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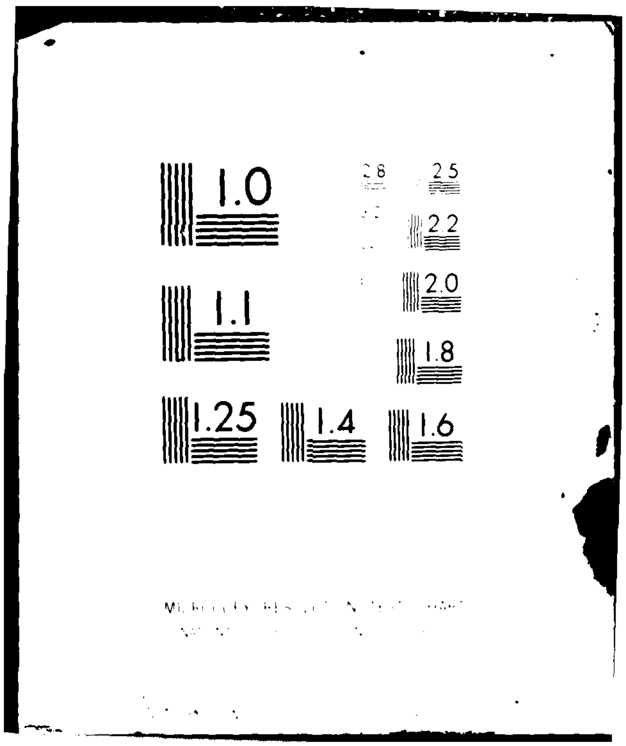
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THE PENNSYLVANIA STATE UNIVERSITY

THE EQUIPMENT ALLOCATION PROBLEM WITH MIXED FLEET

by

SRIKANTH RAO
DEPARTMENT OF MANAGEMENT

OCTOBER 1972

PENNSYLVANIA TRANSPORTATION AND TRAFFIC SAFETY CENTER

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In trying to allocate commodity flow to transport equipment units with a mixed fleet, a set of linear programming transportation models were derived. Included were notes on cooperation, reaches (mutually exclusive subsets), one-way flow with sections on general descriptions of models, input data, mathematical formulation, and model applications. The basic waterway application was the Illinois-Mississippi system.

The Pennsylvania State University

The Graduate School

College of Business Administration

The Equipment Allocation Problem With Mixed Fleet

A Paper in

Business Administration

by

Srikanth Rao

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of the Requirements
for the Degree of

Master of Science

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ABSTRACT

The problem of allocating commodity flows to transport equipment units in a mixed fleet is treated by a set of linear programming transportation models. The models allow for the allocation of commodity flows in a system which may be regarded as one whole entity such as a firm, or as a set of reaches differentiated by equipment characteristics, or as an environment of competing firms, or any combination of these The models were shown to generate feasible, optimal and integer solutions in their application to a hypothetical two commodity example.

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FOREWORD

This paper represents the extension of a previous investigation recorded in Working Paper No. 2: "Allocation of Open Hopper Loads to Empty Covered Hopper Returns" at the Pennsylvania Transportation and Traffic Safety Center. During his labor, the author was supported by the U. S. Army Corps of Engineers, North Central Division, under contract number DACW23-72-C-0009.

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The author's interest in this investigation is solely due to the encouragement supplied by Joseph L. Carroll without whom this work would not have been. The author also gratefully acknowledges Michael S. Bronzini for his constructive criticisms. Ned Shilling and John Dinkel served as committee members and offered valuable evaluations.

Regardless of the assistance of many, the responsibility for any shortcomings, omissions, opinions or conclusions are solely those of the author.

I. INTRODUCTION

The purpose of this study is to investigate the problem of allocating commodity flows to transport equipment units from a mixed fleet. Rather than encroaching upon the realm of scheduling and routing however, the consideration in this paper is geared more towards planning purposes, thus enabling the use of static models. That is, the assumption inherent in the solution of this allocation problem is that the relevant supplies and demands for some fixed time period are known or can be estimated, thus removing the dynamic nature of real time from the problem. Bearing this assumption in mind, a set of linear programming transportation models are presented with initial application being made on the inland waterway system.

The problem traces its origin to the developmental efforts of a computer simulation model at the Pennsylvania Transportation and Traffic Safety Center designed to study the inland waterway transportation systems. Operationally, this simulation model is divided into two parts. The first section is a tow generation program (TOWGEN)[1], which produces a time ordered list of tow departures throughout the system. This list is then processed by a waterway simulation program (WATSIM)[2].

The usefulness of this paper is not limited to the simulation model or the waterway systems in general but in fact the models presented here are applicable to any mode of transportation. Since the initial

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[1] Waterway Simulation Series. Vol. III; TOWGEN: A Tow Generation Model by Michael S. Bronzini.

[2] Waterway Simulation Series, Vol. II; WATSIM: A Waterway Transport Simulator by John H. Gimbel, III.

application of these models is made in the waterway system, however, a brief description of TOWGEN is provided in the next section. The remainder of this paper is devoted to problem definition, a general description of the models, the mathematical formulation, and model applications.

II. TOWGEN

The purpose of TOWGEN is to convert the input commodity O-D (origin to destination) tonnage matrices into a set of O-D movements of tows having known characteristics. In doing so, TOWGEN must exhaust the O-D tonnage matrix for each commodity while simultaneously satisfying the balance principle. This balance principle is stated as follows:

> "The numbers of towboats and barges of each type which arrive at and depart from each port in the system should be equal in the long run."

This is simply a steady state requirement, and says that, for each type of equipment in use on the waterway, input must equal output at every point.

Figure 1 presents a generalized logic diagram of TOWGEN. The left hand side of the figure depicts the various computational operations which occur within the model, while the right hand side shows the source and disposition of the data files which TOWGEN processes. The dashed lines connecting the two sides of the figure represent information flow

The input data required by TOWGEN consist primarily of the following:

(1) O-D tonnage matrix for each commodity;

(2) table of barge data, showing commodities carried, average loading, and dedicated equipment percentage for each type of barge in use on the waterway;

(3) O-D mileage table and other system description data;

(4) frequency distributions of flotilla size (number of barges) vs. towboat horsepower for each commodity or commodity group.

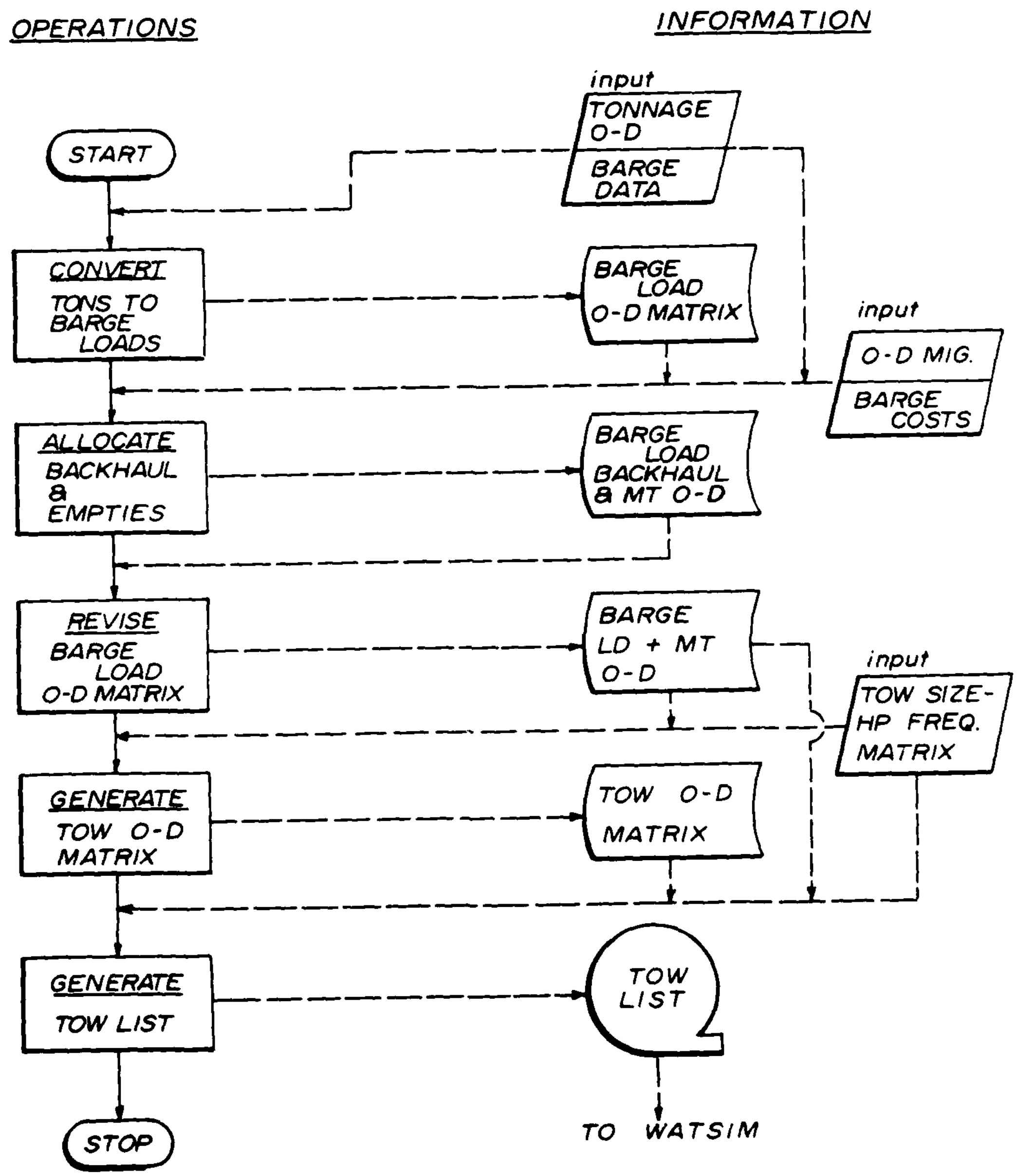

Figure 1 TOWGEN Logical Flow Diagram

The O-D tonnage matrix, which gives the number of tons of each commodity to be shipped from each port to every other port, is perhaps the most important input, and is the prime determinant of the number of tows which will be generated for processing in WATSIM.

TOWGEN processes these data in the following manner: the Commodity O-D matrix is divided through by average barge loadings to produce a matrix of loaded barge O-D movements. Next, the movements of empty barges required to support the loaded barge activity are determined This is done by (1) providing empty movements of dedicated equipment and (2) eliminating any remaining imbalances between barges originating and terminating at each port by providing empty barge movements which minimize total empty barge-miles of travel. The total barge O-D matrix (loaded plus empty barges) is then divided through by average flotilla size to determine the O-D tow movements required. This tow O-D matrix is randomly sampled without replacement, thus determining the origin and destination ports of each tow on the tow list and the sequence in which these tows will be introduced into the system. Additional parameters derived for each tow, via appropriate random sampling procedures, include the following:

(1) time of departure from the origin port, determined by modeling system wide tow departures as a Poisson process;

(2) towboat horsepower and flotilla size;

(3) number of loaded and empty barges of each type;

(4) net tonnage.

In summary, it should be noted that TOWGEN itself is not a simulation model, but rather is one part of a simulation package. Starting with the basic elements of commodity transportation demand and transport fleet supply, TOWGEN uses an array of analytical and Monte Carlo techniques to determine the waterway transport demand in terms of discreet traffic units (i.e., barge flotillas).

III. PROBLEM DEFINITION

The problem considered in this paper begins with the second operation performed in TOWGEN, namely the computation of empty barge movements required to support the loaded barge activity and to satisfy the balance principle. TOWGEN first considers dedicated equipment movements. Any remaining imbalances are then treated as a linear programming transportation problem.

1. Dedicated Equipment

Barges are considered to be dedicated if they move loaded from an origin port to a destination port, and are then returned empty to the origin port. For each barge type, the percentage of loaded movements utilizing dedicated equipment is specified, and TOWGEN calculates the corresponding empty barge movements as,

$$E_{jib} = L_{ijb}\, D_b/100 \qquad i,j = 1, \ldots, k \qquad b = 1, \ldots, B$$

where

L_{ijb} = number of loaded type b barges moving from port i to port j;

E_{jib} = number of empty type b barges moving from port j to port i;

D_b = dedicated equipment percentage for barge type b;

k = total number of ports in the system.

If all D_b were equal to 100, there would be no barge input-output imbalances at any port in the system. In all other circumstances, however, there will normally be further balance requirements to be met.

2. The Balance Problem

To determine what additional empty barge flows are needed, TOWGEN computes the total number of barges of each type which originate and terminate at each port. In the general situation, some ports will originate more barges than they terminate, and hence will have a "demand" for empty barges. The reverse situation will hold true for other ports, which thus will have a "supply" of empties. Also, the total supply and demand will be equal for each barge type. Consequently, a balance between input and output of barges at each port may be achieved by solving the following problem for each barge type: "What movements of empty barges should occur so as to exhaust all supplies and meet all demands?"

The problem posed above is solved in TOWGEN by determining those empty barge O-D movements which minimize total empty barge-miles of travel throughout the entire waterway system, using the linear programming transportation algorithm.

The approach used in TOWGEN neglects, however, the opportunity to reduce costs through barge intermix. That is, a more efficient utilization of equipment is derived through the allocation of loads to barges which, in the absence of available loads, would be required to move empty because of imbalances in the movements of the primary commodity carried. This idea was introduced in two "Working Papers" produced by the project staff.[3] Essentially, this technique involves the utilization of partial crosshaul operations to meet some of the empty barge requirements prior to balancing barge movements via the linear programming transportation algorithm. That is, for three ports A, B, and C,

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[3]These papers have been included in the Waterway Systems Simulation series as Technical Memorandum No. 1.

movements of loaded barges from A to C result in a supply of empty barges at C.[4,5] These empties were then allowed to move commodities from C to B, then travel empty from B back to A. The entire array of such possibilities was formulated as a general linear programming problem, with cost minimization as the objective.

This paper presents five models designed to approach the problem in a systematic way. Four of the models are actually modifications of the first general purpose model, presented to take into account the degree of optimality desired and to simulate various real world situations. Essentially, these models have the following properties:

(1) they present a general multiport, multicommodity, multi-equipment model;

(2) they consider all possible combinations of movements to produce the optimal set while satisfying the supply and demand characteristics at every port;

(3) the technique mentioned above and that used in TOWGEN view the waterway operation as a set of movements from port i to port j to be optimized so as to result in minimum total cost. In addition to that the models presented in this paper view the waterway system from the equipment user's point of view.

Before presenting these models, it may be useful to restate the problem in an example. Consider two commodities, for simplicity's sake, one of which needs to be carried in special equipment, say barge type 1. The

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[4]B is between A and C.

[5]Considers only one stop between any two ports.

other commodity can be carried in the lesser expensive barge type 2 and if convenient in barge type 1. Consideration of the commodity matrix for only barge type 1 results, using the classical linear programming transportation algorithm, in a minimum number of loaded and empty barge movements. The allocation of the second commodity loads to the empty type 1 barges when economical, leads to a reduction in the number of barge type 2 movements. The question is: when is it convenient, i.e., when is it economical, to carry commodity type 2 in type 1 barges?

The following section presents the general description and the theoretical justifications for each model. As mentioned before, these models are not meant to be mutually exclusive and in fact it would not be unusual to combine these models to suit a given situation.

IV. GENERAL DESCRIPTION OF MODELS

A. The General Model

The most efficient operation results when the system is viewed as one unit. The system consists of ports with individual supply and demand characteristics resulting in a prior specification of certain origin-destination commodity movements. However, these movements need not be reproduced as originally specified but can be manipulated around so as to result in a total cost minimization. This total minimization of O-D movements is valid when:

(1) the waterway operation is being viewed from a systems approach with the primary goal being the reduction in congestion or delays without regard for the resultant O-D movements;

(2) or correspondingly when the waterway operation is under the jurisdiction of one corporate entity or to use the more common term monopoly.

Obviously this model is unacceptable as regards to its application to the real world waterway system. However, it serves two useful purposes. First, it provides the general framework from which other models with suitable modifications can be obtained. Second, it provides a good basis, in fact the best considering it is the most efficient, for the comparison of the performance of other models.

This model will also be known as the total cooperation model in view of the fact that the monopolistic firm can cooperate with itself completely. The term "cooperation" will be seen to be useful in describing the other models since it not only refers to the amount of competition present in the system (total cooperation in this context is equivalent to zero competition), but also in fact points out the degree of optimality that can be attained in the solution.

B. The Non-cooperative Model

This model does not refer, as the name may imply, to perfect competition but rather to the fact that the degree of optimality attained is minimal compared to the other models. The model is non-cooperative in the sense that the originally specified O-D movements must be preserved in the final solution, and the allocation of type 2 commodity loads to barge type 1 in the illustrative example mentioned before must be made under this condition. This condition implies that we can no longer consider only the total supply and demand characteristics of every port but in fact must also reproduce the original O-D movements. This model is appropriate when:

(1) the waterway system under consideration is being used by a set of small firms, for example private shippers, who are primarily concerned with their own supply and demand rather than the supply and demand of each port in the system,

(2) there are extraneous and unknown costs involved in manipulating the originally specified O-D movements and when these costs are, for the purpose of this model, prohibitive

This model has limited use in application to the real world phenomenon but is useful, as will be seen in the next section, in illustrating the type of modifications that can be made on the general model.

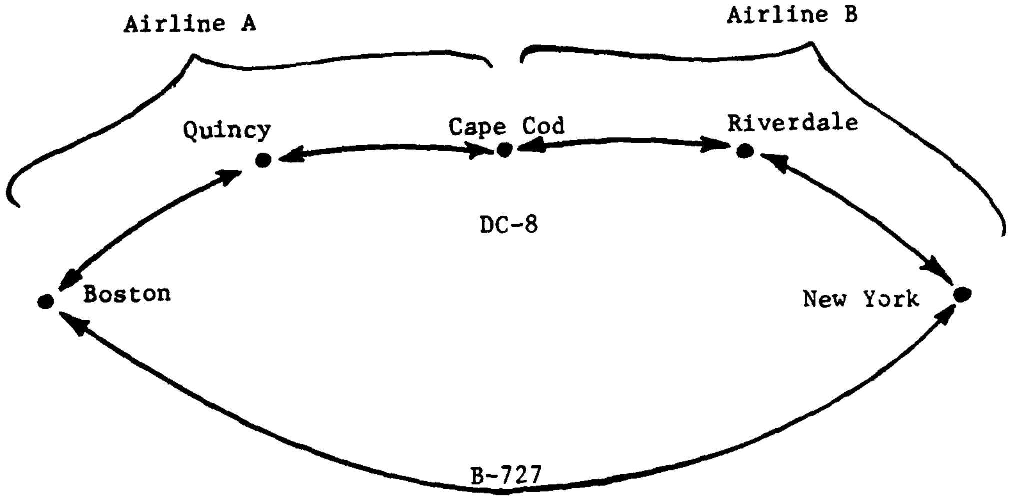
The next two models are known as the partial cooperation models indicating that there is a reduction in the degree of optimality attained (compared to the general model) as a penalty for including some of the real world constraints.

C. The Multi-reach Model

Reach definition: A system may be considered as a set of reaches if and only if the degree of cooperation for intrareach movements is greater than that for interreach movements. In particular, the intra-reach movements must be formulated as the general model (cooperation is total) while the interreach movements must be modeled as non-cooperative.

As an example, consider the hypothetical construction shown in Figure 2. Suppose Airline A has the sole franchise for the charter trips between Boston and Cape Cod while Airline B serves Cape Cod and New York. Both airlines use, to a large extent, DC-8s while the Boston-New York route, served by a number of major domestic trunk carriers utilizes the B-727s.

In the non-dynamic context (i.e., disregarding the time factor), New York originates 720 trips but terminates only 570 trips. Thus New York has a demand for 150 plane trips while similarly Boston has a supply of 130 plane trips. The general model cannot be applied however, since these supplies and demands involve both DC-8s and B-727s. That is the 150 plane trips originating from New York must be distinguished as to their destination between Riverdale <u>and</u> Cape Cod <u>or</u> Boston so that the appropriate type of plane trips can be considered. The non-cooperative model is also inappropriate since the two routes Boston to Cape Cod and Cape Cod to New York can each be modeled as the general model. Thus it becomes convenient to model Boston, Quincy and Cape Cod as one reach and Cape Cod, Riverdale and New York as another reach. All movements except the Boston-New York transits have now been



(a) Schematic Diagram

Destin. / Origin	Boston	Quincy	Cape Cod	Riverdale	New York	Total
Boston	X	20	50	-	500	570
Quincy	30	X	80	-	-	110
Cape Cod	70	10	X	20	40	140
Riverdale	-	-	30	X	30	60
New York	600	-	90	30	X	720
Total	700	30	250	50	570	1600

(b) Trip Matrix

Figure 2. Hypothetical System Construction for the Multi-reach Model

accounted for in one of these two reaches. These transits, utilizing the B-727s would be classified as interreach movements and must occur as specified, requiring an extra backhaul of 100 B-727 vacant plane trips from Boston to New York.

The model is appropriate when:

(1) the reaches are so geographically separated as to make consideration of the whole system as one unit invalid. Movement optimization, therefore, is possible within the reaches but not between reaches;

(2) the commodities being hauled within the reaches are compatible in the sense of making cooperation possible but any movement between reaches involves a different incompatible commodity;

(3) the equipment required for an interreach movement is incompatible (physically or financially) with the equipment used within the reaches. This occurs when small towboats and barges are used within the reaches but larger towboats and barges are used for a movement between the reaches.

(4) Combinations of above. States (1), (2) and (3) above are not mutually exclusive, but this model is usable for combinations of these states as long as the contraints concerning the use of different barges to carry different commodities are prespecified.

D. The Multi-firm Model

This model attempts to approximate, perhaps more than any other model, the real world phenomenon on the waterways in that it considers the waterway system from the user's point of view rather than that of a bystander.

Consider a set of firms operating on the waterway. It would not be unreasonable to assume that a firm seeks to minimize its cost of equipment use while satisfying its own supply and demand characteristics. Therefore a firm with certain originally specified load movements from port i to port j would be expected to arrange its equipment movements so as to result in minimum cost.

The most efficient equipment movements result when all the commodities are being carried by one firm and that firm employs an optimal level of fleet mix to satisfy the supply and demand at each port. This is of course the concept used in the general model. Correspondingly, a number of small firms operating without any interline cooperation would tend to produce a rather inefficient set of equipment movements from a systems point of view. This is precisely the non-cooperative model so that the two extreme points on the "optimality" scale are available for comparison with the other partial cooperation models.

This model therefore, simulates the real world case of a few large firms where optimization within a firm's operation is possible with side benefits in smaller delays and reduced congestion in addition to the minimization of an individual firm's cost of equipment use.

E. The Dedicated Equipment Model

This model is formulated to bring the dedicated equipment concept into the analysis of empty barge movements. Recall that the dedicated equipment concept establishes a minimum value for the number of empty barges, calculated as a specified percentage of loaded barge movements. That is,

$$E_{ijk} = (L_{jik})\ (P_k)/100$$

where

E_{ijk} = number of empty type k barges traveling i to j;

L_{jik} = number of loaded type k barges traveling j to i;

P_k = dedicated equipment percentage for barge type k;

$0 \leq P_k \leq 100$.

This model corresponds to standard practices within the towing industry since, for some commodities, such as liquid chemicals, virtually all shipments utilize integrated tows which move loaded in one direction and empty in the other. For other barge types, the dedicated equipment percentage reflects the operations of private and contract carriers engaged primarily in accommodating specific trades, or of common carriers hauling select commodities.

This model is primarily a mathematically simple modification of the general model. It is useful, however, in allowing the reader to focus on the types of modifications that can be made on the general model. It is further useful in providing a rather simple way to account for a real world situation and points out the fact that many of these modifications presented in this paper can and probably will be performed simultaneously on the general model so as to provide in the final analysis a more complex, and a more realistic simulation of the waterways.

V. INPUT DATA

This section describes the classes of data required by these models. Since the section is all inclusive (i.e , lists all data requirements for all models), the use of a particular model or a particular combination of models might actually require only some of these data. The choice, however, is not difficult to see.

The models require three basic classes of input data:

(1) system data;

(2) barge data;

(3) commodity data.

A. System data

This class of data specifies the size and character of the network of ports on the waterway system under consideration. It also specifies the cost functions used in the mathematical formulation.

1. Ports and Reaches

A port is defined as "any point on the waterway which is an origin and a destination for the movement of goods." This includes system end points. A reach is defined as "any set of contiguous ports." Reaches as mentioned before are used solely to designate portions of the system among which the distributions of equipment or commodity characteristics significantly differ. The number of ports and reaches in a system are limited only by available computer hardware.

2. Cost Functions

These models require as input two cost tables (or matrices) for each barge type. In these tables, the rows and columns represent ports, and each cell of the matrices contains a scalar value. For one table, the scalar value is C_{ijk} = cost of moving one barge load in barge type k from port i to port j; $i \neq j$. For the other table with the same barge type the scalar value is,

C^1_{ijk} = cost of moving an empty barge type k from port i to port j; $i \neq j$.

Therefore, if k is the number of barge types, a total of 2k cost tables are required.

Two points need to be considered here:

a. If the cost is a linear function of distance, then minimizing one in linear programming is equivalent to minimizing the other.

Hence, in this case, the cost tables are easily computed by multiplying an O-D mileage table, which gives the river miles between pair of ports in the system, by a scalar and adding a scalar. That is,

$$C_{ijk} = A_k + B_k d_{ij};\ i \neq j$$

where

d_{ij} = miles from port i to port j found in each cell of the O-D mileage matrix;

k = number of barge types;

A_k, B_k = constants corresponding to the fixed costs and the marginal cost of moving one barge load one mile in barge type k;

C_{ijk} = as defined above.

A similar relationship holds for empty barges and for all other barge types with the substitution of appropriate constants in the function given above.

b. If the concept of firms is brought into the analysis then two conditions can prevail;

i) the cost functions can be assumed independent of the user firm. In this case no change is made.

ii) If the above assumption does not hold then some additional data is needed. If p firms are considered in the system then with k barge types, 2pk cost matrices are required instead of the 2k matrices mentioned above. In these matrices, the rows and columns are again origin and destination ports respectively, however, the cell values are,

S_{ijkp} = cost of moving one load in barge type k from port i to port j for firm p; $i \neq j$.

Similarly,

S^1_{ijkp} = cost of moving an empty barge type k from port i to port j for firm p; $i \neq j$.

B. Barge Data

This class of data includes the following:

(1) the number of barge types used to transport the commodities in the system;

(2) specification of the barge types that can be used to carry each commodity;

(3) list of dedicated equipment percentages; (this could be specified by barge type, commodity, reach, firm, or in general for any O-D movement).

C. Commodity Data

The commodity data provide information concerning the loads to be moved within the system. The commodity data can be given in barge loads where one barge load of commodity k refers to the amount of commodity k carried in one barge, or if the commodity flows are given in tons, then the average loading factor (in tons per barge) must also be specified so that the tonnage data can be converted into barge loads.

The commodity data can include:

1. Total Commodity O-D Matrix

The cells of this matrix contain the total barge loads to be shipped from each port to every other port. Origin ports are specified by the row number, destination ports by the column number (i.e., the number in row i, column j specifies flow from port i to port j). If there are m commodities specified in the system, then m O-D commodity matrices need to be provided. Figure 3 illustrates a commodity O-D matrix for a system with n ports and m commodities where,

X_{ijm} = barge loads of commodity m to be shipped from port i to port j; $i \neq j$.

2. Commodity O-D Matrices for Firms

The introduction of the firm concept into the system necessitates some further data. If p firms are to be considered in the system, then at least p-1 commodity O-D matrices must be specified for each commodity in addition to the total commodity O-D matrix mentioned above. Origin ports are again specified by the row number, destination ports by the column number. The cell values are:

X_{ijmp} = barge loads of commodity m shipped from port i to port j by firm p; $i \neq j$.

Notice that if p-1 O-D matrices are specified for each commodity then the O-D matrix for the p^{th} firm for each commodity can be calculated from the relationship:

Port Destinations

Port Origins		1	2	.	.	.	.	.	.	.	.	n
	1	0	X_{12m}	.	.	.	.	.	.	.	.	X_{1nm}
	2	X_{21m}	0									
	.	.		0								
	.	.			0							
	.	.				0						
	.	.					0					
	.	.						0				
	.	.							0			
	.	.								0		
	.	.									0	
	n	X_{n1m}										0

Figure 3. Total O-D Matrix for Commodity M

$$X_{ijm} = \sum_{r=1}^{p} X_{ijmr} \qquad i \neq j,\ i,j = 1, 2, \ldots, n \text{ ports}$$

where X_{ijm} is the cell value from the total commodity O-D matrix. Ultimately, a system with p firms and m commodities would require pm commodity O-D matrices.

VI. MATHEMATICAL FORMULATION

The procedure used to solve the problem when the appropriate model is mathematically formulated is the linear programming transportation technique. In order to mathematically formulate the model, the input data including the model parameters must be specified. Therefore, consider the seven port, two commodity, two barge types system given below. This system is convenient because solutions using the TOWGEN and the "Working Papers" approaches are available for comparison.

Consider two O-D matrices for covered hopper barge loads and open hopper barge loads (illustrative matrix in Table 1).[6] These two matrices result from the need for carrying certain commodities in special equipment, such as grain in covered hoppers, to prevent damage due to the elements. Other commodities, such as steel, sand and gravel, etc., need not be protected and therefore, open hopper barges will suffice; however, these commodities can be carried in covered hopper barges if it is economical to do so. The cost matrices for covered hopper barges, loaded and empty and open hopper barges, loaded and empty are also given.

The data given above is the minimum amount of data that can be used by one of the models. In fact, it is sufficient for the general and the non-cooperative models. Additional data will be specified for the partial cooperation and the dedicated equipment models to facilitate the mathematical formulation.

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[6]All tables are collected at the end.

The principle variables are formulated as follows:

$XCLC_{ij}$ = loaded covered hopper barges carrying covered hopper loads from port i to port j;

XLC_{ij} = loaded covered hopper barges carrying open barge hopper loads from port i to port j;

XEC_{ij} = empty covered hopper barges moving from port i to port j;

XLO_{ij} = loaded open hopper barges carrying open hopper loads from port i to port j;

XEO_{ij} = empty open hopper barges going from port i to port j;

C_{ij1} = cost of moving a loaded covered hopper barge from port i to port j;

C^1_{ij1} = cost of moving an empty covered hopper barge from port i to port j;

C_{ij2} = cost of moving a loaded open hopper barge from port i to port j;

C^1_{ij2} = cost of moving an empty open hopper barge from port i to port j.

A. The General Model

Since the general model is only concerned with the total supply and demand at every port, the formulation of the linear programming format ignores the originally specified individual movements. The formulation begins with the objective function.

$$\text{Minimize} \sum_{i=1}^{7} \sum_{j=1}^{7} C_{ij1} (XCLC_{ij} + XLC_{ij}) + \sum_{i=1}^{7} \sum_{j=1}^{7} C_{ij2}\, XLO_{ij}$$

$$+ \sum_{i=1}^{7} \sum_{j=1}^{7} C^1_{ij1}\, XEC_{ij} + \sum_{i=1}^{7} \sum_{j=1}^{7} C^1_{ij2}\, XEO_{ij} .$$

The supply constraints from the O-D commodity matrix for covered hoppers are,

$$\sum_{j=1}^{7} XCLC_{1j} = a_1 \qquad \text{where } a_1 = \text{row 1 total;}$$

$$\sum_{j=1}^{7} XCLC_{2j} = a_2 \qquad \text{where } a_2 = \text{row 2 total;}$$

.

.

.

$$\sum_{j=1}^{7} XCLC_{7j} = a_7 \qquad \text{where } a_7 = \text{row 7 total.}$$

The demand constraints from the same table are,

$$\sum_{i=1}^{7} XCLC_{i1} = b_1 \qquad \text{where } b_1 = \text{column 1 total;}$$

$$\sum_{i=1}^{7} XCLC_{i2} = b_2 \qquad \text{where } b_2 = \text{column 2 total;}$$

.

.

.

$$\sum_{i=1}^{7} XCLC_{i7} = b_7 \qquad \text{where } b_7 = \text{column 7 total.}$$

The balance constraints for this commodity are,

$$\sum_{j=1}^{7} [(XCLC_{ij} + XLC_{ij} + XEC_{ij}) - (XCLC_{j1} + XLC_{j1} + XEC_{j1})] = 0;$$

$$\sum_{j=1}^{7} [(XCLC_{2j} + XLC_{2j} + XEC_{2j}) - (XCLC_{j2} + XLC_{j2} + XEC_{j2})] = 0;$$

.

.

.

$$\sum_{j=1}^{7} [(XCLC_{7j} + XLC_{7j} + XEC_{7j}) - (XCLC_{j7} + XLC_{j7} + XEC_{j7})] = 0.$$

These balance constraints simply state that crosshaul movements cancel each other (i.e., the model looks only at net input-output imbalances at each port).

The supply constraints from the O-D commodity matrix for open hoppers are,

$$\sum_{j=1}^{7} (XLO_{1j} + XLC_{1j}) = aa_1 \qquad \text{where } aa_i = \text{row } i \text{ total};$$

$$\sum_{j=1}^{7} (XLO_{2j} + XLC_{2j}) = aa_2;$$

.

.

.

$$\sum_{j=1}^{7} (XLO_{7j} + XLC_{7j}) = aa_7.$$

The demand constraints from this matrix are,

$$\sum_{i=1}^{7} (XLO_{i1} + XLC_{i1}) = bb_1 \qquad \text{where } bb_i = \text{column } i \text{ total};$$

$$\sum_{i=1}^{7} (XLO_{i2} + XLC_{i2}) = bb_2;$$

.

.

.

$$\sum_{i=1}^{7} (XLO_{i7} + XLC_{i7}) = bb_7.$$

The balance constraints for this commodity are,

$$\sum_{j=1}^{7} [(XLO_{1j} + XEO_{1j}) - (XLO_{j1} + XEO_{j1})] = 0;$$

$$\sum_{j=1}^{7} [(XLO_{2j} + XEO_{2j}) - (XLO_{j2} + XEO_{j2})] = 0;$$

.

.

.

$$\sum_{j=1}^{7} [(XLO_{7j} + XEO_{7j}) - (XLO_{j7} + XEO_{j7})] = 0.$$

In order to guarantee that in the final solution the elements along the diagonal line will be zero (i.e., to make sure that the model will not generate positive movements from a port to itself), a few additional constraints are necessary.

$$XCLC_{ij} = XLO_{ij} = XEC_{ij} = XLC_{ij} = XEO_{ij} = 0 \quad \text{for all } i=j,$$

$$i,j = 1,2, \ldots, 7.$$

In running the model on the computer, these variables could simply be omitted from the model thus eliminating these constraints.

Non-negativity constraint,

all $X_{ij} \geq 0$

where X_{ij} refers to all variables with X as the first alphabet.

B. The Non-cooperative Model

This model requires the preservation of the originally specified individual movements. Hence there are 49 constraints (n^2 for a system with n ports) from the covered hopper barge load matrix of the type,

$XCLC_{ij} = N_{ij}$ for all i,j; i,j = 1,2, . . ., 7.

The balance constraints again are,

$$\sum_{j=1}^{7} [(XCLC_{1j} + XLC_{1j} + XEC_{1j}) - (XCLC_{j1} + XLC_{j1} + XEC_{j1})] = 0;$$

$$\sum_{j=1}^{7} [(XCLC_{2j} + XLC_{2j} + XEC_{2j}) - (XCLC_{j2} + XLC_{j2} + XEC_{j2})] = 0;$$

$$\vdots$$

$$\sum_{j=1}^{7} [(XCLC_{7j} + XLC_{7j} + XEC_{7j}) - (XCLC_{j7} + XLC_{j7} + XEC_{j7})] = 0.$$

The balance constraints state that while the individual loaded barge movements must be preserved as originally specified, the allocation of empty barge movements is undertaken with the balancing of supply and demand at every port.

The indivudual movement constraints from the open hopper barge load matrix result in 49 equations of the form,

$XLO_{ij} + XLC_{ij} = M_{ij}$ for all i,j; i,j = 1,2, . . ., 7.

The balance constraints are,

$$\sum_{j=1}^{7} [(XLO_{1j} + XEO_{1j}) - (XLO_{j1} + XEO_{j1})] = 0;$$

$$\sum_{j=1}^{7} [(XLO_{2j} + XEO_{2j}) - (XLO_{j2} + XEO_{j2})] = 0;$$

$$\vdots$$

$$\sum_{j=1}^{7} [(XLO_{7j} + XEO_{7j}) - (XLO_{j7} + XEO_{j7})] = 0.$$

Specification of zero values for the main diagonal elements result in,

$XCLC_{ij} = XLC_{ij} = XEC_{ij} = XLO_{ij} = XEO_{ij} = 0$ for all $i=j$;

$i,j = 1,2, . . ., 7.$

The non-negativity constraints are,

all $X_{ij} \geq 0$

where X_{ij} refers to all variables with X as the first alphabet.

The objective function is of the same form as before.

C. The Multi-reach Model

This model requires as additional data, the specification of reaches and the ports included in each reach. Let the system be arbitrarily defined as consisting of two reaches, one including ports 1, 2, 3 and 4 and the other including ports 5, 6 and 7 keeping in mind that the theoretical justification for this definition must actually come from the real world phenomenon.

Since this model allows the consideration of only the supply and demand characteristics of every port within any reach, Table 1 can be modified to be Table 2. The shaded areas in Table 2 are shown in Tables 3 and 4 respectively. This is simply a mathematical statement of the fact that the load requirement matrix for each reach can be replaced by one cell (or terminal) as far as interreach movements are concerned. The supply constraints from the covered hopper barge load matrix-reach 1 are,

$$\sum_{j=1}^{4} XCLC_{1j} = Z_1;$$

$$\sum_{j=1}^{4} XCLC_{2j} = Z_2;$$

$$\vdots$$

$$\sum_{j=1}^{4} XCLC_{4j} = Z_4.$$

The corresponding demand constraints are,

$$\sum_{i=1}^{4} XCLC_{i1} = W_1;$$

$$\sum_{i=1}^{4} XCLC_{i2} = W_2;$$

$$\vdots$$

$$\sum_{i=1}^{4} XCLC_{i4} = W_4.$$

The balancing constraints are,

$$\sum_{j=1}^{4} [(XCLC_{1j} + XLC_{1j} + XEC_{1j}) - (XCLC_{j1} + XLC_{j1} + XEC_{j1})] = 0;$$

$$\sum_{j=1}^{4} [(XCLC_{2j} + XLC_{2j} + XEC_{2j}) - (XCLC_{j2} + XLC_{j2} + XEC_{j2})] = 0;$$

$$\vdots$$

$$\sum_{j=1}^{4} [(XCLC_{4j} + XLC_{4j} + XEC_{4j}) - (XCLC_{j4} + XLC_{j4} + XEC_{j4})] = 0.$$

The supply constraints from the covered hopper barge load matrix-reach 2 are,

$$\sum_{j=5}^{7} XCLC_{5j} = Z_5;$$

$$\vdots$$

$$\sum_{j=5}^{7} XCLC_{7j} = Z_7.$$

The corresponding demand constraints are,

$$\sum_{i=5}^{7} XCLC_{i5} = W_5;$$

$$\vdots$$

$$\sum_{i=5}^{7} XCLC_{i7} = W_7.$$

The balancing constraints are,

$$\sum_{j=5}^{7} [(XCLC_{5j} + XLC_{5j} + XEC_{5j}) - (XCLC_{j5} + XLC_{j5} + XEC_{j5})] = 0;$$

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$$\sum_{j=5}^{7} [(XCLC_{7j} + XLC_{7j} + XEC_{7j}) - (XCLC_{j7} + XLC_{j7} + XEC_{j7})] = 0$$

The individual movement constraints are of two sets:

$XCLC_{ij} = N_{ij}$ for all i = 1,2,3,4; j = 5,6,7

and $XCLC_{ij} = N_{ij}$ for all i = 5,6,7; j = 1,2,3,4.

The balancing constraints are,

$$\sum_{j=1}^{4} [(XCLC_{5j} + XLC_{5j} + XEC_{5j}) - (XCLC_{j5} + XLC_{j5} + XEC_{j5})] = 0;$$

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.

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$$\sum_{j=1}^{4} [(XCLC_{7j} + XLC_{7j} + XEC_{7j}) - (XCLC_{j7} + XLC_{j7} + XEC_{j7})] = 0.$$

This same procedure now has to be applied to the open hopper load matrices. The supply constraints for reach 1 are,

$$\sum_{j=1}^{4} (XLO_{1j} + XLC_{1j}) = ZZ_1;$$

.

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.

$$\sum_{j=1}^{4} (XLO_{4j} + XLC_{4j}) = ZZ_4.$$

The corresponding demand constraints are,

$$\sum_{i=1}^{4} (XLO_{i1} + LC_{i1}) = WW_1;$$

$$\vdots$$

$$\sum_{i=1}^{4} (XLO_{i4} + XLC_{i4}) = WW_4.$$

The balancing constraints are,

$$\sum_{j=1}^{4} [(XLO_{1j} + XEO_{1j}) - (XLO_{j1} + XEO_{j1})] = 0;$$

$$\vdots$$

$$\sum_{j=1}^{4} [(XLO_{4j} + XEO_{4j}) - (XLO_{j4} + XEO_{j4})] = 0.$$

The supply constraints for reach 2 are,

$$\sum_{j=5}^{7} (XLO_{5j} + XLC_{5j}) = ZZ_5;$$

$$\vdots$$

$$\sum_{j=5}^{7} (XLO_{7j} + XLC_{7j}) = ZZ_7.$$

The corresponding demand constraints are,

$$\sum_{i=5}^{7} (XLO_{i5} + XLC_{i5}) = WW_5;$$

$$\vdots$$

$$\sum_{i=5}^{7} (XLO_{i7} + XLC_{i7}) = WW_7.$$

The balancing constraints are,

$$\sum_{j=5}^{7} [(XLO_{5j} + XEO_{5j}) - (XLO_{j5} + XEO_{j5})] = 0;$$

$$\vdots$$

$$\sum_{j=5}^{7} [(XLO_{7j} + XEO_{7j}) - (XLO_{j7} + XEO_{j7})] = 0.$$

The individual movement constraints are of two sets:

$XLO_{ij} + XLC_{ij} = M_{ij}$ for all $i = 1,2,3,4$; $j = 5,6,7$

and $XLO_{ij} + XLC_{ij} = M_{ij}$ for all $i = 5,6,7$; $j = 1,2,3,4$.

The balancing constraints are,

$$\sum_{j=1}^{4} [(XLO_{5j} + XEO_{5j}) - (XLO_{j5} + XEO_{j5})] = 0;$$

$$\vdots$$

$$\sum_{j=1}^{4} [(XLO_{7j} + XEO_{7j}) - (XLO_{j7} + XEO_{j7})] = 0.$$

Note: A major assumption in the balancing constraints for this model is that equipment used for intrareach movements cannot interchangeably be used for interreach movements. If this assumption is unnecessary, all the balancing constraints noted in this model are to be replaced by the balancing constraints found in the general model.

Finally, the specification of zero values for the main diagonal elements and the non-negativity constraints are,

$$XCLC_{ij} = XLC_{ij} = XEC_{ij} = XLO_{ij} = XEO_{ij} = 0 \qquad \text{for all } i=j;$$

$$i,j = 1,2, \ldots, 7$$

and

$$\text{all } X_{ij} \geq 0$$

where X_{ij} refers to all variables with X as the first alphabet. The objective function is of the same form as before.

D. The Multi-firm Model

In addition to the data used for the general model, the multi-firm model requires the specification of the firms, the cost matrices for the firms and the commodity O-D matrices for the firms. Consider a system with two major firms. The O-D commodity matrices for each firm for each commodity (covered and open hopper barge loads) are given and the cost matrices are also assumed given.

Since the total O-D commodity matrix is the combination of the O-D matrices for each firm, this merely involves two general models, one for each firm. Hence, the constraints given in the general model need to be applied twice. In the first application, the variables $XCLC_{ij}$, XLC_{ij}, XEC_{ij}, XLO_{ij} and XEO_{ij} are replaced by $XCLC_{ij1}$, XLC_{ij1}, XEC_{ij1},

XLO_{ij1} and XEO_{ij1} respectively, where the third subscript indicates that these variables are for firm 1. Similarly, the second application involves the use of variables $XCLC_{ij2}$, XLC_{ij2}, XEC_{ij2}, XLO_{ij2} and XEO_{ij2} respectively. If the general model has K constraints, a multi-firm model with p firms would in general have pK constraints.

Let the cells in the cost matrices be denoted as follows:

C_{ij11} = cost for firm 1 of moving one loaded covered hopper barge from port i to port j;

C^1_{ij11} = cost for firm 1 of moving one empty covered hopper from port i to port j;

C_{ij21} = cost for firm 1 of moving one loaded open hopper from port i to port j;

C^1_{ij21} = cost for firm 1 of moving one empty open hopper from port i to port j;

C_{ij12} = cost for firm 2 of moving one loaded covered hopper from port i to port j;

C^1_{ij12} = cost for firm 1 of moving one empty covered hopper from port i to port j;

C_{ij22} = cost for firm 2 of moving one loaded open hopper from port i to port j;

C^1_{ij22} = cost for firm 2 of moving one empty open hopper from port i to port j.

The objective function then takes the form,

$$\text{minimize} \sum_{i=1}^{7} \sum_{j=1}^{7} [C_{ij11} (XCLC_{ij1} + XLC_{ij1}) + C^{1}_{ij11} XEC_{ij1}$$

$$+ C_{ij21} XLO_{ij1} + C^{1}_{ij21} XEO_{ij1} + C_{ij12} (XCLC_{ij2} + XLC_{ij2})$$

$$+ C^{1}_{ij12} XEC_{ij2} + C_{ij22} XLO_{ij2} + C^{1}_{ij22} XEO_{ij2}]$$

E. The Dedicated Equipment Model

As mentioned before, this model is introduced only to bring the dedicated equipment concept into the analysis. Since this concept merely establishes a lower limit on a particular O-D empty barge movement, this model requires additional constraints of the type:

$$XLC_{ji} + XEC_{ji} \geq P_{ij1} XCLC_{ij}$$

and

$$XEO_{ji} \geq P_{ij2} XLO_{ij}$$

where

P_{ij1} = dedicated equipment percentage for covered hopper barge (barge type 1) movement from port i to port j,

and

P_{ij2} = dedicated equipment percentage for open hopper barge (barge type 2) movement from port i to port j.

The dedicated equipment percentage could be specified, in addition to barge type, by reach and by firm. Hence, the constraints listed above are in addition to all the constraints (supply, demand, balance, etc.) obtained from the particular model being used whether it be general, non-cooperative, multi-reach, multi-firm or a combination of these.

VII. MODEL APPLICATIONS

A. System Description

The TOWGEN and "Working Papers" (to be denoted W#2) techniques are compared in this section with the general and non-cooperative models. The example used for comparison purposes is the two commodity problem mentioned earlier.

The commodity O-D matrices used in the problem are given in Table 5 and 6. The problem assumes a linear (i.e., sequential) seven port system and the distance between ports matrix is given in Table 7. Cost is assumed to be a linear function of distance and the cost functions used are given below.

If C = total cost of moving a barge from port i to port j;

FC = fixed costs;

VC = variable costs, given in dollars/mile;

D = distance between port i to port j;

then $C = FC + VC * D$.

(1) Covered Hoppers Barges

Loaded: $C = 25 + 4D$

Empty: $C = 5 + D$

(2) Open Hopper Barges

Loaded: $C = 20 + 3.6D$

Empty: $C = 4 + .9D$

The cost functions were developed solely to facilitate solving the problem and while they may not truly represent the actual transport costs, the relationship between the cost functions are assumed to parallel the actual relationships.

The final barge movements generated by TOWGEN, W#2, general and the non-cooperative models are shown in Tables 8, 9, 10 and 11 respectively. Each of these tables is completely balanced, i.e., the number of covered hopper barges and the number of open hopper barges originating at each point is identically equal to the number terminating at each point. Each of the cells in the loaded covered hopper barge matrices include barges carrying either covered hopper or open hopper loads. Table 12 summarizes the statistics for total cost, distance in barge-miles and number of barge movements.

B. Analysis

It is seen from Table 12 that, in comparison with TOWGEN total cost, the general model allows a 54% reduction while the W#2 and the non-cooperative models constitute 0.6% and 1% cost reductions respectively. The reason for the low values from the latter techniques is due to the fact that the opportunity for improving empty barge allocation is small. Table 13 indicates the availability of covered hopper empties for carrying open hopper loads. This table is derived from Table 5 by taking the differences between origins and terminations to yield the supply and demand for empty covered hopper barges. Note that the total number of covered hopper empties in this table is the same number generated by the TOWGEN technique. This number is less than 10% of the total covered and open barge load movements indicating that the opportunity for improving empty barge allocation is small.

Despite this aspect of the problem certain trends in the final solutions are evident. The W#2 and the general model generate 180 more covered hopper loads (all in comparison to TOWGEN solution) and therefore

180 less open hopper loads, while the non-cooperative model derives only 150 more covered hopper loads and correspondingly 150 less open hopper loads. The latter however, allocates barge movements more efficiently than the W#2 technique, resulting in a slightly lower cost ($1,118,640 to $1,122,960). The non-cooperative model generates only 70 empty covered barges as opposed to 120 barges by the W#2 technique indicating that more of the 210 available empty covered hopper barges are being used to haul open hopper loads.

In summary the following points are noted:

1. the general and the non-cooperative models generate movements resulting in lower cost than either the TOWGEN or the W#2 techniques;
2. the TOWGEN, W#2 and non-cooperative techniques preserve individual loaded movements and this constraint coupled with the scarce availability of empty covered hopper barges limits the opportunity for cost reduction in this problem.

C. Further Applications

This paper has presented five models each model considering one particular characteristic of the waterway system. The models have been formulated however, so as to allow the derivation of combinations of these models. The strength of this approach, therefore, lies not in any individual model, but rather in its ability to analyze a complex situation by identifying each of the principal characteristics.

An application of these models is currently being contemplated for the Illinois-Mississippi Waterway Subsystem. The Illinois Waterway extends for a distance of approximately 326 miles from the confluence of the Illinois and Mississippi Rivers about 38 miles above St. Louis, Missouri, to Chicago, Illinois. About 56 miles of the Mississippi River, from lock and dam (L&D) 25 above the Illinois through L&D 26 and 27 below it is also included in this study. A schematic diagram of this waterway system is shown in Figure 4. The system is defined to consist of 15 ports, numbered as shown in the figure, a junction point, and the channel segments connecting these points.

All the necessary data will be obtained through the North Central Division (NCD), Army Corps of Engineers. The main limitation encountered in this task is the preparation of input cards for the linear programming computer program. Towards this end, a data post processor program is being written by the author

The procedure undertaken in the application of the transportation models to the Illinois-Mississippi Waterway Subsystem can be indicated in tree diagram fashion as shown in Figure 5. The tree diagram shown is for one commodity only. The commodity O-D matrix is broken down by firms and then by reaches. Note that the non-cooperative model is applied in two cases: (1) the operation of small firms among which there is little barge intermix, and (2) the interreach movements for large firms. The general model is applied to the intrareach movements of the large firms and the dedicated equipment model is applied at the end if dedicated movement is specified. The solution to the problem provides efficient equipment movements that can be compared with observed data to indicate areas of improvement.

Other possible applications include routing air cargo carriers, rail cars, trucks, etc. The transportation models provide a rather simple approach towards increasing the utilization of a mixed fleet.

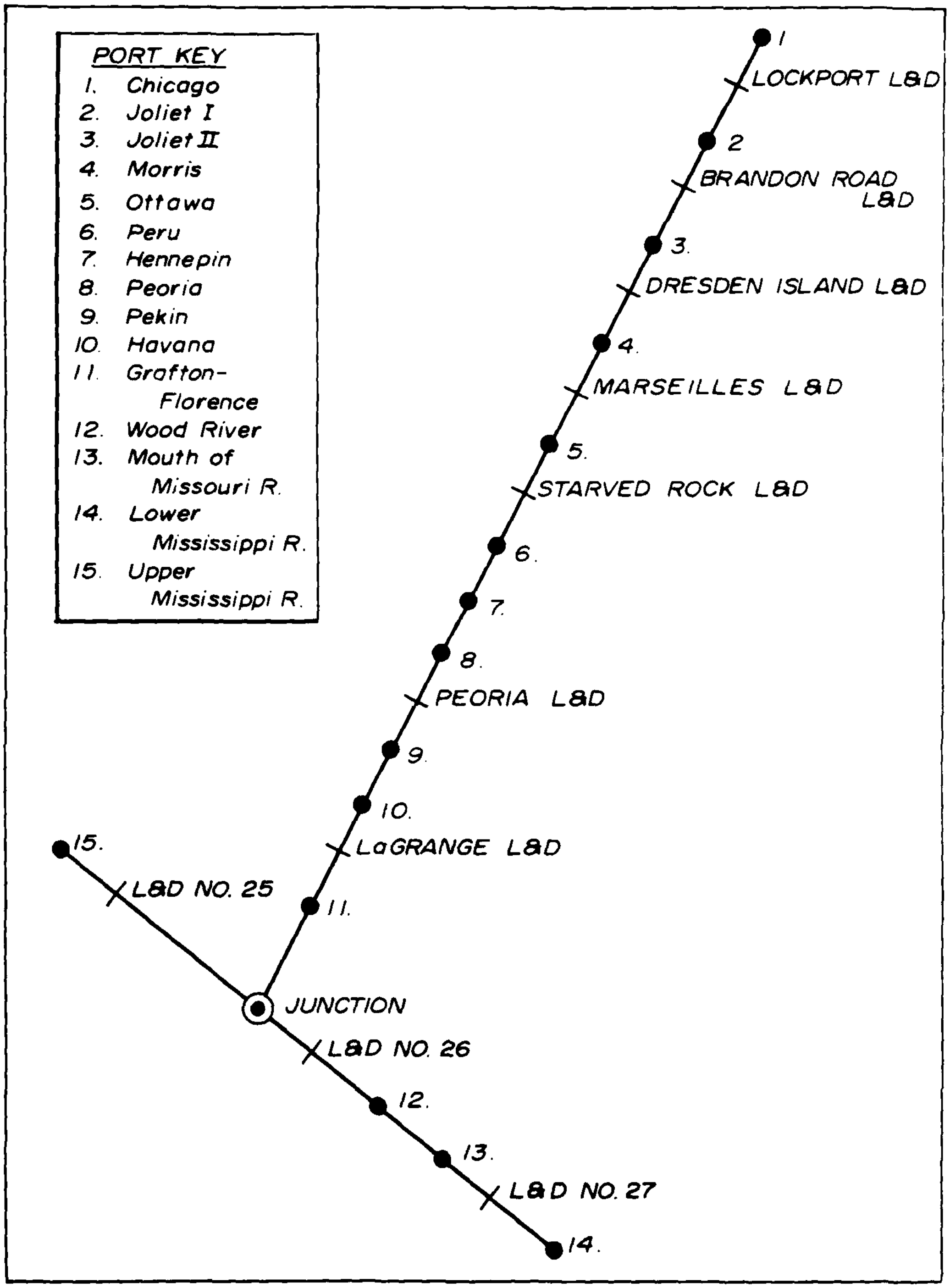

Figure 4. Illinois-Mississippi Ten-Lock Subsystem

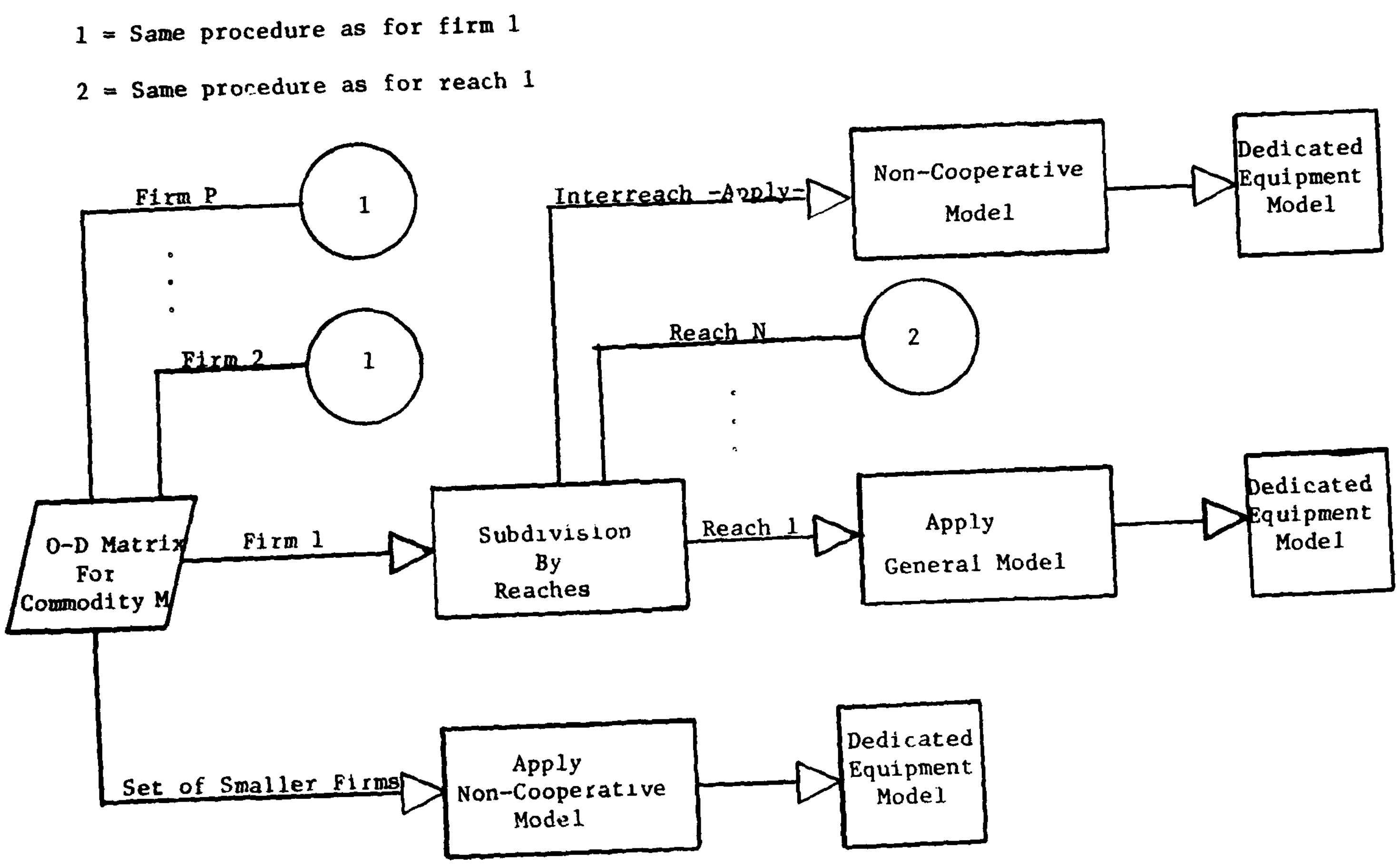

Figure 5. System Application Tree Diagram for Commodity M

In addition to the system characteristics already included, namely, multifirm, multireach, dedicated equipment and cooperation (or lack of it), other constraints on fleet operation such as minimum and/or maximum load limits, specified cargo mixes, etc. can be easily incorporated into the models.

D. *Suggested Modifications*

An inherent assumption in the transportation models is that the demand for commodities in a given period is independent of time. This assumption enables a static solution to the equipment allocation problem. The removal of this assumption involves a considerably greater task in that inventories of equipment and commodities would be required at each node (place of loading and unloading) so that dynamic scheduling and routing can occur.

VIII. CONCLUSION

The problem of allocating commodity flows to transport equipments units with a mixed fleet was addressed in this paper. A set of linear programming transportation models were derived, initially for application on the inland waterway system although not restricted to it.

The models essentially allow for different degrees of optimization in the solution depending on the characteristic of the system being analyzed. The characteristic referred to is the amount of interline cooperation among the different entities operating in the system and this cooperation can further vary by reaches, which are mutually exclusive subsets of the system. Allowance was also made for the one-way flow of certain commodities such as grain and petroleum by including the specification of dedicated equipment.

The performance of the models for a two commodity problem was compared with the solutions obtained using two previous techniques (see footnotes on pages 1 and 6). The models were indeed found to be functional (i.e., generate feasible, optimal, and integer solutions) and further applications to the Illinois-Mississippi Waterway Subsystem is contemplated.

TABLE 1
COMMODITY O-D MATRIX FOR 7 PORT SYSTEM

Port Origins	Port Destinations 1	2	3	4	5	6	7	
1	0	N_{12}	N_{13}	.	.	.	N_{17}	a_1
2	N_{21}	0						a_2
3	.		0					.
4	.			0	N_{ij}			.
5	.				0			.
6	.					0		.
7	N_{71}	N_{72}	N_{73}	.	.	.	0	a_7
	b_1	b_2	.	.	.	.	b_7	

TABLE 2
COMMODITY O-D MATRIX FOR 7 PORT, 2 REACH SYSTEM

Port Destinations

Port Origins	1	2	3	4	5	6	7
1					N_{15}	.	N_{17}
2		Reach 1			.		.
3					.		.
4					N_{45}	.	N_{47}
5	N_{51}	.	.	N_{54}			
6	.			.		Reach 2	
7	N_{71}			N_{74}			

TABLE 3
REACH 1: O-D COMMODITY SUBMATRIX

Port Destinations

Port Origins	1	2	3	4	
1	0	N_{12}	N_{13}	N_{14}	Z_1
2	N_{21}	0	N_{23}	N_{24}	Z_2
3	N_{31}	N_{32}	0	N_{34}	Z_3
4	N_{41}	N_{42}	N_{43}	0	Z_4
	W_1	W_2	W_3	W_4	

TABLE 4
REACH 2: O-D COMMODITY SUBMATRIX

Port Destinations

Port Origins	5	6	7	
5	0	N_{56}	N_{57}	Z_5
6	N_{65}	0	N_{67}	Z_6
7	N_{75}	N_{76}	0	Z_7
	W_5	W_6	W_7	

TABLE 5
COVERED HOPPER BARGE LOADS COMMODITY O-D MATRIX

Port Origins	Port Destinations 1	2	3	4	5	6	7	TOTAL
1	X	80	0	50	20	0	30	180
2	30	X	40	0	30	20	60	180
3	0	60	X	50	0	10	0	120
4	50	0	60	X	0	40	90	240
5	10	40	30	0	X	40	20	140
6	30	20	0	60	40	X	50	200
7	0	30	40	40	10	20	X	140
TOTAL	120	230	170	200	100	130	250	1200

TABLE 6
OPEN HOPPER BARGE LOADS COMMODITY O-D MATRIX

		Port Destinations							
		1	2	3	4	5	6	7	TOTAL
Port Origins	1	X	20	30	0	60	80	50	240
	2	40	X	0	20	70	50	0	180
	3	60	40	X	30	80	20	10	240
	4	90	70	20	X	30	60	40	310
	5	50	80	30	70	X	10	20	260
	6	0	30	50	20	30	X	50	180
	7	80	10	0	90	70	30	X	280
TOTAL		320	250	130	230	340	250	170	1690

TABLE 7
DISTANCES BETWEEN PORTS (MILES)

Port Origins	Port Destinations: 1	2	3	4	5	6	7
1	0	30	70	90	130	180	220
2	30	0	40	60	100	150	190
3	70	40	0	20	60	110	150
4	90	60	20	0	40	90	130
5	130	100	60	40	0	50	90
6	180	150	110	90	50	0	40
7	220	190	150	130	90	40	0

TABLE 8
TOWGEN SOLUTION

(a) Covered Hopper Barge Movements

Origin \ Destination		1	2	3	4	5	6	7	TOTAL
1	LD		80		50	20		30	180
	MT								
2	LD	30		40		30	20	60	180
	MT	50							50
3	LD		60		50		10		120
	MT	10			40				50
4	LD	50		60			40	90	240
	MT								
5	LD	10	40	30			40	20	140
	MT								
6	LD	30	20		60	40		50	200
	MT								
7	LD		30	40	40	10	20		140
	MT					40	70		110
TOTAL	LD	120	230	170	200	100	130	250	1200
	MT	60			40	40	70		210

TABLE 8
TOWGEN SOLUTION (CONTD.)

(b) Open Hopper Barge Movements

Origin \ Destination		1	2	3	4	5	6	7	TOTAL
1	LD		20	30		60	80	50	240
	MT			80					80
2	LD	40			20	70	50		180
	MT			30	40				70
3	LD	60	40		30	80	20	10	240
	MT								
4	LD	90	70	20		30	60	40	310
	MT								
5	LD	50	80	30	70		10	20	260
	MT				40			40	80
6	LD		30	50	20	30		50	180
	MT							70	70
7	LD	80	10		90	70	30		280
	MT								
TOTAL	LD	320	250	130	230	340	250	170	1690
	MT			110	80			110	300

TABLE 9
W#2 SOLUTION

(a) Covered Hopper Barge Movements

Origin \ Destination		1	2	3	4	5	6	7	TOTAL
1	LD		80		50	20		30	180
	MT								
2	LD	30		40		30	20	60	180
	MT			10	40				50
3	LD	30	60		50	30	10		180
	MT								
4	LD	50		60			80	90	280
	MT								
5	LD	10	40	30			40	20	140
	MT				40				40
6	LD	30	20		60	40		50	200
	MT								
7	LD	30	30	40	40	60	20		220
	MT						30		30
TOTAL	LD	180	230	170	200	180	170	250	1500
	MT			10	80		30		

TABLE 9
W#2 SOLUTION (CONTD.)

(b) Open Hopper Barge Movements

Origin \ Destination		1	2	3	4	5	6	7	TOTAL
1	LD		20	30		60	80	50	240
	MT				20				20
2	LD	40			20	70	50		180
	MT			50	20				70
3	LD	30	40		30	50	20	10	180
	MT								
4	LD	90	70	20		30	20	40	270
	MT								
5	LD	50	80	30	70		10	20	260
	MT								
6	LD		30	50	20	30		50	180
	MT							30	30
7	LD	50	10		90	20	30		200
	MT								
TOTAL	LD	260	250	130	230	260	210	170	1510
	MT			50	40			30	120

TABLE 10
NON-COOPERATIVE MODEL SOLUTION

(a) Covered Hopper Barge Movements

Origin \ Destination		1	2	3	4	5	6	7	TOTAL
1	LD		80		50	20		30	180
	MT								
2	LD	70		40		30	20	60	220
	MT				10				10
3	LD	20	60		50		10		140
	MT				30				30
4	LD	50		60			40	90	240
	MT								
5	LD	10	40	30			50	20	150
	MT								
6	LD	30	20		60	40		50	200
	MT								
7	LD		30	40	40	60	50		220
	MT						30		30
TOTAL	LD	180	230	170	200	150	170	250	1420
	MT				40		30		

TABLE 10
NON-COOPERATIVE MODEL SOLUTION (CONTD.)

(b) Open Hopper Barge Movements

Origin \ Destination		1	2	3	4	5	6	7	TOTAL
1	LD		20	30		60	80	50	240
	MT				20				20
2	LD				20	70	50		140
	MT			90	20				110
3	LD	40	40		30	80	20	10	220
	MT								
4	LD	90	70	20		30	60	40	310
	MT								
5	LD	50	80	30	70			20	250
	MT				40				40
6	LD		30	50	20	30		50	180
	MT							30	30
7	LD	80	10		90	20			200
	MT								
TOTAL	LD	260	250	130	230	290	210	170	1540
	MT			90	80			30	200

TABLE 11
GENERAL MODEL SOLUTION

(a) Covered Hopper Barge Movements

Origin \ Destination		1	2	3	4	5	6	7	TOTAL
1	LD		180						180
	MT								
2	LD	170			60				230
	MT								
3	LD	10			120				130
	MT				40				40
4	LD		50	90		100			240
	MT								
5	LD			80			10	50	140
	MT								
6	LD					10		200	210
	MT								
7	LD				20	30	200		250
	MT								
TOTAL	LD	180	230	170	200	140	210	250	1420
	MT				40				

TABLE 11
GENERAL MODEL SOLUTION (CONTD.)

(b) Open Hopper Barge Movements

Origin \ Destination		1	2	3	4	5	6	7	TOTAL
1	LD		240						240
	MT				20				20
2	LD	130							130
	MT			100	20				120
3	LD	130	10			90			230
	MT								
4	LD			100		210			310
	MT								
5	LD			30	230				260
	MT				40				40
6	LD							170	170
	MT								
7	LD						170		170
	MT								
TOTAL	LD	260	250	130	230	300	170	170	1510
	MT			100	80				180

TABLE 12
SUMMARY STATISTICS

	TOWGEN Solution	W#2	Non-cooperative	General
Cost				
Covered, Loaded	$ 465,200	544,000	504,150	264,900
Covered, Empty	10,450	6,200	2,750	1,000
Open, Loaded	637,160	566,600	602,480	249,800
Open, Empty	16,680	6,060	9,260	8,460
Total	$1,129,490	1,122,960	1,118,640	524,160
Barge Movements				
Covered, Loaded	1,200	1,380	1,350	1,380
Covered, Empty	210	120	70	40
Open, Loaded	1,690	1,510	1,540	1,510
Open, Empty	300	120	200	180
Total	3,400	3,130	3,160	3,110
Distance (in barge miles)				
Covered, Loaded	108,800	127,400	117,600	57,600
Covered, Empty	9,400	5,600	2,400	8,000
Open, Loaded	167,600	149,000	158,800	61,000
Open, Empty	17,200	6,200	9,400	8,600
Total	303,000	288,200	288,200	128,000

TABLE 13
SUPPLY AND DEMAND FOR EMPTY COVERED HOPPERS

		Port Destinations				Supply of Mts.
		1	4	5	6	
Port Origins	2					50
	3					50
	7					110
Demand for Mts.		60	40	40	70	210

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