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AN IMPROVED ARCTANGENT CALCULATION ALGORITHM
FOR DIFAR BEARING COMPUTATION IN THE ASP

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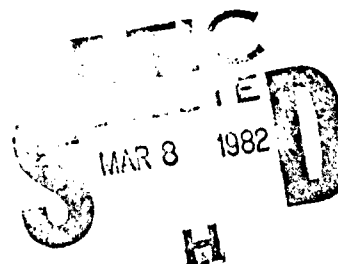
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Arctangent	AU	P-3C															
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APMDD	DIFAR																
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An arctangent calculation algorithm is proposed for the ASP which yields a 2:1 gain in speed over the current algorithm. In addition, the new algorithm is suitable for parallel processing using both ASP Arithmetic Elements (AE's) while the current one is not.																	

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GLOSSARY OF ACRONYMS AND ABBREVIATIONS

AE	Arithmetic Element
AP	Arithmetic Processor
APMDD	Arithmetic Processor Microprogram Description Document
ASP	Advanced Signal Processor
ASW	Anti-Submarine Warfare
AU	Analyzer Unit
DIFAR	Directional Low Frequency Analysis and Recording
ERAPS	Expendable Reliable Acoustic Path Sonobuoy

1.0 BACKGROUND

The ASP arctangent algorithm is used in the computation of DIFAR bearings and also in applications which require calculation of complex phase angles. The algorithm currently available has been implemented in AP microcode and is fully described in reference (a) of Section 2.0. The microprogram name is "BC". The current algorithm is also described in Section 3.0 of this report.

The throughput rate of the "BC" microprogram is low because the current algorithm cannot fully exploit the pipeline features of the AP. The new algorithm provides a 2:1 throughput gain primarily because of its suitability for pipelining. This algorithm was brought to the author's attention by Mr. Morris Plotkin (Code 503) of the Naval Air Development Center.

2.0 APPLICABLE DOCUMENTS

- | | |
|---|---|
| a. CDRL No. A055,
IBM 6259764,
Revision through 2.5.2 | W2140 Computer Microprogram Design
Document, Arithmetic Processor
(APMDD), "BC" Microprogram. |
| b. CDRL No. A008,
IBM Corporation,
30 June 1977 | Advanced Signal Processor Service
Test Model ASP Analyzer Unit
Principles of Operation, Volume 1. |

3.0 DESCRIPTION OF CURRENT ALGORITHM

The current algorithm may be expressed as follows:

$$\theta = \theta_B \text{ +/- } \tan^{-1} (|x|/|y|) \quad (1)$$

where x = East/West Amplitude
 y = North/South Amplitude
 θ_B = Quadrant Base Angle

+/- = "+" for quadrants 1 and 3
 "-" for quadrants 2 and 4

The algorithm executes in the AP of the ASP AU. The AP is a pipelined, micro-programmable unit which has been designed to produce high throughput for signal processing algorithms. To minimize hardware cost, the AP has been provided with limited decision-making capability. It also lacks both hardware divide and inverse tangent functions, since common signal processing algorithms such as Digital Filters and FFT's do not require them.

In the current implementation of the arctangent algorithm, the term $|x|/|y|$ is calculated via iterative loop which yields one additional bit of precision per pass. The loop employs add/subtract operations only. The inverse tangent is approximated with 2 degree precision via an inverse table look-up, implemented by means of a 6-pass binary search. Both the divide and inverse table look-up operations are extremely inefficient.

The alternate implementation to be proposed provides a 2:1 gain in speed by reducing the precision required for the divide and approximating the inverse tangent with a simple polynomial. It also increases the precision of the arctangent from 2 degrees to 1 degree.

4.0 THEORETICAL BASIS FOR THE NEW ALGORITHM

The new algorithm is based on the following trigonometric identity:

$$\tan^{-1}(|x|/|y|) = 45^\circ + \tan^{-1}(r) \quad (2)$$

$$\text{where } r = (|x| - |y|) / (|x| + |y|)$$

This transformation offers two computational advantages for the AP:

- a. $|r|$ requires a dynamic range of only 0 to 1, whereas $|x|/|y|$ requires a range of 0 to infinity. As a result, 7 significant bits in r yield the same precision as 14 significant bits in $|x|/|y|$. Since the calculation time is proportional to the number of significant bits required, r can be calculated twice as fast as $|x|/|y|$.
- b. For $|r| < 1$, $\tan^{-1}(r)$ can be approximated to 1 degree precision by the polynomial:

$$56.31213r - 11.61213r^3 \quad (3)$$

This polynomial can be evaluated in half the time required to perform the binary search currently employed to achieve 2 degree accuracy, primarily because no logical decisions are required.

The computational disadvantages, enumerated below, are small compared to the advantages:

- a. $r = (|x| + |y|) / (|x| + |y|)$ is more difficult to evaluate than $|x|/|y|$.

- b. Since r may be positive or negative and the division technique requires that both numerator and denominator be positive, $|r|$ must be calculated and then complemented as a separate step when $|y| > |x|$. The division technique is described in Section D.8 of reference (b).

5.0 PROPOSED IMPLEMENTATION

The arctangent algorithm expressed by Equation (2) shall be approximated to 1 degree accuracy by:

$$\theta = |\theta_B' + 56.31213r - 11.61213r^3| \quad (4)$$

where $r = (|x| - |y|) / (|x| + |y|)$

and θ_B' = quadrant base angle provided in the Table below

x = East/West Amplitude or imaginary part

y = North/South Amplitude or real part

Quadrant	x	y	θ_B'	θ_B' (Hex)
1	$x > 0$	$y > 0$	$+45^\circ$	002D
2	$x > 0$	$y < 0$	-135°	FF79
3	$x < 0$	$y < 0$	$+225^\circ$	00E1
4	$x < 0$	$y > 0$	-315°	FEC5

Table 1 Quadrant Base Angles

The quantity "r" shall be obtained by calculating $|r|$ via the iterative method described in Section D.8 of reference (b) and then correcting the algebraic sign as required. The 45 degree angle increment arising from the transformation, $\tan^{-1}(|x|/|y|) = 45^\circ + \tan^{-1}(r)$, has been incorporated into the quadrant base. The quadrant-dependent +/-sign in equation 1 has been handled by complementing the quadrant base for quadrants 2 and 4 and then taking the absolute value of the final result. This procedure is more efficient for computation since it eliminates the need for logical decisions.

6.0 SIZING AND TIMING

The microcode required for implementation of the above has been roughly written by the author in order to demonstrate feasibility and obtain sizing and timing estimates. The following results were obtained:

Sizing: 66 microinstructions - new implementation
60 microinstructions - current implementation

Timing: $2.2 + 3.4N$ usec - new implementation
 $26.0 + 7.1 N$ usec - current implementation

where N = number of arctangents to be calculated

The sizing figures indicate that the amount of microstorage required is nearly comparable between the two implementations. With regard to Working Storage utilization, the 48 word tangent table, used currently in conjunction with the binary search for inverse tangent approximation, is no longer needed. The additional Working Storage available under the new implementation may be used to process larger input data sets.

The above timing figures indicate that the execution time is more than halved for the new implementation. An additional timing advantage results from the fact that two AE's may now be used to process a pair of input data sets in parallel. Two AE's may not be utilized with the current implementation because the binary search requires data - dependent branching. Since both AE's must execute the same microinstruction at all times (there is only one AE Controller), data-dependent branching is not possible.

7.0 SCALING CONSIDERATIONS

The multiple constants 56.31213 and -11.61213 must be expressed as 16-bit fixed-point values. For maximum precision, a scaling index, SI=7, shall be used. SI=7 indicates that the decimal point is placed after the seventh bit, counting left to right in the 16-bit word. With this scaling:

56.31213 shall be expressed as 28,832, base 10, which equals hex 70A0.

-11.61213 shall be expressed as -5945, base 10, which equals hex E8C7.

The integer values are obtained by multiplying each constant by 512 and rounding. The negative constant is expressed in hexadecimal as a two's complement.

The ratio, r , shall be calculated by the microprogram with SI=1 and 7 significant bits (plus sign). The low eight bits will always be zero. The arctangent shall be expressed as a pure integer, that is, with SI=16. Therefore, to achieve proper scaling, equation (4) must be implemented as follows:

$$\theta = \theta_B + |(56.31213r/4 - 11.61213r^3)/64| \quad (5)$$

The term $56.31213r$ has $SI = 7 + 1 = 8$. The term $- 11.61213 r^3$ has $SI = 7 + 1 + 1 + 1 = 10$. Division of $56.31213r$ by 4 changes the SI from 8 to 10 so that both terms will be scaled equivalently before addition. Division of the sum by 64 changes the resultant SI from 10 to 16, as required. The hex values of θ_B' in the last column of Table 1 are expressed with $SI = 16$ so no scaling of θ_B' is required.

The divisions by 4 and 64 may be accomplished via AP hardware through the use of a scale factor register (refer to reference (b)). The division by 4 may be achieved via a 2-bit pre-scaler right-shift. The division by 64 may be achieved via a 6-bit post-scaler right-shift.

8.0 IMPACT OF THE NEW ALGORITHM ON EXISTING OPERATIONAL SOFTWARE

If a microprogram employing the new implementation were coded and used in place of the current microprogram, the improvement in performance for two typical operational software packages is described below:

- a. P-3C AU Operational Software/DIFAR Processing Option: Approximately 7.5 ms per PIC would be saved each time ALI output was requested. This savings would not significantly reduce total loading requirements. It would, however, tend to offset the additional load required to implement double-precision integration of the directional channels, required to solve the current DIFAR bearing accuracy problem in the presence of directional noise.
- b. Proposed AU Operational Software for ERAPS: Approximately 1.7% of total throughput capacity would be saved per sensor during operation in the ERAPS bearing resolve mode. This saving, although non-negligible, would not significantly reduce total loading requirements.

9.0 CONCLUSION

The proposed implementation of the Bearing Calculation algorithm for the ASP Analyzer Unit can improve effective execution time by more than 4:1 without a significant increase in microstorage requirements. Its effect on the total loading of current and planned operational software for the AU, however, is small. Nevertheless, it is recommended that a new microprogram be developed for future use. Its effect in reducing total loading could be significant for applications which make more extensive use of DIFAR bearing or complex phase calculations. The phase of any complex quantity may be calculated via the same algorithm.

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