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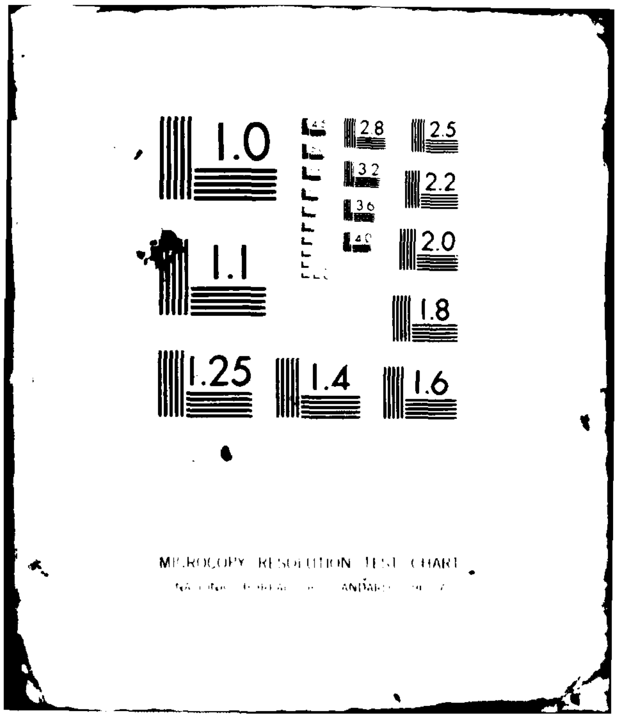
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THE AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

FINAL REPORT

VORTEX BREAKDOWN AND INSTABILITY

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Abstract

Solutions of the full steady axisymmetric Navier-Stokes equations for breakdown in an unconfined viscous vortex have been obtained by Grabowski and Berger[1]. The core Reynolds numbers were varied up to 200 and a two-parameter family of velocity distributions was assumed upstream as the boundary condition. In this report, the Richardson numbers corresponding to the developing solution profiles of Grabowski and Berger are calculated and shown graphically. The inviscid linear stability of the swirling velocity profiles as calculated by Grabowski and Berger is investigated with respect to non-axisymmetric disturbances. This eigen-value problem is solved numerically for various wave numbers and it is shown that the developing velocity profiles lead to instability. These results are believed to be useful in analyzing instabilities for a trailing vortex from an aircraft.

ACKNOWLEDGEMENT

The author would like to thank the Air Force Systems Command, the Air Force Office of Scientific Research and the Computational Aerodynamics Group, at the Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Dayton, Ohio for providing him with the opportunity to do research on Vortex Breakdown.

He would also like to thank Dr. Wilbur Hankey for several suggestions and helpful discussion. The help rendered by Mr. Jeff Graham in digitizing the graphs of Grabowski and Berger and the computational help given by Mr. Jeff Graham and Mr. Steve Scherr is very much appreciated. Finally he would like to acknowledge many helpful discussions with Captain Michael Francis.

Introduction

Vortex flows are encountered on the fore-bodies of aircraft at high angles of attack that suddenly become assymetrical thereby causing departure into spin. Vortex bursting can occur suddenly decreasing the lift and producing adverse results[2]. A great deal of potential hazard is associated with the wake turbulent behind an aircraft. This happens due to the persistence of a trailing vortex system in the wake of a vehicle. Another aircraft flying into this vortex system can experience large perturbations in flight path leading to possible loss in pilot control of this vehicle. The problem of vortex breakdown occuring over delta wings at large angles of attack and in axisymmetric swirling flows in circular pipes has been extensively studied in the past. A number of survey articles has been written on the subject [see Hall[17,18], Leibovich[19], Singh and Hankey[20]].

All the experimental observations indicate that two dimensionless parameters govern the occurence of vortex breakdown both over delta wings and for swirling flows through tubes: (1) the Reynolds number $W_s r_s/\nu$ and (2) the circulatior number $q/W_s r_s$ where W_s and q are the axial velocity and circulation at large radius r_s respectively in cylindrical coordinate system (r,θ,z). ν is the kinematic viscosity and r_s is the characteristic length. When the fluid flows past a lifting wing it produces trailing vorticity which at some distance downstream concentrates into two trailing line vortrices. A characteristic feature of a steady trailing line vortex is the existence of strong axial currents near the axis of symmetry. The link between the azimuthal and the axial velocity components in vortices is provided by the pressure; the radial pressure gradient balances the centrifugal force, and any change in the azimuthal motion in the axial direction downstream produces an axial pressure gradient and consequently axial acceleration. The expression of the velocity distribution for the steady axisymmetric trailing line vortices in cylindrical coordinates (r,θ,z) is obtained[3] as

$$v(r) = q[1-\exp(-\beta r^2)]/r \tag{1}$$

$$w(r) = W_1 + W_s \exp(-\beta r^2) \tag{2}$$

where v and w are the swirl and axial velocity components. The parameters q, β and W_s vary slowly with axial distance z. The swirl velocity v given by (1) resembles a solid-body rotation near the axis $z = 0$ and represents the line-vortex flow away from the axis. The axial velocity w(r) gives the axially-symmetric jet and wake like velocity distribution for a certain combination of W_1 and W_s.

The full Navier-Stokes equations have been numerically integrated by Lavan, Nielsen and Fejer[4], Kopecky and Torrance[5] and Grabowski and Berger[1]. All three sets of investigators assumed the flow to be incompressible and axisymmetric. The initial profiles, imposed at the junction $z = o$ were special cases of (1) and (2) and then the numerical solution was obtained for $z > o$, covering a range of Reynolds numbers from four to several hundred. Lavan et al.[4] treated low-Reynolds-number flow passing from a rigidly rotating circular pipe into a stationary pipe of the same diameter. Thus they dealt with a geometry and a Reynolds number range very different from those in vortex breakdown experiments. On the otherhand, Kopecky and Torrance[5] considered a more realistic situation resembling to those of vortex breakdown experiments in a tube, concerning the geometry, initial conditions and the range of Reynolds number. Calculations performed by Grabowski and Berger[1] in an unconfined region are more extensive and have greater resolution than those of

Kopecky and Torrance. Their solutions exhibit many of the characteristics of vortex breakdown. Taken together the last two results are of great importance to the determination of internal structure in the breakdown region. And the numerical experiments convincingly demonstrate that the Navier-Stokes equations do indeed have solutions with embedded regions of closed stream surfaces which resemble the axisymmetric bubble form of the vortex breakdown.

It is possible that the vortex breakdown may occur from an instability of the mean flow. The inviscid criterion for stability of rotating flows due to axisymmetric disturbances was first derived by Rayleigh[6]. In fact, the "Rayleigh criterion" $(d/dr)(rv)^2 > o$ is shown to be sufficient for stability even with viscosity by Synge[7]. Howard and Gupta[8] have shown that for given mean axial velocity $w(r)$ and swirl velocity $v(r)$, stability of the basic flow to axisymmetric disturbances is guaranteed if the Richardson number criterion $r^{-3}(dw/dr)^{-2}[d(vr)^2/dr] \geq 0.25$ is satisfied. Many other investigators have studied the stability of vortex flows. The stability of a pair of trailing vortices during the early growth stage has been analyzed by Crow[9]. Both symmetric and antisymmetric eigenmodes were shown to be unstable. Widnall and Bliss[10] studied the motion and stability of a vortex filament containing an axial flow in the limit of slender-body theory. Stability of a rotating axisymmetric jet surrounded by a potential vortex to infinitesimal disturbances in the inviscid incompressible fluid approximation was considered by Uberoi, Chow and Narain[11] in the approximation of long and short wavelengths. Lessen, Deshpande and Hadji-Ohanes[12] analyzed the stability of a potential vortex with a rotating and non-rotating jet core and showed that the potential vortex becomes unstable in the presence of a jet. The stability of swirling flows with mean velocity profiles given by (1) and (2) for a trailing vortex from an aircraft has been studied by Lessen, Singh and Paillet[13] and Lessen and Paillet[14] both for inviscid and viscous theory with respect to infinitesimal non-axisymmetric disturbances (with normal modes given by $\exp[i(\alpha z+n\theta-\alpha ct)]$. It is found that in both cases the parameter q governs the stability. Flows are stable to axisymmetric disturbances provided $q > 0.4$, which always seems to be the case in experiments leading to vortex breakdown. For non-axisymmetric disturbances, stability is assured if $q > 1.5$ while instability is obtained for small values. The experiment performed by Singh and Uberoi[15] on trailing tip vortices verifies the presence of laminar instability modes associated with large axial velocities in the vortex core region. Garg[16] in the experimental study of the structure of vortex breakdown in a tube observed that fluctuations are somewhat more regular and more intense in the breakdown region than in the wake, and are stronger in the bubble form of breakdown than they are in the spiral form.

In the present study, the inviscid stability of the developing axial and swirling velocity profiles (as obtained by Grabowski and Berger[1]) is investigated. The analysis of Howard and Gupta has shown that all the vortex flows are stable subject to infinitesimal axisymmetric disturbances provided the Richardson number $J = [(r^2v^2)'/r^3w'^2] \geq 0{\cdot}25$. The Richardson numbers calculated for the developing profiles of Grabowski and Berger show that for the most part the above-mentioned condition is satisfied. That is why only non-axisymmetric disturbances corresponding to $n = 1$ and $n = -1$ (where n is the azimuthal wave number of the Fourier disturbance of the type $\exp[i(\alpha z+n\theta-\alpha ct)]$) are considered. For various profiles, the real and imaginary quantities of the complex phase velocity c are numerically calculated corresponding to the wave number α and these are compared with those given by Lessen, Singh and Paillet[13].

Mathematical Analysis

In this section, first the non-dimensional Navier-Stokes equations in cylindrical coordinates are given. The numerical investigation of vortex breakdown undertaken by Grabowski and Berger[1] is then outlined and their solutions discussed. And finally the linear inviscid stability analysis is formulated.

The conservation of mass and momentum equations for an incompressible flow in terms of cylindrical coordinates (r, θ, z), with corresponding velocity components (u, v, w) are

$$\frac{1}{r}\frac{\partial(ur)}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{\partial p}{\partial r} + \frac{1}{Re}\left[\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right] \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = -\frac{\partial p}{r\,\partial \theta} + \frac{1}{Re}\left[\nabla^2 v - \frac{v}{r^2} + \frac{2}{r^2}\frac{\partial u}{\partial \theta}\right] \tag{3}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \theta} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re}\nabla^2 w \tag{4}$$

where $\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$.

The velocity components have been non-dimensionalized by the free-stream axial velocity W_∞, lengths by a characteristic core radius δ , time t by δ / W_∞ and pressure by ρW_∞^2 after subtraction of p_∞ the uniform static pressure from the vortex. The core Reynolds number is given by $R_e = W_\infty \delta / \nu$. Conditions that approximate those expected in a real vortex flow have been applied by Grabowski and Berger at the boundary of a sufficiently large finite region. At the upstream boundary $z = 0$, it is

$$u(r) = 0 \qquad \text{for } 0 \leq r \leq R \tag{5}$$

$$v(r) = Vr(2-r^2)$$

$$w(r) = \beta + (1-\beta)\, r^2(6-8r+3r^2) \quad \text{for } 0 \leq r \leq 1 \tag{6}$$

$$v(r) = V/r, \text{ and } w(r) = 1 \text{ for } 1 \leq r \leq R \tag{7}$$

At the downstream boundary $z = L$, $0 \leq r \leq R$

$$\frac{\partial u}{\partial z} = 0, \quad \frac{\partial v}{\partial z} = 0, \quad \text{and} \quad \frac{\partial w}{\partial z} = 0 \tag{8}$$

On the axis $r = 0$, $0 \leq z \leq L$

$$u = 0,\ v = 0 \text{ and } \frac{\partial w}{\partial r} = 0 \tag{9}$$

At the radial boundary $r = R$, $0 \leq z \leq L$

$$\frac{\partial (ur)}{\partial r} = 0,\ v = V/R, \text{ and } w = 1. \tag{10}$$

In the above-mentioned boundary conditions β and V are the two parameters, V is the specified circumferential velocity at the core edge; it is equal to the circulation around the core after non-dimensionalization by $2\pi\delta W_\infty$. R and L are much larger than one. The parameter β is the ratio of the axial velocity at the core center to the velocity in the free stream and uniform velocity results when $\beta = 1$. β greater or less than one yields jet-like or wake-like profiles respectively.

The axisymmetric forms of equations (1) to (4) $[\partial/\partial\theta \equiv 0]$ have been solved by Grabowski and Berger[1] subject to the conditions (5) to (10) for V = 0·85, 0·8944, 1·0 and 1·095, Re up to 200 and various values of β. For Re = 200, $\beta = 1$ and V = 1·095, they have shown graphically the stream function contours and comparison with the axisymmetric bubble followed by spiral breakdown (from Sarpkaya[21]) is quite good. This is reproduced in Fig. 1 of this report. Figures 2 and 3 show the axial and swirl velocity profiles for the same parameters.

The linear inviscid stability problem of the velocity profiles obtained by Grabowski and Berger[1] as shown in Figs. 2 and 3 is now formulated by deriving the well-known linearized momentum and continuity perturbation equations. Let u', v', w' be the radial, azimuthal and axial components of velocity perturbation respectively and p', the pressure perturbation. We can then assume

$$\{u', v', w', p'\} = \{i\,G, H, F, P\}(r)\exp[i(\alpha z - \alpha c t) + n i \theta] \qquad (11)$$

where α and n are axial and azimuthal wave numbers, $c = c_r + ic_i$ is the complex phase velocity and F, G, H and P are the complex amplitudes of perturbation. If $c_i < 0$, the disturbances die down and for $c_i > 0$, the perturbation increases without limit and the mean velocity profiles predict instability. On substitution of (11) into the linearized forms of equations (1) to (4), we obtain (see Lessen, Singh and Paillet[13])

$$\alpha r F + (rG)' + nH = 0 \qquad (12)$$

$$r^2\gamma G + 2rVH - r^2P' = 0 \qquad (13)$$

$$r^2\gamma H + r(rV)'G + nrP = 0 \qquad (14)$$

$$r^2\gamma F + r^2 W' G + \alpha^2 r^2 P = 0 \qquad (15)$$

In equations (12) to (15), the coefficients of 1/Re have been taken to be zero to study the inviscid stability. Primes denote differentiation with respect to r and

$$\gamma = \alpha(W - c) + nV/r. \qquad (16)$$

The boundary conditions to integrate (12) to (15) are

$$G(0) = H(0) = 0,\ F(0),\ P(0) \text{ finite for } n = 0, \qquad (17)$$

$$G(0) \pm H(0) = 0,\ F(0) = P(0) = 0 \text{ for } n = \pm 1, \qquad (18)$$

$$G(0) = H(0) = F(0) = P(0) = 0 \text{ for } |n| > 1, \qquad (19)$$

$$G(\infty) = H(\infty) = F(\infty) = P(\infty) = 0 \text{ for all } n. \qquad (20)$$

The requirements are that F and P do not depend on θ at $r = 0$ and all the quantities be finite for all values of r.

For the axisymmetric case $n = 0$, Howard and Gupta[8] derived the sufficient condition for stability that all the vortex flows are stable subject to infinitesimal axisymmetric disturbances provided the Richardson number

$$J \equiv (r^2 v^2)'/(r^3 w'^2) \geq 0 \cdot 25. \tag{21}$$

In figures 4 to 8, we have sketched the Richardson number J vs. r for five velocity profiles v and w of Grabowski and Berger at five plane sections z = constant, as indicated in figures 2 and 3.

Equations (12) - (15) can be reduced to two first-order equations, which are more suitable for numerical integration.

$$G' = \left[\frac{\alpha w' - \alpha^2 r(v' + v/r)/n}{\gamma} - \frac{1}{r}\right] G - (\alpha^2 r^2 + n^2)H/rn \tag{22}$$

$$H' = -[nv' + \alpha(rw' + w - c) + 2nv/r]\, H/\gamma$$
$$-[(v + rw)' G]'/r - nG/r \tag{23}$$

The equations (22) - (23) along with the boundary conditions (17) - (20) constitute an eigen value problem. As a time wise stability problem, an attempt will be made to determine c as an eigen value for given values of α for five profiles v and w. The disturbances are amplified or damped with time depending upon whether $c_i > 0$ or $c_i < 0$ respectively and $c_i = 0$ characterizes neutral disturbances.

Numerical Solution and Results

Lessen, Singh and Paillet[13] have pointed that translation and inversion of the axial velocity profile only affects the frequency and does not alter the amplification factor c_i. That is why in our numerical calculations, the axial velocity w has been replaced by w-1. In this way the asymptotic nature of both the axial and swirl velocity profiles obtained by Grabowski and Berger for large r is similar those used by Lessen, Singh and Paillet. The procedure to find the conditions for integrating equations (22)-(23) has also been discussed by Lessen, Singh and Paillet. The same procedure is adopted here. For large r (say $r = 3)(w - 1) \to 0$ and $v \to V/r$, and (12)-(15) reduce to Bessel's equation in F. The solution valid at large r is the modified Bessel function $K_n(\alpha r)$. Then $G = -F'/\alpha$ and $H = n\, K_n(\alpha r)/\alpha r$, where prime denotes differentiation with respect to r. Starting with these asymptotic values, the solution of (22) and (23) is advanced towards $r = 0$ by numerical integration, and then matched to the known Frobenius series solution at some fixed radius (say $r = 0.2$) near zero.

Verma, Hankey and Scherr[22] described the numerical Fortran program for such problems in detail and for the numerical calculations in this paper, the same program was used. To test the program, we have tried to reproduce two to three eigen values c_i and c_r vs α from Lessen, Singh and Paillet's published results (say Fig. 2). Their results were obtained when the numerical integration is performed starting from $r = 3$ to $r = 0.2$ for $q = 0.02$ and 0.03, $\alpha = 0.2$, $c_i = 0.045$ and 0.112, and $c_r = 0.385$ and 0.402.

The eigen values obtained for the three velocity profiles corresponding the various wave numbers α are similar to those calculated by Lessen, Singh and Paillet. For the azimuthal wave numbers n = 0 and 1, all the disturbances die out just as the results obtained by them for $q \sim 0(1)$. In this report we have calculated the eigen values for n = -1 and -2 only. These are given in Tables 1 and 2 respectively. Thus it is shown that these profiles are unstable corresponding to the non-axisymmetric disturbances. Cases 1, 2 and 3 corresponds to the velocity profiles numbering 2, 3 and 5 respectively in figures 2 and 3.

Table 1

Eigen values for n = -1, V = 1.095, β = 1 and Re = 200

	Case 1		Case 2		Case 3	
α	c_r	αc_i	c_r	αc_i	c_r	αc_i
0.1	0.385	0.056	--	--	--	--
0.2	0.462	0.074	0.295	0.033	--	--
0.3	0.539	0.085	0.378	0.054	0.301	0.023
0.4	0.581	0.091	0.425	0.063	0.338	0.041
0.5	0.630	0.093	0.516	0.069	0.395	0.052
0.6	0.661	0.092	0.563	0.070	0.461	0.052
0.7	0.695	0.087	0.620	0.063	0.533	0.045
0.8	0.721	0.073	0.661	0.048	0.596	0.027
0.9	0.778	0.051	0.695	0.020	--	--

Table 2

Eigen values for n = -2, V = 1.095, β = 1 and Re = 200

	Case 1	Case 2	Case 3
α	αc_i	αc_i	αc_i
0.6	0.084	0.072	0.060
0.8	0.118	0.104	0.091
1.0	0.139	0.122	0.111
1.2	0.141	0.129	0.120
1.4	0.135	0.124	0.101
1.6	0.112	0.099	0.075
1.8	0.052	--	--

REFERENCES

1. Grabowski, W.J. and Berger, S.A., Solution of the Navier-Stokes equations for vortex breakdown, J. Fluid Mech., vol. 75, 525-44, 1976.

2. Kurylowich, G., Analysis relating to Aircraft vortical wakes. AFFDL/FGC-TM-73-23, Feb. 1973.

3. Batchelor, G.K., Axial flow in trailing line vortices, J. Fluid Mech., vol. 14, 593-629, 1962.

4. Lavan, Z., Nielsen, H. and Fejer, A.A., Separation and flow reversal in swirling flow in circular ducts, Phys. Fluids, vol. 12, 1747-57, 1969.

5. Kopecky, R.M. and Torrance, K.E., Initiation and structure of axisymmetric eddies in a rotating stream, Comput. Fluids, vol. 1, 289-300, 1973.

6. Lord Rayleigh (J.W. Strutt), On the dynamics of revolving fluids, Scientific Papers, Cambridge Univ. Press, 6, 447-53, 1916.

7. Synge, J.L., On the stability of a viscous liquid between two rotating coaxial cylinders, Proc. Roy. Soc. Lond. A167, 250-256, 1938.

8. Howard, L.N. and Gupta, A.S., On the hydrodynamic and hydromagnetic stability of swirling flows, J. Fluid Mech. vol. 14, 463-76, 1962.

9. Crow, S.C., A.I.A.A., vol. 8, 2172, 1970

10. Widnall, S.E. and Bliss, D.B., J. Fluid Mech. vol. 50, 335, 1971.

11. Uberoi, M.S., Chow, C.Y. and Narain, J.P., Stability of coaxial rotating jet and vortex of different densities, Phys. Fluids, vol. 15, 1718, 1972.

12. Lessen, M., Deshpande and Hadji-Ohanes, Stability of a potential vortex with a non-rotating and rigid-body rotating top-hat core, J. Fluid Mech., 60, 459, 1973.

13. Lessen, M. Singh, P.J. and Paillet, F., The stability of a trailing line vortex. Part 1, Inviscid theory, J. Fluid Mech., 63, 753-63, 1974.

14. Lessen, M. and Paillet, F., The stability of a trailing line vortex, Part 2, Viscous theory, J. Fluid Mech., vol. 65, 769, 1974.

15. Singh, P.I. and Uberoi, M.S., Experiment on vortex stability, Phys. Fluids, vol. 19, 1858-63, 1976.

16. Garg, A.K., Oscillary behavior in vortex breakdown flows, An experimental study using a Laser Doppler Anemometer, MS thesis, Cornel Univ., Ithaca, 1977.

17. Hall, M.G., Vortex breakdown, Ann. Rev. Fluid. Mech., vol. 4, 195, 1972.

REFERENCES (Continued)

18. Hall, M.G., "The Structure of Concentrated Vortex Cores," Prog. Aeronaut. Sci., vol. 7, 53-110, 1966.

19. Leibovich, S., "The Structure of Vortex Breakdown," Ann. Rev. Fluid Mech., vol. 10, 221-246, 1978.

20. Singh, S.N. and Hankey, W.L., "On the Vortex Breakdown and Instability," AFWAL-TR-81-3021, Wright-Patterson AFB, March 1981.

21. Sarpkaya, T., "On Stationary and Travelling Vortex Breakdowns," J. Fluid Mech., vol. 45, 545-559, 1971.

22. Verma, G.R., Hankey, W.L. and Scherr, S.J., "Stability Analysis of the Lower Branch Solutions of the Falkner-Skan Equations," AFFDL-TR-79-3116, July 1979.

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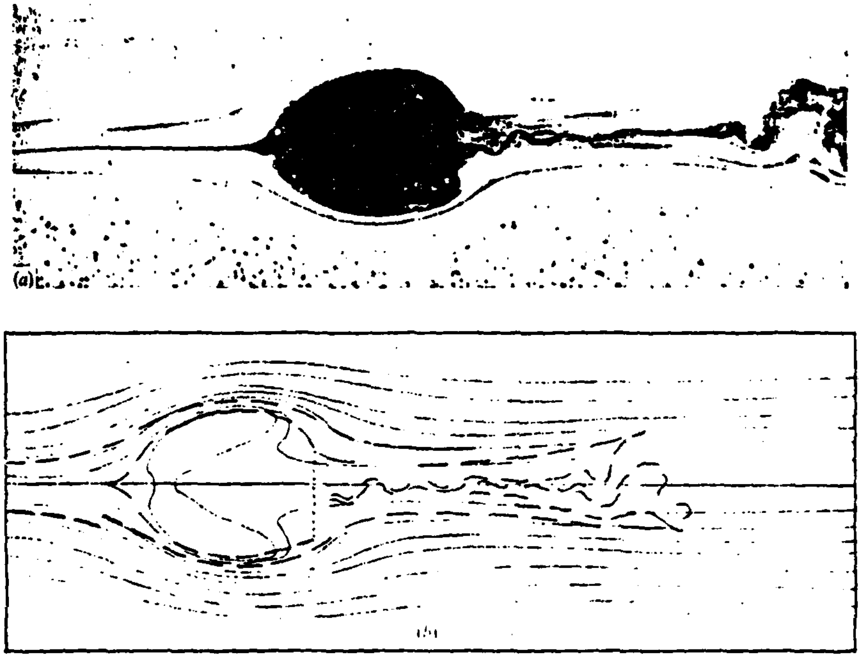

FIGURE 1 Axisymmetric bubble followed by spiral breakdown (from Sarpkaya 1971*a*). (*b*) Comparison of stream function contours, traced from (*a*), with those calculated for $Re = 200$, $\alpha = 1$, $V = 1{\cdot}095$. The latter are uniformly scaled, for best geometric agreement. Location of spiral breakdown roughly coincides with calculated secondary stream surface divergence.

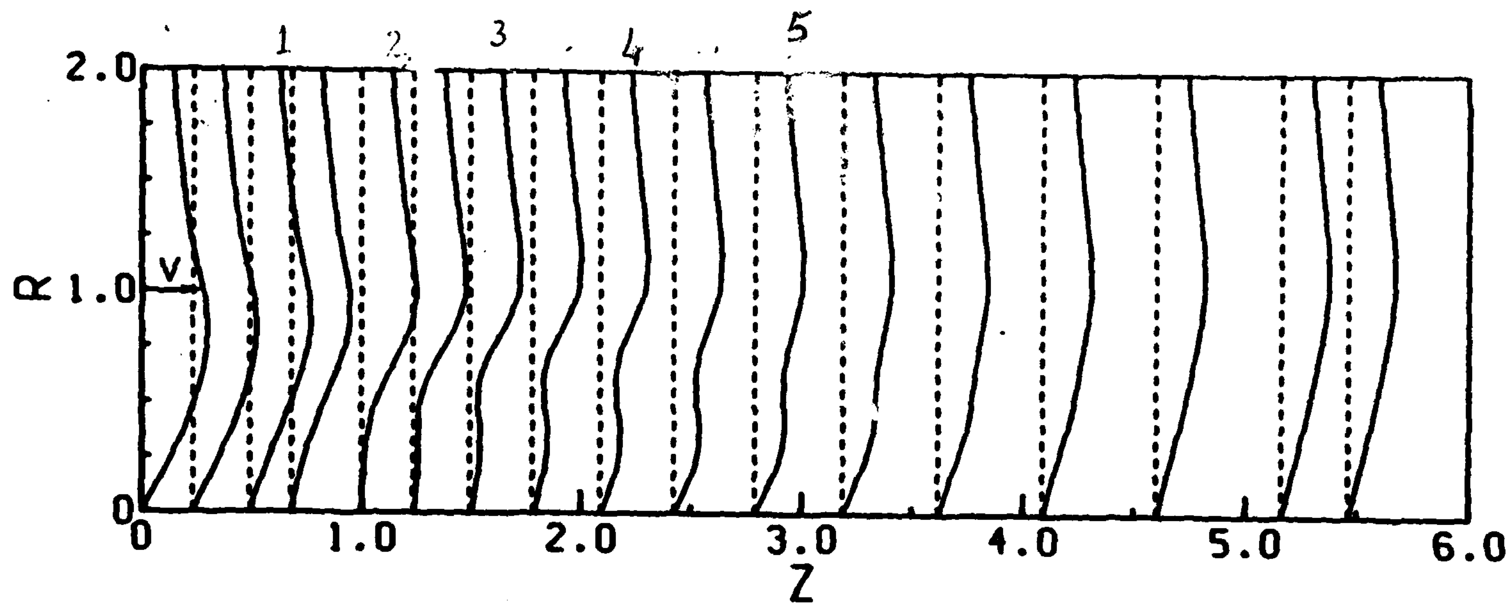

Fig. 2 SWIRL VELOCITY PROFILES -- Re = 200, α = 1.0, V = 1.095

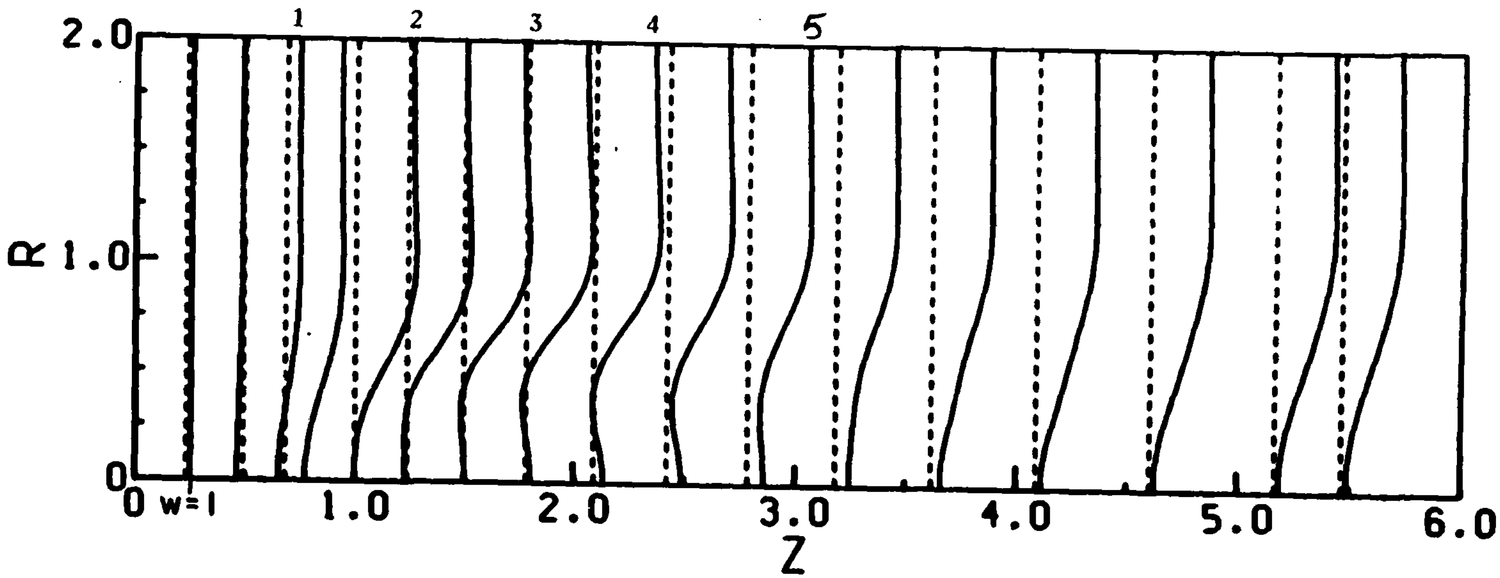

Fig. 3 AXIAL VELOCITY PROFILES -- Re = 200, α = 1.0, V = 1.095

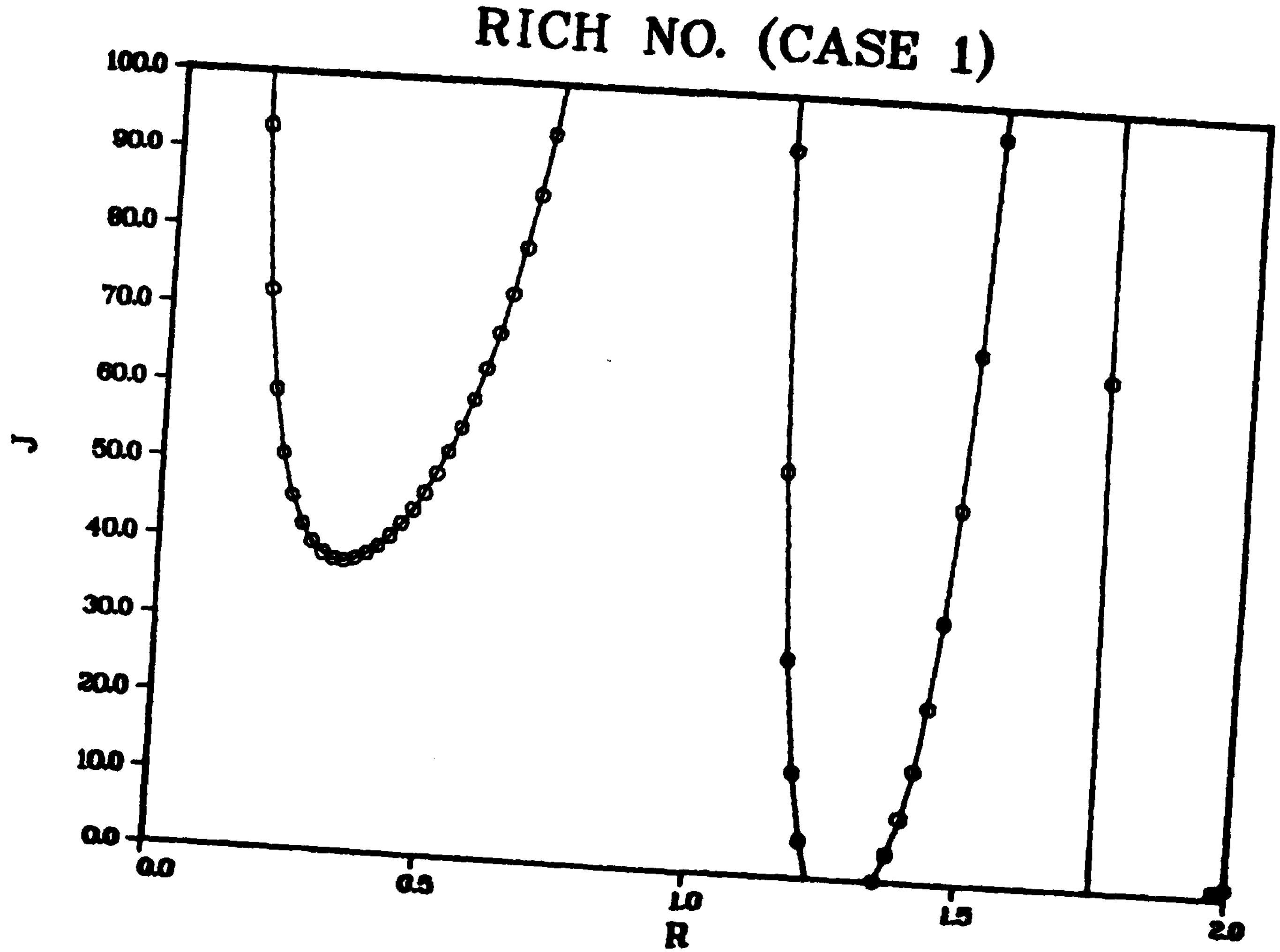
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Figure 4

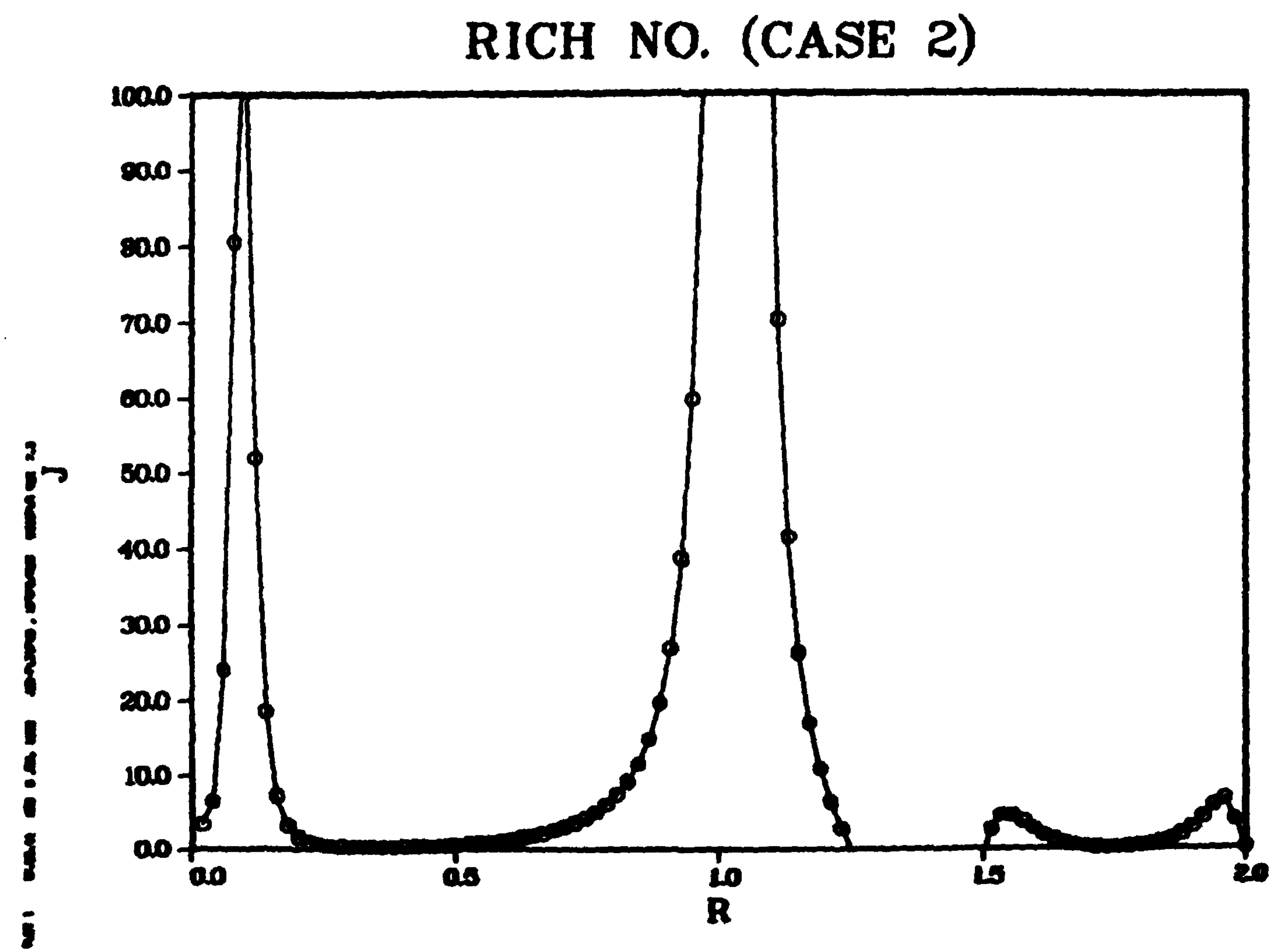
RICH NO. (CASE 2)
100.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
J
0.0
0.5
1.0
1.5
2.0
R

Figure 5

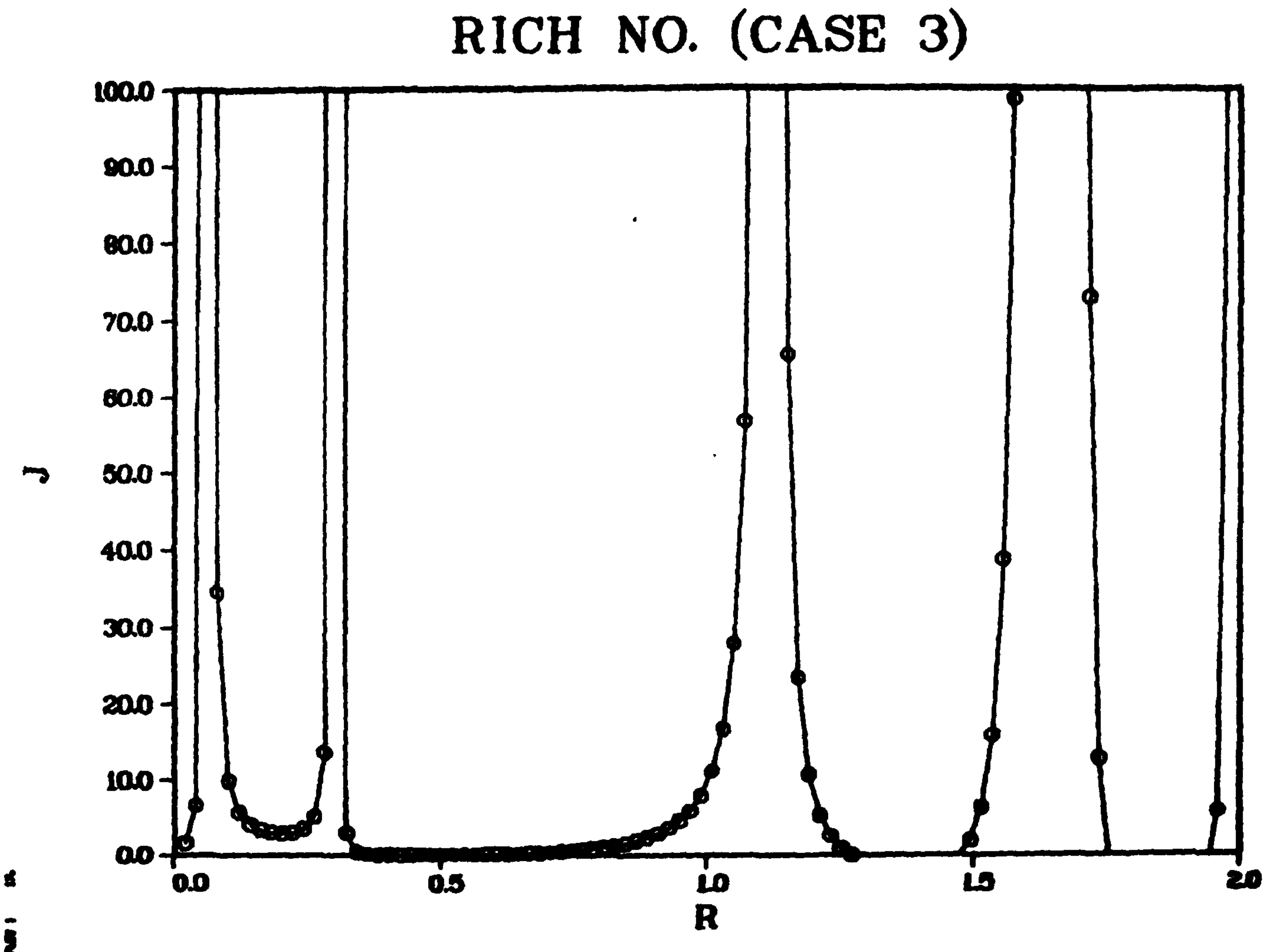
RICH NO. (CASE 3)
J
100.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
0.0
0.5
1.0
1.5
2.0
R

Figure 6

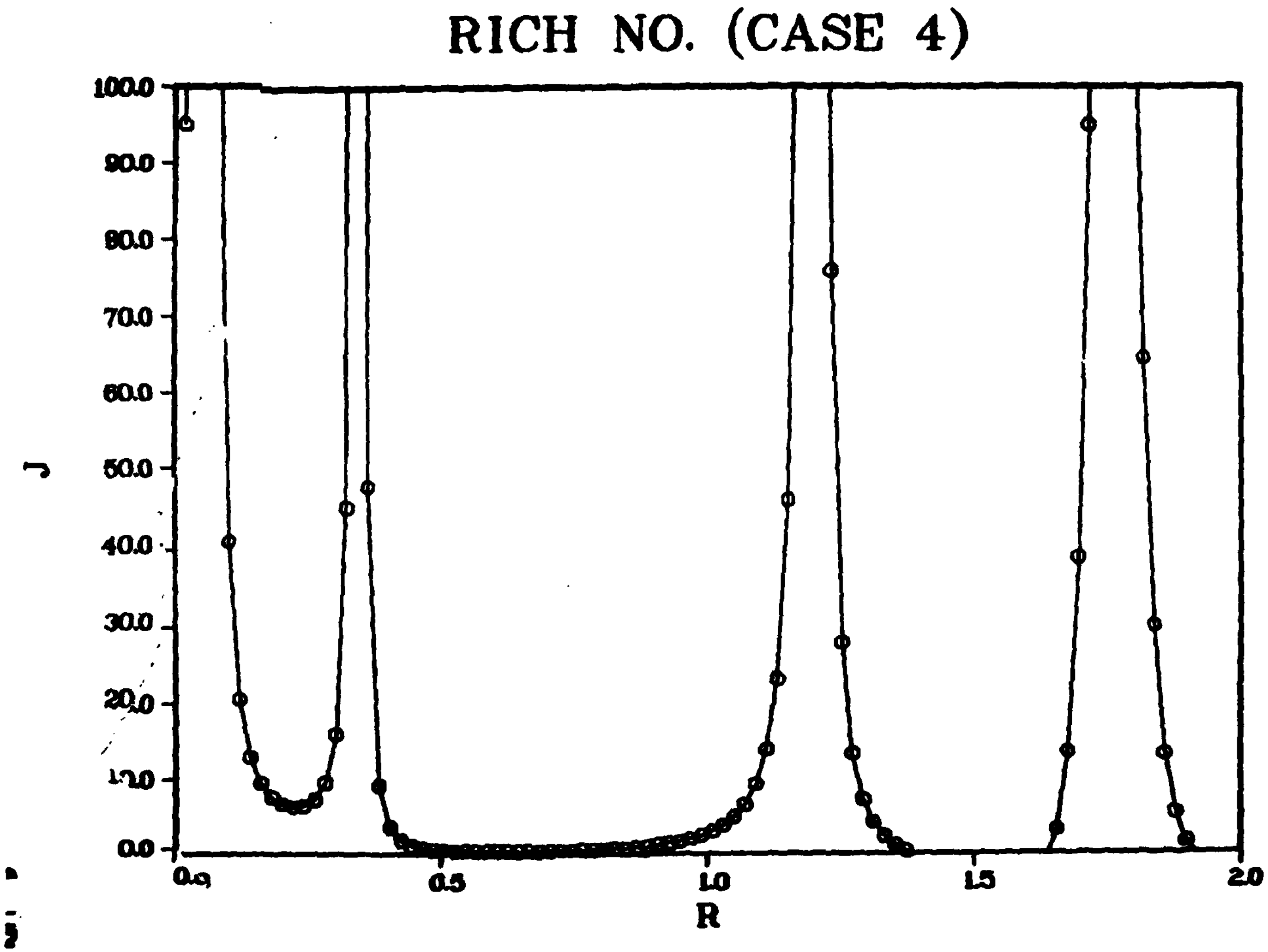
RICH NO. (CASE 4)
J
100.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
0.5
1.0
1.5
2.0
R

Figure 7

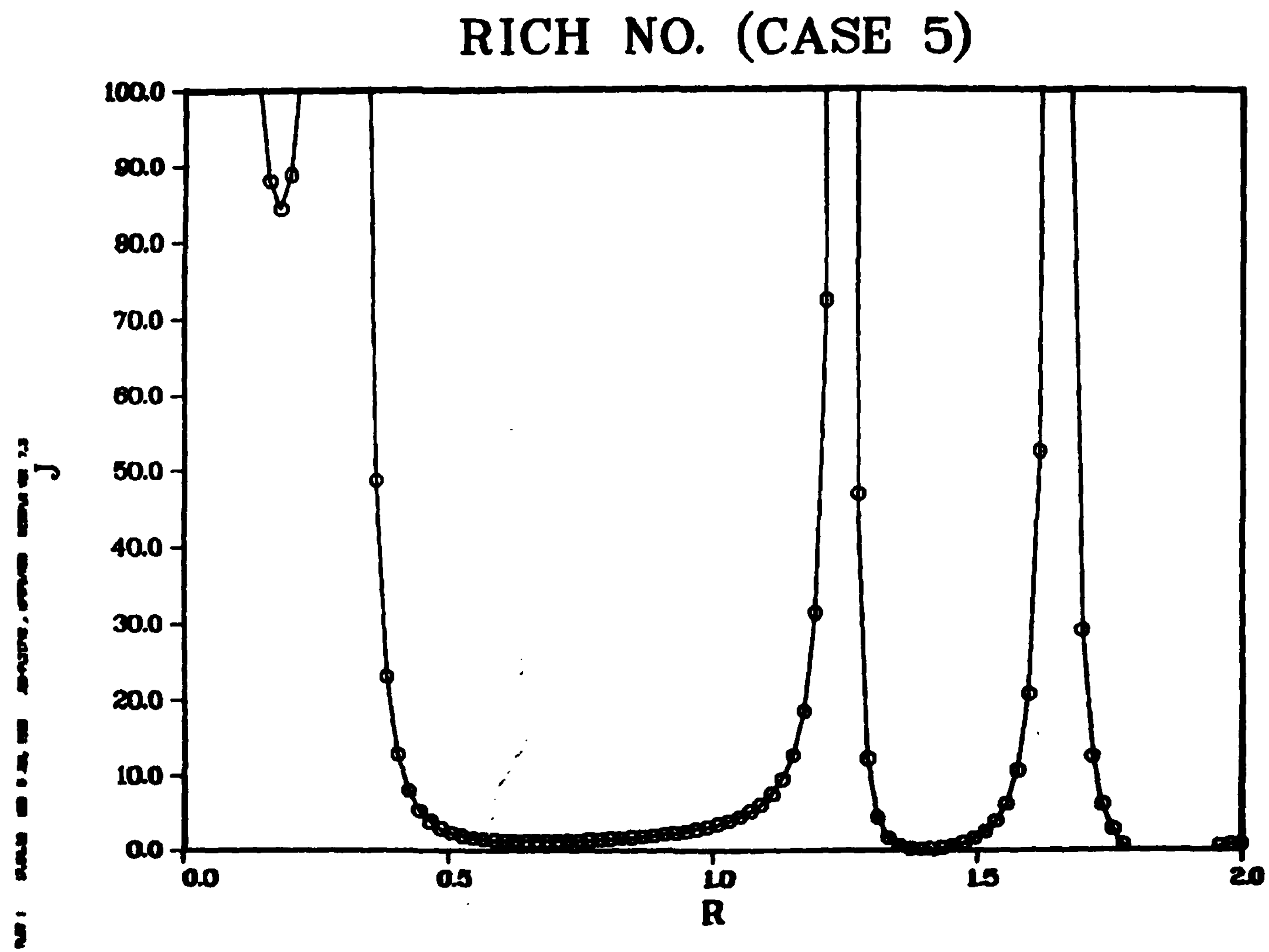
RICH NO. (CASE 5)
J
R
0.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
100.0
0.0
0.5
1.0
1.5
2.0

Figure 8