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T-MATRIX ANALYSIS OF ACOUSTIC WAVE SCATTERING FROM THIN ELASTIC--ETC(U)
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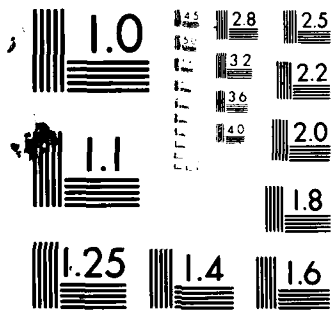

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T-MATRIX ANALYSIS OF ACOUSTIC WAVE SCATTERING FROM THIN ELASTIC SHELLS

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For the Period
May 1, 1980 - April 30, 1982

DEPARTMENT OF THE NAVY
Office of Naval Research
Arlington, Virginia 22217

Contract No. N00014-80-C-0573

October, 1981

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FINAL REPORT

RF Project 712845/762148

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October, 1981

Introduction

In the last five years, much progress has been made in the study of acoustic wave scattering by elastic obstacles immersed in water using the T-matrix approach. Excellent agreement with experiments was demonstrated for the finite elastic cylinder[1,2] with spherical end caps. The T-matrix approach was then extended to the problem of elastic shells[3] immersed in water. Numerical calculations of the frequency dependence of acoustic wave scattering by prolate and oblate spheroidal elastic and viscoelastic shells were presented in Ref. 3. However, since the problem was formulated exactly in the shell region, several numerical difficulties arose as the thickness of the shell was decreased and the frequency increased. It is understood, that in problems of interest to the Navy, scattered field data is required for long, thin bodies of revolution whose wall thickness is very small compared to the wavelength of the incident wave but whose overall *dimensions* are comparable and often larger than the wavelength of the incident wave.

This project was begun to precisely address such problems, taking into account and exploiting the thinness of the shell. We propose to use shell theory equations rather than the full elasticity equations in region I (see figure 1) in order to avoid some of the complications encountered in Ref. 3. Shell theory reduces the parti[illegible]ferential equations in three coordinates to a *higher order* equation in two coordinates. All description of the displacement and stress fields is made with respect to a reference surface in the shell. Now, it is no longer necessary to invoke integral representations to describe the elastic field in this region. One extracts from the shell equations an expression for the impedance matrix of the shell which is then interfaced with the T-matrix approach for acoustic wave scattering problems.

The null field equations

Consider a closed obstacle of piecewise smooth surface S with an outward normal $\hat{n}$ which is immersed in water. A plane harmonic acoustic wave of frequency ω is incident on the obstacle. The total field in the water consists of the incident field ϕ_o and the scattered field ϕ_s. The integral representation of these fields is well known [4,5] and takes the form

$$\left.\begin{aligned} -\phi_o(\vec{r});\ \vec{r} \text{ inside } S \\ \phi_s(\vec{r});\ \vec{r} \text{ outside } S \end{aligned}\right\} = \frac{1}{4\pi} \int_S \Big\{ \big(\hat{n}' \cdot \nabla' g(\vec{r},\vec{r}')\big)\, \phi_+(\vec{r}') - g(\vec{r},\vec{r}')\, \big(\hat{n}' \cdot \nabla'_+ \phi(\vec{r}')\big) \Big\}\, dS(\vec{r}') \tag{1}$$

where $g(\vec{r},\vec{r}')$ is the Green's function of the scalar Helmholtz equation for an infinite medium and ϕ is the velocity potential. The expressions for 'g' are known in both 2-D and 3-D

$$g(\vec{r},\vec{r}') = i\pi\, H_o^{(1)}\, (k|\vec{r}-\vec{r}'|)\ ;\quad \text{2-D} \tag{2}$$

$$g(\vec{r},\vec{r}') = \exp i\, k|\vec{r}-\vec{r}'|\ /\ k|\vec{r}-\vec{r}'|\ ;\quad \text{3-D} \tag{3}$$

In the T-matrix approach we expand the incident and scattered fields in eigen functions of the wave equations as follows

$$\phi_o(\vec{r}) = \sum_n A_n \mathrm{Re}\phi_n(\vec{r}) \tag{4}$$

and

$$\phi_s(\vec{r}) = \sum_n f_n \mathrm{Ou}\phi_n(\vec{r}) \tag{5}$$

where ϕ_n are the circular cylindrical functions in 2-D and the spherical functions in 3-D. The index 'n' is a super index that represents all the required subscripts for the wavefunction. The qualifier 'Re' stands for functions that are regular at the origin and 'Ou' represents functions that are outgoing at infinity. Thus these expansions are consistent with our knowledge of the regularity of the incident field in the region occupied by the obstacle and the radiation conditions satisfied by the scattered field at distances far from s. A detailed description of the wavefunctions for 2-D and 3-D problems may be found in Ref. 4. It may be noted that since only scalar waves are involved, we may use elliptical or spheroidal functions as appropriate for the expansion of the fields[6]. Thus ϕ_n represents any complete, orthogonal set of solutions of the scalar Helmholtz equation.

The expansion of the Green's function is also known and takes the form

$$g(\vec{r},\vec{r}') = i\left(\frac{\pi}{4}\right)\sum_n \text{Ou } \phi_n(r_>) \text{ Re } \phi_n(r_<) \tag{6}$$

where $r_>$ and $r_<$ stand for the greater and lesser of $\vec{r}$ and $\vec{r}'$ respectively. The factor π is used in 2-D and the factor k in 3-D

Substituting eqs. (4)-(6) in eq. (1) we arrive in the usual manner at

$$\begin{Bmatrix} -a_n \\ f_n \end{Bmatrix} = \frac{i}{4\pi}\binom{\pi}{k} \int_S \left[\hat{n}\cdot\nabla\begin{Bmatrix}\text{Ou}\phi_n\\ \text{Re}\phi_n\end{Bmatrix}\phi_+ - \begin{Bmatrix}\text{Ou}\phi_n\\ \text{Re}\phi_n\end{Bmatrix}\left(\hat{n}\cdot\nabla_+\phi\right)\right]dS \tag{7}$$

In order to solve for the known scattered field coefficients 'f_n'

in terms of the known incident field coefficients 'a_n' we must specify ϕ_+ and $\hat{n} \cdot \nabla_+ \phi$ on the surface S of the elastic shell. Since ϕ is the velocity potential, for time harmonic waves, ϕ_+ is related to the surface pressure, P_+ and $\hat{n} \cdot \nabla_+ \phi$ to the normal component of the particle velocity, $\hat{n} \cdot v_+$ on the surface. Thus, since

$$p(\vec{r}) = i \omega \rho \phi (\vec{r}) \tag{8}$$

and

$$\vec{v}(\vec{r}) = \nabla\phi(\vec{r}) \tag{9}$$

equation (7) may be rewritten as

$$\begin{Bmatrix} -a_n \\ f_n \end{Bmatrix} = \frac{i}{4\pi}\begin{pmatrix} \pi \\ k \end{pmatrix} \int_S \left[\left(\hat{n} \cdot \nabla \begin{Bmatrix} Ou \\ Re \end{Bmatrix} \phi_n\right) \frac{-iP_+}{\rho\omega} - \begin{Bmatrix} Ou \\ Re \end{Bmatrix} \phi_n \left(\hat{n} \cdot \vec{v}_+\right)\right] dS \tag{10}$$

where ρ is the mass density of the water. We now proceed to discuss the use of shell theory to represent the pressure and normal velocity on the surface of the shell keeping in mind that the boundary conditions at S are

$$P_+(\vec{r}) = P_-(\vec{r}) \quad ; \quad \vec{r} \text{ on } S \tag{11}$$

and

$$\hat{n} \cdot \vec{v}_+(\vec{r}) = \hat{n} \cdot \vec{v}_-(\vec{r}) \quad ; \quad \vec{r} \text{ on } S. \tag{12}$$

Impedance matrix for elastic shells

Consider a coordinate system on the surface S that is 2-D and tangent to the surface at every point. Let (ξ,η) represent these two coordinates. Let u_i be the components of the displacement field in ζ, ξ and η directions respectively, where ζ is a coordinate normal to the shell surface. If the thickness '2h' of the shell is small compared to its other dimensions, it is appropriate to assume that u, v and w are functions of ξ and η only. Let ψ_n be a complete set of functions that are orthogonal on the surface S. Then we may write

$$u_i(\xi, \ ) = \sum_n \alpha_n^i \psi_n(\xi, \ ) \quad ; \quad i = 1,2,3 \tag{13}$$

The set of coefficients α_n^i can be considered the generalized coordinates of the problem. This approach has been previously used by Junger[7] to analyze the scattering of acoustic wave circular cylindrical shells of infinite length when waves are incident normal to the axis of the cylinder.

We may set up the Lagrangian for the shell as

$$L = T - V \tag{14}$$

where T is the kinetic energy density and V the potential energy density. due to the elastic strain energy of the shell. Hence

$$T = h\rho \int_S [\dot{u}_1^2 + \dot{u}_2^2 + \dot{u}_3^2] dS \tag{15}$$

and

$$V = \int_S F\left\{u_1, u_2, u_3\right\} dS \tag{16}$$

The function F that is quadratic in the displacements u_i may be constructed from shell theory.

In shell theory approximations, one considers various contributions to the elastic strain energy that includes effects such as the membrane effect, bending, rotary inertia, transverse shear etc. These effects must be included as appropriate for the particular problem on hand with consideration given to the frequency of the incident harmonic wave and the exact shape of the shell. At the present time this part of the study is still incomplete. However, this does not prevent us from proceeding further with the formulation.

Lagrange's equations for the system takes the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}_n^1}\right) + \frac{\partial V}{\partial \dot{\alpha}_n^1} = Q_n \quad ; \tag{17}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}_n^2}\right) + \frac{\partial V}{\partial \dot{\alpha}_n^2} = 0 \quad ; \tag{18}$$

and

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}_n^3}\right) + \frac{\partial V}{\partial \dot{\alpha}_n^3} = 0 \tag{19}$$

Where Q_n is the generalized force due to the pressure exerted by the fluid outside the shell. Since the fluid is assumed to be non-viscous, the force acts only in the normal direction i.e., along the coordinate ζ. The generalized force in the other two directions are zero.

An expression for Q_n can be obtained by giving the shell a vertical displacement δu in the ζ direction. If P_+ is the total pressure on the shell surface, then

$$Q_n = - \int_S P_+ \, \psi_n(\xi,\eta) dS \tag{20}$$

The total pressure on S is expanded as follows

$$P_+(\vec{r}) = \sum_m \beta_m \psi_m(\xi,\eta) \quad ; \quad \vec{r} \text{ On } S \tag{21}$$

Thus

$$Q_n = - \sum_m \beta_m \int_S \psi_m(\xi,\eta)\psi_n(\xi,\eta) dS \tag{22}$$

If the ψ_n are an orthonormal set

$$Q_n = - \beta_n \tag{23}$$

We further note using eq. (13) that

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}_n^i}\right) = 2h\rho_s \int_S \ddot{u}_2 \, \psi_n dS = -2h\rho_s \omega^2 \alpha_n^i \tag{24}$$

and

$$\frac{\partial V}{\partial \alpha_n^i} = \sum_{j=1}^{3} \int_S A_n^{ij} \, u_j \psi_n dS = \sum_{j=1}^{3} A_n^{ij} \, \alpha_n \tag{25}$$

where the 3 x 3 matrix A_n^{ij} may be set up for each modal index n from the explicit expressions for the strain energy density of the shell. We repeat that this part of the formulation is incomplete.

Substituting eqs. (24) and (25) in Lagrange's equations as given in eqs. (17) - (19), we obtain

$$-2h\rho_s\omega^2\alpha_n^1 + \sum_j A_n^{1j}\alpha_n^j = -\beta_n \quad ; \tag{26}$$

$$-2h\rho_s\omega^2\alpha_n^2 + \sum_j A_n^{2j}\alpha_n^j = 0 \tag{27}$$

and

$$-2h\rho_s\omega^2\alpha_n^3 + \sum_j A_n^{3j}\alpha_n^j = 0 \tag{28}$$

Equations (26) - (28) may be solved to yield the ratio β_n/α_n^1 which we write as

$$\frac{\beta_n}{\alpha_n^1} = z_n \tag{29}$$

from eqs. (13) and (21) we have

$$P_+(\vec{r}) = \sum_n z_n\alpha_n^1\psi_n(\xi,\eta) \quad ; \quad \vec{r} \text{ on } S \tag{30}$$

Further from the boundary conditions on S as given in eq. (12), we have

$$\hat{n}\cdot\vec{v}_+ = \frac{\partial}{\partial t}u_1(\vec{r}) \quad ; \quad \vec{r} \text{ on } S \tag{31}$$

Again using eq. (13)

$$\hat{n}\cdot\vec{v}_+ = -i\omega\sum_n \alpha_n^1\psi_n(\xi,\eta) \tag{32}$$

The mechanical impedance of the shell can be defined as

$$\frac{P_+}{\hat{n}\cdot\vec{v}_+} = Z \tag{33}$$

and from eqs. (30) and (32) - (33) we can define a modal impedance Z_n. Equations (29) and (34) may now be used in the null field equations (10) to relate the scattered and incident field coefficients.

The T-matrix of a thin elastic shell

Using eqs. (30) and (32) in (10), we have

$$\begin{Bmatrix} -a_n \\ f_n \end{Bmatrix} = \frac{-i}{4\pi}\begin{pmatrix} \pi \\ k \end{pmatrix} \sum_m \left[\int_S \left[\left(\hat{n} \cdot \nabla \begin{Bmatrix} Ou \\ Re \end{Bmatrix} \phi_n \right) (Z_m) - \begin{Bmatrix} Ou \\ Re \end{Bmatrix} \phi_n (i\omega)\right] \psi_n dS \right] \alpha_m^1 \tag{34}$$

where

$$Z_m = -iz_n/\rho\omega \tag{35}$$

Equation (34) may now be solved in the usual manner to yield

$$f_n = \sum_m T_{nm} a_m \tag{36}$$

where

$$T = Q(\overline{Q})^{-1}$$

and

$$\begin{Bmatrix} Q_{nm} \\ \overline{Q}_{nm} \end{Bmatrix} = -\frac{i}{4\pi}\begin{pmatrix} \pi \\ k \end{pmatrix} \int \left[\left(\hat{n} \cdot \nabla \begin{Bmatrix} Re \\ Ou \end{Bmatrix} \phi_n \right) Z_m - i\omega \begin{Bmatrix} Re \\ Ou \end{Bmatrix} \phi_n \right] \psi_m dS \tag{37}$$

Closure

At this time the formulation of the scattering problem using impedance boundary conditions is complete. However, the exact form of the nodal impedance must still be extracted from the shell equations for the strain energy density. After this has been done for shells of various shapes, numerical calculations must be performed for various model shapes, so that they can be compared with available exact calculations (using the full elasticity equations).

Acknowledgement

The authors wish to acknowledge many helpful suggestions from Professor Sunder H. Advani. This work was supported by ONR under Contract No: N00014 - 80 - C - 0573.

References

1. J.H. Su, V.V. Varadan, V.K. Varadan and L. Flax, J. Acoust. Soc. Am. 68, 686 (1980).

2. S.K. Numrich, V.V. Varadan and V.K. Varadan, J. Acoust. Soc. Am., in press.

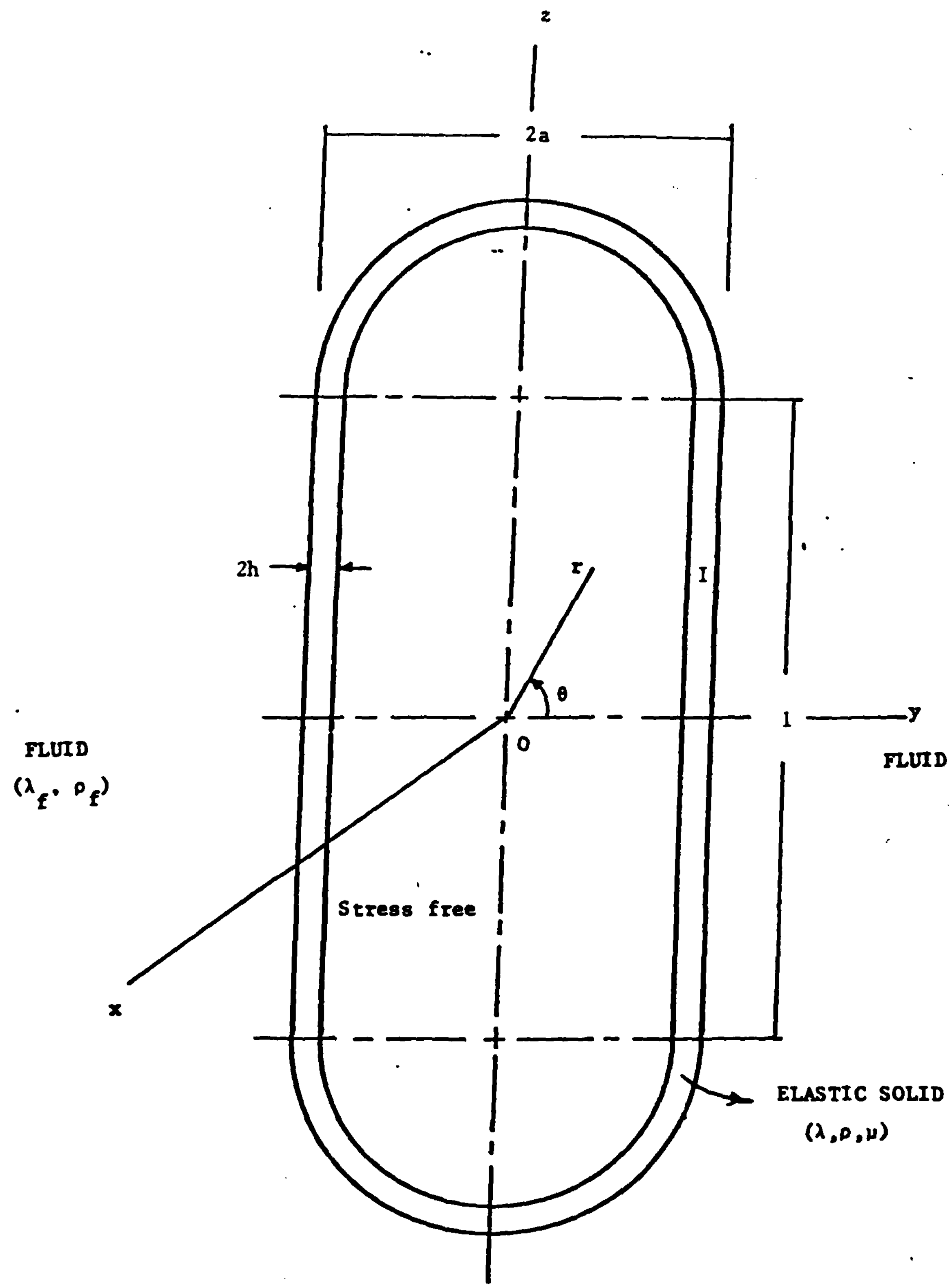
3. B. Peterson, V.V. Varadan and V.K. Varadan, J. Acoust. Soc. Am. 68, 673 (1980).

4. P.C. Waterman, J. Acoust. Soc. Am. 45, 1417 (1969).

5. V.V. Varadan and V.K. Varadan, 'Acoustic, Electromagnetic and Elastic field equations pertinent to scattering problems' in Acoustic, Electromagnetic and Elastic Wave Scattering - Focus on the T-matrix Approach edited by V.K. Varadan and V.V. Varadan, Pergamon Press (1980).

6. V.V. Varadan, 'Elastic Wave Scattering' in Acoustic, Electromagnetic and Elastic Wave Scattering - Focus on the T-matrix Approach edited by V.K. Varadan and V.V. Varadan, Pergamon Press (1980).

7. M.C. Junger, J. Acoust. Soc. Am. 24, 366 (1952).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