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A PROXIMITY EFFECT CORRECTION PROGRAM FOR ELECTRON BEAM LITHOGR--ETC(U)
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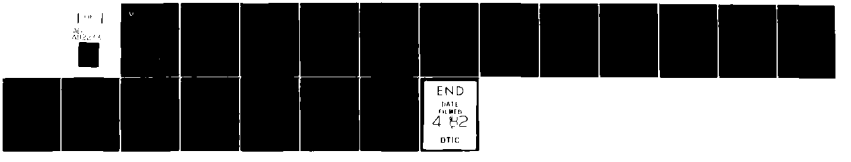

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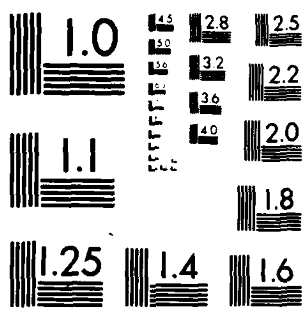

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RESEARCH AND DEVELOPMENT TECHNICAL REPORT

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A PROXIMITY EFFECT CORRECTION PROGRAM FOR ELECTRON BEAM LITHOGRAPHY

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JANUARY 1982

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US ARMY ELECTRONICS RESEARCH & DEVELOPMENT COMMAND
FORT MONMOUTH, NEW JERSEY 07703

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Electron Beam Lithography, Electron Scattering, Proximity Effect, Computer-Aided Design

20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A proximity effect correction program implementing the self-consistent correction algorithm has been developed. The program calculates recommended electron beam exposures given as input, the pattern to be written and the beam scattering parameters. Predicted dose patterns resulting from the recommended exposures are also calculated. Results of sample calculations are shown.

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INTRODUCTION

Electron beam lithography is the highest resolution technique in use for patterning the resists used in integrated circuit processing. The practical resolution of this technique is not limited by the primary electron beam, which can readily be focused to a spot size of less than 1000A[1] or by the inherent resolution of the resist,[2] but rather by scattering of the electrons. This scattering, both within the resist (forward and lateral) and from the substrate, results in a transverse flux of electrons (perpendicular to the beam) whose magnitude depends on the energy of the beam, the resist thickness, and the atomic number of the resist and substrate. This transverse electron flux exposes the resist in regions adjacent to that actually addressed by the e-beam writing pattern which results in a geometrical distortion in going from the e-beam writing pattern to the resulting developed pattern in the resist. This geometrical distortion is commonly known as proximity effect.

DOSE SPREAD

The pattern produced in the resist is the cumulative effect of the electron beam spread, the energy deposition in the resist, and the action of the developer on the resist. Some of the processes have been analyzed in detail. However, for purposes of understanding proximity effect it is sufficient to use a simplified model.

The dose spread is approximated by the sum of two Gaussians with characteristic widths β_f and β_b, related respectively to the spread due to forward- and backward-scattered electrons. The dose delivered to a resist at a point on the resist substrate interface by an electron beam incident at r = 0 is then

$$f(r) = k \left(\frac{1}{\beta_f^2} \exp\left(-\frac{r^2}{\beta_f^2}\right) + \frac{\eta}{\beta_b^2} \exp\left(-\frac{r^2}{\beta_b^2}\right)\right), \qquad (1)$$

where k is a constant and η is the ratio of integrated contribution of backscattered to forward-scattered electrons. The resist is assumed to be developed in any region where the dose exceeds a critical dose D_c. Therefore, the resulting pattern in the resist created by a pencil beam will be that region defined by $f > D_c$. Typical values of the parameters obtained by Monte Carlo calculations of electron transport are shown in Table 1.

1. Murrae J. Bowden, "The Physics and Chemistry of the Lithographic Process," J. Electrochem. Soc. 128, 195 C (1981)
2. Alec N. Broers, "Resolution Limits of PMMA Resist for Exposure with 50 kV Electrons," J. Electrochem. Soc. 128, 166 (1981)

TABLE 1

PARAMETERS THAT DEFINE f (r) EVALUATED AT THE RESIST-SILICON INTERFACE FOR PMMA RESIST OF THICKNESS t.

E-beam (keV)	t	β_f	β_b	η
10	0.5	0.22	0.65	0.51
15	0.5	0.13	1.14	0.51
15	1.0	0.44	1.41	0.52
25	0.5	0.06	2.6	0.51
25	1.0	0.22	2.9	0.49
40	0.5	0.04	6.0	0.42
40	1.0	0.11	6.0	0.45

The parameters are not known to high accuracy. Other estimates, both from calculation and experiment, differ by as much as a factor of two[3] in some cases. Parikh[4] suggests that $\beta_f = .1$, $\beta_b = 2.5$, $\eta = .9$ are useful estimates for PMMA with $t \tilde{\approx} .5\mu$ and electron energy $E_b = 25$ kV. The dose at any point in a plane is obtained by integration of f over all source points addressed by the electron beam. If the incident electron beam is not ideal but has a Gaussian spread of width β beam, Equation (1) is modified by using effective widths β_b^* and β_f^* which are given by

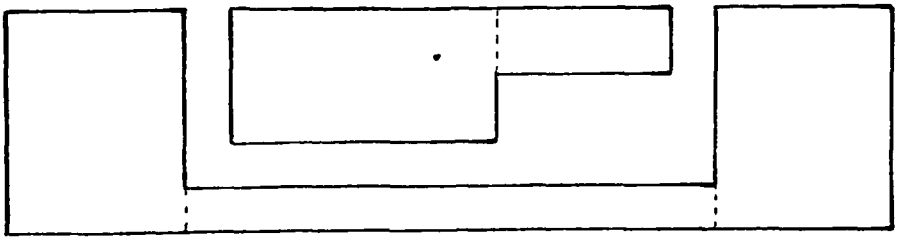
$$\beta_{b,f}^{*2} = \beta_{b,f}^{2} + \beta_{beam}^{2}. \qquad (2)$$

In practice, the beam width will be much smaller than β_b and comparable to β_f, so that only β_f need be replaced by its effective value.

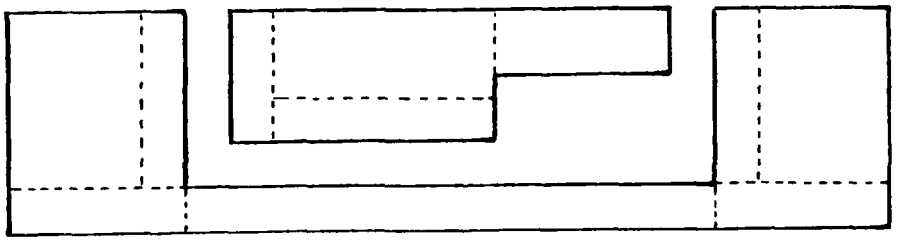
CORRECTION PROCESS

The fundamental problem of a proximity correction process is to arrange that the contour line $D = D_c$ matches a desired pattern. A convenient scheme for accomplishing this is to set the written pattern to the desired pattern and adjust the e-beam exposure within the pattern to achieve the desired contour. Various techniques have been proposed to this end. We will discuss one such technique, the self-consistent technique, proposed by Parikh,[5] which forms the basis of our correction algorithm. In this technique the desired pattern is divided into a collection of N elements, such as shown in Figure 1, each of which is given a uniform exposure. The exposures E_i are chosen so that the average dose in each element is

3. Mihir Parikh and David Kyser, "Energy Deposition Functions in Electron Resist Films on Substrates," J. Appl. Phys. 50, 1104 (1979)
4. Mihir Parikh, "Proximity Effects in Electron Lithography: Magnitude and Correction Techniques," IBM J. Res. Develop. 24, 438 (1980)
5. Mihir Parikh, "Corrections to Proximity Effects in Electron Beam Lithography," J. Appl. Phys. 50, 4371, 4378 (1979)



(a)



(b)

Figure 1. Division of a pattern into (a) 5 and (b) 11 elements.

the same. The average dose in element A is given by the double integral over element areas, S_A and S_i,

$$D_A = \frac{1}{S_A} \Sigma_i kE_i \int_A \int_i dS_A dS_i \left(\frac{1}{\beta_f^{*2}} \exp\left(\frac{-(\vec{r}_A - \vec{r}_i)^2}{\beta_f^{*2}} \right) + \frac{\eta}{\beta_b^2} \exp\left(\frac{-(\vec{r}_A - \vec{r}_i)^2}{\beta_b^2} \right) \right), \qquad (3)$$

where the sum over all elements i include the element A. Setting the average doses equal results in a set of N linear equations for the exposures.

A computer program, PROXR, implementing the self-consistent correction technique, has been written. The program accepts as input the coordinates of the elemental rectangles into which the pattern is partitioned. The specialization to rectangular figures was made for two reasons, (a) the integrals to determine average dose can be done analytically, involving functions no more complex than the sums and products of error functions,[5] and (b) the ET&D Laboratory LEBES-D E-beam System is a vector write machine which most naturally writes in a rectangular pattern. The output of the program is a set of relative exposures for the elemental rectangles. Further diagnostics built into PROXR allow the dose to be calculated at selected points in the pattern, arranged either in an 11 x 11 window or in 101 points equally spaced along a line.

The program is written in interactive form so that the user chooses partitioning of the pattern, and can immediately display predicted doses at selected points in the pattern. For partitioning into N elemental rectangles, the calculation time increases as N^2; hence, there is incentive to partition only as necessary to achieve adequate correction. At the present time it does not seem feasible to do automatic partitioning without exercise of some judgment.

OPERATION OF PROXR

Written in FORTRAN 7, PROXR is an interactive program which implements the self-consistent proximity effect correction scheme. PROXR is currently running on the Fort Monmouth Interdata 8/32 time sharing system.

Files Required

LU0 is assigned to the user's console
LU1 is an input file of a maximum of 100 user-defined pattern rectangles
LU2 is an output file to which simulated dose calculations are written
LU3 is an output file to which the description of the rectangles and their relative exposures are directed.

Any or all of the above may be assigned to the user's console.

Input Required

PROXR reads from LU1, a user-defined file of elemental rectangles written in free-form format. Each rectangle is designated by four numbers indicating x-low, x-high, y-low, y-high, separated by commas, blanks, or carriage returns. The reading sequence is terminated by an END OF FILE or an illegal character.

The proximity parameters BF, BB, ETA, ISO, IREF are requested from LU0, the user's console, and are described as follows:

5. Mihir Parikh, "Corrections to Proximity Effects in Electron Beam Lithography," J. Appl. Phys. 50, 4371, 4378 (1979)

BF and BB are, respectively, the effective forward and backscattering width as defined in Equation (3). All dimensions (the coordinates of the elemental rectangles and the scattering widths) must be in the same units, preferably micrometers.

ETA is the backscatter fraction, ISO is an integer ≥ 0. If ISO is equal to zero then all exposures are set equal to one and no corrections are made.

If ISO is greater than 0 then ISO sets the number of iterations used in solving the linear equations by the subroutine SOLGS. A value of 4 is usually enough to ensure convergence in SOLGS.

IREF is an integer ≥ 0. If IREF = 0 then IREF is the record number of a record in LU1 to which the exposure value 1 will be assigned in normalizing the exposure array EXPO2. If IREF = 0 then EXPO2 is equated to EXPO1. During operation, the program will supply the necessary prompts for all further input.

After reading the rectangle coordinates and the start-up parameters, PROXR calculates A (I,J), the influence matrix, the elements of which are the average dose in Area I due to unit exposure in Area J. Provisions are made to set insignificant matrix elements of A (I,J) to zero. The average doses in the rectangles are equated, and the linear system is solved using a Gauss-Seidel iterative subroutine, SOLGS. The exposures thus calculated are stored in EXPO1, normalized so that the exposure given to large figure is one. They are also stored EXPO2 renormalized so that the exposure in a user-defined figure is unity.

Once the exposures for each elemental rectangle have been determined, PROXR will calculate the dose delivered at an array of points specified by the user by calling one of the two simulation subroutines WINDOW or LINE. If WINDOW is specified, values of the dose are calculated on a grid of 11 x 11 points equally spaced between the limits x-low, x-high, y-low, and y-high. If LINE is selected, the dose is calculated at 101 points equally spaced between the points (x_1, y_1) and (x_2, y_2). In either case, the dose is normalized so that the dose calculated at the center of a large rectangle is $1 + \eta$, as would be expected on giving this feature unit exposure. The simulations continue until the user requests it be terminated, at which time new start-up parameters can be entered or the program terminated. A sample PROXR conversation illustrating the above is included in the Appendix.

RESULTS OF CALCULATIONS

We have used PROXR to calculate exposures and simulate the predicted dose distribution for a variety of test patterns. We present three cases which illustrate situations for which proximity effect corrections are essential.

Recommended exposures for five isolated figures are shown in Table 2 for two values of β_f^*. The wide spread in recommended exposures illustrates the futility of attempting to write patterns with large and small features using a single exposure value. We also note the relative insensitivity of large figures to changes in β_f^*.

TABLE 2. RELATIVE EXPOSURE OF ISOLATED FIGURE CALCULATED FOR $\eta = 0.8$, $\beta_b = 3.0$, $\beta^*_f = 0.1, 0.3$.

Figure		Relative Exposure $\beta_f^* = 0.3$	Relative Exposure $\beta_f^* = 0.1$
Square	20 x 20 μm	1.09	1.08
Square	5 x 5 μm	1.40	1.36
Square	1 x 1 μm	2.51	1.96
Square	0.5 x 0.5 μm	4.05	2.27
Rectangle	1 x 15 μm	1.89	1.68

When the pattern consists of more than one figure with spacing close enough to allow interaction, the predicted exposure depends upon the location of the figure. The exposures for a chessboard pattern with 1 μm squares is shown in Figure 2.

A more complex pattern is shown in Figure 3 where the actual dose distribution along a selected line is shown for corrected and uncorrected exposures. The overall reduction in dose seen in the uncorrected pattern would result in poor feature definition; the degradation being especially severe for the right-most stripe.

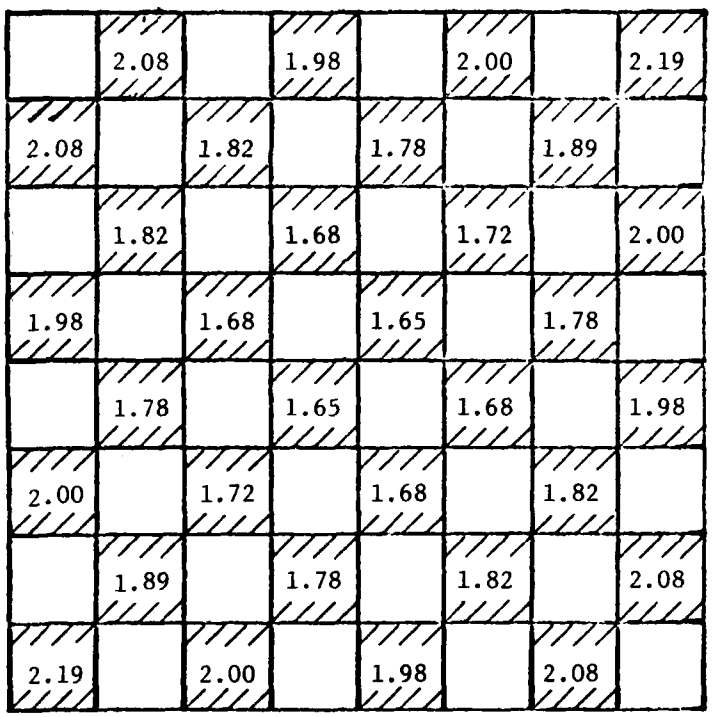


Figure 2. Exposures for a chessboard pattern of 1 μm squares; $\eta = 0.8$, $\beta_b = 3.0$, $\beta_f^* = 0.3$. Exposure for a 1 μm isolated square = 2.51.

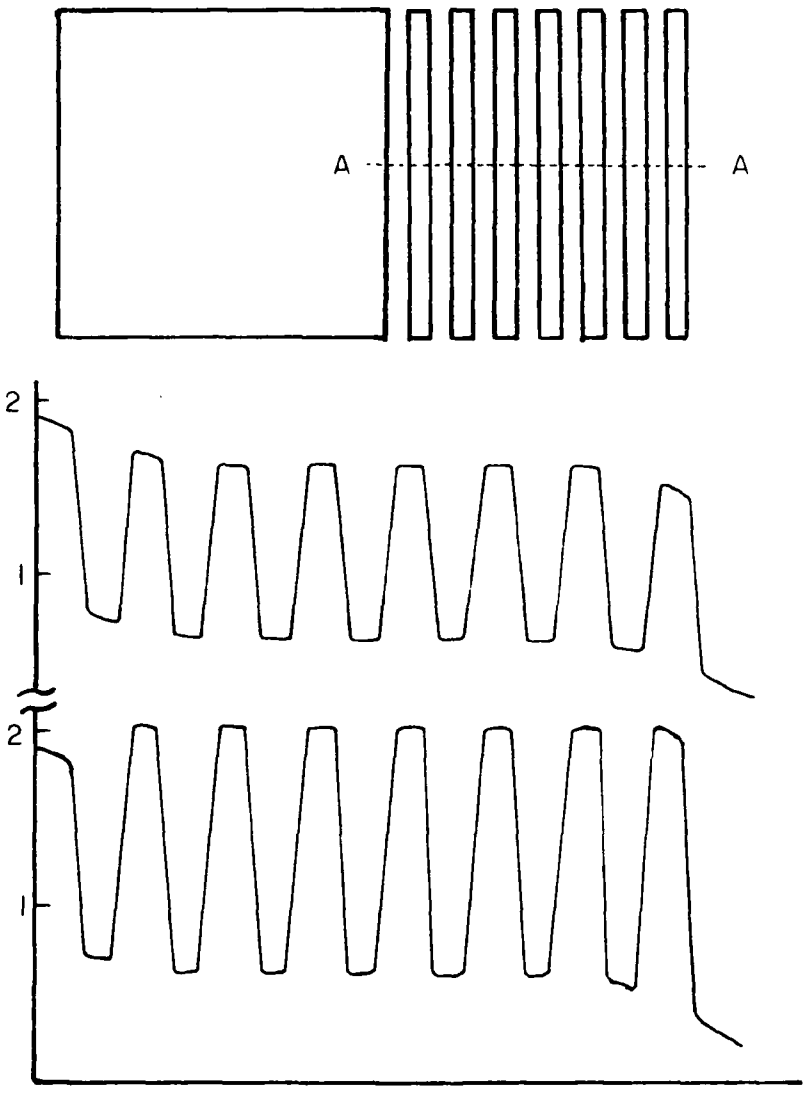


Figure 3. Dose along line A-A for corrected and uncorrected exposures; $\eta = 0.90$, $\beta_b = 2.5$, $\beta_f^* = 0.1$. Corrected exposures are square: 1.09, Stripes: 1.38, 1.39, 1.39, 1.39, 1.38, 1.41, 1.54.

APPENDIX: LISTING OF PROXR

Variable List: MAIN

XY - An array containing the coordinates of the rectangles to be processed in the form

XY(1,*) = x-low

XY(2,*) = x-high

XY(3,*) = y-low

XY(4,*) = y-high

AREA - An array of areas of each of the above rectangles

A - An array of influence coefficients

EXPO1 - An array of exposures normalized so that exposure of a large figure is unity

EXPO2 - An array of exposures normalized so that exposure of a user defined figure is unity

NREF - Index of the reference rectangle in file

NREC - Number of rectangles to be processed

NREC1 - NREC + 1

BF - Effective forward scatter width

BB - Backscatter width

ETA - Backscatter fraction

ISO - Number of times the Gauss - Seidel linear system solver is to be iterated or if ISO = 0 no corrections are made, and doses for equal exposure in all figures can be calculated.

SUBROUTINES AND FUNCTION SUBROUTINES

LINE - Subroutine: calculates the simulated dose at 101 equally spaced points along a user-defined line segment.

WINDOW - Subroutine: calculates the simulated dose at 121 points in a user-defined rectangular window.

ERFXY - Real function: evaluates the integral

$$\int_{x_1}^{x_2} \int_{x'_1}^{x'_2} \frac{d\,x\,dx'}{\beta} \exp - \left(\frac{(x - x')^2}{\beta^2} \right)$$

ERF - Real function: calculates

$$\int_0^x \exp(-t^2)\, dt$$

ERFINT - Real function: calculates

$$\int_0^x \mathrm{erf}(t)\, dt$$

SOLGS - Subroutine: linear system solver using Gauss-Seidel recursive technique.

LISTING OF PROXR

```
      C PROXR.FTN REVISED SEP 8 81
      C FORT MONMOUTH PROXIMITY EFFECT PROGRAM
      C MAIN PROGRAM SECTION
            REAL XY(4,100),,AREA(100),A(100,101),EXPO1(100)
            REAL EXPO2(100)
            DATA EXPO2/100*0./
      C XY(1,*)=X LOW,XY(2,*)=X HIGH,XY(3,*)=Y LOW,XY(4,*)=Y HIGH
      C A IS THE AUGMENTED INFLUENCE MATRIX
      C EXPO1(I) IS THE EXPOSURE OF THE ITH RECTANGLE RELATIVE TO THAT OF
      C A REFERENCE, E.G.IDEAL, ARBITRAIL, LARGE RECTANGLE.
      C EXPO2(I)= EXPO1(I)/ EXPO1(IREF)
      C
            DO 1 I=1,100
            READ(1,*,END=2,ERR=2)(XY(J,I),J=1,4)
            AREA(I)=(XY(2,I)-XY(1,I))*(XY(4,I)-XY(3,I))
      1     CONTINUE
            I=101
      2     NREC=I-1
            NREC1=I
      C NREC IS THE NUMBER OF RECTANGLES TO BE PROCESSED
      200   FORMAT(1X,' ENTER BF,BB,ETA,IID,IREF')
      3     WRITE(0,200)
            READ(0,*)BF,BB,ETA,IID,IREF
            IF (IID<>0)THEN
      C INITIALIZE A
            DO 4 I=1,NREC
            A(I,NREC1)=1
            DO 4 J=1,NREC
      4     A(I,J)=0
      C
      C CALCULATE THE INFLUENCE MATRIX
      C
            DO 6 I=1,NREC
            DO 6 J=I,NREC
            A(I,J)=
           1 -ERFXY(XY(1,I),XY(2,I),XY(1,J),XY(2,J),BF)
           2 +ERFXY(XY(3,I),XY(4,I),XY(3,J),XY(4,J),BF)
           3 +ERFXY(XY(1,I),XY(2,I),XY(1,J),XY(2,J),BB)
           4 -ERFXY(XY(3,I),XY(4,I),XY(3,J),XY(4,J),BB)
           5 *ETA)*AREA(I)*AREA(J)
      6       A(J,I)=A(I,J)
            CALL SOLGI(NREC,NREC1,A,EXPO1,IID)
            DO 7 I=1,NREC
            EXPO1(I)=EXPO1(I)*(1+ETA*AREA(I))
      7     CONTINUE
            IF (IREF==0)THEN
            REF=1.
            ELSE
            REF=EXPO1(IREF)
            ENDIF
            DO 8 I=1,NREC
      8     EXPO2(I)=EXPO1(I)/REF
            ELSE
            DO 10 I=1,NREC
            EXPO2(I)=1.
      10    EXPO1(I)=1.
            ENDIF ; IID <> 0
      300   FORMAT(/1X,' BF=',G10.2,' BB=',G10.2,' ETA=',G10.2,' IID=',I3,
           1 '  NREC=',I3,' IREF=',I3)
      400   FORMAT(6X,'X LOW',T16,'X HIGH',T27,'Y LOW',T36
```

```
     1   ,'Y HIGH',T47,'EXPO1',T57,'EXPO2'
500   FORMAT(1X,6F10.2)
      WRITE(3,300)BF,BB,ETA,IID,NPEC,IREF
      WRITE(3,400)
      DO 11 I=1,NPEC
      WRITE(3,500)(XY(J,I),J=1,4),EXPO1(I),EXPO2(I)
11      CONTINUE
700   FORMAT('   ENTER 1 FOR WINDOW, 2 FOR LINE, OTHERWISE 0')
3      CONTINUE
      WRITE(0,700)
      READ(0,*)IIIM
      IF(IIIM==1)CALL WINDOW(XY,NPEC,BF,BB,ETA,EXPO1)
      IF(IIIM==2)CALL LINE(XY,NPEC,BF,BB,ETA,EXPO1)
      IF(IIIM==1.OR.IIIM==2)GO TO 3
600   FORMAT('      ENTER 1 TO CONTINUE,0 TO END  ')
      WRITE(0,600)
      READ(0,*)NEXT
      IF(NEXT.EQ.1) GO TO 3
      STOP
      END
      SUBROUTINE WINDOW(XY,NPEC,BF,BB,ETA,EXPO1)
C
      REAL XY(4,100),EXPO1(100),XA(11),YA(11),D(11,11)
C SIMULATION SECTION
700   FORMAT(1X,' ENTER THE WINDOW EDGES X1,X2,Y1,Y2')
11    CONTINUE
      WRITE(0,700)
      READ(0,*)X1,X2,Y1,Y2
      DX=(X2-X1)/10
      DY=(Y2-Y1)/10
      DO 9 I=0,10
      XA(I+1)=X1+I*DX
      YA(I+1)=Y1+I*DY
9     CONTINUE
      DO 10 I=1,11
      X=XA(I)
      DO 10 J=1,11
      Y=YA(J)
      D(J,I)=DOSE(X,Y,XY,NPEC,BF,BB,ETA,EXPO1)
10    CONTINUE
800   FORMAT(1X,F6.2,2X,11F6.2)
400   FORMAT(1X,/,T30,'DOSE AT (X,Y)')
900   FORMAT(1X,6X,'Y=',11F6.2,/,1X)
500   FORMAT('     EDGES AT X= F6.2,",",F6.2," AND Y=",F6.2,",",F6.2)
600   FORMAT('   ENTER 1 FOR NEW WINDOW OR 0 TO CONTINUE')
      WRITE(2,500)X1,X2,Y1,Y2
      WRITE(2,400)
      WRITE(2,900)YA
      WRITE(2,800)(XA(I),(D(J,I),J=1,11),I=1,11)
      WRITE(0,600)
      READ(0,*)NEXT
      IF (NEXT.EQ.1)GO TO 11
      RETURN
      END
      SUBROUTINE LINE(XY,NPEC,BF,BB,ETA,EXPO1)
C
       REAL XY(4,100),EXPO1(100),D(10,11)
      DATA D/110*0./
C SIMULATION SECTION
700   FORMAT(1X,'ENTER THE LINE END POINTS X1,Y1  X2,Y2')
```

```
      11    CONTINUE
            WRITE(6,700)
            READ(5,*)X1,Y1,X2,Y2
            DX=(X2-X1)/100
            DY=(Y2-Y1)/100
            DO 10 IJ=0,100
            I=IJ/10 +1
            J=IJ-10*(I-1)+1
            X=X1+DX*IJ
            Y=Y1+DY*IJ
            D(J,I)=BOIE(X,Y,Z,NPEC,EF,EI,ETA,EXPO1)
      10    CONTINUE
      300   FORMAT(1X,I6,1X,10F6.3)
      400   FORMAT(1X,  T30,'BOIE AT POINTS ON LINE',/,10I6)
      500   FORMAT(      F1=',2F6.3,'     F2=',2F6.3,' ')
            WRITE(3,500)X1,Y1,X2,Y2
            WRITE(3,300)(I,I=0,9)
            WRITE(3,300)(10*(I-1),(D(J,I),J=1,10),I=1,11)
      600   FORMAT('   ENTER 1 FOR NEW LINE,0 TO CONTINUE ')
            WRITE(6,600)
            READ(5,*)NEXT
            IF (NEXT.EQ.1)GO TO 11
            RETURN
            END
            REAL FUNCTION ERFX(X1,X2,X3,X4,E)
      C THIS FUNCTION USED TO CALCULATE THE INFLUENCE MATRIX
            ERFX=E*(-ERFINT(X3-X1)+ERFINT(X4-X2)+ERFINT(X3-X2)
           1 -ERFINT(X4-X1))/E
            RETURN
            END
            FUNCTION ERF(X)
      C ERF(-X)=-ERF(X)
            REAL*8 X1,T,U
            X1=ABS(X)
            IF(X1.GE.5.3)THEN
            U=1
            ELSE
            T=1/(1+X1*.47047)
            U=((.7478556*T-.9587938E-1)*T+.3480242)*T
            U=1-U*EXP(-X1*X1)
            ENDIF
            ERF=SIGN(1.,X)*U
            RETURN
            END
            FUNCTION ERFINT(X)
      C ERFINT CALCULATES THE INTEGRAL OF ERF(X).
            REAL*8 X1,T,U
            X1=ABS(X)
            IF(X1.GE.1.5)THEN
            U=X1
            ELSE
            T=1/(1+X1*.47047)
            U=((.7478556*T-.9587938E-1)*T+.3480242)*T
            U=X1-(X1*U-.5641896)*EXP(-X1*X1)
            ENDIF
            ERFINT=U
            RETURN
            END
            SUBROUTINE SOLGS(N,N1,A,X,M)
      C TO USE THIS N1 MUST BE N+1
```

```
      REAL A(100,101),X(100)
C SETUP TRAIL X
      DO 5 I=1,N
      X(I)=0
      DO 6 J=1,N
6     X(I)=X(I)+A(I,J)
5     X(I)=1.*X(I)
      NM1=N-1
C FORWARD  IMPROVE
      DO 4 IM=1,N
      DO 10 I=1,N
      X(I)=A(I,N1)
      DO 20 J=1,N
      IF(I.NE.J.AND.A(I,J).NE.0.)X(I)=X(I)-A(I,J)*X(J)
20    CONTINUE
10    X(I)=X(I)/A(I,I)
C BACKWARD IMPROVE
      DO 30 I=NM1,2,-1
      X(I)=A(I,N1)
      DO 40 J=1,N
40    IF(I.NE.J.AND.A(I,J).NE.0)X(I)=X(I)-A(I,J)*X(J)
30    X(I)=X(I)/A(I,I)
4     CONTINUE
C IMPROVE X(1) AND EXIT
      X(1)=A(1,N1)
      DO 50 I=2,N
50     IF(A(1,I).NE.0)X(1)=X(1)-A(1,I)*X(I)
      X(1)=X(1)/A(1,1)
      RETURN
      END
      REAL FUNCTION DOIE(U,V,XY,N,BF,BB,ETA,EXPO)
      REAL EXPO(100),XY(4,100),BA(2),EA(2),D(2)
      BA(1)=BF
      BA(2)=BB
      EA(1)=.25
      EA(2)=ETA/4.
      DO 1 K=1,2
      D(K)=0.
      DO 2 I=1,N
      U1=(XY(1,I)-U)/BA(K)
      U2=(XY(2,I)-U)/BA(K)
      V1=(XY(3,I)-V)/BA(K)
      V2=(XY(4,I)-V)/BA(K)
      D(K)=D(K)+(ERF(U2)-ERF(U1))*(ERF(V2)-ERF(V1))*EXPO(I)
2     CONTINUE
      D(K)=D(K)*EA(K)
1     CONTINUE
      DOIE=D(1)+D(2)
      RETURN
      END
```

SAMPLE RUN OF PROXR

User inputs are preceeded by >

```
*PROXR_,PROXR
ENTER BF,BB,ETA,ISO,IREF
>.1 2.5 .9 4 1

 BF=  0.10     BB=   2.5     ETA=  0.90     ISO=  4  NPEC=  4 IPEF=  1
     X LOW    X HIGH     Y LOW    Y HIGH     EXPO1     EXPO2
   -50.00    -35.00      0.00     15.00      1.10      1.00
     0.00     15.00      0.00     15.00      1.09      0.99
    16.00     17.00      0.00     15.00      1.40      1.28
    18.00     19.00      0.00     15.00      1.55      1.41
 ENTER 1 FOR WINDOW, 2 FOR LINE, OTHERWISE 0
>1
ENTER THE WINDOW EDGES X1,X2,Y1,Y2
>16 17 0 7.5
   EDGES AT X= 16.00, 17.00 AND Y=  0.00,  7.50

                              DOSE AT (X,Y)
      Y=  0.00  0.75  1.50  2.25  3.00  3.75  4.50  5.25  6.00  6.75  7.50
  X
 16.00    0.68  1.14  1.24  1.30  1.34  1.36  1.37  1.37  1.37  1.37  1.37
 16.10    0.98  1.73  1.82  1.89  1.93  1.94  1.95  1.95  1.95  1.95  1.95
 16.20    1.03  1.84  1.93  1.99  2.03  2.05  2.05  2.06  2.06  2.06  2.06
 16.30    1.03  1.84  1.93  1.99  2.03  2.04  2.05  2.05  2.05  2.05  2.05
 16.40    1.02  1.83  1.92  1.98  2.02  2.04  2.05  2.05  2.05  2.05  2.05
 16.50    1.02  1.83  1.92  1.98  2.02  2.03  2.04  2.04  2.04  2.04  2.04
 16.60    1.02  1.83  1.91  1.97  2.01  2.03  2.04  2.04  2.04  2.04  2.04
 16.70    1.02  1.82  1.91  1.97  2.01  2.02  2.03  2.03  2.03  2.03  2.03
 16.80    1.01  1.82  1.90  1.96  2.00  2.02  2.02  2.03  2.03  2.03  2.03
 16.90    0.96  1.71  1.79  1.85  1.89  1.90  1.91  1.91  1.91  1.91  1.91
 17.00    0.66  1.11  1.20  1.25  1.29  1.31  1.31  1.32  1.32  1.32  1.32
 ENTER 1 FOR NEW WINDOW OR 0 TO CONTINUE
>0
 ENTER 1 FOR WINDOW, 2 FOR LINE, OTHERWISE 0
>2
ENTER THE LINE END POINTS X1,Y1  X2,Y2
>10 7.5  20 7.5
   P1=( 10.00  7.50)    P2=( 20.00  7.50)

                          DOSE AT POINTS ON LINE
        0     1     2     3     4     5     6     7     8     9
   0   2.06  2.06  2.06  2.06  2.06  2.06  2.06  2.06  2.06  2.06
  10   2.06  2.06  2.05  2.05  2.05  2.05  2.05  2.04  2.04  2.04
  20   2.03  2.03  2.03  2.02  2.02  2.01  2.01  2.00  2.00  1.99
  30   1.98  1.98  1.97  1.96  1.95  1.95  1.94  1.93  1.92  1.91
  40   1.91  1.90  1.89  1.88  1.87  1.86  1.85  1.84  1.83  1.74
  50   1.28  0.81  0.72  0.71  0.70  0.70  0.69  0.68  0.68  0.78
  60   1.37  1.95  2.06  2.05  2.05  2.04  2.04  2.03  2.03  1.91
  70   1.32  0.72  0.61  0.60  0.59  0.58  0.58  0.57  0.56  0.67
  80   1.32  1.96  2.07  2.06  2.05  2.03  2.02  2.01  1.99  1.86
  90   1.19  0.52  0.39  0.37  0.36  0.34  0.32  0.31  0.29  0.28
 100   0.26  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00
 ENTER 1 FOR NEW LINE,0 TO CONTINUE
>0
 ENTER 1 FOR WINDOW, 2 FOR LINE, OTHERWISE 0
>0
   ENTER 1 TO CONINUE,0 TO END
>0
STOP
```

END

DATE
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4-82

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