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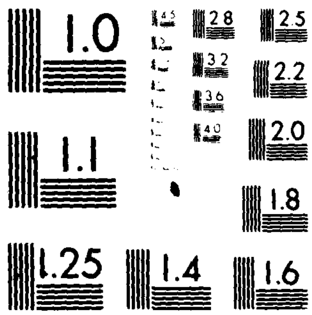
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The Ohio State University

A METHOD OF CHOOSING ELEMENT PATTERNS IN AN ADAPTIVE ARRAY

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16. Abstract (Limit: 200 words)

A procedure for choosing element patterns in an adaptive array is described. The method is useful when the adaptive array must provide sector coverage, i.e., when the desired signal may arrive from anywhere within some sector of space. The procedure is to start with an initial set of elements and then add extra elements one at a time until suitable performance is obtained. The condition that must be satisfied by each new element to obtain maximum performance improvement is derived.

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INTRODUCTION

Adaptive arrays based on the LMS algorithm [1] can, in principle, receive desired signals from any arrival angle. However, the output signal-to-interference-plus-noise ratio (SINR) obtained from the array, as the desired and interference signal arrival angles and polarizations vary, depends critically on the element patterns and spacings used in the array [2].

The purpose of this report is to discuss a procedure that may be used to choose element patterns and spacings in an adaptive array. The method described is appropriate when the desired signal arrival angle will not be known in advance, so the array must provide coverage over some sector of space.

The method is the following. One starts with an array consisting of an initial set of elements, chosen by some means. One evaluates the performance of this array, to find particular signal arrival angles and polarizations where the SINR is poor. One then augments the array, by adding extra elements one at a time, to improve the SINR for these angles and polarizations.

This technique will work properly as long as the new elements added to the array do not affect the patterns of the original elements. That is, the mutual coupling between the new elements and the original elements must be small.

Such a method can be used because, if the original element patterns do not change, the SINR obtained from an adaptive array cannot decrease when additional elements are added. The SINR is a nondecreasing function of the number of elements, for all signal arrival angles and polarizations. Thus, once a set of elements has been found that yields adequate performance over part of a desired sector of space, one does not have to worry that adding new elements will reduce the performance in this region. The new elements can be chosen to improve the SINR in other regions where the original array did not perform well.

The fact that the SINR obtained from an adaptive array cannot decrease when more elements are added is easy to see on physical grounds. It is known that LMS weights in an array yield the maximum SINR that may be obtained from a given set of elements for any given set of signals. (Widrow, et al [1] have shown that LMS weights yield minimum mean-square error signal, and Baird and Zahm [3] have shown that minimum mean-square error weights are equivalent to maximum SINR weights.*) Suppose we have an N-1 element array, to which we add another element to obtain an N element array. Assume a given set of signals is incident on the array. Then the SINR with the N element array must be at least as large as that with the N-1 element array, because the N element array can always achieve the SINR of the N-1 element array by setting the weight on the new element to zero. The validity of this statement depends, of course, on the assumption that the patterns of the original N-1 elements did not change.

Since the SINR cannot decrease when a new element is added, it can only increase. The amount of the increase, however, depends on the pattern and location of the new element. In this report, we determine what condition the new element should satisfy to maximize the SINR increase for the case where a desired signal and one interference signal are incident on the array. We also give an example to illustrate the procedure.

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*Baird and Zahm [3] prove this equivalence under the assumption the signals are narrowband. We shall also make that assumption below. However, in studying element pattern effects in adaptive arrays, the narrowband assumption is automatically satisfied, because pattern effects are most easily studied with CW signals anyway.

FORMULATION

Consider an N-1 element adaptive array. Let the j^{th} element of the array be located at vector position $\bar{r}_j$ with respect to the origin of a spherical coordinate system r,θ,ϕ. Consider first the desired signal. Suppose the desired signal has polarization state P_d* and propagates into the array from angle θ_d,ϕ_d. Let $\hat{k}_d$ be a unit vector parallel to the direction of propagation of the signal. This signal will produce a voltage $\tilde{x}_j(t)$ on the j^{th} array element:

$$\tilde{x}_j(t) = \tilde{s}_d\left(t + \frac{\bar{r}_j \cdot \hat{k}_d}{c}\right) f_j(\theta_d,\phi_d,P_d), \tag{1}$$

where $\tilde{s}_d(t)$ is the desired signal waveform (measured at the coordinate origin, and expressed in analytic form), c is the velocity of propagation, and $f_j(\theta_d,\phi_d,P_d)$ is the pattern response of the j^{th} element to a signal from angle θ_d,ϕ_d with polarization P_d. We may write $\tilde{s}_d(t)$ in the form

$$\tilde{s}_d(t) = u_d(t)e^{j(\omega_0 t+\psi_d)}, \tag{2}$$

where $u_d(t)$ is the complex envelope, ω_0 the carrier frequency, and ψ_d the carrier phase of $\tilde{s}_d(t)$. We assume ψ_d is a uniformly distributed random variable on $(0,2\pi)$.

*We shall not need a complete description of signal polarization here, but we may think of P_d as the point on the Poincare sphere [4] representing the desired signal polarization.

The N-1 element signals $\tilde{x}_j(t)$ may be written as components of an N-1 dimensional desired signal vector $X_d = (\tilde{x}_1(t), \tilde{x}_2(t), --, \tilde{x}_{N-1}(t))^T$ (T denotes the transpose). Under the condition that the signals are sufficiently narrowband, or equivalently, that the dimensions of the array are small enough, we have

$$u_d\left(t + \frac{\bar{r}_j \cdot \hat{k}_d}{c}\right) \approx u_d(t) \tag{3}$$

for every element of the array. In this case the desired signal vector may be written

$$X_d = u_d(t)\, e^{j(\omega_o t + \psi_d)}\, U_d \quad , \tag{4}$$

where U_d is the vector

$$U_d = \begin{pmatrix} f_1(\theta_d, \phi_d, P_d) e^{j\phi_{d_1}} \\ f_2(\theta_d, \phi_d, P_d) e^{j\phi_{d_2}} \\ \cdot \\ \cdot \\ \cdot \\ f_{N-1}(\theta_d, \phi_d, P_d) e^{j\phi_{d_{N-1}}} \end{pmatrix} , \tag{5}$$

and ϕ_{d_j} is the phase of $\tilde{s}_d\left(t + \frac{\bar{r}_j \cdot \hat{k}_d}{c}\right)$ at the j^{th} element, measured with respect to the phase of $\tilde{s}_d(t)$ at the coordinate origin:

$$\phi_{d_j} = \frac{\omega_o}{c}(\bar{r}_j \cdot \hat{k}_d) \quad . \tag{6}$$

Next consider the interference signal. Assume this signal propagates into the array from angle θ_i, ϕ_i with polarization P_i. The interference signal vector will then be

$$X_i = u_i(t)\, e^{j(\omega_o t + \psi_i)}\, U_i \quad , \tag{7}$$

with

$$U_i = \begin{pmatrix} f_1(\theta_i,\phi_i,P_i)\, e^{j\phi_{i_1}} \\ f_2(\theta_i,\phi_i,P_i)\, e^{j\phi_{i_2}} \\ \cdot \\ \cdot \\ \cdot \\ f_{N-1}(\theta_i,\phi_i,P_i)\, e^{j\phi_{i_{N-1}}} \end{pmatrix}, \tag{8}$$

and

$$\phi_{i_j} = \frac{\omega_o}{c}\,(\bar{r}_j \cdot \hat{k}_i) \quad , \tag{9}$$

where the notation is analogous to that for the desired signal. $u_i(t)$ is the complex envelope and ψ_i the carrier phase of the interference, and $\hat{k}_i$ is a unit vector pointing in the direction of propagation. We have again assumed a narrowband signal, so

$$u_i\left(t + \frac{\bar{r}_j \cdot \hat{k}_i}{c}\right) \simeq u_i(t) \tag{10}$$

for every element j. Also, we assume ψ_i is uniformly distributed on $(0,2\pi)$ and independent of ψ_d.

Finally, suppose that the signal from each array element contains a zero-mean thermal noise voltage $\hat{n}_j(t)$ of power σ^2, and that these noise voltages are statistically independent of each other:

$$E\left\{\hat{n}_\ell^*(t)\hat{n}_m(t)\right\} = \sigma^2\delta_{\ell m} \quad , \tag{11}$$

where $E\{\cdot\}$ denotes the expectation and $\delta_{\ell m}$ is the Kronecker delta. Also, we assume the $\hat{n}_j(t)$ are independent of ψ_d and ψ_i.

Under these conditions, it has been shown [2] that the output signal-to-interference-plus-noise ratio (SINR) from an adaptive array may be written

$$\mathrm{SINR}(N-1) = \xi_d\left[U_d^T U_d^* - \frac{\left|U_d^T U_i^*\right|^2}{\xi_i^{-1} + U_i^T U_i^*}\right], \tag{12}$$

where * denotes complex conjugate, and ξ_d and ξ_i are the input desired and interference signal-to-noise ratios, defined by

$$\xi_d = \frac{1}{\sigma^2} E\left\{\left|u_d(t)\right|^2\right\} \quad , \tag{13}$$

and

$$\xi_i = \frac{1}{\sigma^2} E\left\{\left|u_i(t)\right|^2\right\} \quad . \tag{14}$$

In Equation (12), we use the notation SINR(N-1) to indicate that this is the SINR for N-1 elements.

Now suppose another element is added to the array, so there are N elements. (We assume no change in the original N-1 element patterns.) The SINR with N elements will have the same form as Equation (12), except that the vectors U_d and U_i will have N components instead of N-1. To distinguish between the N-1 element array and the N element array, we let U_d and U_i remain defined as in Equations (5) and (8), and define new vectors U_d^N and U_i^N for the N element array:

$$U_d^N = \begin{pmatrix} f_1(\theta_d,\phi_d,P_d)\, e^{j\phi_{d_1}} \\ f_2(\theta_d,\phi_d,P_d)\, e^{j\phi_{d_2}} \\ \vdots \\ f_N(\theta_d,\phi_d,P_d)\, e^{j\phi_{d_N}} \end{pmatrix}, \tag{15}$$

and

$$U_i^N = \begin{pmatrix} f_1(\theta_i,\phi_i,P_i)\, e^{j\phi_{i_1}} \\ f_2(\theta_i,\phi_i,P_i)\, e^{j\phi_{i_2}} \\ \vdots \\ f_N(\theta_i,\phi_i,P_i)\, e^{j\phi_{i_N}} \end{pmatrix}, \tag{16}$$

However, since the first N-1 components of U_d^N and U_i^N are just U_d and U_i, we may write U_d^N and U_i^N as partitioned vectors. For U_d^N, we have

$$U_d^N = \begin{pmatrix} U_d \\ --- \\ U_{dN} \end{pmatrix} = \begin{pmatrix} U_d \\ --- \\ 0 \end{pmatrix} + \begin{pmatrix} \bar{0} \\ --- \\ U_{dN} \end{pmatrix} = V_d + V_{dN}, \qquad (17)$$

where

$$V_d = \begin{pmatrix} U_d \\ --- \\ 0 \end{pmatrix} , \qquad (18)$$

$$V_{dN} = \begin{pmatrix} \bar{0} \\ --- \\ U_{dN} \end{pmatrix} , \qquad (19)$$

($\bar{0}$ denotes an N-1 dimensional vector of zeros) and

$$U_{dN} = f_N(\theta_d, \phi_d, P_d) e^{j\phi_{d_N}} . \qquad (20)$$

It will also be convenient to write U_{dN} in terms of its magnitude and phase,

$$U_{dN} = \rho_d \, e^{j\eta_d} . \qquad (21)$$

Note that η_d may differ from ϕ_{d_N} if $f_N(\theta_d, \phi_d, P_d)$ contains a phase shift.

Similarly, for U_i^N we have

$$U_i^N = \begin{pmatrix} U_i \\ --- \\ U_{iN} \end{pmatrix} = \begin{pmatrix} U_i \\ --- \\ 0 \end{pmatrix} + \begin{pmatrix} \overline{0} \\ --- \\ U_{iN} \end{pmatrix} = V_i + V_{iN} \,, \tag{22}$$

where

$$V_i = \begin{pmatrix} U_i \\ --- \\ 0 \end{pmatrix} \,, \tag{23}$$

$$V_{iN} = \begin{pmatrix} \overline{0} \\ --- \\ U_{iN} \end{pmatrix} \,, \tag{24}$$

and

$$U_{iN} = f_N(\theta_i,\phi_i,P_i)e^{j\phi_{iN}} = \rho_i e^{j\eta_i} \quad . \tag{25}$$

Writing the SINR as in Equation (12), but with vectors U_d^N and U_i^N instead of U_d and U_i, and using the orthogonality relations

$$V_d^T V_{dN}^* = V_d^T V_{iN}^* = V_i^T V_{dN}^* = V_i^T V_{iN}^* = 0, \tag{26}$$

we find

$$\mathrm{SINR}(N) = \xi_d \left[U_d^T U_d^* + \rho_d^2 - \frac{\left| U_d^T U_i^* + \rho_d \rho_i e^{j\eta} \right|^2}{\xi_i^{-1} + U_i^T U_i^* + \rho_i^2} \right], \tag{27}$$

where

$$\eta = \eta_d - \eta_i \quad . \qquad (28)$$

To simplify the algebra below, let us also define

$$A = \xi_i^{-1} + U_i^T U_i^* \quad , \qquad (29)$$

and

$$Be^{j\gamma} = U_d^T U_i^* \quad . \qquad (30)$$

Then Equation (27) may be written

$$\mathrm{SINR}(N) = \xi_d \left[U_d^T U_d^* + \rho_d^2 - \frac{\left| Be^{j\gamma} + \rho_d \rho_i e^{j\eta} \right|^2}{A + \rho_i^2} \right] \quad . \qquad (31)$$

Let us first show that SINR(N) cannot be less than SINR(N-1), regardless of ρ_d, ρ_i or η (i.e., regardless of the signal arrival angles and polarizations and regardless of the pattern and location of the N^{th} element). Substituting Equations (29) and (30) into Equation (12), we have

$$\mathrm{SINR}(N-1) = \xi_d \left[U_d^T U_d^* - \frac{B^2}{A} \right] . \qquad (32)$$

Defining

$$\Delta = \xi_d^{-1} \left[\mathrm{SINR}(N) - \mathrm{SINR}(N-1) \right] \quad , \qquad (33)$$

we find from Equations (31) and (32) that

$$\Delta = \frac{\rho_d^2\, A(A+\rho_i^2) + B^2(A+\rho_i^2) - A|Be^{j\gamma} + \rho_d\rho_i\, e^{j\eta}|^2}{A(A + \rho_i^2)} \; . \qquad (34)$$

The numerator of this expression may be rearranged into the form

$$(A\rho_d - B\rho_i)^2 + 2AB\rho_d\rho_i\left[1 - \cos(\gamma-\eta)\right] , \qquad (35)$$

which is clearly nonnegative because $\cos(\gamma-\eta) \leq 1$. Also, since the denominator of Δ is positive by definition, we have $\Delta \geq 0$ or $SINR(N) \geq SINR(N-1)$ as stated.

Now consider the problem of choosing the pattern and location of the N^{th} element to maximize SINR(N). Since SINR(N) in Equation (31) is unbounded as a function of ρ_d, let us suppose ρ_d is fixed and determine the values of ρ_i and η that maximize SINR(N). Maximum SINR(N) will be obtained when the quantity

$$\frac{|Be^{j\gamma} + \rho_d\rho_i\, e^{j\eta}|^2}{A + \rho_i^2} \qquad (36)$$

is minimized -- i.e., when $Be^{j\gamma}$ and $\rho_d\rho_i e^{j\eta}$ have opposite signs. Hence we first choose

$$\eta = \gamma - \pi \; . \qquad (37)$$

When η has this value,

$$SINR(N) = \xi_d\left[U_d^T U_d^* + \rho_d^2 - \frac{(B-\rho_d\rho_i)^2}{A+\rho_i^2}\right] . \qquad (38)$$

Then the value of ρ_i giving maximum SINR(N) is

$$\rho_i = \frac{B}{\rho_d} \quad . \tag{39}$$

With these values for η and ρ_i, we find

$$\text{SINR}(N) = \xi_d \left[U_d^T U_d^* + \rho_d^2 \right] = \xi_d \left[U_d^{N^T} U_d^{N^*} \right] , \tag{40}$$

which we recognize as the maximum attainable SINR if the response of the new element to the desired signal is ρ_d. This is the SINR that would be obtained with N elements when no interference is present.

Recalling the definitions of ρ_d, ρ_i and η in Equations (21), (25) and (28), we see that the pattern of the N^{th} element should be chosen to satisfy the conditions

$$\left| f_N(\theta_d,\phi_d,P_d)\ f_N(\theta_i,\phi_i,P_i) \right| = \left| U_d^T U_i^* \right| \quad , \tag{41}$$

and

$$\measuredangle f_N(\theta_d,\phi_d,P_d) e^{j\phi_{d_N}} - \measuredangle f_N(\theta_i,\phi_i,P_i) e^{j\phi_{i_N}} = \measuredangle U_d^T U_i^* - \pi \ , \tag{42}$$

to obtain maximum SINR for a given set of signal parameters θ_d, ϕ_d, P_d, θ_i, ϕ_i and P_i. Note that both the phase of $f(\theta,\phi,P)$ and the element location can be varied to satisfy Equation (42).

Equations (41) and (42) do not, of course, define the pattern of the new element completely. They merely specify a relationship between its amplitude and phase response for two signal directions and polarizations. There is considerable flexibility left in the pattern of the new element even after Equations (41) and (42) are satisfied. In a typical design problem, the original N-1 element array may yield poor SINR for several sets of arrival angles and polarizations. One may be able to choose the pattern and location of the new element to satisfy Equations (41) and (42) for more than one set of signals at once. However, even if the new element has been optimized for only one set of signals, it will usually improve the SINR for other signal arrival angles and polarizations as well.

AN EXAMPLE

Now consider a simple example to illustrate the use of Equations (41) and (42). Suppose we have an array of two dipoles located and oriented as shown in Figure 1. To simplify the problem, let us consider only linearly polarized signals whose electric field and direction of arrival lie in the plane of the paper. Let the angle θ be defined as shown in Figure 1. Assume the patterns of the two dipoles are

$$f_1(\theta) = \cos(\theta - 60^0) \quad , \qquad (43)$$

and

$$f_2(\theta) = \cos(\theta + 60^0) \quad . \qquad (44)$$

If we define the coordinate origin to be at the center of dipole 1, then

$$\phi_{d_1} = \phi_{i_1} = 0. \qquad (45)$$

Also, the dipoles are two wavelengths apart, so we have

$$\phi_{d_2} = -4\pi \sin\theta_d, \qquad (46)$$

and

$$\phi_{i_2} = -4\pi \sin\theta_i \quad . \qquad (47)$$

Using these quantities in U_d and U_i, we may compute the SINR for any given θ_d and θ_i from Equation (12).

It is shown in Reference 2 (in particular, see Figure 9 and the accompanying discussion) that this pair of dipoles has a grating null problem when θ_d=22.3^o and θ_i=39^o. The lower curve in Figure 2 shows the SINR as a function of θ_i computed from Equation (12) when θ_d=22.3^o (and for ξ_d=0 dB and ξ_i=40 dB). The effect of the grating null may be seen. Interference at θ=39^o not only causes a null at 39^o, it causes another null at 22.3^o (the grating null). When the desired signal is near the grating null, the SINR is low.

Suppose now we add another dipole to this array to improve the performance for the set of signals θ_d=22.3^o and θ_i=39^o. Let the new dipole have its beam maximum at $\theta=\theta_o$:

$$f_3(\theta) = \cos(\theta-\theta_o), \tag{48}$$

and be located a distance d to the right of dipole 1 in Figure 1. The problem is to determine the proper values of θ_o and d. By computing U_d and U_i for the original 2-dipole array in Figure 1, we find that when θ_d=22.3^o and θ_i=39^o, $U_d^T U_i^*$=.760 (real). Hence, from Equation (41), θ_o must be chosen so

$$|f_3(22.3^o)f_3(39^o)| = |\cos(22.3-\theta_o)\cos(39-\theta_o)| = 0.760. \tag{49}$$

Solving this for θ_o gives two possible values, θ_o=2.8^o and 58.5^o. Since the SINR achieved with the new element included will be $\xi_d \, U_d^T U_d^* + \rho_d^2$ (see Equation (40)), and since $\rho_d = \cos(22.3^o-2.8^o)$ is larger than $\rho_d = \cos(22.3^o-58.5^o)$, we choose θ_o=2.8^o to obtain the largest improvement.

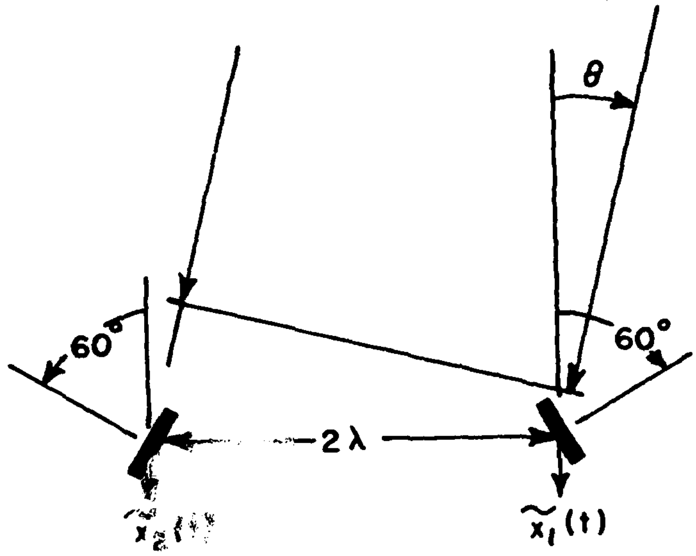

Figure 1. A 2-dipole array.

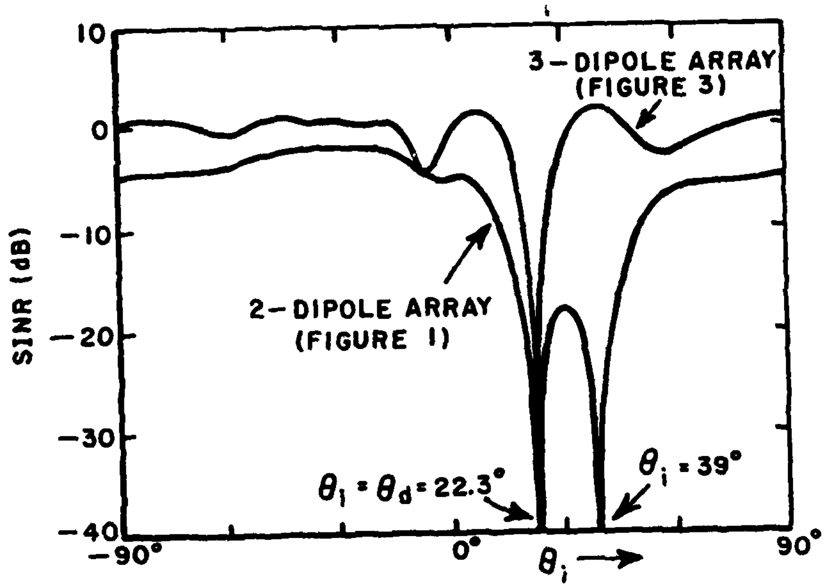

Figure 2. SINR vs. θ_i.
θ_d=22.3^0, ξ_d=0 dB, ξ_i=40 dB

Next, Equation (42) may be used to find d. Since the phase angle of $f_3(\theta)$ with θ_o=2.8^o is zero at both 22.3^o and 39^o, Equation (42) reduces to

$$\frac{2\pi d}{\lambda}\left[\sin(22.3^o) - \sin(39^o)\right] = -\pi , \qquad (50)$$

which yields d=2λ. Adding the third dipole gives us the array shown in Figure 3. Finally, computing the SINR for this 3-dipole array, again with θ_d=22.3^o, we obtain the top curve in Figure 2. It may be seen how the performance has been improved around θ_i=39^o.

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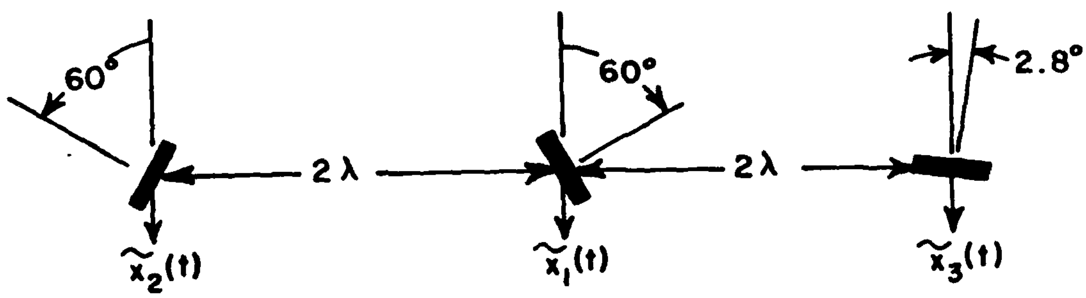

Figure 3. The 3-dipole array.

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