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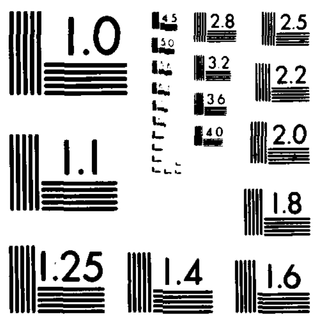

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FURTHER DEVELOPMENT OF THE STREAMLINE METHOD FOR DETERMINATION OF THREE-DIMENSIONAL FLOW SEPARATION

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Abstract

The streamline method for the determination of three-dimensional flow separation has been further exploited with emphasis on improving the friction model. Two approaches (numerical and analytical) for providing the required friction distribution are studied. The close agreement between the calculated line of separation based on numerical friction data and the original three-dimensional boundary-layer solution implies that the streamline method is a valid economical approach for determining the vortex-layer type flow separation if proper friction information is provided. The use of a simple analytical friction formula, on the other hand, only leads to qualitative results.

Introduction

A streamline approach for determining the free vortex-layer type, three-dimensional flow separation was developed recently.[1] This approach is based on the Maskell postulation about separation patterns in three dimensions. The line of separation is determined by the envelope of converging streamlines inside the viscous layer. To simulate the viscous effect, a friction model consisting of inertia terms multiplied by empirical parameters was proposed.

The method has been exploited further with emphasis on improving the friction model. As in the previous paper, a prolate spheroid in an incompressible flow at moderate angle of attack is considered as an illustration. An analytical friction formula as well as numerical friction data, have been employed in place of the previously proposed empirical model. Results of such uses on three-dimensional flow separation patterns over a prolate spheroid of $a/b = 6$ at $\alpha = 10$ deg are presented in the present paper.

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Streamline Method

The streamline method for the determination of the vortex-layer type three-dimensional flow separation consists of three first-order ordinary differential equations. In particular, for the case of a prolate spheroid at an angle of attack, these equations take the axisymmetric form

$$\frac{D\Theta}{DS} = \frac{1}{\gamma M^2 P} \left[\left(\frac{\partial P}{\partial x} - \frac{\partial \tau_x}{\partial z} \right) \sin\Theta - \left(\frac{1}{f} \frac{\partial P}{\partial \phi} - \frac{\partial \tau_\phi}{\partial z} \right) \cos\Theta \right] - \frac{df}{dx} \frac{\sin\Theta}{f} \qquad (1)$$

$$\frac{D\phi}{DS} = \frac{\sin\Theta}{f} \qquad (2)$$

$$\frac{Dx}{DS} = \cos\Theta \qquad (3)$$

where f = radial distance measured from the centerline

S = the distance along a viscous streamline inside the boundary layer (see Fig. 1)

x = the distance along the body surface of a constant ϕ plane

Θ = the local streamline angle measured with respect to the x coordinate

τ = shear stress

ϕ = the azimuthal angle measured from the most windward line (see Fig. 2)

For a prolate spheroid, f is defined by

$$f = b \left[1 - \left(\frac{\bar{x}}{a} - 1\right)^2 \right]^{1/2} \qquad (4)$$

Other symbols have their usual meaning.

For a prolate spheroid at incidence, values of pressure gradients are readily obtained by a closed-form potential-flow solution

$$P = P_\infty + \frac{\rho_\infty V_\infty^2}{2} \left[1 - \frac{u^2}{V_\infty^2} - \frac{v^2}{V_\infty^2} \right] \qquad (5)$$

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The pressure gradients are readily obtained using the relations[1]

$$\frac{\partial P}{\partial x} = -\rho_\infty V_\infty^2 \frac{u}{V_\infty} \left[\frac{2(\frac{\bar{x}}{a}) - (\frac{\bar{x}}{a})^2}{1 - e^2(\frac{\bar{x}}{a} - 1)^2} \right]^{1/2} \frac{\partial}{\partial \bar{x}} (\frac{u}{V_\infty}) \qquad (6)$$

$$\frac{\partial P}{\partial \phi} = -\rho_\infty V_\infty^2 \left[\frac{u}{V_\infty} \frac{\partial}{\partial \phi} (\frac{u}{V_\infty}) + \frac{v}{V_\infty} \frac{\partial}{\partial \phi} (\frac{v}{V_\infty}) \right] \qquad (7)$$

where e is the eccentricity of the ellipsoid. Expressions for u/V_∞ and v/V_∞ given by Wang,[2] are also found in Ref. 1.

With these equations and with the knowledge of the friction distribution for $\partial\tau_x/\partial z$ and $\partial\tau_\phi/\partial z$, the viscous streamlines can be calculated inside the three-dimensional boundary layers. The flow separation line is then determined by tracing the envelope of converging viscous streamlines.[1]

Friction Distribution

The stumbling block in the problem lies, therefore, on the availability of the knowledge of the friction distribution without solving the complex three-dimensional boundary-layer equations. In addition to the earlier attempt,[1] a numerical approach and an analytical approach are used to provide the required friction information. The use of the numerical approach is solely as a check of the accuracy of the method, so that it should not be regarded as a normal course of computations.

Numerical Friction Data

The numerical friction data, provided by Cebeci, are based on the three-dimensional boundary-layer solution procedures described in Ref. 3. The distributions of shear stress and its normal derivative at the wall were computed for a prolate spheroid with body thickness ratio, b/a = 1/6, [illegible] angle of attack α = 10 deg. The Reynolds number was 7.2×10^6 based on [illegible] stream velocity of 54 m/sec (177.17 ft/sec) and a total length of the body, 2a [illegible] m (6.56 ft).

To facilitate the numerical implementation, the friction data were curve fitted in accordance with the least squares method. Fourth-order polynominals were used for fitting the $\partial\tau_x/\partial z$ data and fifth-order polynominals for the $\partial\tau_\phi/\partial z$ data. Discontinuities of the numerical data that occur when $\tau_x \leq 0$ or $\tau_\phi \leq 0$ have been smoothed out by the curve fit. Plots of these curves are shown in Figs. 3 and 4.

Analytical Friction Formula

Attempts were also made to employ approximate analytical expressions for the friction model. As opposed to the balance-of-force type empirical form previously used in Ref. 1, a direct shear stress approach is undertaken in the present work. The shear stresses of interest are those in a surface above the wall where the viscous streamlines are to be calculated. In particular, the surface inside the boundary layer having a velocity $u = 0.707\ u_e$ is chosen for computing the viscous streamlines.

The shear stress for a two-dimensional turbulent boundary layer can be expressed approximately by taking over from the circular pipe equation, Eq. (20.12) of Ref. 4,

$$\tau = 0.0225\ \rho u^{7/4} \left(\frac{\nu}{z}\right)^{1/4} \tag{8}$$

This equation is valid for any wall distance z. Using the 1/10-th-power law velocity distribution (for high Reynolds number flows):

$$\frac{u}{u_e} = \left(\frac{z}{\delta}\right)^{1/10} \tag{9}$$

Equation (8) can be expressed in the form

$$\tau = 0.0225\ \rho u_e^{7/4}\ \delta^{-7/40}\ \nu^{1/4}\ z^{-3/40} \tag{10}$$

Differentiating Eq. (10) with respect to z yields the normal derivative of shear stress in the viscous layer having $u = 0.707\ u_e$

$$\frac{\partial\tau}{\partial z} = -0.075\ \frac{\tau}{z} = -2.4\ \frac{\tau}{\delta} \tag{11}$$

where $z = 0.0312\ \delta$, which corresponds to $u = 0.707\ u_e$ in accordance with Eq. (9), has been used.

The boundary-layer thickness δ is obtained using the flat plate solution with the aid of 1/10-th-power law velocity distribution

$$\delta = 0.453\ (x)\ \left(\frac{\nu}{u_e x}\right)^{1/5} \tag{12}$$

The shear stress at $z = 0.0312\ \delta$ is

$$\tau = 0.0292\ \rho u_e^{\,2}\ \left(\frac{\nu}{u_e \delta}\right)^{1/4} \tag{13}$$

It is assumed that the crossflow is small so that Eq. (11) may be valid in the streamwise direction. Substituting the streamline distance for x and the overall velocity at the edge of boundary layer for u_e, $\partial\tau/\partial z$ will be valid along a streamline. Its components are

$$\frac{\partial \tau_x}{\partial z} = \frac{\partial \tau}{\partial z} \cos \Theta \tag{14}$$

$$\frac{\partial \tau_\phi}{\partial z} = \frac{\partial \tau}{\partial z} \sin \Theta \tag{15}$$

The plots of these components for the case of a prolate spheroid of thickness ratio $b/a = 1/6$ at $\alpha = 10$ deg and $V_\infty = 54$ m/sec (177.17 ft/sec) are shown in Figs. 5 and 6. These conditions are chosen to be exactly the same as for numerical friction data so that a direct comparison can be made.

The small-crossflow assumption that restricts the validity of Eqs. (14) and (15) to bodies at small angle of attack almost prevents the consideration of most flow separation problems. Nevertheless, it is of interest to see how far a simple analytical friction formula can be of use in the subject problem.

Results and Discussion

Results of three-dimensional flow separation over a prolate spheroid with body thickness ratio 1/6 at $\alpha = 10$ deg are presented using: (a) numerical friction data, (b) an analytical friction formula, or (c) zero friction (pure inviscid potential flow). For cases (a) and (b), the viscous streamlines were calculated inside the boundary layers at mean velocities about 0.707 of those at the edge of the boundary layer.

The results of case (a), that is, based on the numerical friction data, are shown in Fig. 7. The calculated line of separation by the present method coincides with the locus of flow separation corresponding to $\tau_x \leq 0$ in the original three-dimensional boundary-layer solution of Cebeci. The close agreement of the presently calculated result with the original solution implies that the streamline method is a valid economical approach for determining the line of separation, if proper friction information is provided. It also sheds some light on the formulation of the analytical friction model, on which the present method mainly relies.

The line of separation found by using the simple analytical form, case (b) on the other hand, lies further downstream; see Fig. 8. The length of the separation line is also reduced considerably. The difference is apparently caused by less severe variations of the shear stress derivatives from the analytical formula than from the numerical data. Particularly, the analytical $\partial\tau_\phi/\partial z$ does not exhibit any change from negative to positive values (see Fig. 6), which are the main driving forces for the flow separation. Therefore, the result can only be regarded as qualitatively correct.

The result is encouraging in the sense that there is plenty of room for improvement to the friction formula, and thus for the accuracy of the line of separation. In particular, the small crossflow assumption can be removed easily either by a correcting procedure or by a more realistic consideration of the crossflow theory.

Finally, for comparison purposes, the pure inviscid flow pattern, case (c), is depicted in Fig. 9. All of the streamlines merge to the rear stagnation point and leave the body surface. Since there is no friction, there is no separation. The curvatures of the inviscid streamlines are noticeably smaller than those of viscous streamlines, which are consistent with similar comparison observed by Han and Patel[5].

Concluding Remarks

The following are conclusions based on the present work

1. The streamline method is a valid economical approach for determining the vortex-layer type, three-dimensional flow separation if proper friction and pressure distributions are provided.

2. The use of a simple analytical friction formula in the streamline method produces only qualitatively correct results on the line of separation.

Acknowledgement

The author wishes to thank T. Cebeci of Douglas Aircraft Company for providing the numerical friction data.

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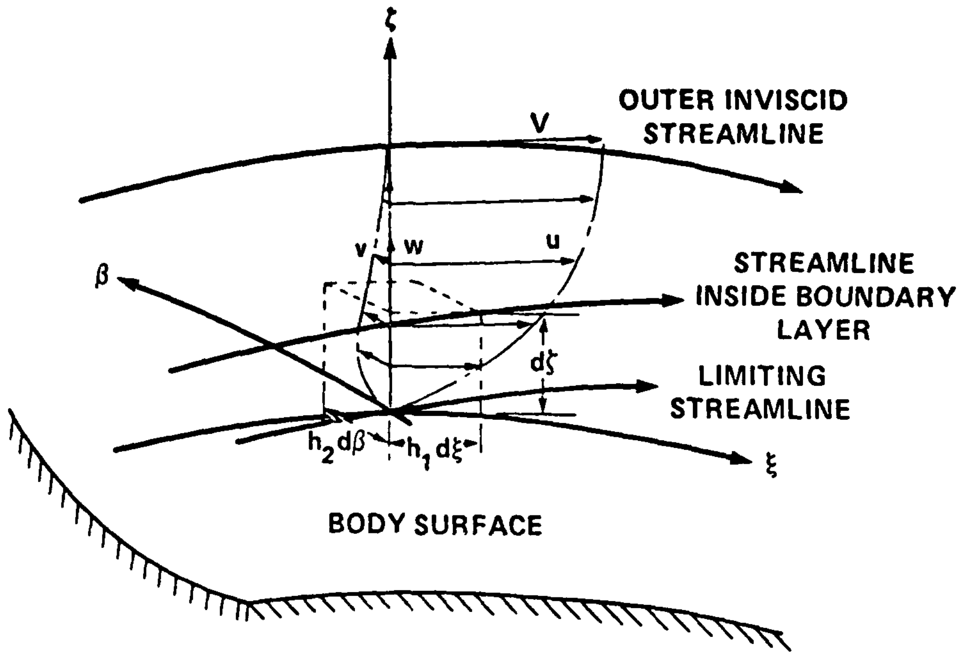


Fig. 1 - Streamline in a Boundary Layer

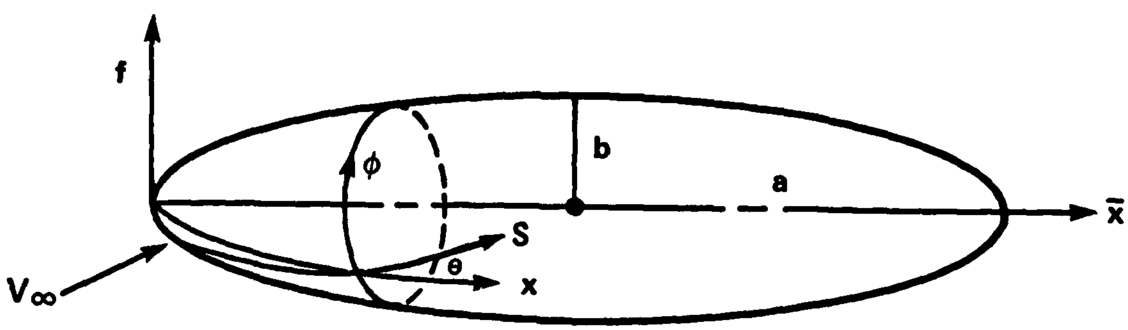


Fig. 2 - Coordinate System for a Prolate Spheroid

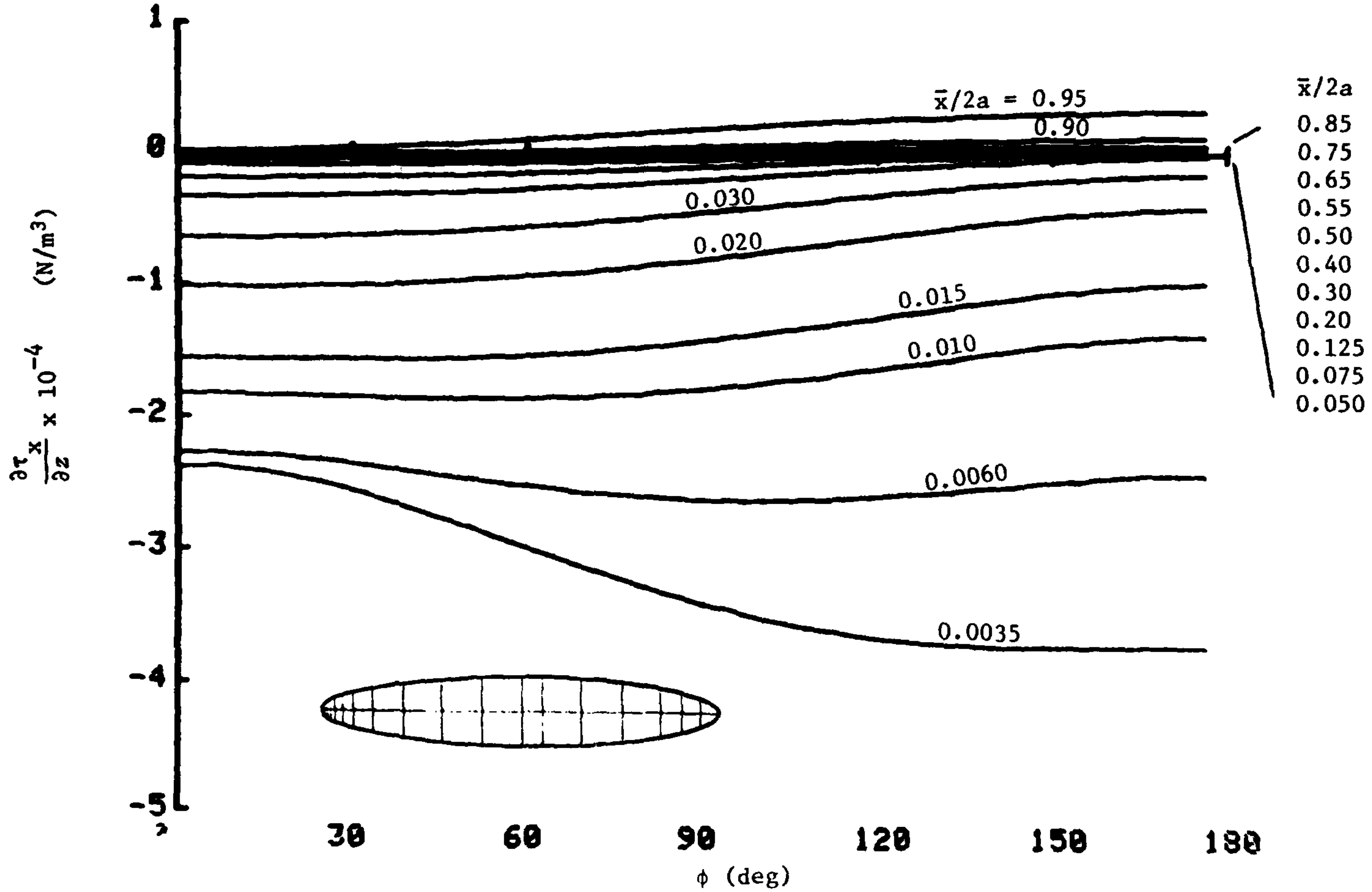


Fig. 3 - Least Squares Fit of Numerical Normal Derivatives of Longitudinal Shear Stress over a Prolate ($a/b = 6$, $\alpha = 10$ deg, and $Re = 7.2 \times 10^6$)

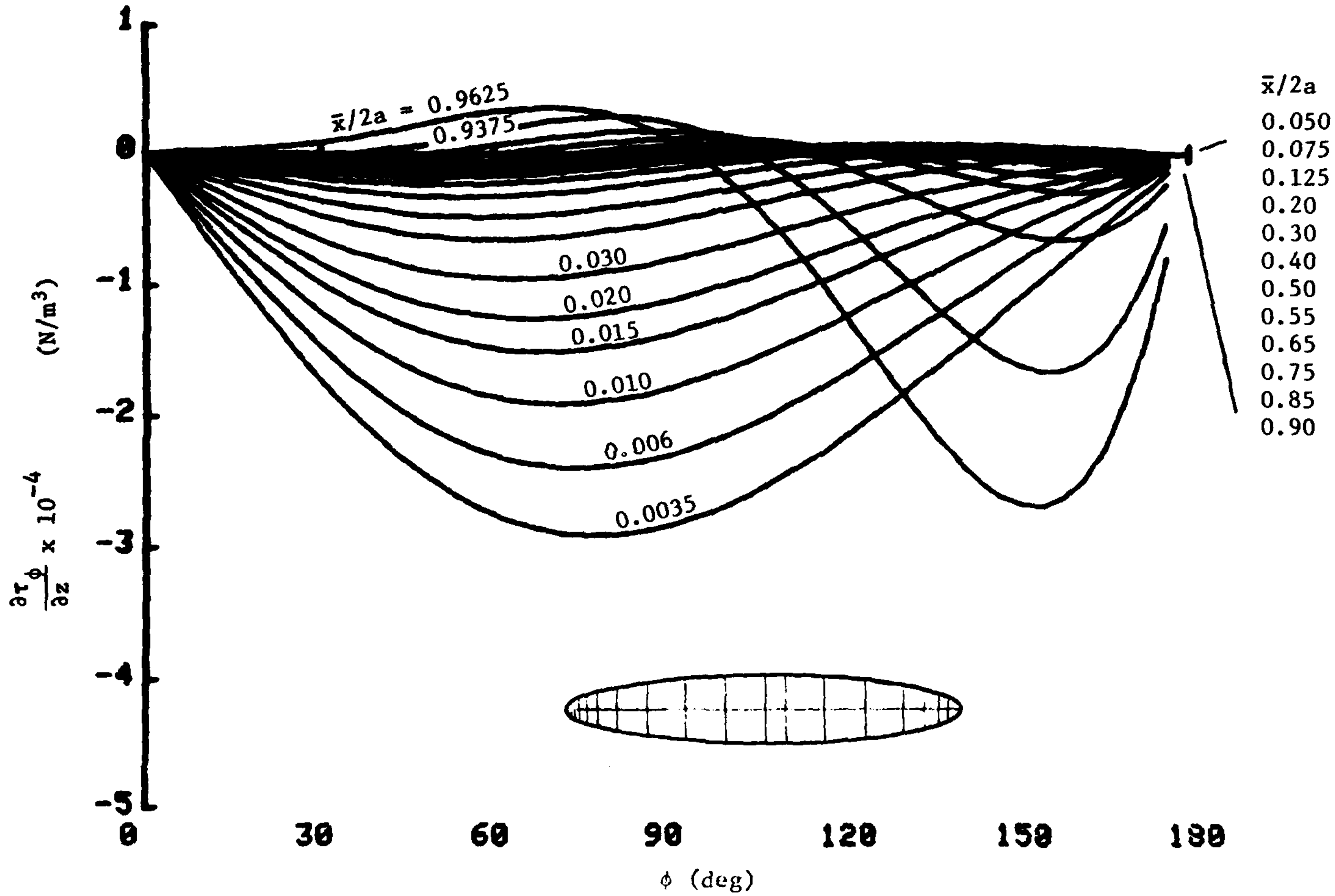


Fig. 4 - Least Squares Fit of Numerical Normal Derivatives of Transverse Shear Stress over a Prolate Spheroid ($a/b = 6$, $\alpha = 10$ deg, and $Re = 7.2 \times 10^6$)

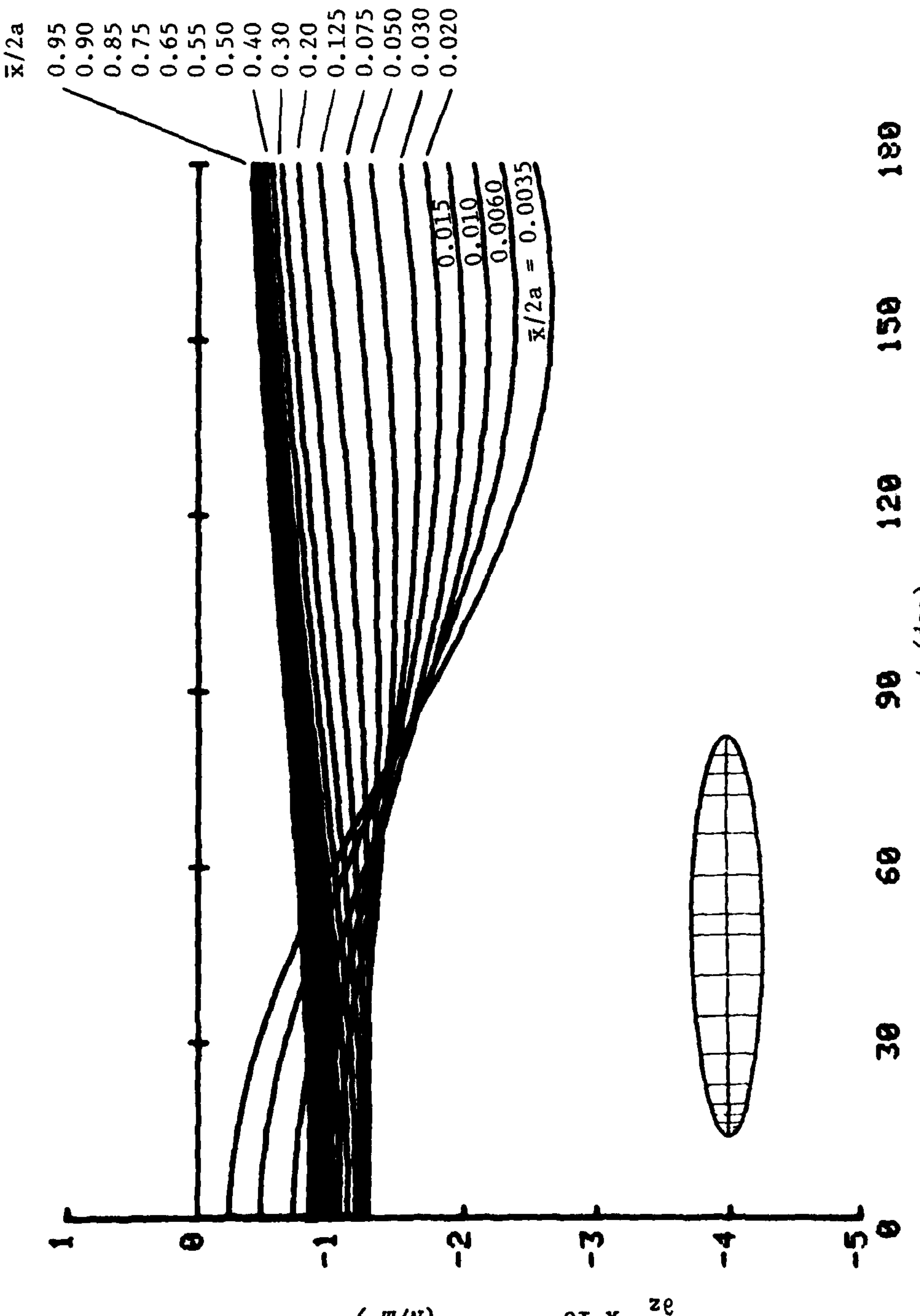


Fig. 5 - Analytical Normal Derivatives of Longitudinal Shear Stress over a Prolate Spheroid ($a/b = 6$, $\alpha = 10$ deg, and $Re = 7.2 \times 10^6$)

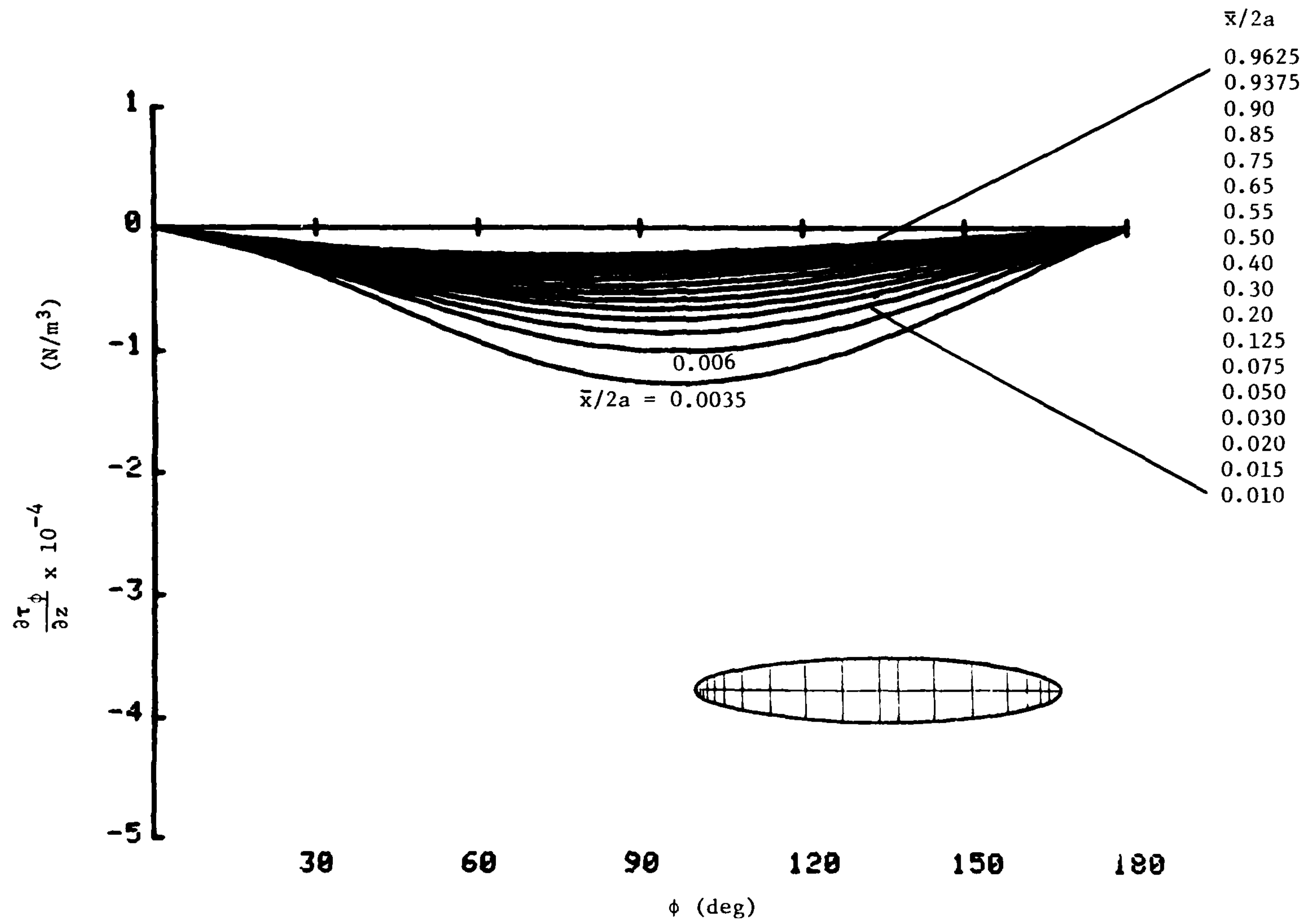
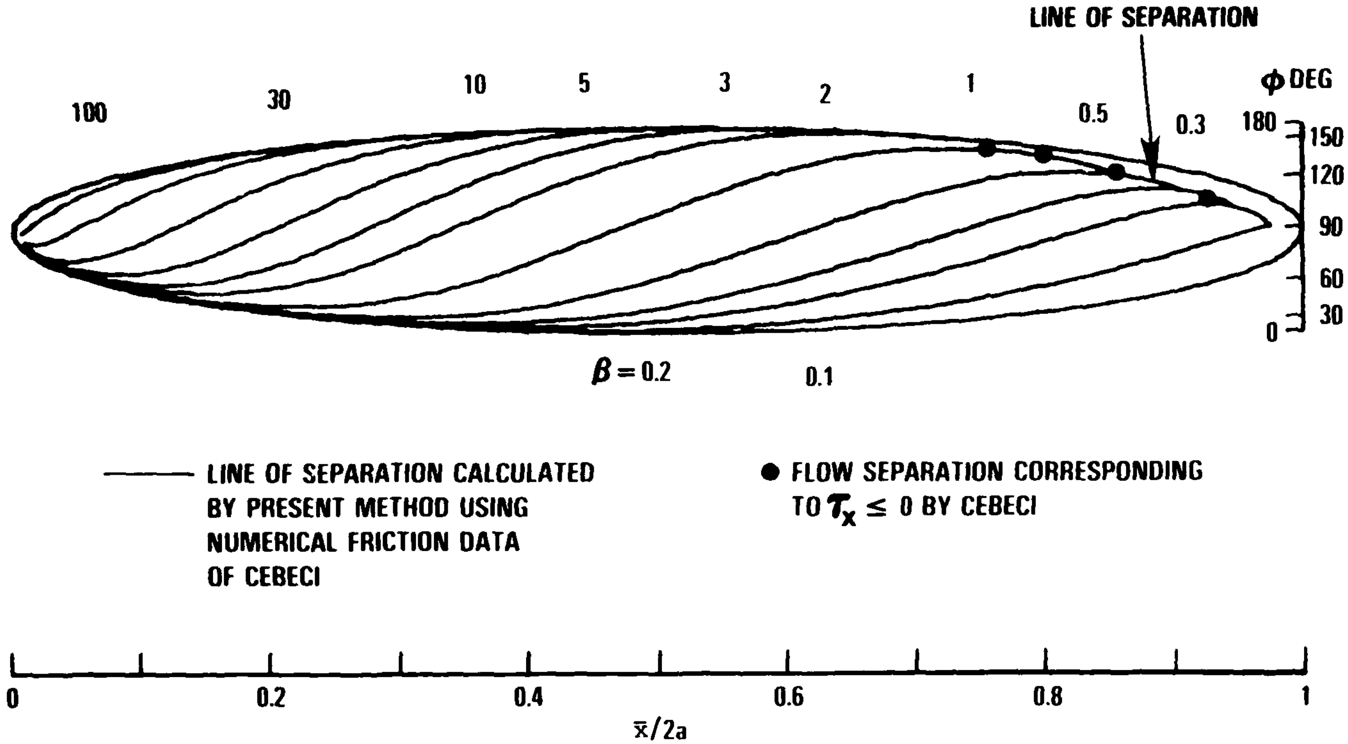
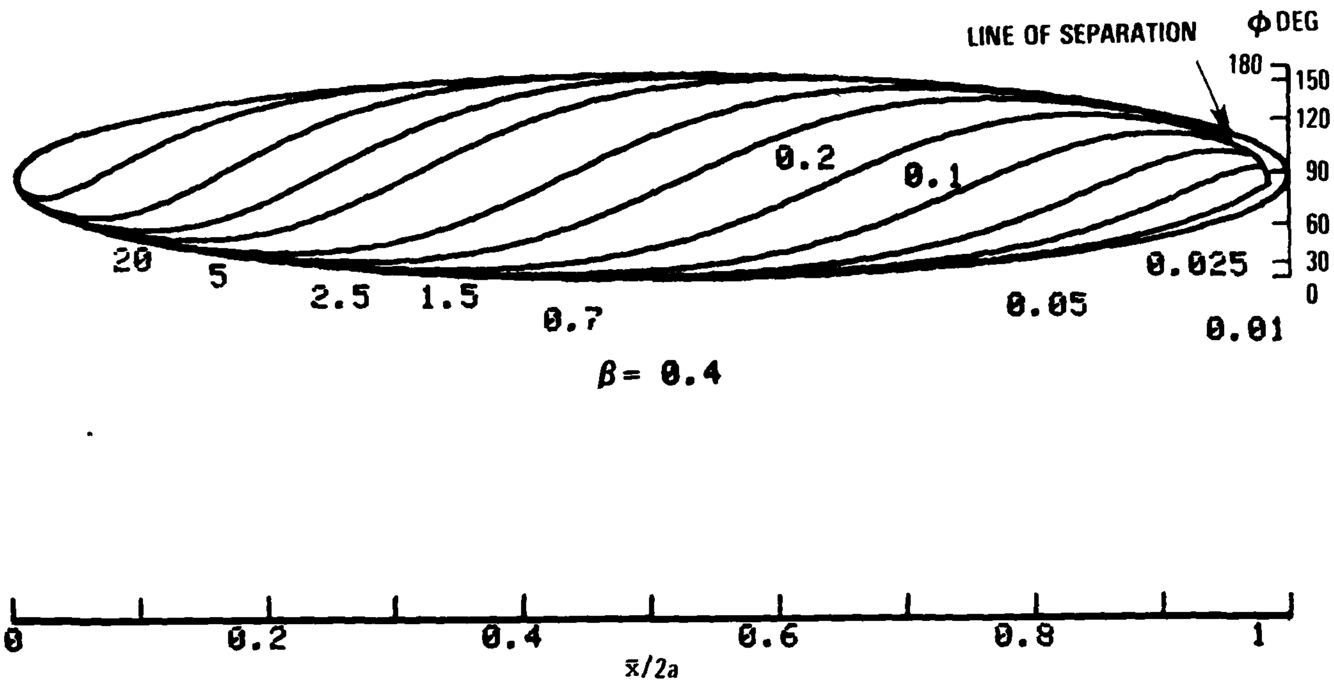


Fig. 6 - Analytical Normal Derivatives of Transverse Shear Stress over a Prolate Spheroid
(a/b = 6, α = 10 deg, and Re = 7.2×10^6)



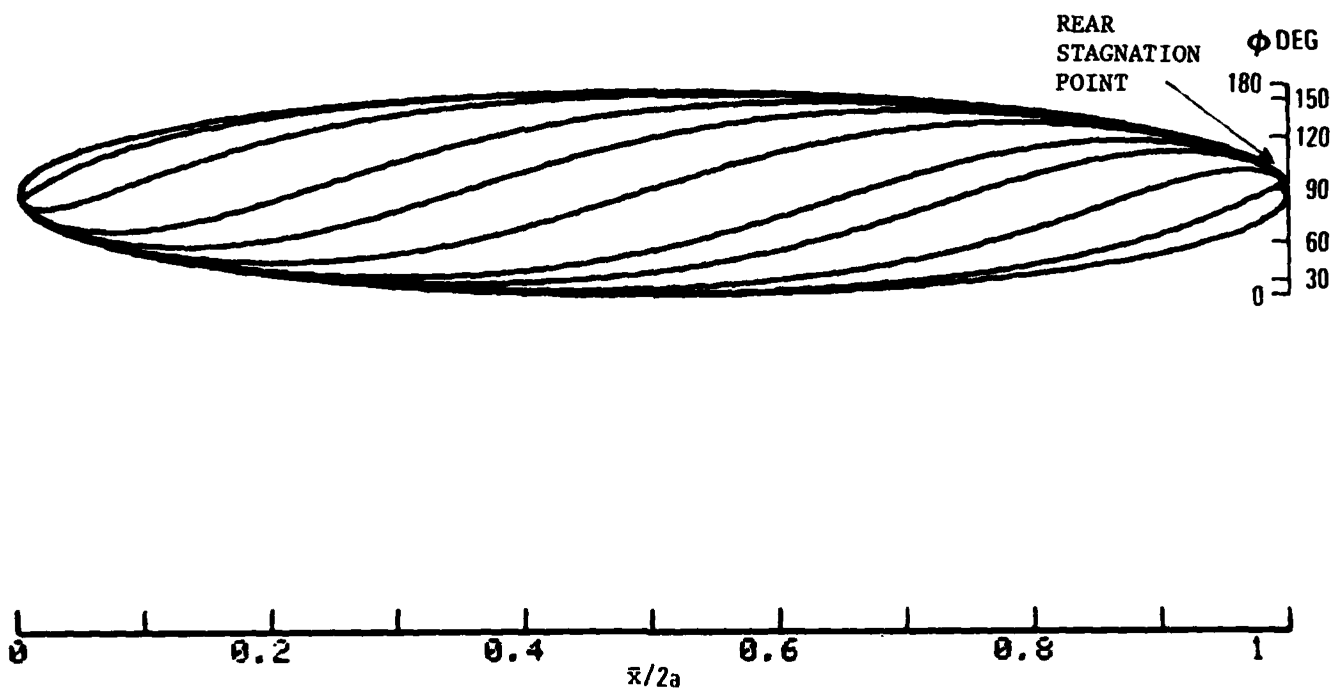
$\alpha = 10$, $a/b = 6$, $V_\infty = 54$ m/sec (177 ft/sec), $(u/u_e)^2 = 0.5$

Fig. 7 - Line of Separation Obtained Using Numerical Friction Data in the Streamline Method



$\alpha = 10$, $a/b = 6$, $V_\infty = 54$ m/sec (177 ft/sec), $(u/u_e)^2 = 0.5$

Fig. 8 - Line of Separation Obtained Using Analytical Friction Formula in the Streamline Method



$\alpha = 10$, $a/b = 6$, $V_\infty = 54$ m/sec (177 ft/sec)

Fig. 9 - Inviscid Flow Pattern

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Fig. 9 - Inviscid Flow Pattern

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