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	C.B. Chang
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FOR THE COMMANDER

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A MODEL FOR SENSOR-INTERCEPTOR TRADE-OFF ANALYSIS

C.B. CHANG

Group 32

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ABSTRACT

In this report, we present an analytical model useful for sensor and interceptor trade-off analysis. Major factors used in this model include sensor measurement accuracy, data rate, interceptor time delay in responding to a command, and interceptor control error in executing a command. Guidance options considered include command guidance and homing guidance whereby the homing sensor accuracy may either be a constant or vary with powers of target range.

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CONTENTS

1. INTRODUCTION

A guided interceptor missile is an essential component of many tactical and strategic defense systems. Evaluation of the performance of an interceptor missile is usually a complicated matter. Before test flights can be performed, exhaustive computer simulation studies are usually conducted for use as guidelines on missile designs and system trade-offs. It is often desired to have a simple analysis technique which can be readily applied to provide expected missile performance and preliminary system trade-offs before a sophisticated and time consuming simulation program can be made available. It is the purpose of this report to discuss such an analysis technique.

The ultimate goal of an interceptor missile is to come as close to the target vehicle as possible to insure that the target will be within the lethal radius of the interceptor warhead. The closest approach between the interceptor and the target is called the miss distance. A perfect interceptor will ideally deliver near zero miss distances. An engagement system consists of many mechanical, electrical, and electronic components; errors in any components will contribute to enlarge the miss distance. Although most of these factors should be included in a detailed simulation, they cannot (and should not) be exactly represented in a "simple" analytical model. In the model described in this report, we include the dominant system/component parameters contributing to the miss distance.

Dominant miss contributors include the guidance sensor measurement error and the interceptor's ability (time and accuracy) to respond to a maneuver command. A given set of parameter values characterizing the above variables will result in a miss distance value. On the other hand, for a given miss distance value, one may find many combinations of parameter values achieving the same miss distance. For example, a poor guidance sensor can sometimes be compensated for with a capable interceptor and vice versa. With this approach, one is therefore able to obtain trade-offs between guidance sensor and interceptor missile.

We emphasize that the work of this report only treats the endgame portion of the engagement. For example, we assume that the target position uncertainty (e.g., the handover basket) is always contained in the interceptor reachable area. The interceptor is therefore not maneuver magnitude limited. Discussion of interceptor engagements from a broader point of view is included in [1], [2]. The handover limit effect is discussed in these reports.

This report is organized as follows. In the next section, we give a derivation which constitutes the fundamental approach of this analytical technique. In section 3, we apply this analysis to the command guidance case. In section 4, we extend our analysis to the homing guidance case. In a homing engagement, the dominant sensor error is in the angle domain. The angle measurement error can be instrumentation error limited (remaining constant as range decreases) or thermal noise limited (varying as a power of target range). All these cases will be discussed. Finally, numerical examples covering several engagement scenarios will be given in Section 5 to illustrate the method and to show how the various error sources contribute to miss-distance.

An Appendix, giving miss distance curves for a variety of systems/component parameters, is included at the end. This appendix can be a handy reference for system analysts evaluating interceptor-sensor trade-off issues.

2. FUNDAMENTAL RESULTS

In this section, we present a set of equations for computing engagement miss distance given sensor measurement accuracy, time interval between two adjacent measurements, missile time constant in responding to a maneuver command and misssile control accuracy in executing a desired acceleration. These results can be easily applied to command guided interceptors as illustrated in the next section. The methodology utilized in deriving these equations is the same for the homing guidance case; this will be done in the fourth section.

The interceptor trajectory dynamics are modeled as a set of second order polynomials. We will first discuss the interceptor tracking problem which is then related to miss distance calculation. Limitations of this analysis are also discussed.

2.1. Interceptor Trajectory Dynamics

The interceptor trajectory is modeled as a second order polynomial in each dimension with the second derivative term (acceleration) representing the interceptor maneuver control.

Let y_k denote true interceptor position at time t_k, then

$$y_k = p + vt_k + (1/2)\ at_k^2 \tag{2.1}$$

where p, v, and a are true interceptor position, velocity and acceleration, at t=0, respectively. Noise corrupted measurements of y_k are represented by

$$z_k = y_k + \xi_k \tag{2.2}$$

where ξ_k is an uncorrelated noise sequence with zero mean and variance σ_z^2.

2.2. The Interceptor Position and Velocity Estimates

The interceptor acceleration a is the command acceleration applied by the command controller. Suppose that the acceleration which the controller wishes to apply to the interceptor is a_n, the interceptor tracking filter can therefore be a constant velocity filter using the following pseudo-measurements.

$$\tilde{z}_k = z_k - \frac{1}{2} a_n t_k^2 \tag{2.3}$$

or

$$\tilde{z}_k = p + vt_k + \mu_k \tag{2.4}$$

$$\mu_k = \xi_k + \frac{1}{2}(a-a_n) t_k^2 \tag{2.5}$$

where the difference, $\Delta a = a-a_n$, is the command acceleration bias error. The pseudo-measurement noise sequence μ_k has mean $1/2\ \Delta a t_k^2$ and variance σ_z^2.

Assuming that the measurements are taken uniformly in time and p, v, and a correspond to the true interceptor dynamics at the center of the data interval, the following least square estimates of position and velocity can be obtained,

$$\begin{bmatrix} p \\ \\ v \end{bmatrix} = \begin{bmatrix} \frac{1}{K} & 0 \\ \\ 0 & \frac{12}{T\ (K-1)K(K+1)} \end{bmatrix} \begin{bmatrix} \sum \tilde{z}_k \\ \\ T \sum \tilde{z}_k (k - \frac{(K-1)}{2}) \end{bmatrix} \tag{2.6}$$

where K is the total number of pulses and T is the time between two adjacent measurements. Using polynomial analysis, we obtain the following results:

(1) The position estimate $\hat{p}$ is <u>biased</u> and the bias is

$$\Delta p = \Delta a \frac{T^2(K-1)(K+1)}{24} \tag{2.7}$$

the variance of $\hat{p}$ is

$$\sigma_{\hat{p}}^2 = \frac{\sigma_z^2}{K} \tag{2.8}$$

(2) The velocity estimate $\hat{V}$ is <u>unbiased</u> with variance

$$\sigma_{\hat{V}}^2 = \frac{\sigma_z^2}{T^2(K-1)K(K+1)} \tag{2.9}$$

(3) Using $\hat{p}$, $\hat{V}$, and a_n, the <u>predicted</u> position $\hat{p}_p$ at time t is also <u>biased</u> and the bias is

$$\Delta p_p = \Delta a \frac{T^2(K-1)(K+1)}{24} + 1/2\, \Delta a\left(t + \frac{T(K-1)}{2}\right)^2 \tag{2.10}$$

The variance of p_p is

$$\sigma_{p_p}^2 = \sigma_z^2 \left[\frac{1}{K}\left(1 + \frac{12\left(t + \frac{(K-1)T}{2}\right)^2}{T^2(K-1)(K+1)}\right)\right] \tag{2.11}$$

where t=0 corresponds to the terminal end of the data interval.

2.3. Optimal Number of Interceptor Tracking Measurements

Examining Eqs. (2.10) and (2.11), it is found that the predicted position estimate bias increases with K while its variance decreases with K. An optimal K would be the one achieving a proper trade-off of these two errors. Assuming that the last interceptor command which can be effectively applied is limited by the time between measurements (T) and the interceptor response time constant (τ), then the optimal K is the K minimizing the following terminal mean-square error,

$$J = \left[\Delta a \frac{T^2(K-1)(K+1)}{24} + \frac{1}{2}\Delta a\left(\tau + T + \frac{T(K-1)}{2}\right)^2\right]^2$$

$$+ \sigma_z^2 \left[\frac{1}{K}\left(1 + \frac{12(\tau + T + \frac{(K-1)T}{2})^2}{T^2(K-1)(K+1)}\right)\right] \qquad (2.12)$$

We note that the interceptor response time (τ) is the time delay in an interceptor in responding to a maneuver command. The partial derivative of J with respect to K is a rather complicated function and it is difficult to obtain a closed-form solution for K. Since the total tracking time (K-1)T is usually longer than the interceptor response time and the time interval between measurements, we use the following approximations in (2.12) to make the problem tractable

1) $KT \gg \tau$
2) $K \gg 1$

Then Eq. (2.12) becomes

$$J = \frac{\Delta a^2 T^4 K^4}{36} + \frac{4\sigma_z^2}{K} \qquad (2.12^*)$$

Taking the partial derivative of J with respect to K and solving for the optimal K for minimizing the mean-square error gives

$$K = \left[\left(\frac{6\sigma_z}{\Delta a T^2}\right)\right]^{2/5} \qquad (2.13)$$

where [ ] denotes the round-off integer of the enclosed quantity. Notice that when $\Delta a \to 0$, $K \to \infty$; this implies that we should use all available tracking pulses when the command bias is zero.

2.4. Target Trajectory Estimation

The same equations can be used to determine the target trajectory parameters. In this case, Δa represents the target acceleration error rather than the interceptor command bias. This generally results from imperfect drag or lift modeling of the target. For the homing case, the sensor is located on the interceptor; it is therefore unnecessary to separate tracking errors into target and interceptor tracks. The control error Δa will become the target to interceptor relative acceleration error which encompasses both interceptor control error and target drag and lift modeling errors.

2.5. Engagement Miss Distance

In the previous sections, we presented formulas for computing the interceptor tracking and prediction accuracy and determining the optimal number of interceptor tracking pulses. In this section, we summarize our results for the miss distance calculation as follows

1) Miss Distance Bias

$$\Delta x = \Delta a \frac{T^2(\hat{K}-1)(\hat{K}+1)}{24} + 1/2\Delta a(\tau + T + \frac{T(\hat{K}-1)}{2})^2 \qquad (2.14)$$

$$\approx \frac{\Delta a \hat{T}_I{}^2}{6} = \left(\frac{\sigma_z{}^4 \Delta a T^2}{6}\right)^{1/5} \qquad (2.14*)$$

where T_I is the interceptor total time interval defined in Eq. (2.17) below

2) Miss Distance Standard Deviation

$$\sigma_x = (\sigma_I{}^2 + \sigma_{Tgt}{}^2)^{1/2} \qquad (2.15)$$

$$\sigma_I = \sigma_z \left[\frac{1}{\hat{K}}\left(1 + \frac{12(\tau + T + \frac{(\hat{K}-1)T}{2})^2}{T^2(\hat{K}-1)(\hat{K}+1)}\right)\right]^{1/2} \qquad (2.16)$$

$$\approx 2\sigma_z \sqrt{\frac{T}{\hat{T}_I}} = \left(\frac{16}{3} \sigma_z{}^4 \Delta a T^2\right)^{1/5} \qquad (2.16*)$$

$$\hat{K} = \left[\left(\frac{6\sigma_z}{\Delta a T^2}\right)^{2/5}\right]$$

$$\hat{T}_I = \left(\frac{6\sigma_z\sqrt{T}}{\Delta a}\right)^{2/5} = T\hat{K} \qquad (2.17)$$

where σ_I and σ_{Tgt} are interceptor and target tracking standard deviations, respectively. As mentioned earlier, the target track should have the same characteristics as the interceptor track, we therefore do not include an equation for σ_{Tgt}. Notice that Eqs. (2.14) and (2.16) are expressed in terms of the number of tracking pulses while Eqs. (2.14*) and (2.16*) are in terms of the total tracking time interval.

3) If we use

$$x = \Delta x + \sigma_x \tag{2.18}$$

then x is the 85% miss distance, i.e., approximately 85% of the miss distances of many random trials will be less than or equal to x.

4) If we use

$$x = \sqrt{\Delta x^2 + \sigma_x^2} \tag{2.19}$$

then x is the RMS (Root-Mean-Square) miss distance.

The above miss distance calculation formulas are derived with the following <u>assumptions</u>:

a) The interceptor guidance time is sufficiently long and the interceptor launch is such that the interceptor maneuver is capable of nulling the <u>predicted</u> miss.

b) The final miss is the predicted position error when the time-to-go is equal to the sum of the interceptor response time and interval between measurements.

2.6. Discussion

The above are only some approximate formulas for quick calculation of engagement miss distances. It is assumed that the dominant miss distance contributors are: 1) position measurement error; 2) interceptor command bias; and 3) interceptor response time. Notice that these formulas are applicable to command guided interceptors and homing interceptors provided that the measurement error (σ_z) is properly represented. When applied to the homing case, the position measurement error (σ_z) becomes the relative error, and the acceleration error (Δa) includes both the command bias error and the target acceleration.

Shortcomings of this analysis are:

1) It is only a one-dimensional analysis. When the dominant error is indeed along a given dimension, it may be a close approximation. Otherwise, one should first calculate miss distance along each independent axis then calculate the total root-sum-square miss distance using these components

2) The command acceleration a_c is calculated using estimated RV and interceptor states. This stochastic feedback process is not modeled explicitly.

3) Detailed dynamic modeling of interceptors and other effects are not included.

If all the above details are included, the results will not be nearly as tractable as those presented above. Because our model is a simplified one, the resulting miss distance should be treated as a lower bound of actual performance. We have compared the above results with those obtained by computer simulation for several cases. The results have always come in good agreement.

Lastly, we emphasize that the miss distance is "defined" as the predicted target-interceptor position error after the last effective maneuver control. For this reason, this model can also be used for warhead fuzing analysis. In this problem, the guidance sensor becomes the fuze sensor and the interceptor response time becomes the warhead reaction time.

3. APPLICATION TO COMMAND GUIDANCE

The equations derived in the previous section are directly applicable to command guidance. In this case, the measurement error σ_z must be properly identified. Notice that in our derivation thus far we have assumed that σ_z is a constant throughout the engagement.

A command guidance system uses sensor(s) located outside the interceptor vehicle tracking both target and interceptor and transmitting maneuver commands to the interceptor. If the tracking sensor is a radar, the dominant error component is angle error. If the target/interceptor optimal tracking interval corresponds to a trajectory length which is much smaller than the target/interceptor range to the tracking sensor, then the measurement error is nearly a constant and can be approximated with

$$\sigma_z = R_o \sigma_{\theta_o} \tag{3.1}$$

where R_o and σ_{θ_o} are mean range and angular measurement error corresponding to the optimal tracking interval.

If a multilateration tracking system is employed, the tracking uncertainty volume is the intersection of (at least) three range uncertainties. Let σ_R denote the largest range uncertainty and β the bistatic angle, then the worst position measurement uncertainty is

$$\sigma_z = \begin{cases} \sigma_R / \sin \frac{\beta}{2} & \beta \leq 45° \\ \sigma_R / \cos \frac{\beta}{2} & \beta \geq 45° \end{cases} \tag{3.2}$$

The above formula should be used as long as the trilateration error is smaller than the smallest cross-range measurement error made by a monostatic radar in the multilateration tracking system.

In the numerical result section, we will evaluate miss distances for a range of σ_z values.

4. EXTENSION TO HOMING GUIDANCE

There are four types of homing sensor guidance errors considered in Ref. [2]. They are

Case 0	σ_P = constant	Glint
Case 1	$\sigma_P = \sigma_\theta R$	Instrumentation
Case 2	$\sigma_P = \sigma_A (R^2/R_A)$	Thermal (passive or semi-active)
Case 3	$\sigma_P = \sigma_A (R^3/R_A{}^2)$	Thermal (active)

where σ_p is the position measurement uncertainty, σ_θ is a range-invariant angle measurement error and σ_A is the angle measurement error at acquisition (or a reference) range R_A.

Notice that the case 0 above is similar to the command guidance case and the equations of Section 2 are readily applicable, we therefore will not discuss it further. All remaining cases require modification of equations of Section 2.

The above equations define an estimation problem with time-varying measurement standard deviations. One can solve this problem by applying the polynomial analysis with time-varying measurement standard deviations.

Let $\sigma_k{}^2$ denote the measurement variance of the k-th measurement $\hat{\xi}_k$ (see (2.3)-(2.5)), one can obtain the following results for the time-varying noise variance case after some manipulations:

(1) The position estimation bias at the center of data window is

$$\Delta p = \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2}\right)^{-1} \left(\sum_{k=1}^{K} \frac{\Delta a t_k^2}{2\sigma_k^2}\right) \tag{4.1}$$

(2) The variance of the position estimate is

$$\sigma_p^2 = \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2}\right)^{-1} \tag{4.2}$$

(3) The variance of the velocity estimate is

$$\sigma_v^2 = \left(\sum_{k=1}^{K} \frac{t_k^2}{\sigma_k^2}\right)^{-1} \tag{4.3}$$

With the same approach as in the second section, the optimum number of tracking pulses is the K minimizing

$$J = [\Delta p + \tfrac{1}{2}\Delta a(\frac{T(K-1)}{2} + T + \tau)^2]^2 + [\sigma_p^2 + \sigma_v^2(\frac{T(K-1)}{2} + T + \tau)^2] \tag{4.4}$$

An analytical expression for determining the optimum K is difficult to obtain. Since K only takes integer values and is limited by

$$\frac{R_a}{V_R T} - \frac{\tau}{T} \geq K \geq 2 \tag{4.5}$$

where R_A is the sensor acquisition range and V_R is the relative velocity, an exhaustive search by stepping through its range of values appears to be a reasonable approach. The above expression is obtained using the fact that (1) the sensor needs at least two measurements to predict the future trajectory and (2) the maximum number of measurements is limited by the total duration of the engagement.

The RMS miss distance is the square root of the minimum J of (4.4).

The evaluation of (4.1)-(4.3) will be discussed individually in the following subsections. In order to simplify the manipulations we will use the following approximation for replacing all summations:

$$\sum_{k=1}^{K} f(x_k) \approx \frac{K}{b-a}\int_a^b f(x)dx \tag{4.6}$$

where f(·) denotes a function of the enclosed variable.

4.1. Instrumentation Limited Measurement Error

In this case, we have

$$\sigma_k = R_k\sigma_\theta \tag{4.7}$$

This equation implies that the angle measurement uncertainty is not a function of range. The target position error is the cross range error.

Examining (4.1)-(4.3), one concludes that there are two specific summations to be evaluated,

$$S_1 = \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \tag{4.8}$$

$$S_2 = \sum_{k=1}^{K} \frac{t_k^2}{\sigma_k^2} \tag{4.9}$$

Using $t_k \approx R_k/V_R$, substituting (4.7) in (4.8) and (4.9), and applying (4.6), one obtains

$$S_1 \approx I_1 = \frac{K}{\sigma_\theta^2 (R_b - R_a)} \int_{R_a}^{R_b} \frac{dR}{R^2} = \frac{K}{\sigma_\theta^2 R_a R_b} \tag{4.10}$$

$$S_2 \approx I_2 = \frac{K}{\sigma_\theta^2 (R_b - R_a) V_R^2} \int_{R_a}^{R_b} dR = \frac{K}{\sigma_\theta^2 V_R^2} \tag{4.11}$$

Using (4.10) and (4.11) in (4.1)-(4.3) yields

$$\Delta P = \frac{\Delta a R_a (KT)^2}{2R_b} \tag{4.12}$$

$$\sigma_P^2 = \frac{\sigma_\theta^2 R_a R_b}{K} \tag{4.13}$$

$$\sigma_V^2 = \frac{\sigma_\theta^2 R_b^2}{K(KT)^2} \tag{4.14}$$

where

$$R_a = V_R(\tau + T) \tag{4.15}$$

$$R_b = V_R(\tau + KT) \tag{4.16}$$

V_R = target-interceptor relative velocity.

The optimum K is the K which minimizes the J of (4.4) with terms defined above; the miss distance is the square root of J.

4.2. Thermal Error Limited Case - Passive and Semi-Active Homing

In this case, the angle measurement standard deviation is inversely proportional to the range to the target through the relationship with signal-to-noise ratio. Let σ_A denote the angle measurement accuracy at a reference range R_A, one then has the cross range measurement error as

$$\sigma_k = \sigma_A \frac{R_k^2}{R_A} \tag{4.17}$$

Applying the same derivation, we obtain these following results.

$$I_1 = \frac{K R_A^2}{\sigma_A^2 (R_b - R_a)} \int_{R_a}^{R_b} \frac{dR}{R^4} = \frac{K R_A^2 (R_b^3 - R_a^3)}{\sigma_A^2 (R_b - R_a) 3 R_a^3 R_b^3} \tag{4.18}$$

$$I_2 = \frac{K R_A^2}{\sigma_A^2 (R_b - R_a) V_R^2} \int_{R_a}^{R_b} \frac{dR}{R^2} = \frac{K R_A^2}{\sigma_A^2 R_a R_b V_R^2} \tag{4.19}$$

$$\Delta p = \frac{\Delta a (KT)^2}{2} \left[\frac{3 (R_b - R_a) R_a^2}{R_b^3 - R_a^3} \right] \tag{4.20}$$

$$\sigma_p^2 = \frac{\sigma_A^2}{K R_A^2} \left[\frac{3 (R_b - R_a) R_a^3 R_b^3}{R_b^3 - R_a^3} \right] \tag{4.21}$$

$$\sigma_v^2 = \frac{\sigma_A^2}{K\ R_A^2} \frac{R_a R_b^3}{(KT)^2} \tag{4.22}$$

Similarly, one applies (4.20)-(4.22) in (4.4) and searches for the minimum to obtain the miss distance.

4.3. Thermal Error Limited Case - Active Homing

In active homing, the square-root of the signal-to-noise ratio is inversely proportional to the range squared. The cross range error is therefore

$$\sigma_k = \sigma_A \frac{R_k^3}{R_A^2} \tag{4.23}$$

Similarly, one obtains the following results:

$$I_1 = \frac{K\ R_A^4}{\sigma_A^2 (R_b - R_a)} \int_{R_a}^{R_b} \frac{dR}{R^6} = \frac{K\ R_A^4 (R_b^5 - R_a^5)}{\sigma_A^2 (R_b - R_a) 5 R_a^5 R_b^5} \tag{4.24}$$

$$I_2 = \frac{K\ R_A^4}{\sigma_A^2 (R_b - R_a) V_R^2} \int_{R_a}^{R_b} \frac{dR}{R^4} = \frac{K\ R_A^4 (R_b^3 - R_a^3)}{\sigma_A^2 (R_b - R_a) 3 R_a^3 R_b^3 V_R^2} \tag{4.25}$$

$$\Delta p = \frac{\Delta a (KT)^2}{2} \left[\frac{5 (R_b^3 - R_a^3) R_a^2}{3 (R_b^5 - R_a^5)} \right] \tag{4.26}$$

$$\sigma_p^2 = \frac{A^2}{K\ R_A^4} \left[\frac{5 (R_b - R_a) R_a^5 R_b^5}{R_b^5 - R_a^5} \right] \tag{4.27}$$

$$\sigma_v^2 = \frac{\sigma_A^2}{K\ R_A^4 (KT)^2}\left[\frac{3(R_b-R_a)R_a^3R_b^5}{R_b^3-R_a^3}\right] \qquad (4.28)$$

The miss distance is the square root of the minimum J of (4.4) with terms defined above.

4.4. Summary

In this section, we have extended the miss distance equations of Section 2 to the homing guidance case. Equations for evaluating estimation bias and variances are given. An integral for approximating summations is used in deriving these results. The root-mean-square miss distance is obtained with an exhaustive search of the optimum number of measurements minimizing the terminal miss distance.

We <u>re-emphasize</u> that when these equations are applied for the homing case, Δa is the target to interceptor relative acceleration uncertainty and it will no longer be necessary to separate miss distance into target track and interceptor track individually.

5. NUMERICAL RESULTS

In this section, we present numerical results illustrating system/component parameter trade-offs. The results are in terms of miss distance vs. measurement error (σ_z or σ_θ) with interceptor response time τ as a parameter.

In Figs. 5.1-5.4, we present several comparisons for various system and design parameters. A comparison of various guidance modes is given in Fig. 5.1. These are for R_A=10 km, V_R=10 km/s, control acceleration error of 1 m/s^2, and pulse repetition rate of 50 (T=.02 s). All pertinent parameter values are shown on the upper right-hand corner of the figures. A set of nominal values used for the interceptor response time is shown on the upper left-hand corner of the figures. Notice that there is an upper limit on the interceptor response time (τ_{max}). It is limited by the total engagment time ($=R_A/V_R$) and the requirement of processing at least two measurements for trajectory prediction. The bottom curve corresponds to the smallest response time indicated in the upper left-hand corner and the upper curve corresponds to the largest response time used for this case. The response time for both curves are labelled. Response times for intermediate curves follow those shown in the upper left-hand corner. One can compare parameter values for achieving a giving miss distance using these results. Notice that for the command guidance case, a faster interceptor cannot compensate for poorer measurement accuracy when τ is less than about .3 second. On the contrary, a responsive interceptor can compensate for a poor sensor for the homing case. Especially for the thermal noise limited cases very poor sensor accuracy can be tolerated with a good interceptor. This is because the cross range measurement error is proportional to target range and sensor angle measurement error. For the thermal noise limited cases, the angle measurement error improves significantly with decreasing range. When target and interceptor are ery close, the cross range measurement error becomes very small even for large angle measurement error at acquisition.

One must keep in mind that any practical homing sensor will have glint, instrumentation and thermal errors which must be combined to obtain the overall sensor accuracy.

Comparing Fig. 5.1(b)-5.1(d), it is seen that the instrumentation limited homing case gives the most strict requirement on both sensor and interceptor. We will therefore concentrate on the instrumentation limited homing case for the remainder of the report.

In Fig. 5.2, we examine the effect of closing velocity for R_A=10 km, Δa=1 m/s^2 and T=.02s. The miss distance decreases with decreasing closing velocity. It is not a strong function of τ until τ becomes comparable with the time-to-go (R_A/V_R). Miss distance is reduced almost linearly with sensor angular error except for very small errors.

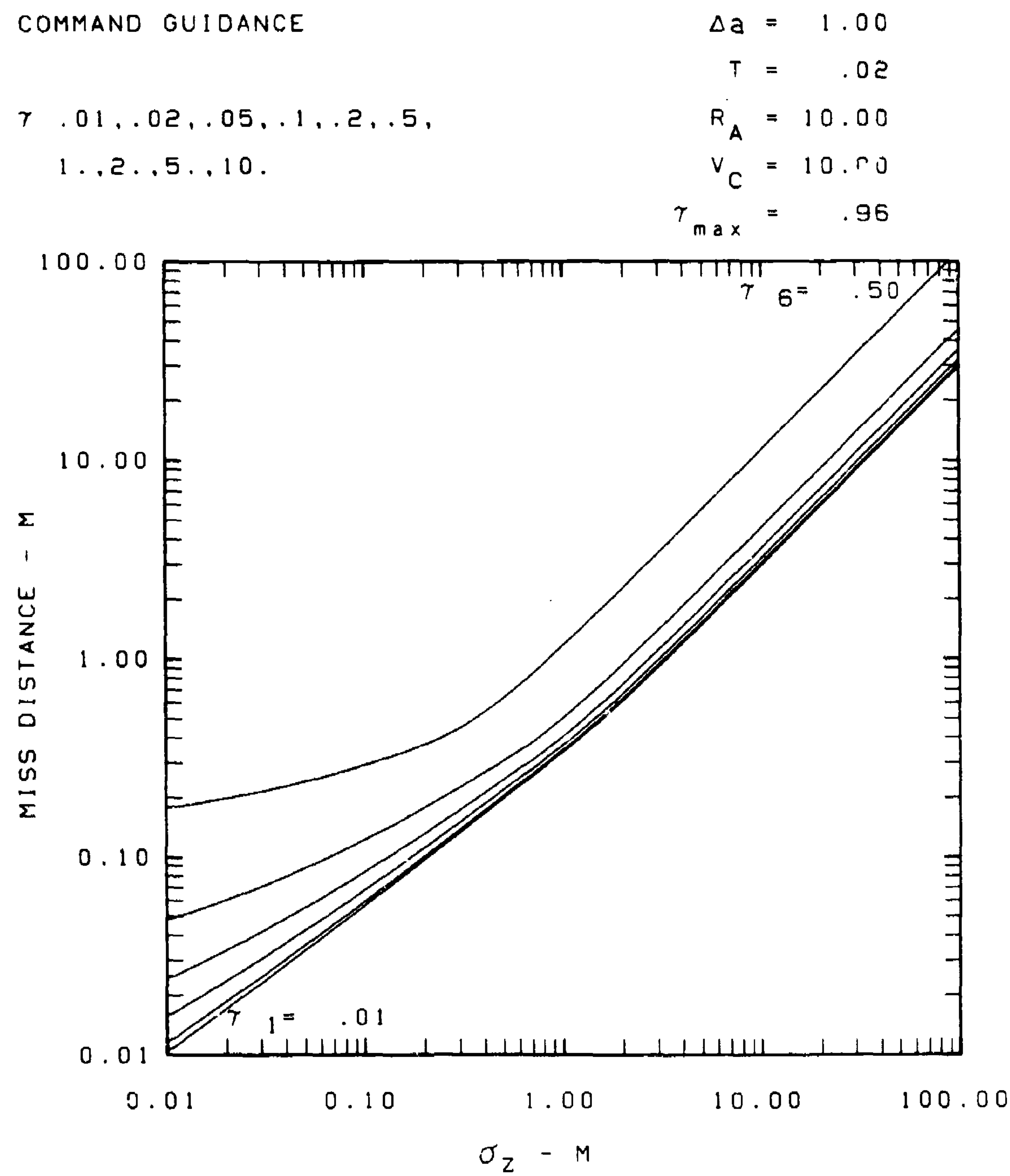


Fig. 5.1(a) Comparison of various guidance modes, command guidance/glint limited homing.

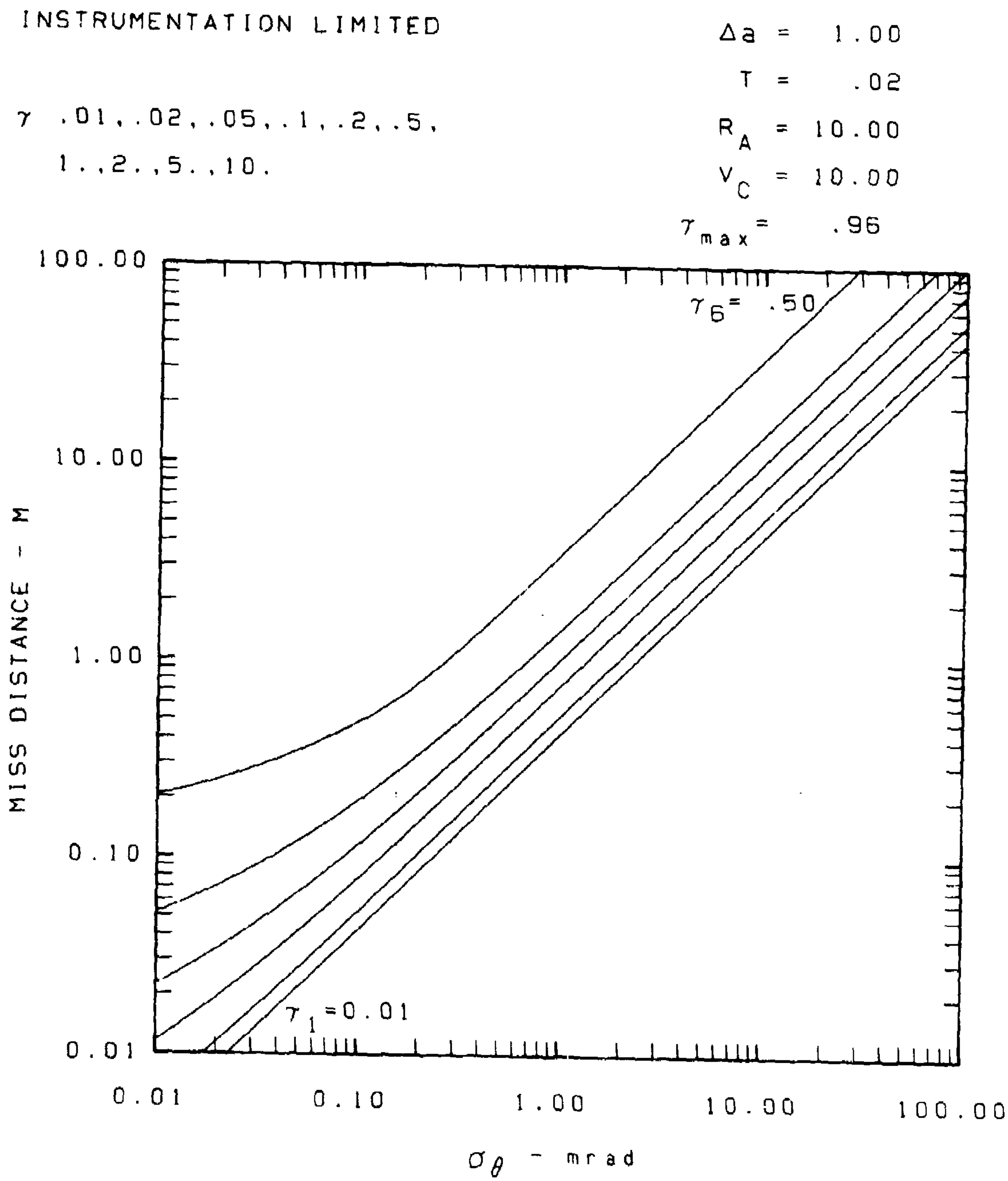


Fig. 5.1(b) Comparison of various guidance modes, instrumentation limited homing.

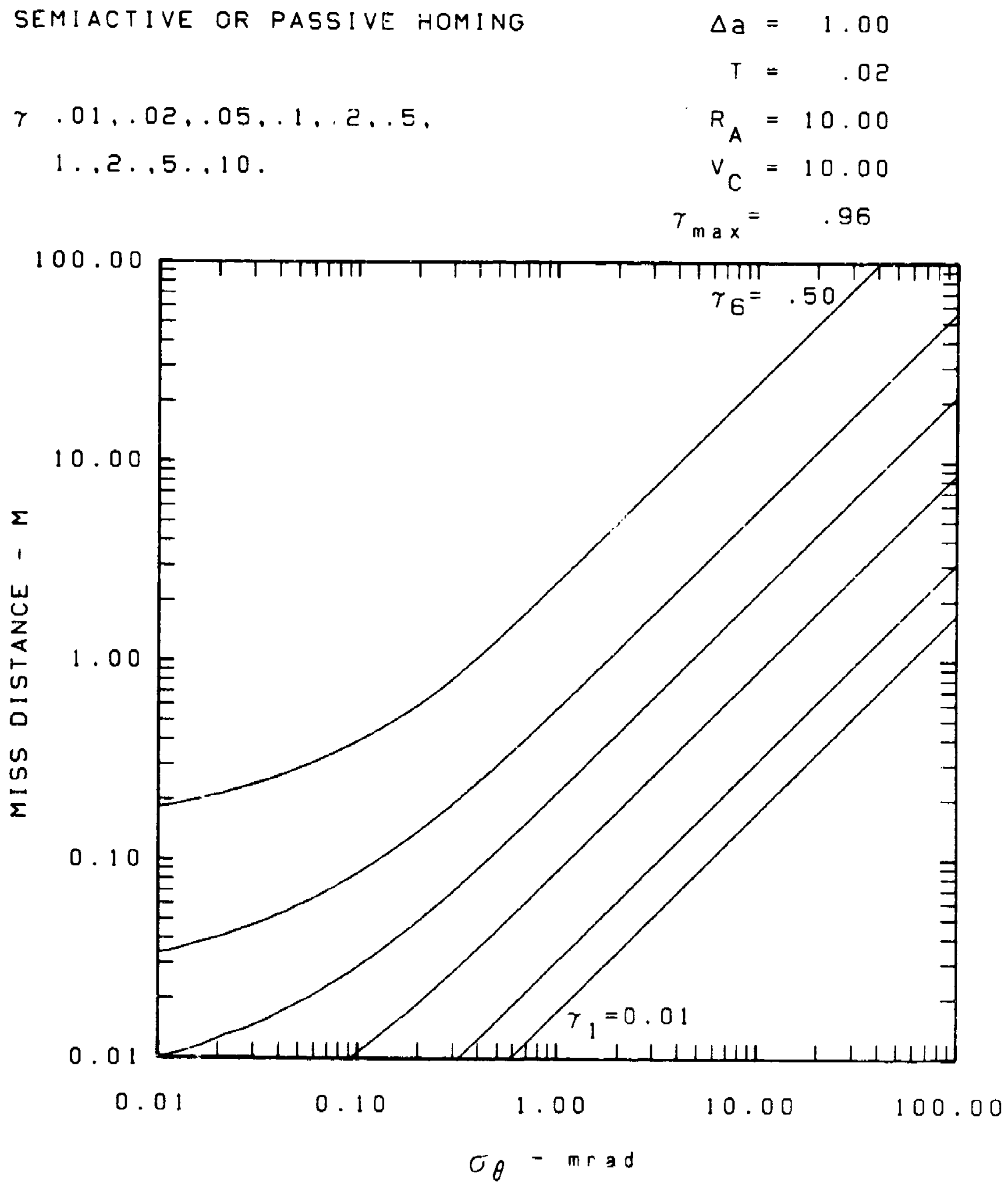


Fig. 5.1(c) Comparison of various guidance modes, passive or semi-active homing.

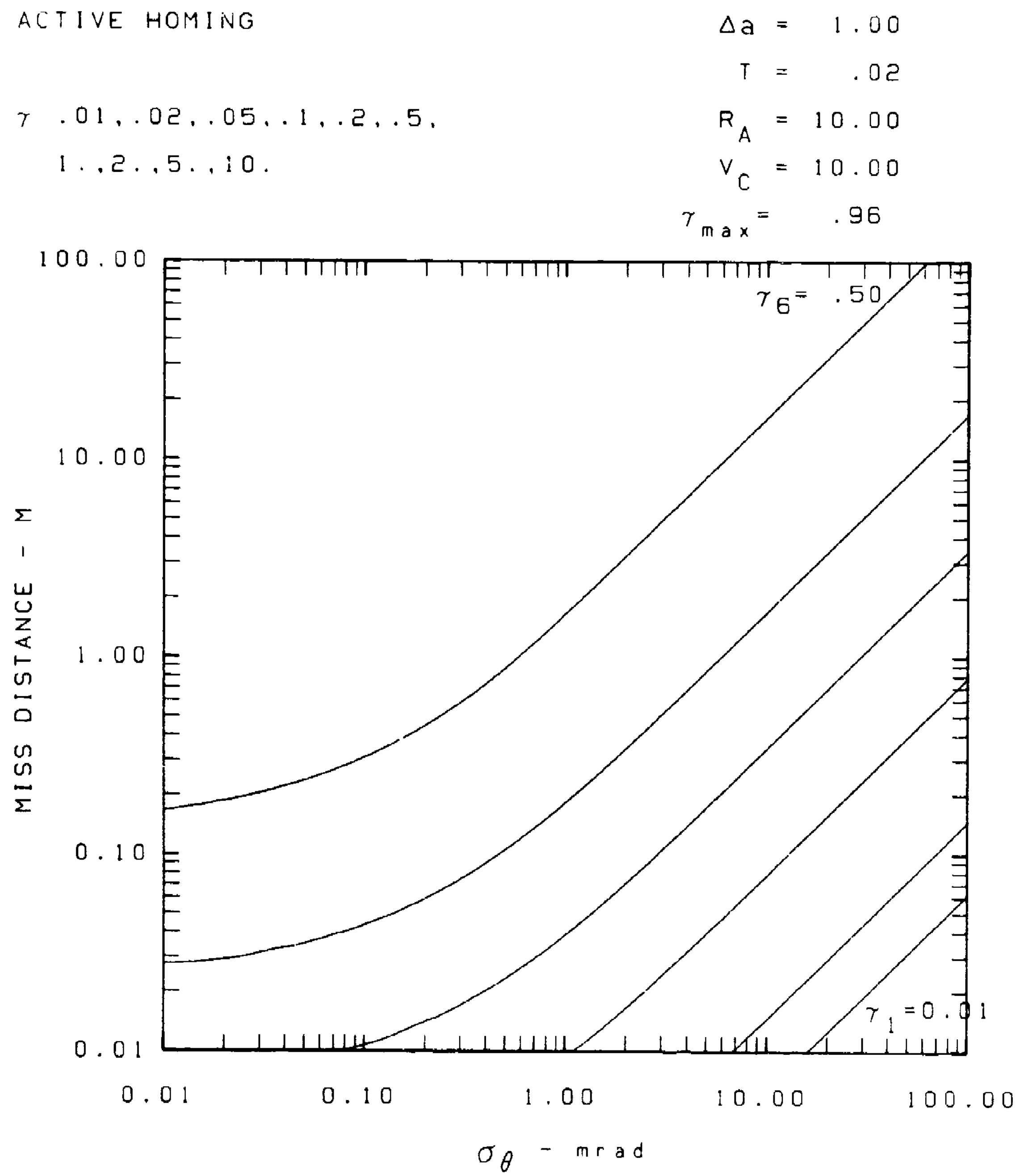


Fig. 5.1(d) Comparison of various guidance modes, active homing.

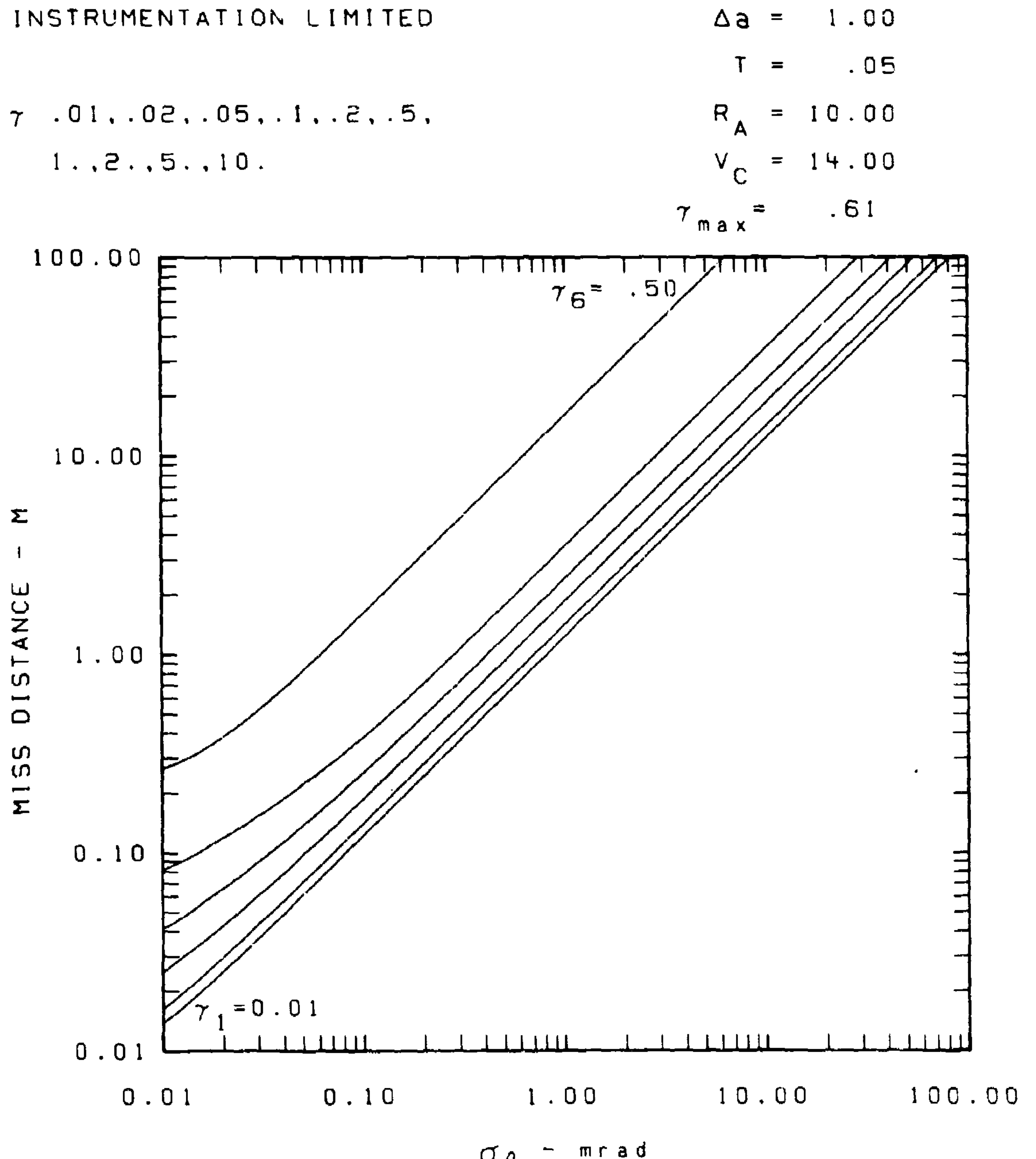


Fig. 5.2(a) Effect of initial time-to-go, R_A=10 km, V_R=14 km/s.

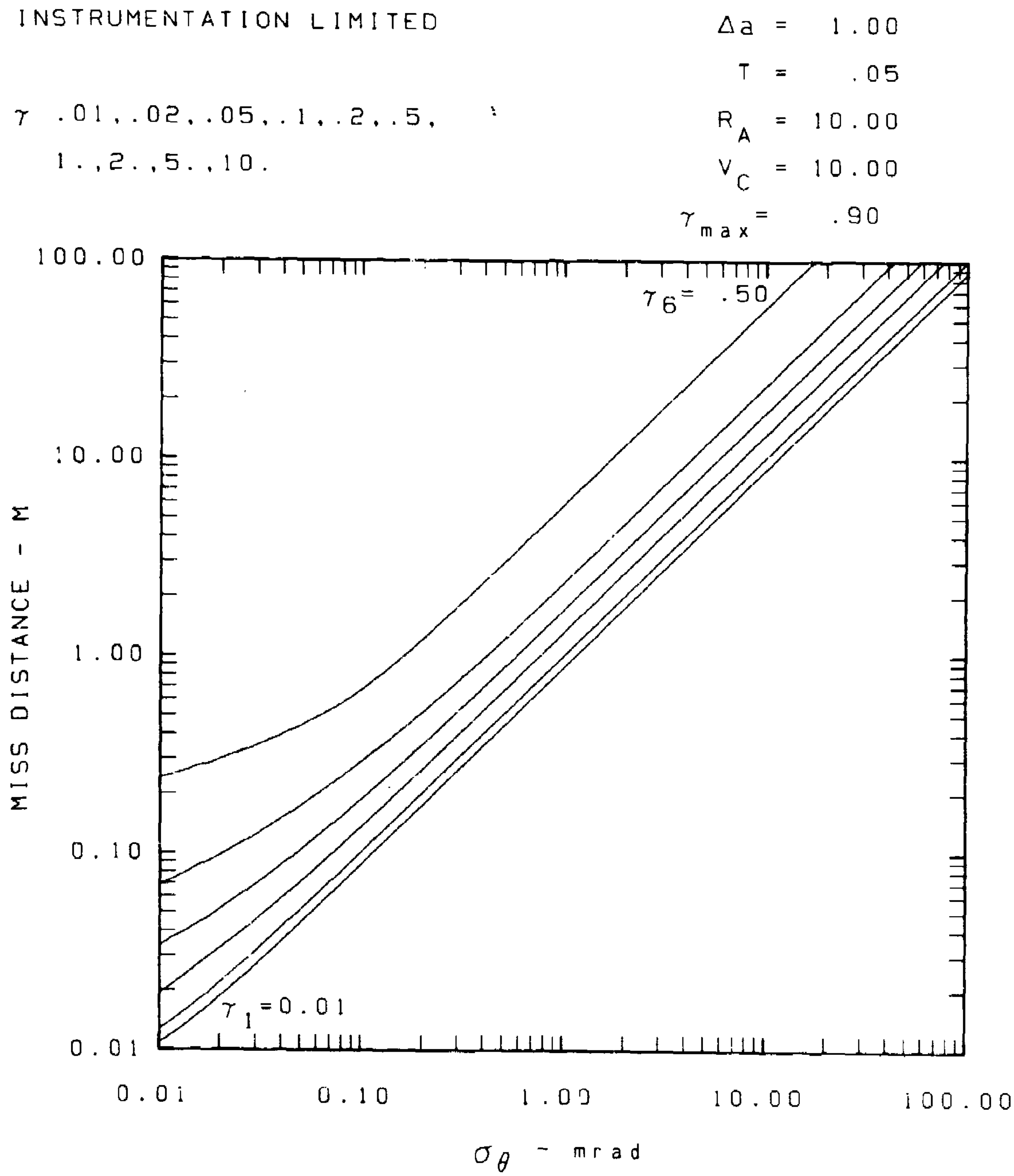


Fig. 5.2(b) Effect of initial time-to-go, R_A=10 km, V_R=10 km/s.

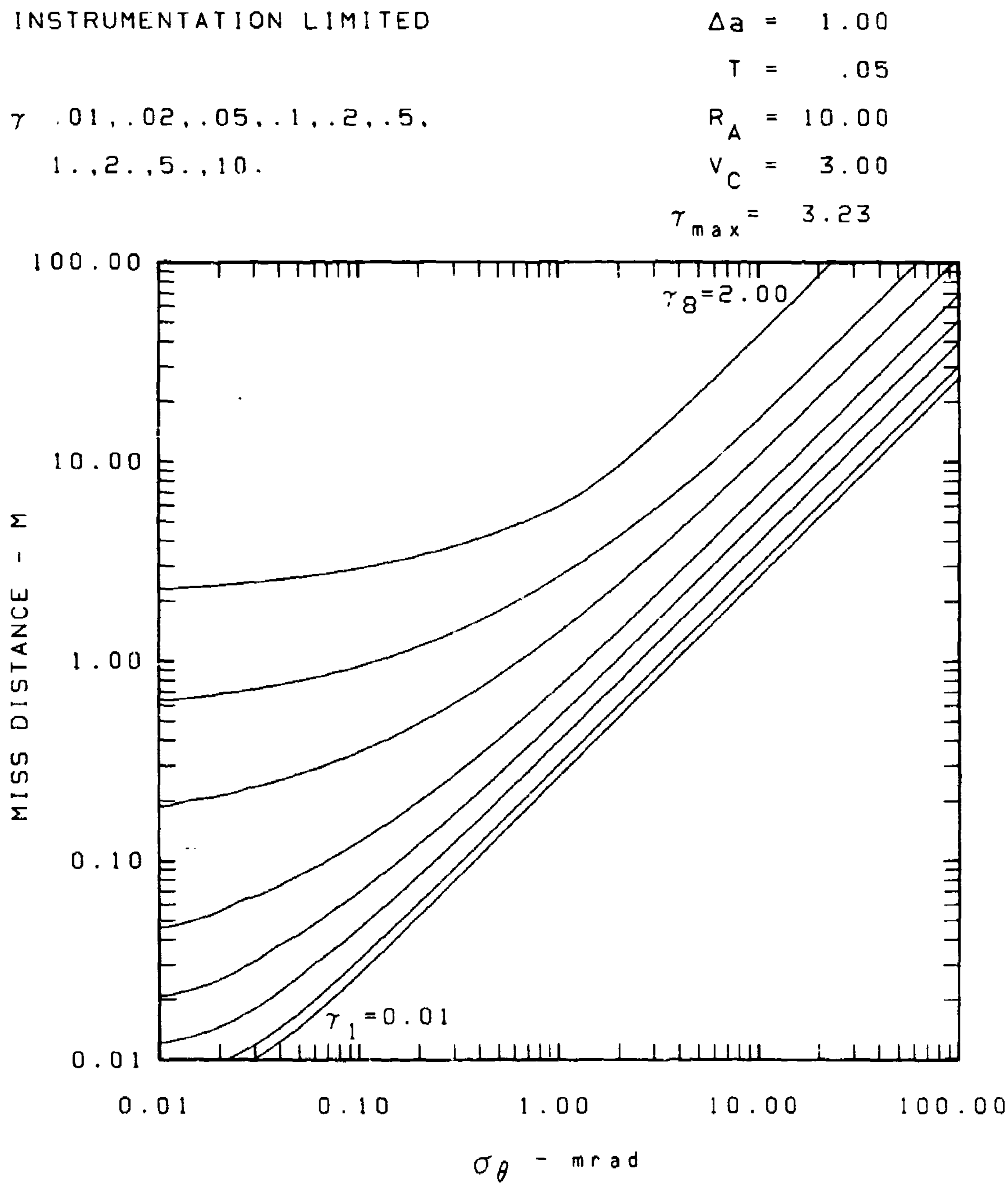


Fig. 5.2(c) Effect of initial time-to-go, R_A=10 km, V_R=3 km/s.

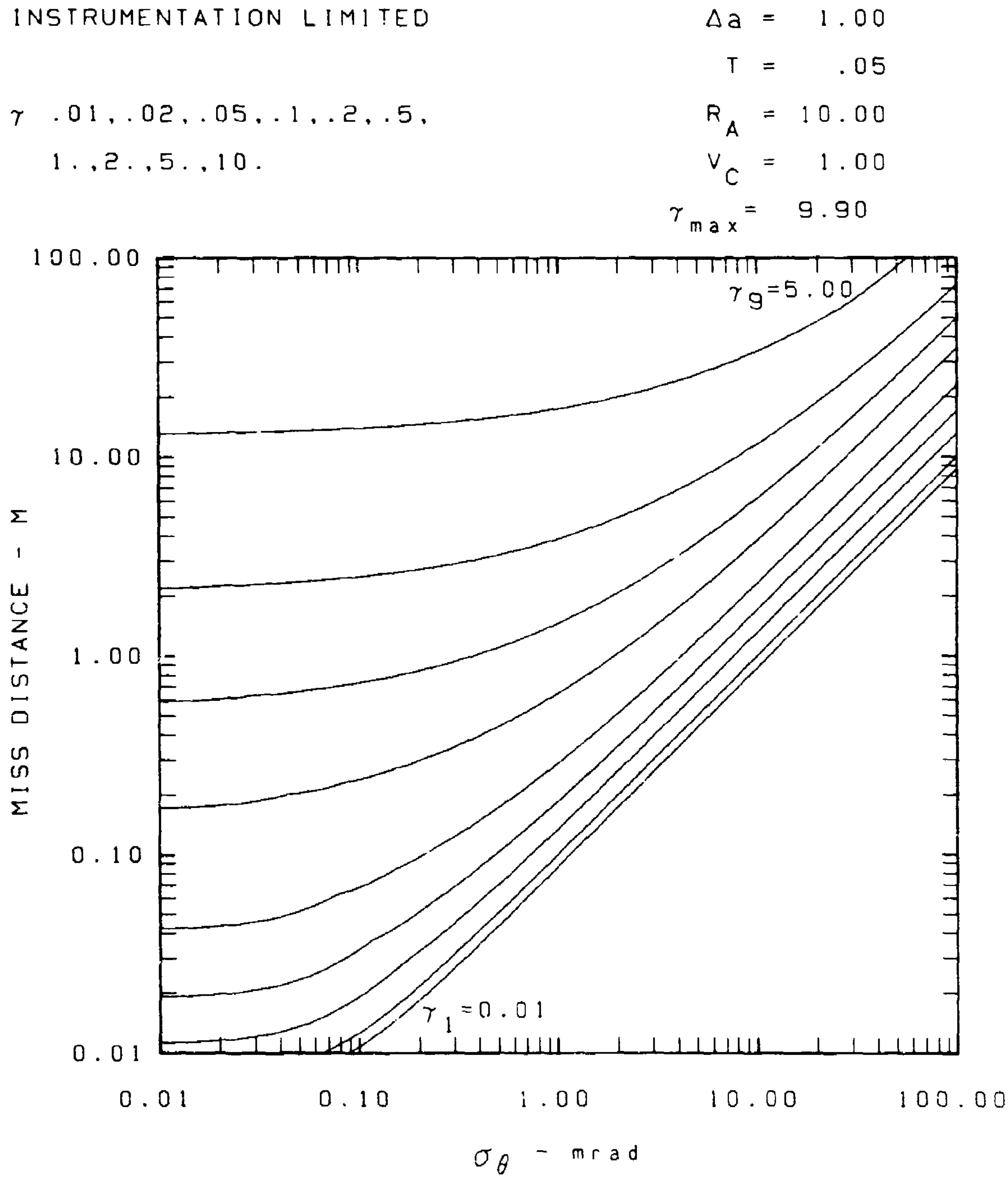


Fig. 5.2(d) Effect of initial time-to-go, R_A=10 km, V_R=1 km/s.

The effect of data rate is shown in Fig. 5.3. These results are for R_A=10 km, V_C=10 km/s, Δa=0.1 m/s^2, and data rates of 20, 50, and 100 measurements per second. There is a uniform reduction of miss distance at higher data rates. Figure 5.4 gives the dependence on control error. For large σ_θ, the miss-distance is independent of Δa but as σ_θ decreases, the dependence on Δa is almost linear. These results are for R_A=10 km, V_C=3 km/s, T=0.01 second, and control error of .1, 1 , and 10 m/s^2. A cross-plot of miss-distance as a function of missile response time, τ, with σ_θ as a parameter is shown in Fig. 5.5. Note that the dependence is approximately $\sqrt{\tau}$ for small τ and large σ_θ. In addition, it shows the effect of τ_{max} which is determined by the available engagement time.

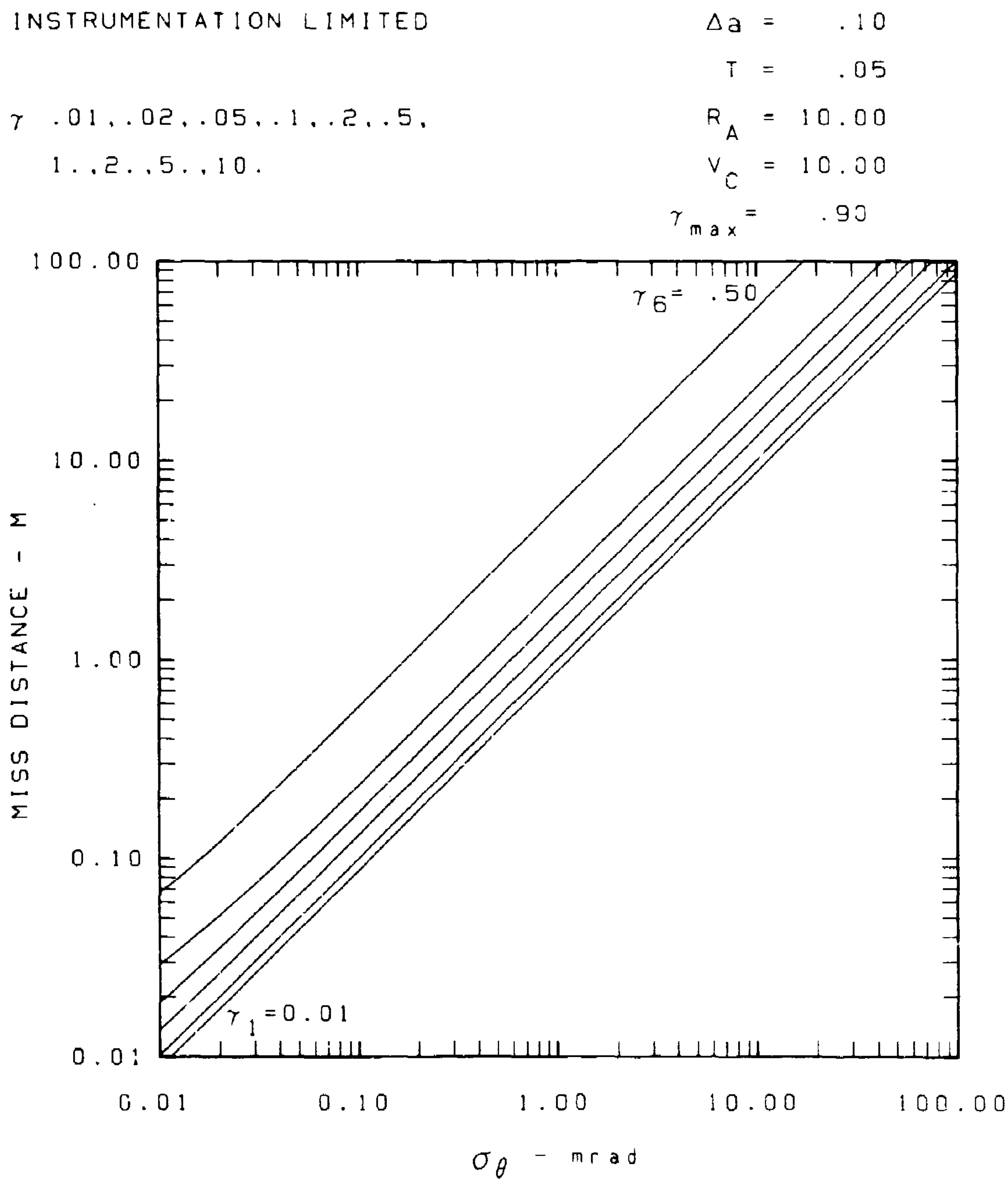


Fig. 5.3(a) Effect of data rate, 20 measurements per second.

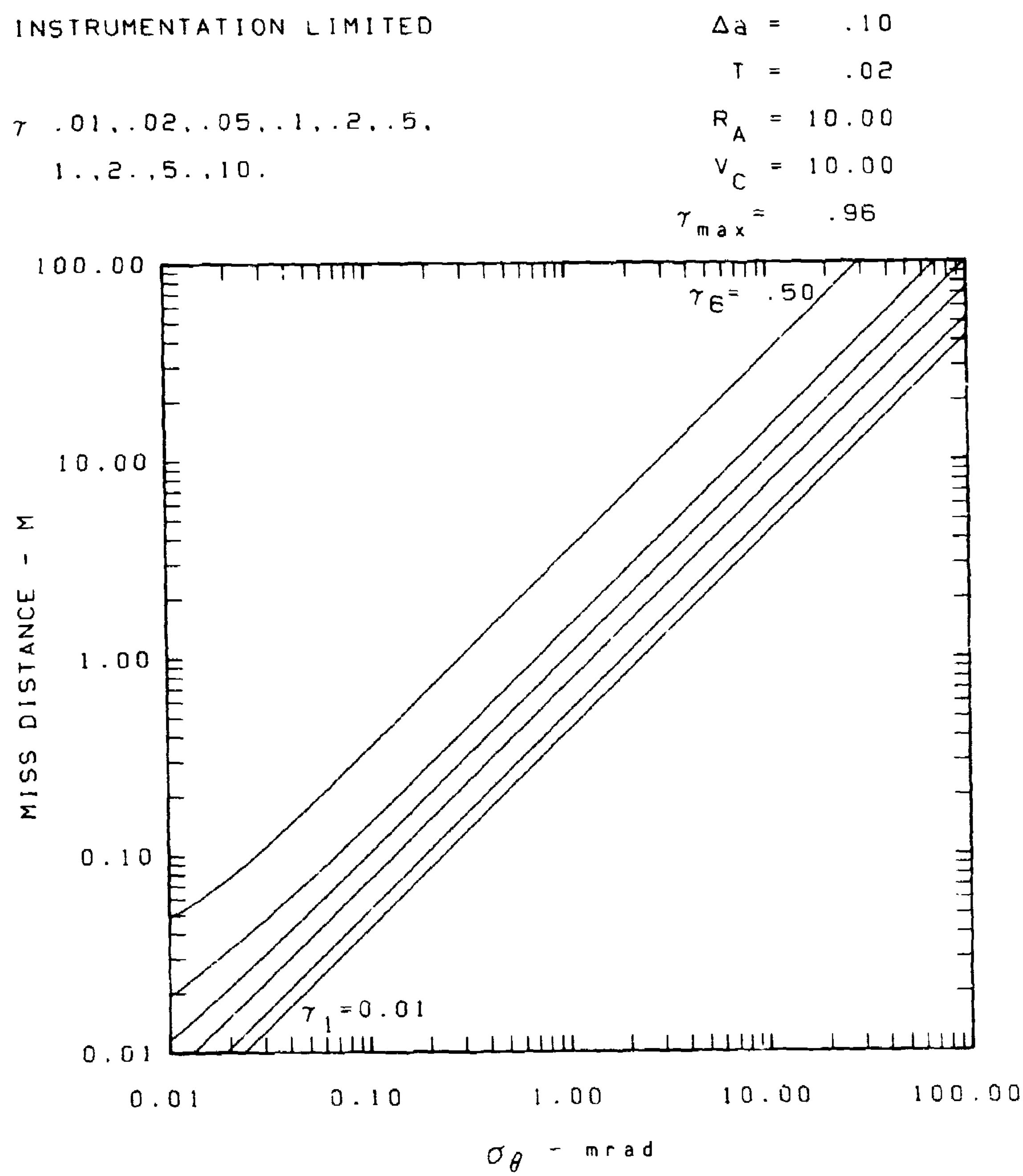


Fig. 5.3(b) Effect of data rate, 50 measurements per second.

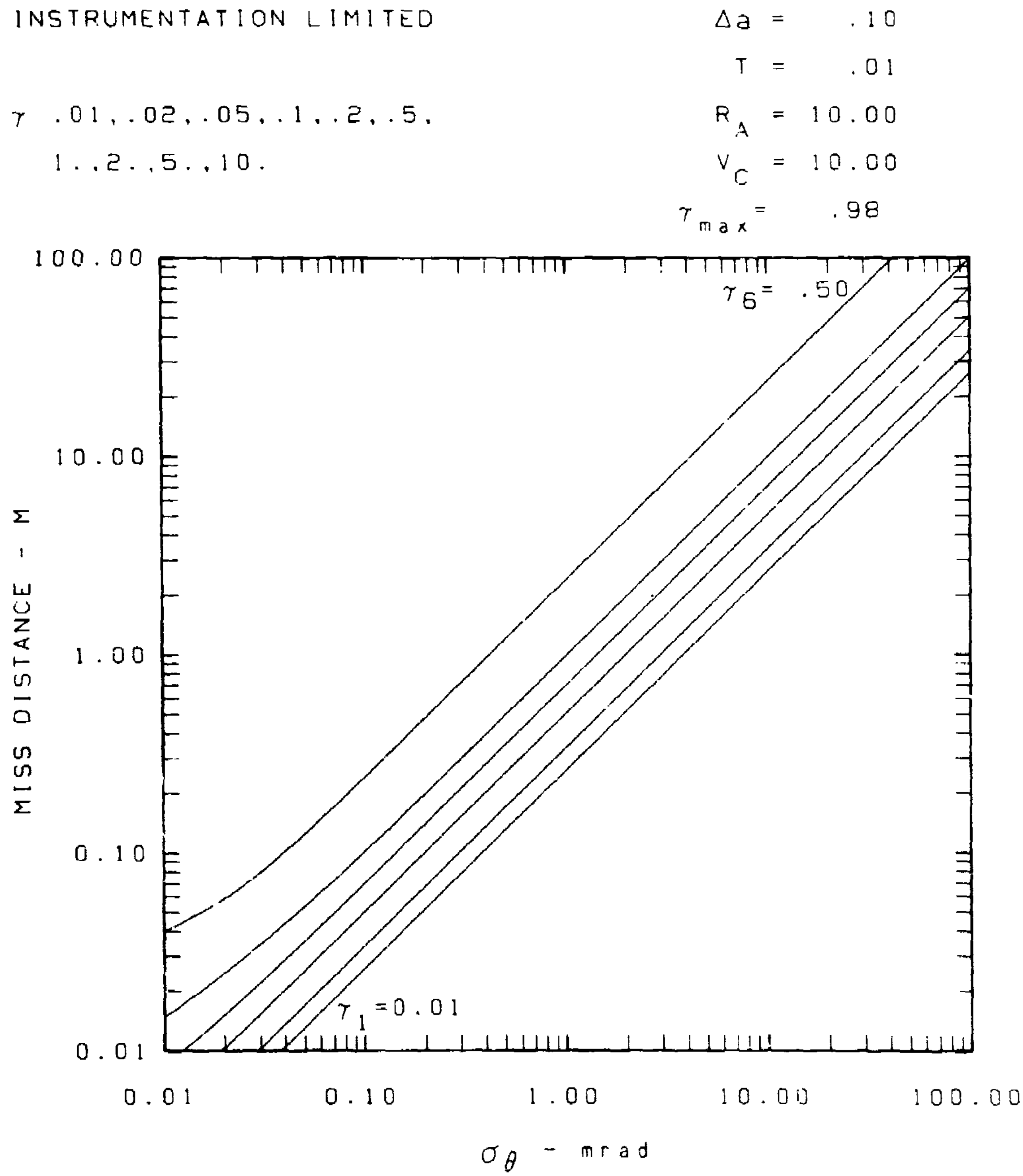


Fig. 5.3(c) Effect of data rate, 100 measurements per second.

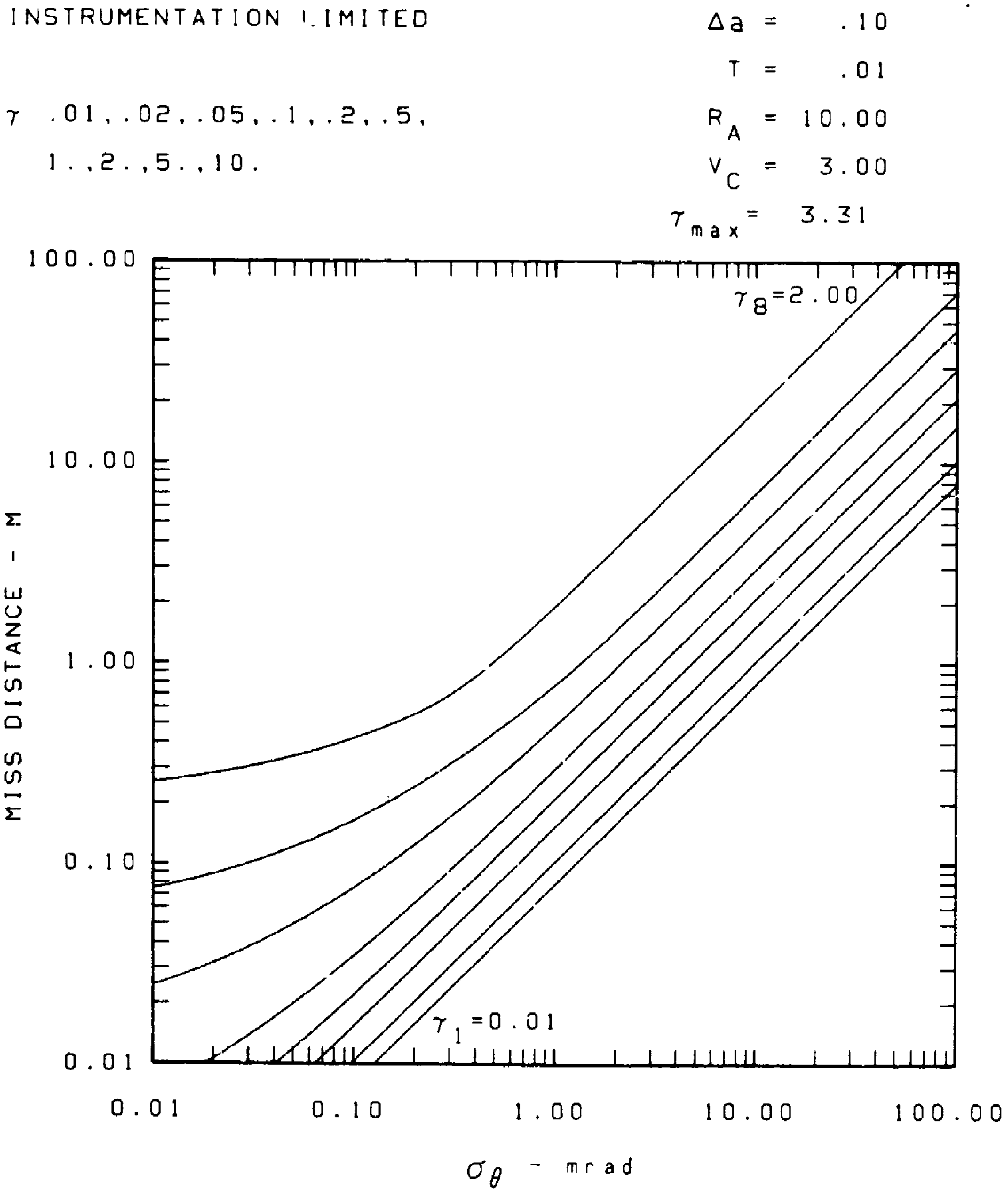


Fig. 5.4(a) Effect of control error, Δa=.1 m/s.

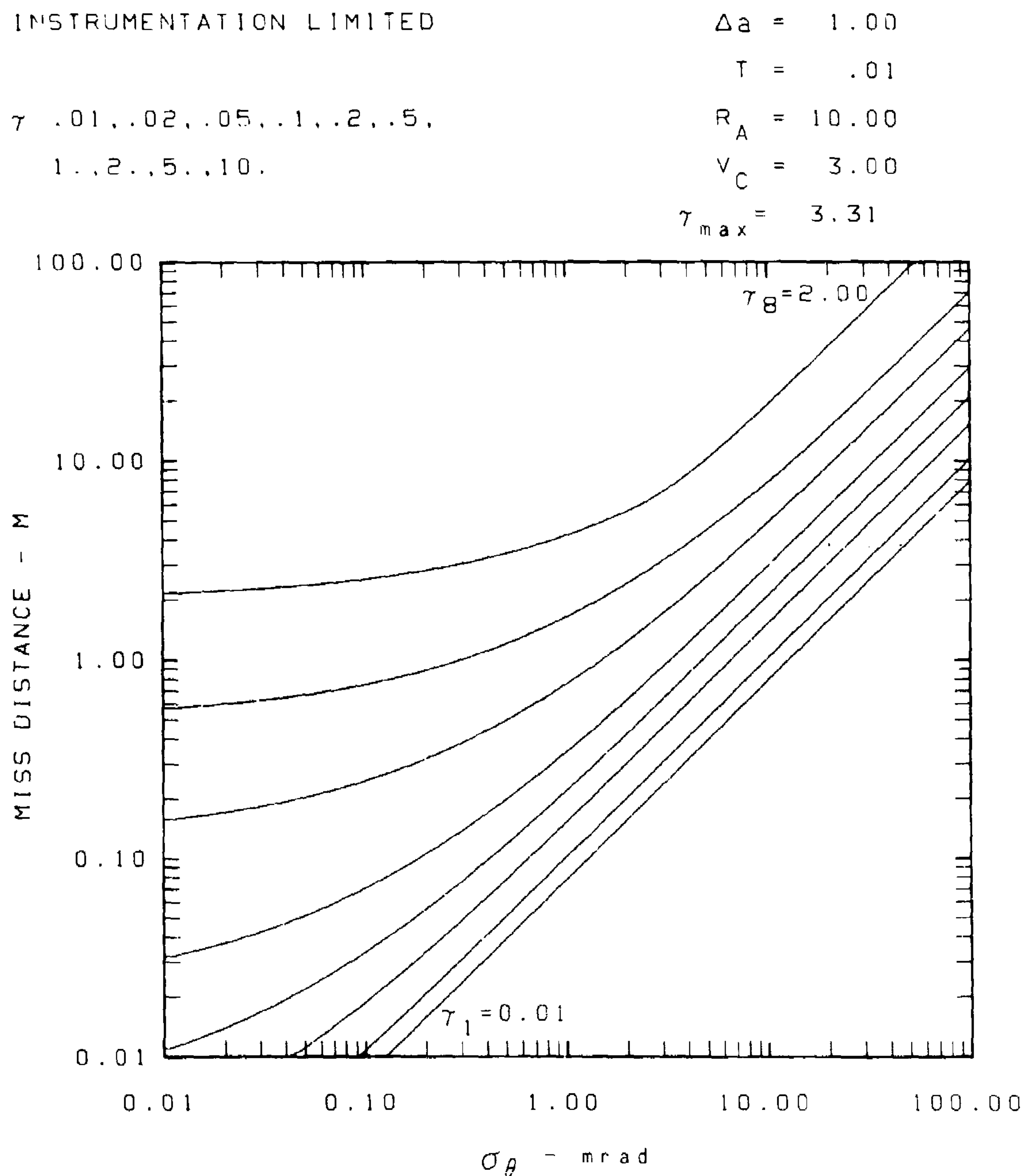


Fig. 5.4(b) Effect of control error, Δa=1. m/s.

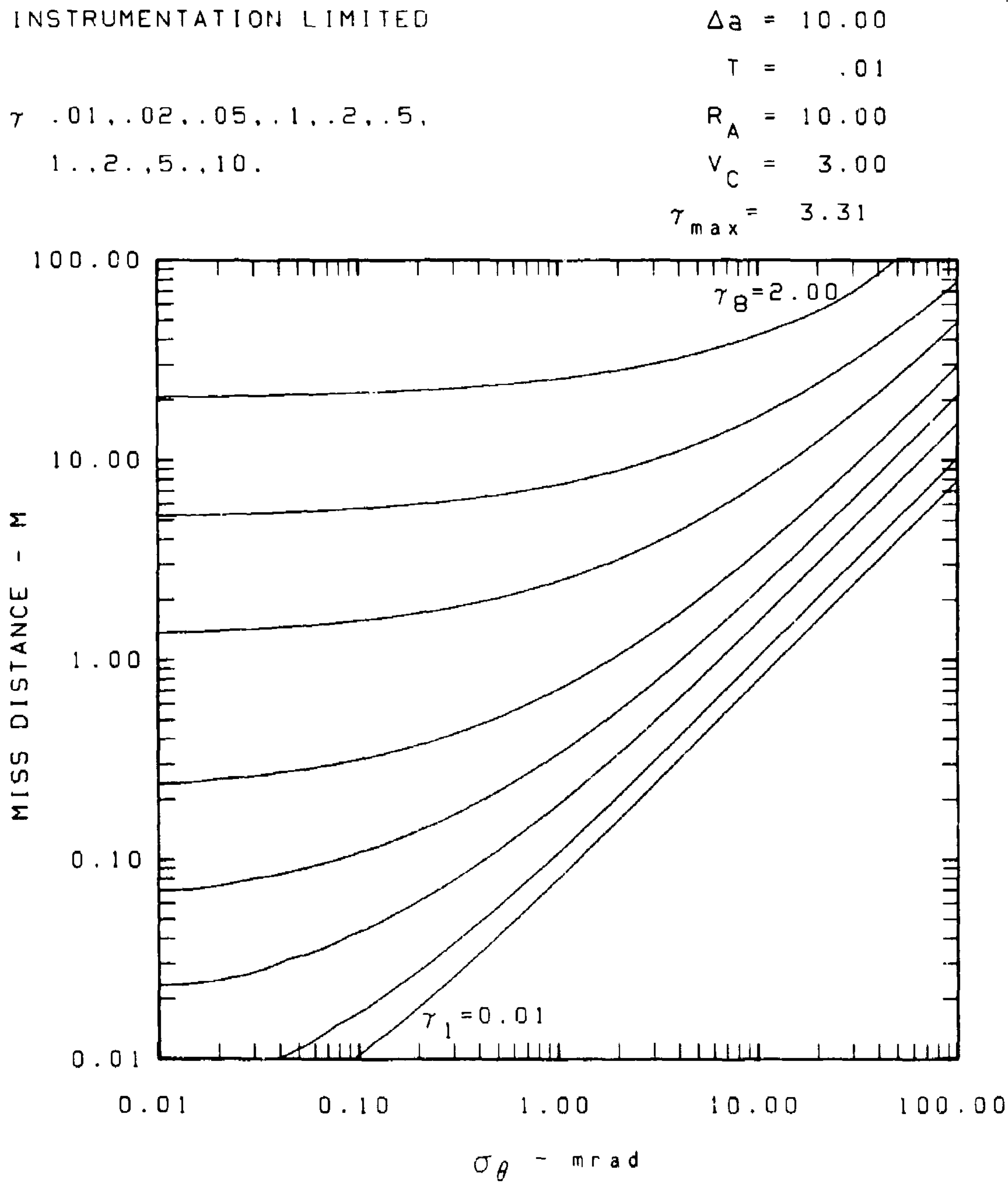


Fig. 5.4(c) Effect of control error, Δa=10 m/s.

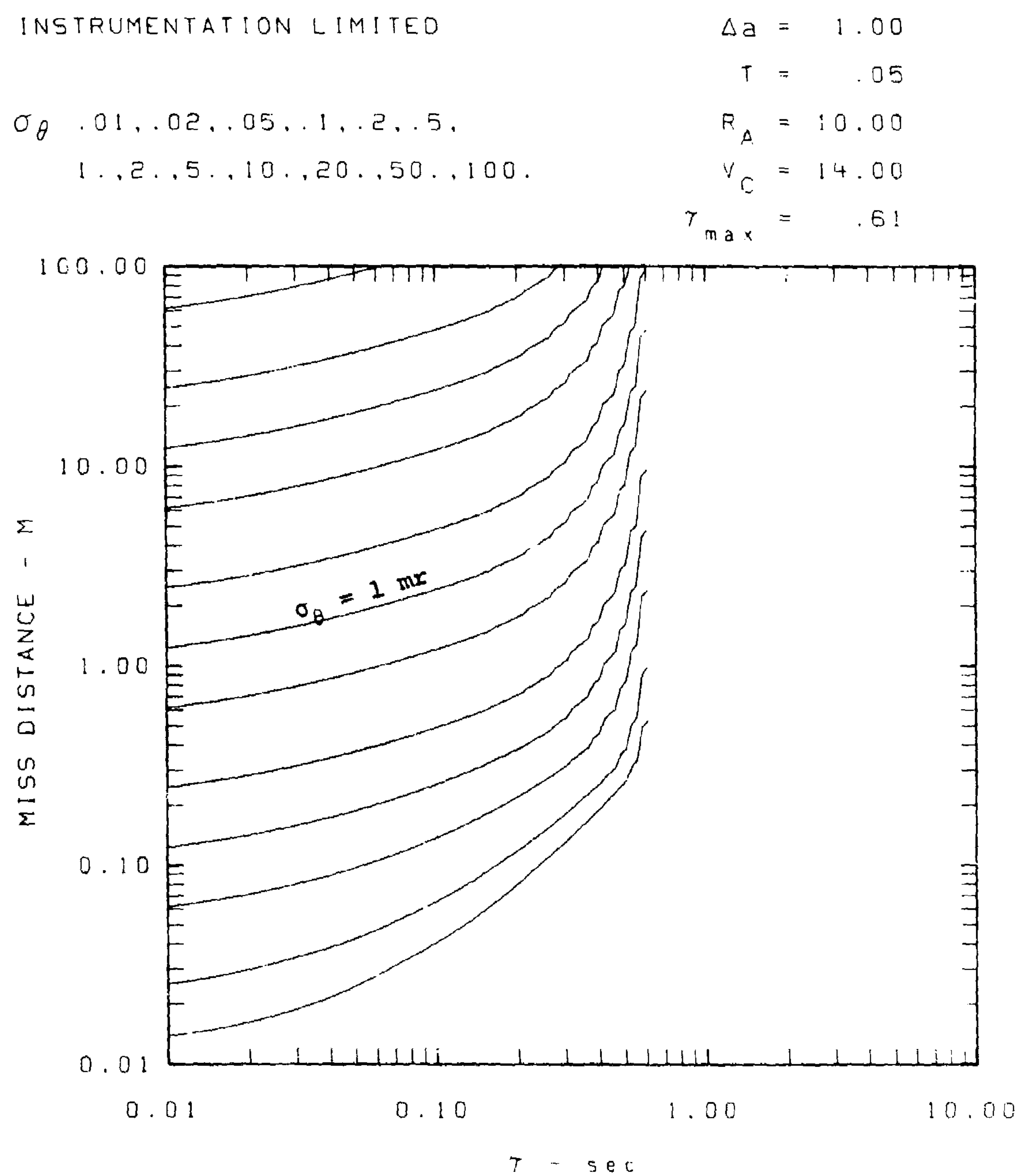


Fig. 5.5(a) Cross reference with response time, R_A=10 km, V_R=14 km/s.

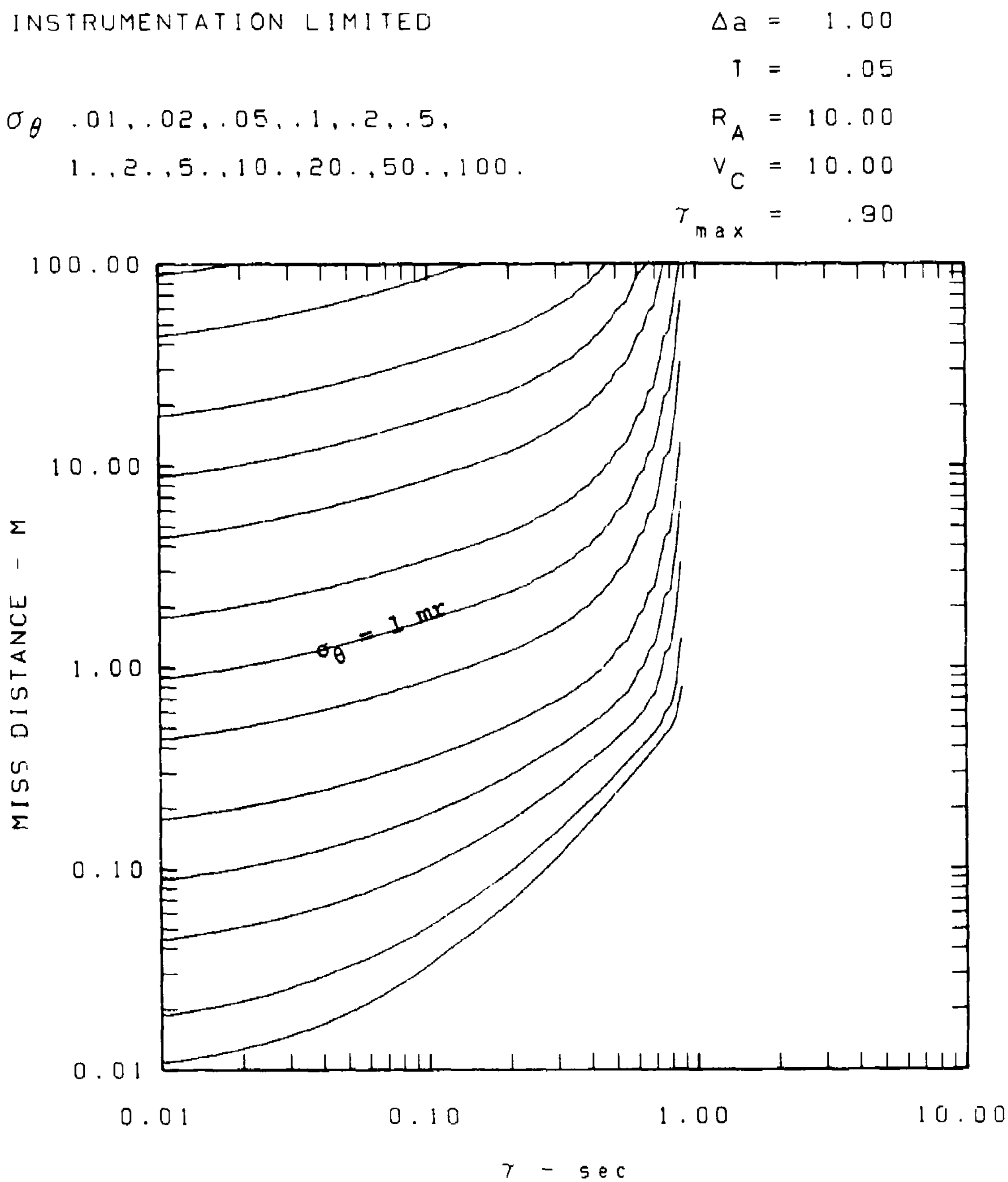


Fig. 5.5(b) Cross reference with response time, R_A=10 km, V_R=10 km/s.

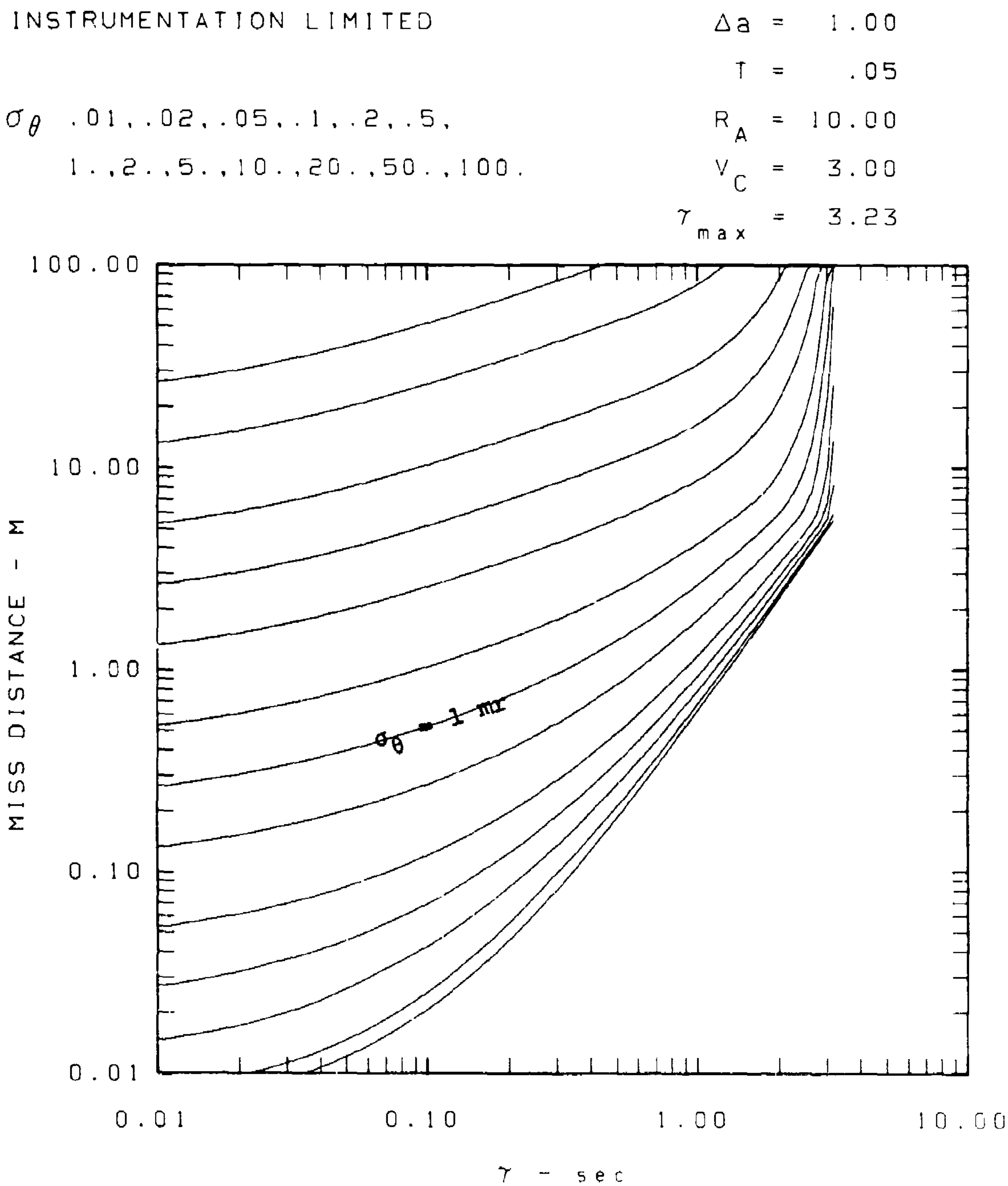


Fig. 5.5(c) Cross reference with response time, R_A=10 km, V_R=3 km/s.

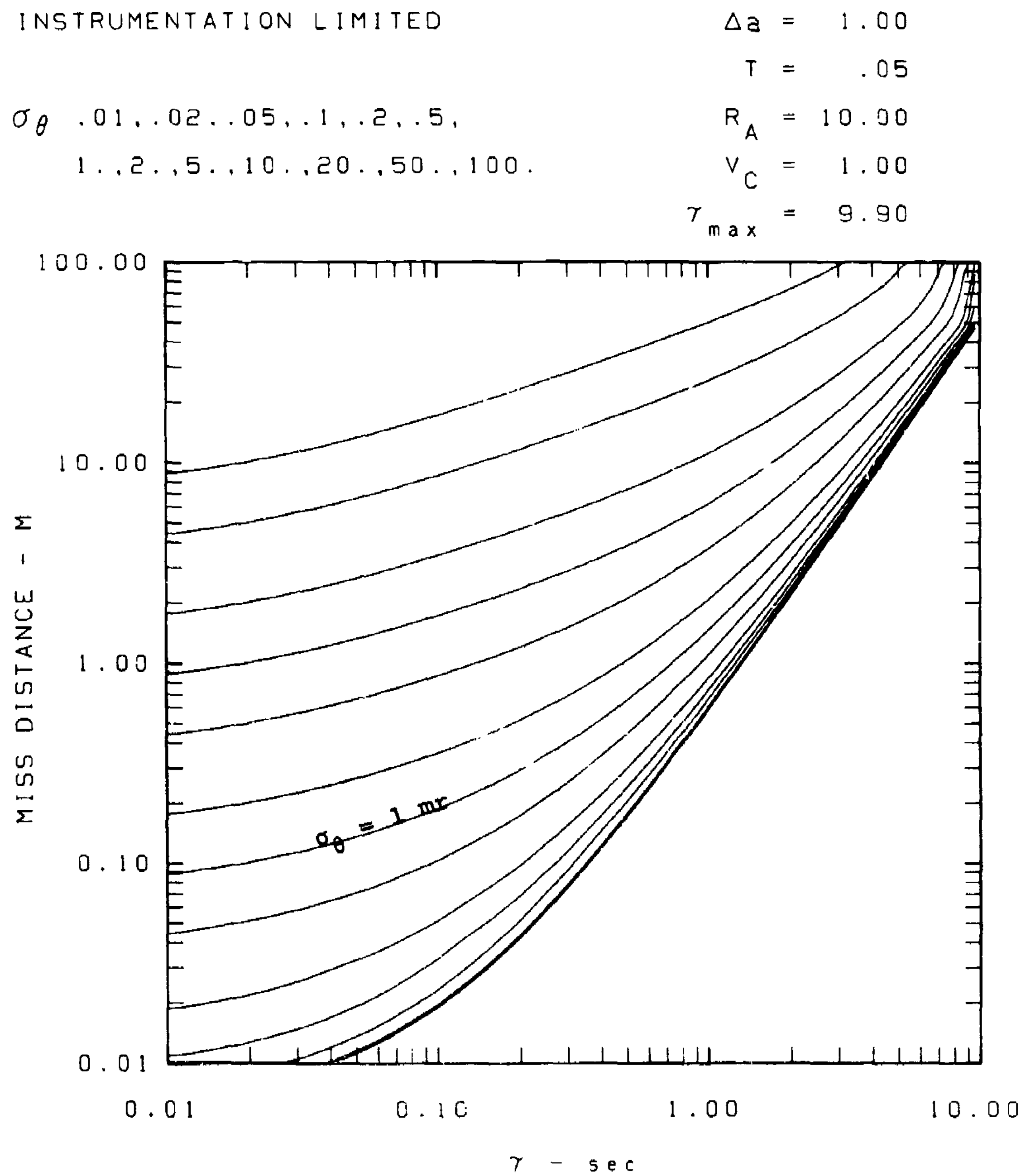


Fig. 5.5(d) Cross reference with response time, R_A=10 km, V_R=1 km/s.

6. SUMMARY

In this report, we have presented an analytical model useful for sensor and interceptor trade-off analysis. This model can treat various guidance modes such as command guidance and homing guidance whereby the homing sensor accuracy may either be a constant or vary with powers of target range.

The advantage of this model is its simplicity. Although simplified, it does include major miss contributors including sensor accuracy, data rate, interceptor time delay in responding to a given command, and interceptor control execution error. The results of this model therefore give a tight lower bound to the actual performance.

This model also assumes that the interceptor is not maneuver force limited and has sufficient fuel for nulling the initial miss. This requirement corresponds to the fact that the pre-guidance sensor provides good handover, thereby eliminating the need of excessively high acceleration maneuvers.

This model can also be used for warhead fuzing analysis. In this application, one replaces the guidance sensor with the fuze sensor and the interceptor response time with the warhead response time.

An appendix containing miss distance results for a wide range of parameters is also given.

We summarize some approximate scaling rules for miss-distance as a function of various engagement parameters below:

Sensor Error:	Miss distance ~ σ_θ unless σ_θ is very small
Missile Response Time:	Miss distance ~ $\sqrt{\tau}$ unless τ is comparable to R_A/V_R
Data Rate:	Miss distance ~ $\sqrt{T}$ unless σ_θ is very small
Closing Velocity:	Miss distance ~ V_R unless τ is comparable to R_A/V_R
Command Bias:	Miss distance ~ Δa when σ_θ is very small

APPENDIX A

A Collection of Miss Distance Results for Homing Guidance with Instrumentation Error Limited Sensor

In this appendix, we give a set of miss distance curves covering a wide range of design parameters for the homing guidance instrumentation error limited case. An index to the figure numbers and parameters used is given in Table A.1.

We re-emphasize that these results apply for the case when the interceptor is not maneuver force limited.

TABLE A.1

INDEX ON CASES EXAMINED

R_A-km		5				10				1
V_R-km/s		3		5		5		10		1
T-sec		.05	.01	.05	.01	.05	.01	.05	.01	.01
Command Error - m/s^2	.0.1	A.1	A.2	A.3	A.4	A.5	A.6	A.7	A.8	A.9
	1.0	A.10	A.11	A.12	A.13	A.14	A.15	A.16	A.17	A.18
	10.0	A.19	A.20	A.21	A.22	A.23	A.24	A.25	A.26	A.27

Note: (1) Entries are figure numbers.

(2) All cases are homing with instrumentation limit.

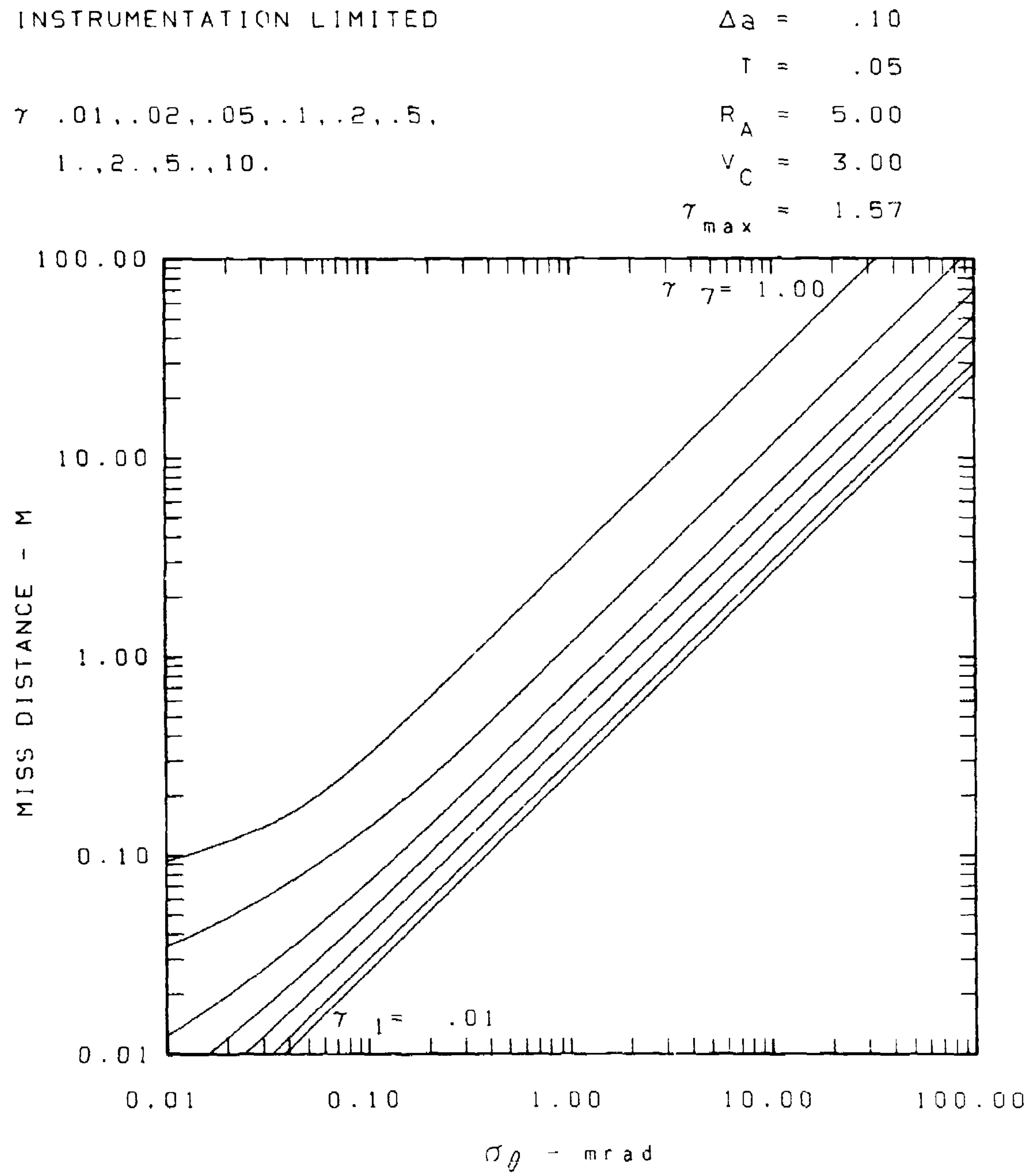
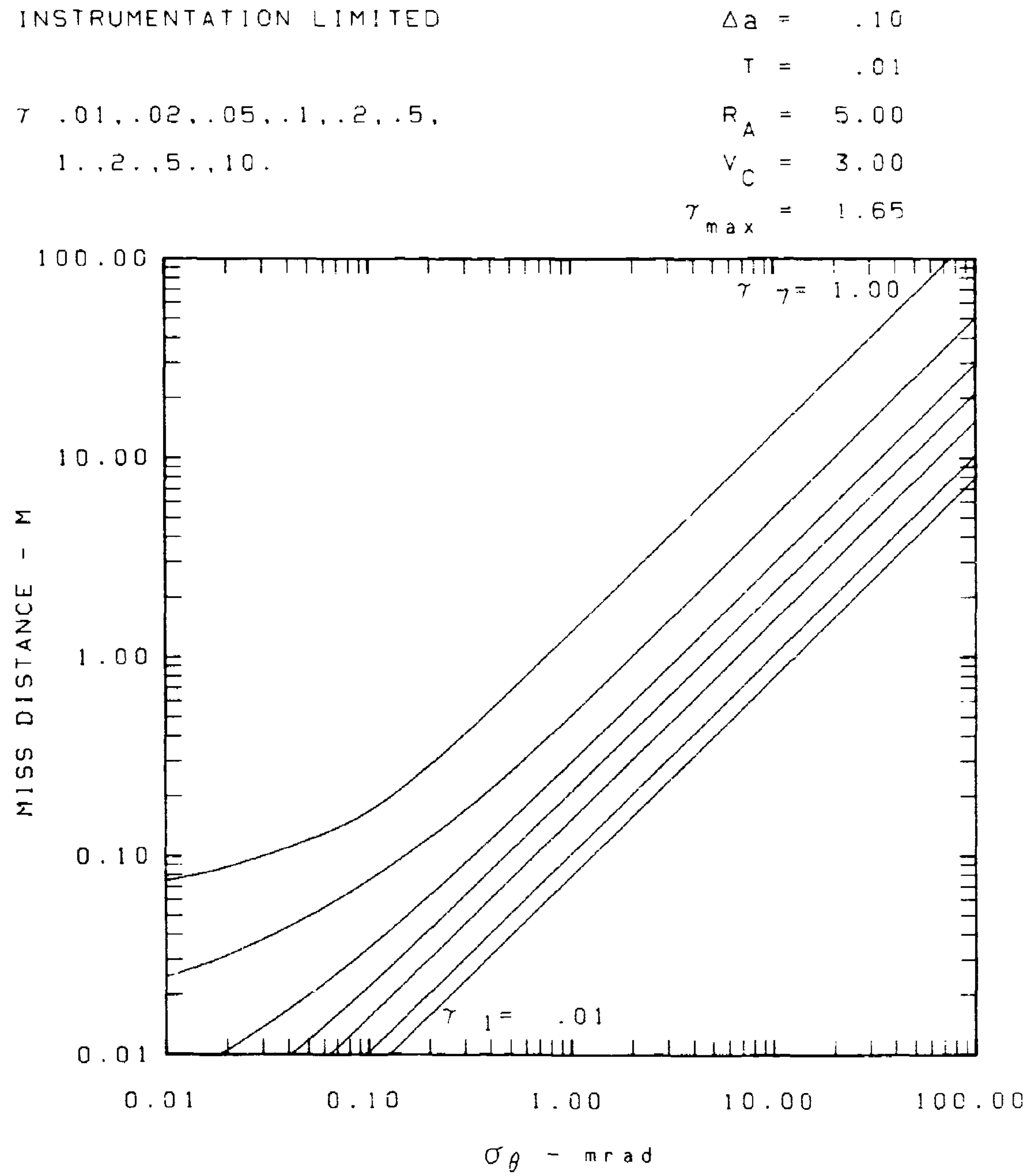
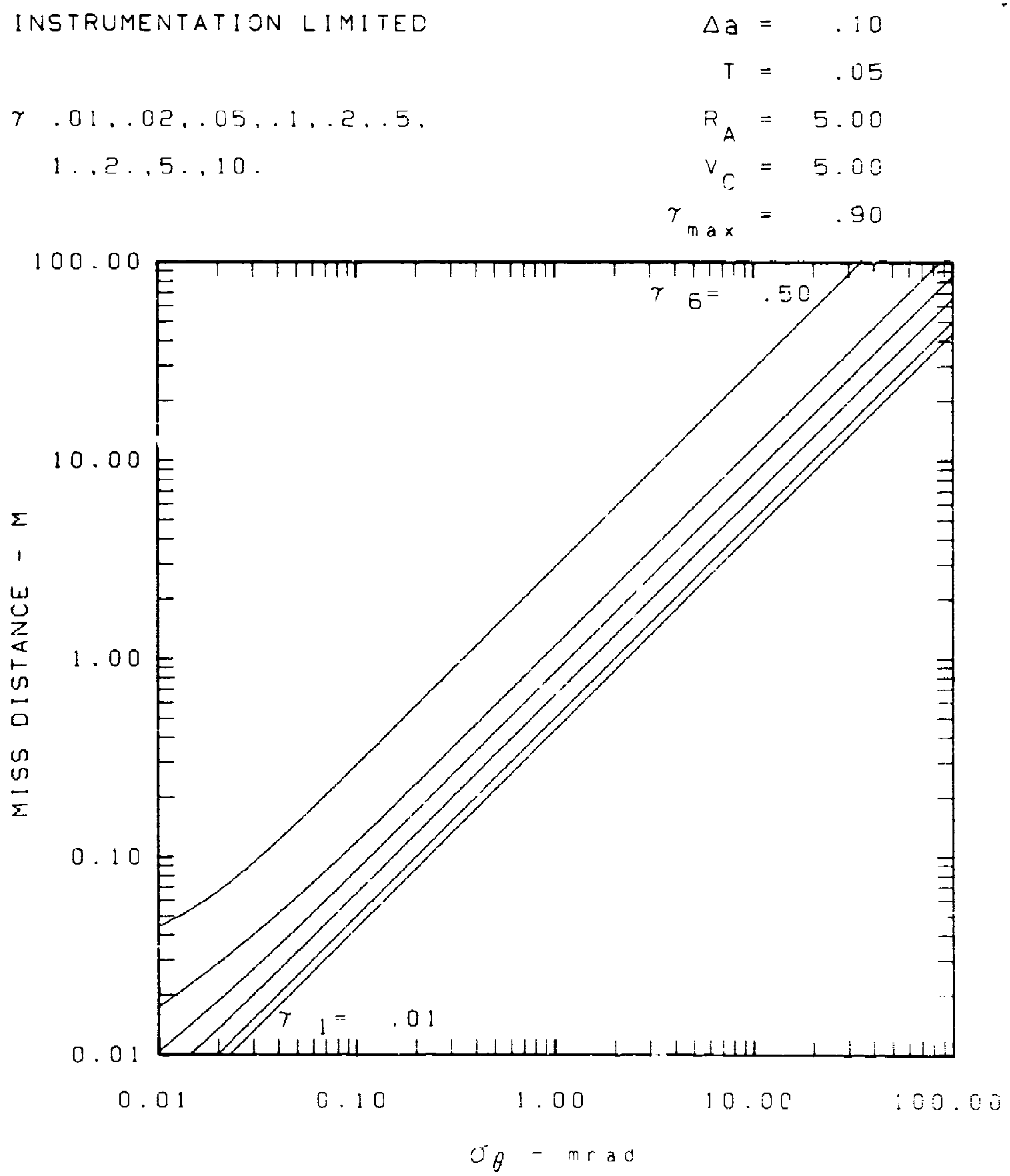

INSTRUMENTATION LIMITED
Δa = .10
T = .05
τ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
R_A = 5.00
V_C = 3.00
τ_{max} = 1.57
τ $_7$= 1.00
τ $_1$= .01
MISS DISTANCE - M
100.00
10.00
1.00
0.10
0.01
0.01
0.10
1.00
10.00
100.00
σ_θ - mrad

Fig. A-1.



INSTRUMENTATION LIMITED
γ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
Δa = .10
T = .01
R_A = 5.00
V_C = 3.00
γ_{max} = 1.65
γ $_7$= 1.00
γ $_1$= .01
100.00
10.00
1.00
0.10
0.01
0.01
0.10
1.00
10.00
100.00
MISS DISTANCE - M
σ_θ - mrad

Fig. A-2.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Δa = .10
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γ .01,.02,.05,.1,.2,.5, 1.,2.,5.,10.
R_A = 5.00
V_C = 5.00
γ_{max} = .90
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
γ_6= .50
γ_1= .01
0.01
0.10
1.00
10.00
100.00
σ_θ - mrad

Fig. A-3.

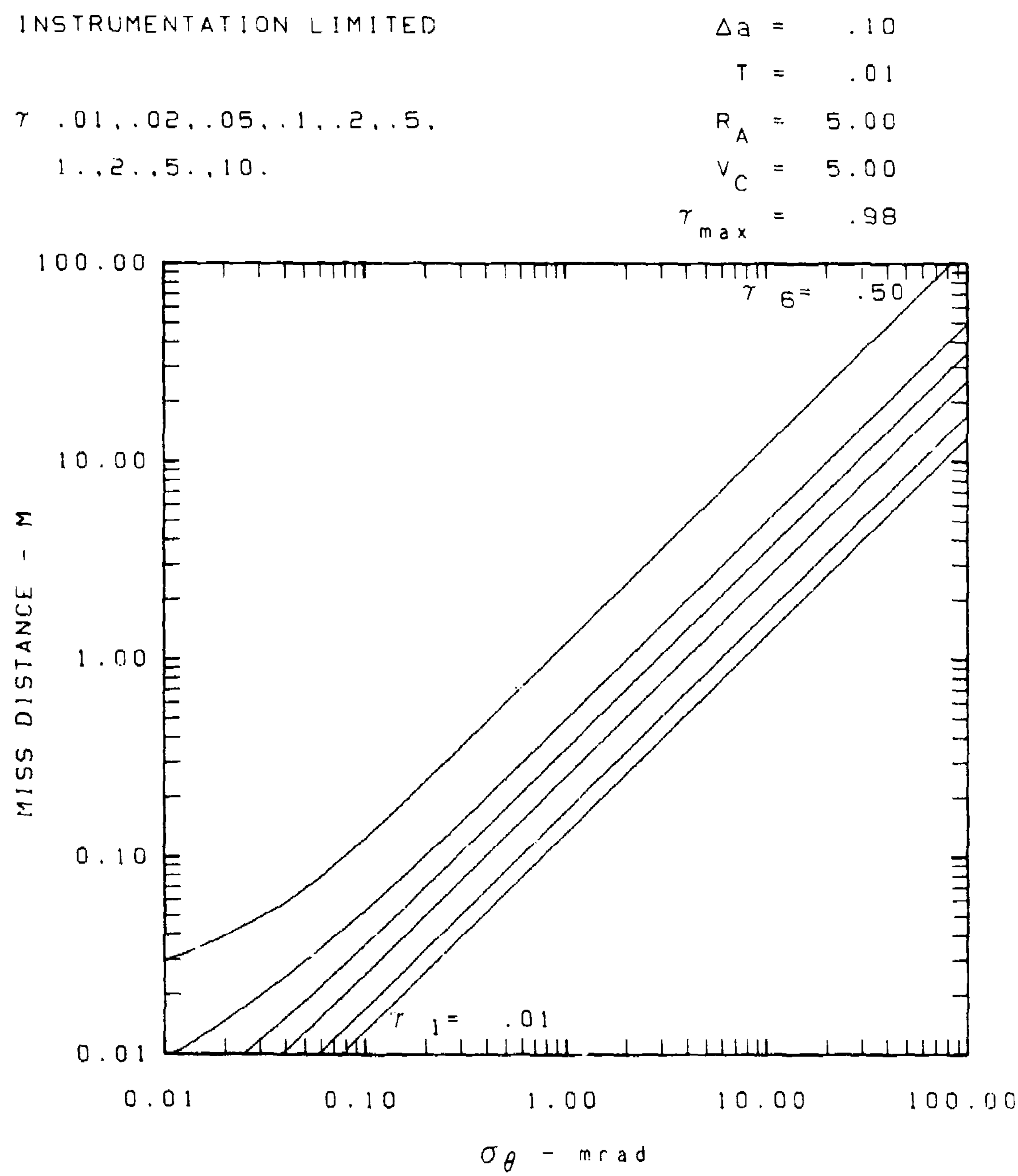

INSTRUMENTATION LIMITED
τ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
Δa = .10
T = .01
R A = 5.00
V C = 5.00
τ max = .98
τ 6= .50
τ 1= .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σθ - mrad

Fig. A-4.

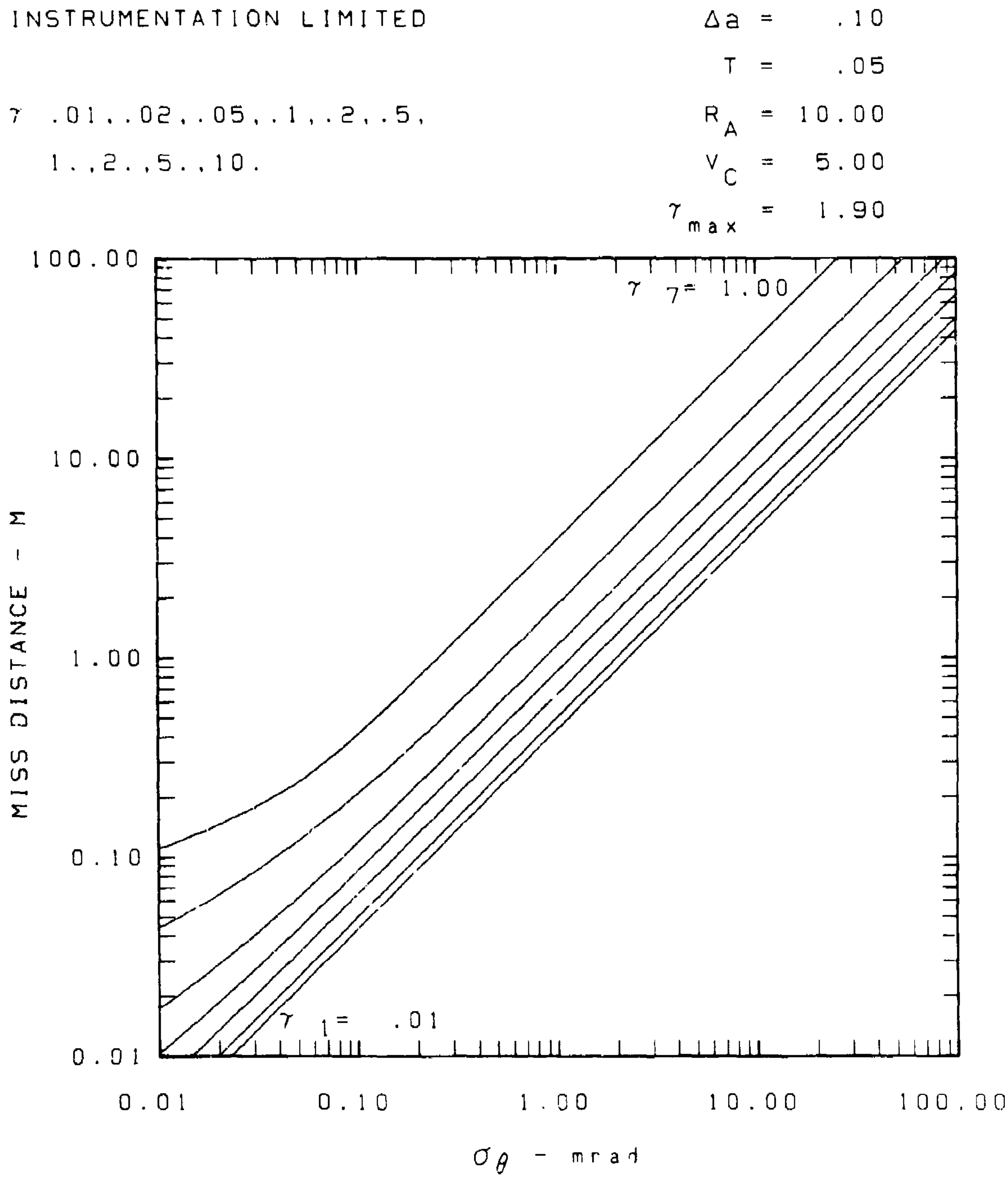
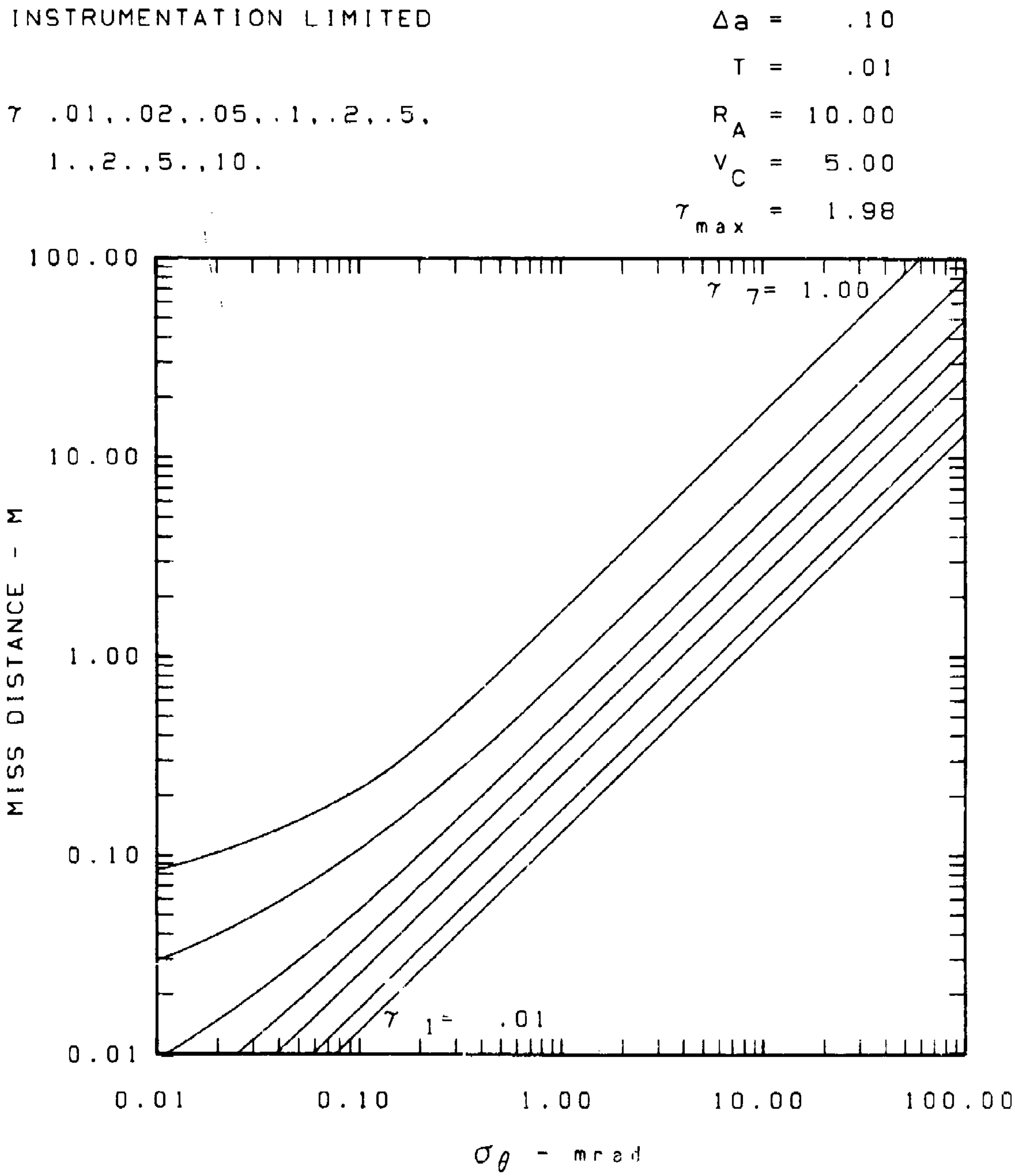
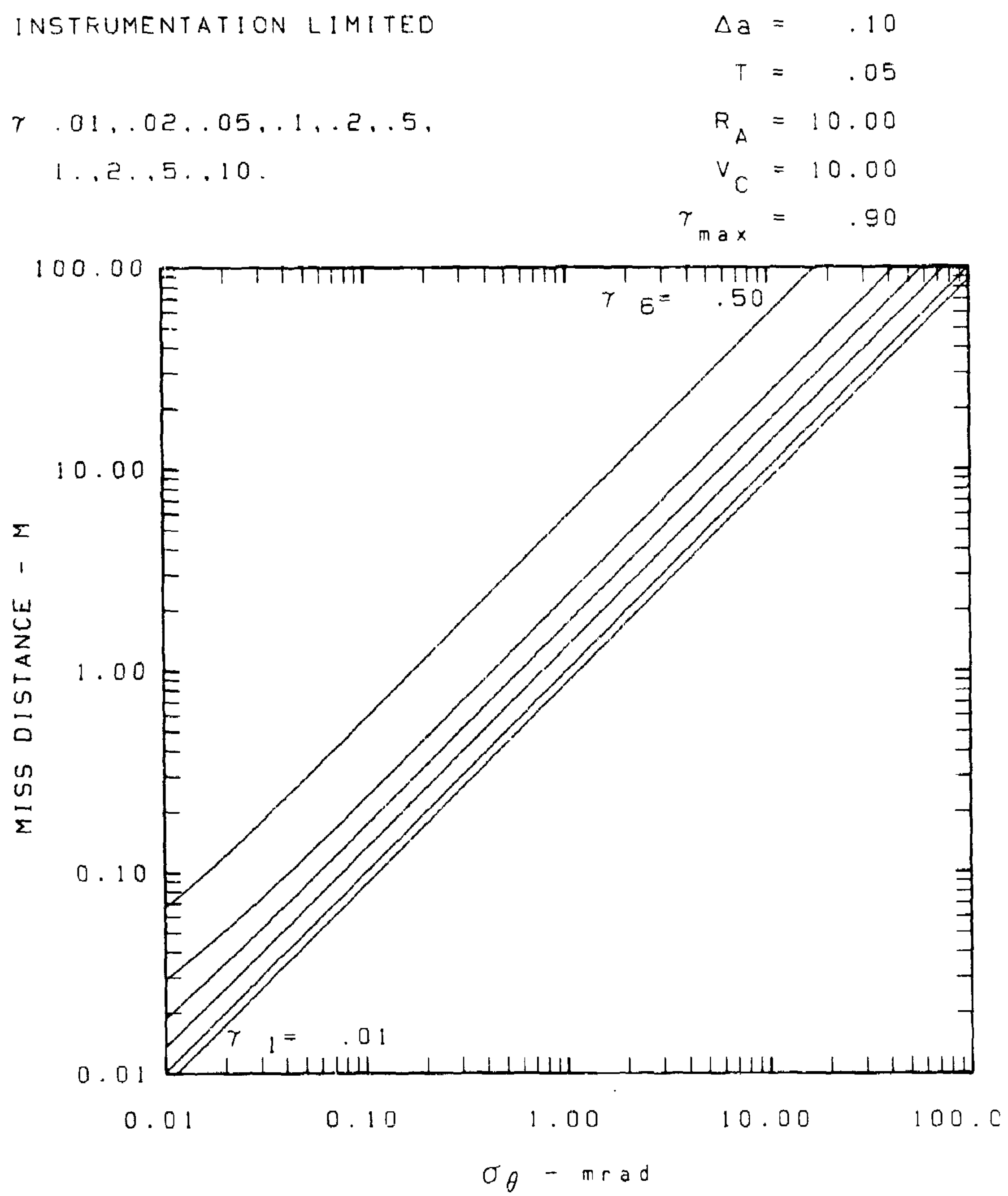

INSTRUMENTATION LIMITED
γ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
Δa = .10
T = .05
R A = 10.00
V C = 5.00
γ max = 1.90
γ 7= 1.00
γ 1= .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σθ - mrad

Fig. A-5.



INSTRUMENTATION LIMITED
Δa = .10
T = .01
γ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
R A = 10.00
V C = 5.00
γ max = 1.98
γ 7 = 1.00
γ 1 = .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σθ - mrad

Fig. A-6.



INSTRUMENTATION LIMITED
γ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
Δa = .10
T = .05
R A = 10.00
V C = 10.00
γ max = .90
γ 6 = .50
γ 1 = .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σθ - mrad

Fig. A-7.

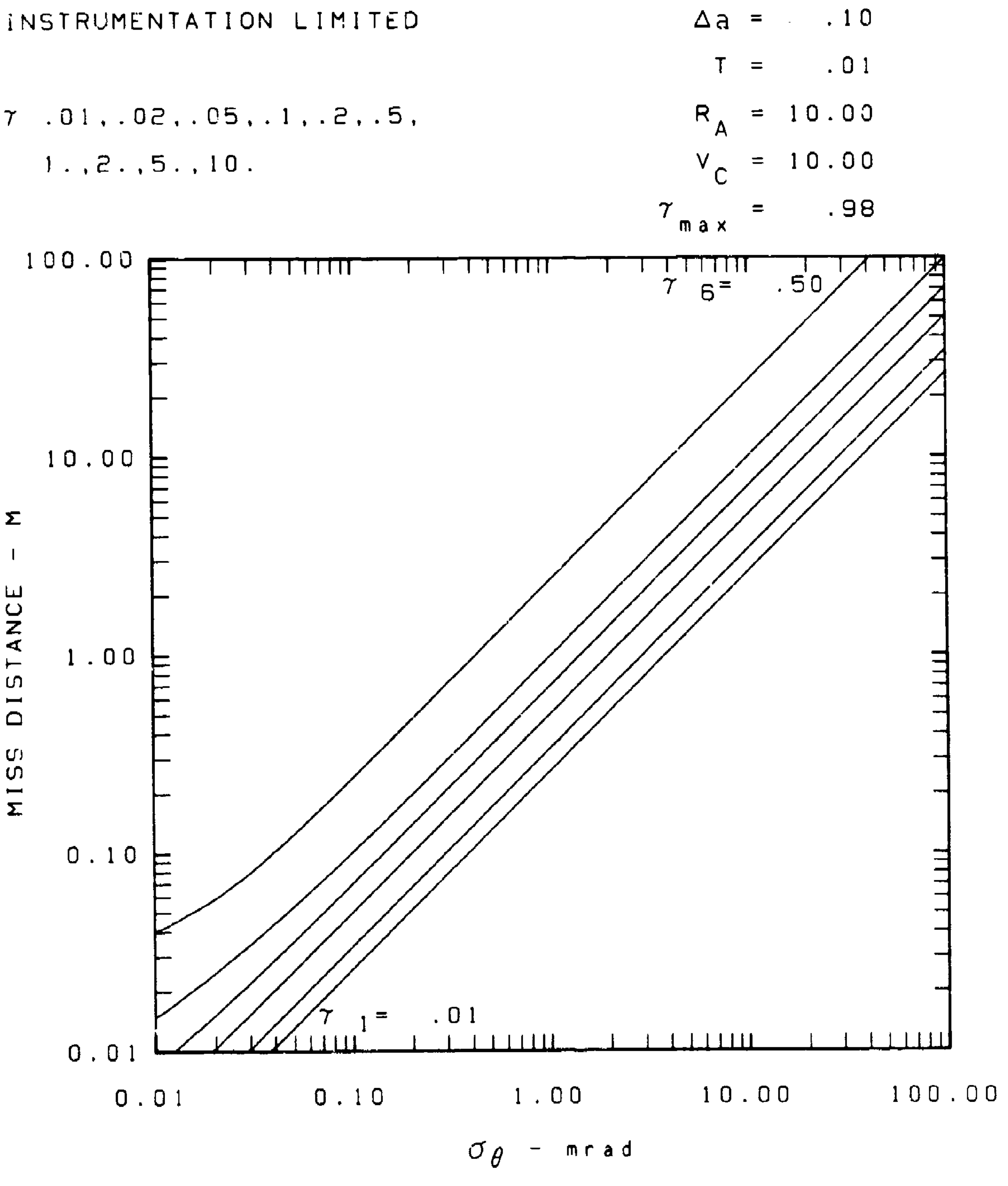

INSTRUMENTATION LIMITED
γ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
Δa = .10
T = .01
R_A = 10.00
V_C = 10.00
γ_{max} = .98
γ_6 = .50
γ_1 = .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σ_θ - mrad

Fig. A-8.

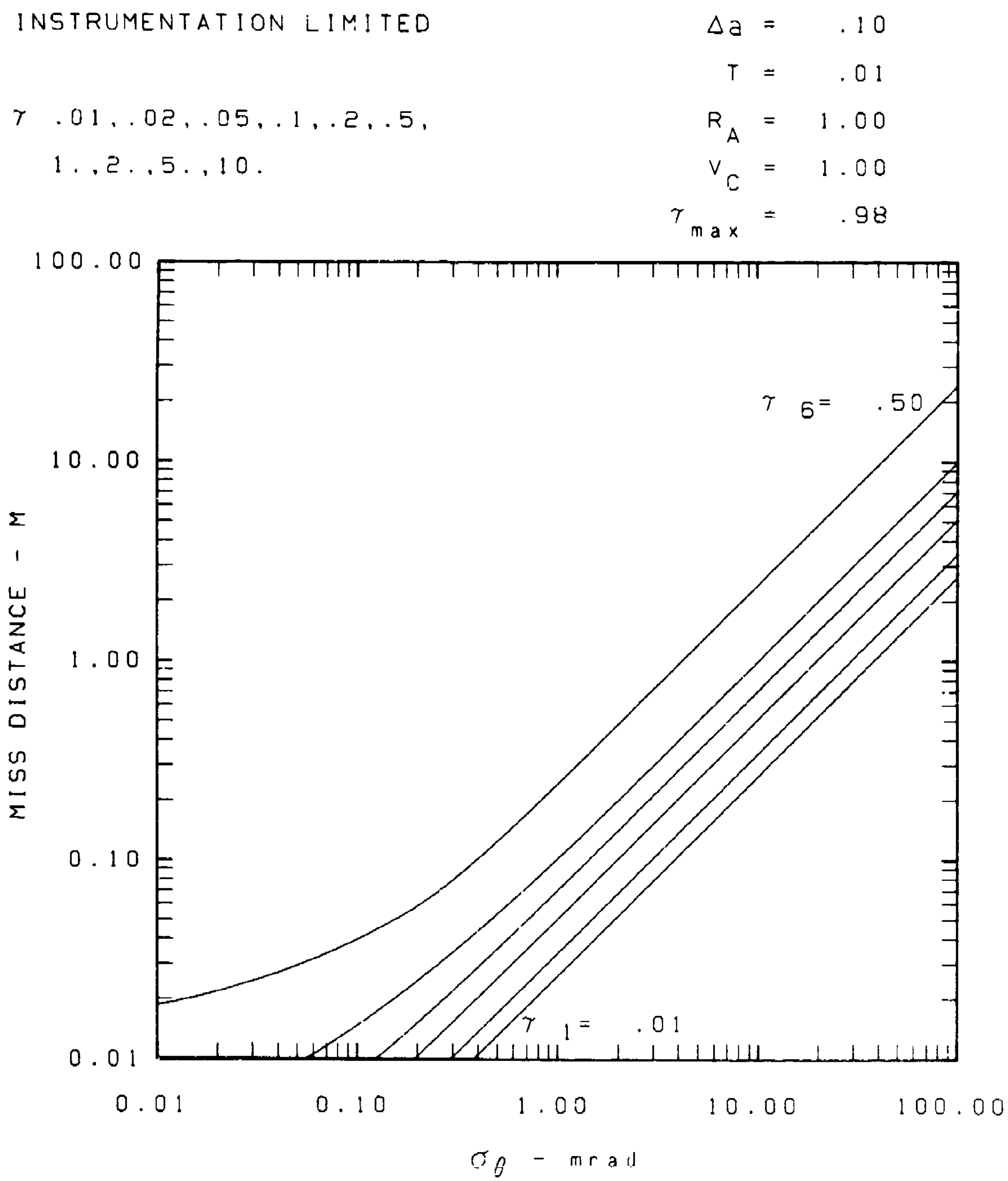
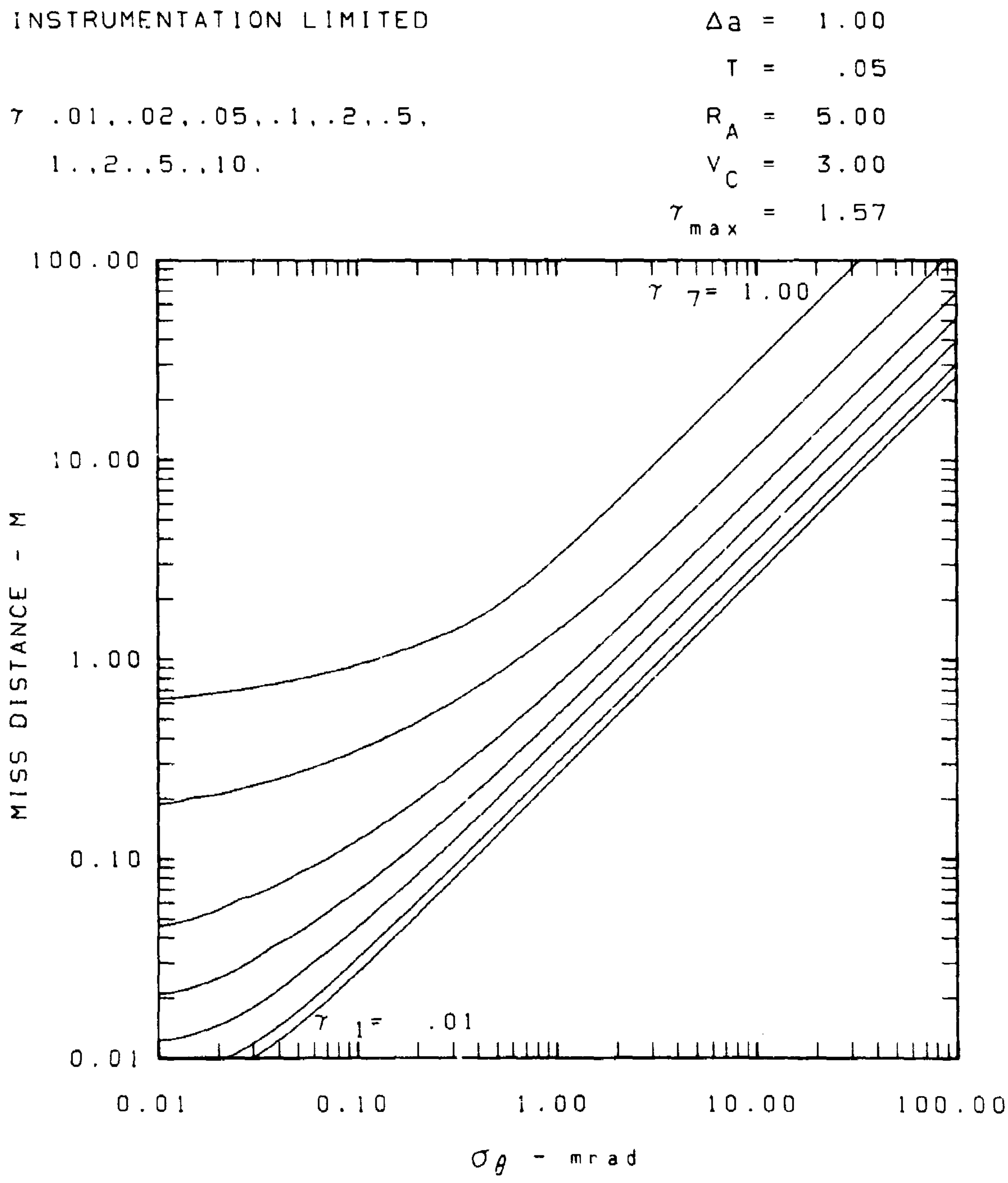
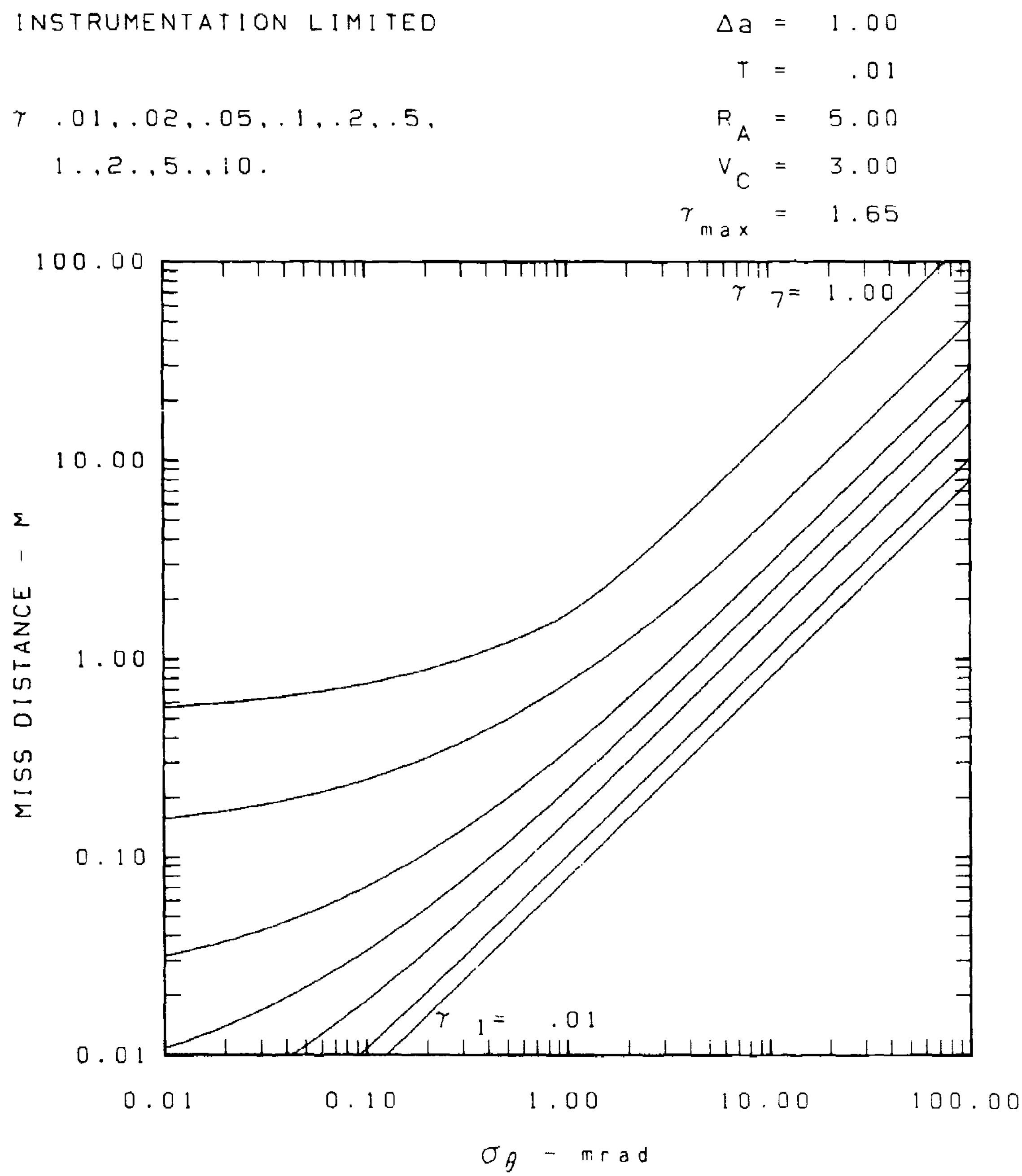

INSTRUMENTATION LIMITED
Δa = .10
T = .01
γ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
R_A = 1.00
V_C = 1.00
γ_max = .98
γ_6 = .50
γ_1 = .01
MISS DISTANCE - M
100.00
10.00
1.00
0.10
0.01
0.01
0.10
1.00
10.00
100.00
σ_θ - mrad

Fig. A-9



INSTRUMENTATION LIMITED
γ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
Δa = 1.00
T = .05
R_A = 5.00
V_C = 3.00
γ_max = 1.57
γ_7 = 1.00
γ_1 = .01
MISS DISTANCE - M
σθ - mrad
100.00
10.00
1.00
0.10
0.01
0.01
0.10
1.00
10.00
100.00

Fig. A-10.


INSTRUMENTATION LIMITED
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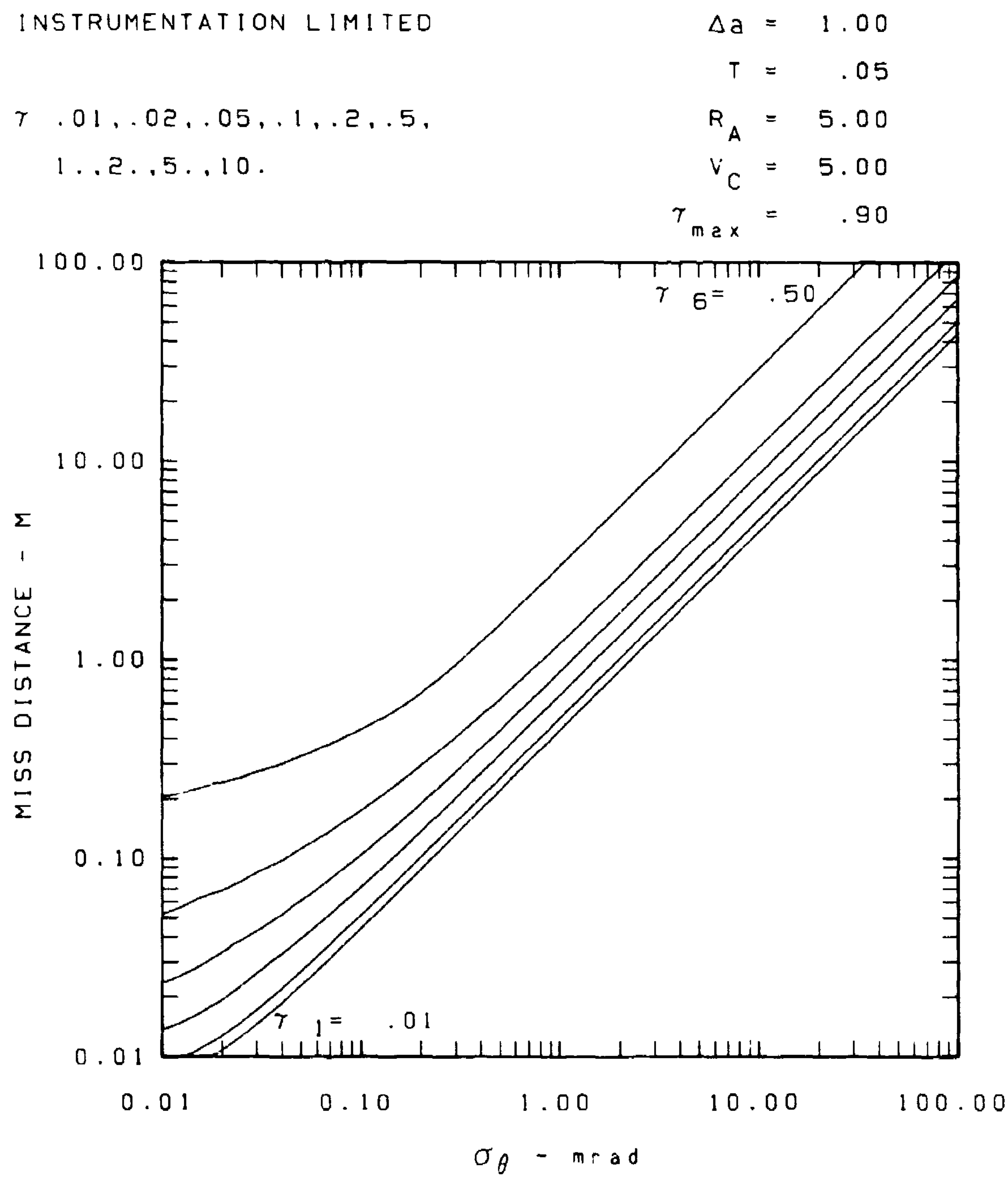
1.,2.,5.,10.
Δa = 1.00
T = .01
R A = 5.00
V C = 3.00
γ max = 1.65
γ 7= 1.00
γ 1= .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σθ - mrad

Fig. A-11.



INSTRUMENTATION LIMITED
τ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
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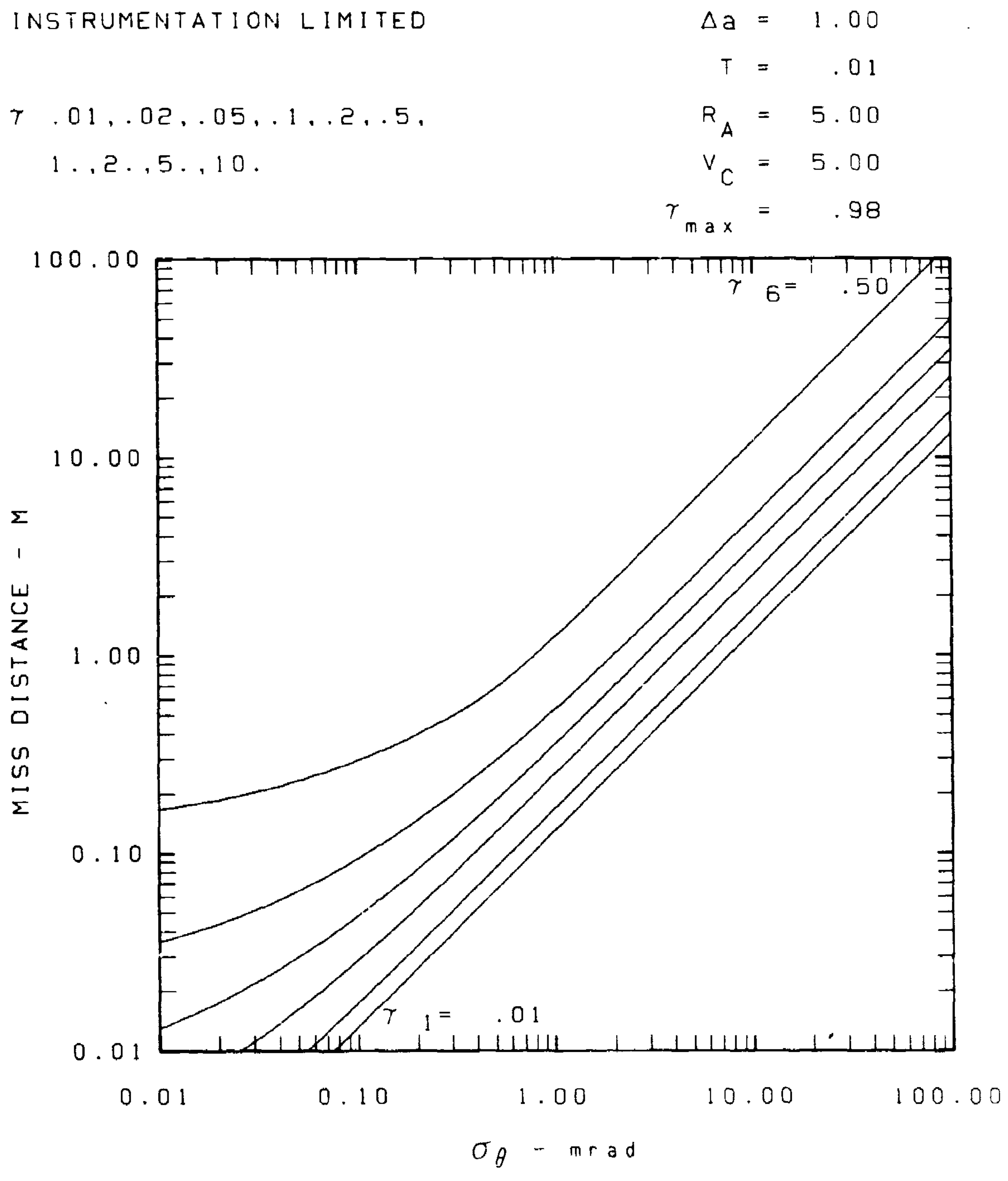
T = .05
R A = 5.00
V C = 5.00
τ max = .90
τ 6= .50
τ 1= .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σθ - mrad

Fig. A-12.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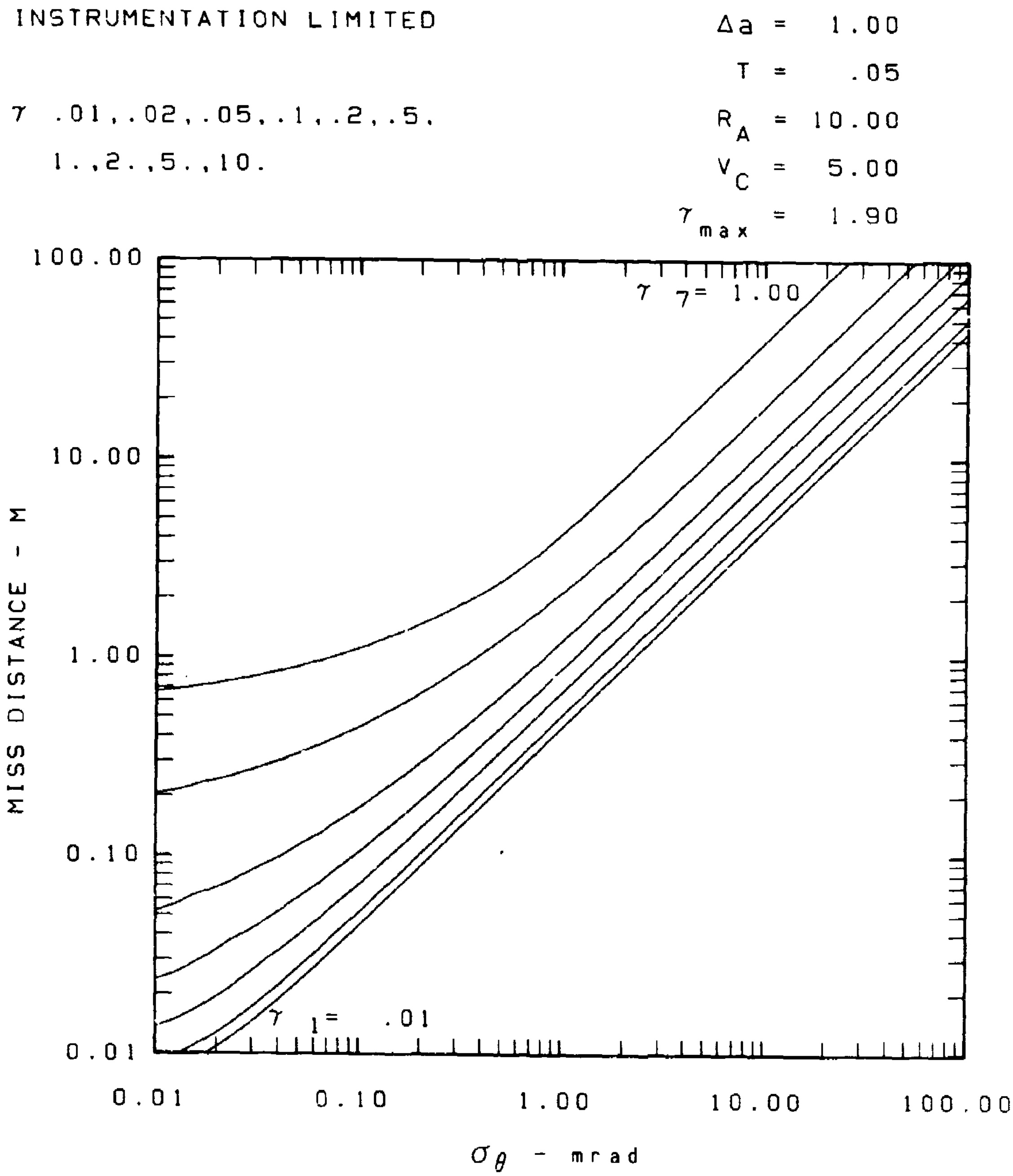
T = .01
τ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
R A = 5.00
V C = 5.00
τ max = .98
τ 6 = .50
τ 1 = .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σθ - mrad

Fig. A-13.



INSTRUMENTATION LIMITED
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1.,2.,5.,10.
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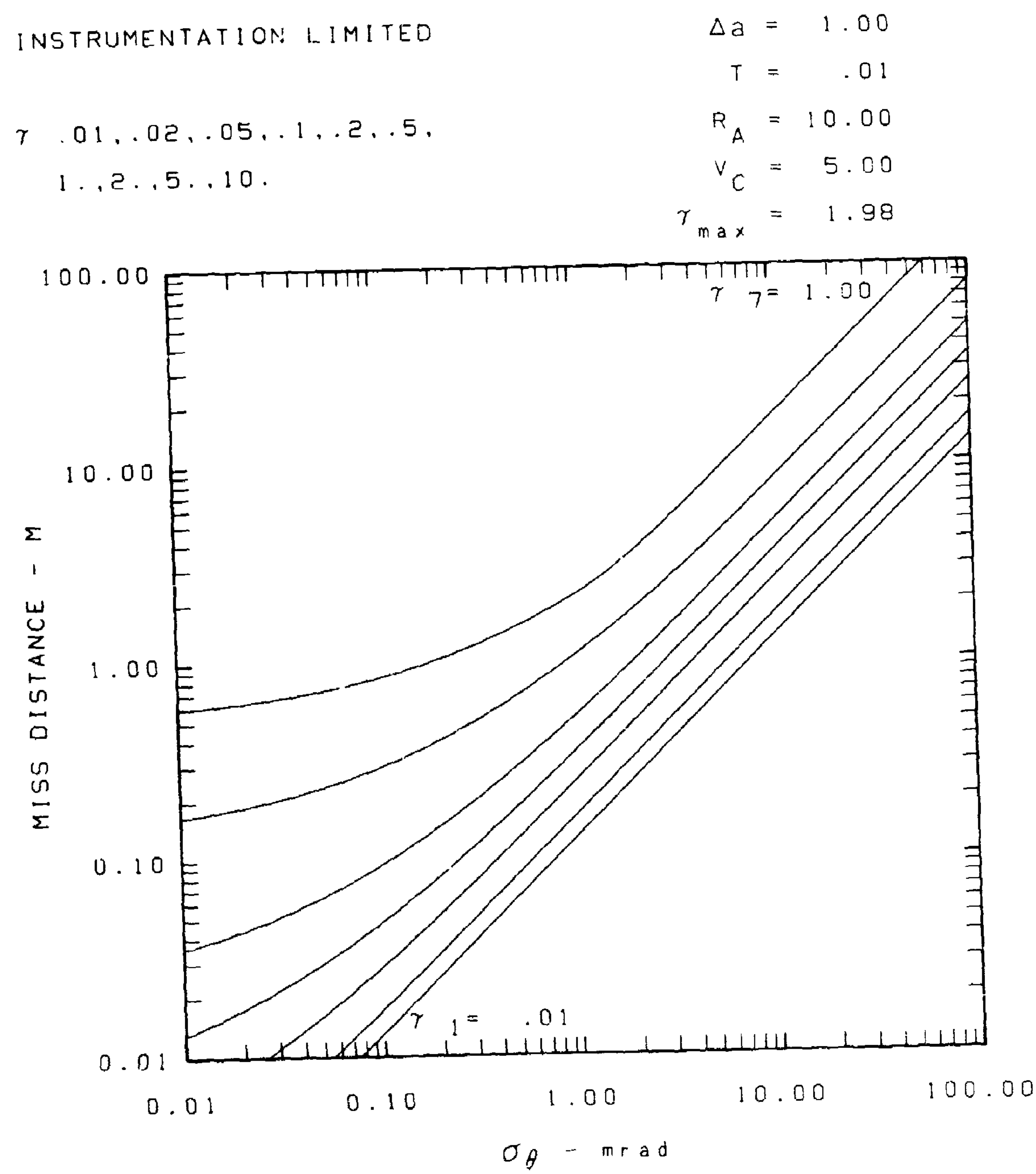
R_A = 10.00
V_C = 5.00
γ_max = 1.90
γ_7 = 1.00
γ_1 = .01
MISS DISTANCE - M
100.00
10.00
1.00
0.10
0.01
0.01
0.10
1.00
10.00
100.00
σ_θ - mrad

Fig. A-14.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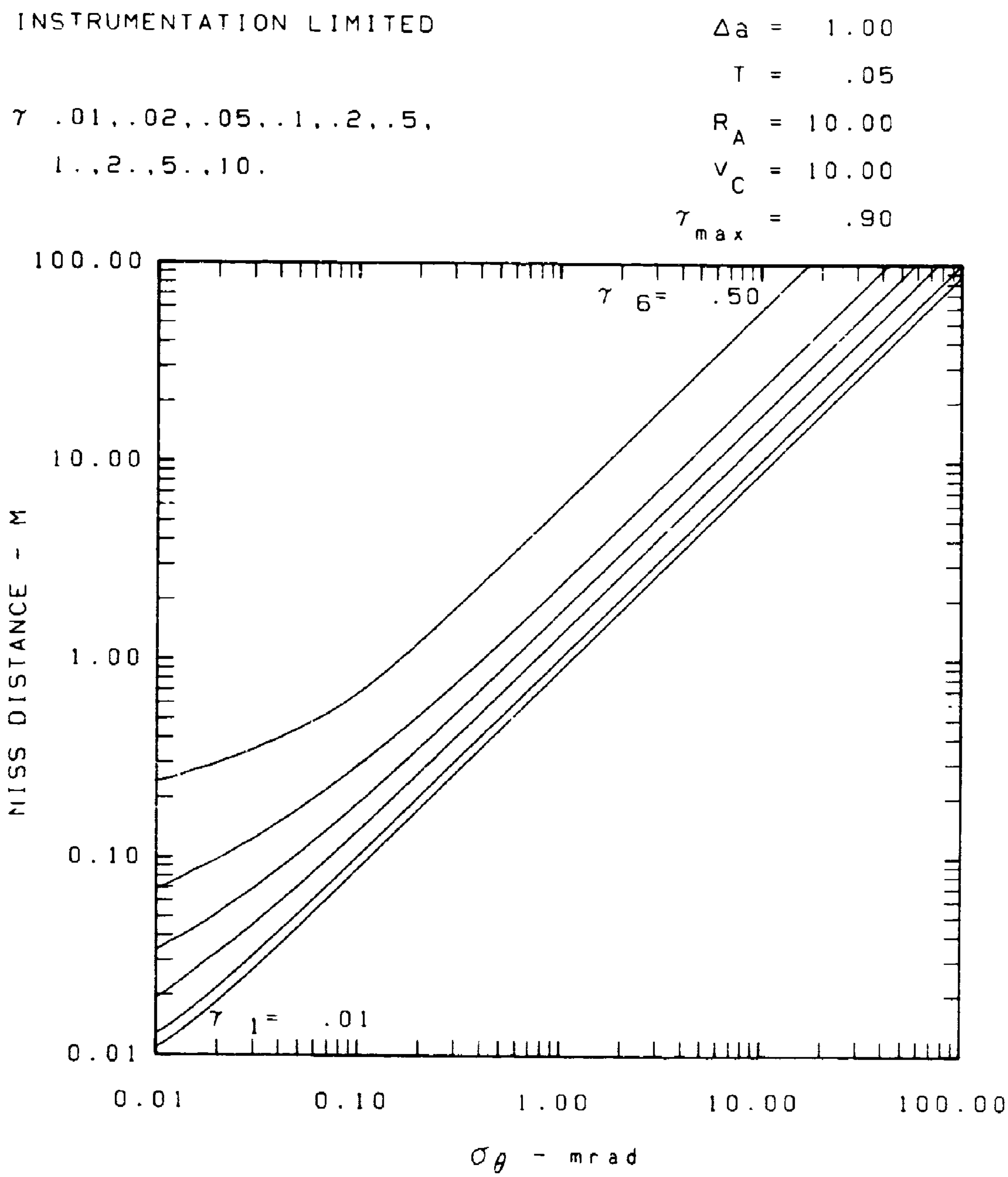
R_A = 10.00
V_C = 5.00
γ_max = 1.98
γ 7= 1.00
γ 1= .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σ_θ - mrad

Fig. A-15.


INSTRUMENTATION LIMITED
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1.,2.,5.,10.
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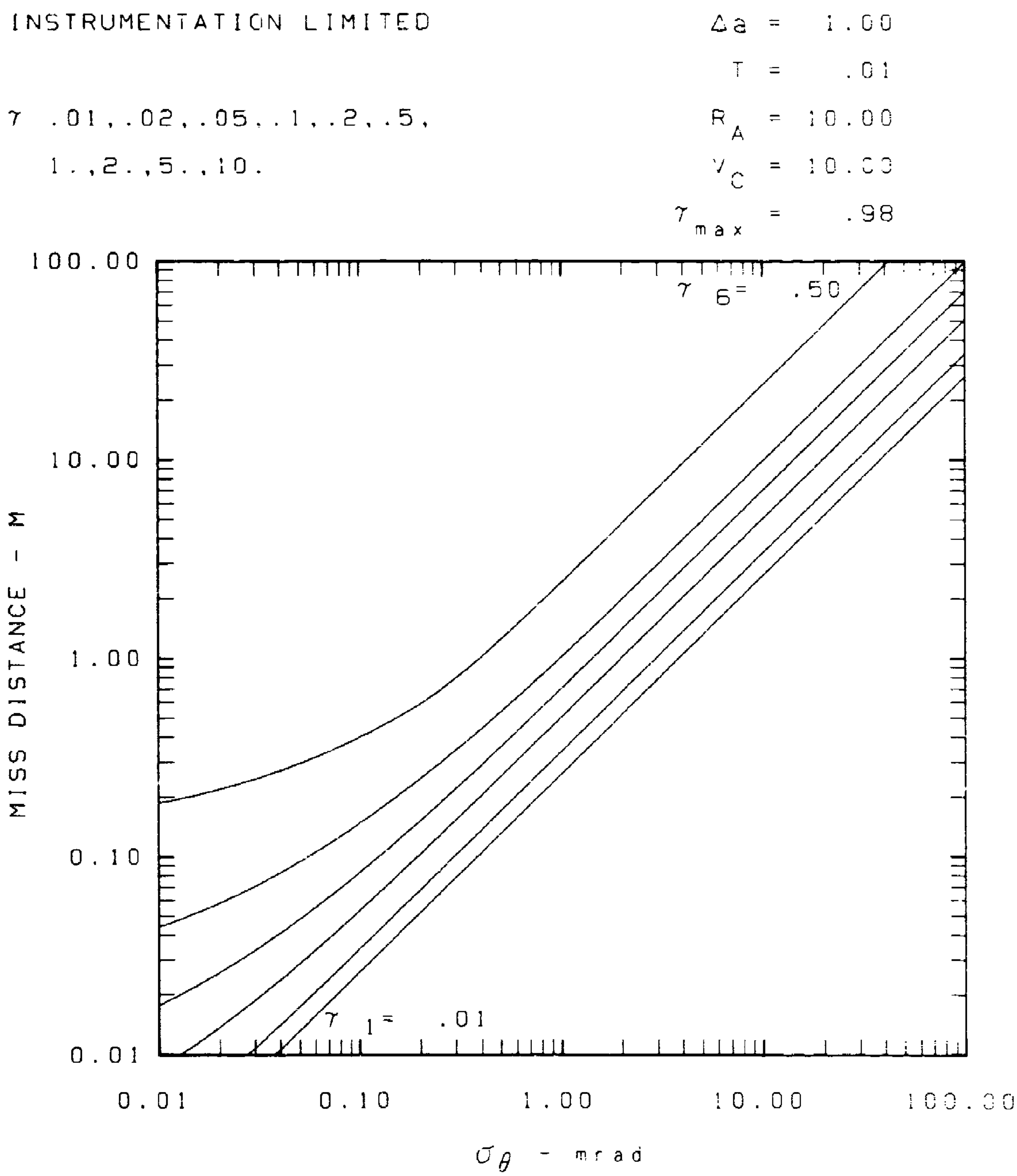
R_A = 10.00
V_C = 10.00
τ_{max} = .90
τ_6 = .50
τ_1 = .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σ_θ - mrad

Fig. A-16.


INSTRUMENTATION LIMITED
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1.,2.,5.,10.
Δa = 1.00
T = .01
R_A = 10.00
V_C = 10.00
γ_max = .98
γ_6 = .50
γ_1 = .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σ_θ - mrad

Fig. A-17.

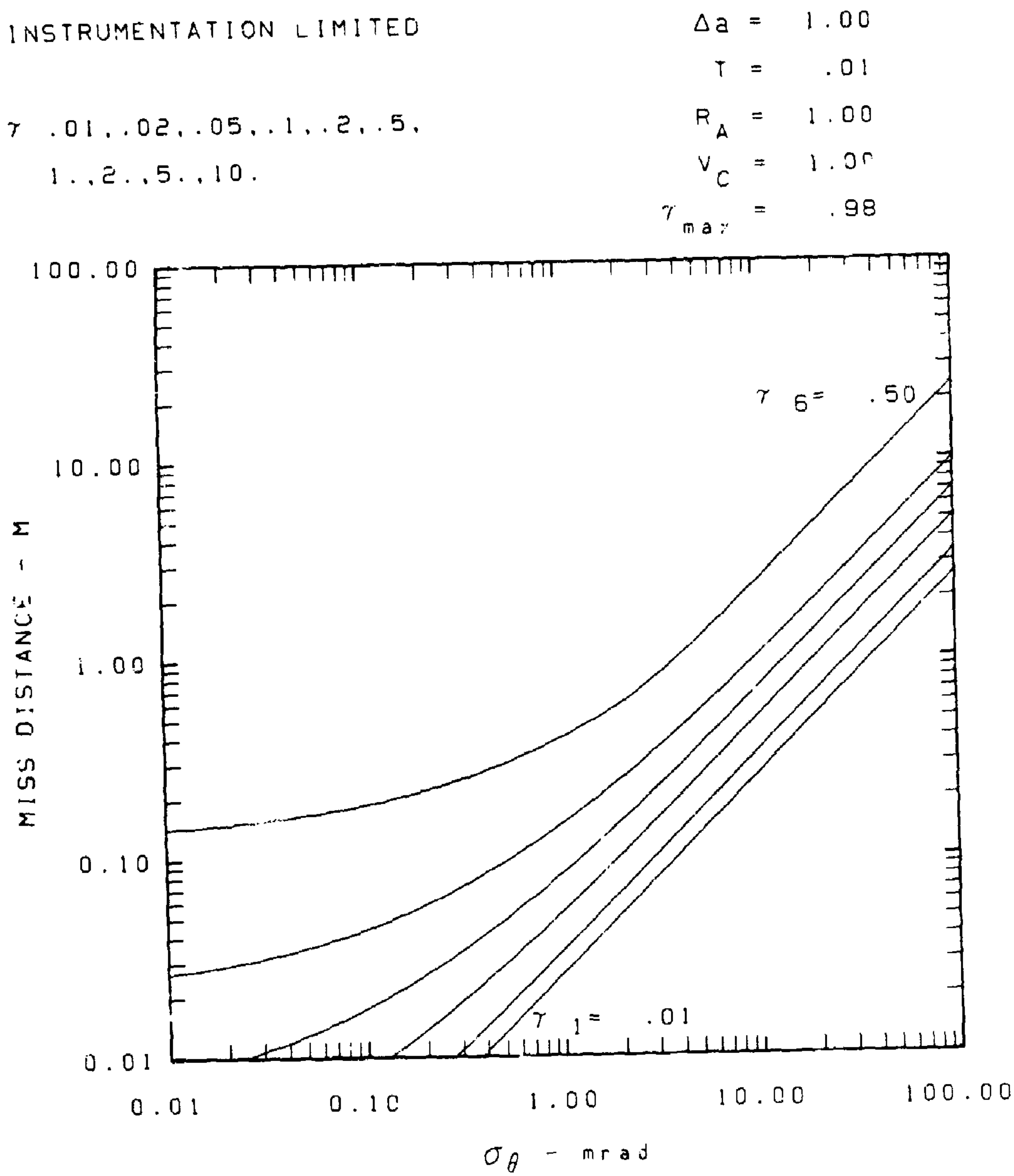
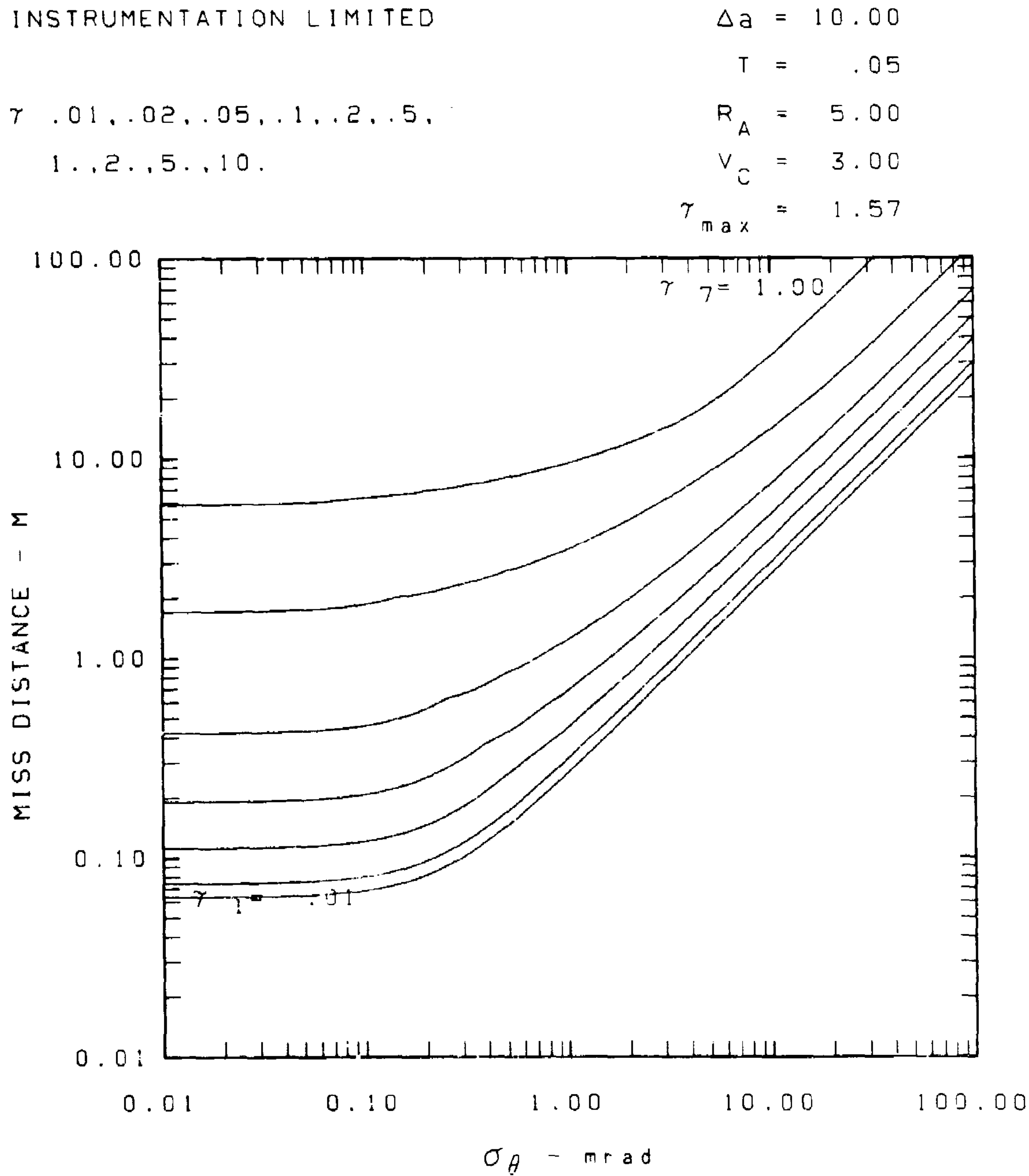

INSTRUMENTATION LIMITED
Δa = 1.00
T = .01
R_A = 1.00
V_C = 1.00
γ_{max} = .98
γ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
γ_6 = .50
γ_1 = .01
MISS DISTANCE - M
σ_θ - mrad
100.00
10.00
1.00
0.10
0.01
0.01
0.10
1.00
10.00
100.00

Fig. A-18.



INSTRUMENTATION LIMITED
γ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
Δa = 10.00
T = .05
R_A = 5.00
V_C = 3.00
γ_max = 1.57
γ = 1.00
γ = .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σθ - mrad

Fig. A-19.

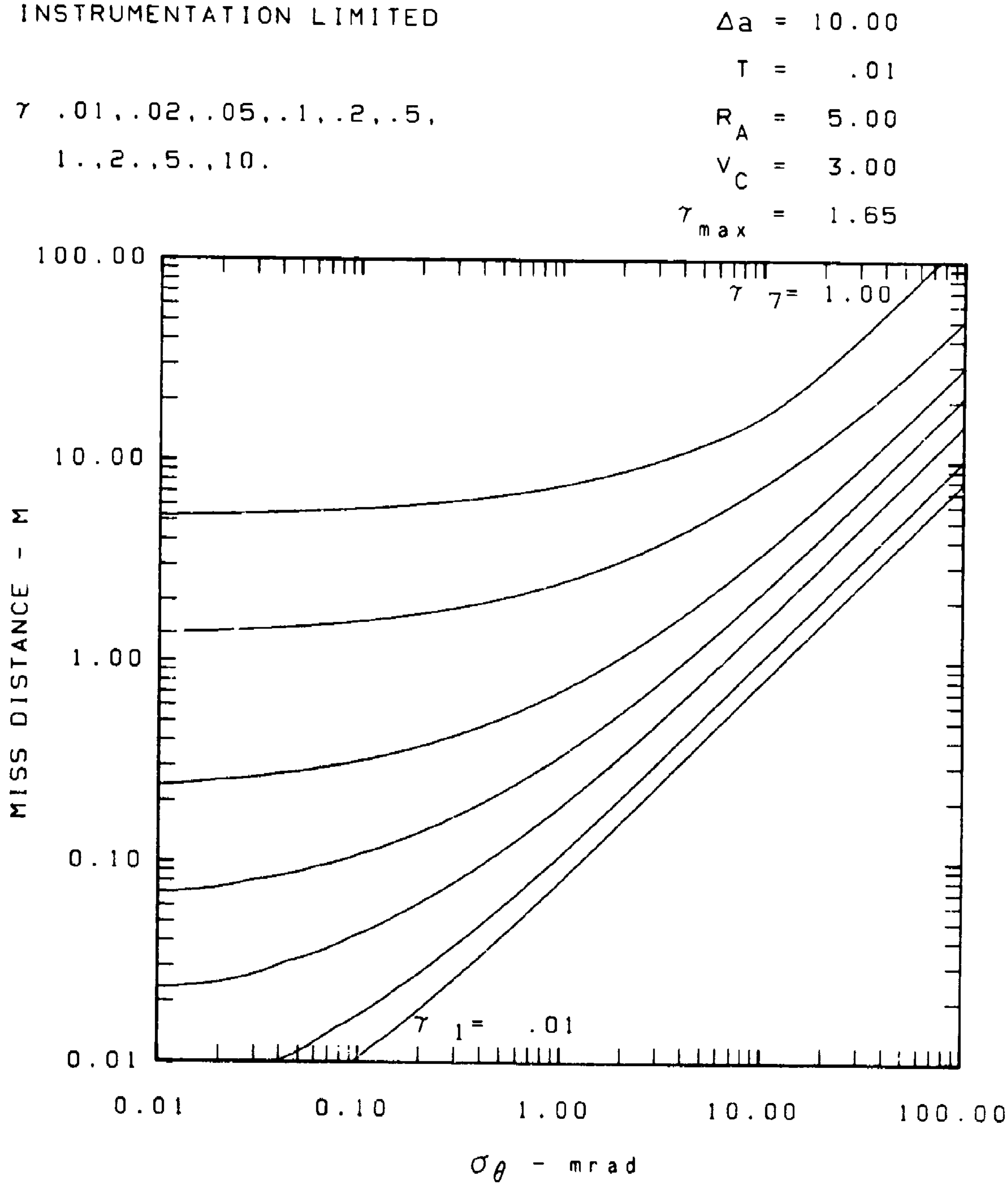

INSTRUMENTATION LIMITED
γ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
Δa = 10.00
T = .01
R_A = 5.00
V_C = 3.00
γ_max = 1.65
γ_7 = 1.00
γ_1 = .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σ_θ - mrad

Fig. A-20.

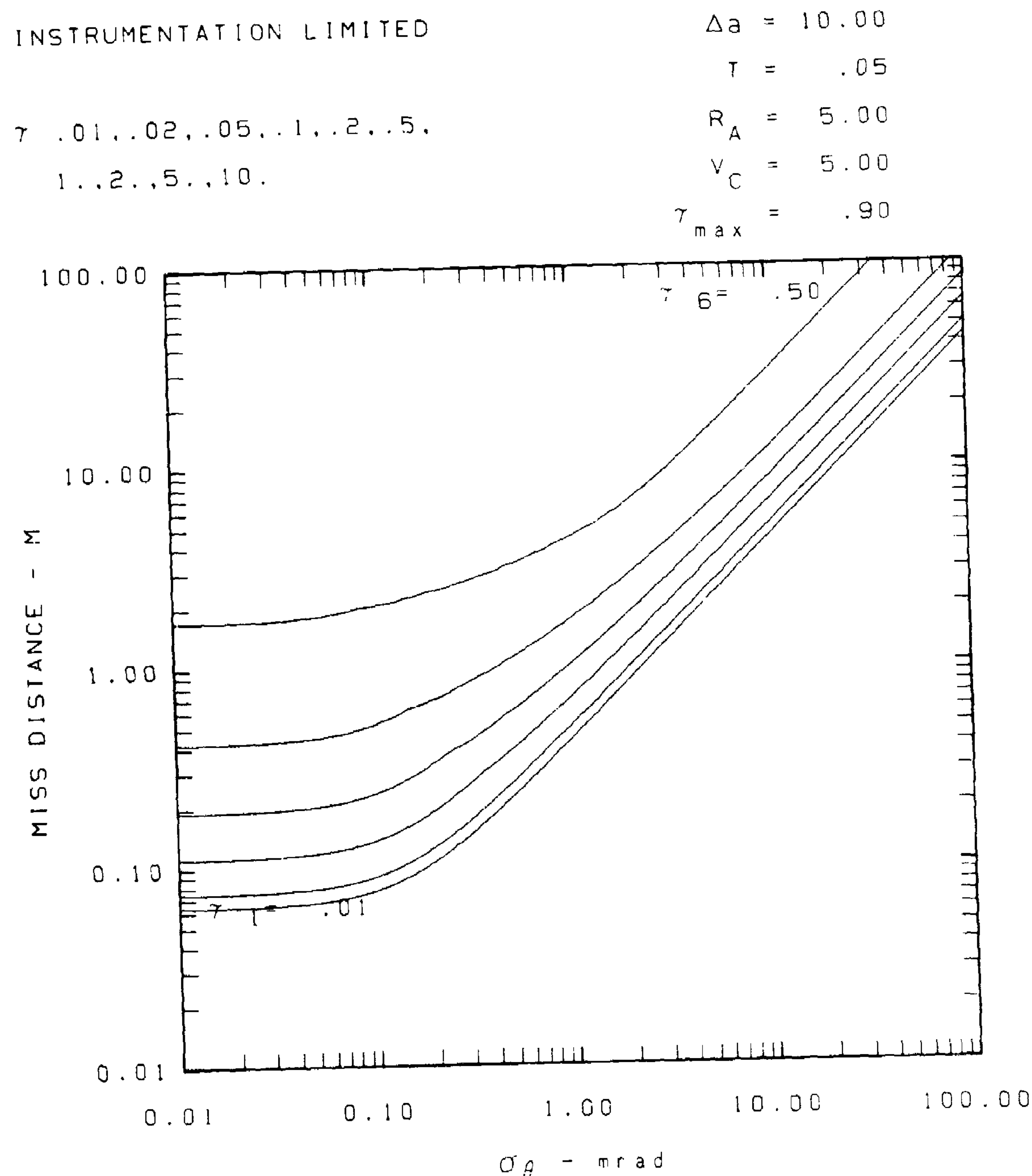

INSTRUMENTATION LIMITED
γ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
Δa = 10.00
T = .05
R_A = 5.00
V_C = 5.00
γ_{max} = .90
γ 6= .50
γ 1= .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σ_θ - mrad

Fig. A-21.

INSTRUMENTATION LIMITED

Δa = 10.00

T = .01

τ .01, .02, .05, .1, .2, .5, 1., 2., 5., 10.

R_A = 5.00

V_C = 5.00

τ_{max} = .98

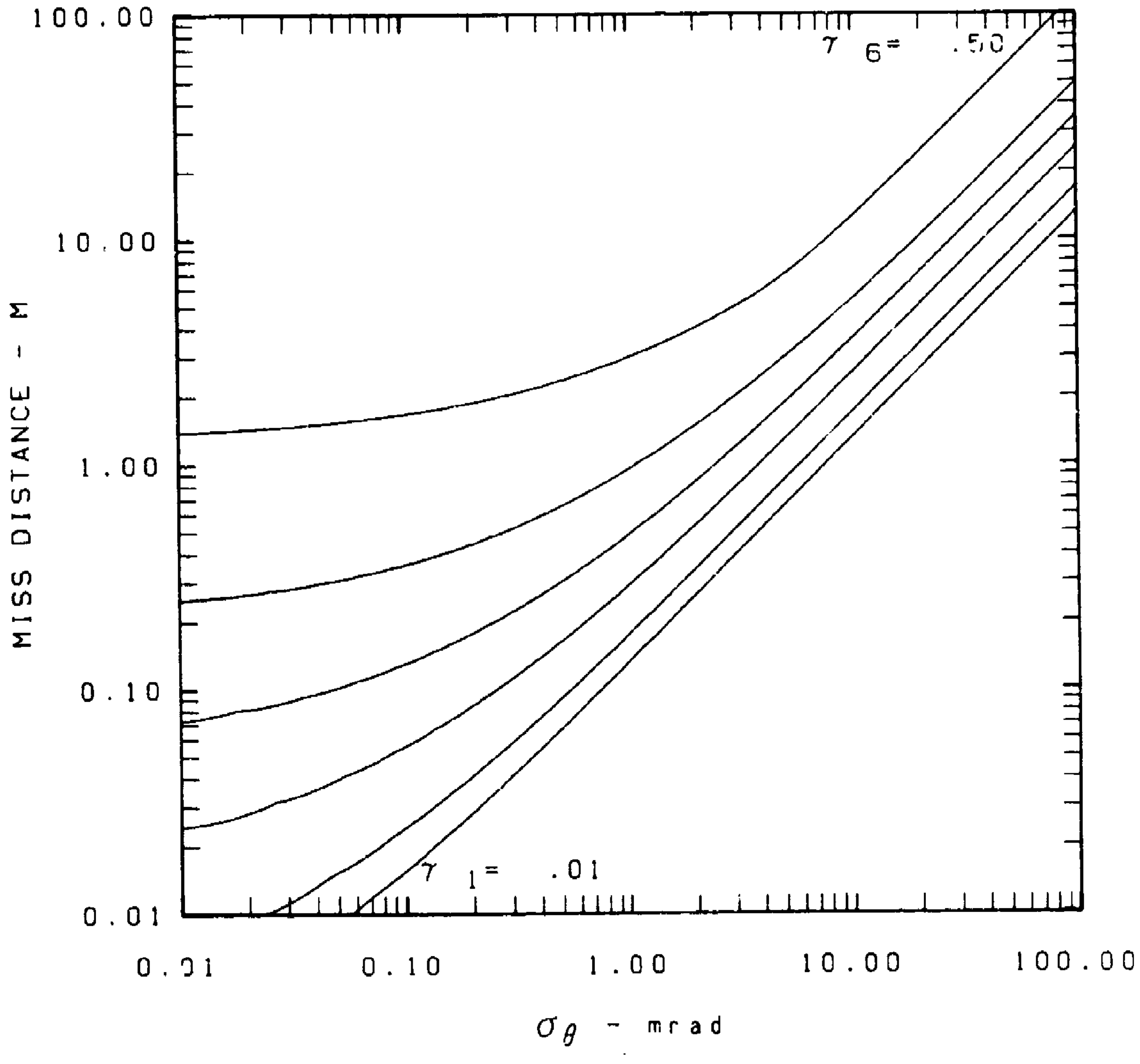


Fig. A-22.

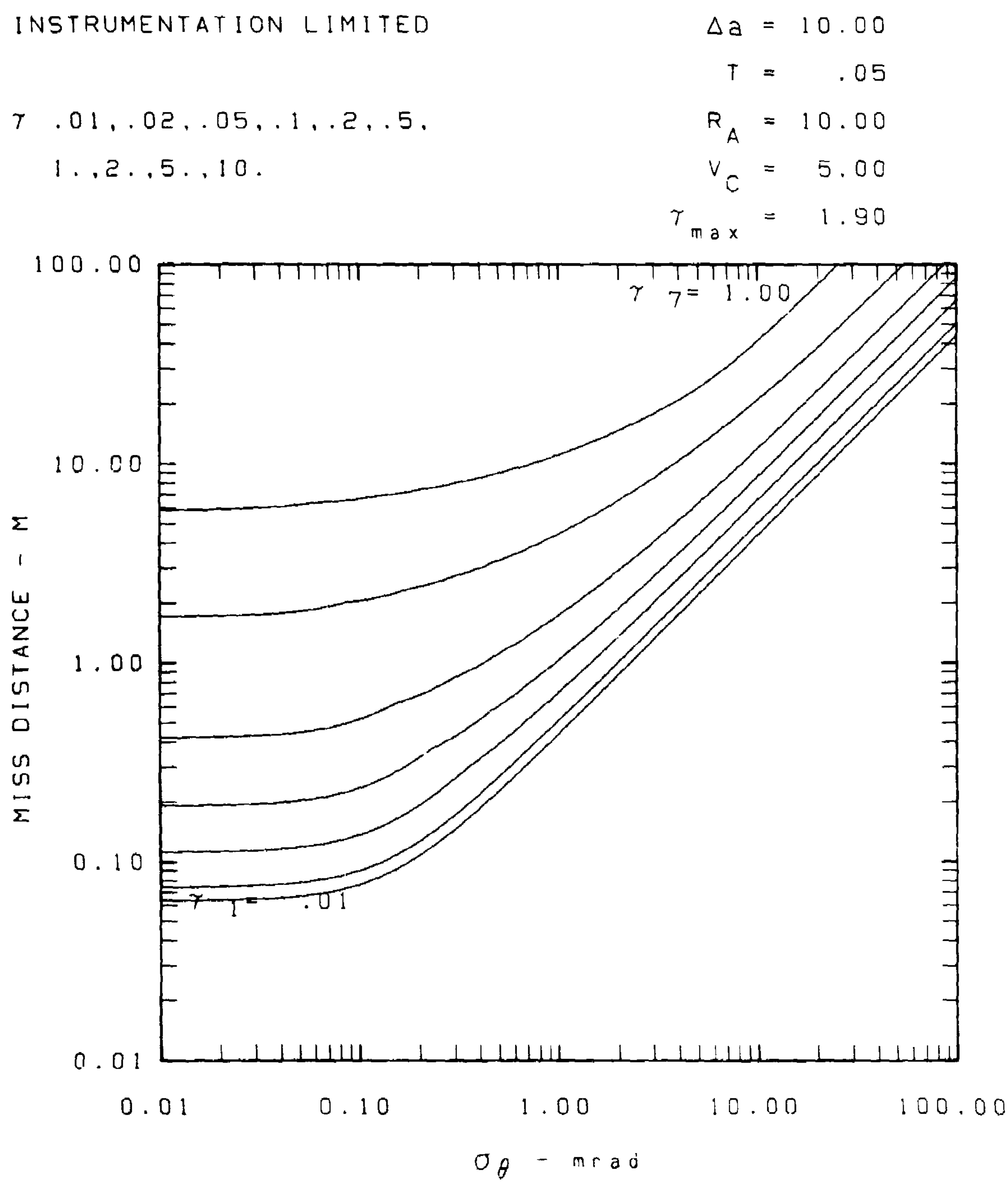

INSTRUMENTATION LIMITED
Δa = 10.00
T = .05
γ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
R A = 10.00
V C = 5.00
γ max = 1.90
γ = 1.00
γ = .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σθ - mrad

Fig. A-23.

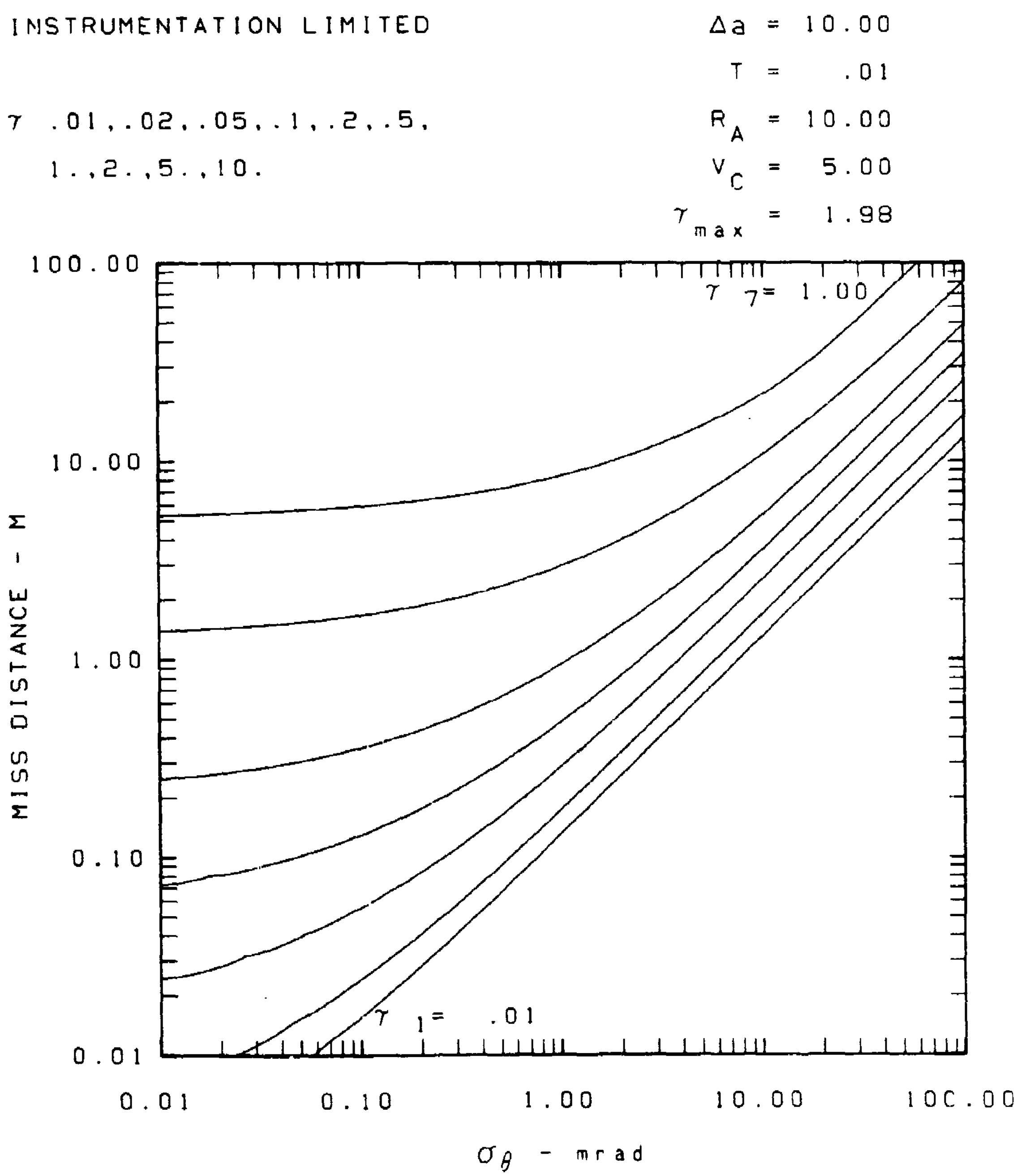
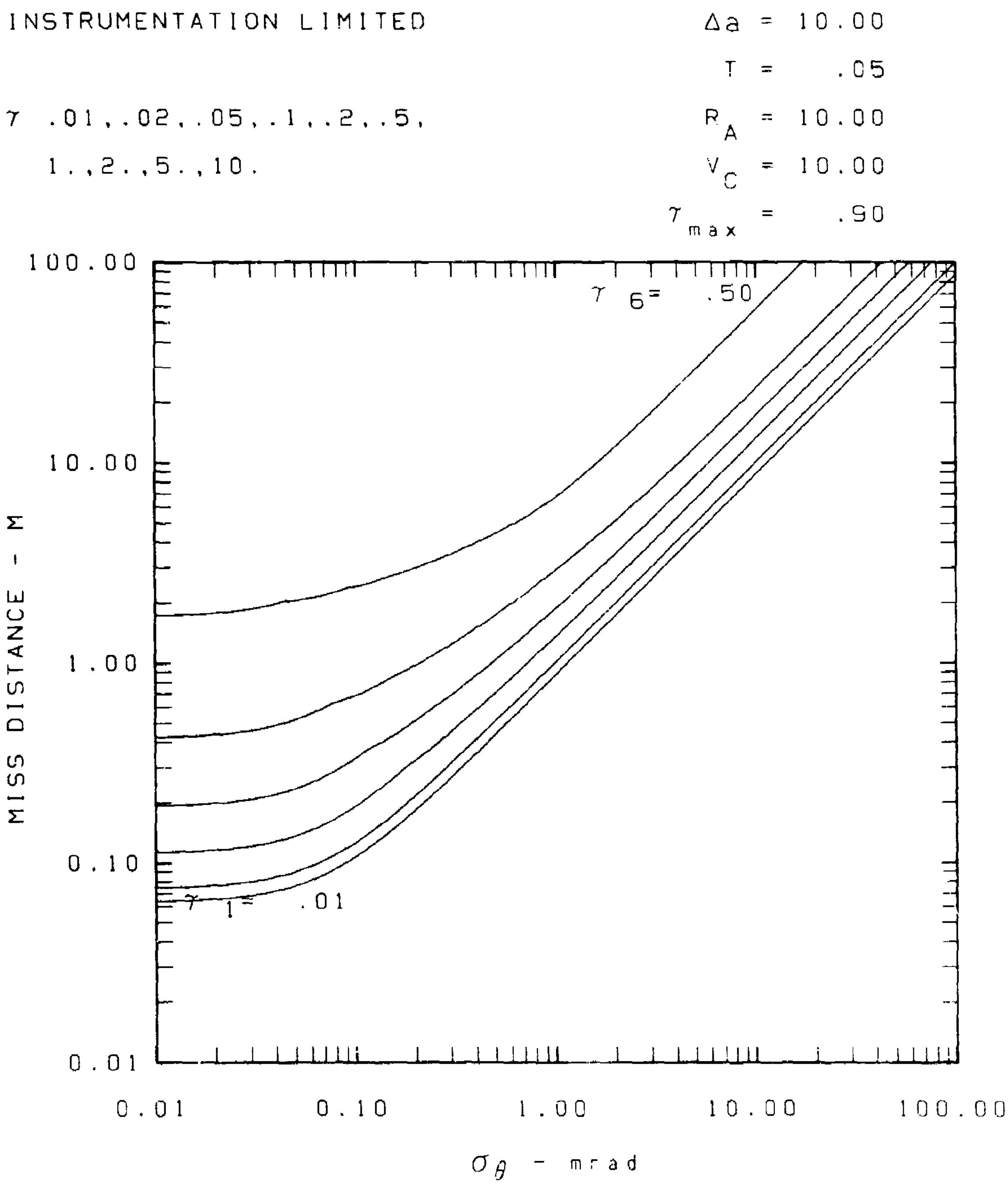
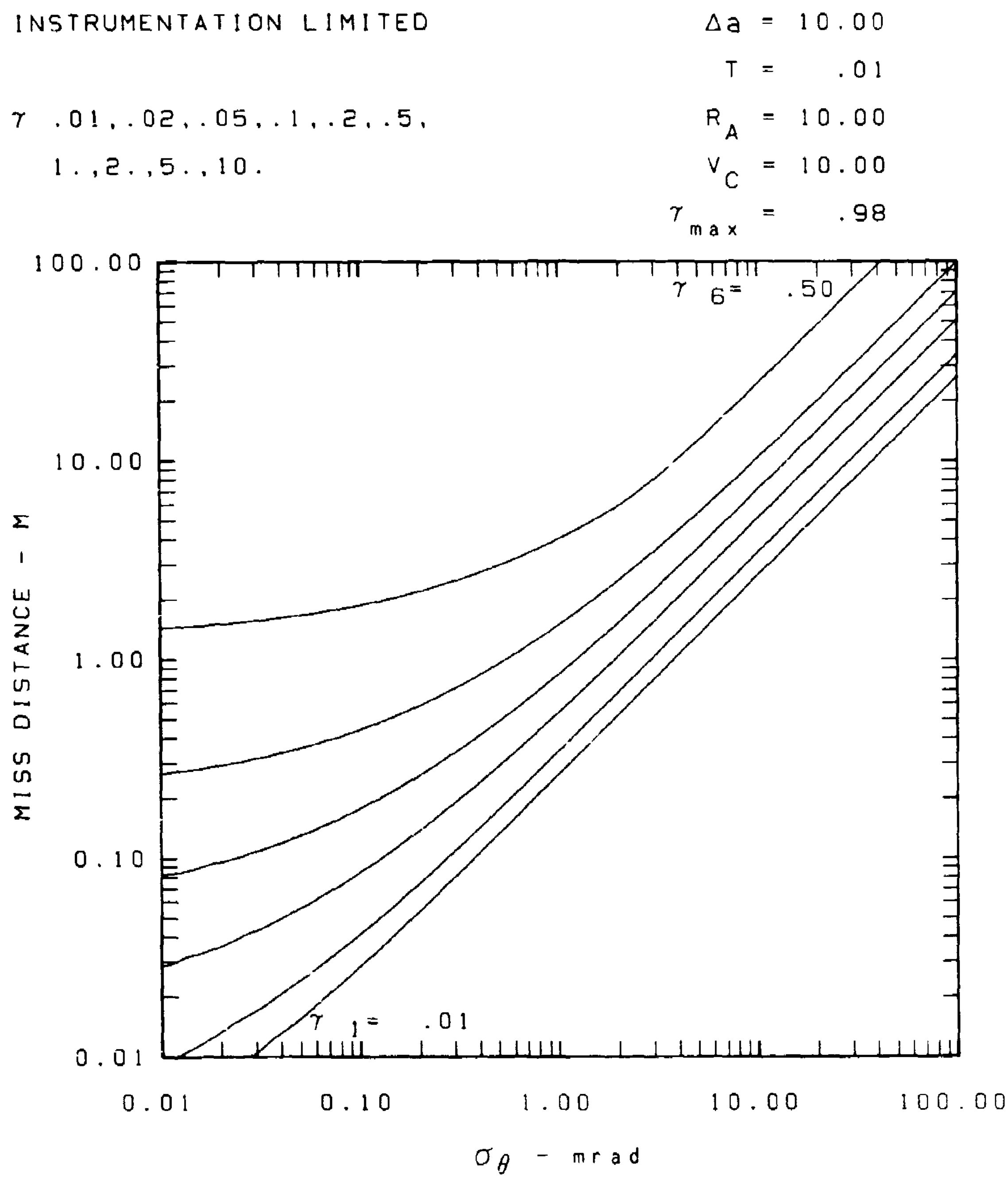

INSTRUMENTATION LIMITED
τ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
Δa = 10.00
T = .01
R_A = 10.00
V_C = 5.00
τ_max = 1.98
τ 7= 1.00
τ 1= .01
MISS DISTANCE - M
100.00
10.00
1.00
0.10
0.01
0.01
0.10
1.00
10.00
100.00
σθ - mrad

Fig. A-24.



INSTRUMENTATION LIMITED
Δa = 10.00
T = .05
R A = 10.00
V C = 10.00
γ max = .90
γ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
γ 6= .50
γ 1= .01
MISS DISTANCE - M
100.00
10.00
1.00
0.10
0.01
0.01
0.10
1.00
10.00
100.00
σθ - mrad

Fig. A-25.



INSTRUMENTATION LIMITED
Δa = 10.00
T = .01
γ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
R A = 10.00
V C = 10.00
γ max = .98
γ 6= .50
γ 1= .01
100.00
10.00
1.00
0.10
0.01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σθ - mrad

Fig. A-26.

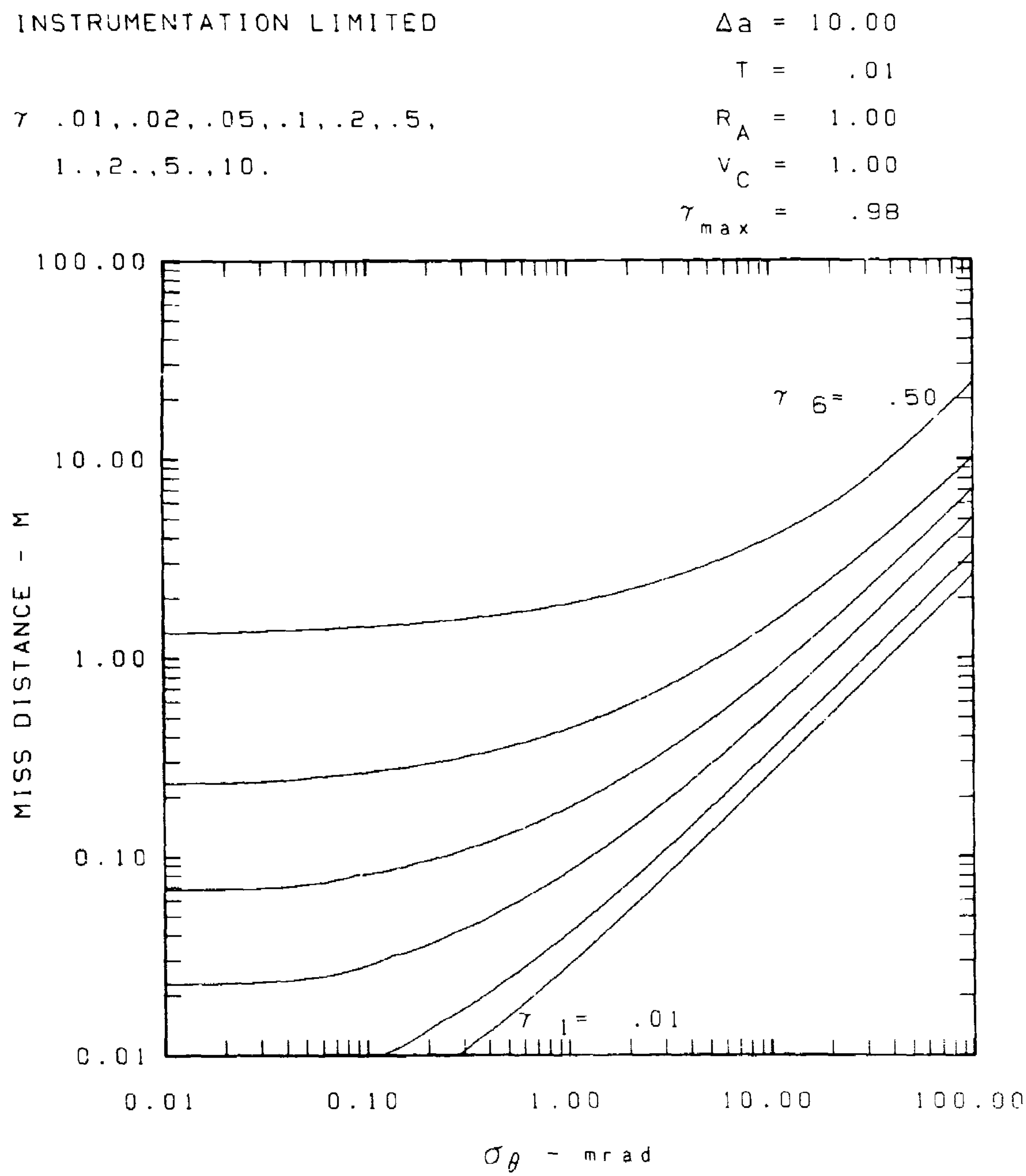

INSTRUMENTATION LIMITED
Δa = 10.00
T = .01
γ .01,.02,.05,.1,.2,.5,
1.,2.,5.,10.
R A = 1.00
V C = 1.00
γ max = .98
100.00
10.00
1.00
0.10
0.01
γ 6= .50
γ 1= .01
MISS DISTANCE - M
0.01
0.10
1.00
10.00
100.00
σ θ - mrad

Fig. A-27.

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REFERENCES

(1) S. D. Weiner, "A Simple Graphical Model for Analyzing Radar Homing Interceptor Engagements", Technical Note 1979-82, Lincoln Laboratory, M.I.T. (17 December 1979), AD-A082460/7.

(2) S. D. Weiner, "A Model of Interceptor Engagements Including Warhead and Fuze Interactions", Technical Note 1980-45, Lincoln Laboratory, M.I.T. (18 September 1980), AD-A093174/1

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interceptor control error
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interceptor time delay
homing guidance

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this report, we present an analytical model useful for sensor and interceptor trade-off analysis. Major factors used in this model include sensor measurement accuracy, data rate, interceptor time delay in responding to a command, and interceptor control error in executing a command. Guidance options considered include command guidance and homing guidance whereby the homing sensor accuracy may either be a constant or vary with powers of target range.

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