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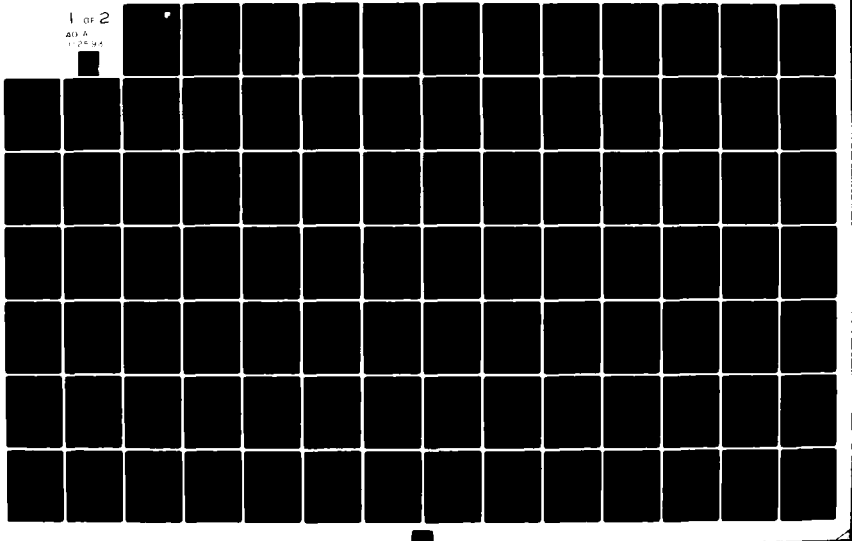

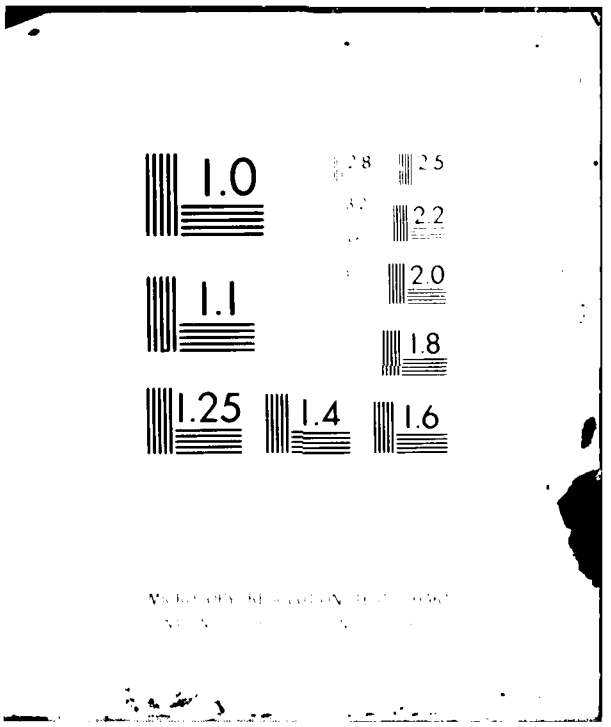
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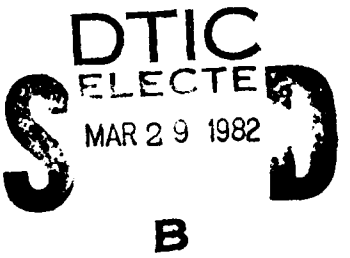
January 1982

NONLINEAR SYSTEM IDENTIFICATION TECHNIQUE VALIDATION

SIGNATRON, Inc.

Dr. Michael Rudko
Dr. Julian J. Bussgang



ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
Griffiss Air Force Base, New York 13441

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This final technical report describes the results obtained by SIGNATRON, Inc. of Lexington MA on Air Force Contract F30602-80-C-0104 for Rome Air Development Center. The objective of this effort is to develop a technique for identifying system response of nonlinear circuits by measurements of output response to known inputs.

The report describes results of a study into the system identification technique based on the pencil-of-function method previously explored by

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Jain (1974) and Ewen (1979). The procedure identified roles of the linear response and is intended as a first step in nonlinear response and is intended as a first step in nonlinear circuit identification. There are serious implementation problems associated with the original approach such as loss of accuracy due to repeated integrations, lack of good measures of accuracy and computational iteration to identify the number of poles.

The new discrete, iterative approach presented here attempts to solve these problems. The approach has three principal features:

Time shifts, rather than integration, are used to form the set of output functionals necessary for identification.

In order to determine the accuracy of the identification procedure, the input poles themselves are also identified and a comparison of the true and identified input pole values is made.

The procedure is iterative with respect to the frequency region being identified; lower frequency poles are identified first. The identified poles are then used as knowns in the identification of higher frequency poles. This facilitates the identification of wideband systems over the frequency range of interest.

FOREWORD

This final technical report describes the results obtained by SIGNATRON, Inc. of Lexington, Mass. on Air Force Contract F30602-80-C-0104 for Rome Air Development Center. The objective of this effort is to develop a technique for identifying system response of nonlinear circuits by measurements of output response to known inputs.

The SIGNATRON Project Engineer for the effort was Dr. Michael Rudko. The project was supervised by Dr. Julian J. Bussgang.

The support and assistance provided by cognizant RADC personnel Mr. D. Kenneally, the technical monitor and Mr. J. Spina, the Section Manager, are gratefully acknowledged.

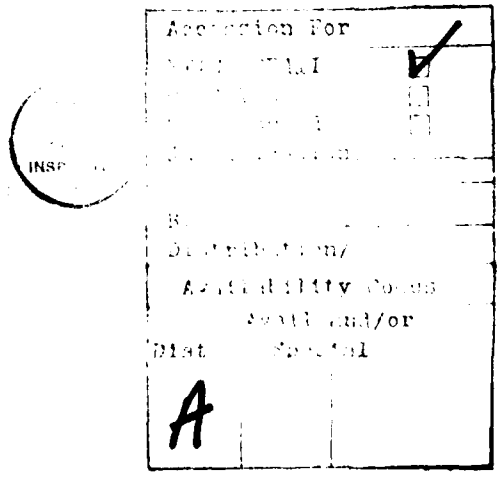


TABLE OF CONTENTS

TABLE OF CONTENTS (CON'T)

TABLE OF CONTENTS (CON'T)

LIST OF FIGURES

LIST OF TABLES

SECTION 1
INTRODUCTION

This report describes the results of a study into nonlinear system identification techniques. The primary objective was to refine previously proposed techniques and make actual circuit measurement easier.

The need for nonlinear system identification is of importance because of the increasingly dense environment of present and future Air Force communication systems. Accurate nonlinear system identification would help in the accurate prediction of such nonlinear effects as, for example, intermodulation and harmonic distortion, thus leading to the determination of RF susceptibility of various electronic equipment.

A promising system identification technique known as the pencil of functions method was originally proposed by Jain [1974] and later applied by Ewen [1975,1979] to the identification of the nonlinear transfer functions of a class of nonlinear systems. Ewen's identification was based on the fact that, for lumped parameter circuits containing zero-memory nonlinearities between circuit nodes, the nonlinear transfer functions have poles which are uniquely determined by the poles of the linear part of the circuit. Such nonlinear transfer functions can therefore be identified by finding the linear transfer function and thus its poles and then identifying the residues or zeros of the nonlinear transfer functions.

Unfortunately, the identification technique, as proposed by Ewen was not readily implementable in practice and numerous questions remained as to the appropriate parameters to be used and the performance to be expected in a realistic, noisy environment.

In fact, Ewen found that for a two pole system, his required sampling rate was 4 to 10 times the highest frequency of the passband of the system. The sampling rate was found to rise unreasonably high as the number of poles was increased. Ewen concluded that current A/D device availability limited the bandwidth of two pole systems that could be analyzed to 10 or, at most, 30 kHz. As the number of poles increased, the sampling rate requirements increased. Systems with more than two poles required a sampling rate which exceeded the Nyquist sampling rate. Thus, additional poles limited further the bandwidth of the systems which could be identified.

Ewen noted that the fastest commercially available A/D converter with 16 bit resolution had the capability of sampling a signal at 125 ksample/sec. He concluded that, at least at present, only two-pole systems with a maximum bandwidth of up to 10K or 30 kHz could be analyzed. For a 4-pole system, the resolution required in an A/D converter is at least 24 bit per sample at 125 ksample/sec in order to achieve a minimum performance. This again indicates that at the present time, this type of nonlinear system analysis would be limited to two-pole systems.

The objective of our study was to solve the implementation problems found by Ewen. The organization of this report is as follows: the problem is first defined in Chapter 2. The pencil-of-functions method and Ewen's technique are then summarized in Section 3.1. The identification parameters and practical difficulties associated with Ewen's method are analyzed in Section 3.2. Based on these factors, Section 3.2 concludes with an outline of the detailed objectives of the present effort.

In order to meet these objectives, a new discrete iterative identification technique is defined in Chapter 4. It has the advantage of being defined totally on discrete samples of the input and output processes, thus eliminating a major difficulty asso-

ciated with Ewen's approach which stems from the numerical evaluation of repeated continuous integrals. Moreover, the new method permits evaluation of the reliability of the identified unknown poles by also identifying at each step the known input poles. The distance between the identified input poles and the true input poles is indicative of the accuracy of the identified unknown system poles. In the identification of wideband systems, iteration proceeds by starting with the identification of the lowest frequency poles, then using previously identified lower frequency poles as knowns to identify the next higher frequency poles.

The problem of identifying a system in the presence of noise is examined in Chapter 5. The effects of noise are investigated and desirable identification parameters are specified using simulation results.

Results of simulations of the discrete iterative identification procedure are presented in Chapter 6 for wideband and narrowband systems containing up to four closely grouped poles. The simulations were performed with varying levels of noise added to the system output. The standard deviation of the noise ranged from 0 to 5% of the maximum system output. The highest sampling frequency used is not greater than the Nyquist rate. In all cases the correct number of poles was identified. The average error in pole location ranged from .9% for the noiseless case to 15% for additive noise with a standard deviation equal to 5% of the maximum value of the system output. The feasibility of achieving identification of this accuracy at sampling rates no higher than the Nyquist rate, greatly simplifies future implementation. The new method substantially decreases the required sampling rate and, because of the greater tolerance to noise, the necessary A/D converter resolution.

Possible performance assessement criteria are outlined in Chapter 7. Two types of criteria are possible. In Section 7.2,

the accuracy of the identified poles and zeros/residues are assessed directly. In Section 7.3, global measures of the accuracy of the identified impulse response or transfer function are discussed. It should be noted that nonlinear transfer function identification makes use of the results of the linear system identification, but requires the additional identification of the zeros/residues of the nonlinear transfer functions which was not undertaken during the current effort. Thus, the impact of the linear system parameter and global error measures on the quality of nonlinear transfer function identification was not evaluated in this study.

Finally, Chapter 8 summarizes the characteristics of the discrete iterative identification technique and presents conclusions as to its expected performance. The main characteristics of the postulated technique are:

- Completely discrete formulation.
- Selective use of the knowledge of the input to test for regions where poles are present and to determine the reliability of the identified poles.
- Iterative decrease of the sampling interval T.
- Use of previously identified poles as knowns in the set defining the Gram determinant during the identification of additional poles.

The advantages of the discrete iterative approach can be summarized as follows:

- It does not require repeated integrations; instead, time shifts are used.

- It permits the determination of regions where system poles are present.
- It aids in determining the number of poles being identified.
- The accuracy of an identified input pole when it is far from system poles gives a measure of the best accuracy that can be expected.
- Identifying the input poles while using the identified poles as knowns gives a measure of the accuracy of the identified poles.
- The use of identified poles as knowns is especially useful for wideband systems.
- Linear identification requires the measurement of only approximately twenty output samples.
- The required sampling frequency is not higher than the Nyquist rate.

Based on simulation results, it is conservatively concluded that the method should be capable in practice of identifying systems containing separate groups of four or fewer closely packed poles in the presence of additive output noise whose standard deviation is of the order of 1% of the maximum system signal output. If the output noise is mainly due to quantization, this should permit the use of 9 or 10 bit quantizers. The required sampling frequency is approximately equal to the frequency of the highest system pole. These values should permit a relatively straightforward implementation.

SECTION 2
PROBLEM DEFINITION

In the increasingly dense communications environment found in many defense communication systems, it is of great interest to be able to predict the amount of intermodulation and harmonic distortion generated in these various equipments. This entails accurate descriptions of the nonlinear characteristics of such equipment.

In the case where the nonlinear behavior exhibits no jumps or hysterisis, the nonlinear system can be represented by a Volterra series. Its output $y(t)$ due to excitation $x(t)$ is then given by

$$y(t) = \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(u_1,\ldots,u_n) \prod_{i=1}^{n} x(t-u_i)du_i$$

$$= \sum_{n=1}^{\infty} y_n(t) \qquad (2.1)$$

where, $y_n(t)$ is the nth-order system output. The system is then characterized, that is its input-output relation is completely specified by the nth-order impulse responses $h_n(u_1,\ldots,u_n)$, $n=1,2,\ldots$.

If the system's nonlinearity is mild, the output is given by the first few, predominantly three first terms of the series:

$$y(t) \tilde{=} \sum_{n=1}^{3} y_n(t). \qquad (2.2)$$

In order to characterize such a system, it is necessary to determine $h_1(u_1)$, $h_2(u_1,u_2)$, $h_3(u_1,u_2,u_3)$ or equivalently, the higher-order transfer functions

$$\begin{aligned} H_1(s) &= \mathcal{L}_1[h_1(u)] \\ H_2(s_1,s_2) &= \mathcal{L}_2[h_2(u_1,u_2)] \\ H_3(s_1,s_2,s_3) &= \mathcal{L}_3[h_3(u_1,u_2,u_3)] \end{aligned} \tag{2.3}$$

where $\mathcal{L}_n$ denotes n-dimensional Laplace transformation.

This problem is, in general, extremely complex since it involves the determination of multidimensional functions. It has, however, been shown that, in the case where the nonlinear system is a lumped parameter circuit with zero-memory nonlinearities between circuit nodes, the equivalent 2nd and 3rd-order transfer function poles can be obtained from the poles of the linear transfer function [Graham and Ehrman (1973); Ewen (1975)].

More precisely, let the transfer function of the linear part of the system be given by

$$\begin{aligned} H_1(s) &= \frac{\prod_{i=1}^{M}(s+q_i)}{\prod_{i=1}^{N}(s+p_i)}, \quad M<N \\ &= \sum_{i=1}^{N}\frac{R_i}{s+p_i} \end{aligned} \tag{2.4}$$

where, $-q_1,-q_2,\ldots,-q_M$; $-p_1,-p_2,\ldots,-p_N$; $R_1,R_2,\ldots,R_N$, are, respectively, the zeros, poles and residues of the transfer function and it has been assumed for notational simplicity that the poles are distinct.

Then, the 2nd and 3rd-order transfer functions are given, respectively, by [Ewen (1975)]

$$H_2(s_1,s_2) = \sum_{k_1=1}^{L} \sum_{k_2=1}^{N} A_{k_1k_2} \frac{s_1+s_2+2a_{k_2}}{(s_1+s_2+a_{k_1}+a_{k_2})(s_2+a_{k_2})(s_1+a_{k_2})} \tag{2.5}$$

$$H_3(s_1,s_2,s_3) = \frac{1}{3} \sum_{k_1=1}^{J} \sum_{k_2=1}^{L} \sum_{k_3=1}^{N} \frac{C_{k_1k_2k_3}}{(s_1+s_2+s_3+(a_{k_1}+a_{k_2}+a_{k_3}))}$$

$$\frac{1}{(s_1+a_{k_3})(s_1+s_3+(a_{k_2}+a_{k_3}))} + \frac{1}{(s_2+a_{k_3})(s_1+s_2+(a_{k_2}+a_{k_3}))}$$

$$+ \frac{1}{(s_1+a_{k_3})(s_1+s_2+(a_{k_2}+a_{k_3}))} + \frac{1}{(s_3+a_{k_3})(s_2+s_3+(a_{k_2}+a_{k_3}))}$$

$$+ \frac{1}{(s_2+a_{k_3})(s_2+s_3+(a_{k_2}+a_{k_3}))} + \frac{1}{(s_3+a_{k_3})(s_1+s_3+(a_{k_2}+a_{k_3}))}$$

where, the quantities J, L, a_{k_1}, a_{k_2} are uniquely determined by the poles of the linear transfer function $H_1(s)$. The complete identification of the nonlinear transfer functions of the system requires, therefore, the identification of the linear transfer function and the determination of the constants $A_{k_1k_2}$, $C_{k_1k_2k_3}$ for the permissible values of k_1, k_2, and k_3.

The poles of the linear transfer function thus play a crucial role in the identification of the linear and nonlinear parts of the system. They not only specify the denominator of $H_1(s)$ but linear combinations of these poles determine the poles of $H_2(s_1,s_2)$ and $H_3(s_1,s_2,s_3)$.

In the next section, an identification technique, known as the pencil of functions method, developed by Jain [Jain (1974)] and applied by Ewen [Ewen (1975), (1979)] to the above problem is presented. The critical parameters of the technique and the practical difficulties involved in applying it are isolated and analyzed. An efficient and practical method for the identification of the poles of a linear system is then developed and illustrated by simulation examples.

SECTION 3

PENCIL OF FUNCTIONS METHOD

3.0 Summary of the Method

A promising technique for determining, from input/output measurements, the poles of the transfer function of a linear system has been developed by Jain [1974]. Known as the pencil of functions method, this technique was then applied, by Ewen [1975, 1979] to the identification of the 2nd and 3rd-order transfer functions of a Volterra system. We next outline the pencil of functions method as originally presented by Jain and used by Ewen. This technique was the starting point of the present contract. Subsequent developments by Jain and Osman [1979] under separate contract No. F30602-75-C-0118, conducted in parallel with the present contract, are presented where appropriate.

Suppose that the transfer function of the linear part of the system is given by

$$H(s) = \frac{\prod_{i=1}^{M} (s+q_i)}{\prod_{i=1}^{N} (s+p_i)} = \frac{Q(s)}{P(s)} \qquad (3.1)$$

$$= \sum_{i=1}^{N} \frac{R_i}{s+p_i} \quad .$$

Excite the system by $x_1(t)$*. With $x_1(t)$ known or measured, measure the system output $y_1(t)$. By successive integrations, compute

---

* Ewen selected $x_1(t)$ exponential for ease of analysis.

$$x_i(t) = \int_0^t x_{i-1}(u)du$$
$$y_i(t) = \int_0^t y_{i-1}(u)du, \quad i=1,2 \ . \tag{3.2}$$

Suppose that N, the number of poles, is known. Form the set

$$\{(y_1(t)-\lambda y_2(t)),(y_2(t)-\lambda y_3(t)),\ldots,(y_N(t)-\lambda y_{N+1}(t)), \tag{3.3}$$
$$x_2(t),\ldots,x_{N+1}(t)\}$$

where λ is constant. Test the set for linear dependence by forming the linear combination

$$c_1(y_1(t)-\lambda y_2(t)) + \ldots + c_n(y_N(t)-\lambda y_{N+1}(t)) + d_1x_2(t) + \ldots + d_Nx_{N+1}(t) = 0 \ . \tag{3.4}$$

The set is linearly independent if the only solution to (3.4) is

$$c_1 = c_2 = \ldots = c_N = d_1 = \ldots = d_N = 0.$$

It is linearly dependent otherwise.

Taking the Laplace transform of (3.3) an equivalent test for linear independence is

$$c_1(Y_1(s) - \lambda Y_2(s)) + \ldots + c_n(Y_N(s) - \lambda Y_{N+1}(s)) + d_1X_2(s) + \ldots + d_NX_{N+1}(s) = 0 \ . \tag{3.5}$$

But, using (3.1) and (3.2),

$$Y_1(s) = \frac{Q(s)}{P(s)} X_1(s)$$

$$X_i(s) = X_1(s) \,/\, s^{i-1} \tag{3.6}$$

$$Y_i(s) = Y_1(s) \,/\, s^{i-1}, \quad i=1,2,\ldots .$$

The test for linear independence in (3.5) therefore becomes

$$Q(s)(s-\lambda)(c_1 s^{N-1} + c_2 s^{N-2} + \ldots + c_N) \tag{3.7}$$
$$+ P(s)(d_1 s^{N-1} + \ldots + d_N) = 0$$

The left hand side of (3.7) contains $2N$ coefficients. If $(s-\lambda)$ is not a factor of $P(s)$ that is, if λ is not a pole of $H(s)$ the polynomial in (3.7) is of degree $2N-1$, containing terms in $1, s, \ldots, s^{2N-1}$, and it has 2^N coefficients. The set is therefore linearly independent.

If λ is a pole of $H(s)$ or, equivalently, $(s-\lambda)$ is a factor of $P(s)$,

$$P(s) = P'(s)(s-\lambda) \tag{3.8}$$

where, $P'(s)$ is of degree $N-1$. Dividing (3.7) by $(s-\lambda)$ results in a polynomial of degree $2N-2$. Since there are still $2N$ coefficients, the set is linearly dependent.

It follows that the poles of $H(s)$ are the values of λ for which the set in (3.3) is linearly dependent. The linear dependence of a set of time functions can be readily checked using the Gram determinant which, for the set of functions

$$\{\omega_1(t), \omega_2(t), \ldots, \omega_k(t)\}, \quad 0<t<T \tag{3.9}$$

is defined as

$$G_k = \det \begin{bmatrix} \overline{\omega_1\omega_1} & \cdots & \overline{\omega_1\omega_k} \\ \overline{\omega_2\omega_1} & \cdots & \overline{\omega_2\omega_k} \\ \vdots & & \vdots \\ \overline{\omega_k\omega_1} & \cdots & \overline{\omega_k\omega_k} \end{bmatrix} \tag{3.10}$$

where the inner product $\overline{\omega_i\omega_j}$ over the interval [0,T] is equal to

$$\overline{\omega_i\omega_j} = \int_0^T \omega_i(\tau)\omega_j^*(\tau)d\tau, \quad i,j=1,2,\ldots,k$$

and where * denotes complex conjugation.

Hence, the poles of H(s) are the values of λ for which the Gram determinant of the set of functions in (3.3) is equal to zero.

Equivalently, it can be shown [Jain (1974)] that the poles are the roots of the polynomial in λ

$$\sum_{i=0}^{N} \lambda^{N-i} \left([G_{2N+1}]_{(i+1,i+1)} \right)^{1/2} = 0 \tag{3.11}$$

where, $[G_{2N+1}]_{(i+1,i+1)}$ is the value of the (i+1,i+1)th co-factor of the Gram determinant of the set

$$\{y_1(t), \ldots, y_{N+1}(t), x_2(t), \ldots, x_{N+1}(t)\} . \tag{3.12}$$

In the above, it was assumed that N, the number of poles, was known. In practice, this number must, of course, be found before determining the poles using (3.11). The number N can be determined by considering the set

$$\{y_1(t), \ldots, y_K(t), x_2(t), \ldots, x_K(t)\} . \qquad (3.13)$$

Using the same approach as above, this set is linearly independent if $K<N$; it is linearly dependent if $K>N+1$. The number of poles can, therefore, be found by calculating the Gram determinant for the set in (3.13) for increasing values of K, $K=2,3,\ldots$. If K' is the smallest K for which the Gram determinant is zero, that is, the first value for which the set is linearly dependent, the number of poles is equal to

$$N = K' - 1 . \qquad (3.14)$$

3.2 Identification Parameters and Practical Difficulties Associated with Ewen's Methods

The identification of an unknown circuit using the pencil of functions approach as proposed by Ewen [Ewen (1975), (1979)] and as described in Section 3.1 therefore involves the following steps:

(1) Excite the system

(2) Measure the system outputs and inputs

(3) Calculate $x_2(t),\ldots, x_k(t)$, $y_2(t)$, ..., $y_k(t)$ as defined in (3.2) by successive integrations.

(4) Calculate the inner products over an interval T.

(5) Calculate the Gram determinant of the set of functions in step (3) for increasing values of K.

(6) Determine the number of poles N as K'-1, where K' is the smallest value of K for which the Gram determinant is zero.

(7) Find the poles using (3.11).

Because of the amount of computation which is necessary, it is more efficient to perform the calculations using a computer. This suggests the identification set-up shown in Figure 3.1 where the double arrow at the input indicates that the input can either be generated in analog form and then converted to digital form or generated by the computer and converted to analog form.

Note that, in addition to the blocks shown in Figure 3.1, a practical identification implementation would necessitate use of a linear output and input amplifier/attenuator with large dynamic range to insure that the system input is of the proper level and that $y(t)$ uses the full range of the analog to digital converter. The contract under which the present effort was performed called originally for the design and the evaluation of a practical implementation of the above technique. This was however abandoned because of the following practical difficulties and unresolved questions associated with the method:

(1) What is the impact of using different excitations? In the original technique only a decaying exponential input was considered.

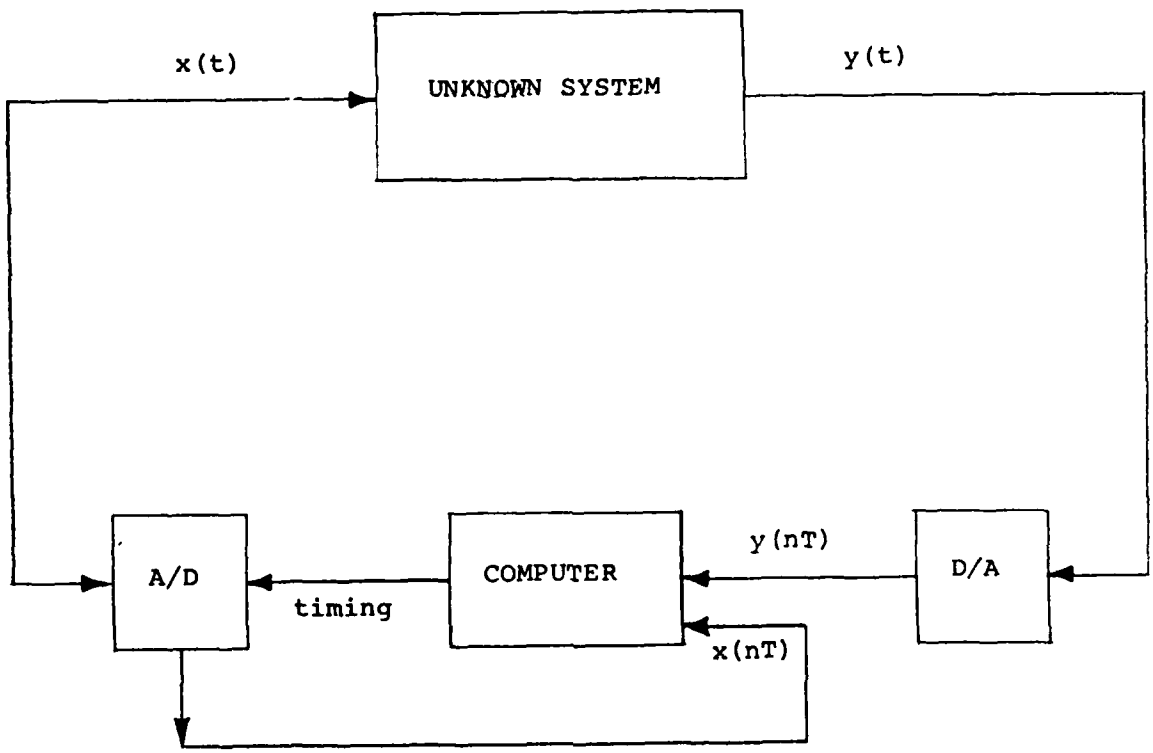


Figure 3.1 Block Diagram of Identification Test Bed

(2) Because of the digital implementation the output signal must be sampled and quantized. A sampling rate must therefore be specified which will assure optimum performance and which will be practical with existing analog to digital converters.

(3) Simpson's rule was used in evaluating the integration of a function as given by

$$\int_a^b y(t)dt = \frac{b-a}{6n}\left[y(0) + 4y(\Delta t) + 2y(2\Delta t) + 4y(3\Delta t) + \right.$$

$$\left. \ldots + 2y((2n-2)\Delta t) + 4y((2n-1)\Delta t) + y(2n\Delta t)\right] .$$

Each integration reduces the number of samples by one half and therefore increases the error. As the number of poles of the system increases, the number of required integrations increases accordingly. Integration by Simpson's rule introduces error at each step. After a number of integrations, therefore, the error becomes excessively large. The accuracy of successive integrals decreases rapidly. This loss of accuracy due to numerical integration can be overcome by increasing bits in the sample and the sampling rate. However, the specifications of hardware devices which implement analog to digital (A/D) and digital to analog (D/A) conversions and the high rate of sampling rapidly exceed the range of current technology as the number of poles of the system increases. For example, Ewen states that if we were to analyze a system of four poles, the A/D converter would be required to have 20-24 bits of resolution. The present commercially available A/D converters are capable of at most a 16 bit resolution.

The performance of successive numerical integrations has a large impact on the necessary sampling rate. Ewen concluded that the sampling rate required is determined by the need for accuracy of numerical integration and not by Nyquist sampling constraints. For a two pole system, the required sampling rate is 4 to 10 times the highest frequency of the passband of the system if we desire a mean square error between the actual and predicted system output to be smaller than the mean square error induced by a 10 percent error in each pole and residue. The sampling rate may rise unreasonably high as the number of poles increases. Considering errors in identified poles, Ewen concluded that the bandwidth of a two pole system that can be analyzed is limited to 10 to 30 kHz by current device availability.

Ewen noted that the fastest commercially available A/D converter with 16 bit resolution had the capability of sampling a signal at 124 kHz. This resulted in his conclusion that, at least at present, only systems with a maximum bandwidth of up to 10 kHz or 30 kHz could be analyzed.

(4) The inner product interval T which, in noisy practical situations, will result in the specified error in the location of the identified poles, cannot be readily determined.

(5) The measured digital output which is used will contain noise due essentially to three sources: quantization noise from the A/D converter, thermal noise originating in the unknown circuit and measurement errors. The inner products which are the elements of the Gram

matrix will therefore be noisy. As a consequence, the Gram determinant will not be zero for any k. This will, of course, make it difficult to determine the number of system poles. Even if the correct value of N is determined errors in the inner products will result in errors in pole locations. These errors in pole locations can, of course, be expected to be increased if the wrong value of N is chosen.

(6) Quality criteria must be studied which will permit to quantify the performance of the identification technique and, after the global identification of the system (the identification of the poles and zeros/residues, traded off against the problems identified in (1)-(5).

After the initial study resulted in the above conclusions, it was decided with the concurrence of RADC to redirect the effort to the study of the aforementioned difficulties and unresolved questions. In Section 4, we describe the identification procedure that was developed to combat these problems. More precisely, the following items were addressed:

- Selection of excitation waveforms and techniques.
- Selection of input and output functionals to be used in forming the Gram determinant.
- Determination of correlation intervals to be used in the calculation of the inner products.
- Investigation of stopping rules for the determination of the number of linear system poles in the presence of system noise, quantization noise and as a function of the relative importance of the poles.

- Evaluation of the effectiveness of the proposed techniques.

- Analysis of the expected errors in pole locations due to quantization noise, underspecification of the number of poles and overspecification of the number of poles.

- Assessment of the class of systems that can be identified given practical sampling rates and quantization accuracies.

- Study of measurement and computational techniques that minimize errors in pole locations by averaging out results of several measurements or of several stages of the identification process.

SECTION 4
DISCRETE ITERATIVE APPROACH

4.1 Principle of the Discrete Method

4.1.1 Basic Method

As discussed in Section 3.2, because of the noise introduced by the numerical calculation of repeated integrals and the ensuing necessity of using very high sampling rates, a primary objective of any practical implementation is to modify the procedure so as to eliminate this difficulty.

This can be accomplished by analyzing the problem directly in the discrete sampled domain instead of implementing a digital technique which emulates analog operations, i.e., the numerical computation of successive integrals.

As in (3.1) suppose that the linear part of the system is characterized by input-output transfer function

$$H(s) = \frac{\prod_{i=1}^{M} (s+q_i)}{\prod_{i=1}^{N} (s+p_i)} , \qquad M<N \tag{4.1}$$

$$= \sum_{i=1}^{N} \frac{R_i}{s+p_i}$$

where, for notational simplicity, it is assumed that the system poles are distinct. If the unknown circuit is excited by the sum of I exponentials

$$x(t) = \sum_{i=1}^{I} a_i e^{-b_i t} , \qquad (4.2)$$

the system output is equal to

$$y(t) = \sum_{i=1}^{N} c_i e^{-p_i t} + \sum_{i=1}^{I} d_i e^{-b_i t} \qquad (4.3)$$

where, the constants c_i and d_i are, respectively, given by

$$c_i = \sum_{j=1}^{I} \frac{R_i}{b_j - p_i}$$

$$d_i = \sum_{j=1}^{N} \frac{R_j}{p_j - b_i}$$

Note that the assumption that the input is the sum of exponentials is equivalent to the assumption that the Laplace transform of $x(t)$ is the ratio of two polynomials in s with the denominator polynomial of higher degree than the numerator polynomial and the denominator polynomial having distinct roots. Sampling $y(t)$ with sampling frequency $f_s=1/T$, sampling interval T, the sampled version of the output, over an interval $[0,LT]$, is given by

$$y_1(nT) = \sum_{i=1}^{N} c_i e^{-np_i T} + \sum_{i=1}^{I} d_i e^{-nb_i T}, \quad 0 \leq n \leq L . \qquad (4.4)$$

Note that the choice of L, the number of samples used, and of the sampling period T are discussed in Section 5.3.2.

By successive shifts, form

$$y_2(nT) = y_1((n+1)T)$$

$$= \sum_{i=1}^{N} c_i e^{-(n+1)p_i T} + \sum_{i=1}^{I} d_i e^{-(n+1)b_i T}$$

$$= \sum_{i=1}^{N} c_i(2) e^{-np_i T} + \sum_{i=1}^{I} d_i(2) e^{-nb_i T}, \quad 0 \le n \le L$$

$$\vdots$$

$$y_K(nT) = y_{K-1}((n+1)T) = y_1((n+K-1)T) \tag{4.5}$$

$$= \sum_{i=1}^{N} c_i(k) e^{-np_i T} + \sum_{i=1}^{I} d_i(k) e^{-np_i T}, \quad 0 \le n \le L$$

where

$$c_i(k) = c_i(k-1) e^{-p_i T} = c_i e^{-(k-1)p_i T}$$

$$d_i(k) = d_i(k-1) e^{-b_i T} = d_i e^{-(k-1)b_i T}, \quad k = 2, 3, \ldots, K .$$

Note that each of the $y_i(nT)$, $i=1,2,\ldots,K$ is a linear combination of the functions e^{-np_iT}, , ..., e^{-np_NT}, e^{-nb_iT}, ..., e^{-nb_IT}. Since these functions are *linearly independent* they can be considered to form a basis for $y_i(nT)$, $i=1,2,\ldots,K$.

Consider the set of shifted versions of the *sampled output*

$$\{y_1(nT), y_2(nT), \ldots, y_K(nT)\} \tag{4.6}$$

and test it for linear independence by forming the *linear combination*

$$a_1y_1(nT) + a_2y_2(nT) + \cdots + a_Ky_K(nT) = 0, 0 \leq n \leq L . \tag{4.7}$$

Since the linear combination in (4.7) has e^{-np_iT}, ..., e^{-np_NT}, e^{-nb_iT}, ..., e^{nb_JT} as its basis, it is a (n+I)-dimensional function. The left-hand side of (4.7) contains K coefficients. It follows that the set is linearly independent if

$$K \leq N+I \tag{4.8}$$

and that it is linearly dependent if

$$K > N+I . \tag{4.9}$$

As in Section 3.1, the linear dependence can be checked by using the Gram determinant.

The set of functions in (4.6) is linearly dependent if the Gram determinant

$$G_K = \det \begin{bmatrix} \overline{y_1 y_1} & \overline{y_1 y_2} & \cdots & \overline{y_1 y_K} \\ \overline{y_2 y_1} & \overline{y_2 y_2} & \cdots & \overline{y_2 y_K} \\ \vdots & & & \\ \overline{y_K y_1} & \overline{y_K y_2} & \cdots & \overline{y_K y_K} \end{bmatrix} = 0 \qquad (4.10)$$

where,

$$\overline{y_i y_j} = \sum_{n=0}^{L} y_i(nT) y_j^*(nT), \quad i,j=1,2,\ldots,K \ .$$

The set is linearly independent if

$$G_K \neq 0.$$

The dimensionality $N+I$ of the basis, and thus the number N of poles, can be determined by calculating the Gram determinant for increasing values of K. If K' is the smallest value of K such that $G_{K'}=0$, then

$$N+I = K'-1 \ . \qquad (4.11)$$

Having determined N+I, the system poles $-p_1, \ldots, -p_N$ and the input poles $-b_1, \ldots, -b_I$ can be found using a similar approach.

Consider the set

(4.12)

$$\{(\lambda y_1(nT)-y_2(nT)),(\lambda y_2(nT)-y_3(nT)),\ldots,(\lambda y_{N+I}(nT)-y_{N+I+1}(nT))\} .$$

Using (4.5), the test for linear independence can be written as

$$a_1\left[\sum_{i=1}^{N} c_i(\lambda-e^{-p_iT})e^{-np_iT} + \sum_{i=1}^{I} d_i(\lambda-e^{-b_iT})e^{-nb_iT}\right] \qquad (4.13)$$

$$+ a_2\left[\sum_{i=1}^{N} c_i e^{-p_iT}(\lambda-e^{-p_iT})e^{-np_iT} + \sum_{i=1}^{I} d_i e^{-b_iT}(\lambda-e^{-b_iT})e^{-nb_iT}\right]$$

$$+ \cdots + a_{N+I}\left[\sum_{i=1}^{N} c_i e^{-(N+I-1)p_iT}(\lambda-e^{-p_iT})e^{-np_iT}\right.$$

$$\left. + \sum_{i=1}^{I} d_i e^{-(N+I-1)b_iT}(\lambda-e^{-b_iT})e^{-nb_iT}\right] = 0 .$$

If $\ell n\lambda/T$ is not one of the system or input poles, that is, if

$$\lambda \neq e^{-p_iT}, \quad i=1,2,\ldots,N \qquad (4.14)$$

and

$$\lambda \neq e^{-b_iT}, \quad i=1,2,\ldots,I ,$$

the left-hand side of (4.13) is of dimension $N+I$ and contains $N+I$ coefficients. It follows that the set in (4.12) is then linearly independent.

If $\ell n\lambda/T$ is one of the system or input poles,

$$\lambda = e^{-p_i T}, \quad 1 \leq i \leq N \tag{4.15}$$

or

$$\lambda = e^{-b_i T}, \quad 1 \leq i \leq I \ ,$$

the dimension of the space in (4.13) is $N+I-1$ but the left-hand side still contains $N+I$ coefficients. It follows that the set is linearly dependent.

Hence, the system and input poles can be determined from the values of λ for which the set in (4.12) is linearly dependent, or equivalently, for which the Gram determinant of the set if zero. Using an approach due to Jain [1974] as in (3.11) it can be shown that these values of λ are the roots of the polynomial

$$\sum_{i=0}^{N+I} \lambda^{N+I-1} \left([G_{N+I+1}]_{(i+1,i+1)} \right)^{1/2} = 0 \tag{4.16}$$

where, $[G_{N+I+1}]_{(i+1,i+1)}$ is the $(i+1,i+1)$th cofactor of the Gram determinant of the set

$$\{y_1(nT), y_2(nT), \cdots, y_{N+I+1}(nT)\} \ . \tag{4.17}$$

If $\lambda_1, \lambda_2, \cdots, \lambda_{N+I}$ represent the $N+I$ roots of the polynomial in (4.16), the system and input poles can be obtained from

$$\ell n \lambda_i / T, \qquad i=1,2,\ldots,N+I \quad . \tag{4.18}$$

Our discrete sampling and shift identification technique is very similar to the pencil of functions method described in Chapter 3. The essential differences are that

- The proposed technique is purely discrete: the successive integrals have been replaced by successive shifts.
- If the system is excited by the sum of I exponential excitations, $(N+I)$ poles are identified based on the determinant of a $(N+I+1)x(N+I+1)$ matrix. The technique in Chapter 3 used a $(2N+1)x(2N+1)$ matrix to identify N poles.

Note that, in the above, the input poles are identified together with the system poles. In the following section the identification method derived here is slightly modified to make use of the knowledge of the input poles.

4.1.2 Use of Input

During the identification of an unknown circuit, the input to the circuit is, of course, known. This knowledge can therefore be used in the identification technique.

Suppose that e^{-nb_1T}, e^{-nb_2T}, ..., e^{-nb_IT} are known and define the set

$$\{y_1(nT), y_2(nT), \cdots, y_k(nT), e^{-nb_1T}, \cdots, e^{-nb_IT}\} \ . \qquad (4.19)$$

Test this set for linear independence by considering

$$a_1y_1(nT) + a_2y_2(nT) + \cdots + a_Ky_K(nT) + a_{K+1}e^{-nb_1T} + \cdots + a_{K+I}e^{-nb_IT} = 0 \ , \quad 0 \leq n \leq L \qquad (4.20)$$

where $y_i(nT)$, $i=1,2,\ldots,K$ is defined in (4.5).

Repeating the argument used to derive (4.8) and (4.9), the set in (4.19) is linearly independent if

$$K \leq N \qquad (4.21)$$

and it is linearly dependent otherwise. It follows that if K' is the smallest value of K for which the Gram determinant of the set in (4.19) is zero, the number of system poles N is given by

$$N = K' - 1 \ . \qquad (4.22)$$

Having determined the number of system poles by increasing K until K' is found, the system poles can be identified by considering the set

$$\{(\lambda y_1(nT) - y_2(nT)), \cdots, (\lambda y_N(nT) - \lambda y_{N+1}(nT)), e^{-nb_1T}, \cdots, e^{-nb_IT}\} \ . \qquad (4.23)$$

Repeating the steps leading to (4.16), the system poles can be found from the roots of the polynomial in λ

$$\sum_{i=0}^{N} \lambda^{N-i} \left([G_{N+I+1}]_{(i+1,i+1)} \right)^{1/2} = 0 \qquad (4.24)$$

where, $[G_{N+I+1}]_{(i+1,i+1)}$ is the $(i+1,i+1)$th cofactor of the Gram determinant of the set

$$\{y_1(nT),\cdots,y_{N+1}(nT),e^{-nb_1T},\cdots,e^{-nb_IT}\} \; . \qquad (4.25)$$

Denoting the roots of the polynomial in (4.24) by $\lambda_1,\lambda_2,\cdots,\lambda_N$, the system poles are given by

$$-p_i = \ell n\lambda_i/T, \quad i=1,2,\ldots,N \; . \qquad (4.26)$$

Equation (4.24) thus permits the identification of the system poles. The only difference between this result and procedure and those presented in Section 4.1.1 is that, in the present case, knowledge of the input is used and only the system poles are identified. Note that in the pencil of functions method, Ewen [1979] and Jain [1980] also use the knowledge of the input. This is further discussed in Section 4.3. Thus, the present method can be considered as a discrete version of the pencil of functions technique which uses shifts.

In the following section, use of the input is applied selectively and an iterative identification technique is defined.

4.2 Iterative Approach

The selective use of the knowledge of the input in the identification procedure can serve as a check for the accuracy of the identified system poles and in the determination of the regions where the poles are located. Moreover, the iterative variation of the sampling interval together with the use as knowns of previously identified poles can be the basis of an iterative identification method.

This approach can best be illustrated through a simple example. Suppose that the system transfer function contains two real poles $-p_1$ and $-p_2$ with p_2 much larger than p_1. Excite the system by an input consisting of a single exponential

$$x(t) = e^{-bt}.$$

Then, using (4.4), the sampled output is given by

$$y_1(nT) = c_1 e^{-np_1T} + c_2 e^{-np_2T} + de^{-nbT}. \qquad (4.27)$$

First, choose a sampling interval T and an input pole $-b$ such that

$$b << p_1 << p_2 \qquad (4.28)$$
$$T \approx 1/b.$$

It follows that

$$bT \approx 1 \qquad (4.29)$$
$$p_1T << 1$$
$$p_2 << 1$$

and

$$y_1(nT) \approx de^{-nbT} \quad . \tag{4.30}$$

Carry out the identification technique without using the knowledge of the input and identify a single pole using the set

$$\{y_1(nT),\ y_2(nT)\}$$

and (4.16) with N+I=1. From (4.30) it follows that this will result in the input pole being identified.

Iteratively decrease T and repeat the step above. When the sampling interval is small enough such that

$$\begin{aligned} p_1T &\approx 1 \\ p_2T &<< 1 \end{aligned} \tag{4.31}$$

the sampled output will, using (4.27), be approximately equal to

$$y_1(nT) \approx c_1e^{-np_1T} + de^{-nbT}. \tag{4.32}$$

Because of the influence of the system pole at $-p_1$ on the output, the identification of a single pole as above will no longer result in the input pole being identified.

The identification of a single pole which is not the input pole thus indicates the presence of a system pole(s) $-p_i$ such that e^{-np_iT}, n=1,2,..., is not negligibly small. Then, this pole(s) can be identified using (4.24) where, knowledge of the input is used.

Suppose that this procedure resulted in $\hat{p}_1$ being identified. The accuracy of $\hat{p}_1$, that is how well $e^{-n\hat{p}_1T}$ models the system contribution to the output, can be checked by using

$e^{-n\hat{p}_1 T}$ as a known and identifying the input pole. That is, use (4.24) with N=1 and the set

$$\{y_1(nT),\ y_2(nT),\ e^{-n\hat{p}_1 T}\} \tag{4.33}$$

to carry out the identification. If, following this procedure, the identified pole is close to the input pole, $\hat{p}_1$ can be judged to be an adequate estimate of p_1. The degree of closeness which is required depends on the amount of noise corrupting the measurements. In the noiseless case, the identified input pole is judged close to the actual input pole if it is within 10% of the true value. In the higher noise level cases considered in the following, the acceptance region increases to 40%.

This last step can then be repeated for decreasing values of T. The identification based on the set in (4.33) will give a good approximation of the input pole when (4.32) is an adequate representation of the system output. When this is not the case, that is, when T is small enough so that

$$p_2 T \approx 1,$$

and the system output is given by (4.27) the input pole will not be identified. Then, the knowledge of the system input and of $\hat{p}_1$ can be used to identify the other poles which contribute to the system output. The identified poles can again be checked by using them as knowns and identifying the input. The above procedure can be continued to identify more poles.

Note that the iterative identification technique was illustrated above for a system which only had two real widely separated poles and an input which was a single exponential. Neither of these restrictions is an inherent part of the technique which can be readily generalized for complex conjugate poles, grouped poles, and multi-exponential inputs.

The iterative discrete identification method described here has the following advantages:

- It permits us to find the regions where the system poles are located.
- It provides a checking procedure for the accuracy of the identified poles.
- It gives a mechanism for determining the number of system poles by finding which poles, when used as knowns, result in an identified input pole(s) which is closest to the true input pole(s). As discussed in Chapter 5, this is especially helpful when the system output contains noise.
- By using previously identified poles as knowns, it allows the separate identification of the poles of a wideband system in different frequency bands.

The only restriction inherent in this technique is that the system excitation be the sum of exponentials.

Note that if the regions where the poles are located and the number of poles in each region are known a priori, the iterative procedure is not even needed. For example, suppose that it is known that the system contains three poles between f_1 and f_2 and two poles between f_3 and f_4, $f_3 >> f_2$. The steps would then be:

(1) Input e^{-tf_2}.

(2) Sample with sampling periof $1/f_2$.

(3) Use the knowledge of the input and identify three poles. The accuracy of the three identified poles can be checked by using them as knowns and identifying the input.

(4) Input e^{-tf_4}.

(5) Sample with period $1/f_4$.

(6) Use the knowledge of the input and of the three identified lower frequency poles to identify two poles.

(7) Check for the accuracy of the identified poles.

Note that this procedure would require the factoring of a polynomial of degree 3 to find the lower frequency poles and of a polynomial of degree 2 to find the higher frequency poles. This may result in large errors in pole locations even though the polynomials which are factored are close in a global sense to the true polynomials. This question is discussed further in Section 7.2. Note also that an iterative technique where the poles are identified one at a time does not require the factoring of polynomials since at each step the polynomial is of degree 1.

Before applying the discrete iterative method in Chapter 5 to the identification of a system in the presence of noise, the following section relates the iterative method to the pencil-of-functions method described in Chapter 3.

4.3 Relation to the Pencil-of-Functions Method

Aside from the selective use of the input and the iterative nature of the procedure, the iterative discrete identification method described in Sections 4.1 and 4.2 is very closely related to the pencil-of-functions method presented in Chapter 3. The principal difference between the two methods lies in the functions included in the set which is tested for linear independence.

More precisely, in the pencil-of-functions method the poles are determined using the polynomial equation in (3.11) and the Gram determinant of the set

$$\{y_1(t), \ldots, y_{N+1}(t), x_2(t), \ldots, x_{N+1}(t)\}$$

whose elements are formed by successive integrations as defined in (3.2).

On the other hand, in the discrete iterative approach the determination of the poles is based on a similar polynomial equation (4.16) but with the set now being defined as

$$\{y_1(nT), \ldots, y_{N+I+1}(nT)\}$$

or

$$\{y_1(nT), \ldots, y_{N+1}(nT), e^{-nb_1T}, \ldots, e^{-nb_IT}\}$$

where, the elements are formed by successive shifts as in (4.5).

Note from (4.5) that successive shifts preserve the dimensionality of the space of $y_1(nT)$. That is, if $y_1(nT)$ can be expressed as the linear combination of the basis functions

$$e^{-np_1T}, \ldots, e^{-np_NT}, e^{-nb_1T}, \ldots, e^{-nb_IT},$$

$y_k(nT)$, obtained through (k-1) successive shift of $y_1(nT)$, can also be expressed as a linear combination of the same functions. This space preservation aspect represents a fundamental advantage of the new approach. The pencil-of-functions method as described in Chapter 3 augments the space of $y_1(t)$ through the use of successive integrals. If $e^{-p_1t}, \ldots, e^{-p_Nt}$ represent the basis of the system response, an integration

$$\int_0^t e^{-p_i u} du = \frac{1}{p_i} [1 - e^{-p_i t}]$$

introduces the additional basis function 1. Successive integrations introduce the additional components $t, t^2, \ldots,$.

The repeated integral and the shift methods of forming the elements of the set are, by no means, unique. In general, the elements of the set can be considered to be the result of a mapping from the input/output functions. Several continuous and discrete domain mappings have been tried. Sarkar [Sarkar, et al., (1980)] suggested using reverse time integration which for exponential inputs does not augment the space. Jain [Jain and Osman (1979), Jain (1980)] successively used the z-domain operator $z/(z-b)$ to form the members of the set. This technique augments the space by introducing additional poles at $z=b$.

Note that the space dimension preservation property of either successive shifts or successive reverse integrations depends on the exponential nature of the input. If the input cannot be expressed as the sum of exponentials, these operations will augment the space. Since an important part of the discrete iterative procedure developed in Section 4.2 is the ability to selectively use the knowledge of the input and since this possibility also depends on the exponential nature of the input, we adopted the successive shift formulation which does not augment the space.

Note also that the selective use of the input can be applied in the pencil-of-functions method. In order to identify the input pole it would suffice to consider that the input pole is part of the unknown system whereas the input is an impulse.

SECTION 5

IDENTIFICATION IN THE PRESENCE OF NOISE

5.1 Introduction

When the identification technique is applied in practice to a real system, the measured discrete output will be contaminated by noise. Then, instead of $y_1(nT)$, the measured quantity can, in general, be expressed as

$$z_1(nT) = y_1(nT) + w_1(nT) \tag{5.1}$$

where $w_1(nT)$ is an additive noise which can include quantization noise, measurement errors and system thermal noise.

Since the linear independence relations permitting the identification of the system poles will then no longer be strictly valid, difficulties can be expected in the determination of the number of poles and the identification of the poles themselves. More precisely, the number of poles is determined from the value of the Gram determinant

$$G_k = \det \begin{bmatrix} \overline{z_1 z_1}, & \cdots & \overline{z_1 z_k} \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ \overline{z_k z_1}, & \cdots & \overline{z_k z_k} \end{bmatrix} \tag{5.2}$$

of the set $\{z_1(nT), \ldots, z_k(nT)\}$ where, the shifted versions of the measured output are again defined as

$$z_i(nT) = z_{i-1}\big((n+1)T\big), \quad 2 < i < k .$$

and

$$\overline{z_i z_j} = \sum_{n=0}^{L} z_i(nT)\, z_j^*(nT) \quad . \tag{5.3}$$

where L is the number of sampling periods. Because of the noise term contained in $z_1(nT)$, the inner products which are the elements of the Gram matrix will also be corrupted by noise. Using (5.1),

$$\overline{z_i z_j} = \overline{y_i y_j} + \overline{y_i w_j} + \overline{w_i y_j} + \overline{w_i w_j} \quad . \tag{5.4}$$

The first term in the sum in (5.4) is the inner product which would be measured in the absence of noise and on which the identification technique, as described in Chapter 4, is based.

Knowledge of the statistics of the noise $w_1(nT)$ permits the calculation of the statistics of the inner products and thus can be used to evaluate the effects of noise on the inner products. The analysis of the effect of noise on the Gram determinant in (5.2), which serves to determine the number of poles and is the basis for the determination of the poles, is highly complicated because of the nature of a determinant. Consequently, the evaluation of the identification procedure and the determination of the parameters to be used in the identification is best carried out through simulation.

5.2 Simulation Program Characteristics

A listing of the simulation program used in the evaluation of the discrete iterative technique and in the determination of the optimum identification parameters is given in Appendix A. The functions performed by the program and the options available can be summarized as follows.

5.2.1 System Output

The program calculates the system output as

$$z_1(nT) = y_1^{(1)}(nT) + y_1^{(2)}(nT) + w(nT), \quad 0 \leq n \leq P \qquad (5.5)$$

where, the output due to one exponential at the input is, using (4.3) and (4.4), given by

$$y_1^{(j)}(nT) = \sum_{i=1}^{N} \frac{R_i}{b_j - p_i} e^{-np_iT} + \sum_{i=1}^{N} \frac{R_i}{p_i - b_j} e^{-nb_jT}, \quad j=1,2 \qquad (5.6)$$

and $w(nT)$ is a real, zero-mean, uncorrelated Gaussian random process.

The number of system poles N, the system poles $-p_1, \ldots -p_N$, the system residues $R_1, \ldots R_N$, the input poles $-b_1, -b_2$, the sampling interval T, the length of the interval of interest L and the standard deviation of the noise are inputs to the program. The choice of T and L is made at each step of the procedure in the manner described in Sections 5.3.2 and 6.1.

Several notes are in order:

- Since the simulated system is linear, the output in (5.1) which is the sum of the outputs due to single exponentials is the same as that due to the sum of the same two input exponentials.

- For simplicity, the input poles are constrained to be complex conjugates of each other. This permits to specify a real exponential input by setting $I_m\{b_1\} = I_m\{b_2\} = 0$; a sinusoidal input by setting $R_e\{b_1\} = R_e\{b_2\} = 0$; a decaying sinusoidal input by specifying b_1 and b_2 ($b_1=b_2^*$) as having non-zero real and imaginary parts.

- The noise sequence $w_1(nT)$ models thermal, measurement and quantization noise. For simplicity, it is assumed to be Gaussian even though $w_1(nT)$, and especially its quantization noise components can be expected to be non-Gaussian in practice. If dithering is used, however, the Gaussian assumption can be expected to be valid. Since the signal-to-noise ratio of $z_1(nT)$ is a function of the length of the interval over which it is defined, the noise standard deviation is defined relative to the maximum value of the signal component $y_1^{(1)}(nT) + y_2^{(1)}(nT)$ instead of being defined in terms of total signal power over the interval. This permits the comparison of results obtained using various interval lengths.

5.2.2 Identification Algorithms

In the case where knowledge of the input is not used and the input poles are identified together with the system poles, the program performs the operations described in Section 4.1.1. It

- forms by successive shifts the set

$$\{z_1(nT),\ldots,z_k(nT)\}, \quad 0 \leq n \leq L, \quad L + k \leq P \quad (5.7)$$

- calculates the inner products

$$\overline{z_i z_j} = \sum_{n=0}^{L} z_i(nT) z_j^*(nT)$$

- computes the Gram determinant

$$G_k = \det \begin{bmatrix} \overline{z_1 z_1} & \cdots & \overline{z_1 z_k} \\ \cdots & & \\ \overline{z_k z_1} & \cdots & \overline{z_k z_k} \end{bmatrix}$$

- finds the (i+1, i+1)th co-factors of the Gram determinant for $i=0,1,\ldots,k-1$

- forms the polynomial equation

$$\sum_{i=0}^{k-1} \lambda^{k-1-i} \left([G_k]_{(i+1,i+1)} \right)^{1/2} = 0$$

- finds the roots $\lambda_1,\ldots,\lambda_{k-1}$ of the polynomial

- determines the corresponding poles as

$$\ell_n \lambda_i / T, \quad i=1,2,\ldots,k-1 \ .$$

These calculations are performed for a specified range of values of k.

In the case where the input and/or previously identified poles are used as knowns, the set in (5.7) is modified to include these quantities. If $a_1, a_2, \ldots, a_I$ denote the known input and/or system poles the set is defined as

$$\{z_1(nT), \ldots, z_k(nT), e^{-na_1T}, \ldots, e^{-na_IT}\} \quad . \qquad (5.8)$$

The operations described above are then carried out for the set in (5.8). Using (4.23), the poles of interest are found from the roots of the polynomial

$$\sum_{i=0}^{k-1} \lambda^{k-1-i} \left([G_{k+I}]_{(i+1,i+1)}\right)^{1/2} = 0 \ .$$

5.3 Parameter Choices

Before carrying out the identification procedure it is necessary to choose the excitation, the sampling rate or interval and the interval over which the inner products which define the Gram matrix elements are calculated so as to insure optimum performance with a minimum of complexity.

5.3.1 Excitation

In the choice of the excitation the following factors must be considered:

(1) The excitation must be easy to implement in a laboratory.

(2) It must excite the system poles. Since the response due to the system is the sum of decaying real or complex exponentials (4.3), the identification is based on the transient system response. This transient response must be excited by the input.

(3) It must be adjustable so that, as a first step, it will essentially only excite the linear part of the system.

(4) It must result in an easily analyzable and identifiable output.

An input consisting of a sum of exponentials satisfies the four objectives stated above. We have selected the following elementary inputs: a single real exponential; a sinusoid; and a decaying sinusoid. They present the advantage of containing one or two input poles which can be readily identified using the discrete iterative approach.

Although they satisfy requirements (1), (3), and (4), the decaying and nondecaying sinusoidal inputs give rise to the following effects with respect to requirement (2):

First, suppose that the system contains one real pole - p_1 and is excited by a decaying sinusoid

$$x(t) = e^{-(b+j\omega)t} + e^{-(b-j\omega)t} \qquad (5.9)$$

$$= 2e^{-bt}\cos\omega t$$

Then, using (4.3), the system output is given by

$$y(t) = \left[\frac{R_1}{b+j\omega-p_1} + \frac{R_1}{b-j\omega-p_1} \right] e^{-p_1 t} \qquad (5.10)$$

$$+ \frac{R_1}{p_1-b-j\omega} e^{-(b+j\omega)t} + \frac{R_1}{p_1-b+j\omega} e^{-(b-j\omega)t} .$$

If b is equal to p_1, the system output becomes

$$y(t) = \frac{R_1}{-j\omega} e^{-(b+j\omega)t} + \frac{R_1}{j\omega} e^{-(b-j\omega)t} \qquad (5.11)$$

and the output does not contain a contribution due to the system pole. Thus, if it is desired to identify p_1, care should be taken not to use a decaying sinusoidal input with decay factor b which can equal p_1. Rather, we should select a value of b which is smaller than the lowest real poles to be identified. On the other hand, suppose that p_K is one of several system poles, that p_{K+1} is the next larger pole, and that p_K is the largest pole that has already been identified. Then, if the system is excited by $e^{-p_K t}\cos\omega t$, the pole at $-p_K$ will not contribute to the output. This will facilitate accurate identification of p_{K+1} by decreasing the contribution to the output of the known pole closest to the unknown pole.

The second effect that should be noted arises in the case where a sinusoidal excitation is used. Suppose that

$$x(t) = 2\cos\omega t \qquad (5.12)$$

$$= e^{j\omega t} + e^{-j\omega t}$$

and, for simplicity, that the system has one pole $-p_1$. Then, using (4.3), the system output becomes

$$y(t) = \left[\frac{R_1}{j\omega - p_1} + \frac{R_1}{-j\omega - p_1} \right] e^{-p_1 t} + \frac{2R_1 p_1}{p_1^2 + \omega^2} \cos\omega t. \qquad (5.13)$$

Note that the system pole, p_1 contributes a decaying exponential to the output whereas the input poles at $\pm j\omega$ contribute a pure sinusoid. It follows that the input contribution will not decay with time and will eventually tend to be greater than the system pole contribution, thus obscuring the pole contribution. Although we give an example in Section 6.2.4 where a pure sinusoidal input was used to yield a successful identification, this is not generally true. In some cases, a steady sinusoidal excitation can be impractical for detection of poles in certain frequency regions.

5.3.2 Correlation Interval and Sampling Rate

An indication of the effect of the sampling rate and the correlation interval can be obtained from the following considerations.

Using (5.4), examine the inner product over an interval [0,L]

$$\overline{z_1 z_1} = \overline{y_1 y_1} + \overline{y_1 w_1} + \overline{w_1 y_1} + \sum_{n=0}^{L} w_1^2(nT) \; . \qquad (5.14)$$

Suppose that the noise $w_1(nT)$ is a zero-mean, stationary, uncorrelated Gaussian random sequence with variance σ^2. Then the expected value of the inner product is equal to

$$E\{\overline{z_1 z_1}\} = \overline{y_1 y_1} + (L+1)\sigma^2 \tag{5.15}$$

where $\overline{y_1 y_1}$ represents the signal component and $(L+1)\sigma^2$ is the bias introduced by the noise. Similarly, the variance of $\overline{z_1 z_1}$ can be expressed as

$$\text{Var}[\overline{z_1 z_1}] = 2\sigma^2 \overline{y_1 y_1} + 2(L+1)\sigma^4. \tag{5.16}$$

Note that both the mean and the variance of the inner product contain a noise component which increases linearly with the interval length L. It follows that it is desirable to use a short correlation interval over which $\overline{y_1 y_1}$ includes a large portion of the total signal power and the noise contribution is as small as possible.

The effects of the sampling interval or rate can be observed by considering the following example. Suppose that the noisy system output is given by

$$z_1(nT) = e^{-nT} + e^{-5nT} + w_1(nT) \tag{5.17}$$

and it is desired to identify the two poles 1 and 5. Suppose that a high sampling rate equivalent to ten times the highest signal frequency (5) contained in (5.17) is used. Then,

$$T = \frac{1}{50} = .02 \tag{5.18}$$

and

$$\begin{aligned} z_1(nT) &= (e^{-.02})^n + (e^{-.1})^n + w_1(nT) \\ &= (.98)^n + (.905)^n + w_1(nT) \\ &= y_1(nT) + w_1(nT) \end{aligned} \tag{5.19}$$

where

$$y_1(nT) = (.98)^n + (.905)^n \; .$$

Consider the Gram determinant

$$|G_2| = \det \begin{bmatrix} \overline{y_1 y_1} + N_{11} & \overline{y_1 y_2} + N_{12} \\ \overline{y_2 y_1} + N_{21} & \overline{y_2 y_2} + N_{22} \end{bmatrix} \qquad (5.20)$$

where, N_{ij}, i,j=1,2 represent the noise contribution to the Gram determinant elements. If the correlation interval is sufficiently long so that the $\overline{y_i y_j}$, i,j=1,2, contain all the signal power, for $p_1 = -1$, $p_2 = -5$, and T = .02, the Gram determinant in (5.20) is equal to

$$|G_2| = \det \begin{bmatrix} 48.46 + N_{11} & 46.41 + N_{12} \\ 46.41 + N_{12} & 44.46 + N_{22} \end{bmatrix} \; . \qquad (5.21)$$

Note that the signal components in the matrix elements in (5.21) are within 10% of each other. This indicates that the determinant is close to being ill-conditioned and will be severely affected by the noise components.

Suppose that a sampling rate equivalent to the highest frequency contained in (5.17) is used. Then

$$T = \frac{1}{5} = .2$$

and

$$z_1(nT) = (e^{-.2})^n + (e^{-1})^n + w_1(nT)$$

$$= (.82)^n + (.37)^n + w_1(nT) \quad . \tag{5.22}$$

Then, for a large correlation interval, the Gram determinant in (5.20) is equal to

$$|G_2| = \det \begin{bmatrix} 7.08 + N_{11} & 4.64 + N_{12} \\ 4.64 + N_{12} & 3.08 + N_{22} \end{bmatrix} \tag{5.23}$$

The relative spread of the element signal components in (5.23) is much greater than in (5.21). It can be expected therefore that (5.23) will be better behaved than (5.21) in the presence of noise and would lead to a more accurate identification. Note that identification is not based on the $[G_2]$ matrix in (5.20) but would be based on the $[G_3]$ matrix defined in (4.10). The $[G_2]$ matrix is then one of the co-factors of the $[G_3]$ matrix. Conclusions similar to the above can be drawn regarding the $[G_3]$ matrices resulting from using a sampling interval of .02 and .2.

The conclusions as to the desirable sampling interval and correlation interval can be checked using the simulation results presented in Table 5.1. These results were obtained for the output given in (5.12) without additive noise and for various levels

TABLE 5.1

IDENTIFICATION RESULTS FOR TWO POLE CASE

L	T	p_1	p_2	$\hat{p}_1$	$\hat{p}_2$	σ/y_{max}
10	1	1	2	.998	2.0005	0
10	1	1	2	COMPLEX	CONJUGATE	.01
10	1	1	2	COMPLEX	CONJUGATE	.005
10	1	1	2	1.074	1.797	.001
50	1	1	2	COMPLEX	CONJUGATE	.001
10	0.5	1	2	1.0049	1.98	.001
50	0.5	1	2	1.028	1.94	.001
10	0.5	1	2	1.11	1.78	.005
50	0.5	1	2	COMPLEX	CONJUGATE	.005
50	0.1	1	2	1.05	1.979	.001
10	0.05	1	2	1.016	1.98	0
10	0.05	1	2	1.026	2.065	0.001
10	0.05	1	2	COMPLEX	CONJUGATE	0.005
10	0.2	1	5	99988	5.00013	0
10	0.2	1	5	1.0048	4.968	0.001
10	0.2	1	5	1.085	4.727	0.005
10	0.2	1	5	1.33	4.2	0.01
50	0.2	1	5	COMPLEX	CONJUGATE	0.01
10	0.05	1	5	COMPLEX	CONJUGATE	0.01

L = CORRELATION INTERVAL

T = SAMPLING INTERVAL

p_1, p_2 SYSTEM POLES

$\hat{p}_1, \hat{p}_2$ = ESTIMATED POLES

σ/y_{max} STANDARD DEVIATION/MAX. OF OUTPUT OF ADDITIVE GAUSSIAN NOISE

of additive noise. The noise present for $\sigma=0$ is due to computer round-off errors. The best results can be observed to be obtained for a small correlation interval length and a large sampling interval (sampling rate approximately equal to the highest frequency contained in the output).

For complex conjugate poles, two cases present themselves. If the imaginary part of the pole is smaller than the real part the conclusions made above as to the desirable sampling and correlation intervals remain true. If the imaginary part is larger than the real part the sampling rate must be proportional to the imaginary part in order to prevent aliasing.

5.3.3 Noise Reduction

In a practical application, the noise level may, in some cases, be too high for the proper operation of the identification technique. Then several methods may be used to diminish the effects of noise.

If the primary component of noise and signal output are uncorrelated and output noise is zero mean useful methods of noise reduction include the measurement and the averaging of responses to periodic repetitions of the input and the averaging of the Gram matrix elements obtained from repeated measurements.

In cases where the largest measurement error is due to quantization error induced by the A/D converter, dithering of the A/D input signal could be especially helpful. This is accomplished by adding a pseudorandom noise to the output of the black box to cause a uniformly distributed amplitude dither before the A/D converter. Digital samples at the output of the A/D converter can then be taken repetitively and corresponding data points averaged to eliminate effects of the dither and establish the true value of the signal. If quantization levels and timing re-

mained constant for all the repeated responses, this method could be used to improve the A/D accuracy. Alternatively, the timing can be dithered, within a small range so that samples are first taken on one set of points on the waveform and then on another. In this manner, the quantization error can be averaged out to a certain extent.

The noise effects will manifest themselves during the practical implementation of the identification technique. The adequacy of the proposed noise reduction techniques can only be truly ascertained in practice where constraints as to timing effects are included and the predominant noise sources are identified.

5.4 Determination of the Number of Poles

As described in Section 4.1.1 the number of system poles can be determined from the order of the lowest Gram determinant to vanish. That is, if K' is the lowest value of K for which G_K approaches zero, the number of poles is given by

$$N = K' - 1.$$

In the case where the system output is not corrupted by noise, this result is valid as illustrated by the example presented in Figure 5.1. The parameters are, in this case:

System Poles:	.033, .080
System Residues:	-.06112, .036
Input Pole:	.12
Sampling Interval T:	5
Correlation Interval L:	10.

In this example, there are two system poles and one input pole. The system output, therefore, contains three exponentials. From Figure 5.1, the sudden drop in the value of the Gram determinant is observed at K'=4 and the number of poles is therefore accurately determined as 3. Note that because of computer round-off errors the Gram determinant never equals exactly zero.

When the system output is corrupted by noise, however, the determination of the number of poles from the value of the Gram determinant for increasing values of K becomes more problematical. This is illustrated in Figure 5.2 which was obtained for the same parameters as those used for Figure 5.1 but with additive noise whose standard deviation ranged from .1% to 5% of the maximum value of the output. Clearly, in the presence of a substantial noise component the determination of the number of poles is very difficult and from the results of Figure 5.2 would appear to be impossible for noise levels higher than .1% of the maximum output.

This suggests the advisibility of a different criterion for determining N. In particular, we explored the possibility of identifying the system input, which is known, as a check on whether the number of unknown system poles has been properly identified.

We believe that such an approach is possible because when the number of poles is incorrectly specified, the poles which are identified are, in general, very different from the true poles and vary widely.

As discussed in Chapter 7, the identified linear transfer function may be relatively close, in a global sense, to the true transfer function even if the location of the identified poles are quite inaccurate. For this reason, we cannot be satisfied simply by examining the linear transfer function and eventually the acceptability of results will depend on the effect of linear pole location errors on the identification of the nonlinear transfer functions themselves.

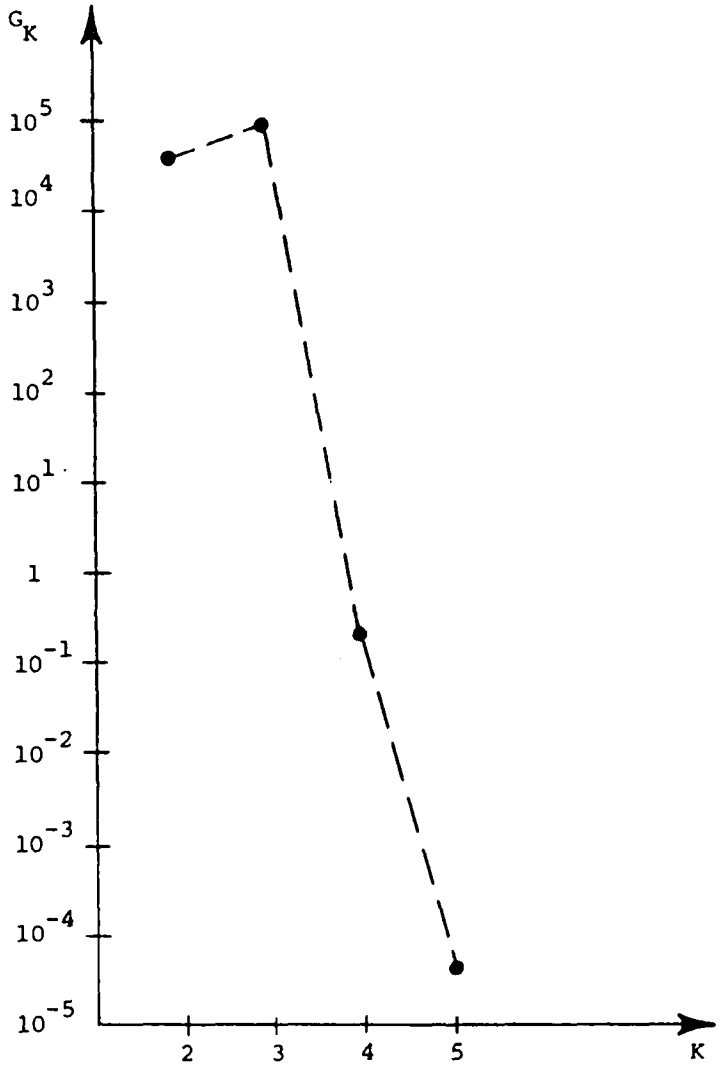


Figure 5.1 Determination of Number of Poles in Absence of Noise for Three Pole Output (points connected for convenience)

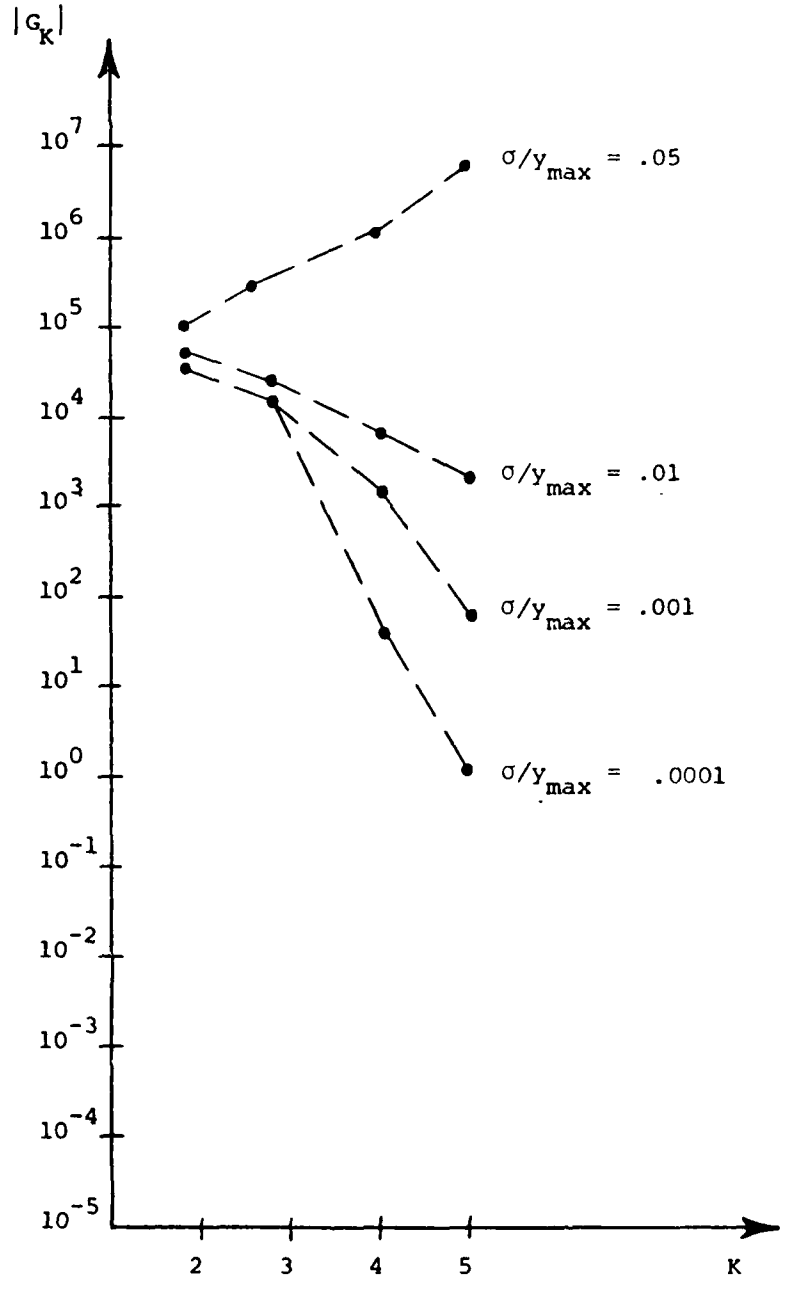


Figure 5.2 Determination of Number of Poles in Presence of Noise for Three Pole Output. (points connected for convenience)

SECTION 6
SIMULATION RESULTS

6.1 Introduction

In the identification of a system, two cases may present themselves. In the first case, the regions where poles are present and the number of poles in each region are known a priori. In the second case, this information is not available and it is desired to find the poles in a region of interest, let us say between frequencies f_1 and f_2. In the first case, as discussed in Section 4.2, the iterative aspect of the identification may not be needed. The identification technique then becomes very similar to the one proposed by Jain and Osman [1980]. The additional feature of the proposed technique is the possibility for checking the accuracy of the identified poles by taking them as knowns and identifying the input poles.

In the case where the regions where the poles are located and/or their number is not known and it is desired to identify the system between frequencies f_1 and f_2, the steps of the iterative discrete identification procedure, as described in Sections 4.1 and 4.2, are as follows for a real exponential input:

1. Excite the system with

$$x(t) = e^{-bt},$$

b small (smaller than f_1 the smallest system pole expected to be found). Using a sampling interval in the approximate range $\frac{1}{5b} < T < \frac{1}{b}$, identify one pole using (4.16) with N + I = 1. Since at least one pole, the input pole, contributes to the output, the Gram determinant is not ill-posed for N + I = 1.

2. If the identified pole is close to -b, increase b, decrease proportionally T and repeat (1). Based on experience with several numerical examples, in this step and in step 5 below, the identified input pole is judged to be close to the known input pole if it is within 10% of the true value in low noise cases and 40% in higher noise cases. Low noise conditions are said to exist if $0 < \sigma/y_{max} < .005$; high noise conditions are said to exist if $.005 < \sigma/y_{max} < .05$.

3. If the identified pole is not close to -b, use -b as a known and identify the pole(s) contributing to the system output using (4.24). This can be done in conjunction with the information as to the number of poles given by the value of G_K for increasing K.

4. Check the identified poles by using them as knowns and looking for the input pole using (4.24) with N=1.

5. When the identification is found to be satisfactory, use the identified pole(s) as knowns, increase b, decrease proportionally T and again look for the input.

6. Continue this procedure until all poles in the region of interest are determined.

As noted in Section 5.1, a decaying sinusoidal input, a sinusoidal input or an input consisting of the sum of exponentials could also be used.

In certain cases, the identification may not be found to be satisfactory for any of the identified poles. This may arise for three principal reasons. The noise may be too large for satisfactory identification. In this case, the noise would need to be diminished before proceeding with the identification. In prac-

tice, this may be accomplished by sampling of repeated replicas of the output and averaging the samples.

A second cause for unsatisfactory identified poles may be poorly chosen values of b and T. This would occur, for example, if b is too large ($T \approx 1/b$). Then, too many poles may be identified at once or the poles may be much smaller than b, resulting in a poorly conditioned Gram determinant.

Finally, unsatisfactory results may be obtained if complex conjugate poles, say $-p_1+jw_1$, $-p_1-jw$, with $w_1 >> p_1$ are being identified with $T \approx 1/p_1$. Sampling at the rate 1/T would then result in substantial aliasing leading to erroneous results. These could be improved by decreasing T.

Note that these three cases may, just as well, arise in the non-iterative pencil-of-functions method. Note also that in the non-iterative pencil-of-functions method the sampling rates to be used are not known unless the regions where poles are located are known a priori. One sampling rate corresponding to the highest frequency of interest f_2 cannot be used for wideband systems because of the large dynamic range of the possible poles between f_1 and f_2. In all cases where the number of poles is not known the pencil-of-functions method involves iterations. Only one input is used but iterations are performed in order to estimate the number of poles.

The general question of what represents a satisfactory or unsatisfactory identified pole is closely tied to the problem of defining a quality criterion for the identification results. This question will be addressed in more detail in Section 7. In the present study, we concentrated on the location of poles only and not location of zeros or the value of residues. Both poles and zeros are needed to judge the accuracy of transfer function identification. In view of this, the most obvious criterion of accuracy is, of necessity, simply the percentage mean square error in the value of the identified poles.

6.2 Simulation Results

6.2.1 One Pole Case

In order to evaluate the effect of noise on the identification of a very elementary system consider the one pole case where the impulse response is given by:

$$h(t) = e^{-t}.$$

At time $t=0$, excite the system with a single real exponential

$$x(t) = e^{-bt}.$$

The system output is then given by

$$y(t) = \frac{1}{b-1}\left[e^{-t} - e^{-bt}\right].$$

If the input is used as a known, one pole needs to be identified. If knowledge of the input is not used, the problem is equivalent to that of identifying two poles. The number of poles to be identified was assumed known in both cases.

The results of the simulation are presented in Table 6.1. A correlation interval of $L=10$ (10 samples) was used in all cases. The identified parameters are denoted as $\hat{b}$ and $\hat{p}$. The following conclusions can be drawn from these results.

- Noise tolerance is considerable when only one pole is being identified. Noise has much more effect in the two pole case. In general, identification quality can be expected to be inversely proportional to the number of poles being identified. The best results are obtained when b is small; that is, for an input approaching a step.

TABLE 6.1

IDENTIFICATION RESULTS FOR ONE POLE CASE

IN THE PRESENCE OF NOISE

CORRELATION INTERVAL L=10

T	σ/y_{max}	b	p_1	$\hat{p}_1$
1.	.005	.001	1.	1.013
1.	.05	.001	1.	.996
1.	.5	.001	1.	.72
1.	1.	.001	1.	.67
.5	.005	2.0	1.	.988
.5	.05	2.0	1.	.87
.5	.5	2.0	1.	.303
.2	.005	5.0	1.	.994
.2	.05	5.0	1.	.94
.2	.5	5.0	1.	.523
.002	.05	500.0	1.	.96
.002	.05	500.0	1.	.62

(a) Knowledge of input used, b not identified.

T	σ/y_{max}	b	p_1	$\hat{b}$	$\hat{p}_1$
.5	.005	2.	1.	1.78	1.11
.5	.01	2.	1.	1.36	± j .4
.5	.05	2.	1.	.76	± j 1.5
.2	.005	5.	1.	4.73	1.085
.2	.01	5.	1.	4.2	1.33
.2	.05	5.	1.	1.7	± j 2.9
.002	.005	500.	1.	474.68	8.61
.002	.01	500.	1.	428.	28.16
.001	.005	1000.	1.	949.2	16.3
.001	.01	1000.	1.	855.2	55.7

(b) Knowledge of input not used, b identified.

- In the identification of two poles, noise tends to generate complex conjugate poles when the two poles are close together; it results in relatively large errors when they are far apart.

6.2.2 Wideband Four-Pole Transistor Amplifiers Circuit

In this section the discrete iterative approach is applied to the wideband four-pole transistor amplifier circuit previously studied by Jain and Osman [1979]. The schematic diagram of this circuit is shown in Figure 6.1. The transfer function of the amplifier is equal to

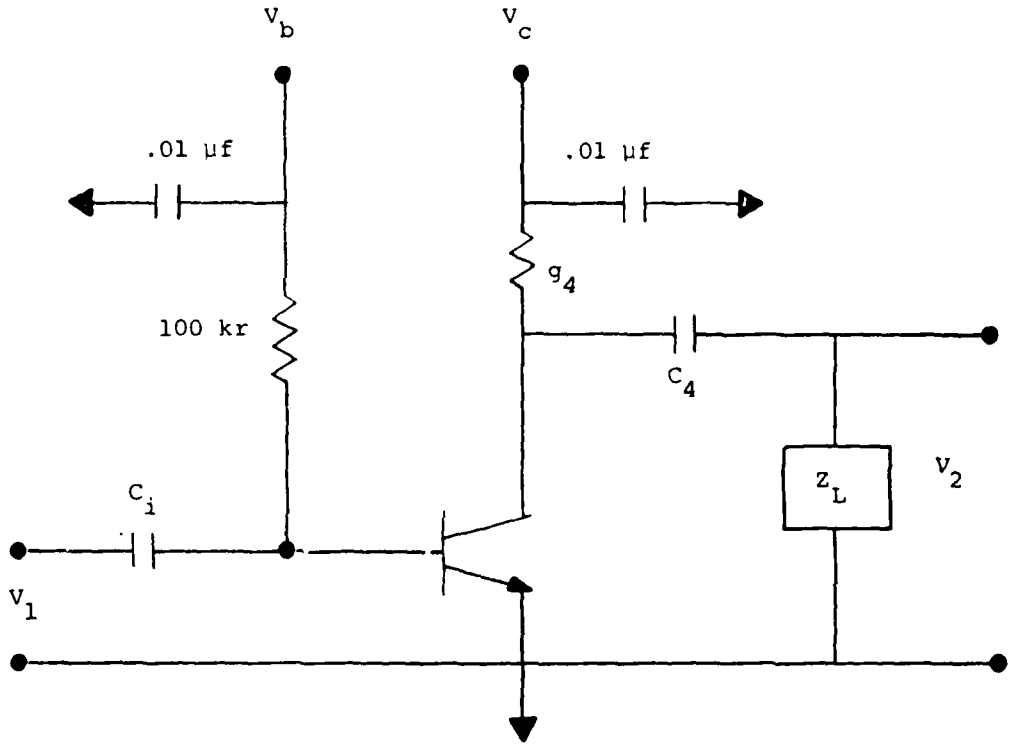
$$H(s) = \frac{8(10^7)s^2(S - 8000(10^6))}{(S + .033(10^6))(S + .080(10^6))(S + 25.2(10^6))(S + 1205.1(10^6))} .$$

It contains two low-frequency poles, one mid-frequency pole and one high-frequency pole. The system frequency response is shown in Figure 6.2.

The iterative identification procedure steps are as follows for, initially, the case where no noise is added to the output. Note that the inner product correlation interval, L, is equal to 10 in all cases. It is supposed that the normalized frequency band of interest is between .001 and 1500.

1. Excite the system with an exponential with a long time constant, say, $T_c = 1000$,

$$x(nT) = e^{-(.001)nT}.$$



C_1 = 0.01 μf C_3 = 5pf g_1 = 4 mΩ g_3 = 40 mr

C_2 = 50.0 pf C_4 = 0.01 μf g_2 = 1 mΩ g_4 = 0.5 mr

Z_L = 1 KΩ

Figure 6.1 Wide-band Transistor Amplifier Circuit
(Jain and Osman [1979])

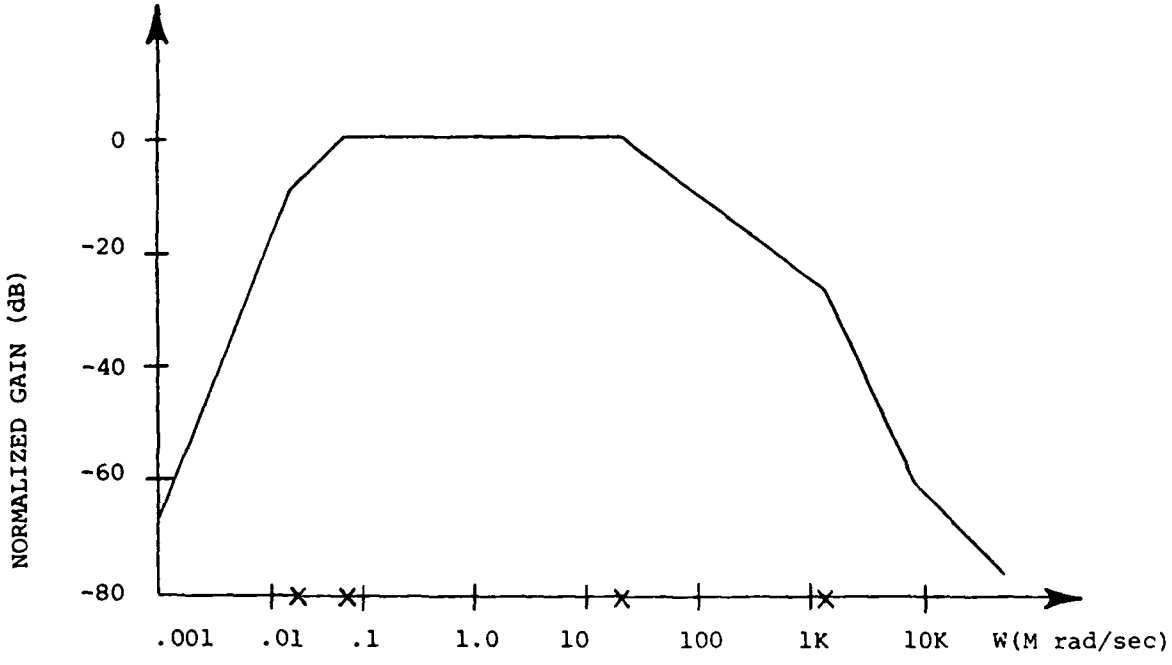


Figure 6.2 Frequency Response of Wide-band Transistor Amplifier Circuit in Figure 6.1

Use a sampling period of T=1000, so that $T/T_c=1$. Do not use x(nT) as a known in forming the Gram determinant. Identifying one pole, we find,

$$\hat{b} = .00099999$$

which is very close to the input. It can be concluded that since the system poles did not perturb the input time constant they are all larger than .001.

2. Increasing the time constant and decreasing T, excite the system with

$$x(nT) = e^{-(.01)nT}$$

with T=100. Identify the input pole using (4.16) with N + I = 1; we get,

$$\hat{b} = .02 .$$

3. Since .02 is different from .01, it is concluded that system poles perturbed the identification of input exponential. Look for the dominant system pole (the one closest to .01) by using the same excitation and T but now using the input as a known in forming the Gram determinant in (4.24) with N + 1. Find,

$$\hat{p}_1 = .0328 .$$

4. Test by exciting the system with

$$x(nT) = e^{-(.01)nT}, \quad T=100$$

and using .0328 in forming the Gram determinant and looking for the input time constant. Find,

$$\hat{b} = .00998$$

which is close to the input time constant which is, of course, known. It is concluded that $\hat{p}_1 = .0328$ is a good estimate of the dominant pole, that is, the pole closest to b.

5. The procedure is now repeated at each step looking successively for the next higher frequency pole. The following results are obtained:

i. Input exponent b = .05, T=20, .0328 used as known, find $\hat{b} = .0012$. Since $\hat{b}$ is not close to b, another pole is present in the vicinity of .05.

ii. Input exponent b = .05, T=20, .0328 and .05 used as knowns. Find $\hat{p}_2 = .0803$.

iii. Input exponent b = .05, T=20, .0328 and .0803 used as knowns. Find $\hat{b}$= .04998 which is close to input exponent. The estimate $\hat{p}_2 = .0803$ is thus assumed to be correct.

iv. Input exponent b = 1., T=1., .0328 and .0803 used as knowns. Find .9999. Since, $\hat{b}$ is close to b, we conclude that the system contains no poles between .0803 and approximately 3.

v. Input exponent b = 106, T=.1, .0328 and .0803 used as knowns. Find $\hat{b}$ = 7.7. The input is perturbed by a system pole.

vi. Input exponent 10., T=.1, .0328, .0803, 10. used as knowns. Find $\hat{p}_3$ = 25.12 .

vii. Input exponent 10., T=.1, .0328, .0803, 25.12 used as knowns. Find 10.006. Thus, $\hat{p}_3$ is judged to be a good estimate of the third pole.

viii. Input exponent 100., T=.01, .0803, 25.12 used as knowns. Find 99.9997.

Hence, the system has no poles between 25.12 and approximately 300. Note that only .0803, and not both .0803 and .0328, was used as a known. Since (.0803)(.01) ≈ (.0328)(.01) use of both results in an ill-conditioned Gram determinant.

ix. Input exponent 1000., T=.001, .0803 and 25.12 used as knowns. Find 600.

x. Input exponent 1000., T=.001, .0803, 25.12, and 1000 used as knowns. Find $\hat{p}_4$ = 1204.66 . .

xi. The estimate can be refined. Input exponent 1204.66, T=.001, .0803, 25.12 and 1204.66 used as knowns. Find $\hat{p}_4$ = 1205.1 .

xii. Input 1000., Δt=.001, use 1205.1, 25.12, .0803 as knowns. Find 999.87. Thus, $\hat{p}_4$ is a good estimate of the fourth pole.

The procedure was repeated for the cases where the output was corrupted by additive zero-mean white Gaussian noise with standard deviation equal to .05 and .5 of the maximum value of the noise free output. The results appear in Table 6.2.

For comparison, the results obtained by Jain and Osman [1979] in the absence of noise are also included in the table.

Note that the identification quality remains good for a noise standard deviation equal to 5% of the maximum output. The average percentage error in pole location is .25%, 5.6% and 18.5%, respectively, for $\sigma/y_{max} = 0$, .005, and .05 where the percentage error of the identified pole $\hat{p}$ with respect to the true pole p is defined as

$$E(\hat{p}) = \frac{|p-\hat{p}|}{|p|} \times 100 .$$

The results obtained using the discrete iterative procedure with an additive noise of $\sigma/y_{max} = .005$ are essentially of the same quality as those given by Jain and Osman [1979] in the absence of noise.

6.2.3 One Real, Two Complex Pole Case

Consider a system whose transfer function is given by

$$H(S) = \frac{(S^2 + .08)}{(S + .1625)(S + .0823 + j .306)(S + .0823 - j .306)} .$$

TABLE 6.2

IDENTIFICATION OF WIDEBAND TRANSISTOR AMPLIFIER CIRCUIT

RESULTS:

σ/y_{max}	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$
0.	.0328	.0803	25.12	1205.1
.005	.0324	.0801	25.5	1169.
.05	.0216	.0713	21.05	1053.2
JAIN & OSMAN .0	.034	.075	24.9	1139.5

TRUE VALUES: p_1=.033, p_2=.080, p_3=25.2, p_4=1205.1

The system contains three poles: one real pole at -.1625 and a complex conjugate pair at -.0823 ± j .306. Note that the imaginary part of the complex poles is larger than the real part. The simulation results for this case are presented in Table 6.3 for a noise standard deviation equal to 0, .5%, 1% and 5% of the maximum noise free output. The identified poles can be observed to be very close to the true values in the first three cases. Only for the highest noise level do the identification poles really differ from the system poles. But, even in this case where the noise level is high, the results are of reasonable accuracy. The average percentage error in pole location is, in this case, .26%, 1.8%, 4.4%, and 20.6%, respectively, for σ/y_{max} = 0, .005, .01 and .05.

In order to illustrate further the performance of the discrete iterative approach and to demonstrate the use of a decaying sinusoidal input it is instructive to examine the steps used to obtain these results. It is supposed in this case that the frequency band of interest is between .01 and .15.

The system is excited by

$$x(nT) = e^{-(b_1+jb_2)nT} + e^{-(b_1-jb_2)nT} .$$

For σ=0, the steps are:

- Input T=100, $b_1=b_2=.005$. Looking for two poles, find $\hat{b}_1=.0050002$, $\hat{b}_2=.004999$. Therefore, conclude that no system poles contribute to the output.

- Input T=25, $b_1=b_2=.02$. Looking for two poles, find $\hat{b}_1=.019785$, $\hat{b}_2=.019856$. Again, conclude that no system poles contribute to the output.

TABLE 6.3

IDENTIFICATION RESULTS FOR THREE POLE CASE

(ONE REAL POLE AND COMPLEX CONJUGATE PAIR)

σ/y_{max}	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$
0.	.1613	.082352 + j .30607	.082352 - j .30607
.005	.1577	.08106 + j .3094	.08106 - j .3094
.01	.147	.0866 + j .31	.0866 - j .31
.05	.135	.037 + j .361	.037 - j .361

TRUE POLES ARE: $p_1 = .1625$, $p_2 = .0823 + j\ .306$

$p_3 = .0823 - j\ .306$

- Input T=5, $b_1=b_2=.1$. Looking for two poles, find $\hat{b}_1=.02$, $\hat{b}_2=.13$. Thus system poles contribute to the system output. However, looking for five poles, find .16, .1 ± j.1, .0823 ± j .306 which contains the input poles.

Conclude that .16, .0823 ± j .306 could be the system poles.

- As a check input T=2.5, $b_1=b_2=.2$ and use .16, .0823 + j .306, .0823 - j .306 as knowns. Looking for two poles, find $\hat{b}_1 = .19958$, $\hat{b}_2 = .19972$. Thus, conclude that .0823 ± j .306, .16 represent the system poles.

Similarly, for $\sigma=.005$, the steps are:

- T=100, $b_1=b_2=.005$. Find $\hat{b}_1 = .00501$, $\hat{b}_2 = .00501$.
- T=25, $b_1=b_2=.02$. Find $\hat{b}_1 = .0198$, $\hat{b}_2 = .0199$.
- T=5, $b_1=b_2=.1$. Find $\hat{b}_1 = .02$, $\hat{b}_2 = .13$.
- T=5, $b_1=b_2=.1$. Use input .1 + j .1, .1 - j.1 as knowns.

Find .17 for N=1; .074 ± j .256 for N=2; .18, .0859 ± j .308 for N=3.

- T=5, $b_1=b_2=.1$, use .17 as known, find $\hat{b}_1 = .0656$, $\hat{b}_2 = .204$. Conclude that .17 does not represent the system poles.

- T=5, $b_1=b_2=.1$, use .074 + j .256, .074 - j .256 as knowns. Find $\hat{b}_1 = .0828$, $\hat{b}_2 = .107$. Conclude that .074 ± j .256 may represent the system poles.

- T=5, $b_1=b_2=.1$, use .18, .0859 + j .308, .0859 - j .308 as knowns. Find $\hat{b}_1 = .1008$, $\hat{b}_2 = .106$. Thus conclude that .18, .0859 ± j .308 represent the system poles.

This same procedure applies to the case where $\sigma/y_{max}=.01$. The noise level is, however, in this case, sufficiently high not to permit to properly check for the accuracy of the identified poles using a decaying sinusoidal input. Instead, using the real input

$$x(nT) = e^{-nbT}$$

the steps are as follows:

- Input T=100, b=.01. Find $\hat{b}=.0074$.
- Input T=20, b=.05. Find $\hat{b}=.035$.
- Input T=10, b=.1. Find $\hat{b}=.0198$.
- Input T=10, b=.1. Use knowledge of input.

Find .135

- Check inputting T=10, b=.1 and using .135.

Find $\hat{b}=.119$. Thus, accept $\hat{p}_1=.135$.

- Input T=2, b=.5 and use .135. Find .0288.
- Input T=2, b=.5 and use .135 and .5.

Find .037 ± j .361.

- Check by inputting T=2, b=.5 and using .135, .037 ± j .361. Find $\hat{b}$ = .21. The identified $\hat{b}$ indicates that the identified poles are in error but is judged sufficiently close to b so that they are acceptable.

Note that the case treated in this section is more difficult than the four pole case identified in Section 6.2.1. Although the system only contains three poles, these are not as readily separable as in the wideband four pole case.

6.2.4 Narrowband Four Pole Filter

As a final example, consider the case where the system transfer function is given by:

$$H(S) = \frac{S^2}{(S+1)(S+2)(S+3)(S+4)}$$

which represents a narrowband filter. The example serves to demonstrate the capability of the identification technique when the effects of the various poles cannot be readily isolated through use of different sampling intervals.

The identification results are presented in Table 6.4. Their accuracy is acceptable for noise levels up to 5% of the maximum system output. The average percentage error in pole location ranges from 2.35% in the noiseless case to 13% in the case where σ/y_{max} = .05. Two sets of results are presented for σ/y_{max}=.01. The first set was obtained using a decaying sinusoidal input; the second using a nondecaying sinusoidal input. The identification was performed by identifying one system pole

at a time. The simultaneous identification of all four poles would result in complex conjugate, not real, identified poles.

The procedure used to generate the results in Table 6.4 can be illustrated by examining the steps involved in obtaining the results for σ/y_{max}=.01 using the nondecaying sinusoidal input

$$x(nT) = 2 \cos n\omega T$$
$$= e^{-jn\omega T} + e^{jn\omega T} .$$

The frequency band of interest is supposed to be between .01 and 5.

- Input T=100, ω=.005. Identify two poles: .000036 ± j .0049 ≈ ± jω.

- Input T=10, ω=.05. Identify two poles. .0013 ± j .048 ≈ ± jω.

- Input T=5, ω=.1. Identify two poles: .17, .019≠ ± jω.

- Input T=5, ω=.1 and use knowledge of the input poles ± j .1. Identifying one pole, find $\hat{p}_1$ = .59.

- Test inputting T=5, ω=.1 and using .59 as known. Identify two poles: .0038 ± j .098 ≈ ± jω. Thus, accept $\hat{p}_1$ = .59.

- Input T=2.5, ω=.2, use .59 as known. Identify two poles: .025 ± j .196 ≈ ± jω.

- Input T=1., ω=.5, use .59 as known. Identify two poles: .34 ± j .37 ≠ ± jω.

TABLE 6.4

NARROWBAND FOUR POLE FILTER

σ/y_{max}	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$
0.	1.019	1.93	3.09	3.96
0.01(1)	.696	2.01	2.6	4.18
.01(2)	.59	2.05	2.6	3.95
0.05	.7036	1.815	3.278	3.897

(1) Decaying sinusoidal input.

(2) Nondecaying sinusoidal input.

- Input T=1., ω=.5, use .59 and input poles $\pm$ j .5 as knowns. Identifying one pole, find $\hat{p}_2 = 2.05$.

- Test inputting T=..., ω=.5 and using .59 and 2.05 as knowns. Identify two poles: .015 $\pm$ j .498 $\approx \pm j\omega$. Thus, accept $\hat{p}_2 = 2.05$.

- The remaining two poles are found by continuing this procedure.

SECTION 7
IDENTIFICATION PERFORMANCE ASSESSMENT

7.1 Introduction

In order to assess the practicality of the identification technique, it is very important to develop an accurate method of measuring the quality of the identification results. Two fundamental approaches to this problem are possible: a measure of the accuracy of the identified parameters (poles and zeros/ residues) or a global measure of the difference between the impulse responses or transfer functions of the identified and actual systems.

The choice of the quality criterion is, of course, dictated in practice by the purpose to which the identification results will be used. In a control systems application, the accuracy of the identified parameters may be very important in the design of a compensator. In a speech synthesis system, on the other hand, an accurate identification of the impulse response of the synthesis filter may be sufficient.

Note that the parameter accuracy approach is much more restrictive than the global approach. An identified pole can only be accurate if it is close to the true pole. The representation of an impulse response as a function of poles and residues is not unique, however. An identified impulse response may be accurate even though the underlying poles and residues are quite far from the true values.

The purpose of the linear system identification carried out in this effort is to serve as a first step to the characterization of the nonlinear transfer functions of the system. A final choice of a quality criterion for this problem cannot therefore be made until the impact of this choice on nonlinear transfer function identification is analyzed.

The present effort was limited to the identification of the poles of the linear transfer function. As such, only the parameter accuracy of the identified poles could be determined. A choice between parameter and global accuracy measures depends on their respective impact on the identification of the nonlinear parts of the system. Therefore, a choice cannot be made at this time and only the principles of the parameter and global accuracy measures are discussed in the following.

7.2 Parameter Accuracy Measure

A reasonable parameter accuracy measure is the ratio of the difference squared between the true and identified parameter to the true parameter or the percentage error in pole location. Thus, if p and $\hat{p}$ represent, respectively, the true and identified values of a pole, the parameter accuracy of $\hat{p}$ can be defined as:

$$p_a(\hat{p}) = \frac{|p-\hat{p}|^2}{|p|^2} \quad . \tag{7.1}$$

and the percentage error as:

$$P(\hat{p}) = \frac{|p-\hat{p}|}{|p|} \times 100 \quad . \tag{7.2}$$

Note that in the preceding, the error measure in (7.2) was used. Such an accuracy measure is easy to implement during the simulation testing of the identification procedure. It has two major disadvantages, however. If the order of the system is erroneously determined, it does not readily permit a measure of the effect of extra or missing poles and residues. Secondly, in the practical application of the identification technique the true parameter values are unknown. Consequently, such an accuracy measure cannot be applied.

7.3 Global Accuracy Measure

The standard global accuracy figure measures the distance between the impulse responses of the true and identified impulse responses normalized with respect to the total energy in the system impulse response:

$$G_a(\hat{h}(t)) = \int_{-\infty}^{\infty} \left|h(t) - \hat{h}(t)\right|^2 dt \Big/ \int_{-\infty}^{\infty} \left|h(t)\right|^2 dt. \qquad (7.3)$$

Equivalently, the accuracy figure in (7.3) can be expressed in the frequency domain as

$$G_a(\hat{h}(t)) = G_a(\hat{H}(f)) = \int_{-\infty}^{\infty} \left|H(f) - \hat{H}(f)\right|^2 df \Big/ \int_{-\infty}^{\infty} |H(f)|^2 df. \qquad (7.4)$$

Such a global accuracy figure is only suitable during the simulation evaluation of the identification technique. In practice the true impulse response or frequency response are not known. The true system response can be measured but not to an impulse since a perfect impulse cannot be generated in practice. Measurement of the true frequency response would not be practical.

The global accuracy measure in (7.3) can, however, be readily modified to account for a non-impulsive input used to measure the true system output. This is accomplished by weighting the performance measure by the test input g(t). A practically implementable test input which is as close as possible to an impulse should be chosen. Note that the test input should be different from the input used to identify the system. Consequently, the weighed performance measure is defined as:

$$P_a(\hat{h}(t)) = \frac{\int_{-\infty}^{\infty} [[h(t)-\hat{h}(t)] * x(t)]^2 dt}{\int_{-\infty}^{\infty} [h(t) * x(t)]^2 dt}$$

$$= \frac{\int_{-\infty}^{\infty} |H(f) - \hat{H}(f)|^2 |X(f)|^2 df}{\int_{-\infty}^{\infty} |H(f)|^2 |X(f)|^2 df} . \qquad (7.5)$$

Equation (7.5) can easily be modified to measure the performance of a sampled system:

$$P_a(\hat{h}(n)) = \frac{\sum_{n=-\infty} [(h(n)-\hat{h}(n)) * x(n)]^2}{\sum_{n} [h(n) * x(n)]^2}$$

$$= \frac{\int_{-\pi}^{\pi} \left|H(e^{jw})-\hat{H}(e^{jw})\right|^2 \left|X(e^{jw})\right|^2 dw}{\int_{-\pi}^{\pi} \left|H(e^{jw})\right|^2 \left|X(e^{jw})\right|^2 dw} \qquad (7.6)$$

where $X(n)$ is the sampled version of $x(t)$ and w is the frequency normalized with respect to the sampling rate.

As discussed in Section 7.1, a definitive choice of a performance measure must follow an analysis of the effects of linear system identification errors measured in a parameter or global sense on the identification of the nonlinear parts of the system. It should be noted that if a good global performance measure is sufficient to insure adequate nonlinear transfer function identification, a question as to the appropriate linear system identification technique arises since a good global performance measure does not necessarily require that the identified poles be identified with an equally good rms accuracy. Then, it may be sufficient to identify a system frequency response which is close to the true frequency response and, from it, derive the system

poles and zeros. The linear system frequency response can, for example, be estimated from steady-state sinusoidal measurements. Such a technique, although potentially requiring more measurements, should be more resistant to noise than the techniques discussed in this report which first estimate the system poles and zeros.

To illustrate the difference between the global and parameter accuracies and the relation of these measures to the identification technique, consider the following example. Suppose that it is known that the transfer function of the system is given by

$$H(s) = \frac{1}{P(s)} \tag{7.7}$$

that is, that the system has no finite zeros. It is desired to identify H(s). Suppose that the identification is based on a non-iterative technique and that the poles of H(s), that is, the roots of P(s), are identified as the roots of the polynomial of the form (3.11):

$$\sum_{i=0}^{M} \lambda^{N-1} \left([G_{2N+1}]_{i+1,i+1}\right)^{1/2} = 0 \quad . \tag{7.8}$$

This implies that P(s) is identified as

$$\hat{P}(s) = \sum_{i=0}^{N} s^{N-i} \left([G_{2N+1}]_{i+1,i+1}\right)^{1/2} \quad . \tag{7.9}$$

The relation between the global and parameter accuracy measures is now apparent. The global accuracy depends on the difference between P(jw) and the polynomial in (7.8) with λ=jw. The parameter accuracy depends on the distance between the roots of P(s),

and those of the polynomial in (7.8). Note that the two measures are equivalent when one pole is identified at a time, that is when N=1.

In order to ensure a good parameter accuracy in the general case not only must the polynomial in (7.9) be accurate in a global sense but the coefficients of the polynomial (the Gram determinants) must also be accurate. Note that a good global accuracy can be obtained even if the estimate of the number of poles is wrong especially if the estimated number is larger than N. Note also that the estimation of P(jw) is essentially spectral estimation. It follows that an accurate identification in the global sense is much easier to perform than an accurate identification in a parameter sense. However, the adequacy of the global measure in ensuring an accurate nonlinear transfer function identification is difficult to quantify.

SECTION 8
CONCLUSIONS

The starting point for our effort was "the pencil-of-functions" transfer function identification method originally proposed by Jain [1974] for the identification of linear systems and later applied by Ewen [1975, 1979] also to the identification of nonlinear transfer functions of a circuit. Because of the limitations and difficulties encountered by Ewen [1979] in the practical application of this method, the objective of the present study was to devise a modified approach to system identification that would lend itself more readily to actual laboratory measurements.

As a result of this investigation, the discrete, pencil-of-functions identification (DI) method described in Chapters 4 and 5 was formulated. Its main characteristics are:

- Completely discrete formulation directly applicable to sampled data.
- Selective use of the knowledge of the input poles to test for regions where poles are present and to determine the reliability of the identified poles.
- Iterative search for poles starting from the lowest frequencies and progressively moving up to higher frequency poles with each iteration. Adjustment of the sampling interval T to match the pole being sought.
- Use of previously identified poles as knowns in the set defining the Gram determinant during the identification of additional poles.
- Use of time shifts to create new members of the equivalent to Jain's pencil-of-functions set.

The advantages of the discrete iterative approach can be summarized as follows.

- Repeated integrations are not required since time shifts replace integration.
- The accuracy of an identified input pole when it is far from system poles gives a measure of the best accuracy that can be expected.
- Regions where system poles are present can be determined iteratively.
- The number of poles to be identified is determined by stopping at the highest significant pole as opposed to iterative predetermination with a Gram determinant or spectral analysis.
- Re-identification of the input poles while using the identified poles as knowns gives a measure of the accuracy of the identified poles.
- The use of identified poles as knowns is especially useful for wideband systems.
- Each stage of iteration requires the measurement of approximately only twenty output samples.
- The highest required sampling frequency is equal to the Nyquist rate.

The discrete iterative identification (DI) technique was tested by simulation for the case where the output signal is corrupted by additive noise. The results of these tests were presented in Chapter 6. The identification was successful for both narrowband systems containing up to four closely grouped poles and for a wideband system when output noise standard deviation levels did not exceed 5% of the noise free maximum system output. Additive output noise level can generally be decreased to the desired level by averaging out sample values in repeated ex-

periments, but an individual sample has to be matched by the number of bits in the A/D quantizer.

Based on these results, it is concluded that the DI method should be capable in practice of identifying systems containing separate groups of four or fewer even closely packed poles in the presence of additive output noise whose standard deviation is of the order of 1% of the maximum system signal output. If the output noise is mainly due to quantization, this should permit use of 9 or 10 bit quantizers. As noted above, the highest required sampling frequency is equal to the Nyquist rate. This should permit a relatively straightforward implementation provided we are not dealing with a very high Nyquist rate.

These results compare very favorably with those obtained by Ewen [1979] who was able to apply his method to identification of only a two pole circuit and who concluded that for his method, a two pole system required the sampling rate of 4 to 10 times the highest frequency of the passband of the system; that is, two to five times the Nyquist rate, and an A/D converter with a resolution of not less than 16 bits. Furthermore, the identification of a circuit containing a larger number of poles would have required an A/D converter with a greater resolution. Because 16 bit (or higher) A/D converters cannot be clocked at high rates, Ewen's method was correspondingly very limited as to the bandwidth and the order (number of poles) of the system which could be identified. Moreover, the method had no special provisions for the identification of wideband systems to take advantage of the large spread in pole locations even in the cases where the total number of poles was relatively small.

The discrete iterative technique described in this report has the following comparative advantages. The DI method is more noise tolerant and this permits the use of an A/D converter with less resolution. Since the availability of high speed 10 bit A/D

converters (output noise less than 5%) is much greater than that of 16 bit converters, the DI method can be employed at higher sampling rates. This characteristic added to the fact that the present technique requires a sampling rate equal to the Nyquist rate implies that the DI method will permit identification of higher frequency circuits. For example, if the fastest commercially available 16 bit converter were of rate 125 kHz, the maximum bandwidth of a system which could be identified using Ewen's method would be limited to 125/10 or 125/4, i.e., 10 to 30 kHz. If, on the other hand, a commercially available 10 MHz 10 bit A/D converter were used with the discrete iterative approach, the bandwidth of the system could be as high as 10 MHz.

The advantage just cited appears to offset the primary disadvantage of the DI technique which is that because of its iterative nature, more input cases are needed. The sampling rate is, however, no greater in each case than the Nyquist rate. Moreover, the iterative nature of the method and the selective identification of the input poles as system poles permit the identification of wideband systems without the requirement that the regions where the poles are located and the number of poles be known a priori. In defense of iteration, it should also be pointed out that the first stage of the Ewen procedure, which is the determination of the number of poles, is itself also an iterative procedure. Furthermore, there is little guidance to the experimenter what sampling rate should be used in that first stage.

Note, finally, that the system identification approach discussed in this report is not being presented as the ultimate solution for all circuits and may still be impractical for the case of the most complex, widest-band circuit. The DI method is, however, another step forward, hopefully significant, for identification of circuits with relatively few poles and hopefully contributes some enlightment for the analysis even of circuits with many poles.

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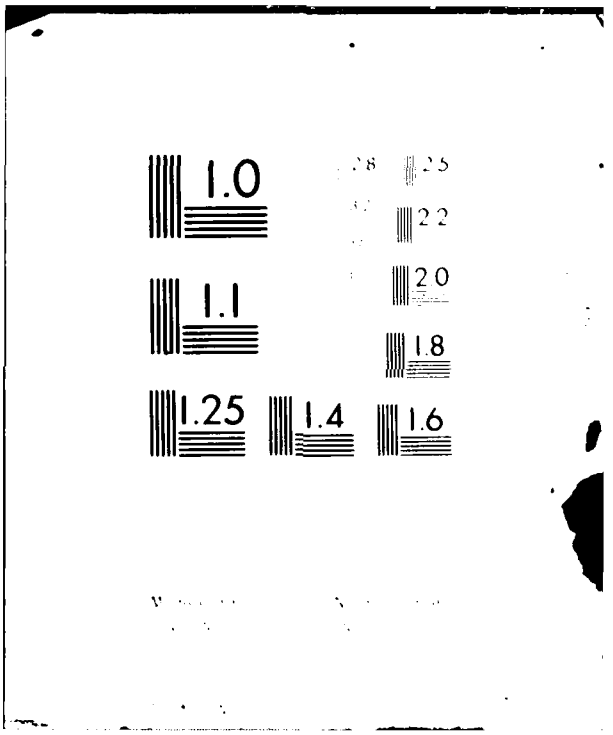
APPENDIX A
PROGRAM LISTING

AD-A112 593 SIGNATRON INC LEXINGTON MA F/G 17/2
NONLINEAR SYSTEM IDENTIFICATION TECHNIQUE VALIDATION.(U)
JAN 82 M RUDKO, J J BUSSGANG F30602-80-C-0104
UNCLASSIFIED A276-4 RADC-TR-81-391 NL

2 OF 2


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5

```
 PIP T1:=IDENTC.FTN
C IDENTIFICATION PROGRAM   A276
C LESLIE MATES 11-17-80
C
C MAIN PROGRAM
      INCLUDE 'IDENTC.CMN'
C
      IXXV(1)=0
      IXXV(2)=0
C
      CALL INPUT
      CALL NORM1
      DO 600 MSAVE=MMM,MSTOP
      DO 500 RANFTR=1,RANF
        IXX(1)=IXXV(RANFTR*2-1)
        IXX(2)=IXXV(RANFTR*2)
        CALL NOISE
        MM=MSAVE
        MT=MM-1
        WRITE (4,2000) MM
2000    FORMAT(/1X,'M=',I3)
        CALL MATRX1
        IF (MAT2.NE.0) CALL MATRX2
        CALL COFCTR
        CALL POLES(IER)
500   CONTINUE
      MM=MSAVE
      MT=MM-1
      IF (RANF.GT.1) CALL AVERAG
600   CONTINUE
C
      CALL EXIT
      END
C
      SUBROUTINE INPUT
      BYTE FNAME(40)
      COMPLEX EX1,EX2,FACTR1,FACTR2,SUM1,SUM2,SUM3
      INCLUDE 'IDENTC.CMN'
C
      TYPE *,'INPUT FILENAME:'
      ACCEPT 1100,FNAME
1100  FORMAT(40A1)
      FNAME(40)=0
      OPEN(UNIT=1,NAME=FNAME,TYPE='OLD')
      READ (1,1000) NN,FF,KK,MMM,MSTOP,ISHFT
1000  FORMAT(20I4)
      READ (1,2000) MU1,DELTAT,SIGMA
2000  FORMAT(20F16.7)
      MU2=CONJG(MU1)
      READ (1,2000) (OMEGA(I),I=1,NN)
      READ (1,2000) (R(I),I=1,NN)
      READ (1,1000) MAT2,IDEF,RANF,JDUAN
      IF (RANF.GT.0) READ (1,1000) (IXXV(I),I=1,RANF*2)
-1-   IF (JDUAN.GT.0) READ (1,2000) (W(I),I=1,JDUAN)
      CLOSE (UNIT=1)
      I=IDEF*JDUAN*2
      IF (MMM.LT.I) MMM=I
      IF (MSTOP.LT.MMM) MSTOP=MMM
      WRITE (4,4000) NN,FF,KK,MMM,MSTOP,ISHFT
4000  FORMAT(1X,'N=',I3/1X,'F=',I5/1X,'K=',I4/1X,'STARTING M=',I3,
     1    /1X,'FINAL M=',I3,/1X,'SHIFT PARAMETER=',I3)
      WRITE (4,4500) MU1,MU2,DELTAT,SIGMA
4500  FORMAT(1X,'MU1=',F16.7,',',F16.7,3X,'MU2=',F16.7,',',F16.7/
```

```
      WRITE (4,5000) (OMEGA(I),I=1,NN)
5000  FORMAT(1X,'OMEGA=',2(F16.7,',',F16.7,2X),
     1   (/1X,6X,2(F16.7,',',F16.7,2X)))
      WRITE (4,6000) (R(I),I=1,NN)
6000  FORMAT(1X,'R=',2(F16.7,',',F16.7,2X),
     1   (/1X,2X,2(F16.7,',',F16.7,2X)))
      IF (MAT2.EQ.0) WRITE (4,6500)
      IF (MAT2.NE.0) WRITE (4,6600)
      WRITE (4,6700)
6500  FORMAT('$','SKIP')
6600  FORMAT('$','DO')
6700  FORMAT('4',' UNBIASED DIAG ELEMENTS FOR GRAM DET')
      IF (IDDEF.EQ.0.AND.JDUAN.EQ.0) WRITE (4,7000)
7000  FORMAT (1X,'REGULAR DEF OF I/O FUNCTIONS')
      IF (IDDEF.NE.0.AND.JDUAN.EQ.0) WRITE (4,7100)
7100  FORMAT(1X,'DEFINE Y(M-1,N)=X1(N)   Y(M,N)=X2(N)')
      IF (IDDEF.NE.0.AND.JDUAN.NE.0) WRITE (4,7200)
7200  FORMAT(1X,'DEFINE Y(M-J-1,N)=X1(N)   Y(M-J,N)=X2(N)')
      IF (JDUAN.NE.0) WRITE (4,8000) JDUAN,(W(I),I=1,JDUAN)
8000  FORMAT(1X,'DEFINE Y(M-J+1...M,N)=Z(N)'/1X,'J=',I3/
     1   1X,'INPUT W=',2(F16.7,',',F16.7,2X),
     1   (/1X,8X,2(F16.7,',',F16.7,2X)))
      IF (RANF.EQ.0) WRITE (4,9000)
      IF (RANF.NE.0) WRITE (4,9100) RANF,(IXXV(I),I=1,RANF*2)
9000  FORMAT(1X,'NO INPUT TO RANDOM NUMBER GENERATOR')
9100  FORMAT(1X,I2,' INPUTS TO RANDOM NUMBER GENERATOR:'/
     1   1X,10(3X,I5,','),I5))
C
      KT=KK-1
C CALCULATE X(N),YH(N)
      DO 200 N=1,PP
            EX=(-1.)*DELTAT*N
            EX1=EX*MU1
            EX2=EX*MU2
            X1(N)=EXP(EX1)
            X2(N)=EXP(EX2)
D     WRITE (4,1001) N,X1(N),X2(N)
D1001 FORMAT(1X,'X(',I3,')=',E14.7,',',E14.7,2X,E14.7,',',E14.7)
            SUM1=(0.,0.)
            SUM2=(0.,0.)
            SUM3=(0.,0.)
            DO 100 I=1,NN
                  FACTR1=R(I)/(MU1-OMEGA(I))
                  FACTR2=R(I)/(MU2-OMEGA(I))
                  EX1=(-1.)*OMEGA(I)*DELTAT*N
                  SUM1=SUM1+(FACTR1+FACTR2)*EXP(EX1)
                  SUM2=SUM2+FACTR1
                  SUM3=SUM3+FACTR2
100         CONTINUE
            YH(N)=SUM1-(X1(N)*SUM2)-(X2(N)*SUM3)
      WRITE (4,2001) N,YH(N)
2001  FORMAT(1X,'Y(',I3,')=',E14.7,',',E14.7)
200   CONTINUE
C
      IF (JDUAN.EQ.0) GO TO 700
C INIT ARRAYS YYS AND LAMDAS FOR AVERAGING
700   DO 770 I=1,13
        LAMDAS(I)=(0.,0.)
        DO 750 J=1,13
-2-           YYS(I,J)=(0.,0.)
750     CONTINUE
770   CONTINUE
      RETURN
      END
C
```

```
C
      COMPLEX Z,EX
      INCLUDE 'IDENTC.CMN'
C
      EX=(-1.)*W(I)*DELTAT*N
      Z=EXP(EX)
      Z=Z*10./ZDIV(I)
      RETURN
      END
C
      SUBROUTINE NORMI
C
      COMPLEX Z
      INCLUDE 'IDENTC.CMN'
C
C FIND MAX Y
      XXD1=ABS(REAL(X1(1)))
      XXD2=ABS(REAL(X2(1)))
      YYD=ABS(REAL(YH(1)))
      DO 150 N=2,KK
            YYDT=ABS(REAL(YH(N)))
            IF (YYDT.GT.YYD) YYD=YYDT
150   CONTINUE
      DO 200 N=1,PP
            X1(N)=X1(N)*10./XXD1
            X2(N)=X2(N)*10./XXD2
            YH(N)=YH(N)*10./YYD
D     WRITE (4,3001) N,X1(N),N,X2(N),N,YH(N)
D3001 FORMAT(1X,'X1(',I3,')=',E14.7,',',E14.7,' X2(',I3,')=',E14.7,
D    1   ',',E14.7,' Y(',I3,')=',E14.7,',',E14.7)
200   CONTINUE
      IF (JOUAN.EQ.0) GO TO 900
      DO 500 I=1,JOUAN
        Z=EXP((-1.)*W(I)*DELTAT)
        ZDIV(I)=ABS(REAL(Z))
        DO 300 N=2,KK
            Z=EXP((-1.)*W(I)*DELTAT*N)
            ZZDT=ABS(REAL(Z))
            IF (ZZDT.GT.ZDIV(I)) ZDIV(I)=ZZDT
300     CONTINUE
500   CONTINUE
900   RETURN
      END
C
      SUBROUTINE NOISE
C
      INCLUDE 'IDENTC.CMN'
C
      DO 100 N=1,PP
            CALL GAUSS(YNOISE,SIGMA,0,IXX)
            Y(N)=YH(N) + YNOISE
D           WRITE (4,2001) YNOISE,N,Y(N)
D2001       FORMAT(1X,'YNOISE=',F15.6,3X,'Y(',I3,')=',E14.7,',',E14.7
100   CONTINUE
      RETURN
      END
      SUBROUTINE MATRIX
C
-3-   COMPLEX YI,YJ,SUM1,DET
      INCLUDE 'IDENTC.CMN'
C
C FILL IN MATRIX
      IBREAK=MM + 1 - IDEF*2 - JOUAN
      DO 400 I=1,MM
        DO 300 J=1,MM
```

```
                    DO 200 N=1,NN
                      YI=Y(N+I-J+1SHFT)
                      IF (I.LT.IBREAK) GO TO 80
                      IF (IODEF.EQ.0.OR.I.GT.IBREAK+1) GO TO 50
                      IF (I.EQ.IBREAK) YI=X1(N)
                      IF (I.EQ.IBREAK+1) YI=X2(N)
                      GO TO 80
   50                 II=I-MM+JOUAN
                      CALL GETZ(YI,II,N)
   80                 YJ=Y(N+J-J+1SHFT)
                      IF (J.LT.IBREAK) GO TO 120
                      IF (IODEF.EQ.0.OR.J.GT.IBREAK+1) GO TO 100
                      IF (J.EQ.IBREAK) YJ=X1(N)
                      IF (J.EQ.IBREAK+1) YJ=X2(N)
                      GO TO 120
  100                 JJ=J-MM+JOUAN
                      CALL GETZ(YJ,JJ,N)
  120                 SUM1=SUM1+YI*CONJG(YJ)
C                     WRITE (4,1001) I,J,N,YI,YJ,SUM1
C1001                 FORMAT(1X,'I=',I2,' J=',I2,' N=',I3,
C     1                ' YI=',F12.6,',',F12.6,' YJ=',F12.6,',',F12.6,
C     1                ' SUM1=',F14.6,',',F14.6)
  200               CONTINUE
                    YY(I,J)=SUM1
                    YY(J,I)=CONJG(SUM1)
C SAVE VALUES FOR AVERAGING
                    YYS(I,J)=YYS(I,J) + YY(I,J)
                    IF (I.NE.J) YYS(J,I)=YYS(J,I) + YY(J,I)
  300             CONTINUE
  400           CONTINUE
                DO 555 I=1,MM
                WRITE (4,1111) (YY(I,J),J=1,MM)
 1111           FORMAT(1X,4(F14.6,',',F14.6))
  555           CONTINUE
                CALL DETRM(-1,DET)
                WRITE (4,1000) DET
 1000           FORMAT(1X,'GRAM DETERMINANT',1X,E14.7,',',E14.7)
                RETURN
                END
C
                SUBROUTINE MATRX2
C
                COMPLEX DET
                INCLUDE 'IDENTC.CMN'
C
C FILL IN DIAGONAL WITH UNBIASED ELEMENTS
                VALUE=NN*(SIGMA**2)
                IEND=MM
                IF (IODEF.NE.0) IEND=MM-1
                DO 100 I=1,IEND
                      YY(I,I)=YY(I,I)-VALUE
  100           CONTINUE
C               DO 555 I=1,MM
C               WRITE (4,1111) (YY(I,J),J=1,MM)
C1111           FORMAT(1X,4(F14.6,',',F14.6))
C555            CONTINUE
                CALL DETRM(-1,DET)
                WRITE (4,1000) DET
 1000           FORMAT(1X,'GRAM DET, UNBIASED DIAG ELEMENTS ',E14.7,',',E14.7)
                RETURN
                END
C
-4-             SUBROUTINE COFCTR
C
                COMPLEX SUM1,AVG
```

```
C LOOP FOR COFACTOR
      SUM1=(0.,0.)
      DO 500 I=1,MM
            CALL DETRM(I,COFACT(I))
            WRITE (4,1000) I,COFACT(
1000        FORMAT(1X,'COFACTOR',I3,'=',E14.7,',',E14.7)
            SUM1=SUM1+COFACT(I)
500   CONTINUE
      AVG=SUM1/MM
      WRITE (4,2000) AVG
2000  FORMAT(1X,'AVERAGE OF COFACTORS ',E14.7,',',E14.7)
      RETURN
      END
C
      SUBROUTINE POLES(IER)
C
      INCLUDE 'IDENTC.CMN'
C
C CALCULATE COEF OF POLYNOMIAL FROM COFACTORS
C ADJUST MM AS NECESSARY
      MM=MM-IODEF-JOUAN
      MT=MT-IODEF-JOUAN
20    DO 100 I=0,MT
            COEF(I)=COFACT(I+1)
            IF (COEF(I).EQ.(0.,0.)) GO TO 50
            COEF(I)=(SQRT(COEF(I)))*((-1)**I)
50          CONTINUE
D     WRITE (4,1001) I,COEF(I)
D1001 FORMAT(1X,'COEF(',I2,')=',F15.6,',',F15.6)
100   CONTINUE
      CALL ROOTCP(COEF,COF,MT,LAMDA,IER)
      IF (IER.NE.0) GO TO 300
      DO 200 I=1,MT
            A=REAL(LAMDA(I))
            B=AIMAG(LAMDA(I))
            POLEI=(ATAN(B/A))/DELTAT
            POLER=(ALOG(A**2+B**2))/(2.*DELTAT)
            WRITE (4,1000) I,LAMDA(I),POLER,POLEI
1000        FORMAT(1X,'LAMDA(',I3,')=',E14.7,',',E14.7,/
     1        2X,' POLER=',E14.7,' POLEI=',E14.7)
            LAMDAS(I)=LAMDAS(I) + LAMDA(I)
200   CONTINUE
      GO TO 900
300   WRITE (4,2000) IER
2000  FORMAT(1X,'ERROR ON ROOTCP ',I3)
C FROM NOW ON USE MT INSTEAD OF MM BECAUSE ONLY M-1 ROOTS
900   CONTINUE
      RETURN
      END
C
      SUBROUTINE AVERAG
C
      COMPLEX DET
      INCLUDE 'IDENTC.CMN'
C
      MTT=MT-IODEF-JOUAN
      DO 100 I=1,MTT
            LAMDA(I)=LAMDAS(I)/RANF
-5-         A=REAL(LAMDA(I))
            B=AIMAG(LAMDA(I))
            POLEI=(ATAN(B/A))/DELTAT
            POLER=(ALOG(A**2+B**2))/(2.*DELTAT)
            WRITE (4,1000) I,LAMDA(I),POLER,POLEI
1000        FORMAT(1X,'AVG LAMDA(',I3,')=',E14.7,',',E14.7,/
```

```
100     CONTINUE
C
        DO 250 I=1,MM
          DO 200 J=1,MM
                YY(I,J)=YYS(I,J)/RANF
200       CONTINUE
250     CONTINUE
        DO 555 I=1,MM
        WRITE (4,1111) (YY(I,J),J=1,MM)
1111    FORMAT(1X,5(E14.7,',',E14.7))
555     CONTINUE
        CALL DETRM(-1,DET)
        WRITE (4,2000) DET
2000    FORMAT(1X,'AVG GRAM DETERMINANT',1X,E14.7,',',E14.7)
        CALL COFCTR
        CALL POLES
        RETURN
        END
        SUBROUTINE DETRM(IFLAG,DET)
        COMPLEX DET,DETT(2)
        INCLUDE 'IDENTC.CMN'
C
        IF (IFLAG.LT.0) GO TO 600
C MOVE MATRIX FOR COFACTOR I INTO WORKING MATRIX
        IY=1
        DO 200 I=1,MT
                IF (IY.EQ.IFLAG) IY=IY+1
                JY=1
                DO 100 J=1,MT
                  IF (JY.EQ.IFLAG) JY=JY+1
                  WYY(I,J)=YY(IY,JY)
                  JY=JY+1
100             CONTINUE
                IY=IY+1
200     CONTINUE
        IM=MT
        GO TO 700
C DOING WHOLE MATRIX--MOVE IT ALL
600     IM=MM
        DO 650 I=1,MM
          DO 630 J=1,MM
                WYY(I,J)=YY(I,J)
630       CONTINUE
650     CONTINUE
700     CONTINUE
D       WRITE (4,1001)
D1001   FORMAT(1X,'DETRM MATRIX')
D       DO 755 I=1,IM
D       WRITE (4,1111) (WYY(I,J),J=1,IM)
D1111   FORMAT(1X,4(F14.6,',',F14.6,1X))
D755    CONTINUE
        CALL CGECD(WYY,13,IM,IPVT,RCON,COF)
CD      TYPE *,'REVISED MATRIX'
CD      DO 756 I=1,IM
CD      WRITE (4,1111) (WYY(I,J),J=1,IM)
CD756   CONTINUE
D       WRITE (4,1112) RCON
D1112   FORMAT(1X,'RCON=',E14.8)
CC      IF ((1.+RCON).EQ.1.) GO TO 800
        IF (RCON.EQ.0.) GO TO 800
        CALL CGEDI(WYY,13,IM,IPVT,DETT,COF,10)
-6-     EX=REAL(DETT(2))
        DET=DETT(1)*(10.**EX)
D       WRITE (4,1113) DETT(1),DETT(2),DET
D1113   FORMAT(1X,'DET=',3(E14.6,',',E14.6/))
```

```
800     DET=10.**0.
        WRITE (4,2001)
2001    FORMAT(1X,'SET DET TO 0.')
900     RETURN
        END
C
>TOP
```

```
>FIP T1:=IDENTC.CMN
C COMMON FOR IDENT PROGRAM  A276
C INPUT
        INTEGER NN,PP,KK,MMM,MSTOP,MAT2,ISHFT,IODEF
        INTEGER RANF,JOUAN,IXXV(20)
        REAL DELTAT,SIGMA
        COMPLEX MUI,OMEGA(10),R(10),W(10)
C CALCULATED
        INTEGER MM,MT,KT,IXX(2),IPUT(14)
        REAL ZDIV(10)
        COMPLEX MU2
        COMPLEX X1(0:200),X2(0:200),Y(0:200),YH(0:200)
        COMPLEX YY(13,13),YYS(13,13),COFACT(13),WYY(13,13)
        COMPLEX LAMDA(13),LAMDAS(13),COEF(0:12),COF(14)
C COMMON
        COMMON NN,PP,KK,MMM,MSTOP,MAT2,ISHFT
        COMMON IODEF,RANF,JOUAN,IXXV
        COMMON MM,MT,KT,IXX,IPUT
        COMMON DELTAT,SIGMA,ZDIV
        COMMON MUI,OMEGA,R,W,MU2
        COMMON X1,X2,Y,YH,YY,YYS,COFACT,WYY
        COMMON LAMDA,LAMDAS,COEF,COF
>
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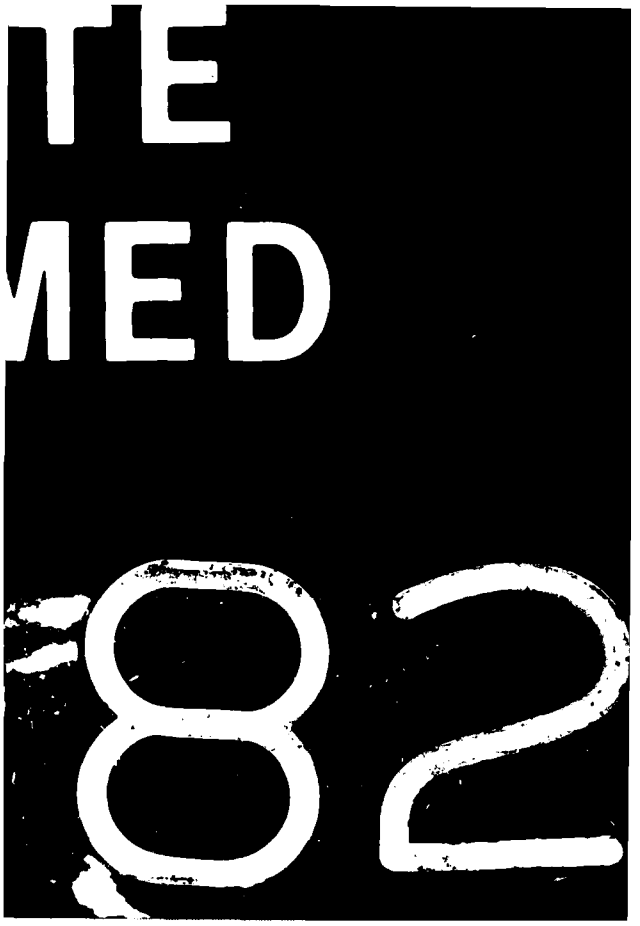
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