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DETECTION, ESTIMATION, AND CONTROL ON GROUP MANIFOLDS.(U)
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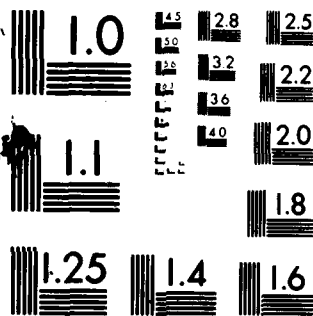
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This final report covers work carried out by the principal investigator and a graduate research assistant at the Department of Mathematics during the 13 months period from 1 August 1980 to 31 August 1981, under the Grant AFOSR-80-0241. The progress has resulted in six technical papers listed within.		

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AFOSR-TR- 82 - 0267

Final Scientific Report
on
Detection, Estimation, and Control
on Group Manifolds

TO: Air Force Office of Scientific Research
Air Force Systems Command
USAF

FROM: James T. Lo
Department of Mathematics
University of Maryland Baltimore County
Baltimore, Maryland 21228

SUBJECT: Grant No. AFOSR-80-0241
Detection, Estimation, and Control
on Group Manifolds

covering the period:
August 1, 1980 - August 31, 1981

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I. General

This final report covers work carried out by the principal investigator and a graduate research assistant at the Department of Mathematics during the 13 months period from Aug. 1, 1980 to Aug. 31, 1981 under the Grant ASFOR-80-0241.

The progress has resulted in six technical papers listed in Section II. The first five of them have been submitted for journal publication. The last one was presented at the 19th Annual Allerton Conference on Communication, Control, and Computing, Monticello, Illinois, Sept. 30, 1981.

During the report period, the following people contributed to the project: Professor James T. Lo and Dr. Sze-Kui Ng.

II. Publications

- (1). Positive-Definiteness of Fourier Transforms of Probabilities on Compact Groups, submitted for publication (with S.-K. Ng).
- (2). Optimal Finite-Order Filtering I - General Theory, submitted for publication (with S.-K. Ng).
- (3). Optimal Finite-Order Filtering II - Multivariable Systems, submitted for publication (with S.-K. Ng).
- (4). Series Representation of Moments and Powers of Ito Diffusion Processes, submitted for publication (with S.-K. Ng.).
- (5). Optimal Finite-Order Filtering III - Design Considerations, submitted for publication (with S.-K. Ng).
- (6). Volterra Series Approach to Filtering, presented at The 19th Annual Allerton Conference on Communication, Control, and Computing, Monticello, Illinois, September 30, 1981 (with S.-K. Ng.)

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 MATTHEW J. KERPNER
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III. Summary of Progress

(1). Positive-Definiteness of Fourier Transforms of Probabilities on Compact Groups:

The positive definiteness of both a sequence of matrices and a sequence of matrix functions is defined. It is shown that the positive definiteness is a necessary and sufficient condition for the sequence to be the Fourier transform of a positive measure or a transition probability semigroup respectively, on a compact group.

(2). Optimal Finite-Order Filtering I: General Theory:

Projecting a Bayesian-type representation of the minimum variance estimate onto the Hilbert subspace generated by the n -th order Volterra series driven by the observation yields a minimum variance Volterra representation $\hat{x}^n$ of the same order. Its kernels are characterized by a Fredholm integral equation of the second kind, which can be viewed as "the Wiener-Hopf equation of the n -th order".

By one more projection, from $\hat{x}^n$ onto the subspace generated by Volterra series with separable kernels, a finite dimensional recursive filter is obtained. It is optimal for its kind. There is much flexibility in the choice of the kernels, which promises great opportunity in the design of the optimal finite-order filters.

(3). Optimal Finite-Order Filtering II: Multivariable Systems:

The Volterra series approach to approximate filtering, introduced in Part I of this series, is developed for the multivariable systems. Generalizations are made of (a) a product-to-sum formula for iterated stochastic integrals; (b) a recipe to calculate the Volterra kernels for the unnormalized conditional moments; (c) the n -th order Wiener-Hopf equation.

(4). Series Representation of Moments and Powers of Ito Diffusion Processes:

A straightforward method is presented to construct a series representation of the moments and a Volterra representation of the powers of the Ito diffusion process. The drift and diffusion coefficients of the stochastic differential equation are assumed analytic. The main idea is that of the Carleman linearization.

(5). Optimal Finite-Order Filtering III: Design Considerations:

The minimal variance estimate is projected directly onto the Hilbert subspace of all Fourier-Hermite (FH) series, driven by the observations, with the same index set. The projection results in a system of linear algebraic equations for the FH coefficients, the parameters of the desired approximate estimator.

The estimator consists of finitely many Wiener integrals of the observations and a memoryless nonlinear postprocessor. The postprocessor is an arithmetic combination of the Hermite polynomials evaluated at the Wiener integrals.

If a complete orthonormal set (CONS) of step functions is chosen for the FH series, the approximate estimator uses only the observations at discrete times. A discrete-time recursive algorithm is given in which the CONS of Rademacher functions is employed.

(6). Volterra Series Approach to Filtering:

Projecting a Bayesian-type representation of the minimum variance estimate onto the Hilbert subspace generated by the n -th order Volterra series driven by the observation yields a minimum variance Volterra representation $\hat{x}$ of the same order. Its kernels are characterized by a Fredholm integral equation of the second kind, which can be viewed

as "the Wiener-Hopf equation of the n -th order".

By one more projection, from $\hat{x}^n$ onto the subspace generated by Volterra series with separable kernels, a finite dimensional recursive filter is obtained. It is optimal for its kind. There is much flexibility in the choice of the kernels, which promises great opportunity in the design of the optimal finite-order filters.

The minimal variance estimate can also be projected directly onto the Hilbert subspace of all Fourier-Hermite (FH) series, driven by the observations, with the same index set. The projection results in a system of linear algebraic equations for the FH coefficients, the parameters of the desired approximate estimator.

The estimator consists of finitely many Wiener integrals of the observations and a memoryless nonlinear postprocessor. The postprocessor is an arithmetic combination of the Hermite polynomials evaluated at the Wiener integrals.

If a complete orthonormal set of step functions is chosen for the FH series, the approximate estimator uses only the observations at discrete times.