the operator must now take the difference of the two readings for each element, for each concentration (0 (offset), 50 ppm, 100 ppm) or 0 (offset), 50 ppm, 300 ppm). The magnitude d_2 of this difference (again ignoring whether it is positive or negative) then is compared with the appropriate entry in Table 2, for each element to be analyzed that day. If each computed magnitude does not exceed the tabled value, the daily standardization test is passed and the machine is ready to use. If one or more computed magnitudes exceeds the tabled limit, two additional burns must be made at the same concentration; these two burns must then be used to repeat both the accuracy check in (a) and this precision check. If either of these two checks are failed for one or more elements of interest at one or more concentrations, the operator must perform a complete standardization procedure.

The following example illustrates this suggested procedure.

Example

Laboratory A expects to analyze only for Fe, Ag, Al, Cr, Cu and Mg on a given day, with typical sample readings between 10 and 60 ppm.

	Ag	Al	B	Ba	Be	Cd	Cr	Cu	Fe	Mg	Mn	Mo	Na	Ni	Pb	Si	Sn	Ti	V	Zn
0	0.7	1	1	0.8	0.7	1.6	1	0.7	0.8	0.9	1	1.6	1.6	1	1.6	1.1	1.6	1.1	2.5	0.9
(offset)																				
50	8	6.5	7.5	6	6	6.5	6.5	6.5	7	7.5	6.5	8	7.5	6	6.5	6	6.5	8.5	7	11.5
100	15	12	15	10	10	15	11	11	11	15	11	15	21	11	12	10	11	15	10	21
300	70	50					70	70	55	50		60	90	60	45	47	35	55		90

Limits for d_1 = Average of two, less true ppm

Table 1

	Ag	Al	B	Ba	Be	Cd	Cr	Cu	Fe	Mg	Mn	Mo	Na	Ni	Pb	St	Sn	Ti	V	Zn
0 (offset)	0.3	1	0.8	0.5	0.3	2	1	0.5	1	1	0.3	2	1.4	1	2	1	1.4	0.8	2	1
50	6	6	6	5	6	7	5	5	7	7	6	7	7	5	5	5	5	7	5	14
100	17	11	14	7	11	14	7	11	14	14	11	17	17	7	7	7	7	17	7	34
300	70	42					38	64	60	60		60	92	41	31	40	31	56		65

Limits for d_2 = difference of two

Table 2

Thus for his daily standardization check he will make two burns of standards at the 0, 50 and 100 ppm level, setting the machine to the offset position for the 0 ppm standard. Only the readings for the six elements listed above will be monitored. Assume the results of the 6 burns are as listed in Table 3, which also gives the computations for the accuracy check (a). Since for each of the six elements, for all 3 concentrations, the value for d_1 is smaller than the entry in Table 1, the accuracy check is passed.

Table 4 repeats the same basic data (two burns each at 0, 50, 100 ppm, readings for the same 6 elements) and illustrates the computation of the d_2 values for the precision check. These d_2 values are compared with the values in Table 2. Notice that each of them is no larger than the Table 2 value, except the value of 1.1 for Cr at 0 ppm, which exceeds the tabled value of 1. Thus this laboratory must make two new burns at 0 ppm (offset) and again go through both the accuracy and precision checks for 0 ppm. The two new burns are given in Table 5, which also presents the accuracy and precision computations. Again the d_1 values are compared with Table 1 while the d_2 values are compared with Table 2. Each magnitude now does not exceed the tabled value (although $d_1 = 1$ for Cr equals the tabled value) so the machine has now passed the daily standardization check and the complete standardization is not required.

If each machine is properly standardized each day, by either the recommended daily check, or if necessary the full standardization procedure, it should be in proper operating condition. It follows that the laboratories require prepared oil standards at only the 0, 50 and 100 ppm levels (or 0,

True Concentration	Fe	Ag	Al	Cr	Cu	Mg
0	50.0	50.3	50.3	50.4	50.0	50.0
0	49.3	50.1	50.0	49.3	49.9	49.8
A = Average of 2 = $\text{sum} \div 2$	49.65	50.2	50.15	49.85	49.95	49.9
A - 50	-.35	.20	.15	-.15	-.05	-.10
d_1 = magnitude	.35	.20	.15	.15	.05	.10
50	50.5	51.1	50.2	52.4	50.1	50.6
50	51.9	51.0	50.4	53.0	51.0	52.5
A = Average of 2 = $\text{sum} \div 2$	51.2	51.05	50.3	52.7	50.55	51.55
A - 50	1.20	1.05	.30	2.70	.55	1.55
d_1 = magnitude	1.20	1.05	.30	2.70	.55	1.55
100	97	99	98	97	97	94
100	101	108	102	100	102	100
A = Average of 2	99	103.5	100	98.5	99.5	97
A - 100	-1.0	3.5	0	-1.5	-.5	-3.0
d_1 = magnitude	1.0	3.5	0	1.5	0.5	3.0

Accuracy check for example

Table 3

True Concentration	Fe	Ag	Al	Cr	Cu	Mg
0	50.0	50.3	50.3	50.4	50.0	50.0
0	49.3	50.1	50.0	49.3	49.9	49.8
D = difference of 2	0.7	0.2	0.3	1.1	0.1	0.2
d_2 = magnitude	0.7	0.2	0.3	1.1	0.1	0.2
50	50.5	51.1	50.2	52.4	50.1	50.6
50	51.9	51.0	50.4	53.0	51.0	52.5
D = difference of 2	-1.4	0.1	-0.2	-0.6	-0.9	-1.9
d_2 = magnitude	1.4	0.1	0.2	0.6	0.9	1.9
100	97	99	98	97	97	94
100	101	108	102	100	102	100
D = difference of 2	-4	-9	-4	-3	-5	-6
d_2 = magnitude	4	9	4	3	5	6

Precision check for example

Table 4

True Concentration	Fe	Ag	Al	Cr	Cu	Mg
0	49.7	49.9	49.9	49.1	49.9	49.7
0	49.5	50.2	49.9	48.9	49.8	49.8
A = Average of 2	49.6	50.15	49.9	49.0	49.85	49.75
A - 50	-.4	0.15	-0.1	-1.0	-0.15	-0.25
d ₁	0.4	0.15	0.1	1.0	0.15	0.25
D = difference of 2	0.2	-0.3	0	0.2	0.1	-0.1
d ₂	0.2	0.3	0	0.2	0.1	0.1

Accuracy and precision checks for
second set of burns at 0 offset.

Table 5

50 and 300 ppm levels) for their daily operation; provision of prepared standards at other levels (3, 10 and 30 ppm) are not required. If during the course of a day's work it is desired to check the accuracy of the instrument (as may frequently occur) the daily standardization check may be repeated at any one, two or all three of the concentration levels employed.

2.2 Origin of Tables 1 and 2

The numbers presented in Tables 1 and 2 are derived from a mixture of theoretical model constructions of how the AE-3 readings should behave, both from day to day and from one instrument to another, and from an examination of actual AE-3 readings from several instruments burning the same samples on two different days.

The 0 (offset), 50 ppm and 100 ppm readings were taken from data collected by LCDR D. C. Hatcher for his thesis "Accuracy and Repeatability Indices for JOAP Data", MS in Operations Research, September 1979, Naval Postgraduate School, Monterey, Ca. Twenty six different laboratories participated in LCDR Hatcher's study. Each participating laboratory burned the same oil sample 15 times, on each of two days, on its own instrument, monitoring up to 20 different contaminants (listed in Table 1); samples at the 0, 3, 10, 50 and 100 ppm levels were employed in his study. The 300 ppm values used came from a special separate study in which each of 10 different laboratories burned the same 300 ppm sample 10 times on two successive days,

monitoring the 14 contaminants identified in Table 1. Three of these laboratories were also included in the done by Hatcher,

For each element-ppm combination of interest, the same basic type of underlying model was assumed, frequently called a components of variance model; this type of model allows one to estimate the contributions of the different laboratories (instruments), of the days within laboratories, and of the inherent instrument variability to the observed variability in the numbers recorded. Specifically, if we let y_{ijk} represent the k^{th} reading made by instrument i on day j (for a given element-ppm combination, say iron at 50 ppm), we can assume that

$$y_{ijk} = \mu + \lambda_i + \delta_{ij} + \epsilon_{ijk}.$$

The parameter μ represents the "true" concentration level, λ_i represents the contribution from laboratory i , δ_{ij} represents the effect of day j within laboratory i , and ϵ_{ijk} represents the observation error, the inherent instrument variability over repeated burns of the same sample. Furthermore, we can assume the λ_i values are normal, independent, mean 0, variance σ_λ^2 , the δ_{ij} values are normal, independent, mean 0, variance σ_δ^2 and the ϵ_{ijk} values are normal, independent, mean 0, variance σ^2 ; the specification of the model is completed by assuming all λ_i 's, δ_{ij} 's, ϵ_{ijk} 's are mutually independent. It then follows that any individual y_{ijk} is normal, mean μ , variance $\sigma^2 + \sigma_\delta^2 + \sigma_\lambda^2$. The observed data can be used to estimate μ , σ^2 , σ_δ^2 , σ_λ^2 . The simplest procedure to use (and the one employed) to estimate the variance parameters is to perform an analysis of variance and use linear functions of the observed mean squares to construct unbiased

estimators for σ^2 , σ_δ^2 , σ_λ^2 ; this procedure is described in many textbooks, such as Statistics in Research, by Ostle and Mensing, Iowa State University Press, Third Edition, 1975. As mentioned in this text, it is possible that this procedure may give negative estimates for certain variances, which are clearly nonsensical; in such cases it is prudent to replace the negative value by zero. In 5 out of the 148 cases in which a negative value could occur in this study, the procedure produced a negative variance estimate; each of these was replaced by zero for the subsequent computations.

Granted that the data described can be used to estimate σ^2 , σ_δ^2 , σ_λ^2 , what then is done with these values to derive reasonable limits for the average (or difference) of two successive readings of the same sample on the same instrument (as given in Tables 1 and 2)? We shall first discuss the derivations for the average of two such readings.

For simplicity, assume the readings are done at laboratory 1 (so $i = 1$) on day 1 (so $j = 1$) and the readings are the first and second of that day ($k = 1, k = 2$). Thus our two observed readings are y_{111} and y_{112} (for the given element-ppm combination). The average of the two is

$$\bar{y}_{11} = \frac{y_{111} + y_{112}}{2} = \mu + \lambda_1 + \delta_1 + \frac{e_{111} + e_{112}}{2}$$

which, from our model, is normal, mean μ , variance $\frac{\sigma^2}{2} + \sigma_\delta^2 + \sigma_\lambda^2$. The methodology described in Ostle and Mensing, quoted above, leads to using

$$V = aX_1 + bX_2 + cX_3$$

as an estimator for $\frac{\sigma^2}{2} + \sigma_\delta^2 + \sigma_\lambda^2$, where

X_1 = error mean square

X_2 = days within labs mean square

X_3 = lab mean square

from the analysis of variance for the given element-ppm combination; the constants a, b and c depend on the number of laboratories, number of burns per day, and number of days per laboratory available. It is a simple consequence of our model that $\bar{y}_{11.}$ and V are independent random variables, as are X_1 , X_2 and X_3 . It also follows that V is approximately a normal random variable with mean $\frac{\sigma^2}{2} + \sigma_S^2 + \sigma_\lambda^2$ and thus

$$(a) \quad P\left[|\bar{y}_{11.} - \mu| \leq 2.24 \sqrt{\frac{\sigma^2}{2} + \sigma_S^2 + \sigma_\lambda^2}\right] = .975$$

$$(b) \quad P\left[\frac{\sigma^2}{2} + \sigma_S^2 + \sigma_\lambda^2 \leq V + 1.96\sigma_V\right] = .975 .$$

It is a simple matter to use the observed data to estimate σ_V ; call the estimate $\hat{\sigma}_V = \sqrt{\hat{\sigma}_V^2}$ ($\hat{\sigma}_V^2$ is a quadratic function of the three mean squares mentioned above). Then from (b) it should be approximately true that

$$(c) \quad P\left[\frac{\sigma^2}{2} + \sigma_S^2 + \sigma_\lambda^2 \leq V + 1.96\hat{\sigma}_V\right] = .975 .$$

The independence results mentioned earlier, together with (a) and (c), lead to

$$P\left[|\bar{y}_{11.} - \mu| \leq 2.24 \sqrt{V + 1.96\hat{\sigma}_V}\right] = (.975)^2 = .95 .$$

Thus we would expect about 95% of the individual readings to be within $2.24 \sqrt{V + 1.96\hat{\sigma}_V}$ of the true sample content (if the instrument is

working correctly). Thus, our model-based approach leads to the computation of $2.24 \sqrt{V + 1.96\hat{\sigma}_V}$, from the observed data, as reasonable 95% bounds on how close the average of two successive burns, by the same instrument on an arbitrary day, should be to the actual true concentration of the sample (for a given element).

A second quite different approach may also be employed to derive reasonable 95% limits for the difference $|\bar{y}_{11} - \mu|$. This is best described by referring to one actual case, say iron at 50 ppm. For this case there were 26 laboratories (instruments) involved, each of which analyzed the same sample 15 times on each of two days ($30 \times 26 = 780$ analyses in total). Within each day we can pair together successive analyses (1 and 2, 3 and 4, ..., 13 and 14, drop 15) and average these pairs; thus we can compute 7 averages for each lab for each day, a total of $7 \times 2 \times 26 = 364$ independent averages of pairs (for this element-ppm combination). We can then rank these 364 averages from smallest to largest and ask how far below 50 ppm, and how far above 50 ppm, must we go to include the 346 ($.95 \times 364$) central averages. The distance we must go should be roughly the same as $2.24 \sqrt{V + 1.96\hat{\sigma}_V}$ derived from our model and indeed the agreement is quite good. The numbers given in Table 1 were derived by looking at both these approaches, for each element-ppm combination.

An entirely analogous procedure was used for deriving limits for the difference of two successive readings, given in Table 2. From our model, using the notation discussed earlier,

$$\begin{aligned} y_{111} - y_{112} &= (\mu + \lambda_1 + \delta_{11} + e_{111}) - (\mu + \lambda_1 + \delta_{11} + e_{112}) \\ &= e_{111} - e_{112} \end{aligned}$$

which is normal with mean 0, variance $2\sigma^2$ (both the lab effect and the day effect, as well as the true concentration μ , cancel in forming the difference). The estimate of $2\sigma^2$, for any element-ppm combination, is $W = 2X_1$, two times the error mean square in the analysis of variance mentioned earlier. Again, W and $y_{111} - y_{112}$ are independent and

$$P(|y_{111} - y_{112}| \leq 2.24\sqrt{2}\sigma) = .975$$

$$P(2\sigma^2 \leq W/(1 - 1.96d)) = .975,$$

where $d = \sqrt{\frac{2}{v}}$, v = error mean square degrees of freedom.

These two equations, and independence, lead to

$$P\left(|y_{111} - y_{112}| \leq 2.24 \sqrt{\frac{W}{1 - 1.96d}}\right) = .95,$$

so our model gives $2.24 \sqrt{\frac{W}{1 - 1.96d}}$ as reasonable 95% limits for the difference of two successive analyses on the same instrument; these quantities were computed for each element-ppm combination. As with the averages of pairs, differences of successive analyses were computed, ranked, and used to find how far above and below zero one must go to bracket 95% of the observed differences. Again, the agreement with the model-based value of $2.24 \sqrt{\frac{W}{1 - 1.96d}}$ was quite good, resulting in the values given in Table 2.

The values given in Tables 1 and 2 should allow a correctly calibrated instrument to pass the daily standardization check about 95% of the time. It follows that about 5% of the time the full standardization procedure will

unnecessarily be required. This proportion (5%) can be decreased by increasing the values presented in the two tables. Such an increase would also increase the probability that an incorrectly calibrated instrument, which should employ the full standardization procedure, will pass the daily standardization check.

3. DETERMINATION OF CONVERSION FACTORS FOR READINGS FROM ATOMIC ABSORPTION AND ATOMIC EMISSION SPECTROMETERS

The joint Oil Analysis Program was established in 1976 for the purpose of utilizing spectrometric oil analysis to detect impending failures of aircraft engines, gear boxes, hydraulic systems etc. Presently, there are nearly 180 spectrometric laboratories participating in the program. Although a wide majority of the laboratories use atomic emission spectrometers (A/E35U-3), a few of them still perform the oil analyses on atomic absorption spectrometers. Because of the differences in the operating principles of the two types of spectrometers the analytic results obtained from the two machines tend to be different even for the same oil sample. In fact, measurements from an absorption spectrometer tend to be lower than those for an emission spectrometer.

The spectrometric laboratories depend on wearmetal evaluation criteria to determine the appropriate action to be taken based on the oil sample analytic results. The criteria are presented in tables of wearmetal ranges and trends that are to be considered normal, marginal, high or abnormal for a given piece of equipment. The criterion values are determined from historical records and statistical distributions, based on a large number of sample analytical results. The required statistical data is obtained by pooling all available analytic results on a given piece of equipment. In order for this method to be successful, it is important that all the laboratories analyzing oil samples for a particular type of equipment obtain similar measurements for similar samples.

Ensuring uniformity in analytic results for all laboratories using emission spectrometers is a relatively easy problem; proper calibration of the spectrometer on a regular basis should achieve uniformity. However,

as stated earlier, emission and absorption spectrometers do not provide comparable analytic results and one approach to correct this is to establish formulas to convert analytic results from absorption spectrometers into comparable results for emission spectrometers. Such formulas would also be useful in maintaining consistent records (unit failure histories) on transient aircraft that move from one base to another and their spectrometers are different.

The Naval Weapons Engineering Support Activity (NAVWESA), Washington D.C. in 1975 demonstrated the feasibility of using statistical regression techniques to determine the Conversion formulas. The investigators Lynch and Short [3] used limited data from the interlaboratory correlation program to demonstrate the techniques and recommended a more extensive study to establish the conversion formulas. Cueller, Jr [2] of the Southwest Research Institute also studied this problem in 1975 and recommended the use of the following conversion formulas:

$$A/E35U-3 = 2AA = \frac{4}{3} A/E35U-1$$

Whereas the NAVWESA study indicated the need for a separate formula for different wearmetals, the Southwest Research Institute results are to be applied uniformly to all wearmetals. One of the tasks of our research project is a further investigation of the problem for the purpose of establishing firm conversion formulas. We extracted the data for our analysis from the interlaboratory correlation test results for the period Mar. 1980 - Feb. 1981. We selected six elements viz., iron, silver, aluminum, chromium, copper and magnesium for this study. Approximately 140 AE laboratories and 25 AA laboratories contributed to the data base. Each month the laboratories analyzed two independent oil samples. The

results were summarized by averaging over all AE laboratories and over all AA laboratories, respectively. Thus, the data for our analysis consisted of 24 observations (2 per month for 12 months) for the AE laboratories and 24 observations for the AA laboratories (Table 6) for each of the six elements.

We used regression analysis to fit a straight line to represent the relationship between the AA and the AE readings and the results were surprisingly good. With the exception of copper, a straight line provided a very good model with coefficient of determination R^2 values above .96; for copper R^2 was .75. The results of the regression analysis are presented in Table 7 and Figures 1-6 are scatter plots of the data and the goodness of fit of linear models is quite apparent from these plots. We are of the opinion that the linear models described in Table 7 can be used safely to convert an AA or AE reading into an equivalent AE or AA reading for the elements iron, silver, aluminum, chromium and magnesium. For copper also we suggest the use of the linear model although it may not be totally satisfactory for converting an AE reading into an equivalent AA reading.

To make it more convenient for spectrometric laboratory personnel to use the results we have prepared for each of the six elements, ready lookup tables converting AA data sets into equivalent AE data sets and vice versa. These are presented in Tables 8 and 9.

Under the interlaboratory correlation program the laboratories also analyze two synthesized samples each month. We examined the relationship between the measurements on AA machines and those from the AE machines. Here again, we found that linear models provide a practically perfect fit with an R^2 value greater than .98 for each of the six elements. The data and the regression results are presented in Table 10 and 11.

TABLE OF MEANS: USED OIL SAMPLES

MONTH	FE		AG		AL		CR		CU		MG	
	AA	AE	AA	AE	AA	AE	AA	AE	AA	AE	AA	AE
MAR 80	83.4	105.0	0.1	0.0	18.2	24.5	4.6	8.4	8.6	10.5	11.2	17.3
	73.2	90.9	0.4	0.0	16.1	21.5	4.0	7.1	7.4	9.0	9.4	14.9
APR	12.2	26.7	0.0	0.0	1.1	0.3	0.0	0.4	12.7	24.5	0.7	1.0
	11.3	25.5	0.0	0.0	0.9	0.3	0.0	0.3	12.0	23.1	0.6	1.0
MAY	156.0	183.0	0.6	0.1	18.9	22.4	5.3	7.7	10.3	12.7	18.9	24.5
	149.7	174.0	0.4	0.5	17.9	21.7	5.0	7.6	9.6	12.4	18.0	23.7
JUN	9.3	18.8	0.3	0.0	0.4	0.2	1.2	1.9	1.1	0.1	1.8	2.1
	8.1	16.5	0.0	0.0	0.2	0.1	1.0	1.7	1.0	0.0	1.8	1.9
JULY	4.8	12.1	0.3	0.8	0.6	0.2	0.3	0.7	0.7	1.3	0.2	0.9
	4.5	11.3	0.3	0.6	0.6	0.2	0.1	0.6	0.6	1.2	0.1	0.7
AUG	86.2	101.7	0.4	0.1	7.7	10.4	3.1	4.2	5.5	6.8	30.4	38.1
	79.5	92.7	0.5	0.2	6.8	9.2	2.7	3.7	4.8	5.9	27.7	34.4
SEPT	9.1	17.3	3.0	5.8	2.8	2.5	3.7	4.8	5.1	9.2	4.9	10.6
	8.8	17.0	2.9	5.4	2.6	2.2	3.4	4.3	4.7	8.4	4.4	9.7
OCT	87.6	109.7	0.9	0.4	9.5	14.2	3.3	4.6	8.3	5.5	152.8	189.5
	80.8	98.7	1.0	0.5	8.7	13.0	3.1	4.3	7.5	5.0	140.3	170.0
NOV	4.0	8.2	2.0	3.6	1.8	1.6	2.3	3.7	2.9	6.4	3.0	6.9
	4.0	8.2	2.0	3.5	1.8	1.6	2.3	3.6	3.0	6.3	2.9	6.7
DEC	47.3	55.5	0.3	0.1	7.9	10.6	2.3	2.1	4.5	3.4	33.1	40.1
	44.7	50.2	0.2	0.0	7.4	9.6	2.1	2.0	4.2	3.0	30.6	36.6
JAN 81	21.2	41.2	1.2	2.7	31.1	40.4	36.5	40.6	4.0	7.2	4.2	8.1
	20.2	38.2	1.1	2.5	28.6	36.9	33.6	36.6	3.8	6.4	3.9	7.4
FEB	5.8	11.1	6.6	10.1	7.2	10.7	8.4	10.8	3.2	7.1	1.4	3.1
	5.5	10.2	6.2	9.3	7.0	9.5	8.0	9.9	3.1	6.4	1.4	2.6

TABLE 6

USED OIL SAMPLE MEANS
 LEAST SQUARES FIT OF A STRAIGHT LINE
 $Y = \alpha + \beta X$ (X = AA MEAN; Y = AE MEAN)

ELEMENT	INTERCEPT PARAMETER α	SLOPE PARAMETER β	COEFFICIENT OF DETERMINATION R^2
Fe	7.6875	1.1194	.993
Ag	-0.1497	1.6219	.961
Al	-0.2477	1.3107	.993
Cr	0.9891	1.0848	.992
Cu	-0.4907	1.5053	.748
Mn	1.6897	1.2135	.999

TABLE 7

TABLE FOR CONVERSION OF AA READINGS
INTO EQUIVALENT AE READINGS (PPM)

AE READING AA READING	Fe	Ag	Al	Cr	Cu	Mg
0	7.69	0.00	0.00	.99	0.00	1.69
2	9.93	3.09	2.37	3.16	2.52	4.12
4	12.17	6.33	5.00	5.33	5.53	6.54
6	14.40	9.58	7.62	7.50	8.54	8.97
8	16.64	12.83	10.24	9.67	11.55	11.40
10	18.88	16.07	12.86	11.84	14.56	13.82
12	21.12	19.31	15.48	14.01	17.57	16.25
14	23.36	22.56	18.10	16.18	20.58	18.68
16	25.60	25.80	20.72	18.35	23.59	21.11
18	27.84	29.04	23.34	20.52	26.60	23.53
20	30.08	32.29	25.97	22.69	29.62	25.96
22	32.31	35.53	28.59	24.85	32.63	28.39
24	34.55	38.78	31.21	27.02	35.64	30.81
26	36.79	42.02	33.83	29.19	38.65	33.24
28	39.03	45.26	36.45	31.36	44.66	35.67
30	41.27	48.51	39.07	33.53	44.67	38.09
35	46.87	56.62	45.63	38.96	52.19	44.16
40	52.46	64.73	52.18	44.38	59.72	50.23
45	58.06	72.84	58.73	49.81	67.25	56.30
50	63.66	80.95	65.29	55.23	74.77	62.36
55	69.25	89.05	71.84	60.65	82.30	68.43
60	74.85	97.16	78.39	66.08	89.83	74.50
65	80.45	105.27	84.95	71.50	97.35	80.57
70	86.05	113.38	91.50	76.93	104.88	86.63
75	91.64	121.49	98.05	82.35	112.41	92.70
80	97.24	129.60	104.61	87.77	119.93	98.77
85	102.84	137.71	111.16	93.20	127.46	104.84
90	108.43	145.82	117.72	98.62	134.99	110.90
95	114.03	153.93	124.27	104.05	142.51	116.97
100	119.63	162.04	130.82	109.47	150.04	123.04

TABLE 8

TABLE FOR CONVERSION OF AE READINGS
INTO EQUIVALENT AA READINGS (PPM)

AE READING AA READING	Fe	Ag	Al	Cr	Cu	Mg
0	0.00	0.14	0.25	0.00	1.59	0.00
2	0.00	1.32	1.76	0.97	2.59	0.28
4	0.00	2.51	3.28	2.80	3.58	1.93
6	0.00	3.69	4.79	4.63	4.58	3.57
8	0.56	4.88	6.31	6.46	5.57	5.22
10	2.33	6.06	7.82	8.29	6.56	6.86
12	4.11	7.25	9.34	10.12	7.56	8.51
14	5.88	8.43	10.85	11.94	8.55	10.16
16	7.66	9.62	12.37	13.77	9.55	11.80
18	9.43	10.80	13.88	15.60	10.54	13.45
20	11.21	11.99	15.40	17.43	11.53	15.10
22	12.98	13.17	16.91	19.26	12.53	16.74
24	14.76	14.36	18.43	21.09	13.52	18.39
26	16.53	15.54	19.94	22.92	14.52	20.03
28	18.31	16.73	21.46	24.75	15.51	21.68
30	20.08	17.91	22.97	26.58	16.50	23.33
35	24.52	20.88	26.76	31.15	18.99	27.44
40	28.96	23.84	30.55	35.73	21.47	31.56
45	33.39	26.80	34.33	40.30	23.96	35.67
50	37.83	29.76	38.12	44.87	26.44	39.79
55	42.27	32.73	41.91	49.45	28.93	43.90
60	46.70	35.69	45.69	54.02	31.41	48.02
65	51.14	38.65	49.48	58.59	33.90	52.14
70	55.58	41.61	53.27	63.17	36.38	56.25
75	60.02	44.58	57.06	67.74	38.87	60.37
80	64.45	47.54	60.84	72.31	41.35	64.48
85	68.89	50.50	64.63	76.89	43.84	68.60
90	73.33	53.46	68.42	81.46	46.32	72.71
95	77.76	56.43	72.20	86.04	48.81	76.83
100	82.20	59.39	75.99	90.61	51.29	80.94

TABLE 9

TABLE OF MEANS: SYNTHESIZED SAMPLES

MONTH	FE		AG		AL		CR		CU		MG	
	AA	AE	AA	AE	AA	AE	AA	AE	AA	AE	AA	AE
MAR 80	12.2	12.3	53.4	50.0	50.6	47.5	18.9	20.3	12.0	12.4	56.5	49.0
	11.6	11.7	51.1	48.0	47.9	45.1	17.9	19.1	12.0	11.7	53.6	48.1
APR	80.4	83.7	41.8	40.4	9.2	12.0	10.3	11.4	80.3	77.8	41.3	41.3
	69.6	73.1	36.3	35.3	8.5	10.4	8.8	9.7	70.0	67.9	36.2	36.7
MAY	85.2	86.0	45.1	41.1	11.1	11.8	9.9	11.0	83.1	79.1	44.2	42.2
	75.2	73.9	39.1	35.5	9.3	9.9	8.8	10.4	75.5	68.4	38.8	36.3
JUN	15.5	15.4	10.4	10.4	29.8	29.6	20.3	22.0	29.0	28.9	10.3	11.1
	14.6	14.5	9.7	9.8	27.2	27.3	18.5	20.1	27.2	26.8	9.3	10.3
JULY	30.1	31.0	47.8	46.2	19.9	21.4	29.4	33.9	21.0	21.0	30.0	30.0
	28.3	29.7	45.5	44.4	18.5	20.3	27.8	32.3	19.3	19.8	28.2	29.2
AUG	52.6	52.5	31.3	30.3	18.5	20.3	17.8	19.4	51.3	51.1	28.5	29.3
	48.1	47.5	28.5	29.3	17.0	18.3	16.0	17.6	47.5	46.7	26.0	26.9
SEPT	21.7	22.2	15.4	15.9	30.4	30.2	31.1	32.0	21.8	23.0	14.0	15.1
	20.6	21.1	14.5	15.1	28.8	29.5	30.5	20.9	21.8	21.8	13.2	14.5
OCT	100.0	110.3	11.7	11.6	12.6	15.2	11.3	13.0	102.3	97.2	13.0	14.0
	91.6	99.8	10.5	10.5	11.1	13.6	10.2	11.6	94.2	88.2	11.8	12.5
NOV	27.7	30.8	27.0	28.1	40.7	42.6	9.0	10.7	27.5	30.2	45.5	48.1
	25.3	28.0	24.2	25.5	37.8	38.6	8.2	9.5	25.0	27.3	41.5	44.7
DEC	8.9	9.0	3.8	3.6	15.9	16.0	17.1	16.0	9.0	9.5	3.2	3.3
	8.8	9.1	3.6	3.6	15.5	15.9	16.9	16.0	8.6	9.4	3.1	3.7
JAN 81	34.2	36.5	33.9	36.4	59.8	58.6	55.4	59.0	33.3	36.4	33.9	39.2
	31.2	32.5	31.2	32.3	53.7	52.2	50.3	52.6	30.2	32.4	30.6	35.1
FEB	79.7	79.6	44.3	43.3	49.7	49.4	49.5	51.7	43.6	44.0	67.0	64.6
	69.7	68.8	38.7	37.4	42.0	42.3	43.2	44.6	38.1	38.0	58.9	56.4

TABLE 10

SYNTHESIZED OIL SAMPLE MEANS
 LEAST SQUARES FIT OF A STRAIGHT LINE
 $Y = \alpha + \beta X$ ($X = \text{AA MEAN}$; $Y = \text{AE MEAN}$)

ELEMENT	INTERCEPT PARAMETER α	SLOPE PARAMETER β	COEFFICIENT OF DETERMINATION R^2
Fe	0.485	1.0459	.994
Ag	1.2131	0.9372	.992
Al	2.3827	0.9308	.995
Cr	0.7487	1.0379	.994
Cu	2.5276	0.9231	.996
Mg	2.4419	0.9273	.981

TABLE 11

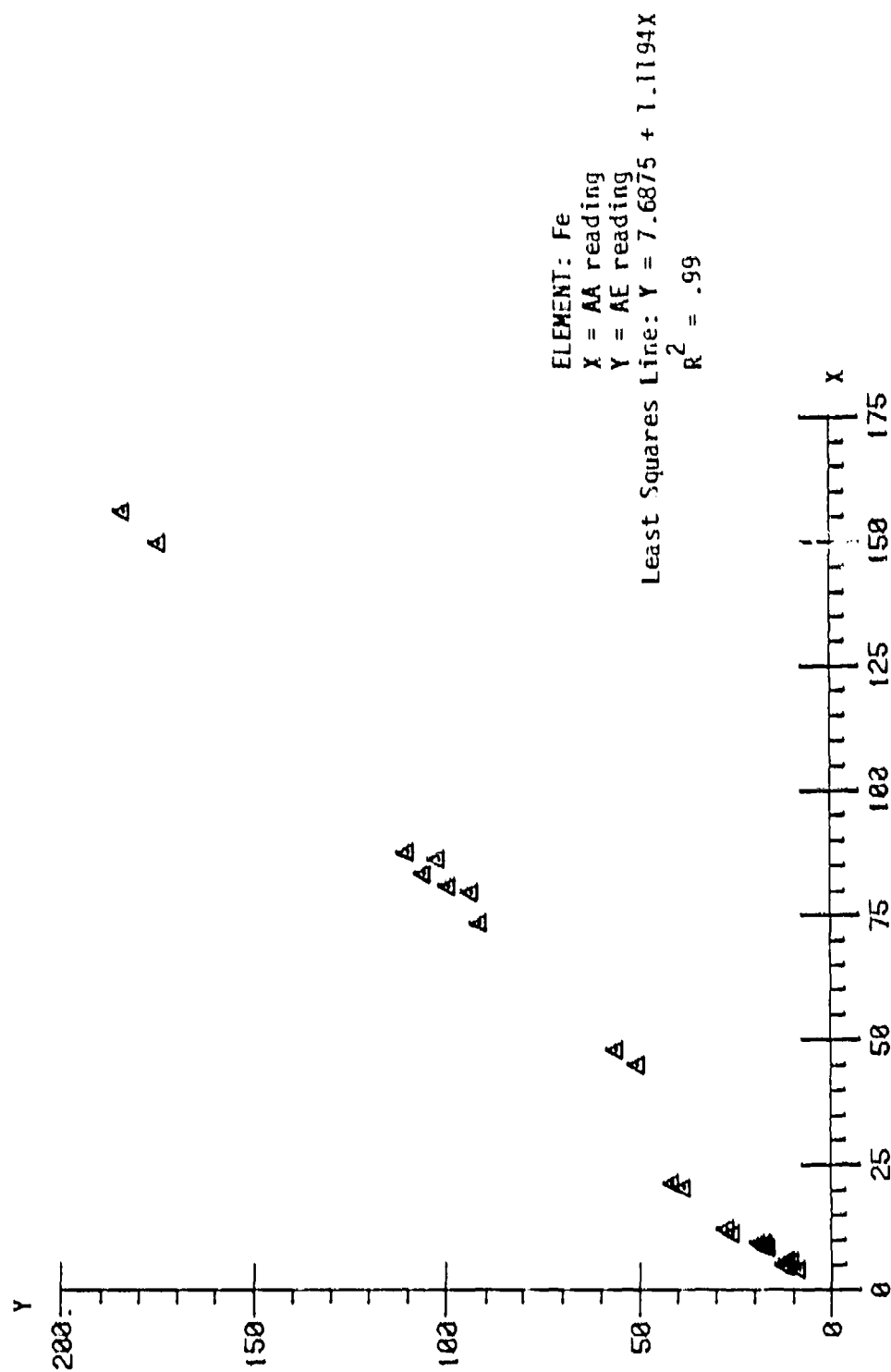


FIGURE 1

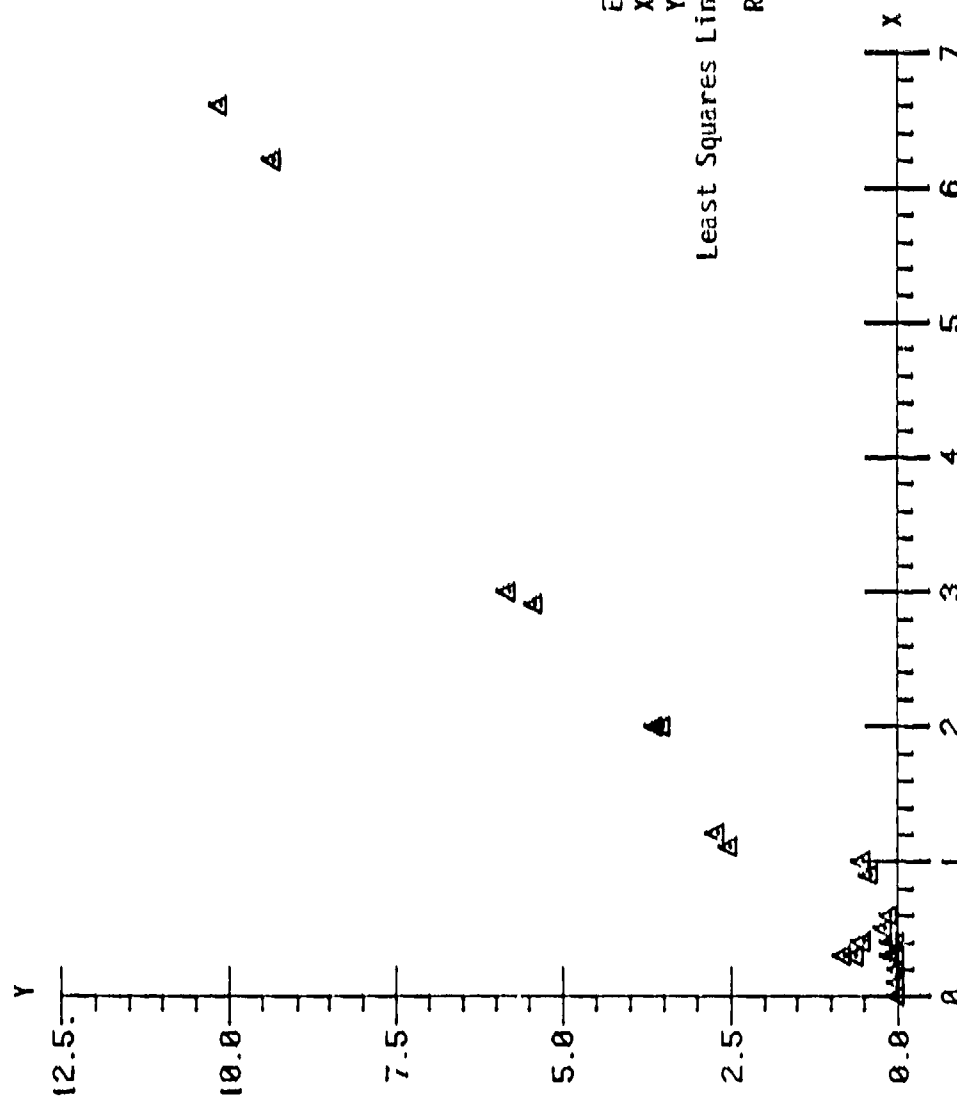
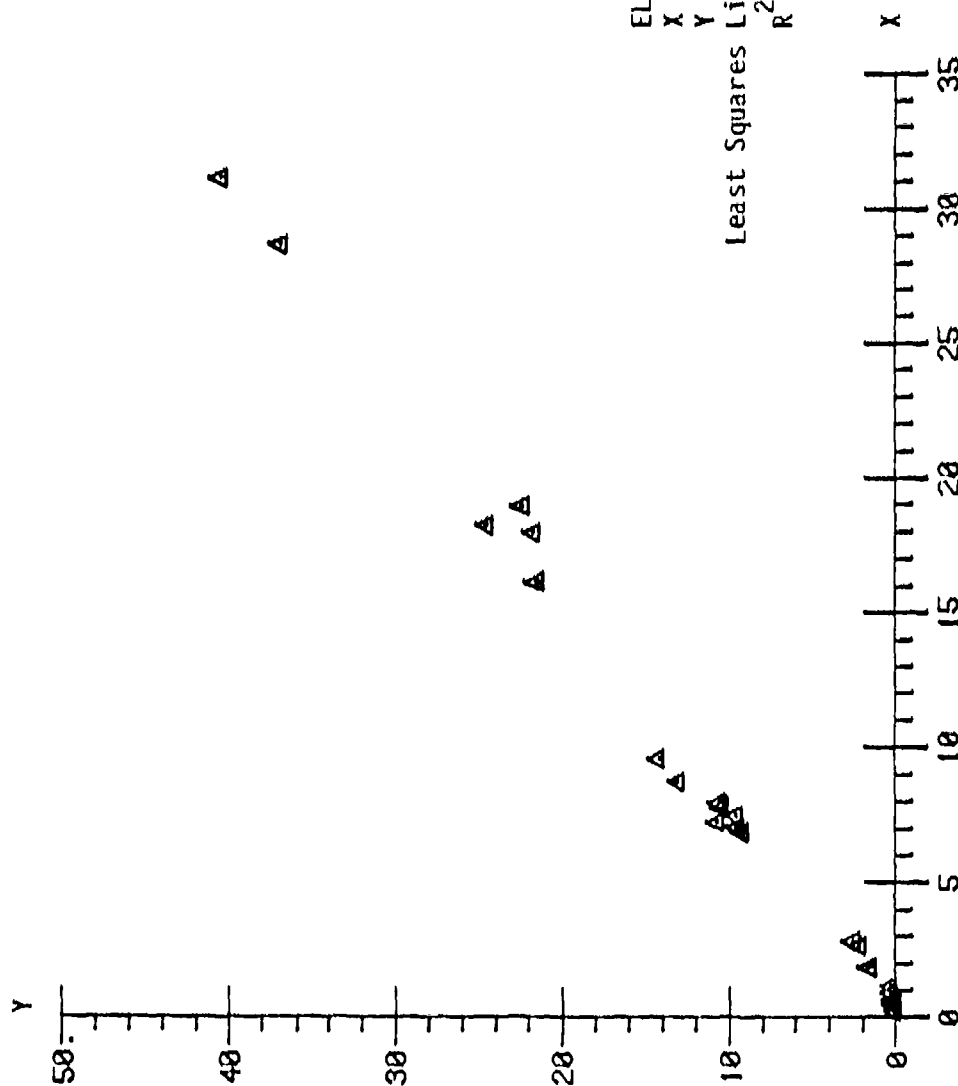


FIGURE 2

ELEMENT: Ag
 X = AA reading
 Y = AE reading
 Least Squares Line: $Y = -0.1497 + 1.6219X$
 $R^2 = .96$



ELEMENT: A1
 X = AA reading
 Y = AE reading
 Least Squares Line: $Y = -0.2477 + 1.3107X$
 $R^2 = .99$

FIGURE 3

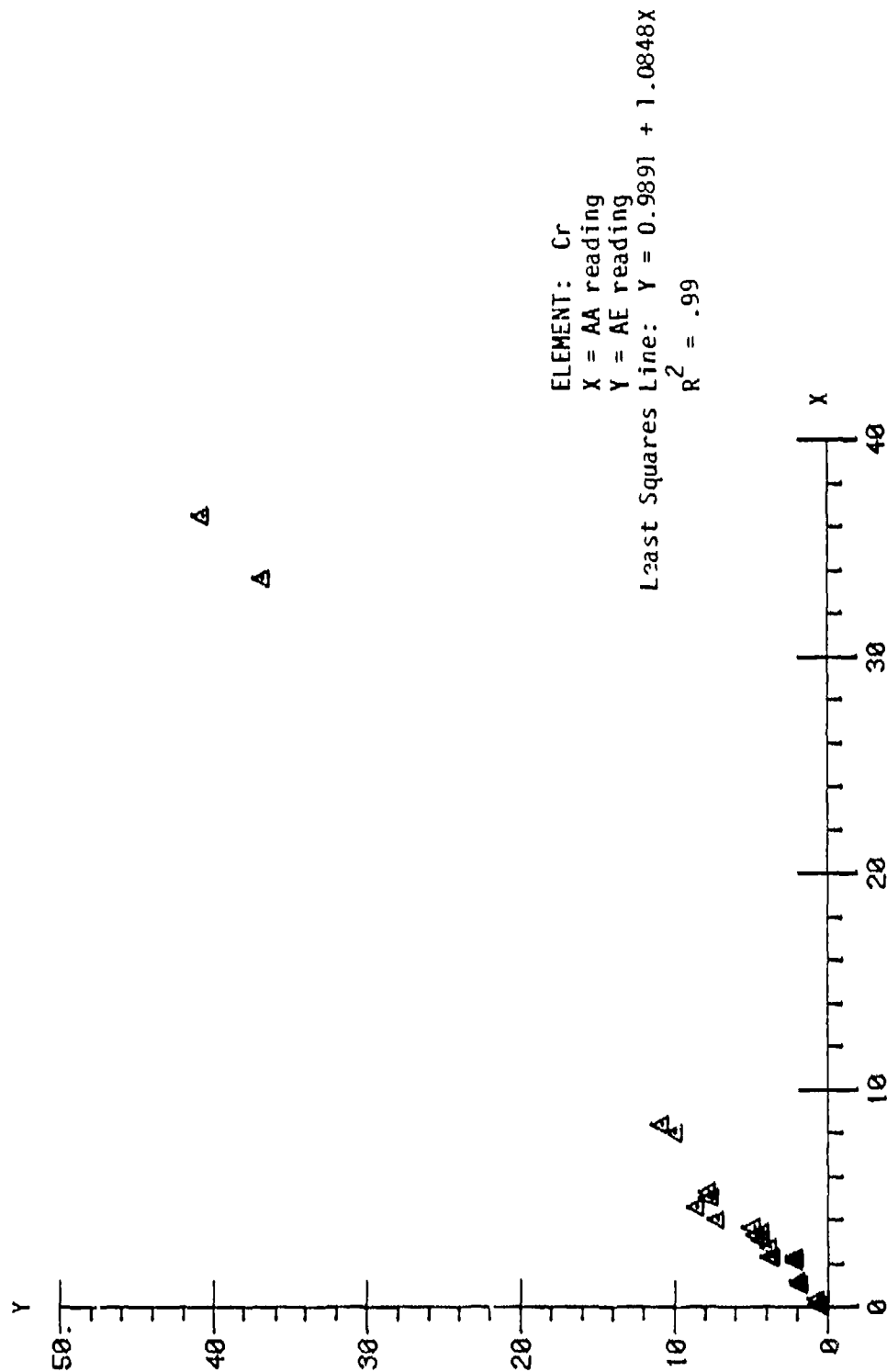


FIGURE 4

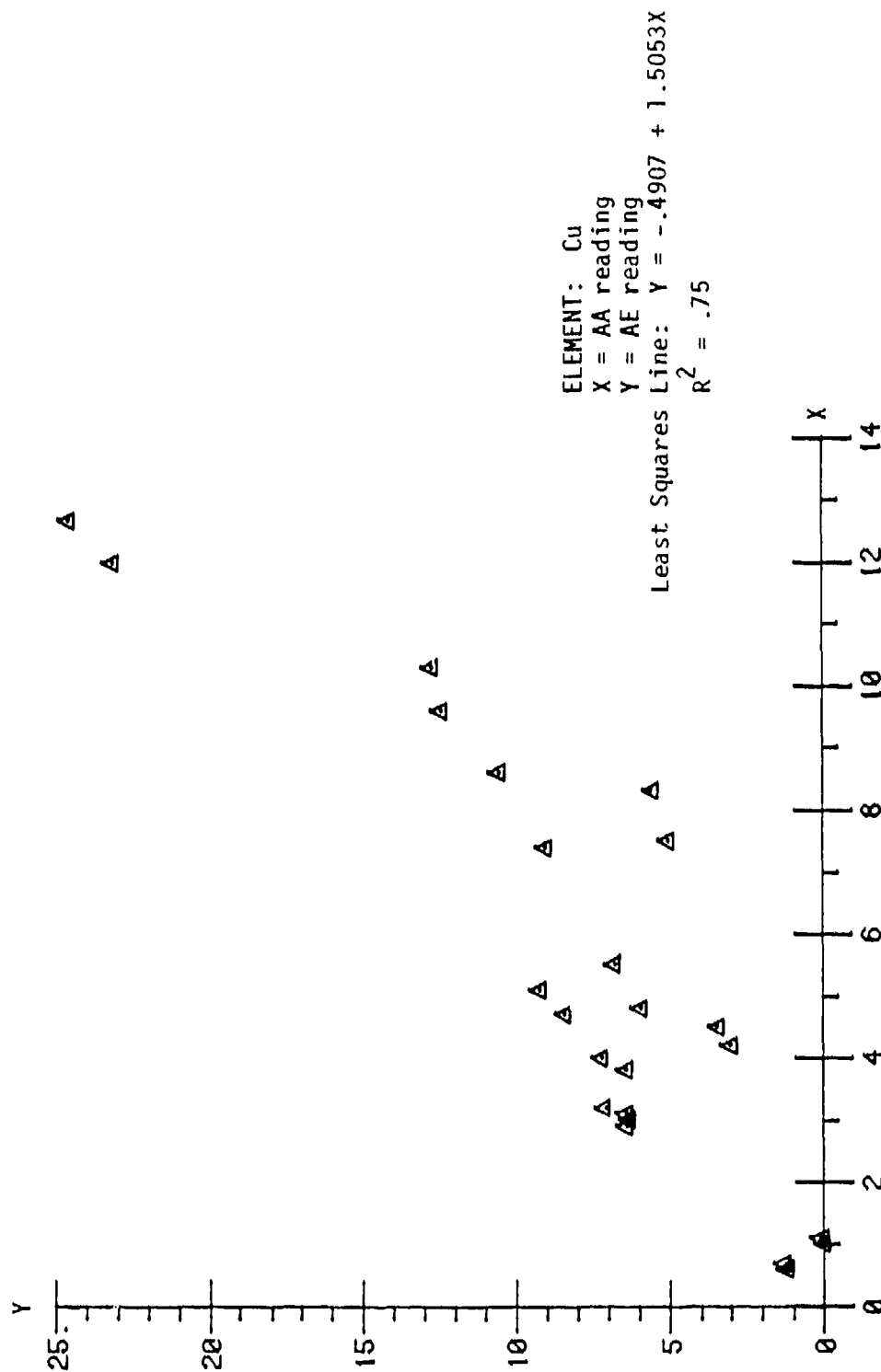
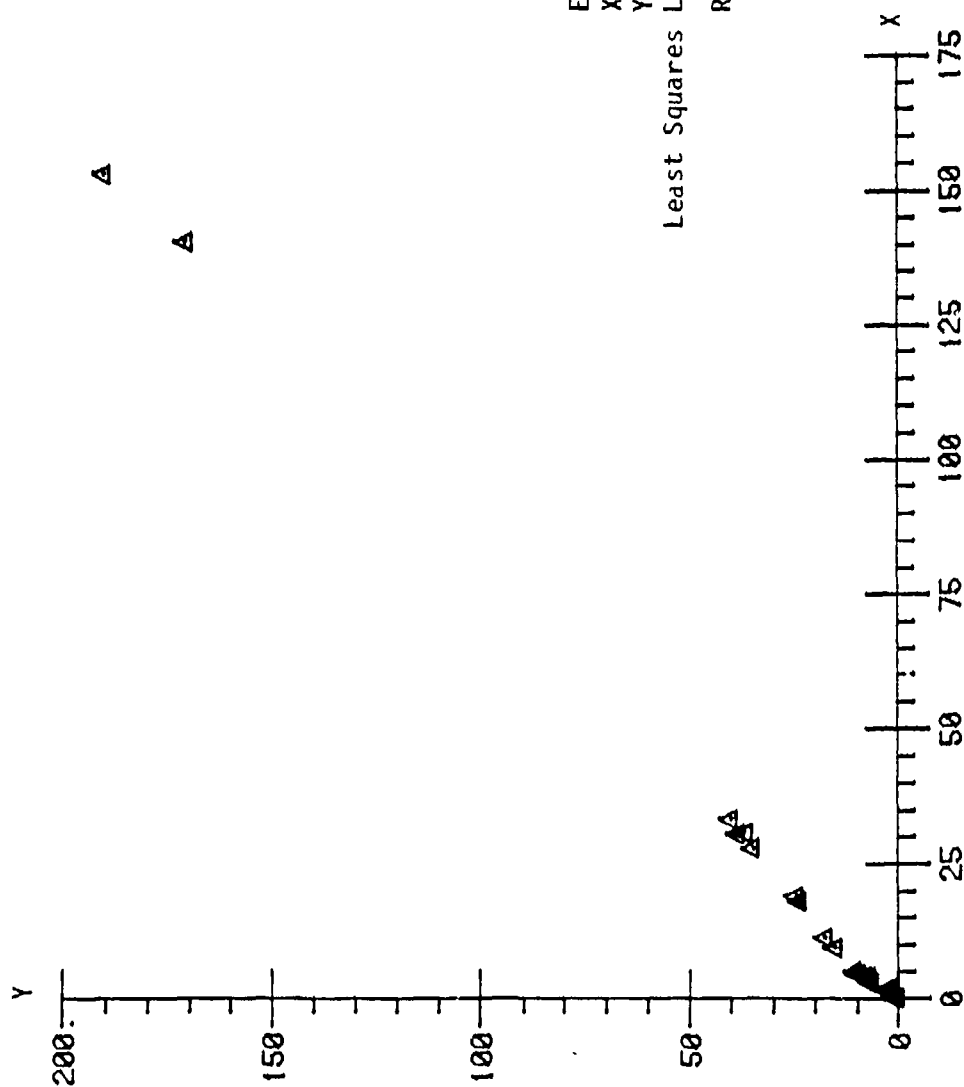


FIGURE 5



ELEMENT: Mg
 X = AA reading
 Y = AE reading
 Least Squares Line $Y = 1.6897 + 1.2135X$
 $R^2 = .99$

FIGURE 6

4. STATISTICAL CRITERIA FOR THE EVALUATION OF OIL SAMPLES

4.1. Description of the procedure.

The current wearmetal methodology consists of a wearmetal evaluation criteria and trend tables as well as a decision making guidance table for each type of equipment included in the JOAP program [1]. The evaluation criteria and the trend tables have been derived using historical data on each type of equipment. When a new sample is received and analyzed the evaluator uses the evaluation criteria and trend tables to make one of several available recommendations such as:

- a) Continue routine sampling
- b) Submit another sample to confirm "abnormality"
- c) Increase the sampling frequency or
- d) Take a prescribed maintenance action immediately.

It is true that historical data can be used effectively to construct the tables described above. However, we believe that each individual piece of equipment will exhibit a unique "signature" of wearmetal buildup that can also be exploited to improve the decision process. In an earlier technical report [2] a regression based approach to monitoring the wear-metal buildup rate for a given piece of equipment was discussed. We now describe a modified version of the procedure that is capable of

- a) recognizing the trace or signature of metallic contamination which is normal for a piece of equipment in good operating condition and

- b) is capable of detecting changes in the signature that may indicate an abnormal condition. The basic assumption behind the methodology is that an engine in proper working order accumulates metallic contaminants in its lubricating fluid in some well behaved pattern and any major deviation from the pattern could be attributable to an abnormal condition that should be corrected.

Let K be the number of wearmetals to be monitored by the procedure. While the equipment is in good working order the lubricating fluid is sampled at prescribed intervals (such as after each flight of an aircraft) and analyzed on a spectrometer. Whenever an oil sample is taken the number of accumulated hours since the last oil change is recorded. The initial data for the "moving regression methodology" consists of N observations taken at times $t_1 \leq t_2 \leq \dots \leq t_N$ hours since the previous oil change. The observations are the spectrometer readings Y_{ij} , $i = 1, 2, \dots, N$; $j = 1, 2, \dots, K$ on the K wearmetals. We shall assume that

$$Y_j(t_i) = \alpha_j + \beta_j \cdot t_i + e_{ij} ;$$

that is, each of the K elements may change linearly in concentration with flight times; β_j represents the rate of change in concentration for element j and α_j represents the initial concentration of element j in the fluid. The e_{ij} are measurement errors, caused by inhomogeneities of the fluid and the inherent randomness of the spectrometer readings etc. It is also assumed that, for each element the e_{ij} 's are independent, normal, each with variance σ_j^2 .

Standard linear regression methodology can be used to estimate the parameters α_j , β_j and σ_j^2 ; these estimates are denoted by a_j , b_j and s_j^2 respectively. Rather than examining the regression results on the wearmetals individually we propose to find a single linear combination $Y(t) = \sum c_j Y_j(t)$ which is most informative, in a sense, regarding rates of accumulation of the contaminants and regarding changes in rates.

For any set of constant weights, $c_1, c_2, \dots, c_K$ our assumed model implies

$$\begin{aligned} Y(t_i) &= \sum c_j Y_j(t_i) \\ &= \sum c_j \alpha_j + t_i \sum c_j \beta_j + c_j e_{ij} . \end{aligned}$$

That is, we would expect $Y(t)$ to also be a linear function of the time of observation t . If a new oil sample is received taken T hours after oil change, the value we would expect for $Y(T)$, based on the initial data set is $\hat{Y}(T) = \sum c_j a_j + T \sum c_j b_j$. The estimated signal-to-noise ratio, frequently employed in many technical areas, is, for a sample taken T hours after oil change,

$$\frac{\hat{Y}(t)}{\sqrt{\text{Var } \hat{Y}(t)}} = \frac{\sum c_j a_j + T \sum c_j b_j}{\sqrt{\text{Var } \hat{Y}_j(t)}}$$

choosing the c_j 's to maximize this quantity gives the "most sensitive" linear combination $Y(T)$ in some sense. It was shown in the NPS Technical Report [2] cited earlier that the optimal coefficients are

$$c_j = \frac{a_j + T b_j}{\sqrt{\text{Var } \hat{Y}_j(T)}}$$

where $\text{Var } Y_j(T) = S_j^2 \left[\frac{N+1}{N} + \frac{(T - \bar{t})^2}{\sum (t_i - \bar{t})^2} \right]$

To keep the range of the $Y(T)$ values on roughly the same scale as the original readings, it is convenient to normalize the weights c_j so that they add to one; this is done by letting the modified coefficients

$$C_j = \frac{c_j}{\sum c_j}$$

Granted the weights, $C_1, C_2, \dots, C_k$ have been determined, they can be used to determine upper bounds which should not be exceeded (with a specified probability) for the composite reading taken T hours after oil change. We propose the use of a student's - t approximation to the distribution of $Y(T) = \sum C_j Y_j(T)$ to determine the bounds; the resulting bounds will be of the form

$$\hat{Y}(T) + t_{\gamma} \sqrt{\sum C_j^2 \text{Var } Y_j(t)}$$

where t_{γ} is the upper γ^{th} percentile of a student's t -distribution with $N-2$ degrees of freedom.

For any given serial number, known to be in good working order, oil samples are analyzed at specified time intervals (such as after each flight of an aircraft) giving N sets of contaminant readings. The regression estimates a_j, b_j, s_j^2 are computed as are the weights $C_1, C_2, \dots, C_k$. For a new sample taken T hours since oil change, confidence bounds $Y_{.9}(T), Y_{.95}(T)$ and $Y_{.99}(T)$ are computed. These bounds are the values which would be exceeded only 10 percent, 5 percent, and 1 percent of the time respectively if the equipment is in good order and may be used for making decisions as to whether the new sample is to be considered unusual or not. For example, a 'K' code may be assigned if $Y(T)$ is between the 95 and 99 percent bounds, a 'T' code is assigned if $Y(T)$ exceeds the 99 percent bound, increase the sampling frequency if $Y(T)$ is between the 90 and 95 percent bounds and continue normal sampling if $Y(T)$ is below the 90 percent bound. If a decision is made not to recommend a special maintenance action and to continue sampling the initial data base is updated pending the arrival of the

next sample. The update is performed by deleting the oldest observation corresponding to the sampling time t_1 and adding the most recent observation obtained at time T to the data base. Once a new sample is analyzed the regression computations are reinitiated to determine if this sample is to be treated as unusual.

If the decision is to take a special maintenance action such as an overhaul then we recommend that a completely new data base consisting of N observations be created before initiating the regression methodology.

It can happen that even when a given piece of equipment appears to be in good working order, the oil gets changed as part of routine maintenance. If this happens, the contaminant readings in samples taken prior to the oil change and subsequent to the oil change tend to be quite different, the latter ones being much smaller. To achieve compatibility between the two observations it is proposed that the post oil change readings and the observation times be added to the last observation prior to oil change and the corresponding observation time respectively before performing the regression analysis. This special handling is to be repeated with each of the first $N-1$ observations subsequent to the oil change. At this juncture a new set of N observations all taken subsequent to the oil change will be available as initial data and normal processing can be resumed. This special handling of the data after an oil change can add a systematic bias if the new oil already has nonzero levels of the contaminants. Ideally, it would be preferable to measure these new oil contaminant levels and subtract them from the new observations before initiating the special treatment.

Details of the computational steps involved in applying the moving regression methodology are presented in Section 2 and Section 3 contains an example illustrating the computations and the possible decision actions.

If this methodology were to be implemented operationally it would be necessary for the SOAP laboratory personnel to have access to some type of micro-computer. Then, a computer program can be written and stored which can be invoked whenever the methodology is to be applied. In fact, the computer calculations can be simplified considerably by using vector and matrix notation thereby handling all the wearmetals simultaneously. Once an appropriate computer program has been developed, all that the laboratory personnel need to do is input each new observation and invoke the program.

The methodology was applied to several sets of unit failure history data (December 1979) for various aircraft and in all but one case the moving regression put out an "alert" at the same time or earlier as the JOAP laboratory recommended teardown (Tables 13-21).

4.2. Computational Procedures for the Moving Regression Methodology

The moving regression methodology provides a means to track the "signature" exhibited by a sequence of observations (spectrometric results) of a piece of equipment in the SOAP program. The observed signature is then used to determine if a new observation is to be considered "extreme" and warranting possibly some maintenance action. The notation and the computational formulae are described below.

Notation

K is the number of wearmetals under scrutiny. For example, if the wearmetals of interest are iron, silver, aluminum, chromium, copper, and magnesium, then $K = 6$.

N is the number of observations to be used in the moving regression.

i is the subscript used to index the observations or samples. If $N = 10$ observations are to be used in the moving regression then i takes the values 1, 2, ..., 10.

j is the subscript used to index the K wearmetals. If $K = 6$, then j takes the values 1, 2, ..., 6.

The initial data consists of K sets (one for each wearmetal) of N observations $Y_1, Y_2, \dots, Y_N$. To avoid cumbersome notation the wearmetal subscript j has been omitted temporarily. After some of the initial computations are completed the wearmetal subscript j will be introduced. The initial computations described below will be exactly the same for each of the K wearmetals, using the appropriate wearmetal data set.

The above observations are obtained at times $t_1, t_2, \dots, t_N$ where the times t_i are measured in "hours since previous oil change". It is assumed that these times form a non-decreasing sequence, i.e., $t_1 \leq t_2 \leq \dots \leq t_N$. Note that these times t_i will be the same for each of the K wearmetals.

Initial Computations

For each of the K wearmetals compute the following:

$$(1) \quad \sum_{i=1}^N t_i; \quad \sum_{i=1}^N t_i^2; \quad \sum_{i=1}^N Y_i; \quad \sum_{i=1}^N Y_i^2; \quad \sum_{i=1}^N t_i Y_i$$

$$(2) \quad \bar{t} = \frac{1}{N} \sum_{i=1}^N t_i; \quad \bar{Y} = \frac{1}{N} \sum_{i=1}^N Y_i$$

$$S_{11} = N \sum_{i=1}^N t_i^2 - \left(\sum_{i=1}^N t_i \right)^2$$

$$S_{22} = N \sum_{i=1}^N Y_i^2 - \left(\sum_{i=1}^N Y_i \right)^2$$

$$S_{12} = N \sum_{i=1}^N t_i Y_i - \left(\sum_{i=1}^N t_i \right) \left(\sum_{i=1}^N Y_i \right)$$

(3) Regression coefficients and variance estimates:

$$\text{Slope parameter } b = S_{12}/S_{11}$$

$$\text{Intercept parameter } a = \bar{Y} - b\bar{X}$$

$$\text{Variance estimate } S_e^2 = \frac{S_{11} \times S_{22} - S_{12}^2}{N(N-2) S_{11}}$$

The wearmetal subscript j will be introduced at this stage. At the end of the above calculations there will be available K sets of regression estimates (a_j, b_j, S_{ej}^2) $j = 1, 2, \dots, K$.

Main Computations

A new observation or a sample taken T hours since oil change is available and it is to be decided whether this observation is to be considered "extreme" or "abnormal". The new observation will consist of the spectrometer readings on each of the K wearmetals $Y_{T1}, Y_{T2}, \dots, Y_{TK}$. Instead of examining the individual wearmetal readings separately the methodology determines an "optimal linear combination" over the wearmetals to be used as a decision function. The necessary computations are listed below.

(4) Check if $T \geq t_N$ the time of the immediately preceding observation. This should be true unless an oil change occurs subsequent to the previous observation. If $T < t_N$ proceed to step (5); otherwise go to step (6).

(5) Define:

$$T' = T + t_n \quad \text{and}$$

$$Y'_{Tj} = Y_{Tj} + Y_{Nj} \quad j = 1, 2, \dots, K$$

Proceed to the next step (6) and perform the calculations replacing T with T' and Y_{Tj} with Y'_{Tj} . This step artificially creates continuity in the data by assuming that had not the oil change occurred the new observation time would have been T' and the new observation Y'_{Tj} , $j = 1, 2, \dots, K$.

Each time an oil change occurs this step is to be repeated with the first N observations at the end of which a new set of initial data all obtained after the oil change will be available for normal processing.

(6) Compute:

$$g_j = a_j + Tb_j$$

$$h_j = s_{ej}^2 \left[\frac{N+1}{N} + \frac{N(T - \bar{t})^2}{s_{11}} \right]$$

$$c_j = g_j/h_j$$

and
$$C_j = c_j / \sum_{j=1}^K c_j$$

for $j = 1, 2, \dots, K$.

The C_j are the coefficients in the optimal linear combination.

(7) Compute the observed value of the optimal linear combination (OLC)

$$Y_T = \sum_{j=1}^K C_j Y_{Tj} ,$$

the expected value of the OLC

$$\hat{Y}_T = \sum_{j=1}^K C_j (a_j + Tb_j) ,$$

the variance of $\hat{Y}(T)$

$$s_T^2 = \sum_{j=1}^K C_j^2 h_j$$

and the upper confidence bounds

$$\hat{Y}_T + t_{.90} S_T$$

$$\hat{Y}_T + t_{.95} S_T$$

$$\hat{Y}_T + t_{.99} S_T$$

where $t_{.90}$, $t_{.95}$ and $t_{.99}$ are the upper 90th, 95th and 99th percentile of a student's t distribution with $N - 2$ degrees of freedom; these values can either be looked up from appropriate tables or computed directly using an appropriate program.

(8) Compare the observed value of the OLC, Y_T with the upper confidence bounds to make decisions on the appropriate laboratory recommendation. As an example, if Y_T is below the 90 percent bound assign an A code to continue routine sampling, increase the sampling frequency for closer monitoring of the equipment if Y_T is between the 90 percent and 95 percent bounds; if Y_T is between the 95 percent and 99 percent bounds, assign a 'K' code calling for a retagged sample and if Y_T exceeds the 99 percent bound assign a 'T' code recommending immediate grounding for appropriate maintenance action.

(9) Update the initial data for the moving regression as follows:

If the current observation did not lead to a special maintenance action add the current data to the initial data base deleting the oldest records. Thus, the sampling times which were $t_1, t_2, \dots, t_N$ would become $t_2, t_3, \dots, t_N, T$. Each wearmetal concentration is treated the same way; for any given wearmetal (suppressing the j subscript) the records which were $Y_1, Y_2, \dots, Y_N$ become $Y_2, Y_3, \dots, Y_N, Y_{N+1}$ where Y_{N+1} is the latest observed spectrometer reading for the wearmetal.

If the current observation is sufficiently unusual to warrant a special maintenance action, as stated in Section 1, accumulate a new data base consisting of N observations all taken subsequent to the maintenance action before invoking the moving regression.

4.3. An Example

An example illustrating the computational steps is presented below. The example starts with an initial data set and each time a new observation is made available the moving regression computations are initiated. The data for the example (Table 1) uses the unit failure history of a T-38B aircraft (serial number 400287) for the month of December 1979. The results of each of the computational steps will be shown for the first iteration only. After that only the results of the most significant steps will be presented. Six wearmetals viz., iron, silver, aluminum, chromium, copper, and magnesium will be tracked in the example. It should be pointed out that in this example all the aluminum readings are identically zero. However, we did include this wearmetal since it is one of the more commonly occurring wearmetal in the SOAP program and also to illustrate that its inclusion has no effect on the analysis.

Number of samples in the moving regression $N = 5$

Number of wearmetals $K = 6$

INITIAL DATA						
Time since oil change	Spectrometer Readings					
t_i	Fe Y_{i1}	Ag Y_{i2}	Al Y_{i3}	Cr Y_{i4}	Cu Y_{i5}	Mg Y_{i6}
86	6	1	0	1	2	2
106	5	0	0	1	2	1
128	7	1	0	2	3	2
205	9	0	0	2	6	2
223	10	0	0	3	6	3

Unit Failure History for December 1979

Aircraft Type: T-37B

Serial Number: 400287

Time Since Oil Change	Fe	Ag	Al	Cr	Cu	Mg
86	6	1	0	1	2	2
106	5	0	0	1	2	1
128	7	1	0	2	3	2
205	9	0	0	2	6	2
* 223	10	0	0	3	6	3
244	9	1	0	2	7	3
264	11	1	0	4	9	4
** 1	4	0	0	1	2	1
24	13	0	0	1	2	1
44	23	0	0	2	3	3
64	27	0	0	2	3	2
86	32	0	0	2	3	2
105	37	0	0	2	4	2
*** 108	40	0	0	3	4	2

* The first five rows are used as the initial data base in the example

** It appears that an oil change occurred just prior to this observation

*** The unit failure history indicates a JOAP recommended teardown and the auxillary drive FT inlet was found to be worn out.

Table 12

Computational Results

			<u>Fe</u>	<u>Ag</u>	<u>Al</u>	<u>Cr</u>	<u>Cu</u>	<u>Mg</u>
Step 1	$\sum t_i$	=	748	748	748	748	748	748
	$\sum t_i^2$	=	126,770	126,770	126,770	126,770	126,770	126,770
	$\sum Y_i$	=	37	2	0	9	19	10
	$\sum Y_i^2$	=	291	2	0	19	89	22
	$\sum t_i Y_i$	=	6017	214	0	1527	3336	1613
Step 2	$\bar{t}$	=	149.6	149.6	149.6	149.6	149.6	149.6
	$\bar{Y}$	=	7.40	.40	0	1.8	3.8	2.0
	S_{11}	=	74346	74346	74346	74346	74346	74346
	S_{22}	=	86	6	0	14	84	10
	S_{12}	=	2409	426	0	903	2468	585
Step 3	A_j	=	2.55	1.26	0	-0.02	-1.17	0.82
	b_j	=	0.032	-0.006	0	0.012	0.033	0.088
	S_{ej}^2	=	0.775	0.398	0	0.357	0.077	0.202

A new observation taken at $T = 244$ hours since oil change is available and the data is:

Y_{Tj}	=	9	1	0	2	7	3
----------	---	---	---	---	---	---	---

Fe Ag Al Cr Cu Mg

Step 4 Since $T = 244 > t_5 = 223$ we proceed to step 6

Step 6 g_j = 10.459 -0.141 0 2.947 6.934 2.743

h_j = 0.953 0.427 0 0.364 0.249 0.647

c_j = 10.97 -0.33 0 8.10 27.85 4.24

C_j = 0.216 -0.006 0 0.159 0.548 0.083

Step 7 Observed value of the optional linear combination (OLC) is 6.341

Expected value of OLC = 6.756

The 90, 95, and 99 percent confidence bounds are 7.353, 7.614, 8.405

Step 8 The observed value of OLC does not exceed any of the three bounds; no maintenance recommendation is made.

Step 9 Update the initial data base to

<u>t_i</u>	<u>Y_{i1}</u>	<u>Y_{i2}</u>	<u>Y_{i3}</u>	<u>Y_{i4}</u>	<u>Y_{i5}</u>	<u>Y_{i6}</u>
106	5	0	0	1	2	1
128	7	1	0	2	3	2
205	9	0	0	2	6	2
223	10	0	0	3	6	3
244	9	1	0	2	7	3

Note that the first row of the original initial data base has been deleted and the last row of the new data base is the latest observation.

The next observation is

			<u>Fe</u>	<u>Ag</u>	<u>Al</u>	<u>Cr</u>	<u>Cu</u>	<u>Mg</u>
T = 264	Y_{Tj}	=	11	1	0	4	9	4
	a_j	=	2.481	0.282	0	0.562	-1.631	-0.010
	b_j	=	0.030	0.001	0	0.008	0.035	0.012
	c_j	=	0.098	0.008	0	0.054	0.724	0.115

Observed value of OLC = 8.286

Expected value of OLC = 7.517

Confidence bounds are 7.639, 7.849, 8.487

The observed OLC exceeds the 90 and 95 percent bounds but not the 99 percent bounds; assign a 'K' code calling for a retagged sample (the data for this example has been taken from the December 1979 unit failure history records; hence no retagged sample would have been submitted.) Since no special maintenance action has been recommended the new initial data would consist of

<u>t_i</u>	<u>Y_{i1}</u>	<u>Y_{i2}</u>	<u>Y_{i3}</u>	<u>Y_{i4}</u>	<u>Y_{i5}</u>	<u>Y_{i6}</u>
128	7	1	0	2	3	2
205	9	0	0	2	6	2
223	10	0	0	3	6	3
244	9	1	0	2	7	3
264	11	1	0	4	9	4

The next observation and the associated computations are as follows:

$$T = 1 \quad Y_{Tj} = \quad \quad \quad 4 \quad 0 \quad 0 \quad 1 \quad 2 \quad 1$$

$T = 1$ implies that an oil change occurred subsequent to the previous observation. Hence, we proceed to step 5 and let

$$\begin{aligned} T' &= 264 + 1 & Y'_{Tj} &= & 11+4 & 1+0 & 0+0 & 4+1 & 9+2 & 4+1 \\ &= 265 & &= & 15 & 1 & 0 & 5 & 11 & 5 \end{aligned}$$

The application of the moving regression methodology results in

$$C_j = \quad \quad \quad 0.349 \quad 0.025 \quad \quad 0 \quad 0.078 \quad 0.354 \quad 0.194$$

Observed OLC = 10.521

Expected OLC = 7.557

Confidence bounds 8.261 8.569 9.503

The observed OLC exceeds all three bounds; the appropriate recommendation would be to ground the equipment for appropriate special maintenance action. In the actual unit failure history data used in the example, routine sampling was continued. We will therefore assume that a judgemental decision not to take special maintenance action has been made. The data base will be updated by adding the data T' and Y'_{Tj} and deleting the data corresponding to the sampling time $t = 128$.

The next 4 observations are:

$T = 24$	13	0	0	1	2	1
$T = 44$	23	0	0	2	3	3
$T = 64$	27	0	0	2	3	2
$T = 86$	32	0	0	2	3	2

Proceeding through step 5 the regression results will be:

<u>T</u>	<u>Observed OLC</u>	<u>Confidence Bounds</u>		
24	5.257	5.735	6.036	6.950
44	7.251	7.860	8.250	9.434
64	10.727	13.825	14.370	16.022
86	12.221	14.948	15.439	16.929

In each one of these cases the observed value of the OLC does not exceed any of the confidence bounds. At this stage, five observations all obtained subsequent to the oil change are available and (until another oil change occurs) step 5 should be bypassed in future computations, with the new initial data base as:

<u>t_i</u>	<u>Y_{i1}</u>	<u>Y_{i2}</u>	<u>Y_{i3}</u>	<u>Y_{i4}</u>	<u>Y_{i5}</u>	<u>Y_{i6}</u>
1	4	0	0	1	2	1
24	13	0	0	1	2	1
44	23	0	0	2	3	3
64	27	0	0	2	3	2
86	32	0	0	2	3	2

The next two observations are:

T = 105	Y _{Tj} =	37	0	0	2	4	2
T = 108	Y _{Tj} =	40	0	0	3	4	2

and the regression results are

<u>T</u>	<u>Observed OLC</u>	<u>Confidence Bounds</u>		
105	6.821	7.874	8.203	9.202
108	10.156	10.460	10.836	11.974

The unit failure history indicates that at this point the SOAP laboratory recommended a teardown and it was observed that the auxillary drive FT inlet was worn out resulting in a "JOAP hit". Although the regression methodology did not identify the final sample as unusual the method did put out an alert earlier at $T = 1$.

SERIAL NUMBER: 232510

AIRCRAFT TYPE: T-38A

HOURS SINCE OIL CHANGE	Fe	Ag	Al	Cr	Cu	Mg	OBSERVED OLC	90%		95%		99%		SIGNIFICANT AT
								CONF	BOUND	CONF	BOUND	CONF	BOUND	90% * 95% ** 99% ***
255	26	0	0	1	0	4								
275	31	0	0	2	1	5								
296	27	0	0	1	1	4								
302	33	0	0	2	1	5								
325	28	0	0	1	1	4								
345	33	0	0	2	1	5	5.68	6.35		6.81		8.18		
366	30	0	0	2	1	4	8.07	10.27		11.02		13.28		
386	35	0	0	1	1	5	8.20	9.17		9.81		11.75		
400	36	1	0	1	1	6	10.82	11.10		11.80		13.93		
416	39	0	0	2	1	6	14.12	15.10		15.77		17.82		
436	40	0	0	2	1	6	15.01	16.85		17.61		19.93		
454	42	0	0	3	1	6	23.29	25.82		26.46		28.40		
459	47	0	0	3	1	6	31.45	29.56		29.92		31.01		** *
✓465	51	0	0	3	1	7	14.80	14.75		15.22		16.66		*

✓JOAP HIT: Number two Bearing

Table 13

AIRCRAFT TYPE: T-38A

SERIAL NUMBER: 230547

HOURS SINCE OIL CHANGE	Fe	Ag	Al	Cr	Cu	Mg	OBSERVED OLC	90%		95%		99%		SIGNIFICANT AT
								CONF	BOUND	CONF	BOUND	CONF	BOUND	90% * 95% ** 99% ***
261	22	0	0	1	1	3								
262	23	0	0	1	1	3								
262	22	0	0	1	1	3								
262	24	0	0	1	1	5								
267	29	0	0	1	1	4								
270	24	0	0	1	1	4	22.27	32.40	33.40	36.46				
272	25	0	0	2	1	4	13.27	17.43	18.82	23.03				
274	23	0	0	1	1	3	5.17	7.61	8.15	9.80				
277	22	0	0	1	1	3	4.60	5.68	6.06	7.20				
282	25	0	0	1	1	3	8.97	7.96	8.50	10.13				*
✓289	34	0	0	2	1	3	14.95	12.35	13.17	15.66				*

✓ JOAP HIT: Aux Dr GRBX Transer Bearing

Table 14

AIRCRAFT TYPE: T-38A

SERIAL NUMBER: 231270

HOURS SINCE OIL CHANGE	Fe	Ag	Al	Cr	Cu	Mg	OBSERVED OLC	90%		95%		99%		SIGNIFICANT AT
								CONF	BOUND	CONF	BOUND	CONF	BOUND	90% *
351	40	0	0	2	3	21								95% **
357	39	0	0	2	2	20								99% ***
363	40	0	0	2	2	20								
366	42	0	0	2	2	19								
372	42	0	0	2	3	19								
377	42	0	0	3	2	21	25.19	24.18	24.53	25.62				**
383	48	0	0	3	2	21	30.38	29.04	29.55	31.13				**
388	48	0	0	3	2	21	15.96	17.23	17.68	19.05				
394	55	0	0	3	3	19	17.60	19.46	19.89	21.19				
✓397	56	0	0	3	3	19	20.07	21.56	22.14	23.90				

✓ JOAP MISS - No Defects

Table 15

SERIAL NUMBER: 430370

AIRCRAFT TYPE: F-4E

HOURS SINCE OIL CHANGE	Fe	Ag	Al	Cr	Cu	Mg	OBSERVED OLC	90%		95%		99%		SIGNIFICANT AT
								CONF	BOUND	CONF	BOUND	CONF	BOUND	90% * 95% ** 99% ***
312	5	0	0	1	8	3								
313	5	0	0	1	7	3								
324	7	1	0	2	11	5								
330	8	1	0	2	14	5								
331	7	1	0	2	11	5								
334	7	1	0	2	11	5	3.65	4.54	4.68	5.11				
335	8	1	0	2	11	5	3.63	4.26	4.42	4.90				
339	7	1	0	2	11	5	7.71	9.72	10.30	12.05				
343	7	1	0	2	13	7	8.44	9.06	9.72	11.71				
345	8	1	0	2	11	6	8.81	10.17	10.53	11.62				
346	20	1	0	2	13	6	13.52	9.38	9.76	10.94				**
✓348	30	2	0	3	14	8	11.14	10.82	11.38	13.09				*

✓ JOAP HIT - Aux Const Speed Dr Bearing

Table 16

SERIAL NUMBER: 439742

AIRCRAFT TYPE: F-4D

HOURS SINCE OIL CHANGE	Fe	Ag	Al	Cr	Cu	Mg	OBSERVED OLC	90%		95%		99%		SIGNIFICANT AT
								CONF	BOUND	CONF	BOUND	CONF	BOUND	90% * 95% ** 99% ***
93	11	0	0	5	5	5								
98	10	0	0	5	5	5								
100	11	0	0	5	5	5								
105	9	0	0	5	4	4								
110	9	0	0	5	4	4								
111	9	0	0	5	5	4	5.43	5.35	5.56	5.35	5.56	6.16		*
112	9	0	0	5	5	4	5.34	5.52	5.73	5.52	5.73	6.38		
113	7	0	0	4	4	2	4.39	5.90	6.12	5.90	6.12	6.77		
119	9	0	0	5	4	4	5.10	5.39	5.74	5.39	5.74	6.81		
123	27	0	1	5	5	5	8.44	5.81	6.20	5.81	6.20	7.40		**
124	34	0	1	6	6	5	5.37	5.09	5.37 ⁺	5.09	5.37 ⁺	6.22		*
✓124	35	0	1	6	6	5	5.09	5.32	5.57	5.32	5.57	6.35		

✓JOAP HIT - Aux Dr Acc Oil Pump

Table 17

AIRCRAFT TYPE: C-130E

SERIAL NUMBER: 103975

HOURS SINCE OIL CHANGE	Fe	Ag	Al	Cr	Cu	Mg	OBSERVED OLC	90%		95%		99%		SIGNIFICANT AT
								CONF	BOUND	CONF	BOUND	CONF	BOUND	90% * 95% ** 99% ***
41	7	0	0	0	5	19								
118	8	0	0	0	5	24								
206	13	0	0	1	8	28								
290	12	0	1	1	10	29								
366	13	0	2	0	11	32								
444	19	0	0	0	17	44	23.19	19.92	20.49	22.21	22.21	22.21	22.21	* *
479	21	0	5	0	17	49	22.51	22.48	23.53	26.70	26.70	26.70	26.70	*
✓479	23	1	6	1	19	49	27.22	27.14	28.32	31.90	31.90	31.90	31.90	*

✓ JOAP MISS - No Defects

Table 19

AIRCRAFT TYPE: B-52G

SERIAL NUMBER: 627986

HOURS SINCE OIL CHANGE	Fe	Ag	Al	Cr	Cu	Mg	OBSERVED OLC	90%		95%		90%		Significant AT	
								CONF	BOUND	CONF	BOUND	CONF	BOUND	90%	95%
1632	6	1	0	0	2	4								*	
1669	7	1	0	1	3	8								**	
1700	10	1	0	1	4	8								***	
1763	6	0	0	0	3	4									
1800	8	1	0	1	3	3									
1833	9	0	0	0	3	30	4.22	4.43	4.94	4.43	4.94	4.43	4.94	6.48	
1833	8	0	0	1	3	27	3.64	4.44	4.83	4.44	4.83	4.44	4.83	6.03	
✓ 14	0	0	0	0	0	5	3.38	3.66	3.89	3.66	3.89	3.66	3.89	4.60	
21	2	0	0	0	0	5	9.37	9.77	10.33	9.77	10.33	9.77	10.33	12.04	
30	3	0	0	0	1	10	20.91	22.88	24.19	22.88	24.19	22.88	24.19	28.16	
102	8	1	0	1	3	29	17.73	19.68	20.77	19.68	20.77	19.68	20.77	24.08	
✓ 105	8	1	0	1	3	30	13.50	14.06	14.29	14.06	14.29	14.06	14.29	14.97	

✓ Oil Change

✓ JOAP HIT - Aux Dr GRBX Ft Inlet
Table 20

AIRCRAFT TYPE: B-52G

SERIAL NUMBER: 627986

HOURS SINCE OIL CHANGE	Fe	Ag	Al	Cr	Cu	Mg	OBSERVED OLC	90%		95%		99%		SIGNIFICANT AT		
								CONF	BOUND	CONF	BOUND	CONF	BOUND	90%	95%	99%
642	29	1	0	3	9	5										
645	32	1	0	3	8	5										
663	32	1	0	3	8	5										
681	37	0	0	3	8	5										
682	35	0	0	3	7	4										
683	35	0	0	3	6	3	11.52	13.75	14.13	15.30						
687	44	1	0	3	7	4	28.26	25.24	25.95	28.13				*	*	*
687	42	0	0	3	7	4	11.07	11.88	12.47	14.28						
690	43	0	0	3	7	4	17.04	19.98	20.82	23.36						
✓ 1	20	0	0	1	3	1	16.38	13.01	13.42	14.68				*	*	*
1	21	0	0	1	3	2	6.65	6.05	6.29	7.05				*	*	*
✓/3	20	0	0	1	4	2	7.15	8.20	8.60	9.80						

✓ Oil Change

✓/ JOAP HIT - Const Speed Drive

Table 21

REFERENCES

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