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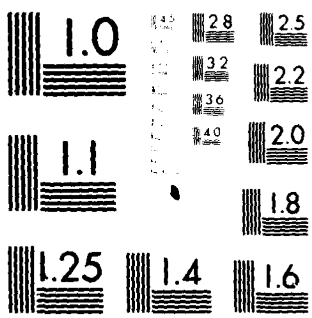

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LEARNING AND THE COST OF PRODUCTION*

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I. Introduction

The airframe industry is usually recognized as being quite different from most manufacturing industries. These differences, which are characterized by the small number of units produced and the frequency of design changes, have been evident for many years. This unique situation, coupled with political considerations, places unusual demands on cost estimators. This has been particularly true in recent years where large cost overruns have generated Congressional demands for better cost estimates.

Both learning curves and neoclassical cost functions have been used in attempts to model this unique production situation, but it has only been in recent years that the two approaches have been combined. Much of the early work that acknowledges a need to combine the two modeling situations has been lacking in terms of theoretical rigor (see 1, 5 and 6). Also, early empirical studies were lacking in that they were dominated by pure empiricism in lieu of being firmly grounded in economic theory (see 2 and 7).

Most recently, significant advances have been made in modeling the made to order production situation. Theoretical contributions by Rosen (8) and Washburn (10) represented the first attempts to actually model the situation as a dynamic optimization problem. Each of these works added to our theoretical understanding of the situation, but neither yielded models definitive enough for empirical application. Recently, a more definitive model has appeared (14). The model was developed for a firm producing to an order which specifies a quantity and delivery date for output. The model augments a neoclassical production function with a learning hypothesis, and the discounted cost of production is minimized to yield optimal time paths of both production and costs.

The model has been applied to the C141 airframe program with good results (11). In that application the process of transferring the theoretical model into a statistical model of the C141 data required assumptions about the details of the production process that the theoretical model did not address. The purpose of this paper is to apply the model to a data environment that is more compatible with the production issues addressed by the model. Here, the model is stated in more general form, data collected on the F102 airframe program is presented, the parameters of the model are estimated, and the economic implications are discussed.

The Model

The model uses a neoclassical pr[illegible] [illegible]n function to yield the time path of output rate as a function of [illegible] requirement rate of a single variable composite resource. The assumption is that the relative prices of the resources contained in the composite resource do not change, and cost is measured in units of the variable resource. The variables of the model are described below:

$q(t)$ = output rate on the program at time t,

$\ell(t)$ = rate of experience at time t,

$Q(t) = \int_0^t q(\tau)d\tau$ = cumulative output at time t,

$L(t) = \int_0^t \ell(\tau)d\tau$ = cumulative stock of knowledge at time t,

δ = a parameter describing learning,

γ = a returns to scale parameter,

α = a learning/production proportionality parameter,

C = discounted variable program cost,

T = time horizon for the production program,

V = volume of output to be produced by T,

K = initial stock of knowledge,

ρ = discount rate

A = constant term

The output function is assumed to be a simpliciation of the general class of CET production functions while the input function is assumed to be of the Cobb-Douglas type. The specification is

$$[q^2(t) + \ell^2(t)]^{1/2} = Ax^{1/\gamma}(t)L^{\delta}(t). \tag{1}$$

The learning parameter δ is assumed to fall between zero and one, and the scale parameter is assumed to be greater than one. The restriction on γ assures that the production function exhibits decreasing returns to the variable factor. Also, neutral technological change is assumed to avoid the problem of having to specify a different learning hypothesis for each resource contained in the composite resource.

The solution to the cost minimization model yields a time path of minimum discounted program cost subject to the production function constraint, that is,

$$\text{Min } C = \int_0^T x(t)e^{-\rho t}dt \tag{2}$$

subject to:

$$q^2(t) + \ell^2(t) = A^2x^{2/\gamma}(t)L^{2\delta}(t), \tag{3}$$

$$Q(0) = 0, \tag{4}$$

$$Q(T) = V, \tag{5}$$

$$L(0) = K, \tag{6}$$

$$L(T) = \text{free} \tag{7}$$

where the terminal time T and terminal cumulative output V are assumed known. The initial stock of knowledge is known, but the terminal stock of knowledge is assumed to be unknown.

The resource requirement function is found by solving (3) for the variable composite resource. The function is

$$x(t) = A^{-\gamma}L^{-\delta\gamma}(t)[q^2(t)+\ell^2(t)]^{\gamma/2}. \tag{8}$$

The problem is restated by substituting (8) into the objective functional (2), that is,

$$\text{Min } C = \int_0^T A^{-\gamma}L^{-\delta\gamma}(t)[q^2(t) + \ell^2(t)]^{\gamma/2}e^{-\rho t}dt. \tag{9}$$

The necessary conditions for an optimal solution require that the Lagrange-Euler equations be equated with zero. These conditions are stated as

$$\gamma A^{-\gamma}L^{-\delta\gamma}(t)q(t)[q^2(t) + \ell^2(t)]^{\gamma/2-1}e^{-\rho t} = k, \tag{10}$$

$$-\delta\gamma A^{-\gamma}L^{-(\delta\gamma+1)}(t)[q^2(t)+\ell^2(t)]^{\gamma/2}e^{-\rho t} - d/dt[\gamma A^{-\gamma}L^{-\delta\gamma}(t)[q^2(t) + \ell^2(t)]^{\gamma/2-1}e^{-\rho t} = 0. \tag{11}$$

This system of differential equations is second order and nonlinear, and at this point in time the analytical solution for Q(t) and L(t) is unknown.

Nevertheless, this formulation of the problem is instructive. In this model experience rate is treated as a decision variable. Resources are diverted from production in order to produce learning, but this produced experience re-enters the production process as enhanced knowledge with the potential of reducing discounted cost at some later instance. The suggestion is that there is some optimal trade-off between learning and output rate.

III. The Restricted Model

Since the solution of the dynamic quadratic model is unknown, additional information is obtained by examining a restricted model. Suppose the model is stated with an additional constraint. The objective is

$$\text{Min } C = {}_0\int^T x(t)e^{-\rho t}dt \tag{12}$$

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subject to:

$$q^2(t) + \ell^2(t) = A^2 x^{2/\gamma}(t) L^{2\delta}(t), \tag{13}$$

$$L(t) = \alpha Q(t) + K \quad \text{or} \quad \ell(t) = \alpha q(t), \tag{14}$$

$$Q(0) = 0, \tag{15}$$

$$Q(T) = V, \tag{16}$$

$$L(0) = K, \tag{17}$$

$$L(T) = \alpha V + K. \tag{18}$$

Notice that the initial stock of knowledge for this model is assumed to be some constant K. This definition, along with the constraint on L(t), defines the terminal condition on L(T). The problem is now expressed as a fixed endpoint problem. The additional constraint defines a specific relationship between experience rate and output rate, i.e.,

$$\ell(t) = \alpha q(t). \tag{19}$$

The parameter α defines a particular location on the production possibility curve. Any expansion in output must occur along a ray with slope α. This restriction is stringent, but it permits solving the problem by using the calculus of variations.

The solution is obtained by performing a series of transformations. First, substitute (14) into (13) and solve for x(t). This yields the following resource requirement function:

$$x(t) = A^{-\gamma}(1+\alpha^2)^{\gamma/2} q^{\gamma}(t)[\alpha Q(t)+K]^{-\delta\gamma}. \tag{20}$$

Let

$$Z(t) = \alpha Q(t) + K. \tag{21}$$

It follows that

$$z(t) = dZ/dt = \alpha q(t). \tag{22}$$

The problem may now be stated as one in the new state variable Z(t). The objective is

$$\text{Min } C = {}_0\int^T A^{-\gamma}(1+\alpha^2)^{\gamma/2}\alpha^{-\gamma} z^{\gamma}(t) Z^{-\delta\gamma}(t) e^{-\rho t} dt \tag{23}$$

subject to:

$$Z(0) = K, \tag{24}$$

$$Z(T) = \alpha V + K. \tag{25}$$

The necessary condition for an optimum is that the Lagrange-Euler equation be equated with zero. This condition is stated as

$$\partial I/\partial Z - d/dt(\partial I/\partial z) = 0 \qquad (26)$$

where

$$I(z,Z,t)=A^{-\gamma}(1+\alpha^2)^{\gamma/2}K^{-\gamma}z\ (t)Z^{-\delta\gamma}(t)e^{-\rho t}dt \qquad (27)$$

is the intermediate function for the problem.

The differential equation (26) is nonlinear and second order. However, an additional transformation leads to a straight-forward analytical solution. Let

$$Y(t) = Z^{1-\delta}(t)/(1-\delta). \qquad (28)$$

This implies that

$$y(t) = dY/dt = Z^{-\delta}\ (t)z(t). \qquad (29)$$

Substituting into (23) and redefining and boundary conditions leads to a third representation of the cost minimization model. The objective is

$$\text{Min } C = \int_0^T A^{-\gamma}(1+\alpha^2)^{\gamma/2}\alpha^{\gamma}y^{\gamma}(t)e^{-\rho t}dt \qquad (30)$$

subject to:

$$Y(0) = K^{1-\delta}/(1-\delta), \qquad (31)$$

$$Y(T) = (\alpha V+K)^{1-\delta}/(1-\delta). \qquad (32)$$

Since the intermediate function for the transformed problem does not depend explicitly on Y(t), the Lagrange-Euler equation for this problem integrates to a constant, that is,

$$A^{-\gamma}(1+\alpha^2)^{\gamma/2}\alpha^{-\gamma}y^{\gamma-1}(t)e^{-\rho t} = k_1. \qquad (33)$$

The solution for optimal y(t) is

$$y(t) = k_2e^{\rho t/(\gamma-1)} \qquad (34)$$

where

$$k_2 = k_1^{1/(\gamma-1)}\gamma^{-1/(\gamma-1)}A^{\ /(\gamma-1)}(1+\alpha^2)^{-\gamma/[2(\gamma-1)]}. \qquad (35)$$

It also follows from (34) that

$$Y(t) = k_2\int_o^t e^{\rho t/(\gamma-1)}dt + k_3. \qquad (36)$$

After integrating (36) and imposing the boundary conditions, (31) and (32), the following expression is found for optimal Y(t):

$$Y(t)=[e^{\rho T/(\gamma-1)}-1]^{-1}[e^{\rho t/(\gamma-1)}-1]$$
$$[(\alpha V+K)^{1-\delta}/(1-\delta)-K^{1-\delta}/(1-\delta)]+K^{1-\delta}/(1-\delta). \tag{37}$$

Also, since $Y(t)=Z^{1-\delta}(t)/(1-\delta)$, optimal $Z(t)$ is

$$Z(t)=\{[e^{\rho T/(\gamma-1)}-1]^{-1}[e^{\rho t/(\gamma-1)}-1]$$
$$[(\alpha V+K)^{1-\delta}-K^{1-\delta}]+K^{1-\delta}\}^{1/(1-\delta)}. \tag{38}$$

By applying (21) to (38) the optimal time path for cumulative output is obtained. The optimal time path is

$$Q(t)=a^{-1}\{[e^{\rho T/(\gamma-1)}-1]^{-1}[e^{\rho t/(\gamma-1)}-1]$$
$$[(\alpha V+K)^{1-\delta}-K^{1-\delta}]+K^{1-\delta}\}^{1/(1-\delta)}-K\alpha^{-1}. \tag{39}$$

Differentiate (39) with respect to time to obtain the following expression for optimal production rate:

$$q(t)=\alpha^{-1}\{\rho/[(1-\delta)(\gamma-1)]\}[(\alpha V+K)^{1-\delta}-K^{1-\delta}] \tag{40}$$
$$[e^{\rho T/(\gamma-1)}-1]^{-1}\{[e^{\rho T/(\gamma-1)}-1]^{-1}$$
$$[e^{\rho t/(\gamma-1)}-1]\ [(\alpha V+K)^{1-\delta}-K^{1-\delta}]$$
$$+K^{1-\delta}\}^{\delta/(1-\delta)}e^{\rho t/(\gamma-1)}.$$

Substitute (39) and (40) into (20) to determine the resource use rate. The optimal resource requirement function is

$$x(t)=A^{-\gamma}(1+\alpha^2)^{\gamma/2}\alpha^{-\gamma}\{\rho/[(1-\delta)(\gamma-1)]\}^{\gamma}\ K_4^{\gamma}\ e^{\rho\gamma t/(\gamma-1)} \tag{41}$$

where

$$K_4=[e^{\rho T/(\gamma-1)}-1]^{-1}[(\alpha V+K)^{1-\delta}-K^{1-\delta}]. \tag{42}$$

The optimal resource requirement function is inserted into the objective functional to obtain program cost as a function of time. Using (2), the relevant integral is

$$C(t)=\int_0^t A^{-\gamma}(1+\alpha^2)^{\gamma/2}\alpha^{-\gamma}\{\rho/[(1-\delta)(\ -1)]\}^{\gamma}K_4^{\gamma}e^{\rho\tau/(\gamma-1)}d\tau. \tag{43}$$

After performing the integration, the optimal expression for total discounted program cost is

$$C(t)=A^{-\gamma}(1+\alpha^2)^{\gamma/2}\alpha^{-\gamma}\{\rho/[(1-\delta)(\gamma-1)]\}^{\gamma} \tag{44}$$
$$[e^{\rho/(\gamma-1)}-1]^{-\gamma}[(\alpha V+K)^{1-\delta}-K^{1-\delta}]^{\gamma}$$
$$[(\gamma-1)/\rho][e^{\rho t/(\gamma-1)}-1].$$

The control format is particularly pertinent for solving the problem. There is always the question of how to measure learning and hence the stock of knowledge. For that matter, with respect to airframe production, there is always a question about how to measure production rate. The control formulation eliminates both of these problems since experience rate and production rate are "optimized" out of the problem, i.e., they are both decision variables. The solution yields discounted program cost as a function of time.

With this optimal expression for cost, numerous hypothetical policy simulations are possible. The cost impacts of primary interest include exogenous changes in production rate via changes in delivery schedules. This information would be particularly helpful in updating cost estimates during the production period of an airframe program.

While the restricted model is stated in more general form than the 1979 model, the addition of the restriction (14) alters the model in such a way that the multiple output technology is transformed into a single output technology. This is seen by examining the resource requirement function (20), i.e., notice that the single input x is a function of the single output q. As a result, the cost function of the 1979 model and (44) are identical up to a constant. Our future papers will explore the relaxation of the constraint at (14).

The cost function (44) is the estimable function for the empirical section of this paper. Given a set of parameter estimates, the model is capable of exploring the impact on discounted program costs of not only exogenous changes in production rate and delivery schedules (V and T), but also changes in α. Since α determines the proportion of resources that are diverted to learning, the model can examine the relative impact on discounted cost of a change in this proportion.

Intuition suggests that at least initially total discounted cost will rise with α since there is a cost associated with learning. However, if the total cost function (44) is examined, the implications of changing α are not clear since the impact on cost depends on the magnitudes of the parameters. The cost impact of a change in α is an econometric problem that can only be resolved empirically.

IV. The F102 Airframe Program

The F102A and the TF102 airframe programs provide the cost data for the applications section of this paper. The F102A is a single-seat, supersonic, delta wing, all weather fighter interceptor, and the TF102 is a two seated trainer version of the F102A. The F102 program was the overall responsibility of General Dynamics-CONVAIR with assistance from General Dynamics-Forth Worth. The support from Forth Worth was mainly on the TF102 nose and miscellaneous components.

The "F102 Program Cost History" (4) is a comprehensive document that includes numerous cost breakdowns by individual airframe on both the F102A and the TF102. These cost breakdowns provide the data for this study.

The F102 program was comprised of 1000 aircraft that were constructed during the years 1953 through 1958. Of these 1000, 889 are F102A interceptors, and 111 are TF102 trainers. The variable of primary interest in the data base is direct labor hours for each airframe. Data is available for all 1000 airframes, but not all of the data is in the proper form to be used with the model specified in this paper. Care is taken to resolve all data problems, and the data is reorganized so that it is compatible with the previously specified theoretical model.

One problem with this data is the cost differential between the F102 and the TF102 airframes. The largest part of this difference is caused by the additional nose cost for the TF102. If both airframes are included in the analysis, some data adjustment is required to compensate for the cost difference. One possible adjustment would be to delete the TF102 observations and complete the analysis using the 889 F102 observations. This procedure is not desirable since the learning on both of the airframes contributes to the cost behavior of each of the airframes. A more appropriate adjustment is the deletion of the additional nose cost from each of the TF102 airframes. For the TF102, the two-seat fuselage components were constructed in Fort Worth and shipped to San Diego for final assembly. The basic difference in the hours required on both models is due to the additional hours at Fort Worth. Although there are other cost differences between the two models, this adjustment appears to be reasonable with respect to the available data.

Additional examination indicates that the data organization is not compatible with the theoretical model. The data presented in the cost history is cost by airframe. The data that is needed for estimation purposes is cost per unit time. The ideal data would be cost per airframe per month, but this data is not available. The next best alternative is cost by lot per month. The F102 cost history provides information that makes it possible to assign airframes to lots. The delivery date for each airframe is known, so if the lot release dates were known, and the monthly completion distribution for each lot were known, then it would be possible to generate a cost per month value to use as the dependent variable in the nonlinear regression.

Unfortunately, the lot release dates are unknown. However, there is still some available data that makes it possible to approximate the lot release dates for eight of the nineteen lots. Tables 1 and 2 give percent completion by lot by month. The tables are segregated into two sections: details and assemblies. These sections are clarified by the information reproduced in Table 3. This table gives the production labor hour summary for the F102 and TF102 by contract. The fabrication hours in Table 3 concern the details that are presented in Table 1. For the F102, details or fabrication hours comprised approximately 20% of the total hours expended. Assemblies in Table 2 are comprised of the four assembly categories in Table 3: major assembly, sub assembly, primary assembly, and final assembly. For the F102, assembly hours accounted for approximately 61% of the total hours expended. The remainder of Table 3, field operations and electronics, are activities that occurred outside of the factory.

The first stage in the data adjustment requires that the total direct manhours per airframe be segregated into three parts: details, assemblies, and outside of the factory. The percentages by contract provided by Table 2 are used to segregate the data. These percentages are presented in Table 4. Next, the percentage that is due to activities outside of the factory is deleted. The reason for this deletion is that there is no information about the monthly distribution of the work that occurred outside of the factory. This leaves total cost by airframe that is due to details and assemblies.

After aggregating into lots, total lot cost is spread over the months using the data in Tables 1 and 2. Implicit in this procedure is the assumption that the lot release date for lots four through eleven may be represented by the first month that activity occurs in the respective lot. Lot cost is multiplied by the percentages in Tables 1 and 2 to obtain a monthly cost figure.

Since the model considers cost over the complete project, as a final adjustment, cost must be aggregated by month. This gives a monthly cost value for lots four through eleven which is used as the dependent variable for the analysis. There are clearly some problems with this data adjustment procedure. The initial months over which learning has an important impact on cost are deleted. Also, the deletion of the latter months of the project interjects bias since the production of lots four through eleven is influenced by anticipation of additional production activity in later lots. For example, a close examination of Tables 1 and 2 shows that after August, 1956 not only is production activity taking place on lots ten and eleven, but it is reasonable to assume that production has already began on lot twelve. The implication is that what production occurs in September and October is not likely to be independent of what happens in later lots. Also, the monthly cost will be severly understated. The only reasonable assumption is to delete all activity past August, 1956, and for estimation purposes use the planned volume and the terminal time for the complete project. The cost per month values that are used in the estimation are presented in Table 5. There is no doubt that this is a severe restriction on the empirical model, but additional data on the latter months of the project is just not available.

1955

	J	F	M	A	M	J	J	A	S	O	N	D
lot 4	20	20	9	9	17	10	13	2				
lot 5		2	15	8	20	15	25	7	4	2	1	1
lot 6					3.4	8.6	12	5	18	23	17	5
lot 7									4	4	31	24
lot 8												
lot 9												
lot 10												
lot 11												

1956

	J	F	M	A	M	J	J	A	S	O	N	D
lot 4												
lot 5												
lot 6	4	4										
lot 7	20	10	5	2								
lot 8	8	25	40	25	2							
lot 9				8	25	40	26	1				
lot 10					5	30	40	20	8			
lot 11								10	32	48	10	

Table 1. F102 details percent completion by lot, by month

1955

	J	F	M	A	M	J	J	A	S	O	N	D
lot 4		5	15	10	12	23	15	3	2	10	5	
lot 5				5	15	15	20	10	15	10	5	3
lot 6						5	5	15	15	20	15	10
lot 7											10	20
lot 8												
lot 9												
lot 10												
lot 11												

1956

	J	F	M	A	M	J	J	A	S	O	N	D
lot 4												
lot 5	2											
lot 6	10	5										
lot 7	25	20	15	10								
lot 8	2	25	35	30	7							
lot 9				4	20	40	30	6				
lot 10						20	40	30	10			
lot 11							5	35	45	15		

Table 2. F102 assemblies percent completion lot, by month

	Contract			
	23903	29264	31774	33685
Fabrication	546448	1022080	2539461	524060
Sub Assembly	328331	641843	2626727	531686
Major Assembly	1082028	1928816	6155937	959650
Primary Assembly	194969	282128	804416	126318
Final Assembly	149095	268258	853890	130216
Field Operations	112895	431869	1639303	328418
Electronics	72712	239996	1134832	236752
Total	2,486,478	4,814,990	15,754,566	2,836,600

Table 3. Production labor hour summary for the F102 and TF102 by contract

	Contract				
	5942*	23903	29264	31174	33965
Fabrication	19.45%	21.98%	21.23%	16.12%	18.47%
Assembly	65.82%	70.56%	64.82%	66.27%	61.62%
Outside of Factory	14.73%	7.46%	13.95%	17.61%	19.91%

* Data on contract number 5942 is not available. The numbers presented are based on averages over the remaining contracts.

Table 4. Percent of total manhours allocated to specific activities by contract.

Cost	Month
87588.960120	1
169255.12250	2
319441.75736	3
283989.79977	4
538366.96822	5
724143.06890	6
735400.38838	7
395148.28286	8
487693.18317	9
605107.55834	10
639831.67327	11
550857.44587	12
638146.06308	13
790801.39694	14
811728.47136	15
756117.01045	16
740850.61671	17
1618970.9276	18
1618970.9276	18
1684015.1252	19
993591.09883	20

Table 5. Monthly data on direct manhours (cost) for lots four through eleven.

Parameter Estimation

As an empirical application, nonlinear least squares is used to estimate the parameters in (44). The quantity that is observable in the data is

$$C(t_1) - C(t_o) = \int_{t_o}^{t_1} x(t)dt. \tag{45}$$

Notice that the discount factor is deleted. This is to make this model directly comparable with other models that are not formulated in terms of discounted cost. After performing the necessary integration and taking derivatives with respect to each of the parameters, the coefficients were estimates using Marquardt's compromise using Statistical Analysis System software (9).

The results indicate that there are more parameters in the model than are necessary to represent the data adequately. The asymptotic correlation matrix for the estimated parameters shows that there is extremely high correlation between the estimated discount rate ($\hat{\rho}$) and the scale parameter ($\hat{\gamma}$). Also, there is a high correlation between the constant term ($\hat{A}$) and the learning parameter ($\hat{\delta}$). The suggestion is that a reparameterized model involving fewer parameters is more appropriate. This does not mean that the original model is inappropriate, but the given set of data is not adequate for estimating all of the parameters.

A close examination of the model shows that the model is of the following form:

$$C(t_1) - C(t_0) = \beta_o \, [e^{\beta_1 t_1} - e^{\beta_1 t_0}]. \tag{46}$$

If the model is reparameterized in this form, $\beta_1 = \rho\gamma/(\gamma-1)$, and β_0 is a constant term. If ρ is fixed at a suitable value, an estimation of (46) yields some information about the scale parameter γ. Unfortunately, the reparameterization required by the data makes it impossible to obtain any information about α and δ. Prior to estimation, both independent and dependent variables are divided by constants so that the units are more manageable, i.e., cost is divided by 1,000,000 and time (months) is divided by 100. The discount rate is fixed at .00008333, a number that is consistent with an assumed 10% annual rate. The parameters were estimated using nonlinear least squares, and convergence was obtained in 4 iterations after using a grid search procedure to establish the initial parameter values. The results of the estimation on (46) are presented in Table 6.

These results support previously presented research (10, 11, 12, 13, 14) which stresses the importance of the cost impact of production rate changes in airframe programs. An asymptotic 95% confidence interval shows that the scale parameter is significantly greater than one, indicating *diminishing returns* to the variable factor.

Source	DF	Sum of Squares	Mean Square
Regression	2	11.309	5.654
Residual	18	.994	.055
Total*	20	12.303	

Parameter	Estimate	Asymptotic S.E.
β_o	.853	.554
γ^o	1.00000982	.00000180

Asymptotic 95% Confidence Interval

	Lower	Upper
β_o	-.31068113	2.01655183
γ^o	1.00000605	1.00001360

* = uncorrected

Table 6. Nonlinear Least Squares Summary Statistics

Conclusion

A dynamic cost minimizing model is presented to model the cost behavior of airframe programs. Given an assumption about the relationship between learning and output, this model reduces to a specification that has been previously presented without empirical verification. Data is organized on the F102 airframe program in such a way that it is compatible with the specification, and the parameters are estimated via nonlinear least squares. The results of numerous estimations on different reparameterizations indicate that the given data is not adequate for estimating all of the parameters in the model. A simple reparameterized model containing fewer parameters is estimated, and the results yield information about a single parameter in the model. Unfortunately, this set of data yields no information about the learning parameter or the optimal proportion of resources diverted to learning.

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Previous work (14) has developed theoretical production models for airframes. The model was developed for a firm producing to an order which specifies a quantity and delivery date for output. The model augments a neoclassical production function with a learning hypothesis, and the discounted cost of production is minimized to yield optimal time paths of both production and costs.

The model has been applied to the C141 airframe program with good results

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(11). In that application the process of transferring the theoretical model into a statistical model of the C141 data required assumptions about the details of the production process that the theoretical model did not address. The purpose of this paper is to apply the model to a data environment that is more compatible with the production issues addressed by the model. Here, the model is stated in more general form, data collected on the F102 airframe program is presented, the parameters of the model are estimated, and the economic implications are discussed.

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