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A NUMERICAL INVESTIGATION OF THE DYNAMICS OF LIQUID SPRAY.(U)
MAR 82 P WEINACHT, J BUCHLIN AFOSR-81-0117

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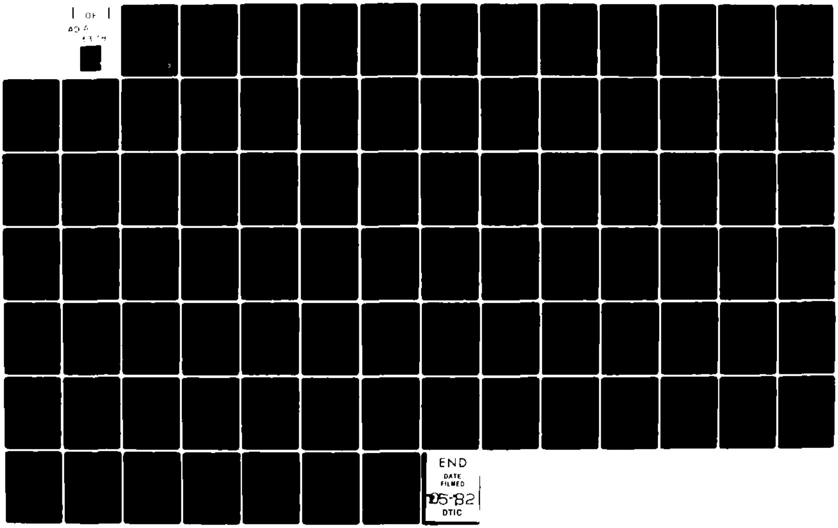

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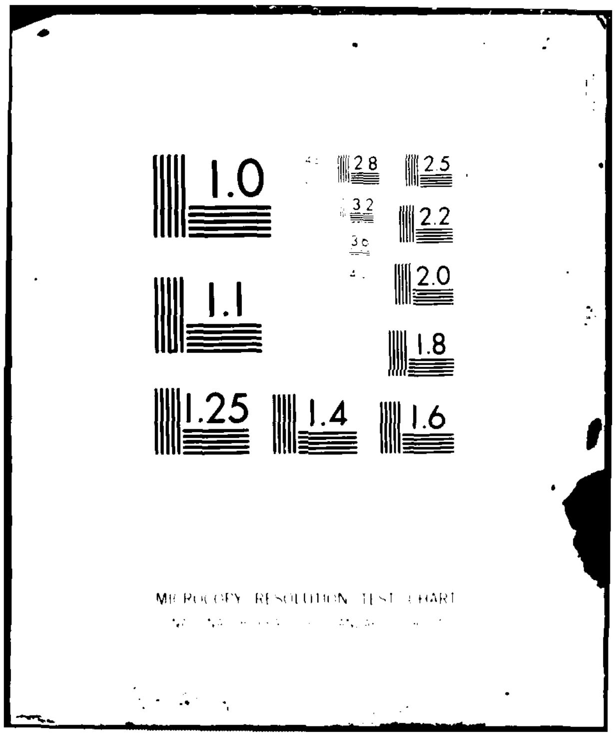
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1. Report Number	2. Govt Accession No.	READ INSTRUCTIONS BEFORE COMPLETING FORM 3. Recipient's Catalog Number
EOARD-TR-82-6		

4. Title (and Subtitle)	5. Type of Report & Period Covered
A numerical investigation of the dynamics of liquid spray	Final Scientific Report 1 Feb 81 - 31 Jan 82 6. Performing Org. Report Number
7. Author(s) Paul WEINACHT & J-M. BUCHLIN	8. Contract or Grant Number Grant AFOSR 81-0117
9. Performing Organization Name and Address von Karman Institute for Fluid Dynamics Chaussée de Waterloo, 72 B-1640 Rhode Saint Genèse, Belgium	10. Program Element, Project, Task Area & Work Unit Numbers P.E. 61102F Proj/Task 2301/D1 W.U. 121
11. Controlling Office Name and Address European Office of Aerospace Research and Development/LNT Box 14 FPO New York 09510	12. Report Date March 1982 13. Number of Pages 86
14. Monitoring Agency Name and Address European Office of Aerospace Research and Development/LNT Box 14 FPO New York 09510	15.

16. & 17. Distribution Statement

Approved for public release; distribution unlimited.

18. Supplementary Notes

19. Key Words

Liquid spray;Gas entrainment;Combustion chamber;Pollutant dispersal;Cooling Tower performance application

20. Abstract

An axisymmetric numerical model of a liquid spray is presented. The model consists of coupled sets of partial differential equations for the gas phase and ordinary differential equations for the liquid phase. A numerical method based on the MAC method is presented and calculations show good agreement with experiment. The effect of a ceiling on the entrainment performance of a spray is found to be negligible. The examination of the calculated pressure field shows that pressure does not play an important role in the flow and may be ignored. This is in agreement with experiment and contributes additional validation to existing models which ignore this effect.

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This technical report has been reviewed and is approved for publication.

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ACKNOWLEDGEMENTS

I would like to express my appreciation to my supervisor, Prof. J-M. Buchlin, for his help and encouragement during the course of this project and to the US Air Force for supporting this research.

G. Borriello's ideas and experience were of much help in completing this work.

Also thanks go to Mme Toubeau and Miss Klinkenbergh for their help during the year in the library.

As always I would like to thank my parents for their constant love and encouragement.

Finally, special thanks to Jan Wooten for helping draw the arrow heads on the flow field figures and for making sure I got enough exercise chasing the last train to Uccle-Calevoet.

TABLE OF CONTENTS

ABSTRACT

An axisymmetric numerical model of a liquid spray is presented. The model consists of coupled sets of partial differential equations for the gas phase and ordinary differential equations for the liquid phase. A numerical method based on the MAC method is presented and calculations show good agreement with experiment. The effect of a ceiling on the entrainment performance of a spray is found to be negligible. The examination of the calculated pressure field shows that pressure does not play an important role in the flow and may be ignored. This is in agreement with experiment and contributes additional validation to existing models which ignore this effect.

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

C_D	drag coefficient of a sphere
C_N	nozzle design parameter
d_0	inside diameter of nozzle (mm)
D	droplet diameter (mm)
D_t	diameter of spray envelope (m)
f_p	fraction of mass flow through the nozzle assigned to given trajectory
f_r	radial component of drag force acting on droplet (N)
f_z	axial component of drag force acting on droplet (N)
F_r	radial volume force term of momentum source (N/m^3)
Fr_{av}	average radial force acting on a particle in source cell (N)
F_z	axial volume force term of momentum source (N/m^3)
g	acceleration of gravity (m/s^2)
H_ℓ	spray height (m)
m	droplet mass (kg)
N	unit normal to boundary
N_p	number of particles in source cell at given instant in time
P	pressure (N/m^2)
ΔP_N	water delivery pressure (N/m^2)
Q_a	volume flow of air entrained into spray (m^3/min)
Q_w	volume flow of liquid through spray nozzle (ℓ/min)
r	variable of integration inside spray envelope
r_0	radial coordinate of particle trajectory
R	radial coordinate
Re	Reynolds number
RE	Reynolds number of nondimensionlization

t	time	(sec)
u_r	radial velocity of particle	(m/s)
u_z	axial velocity of particle	(m/s)
U_0	particle injection velocity	(m/s)
V_r	radial component of velocity	(m/s)
V_z	axial component of velocity	(m/s)
Z	axial coordinate	(m)
Z_0	axial coordinate of particle trajectory	(m)
ρ	density of gas	(kg/m^3)
ρ_L	density of liquid	(kg/m^3)
θ	initial angle of ejection of aprticle	(degrees)
θ_{max}	maximum initial angle of particle ejection	(degrees)
η	efficiency of spray	
δs	arc length of trajectory in source cell	
δt	time for particle to pass through source cell	
ν	kinematic viscosity of gas	(m^2/s)
Δ	indicates incremental quantity	

1. INTRODUCTION

The application of liquid sprays is made in many different fields. Examples of a few are fire suppression, combustion, ventilation,and the dispersion of heavy, possibly toxic or flammable, gases. Interest at VKI in the applicability of water sprays for use as water curtains for the dispersion of heavy gases has mandated a need for better prediction methods for the analysis/design of water sprays.

The configuration for the spray to be analysed in this study is shown in figure 1. Liquid particles are ejected through a nozzle into an environment which is at rest in the absence of the spray. The aerodynamic drag acting on the liquid particles results in a loss in momentum of the particles. Since momentum must be conserved, the momentum loss is translated into a momentum gain by the fluid, causing the fluid to be set in motion. Air is thereby entrained into the spray (in much the same way as a jet) resulting in a two phase plume.

Modeling of this phenomenon has been done in the past. A one dimensional model (Refs. 3,5) has been developed at VKI and has shown good results though its applicability for the inclusion of boundary effects is limited. Two dimensional models, existing in the literature (Refs. 1,4,7), are much more able to handle the effects of boundaries. In this light, it is found desirable to have such a two dimensional model at VKI to complement the range of applicability of the existing one dimensional model. Thus, this project concerns the implementation of an axisymmetric numerical model into a computer code. The basic approach is similar to that followed in references 1 and 4.

In this report the model used is outlined, the numerical method for the solution of the model equations is described, and, finally, the results presented and discussed. The implemented computer code is included as an appendix to this report.

2. THE SPRAY MODEL

2.1 Introduction

The model of the spray consists of two distinct sets of equations, one set governing the gas phase and another governing the liquid phase. Linkage between these two sets of equations accounts for the following two physical phenomena : the first concerns the momentum transfer between liquid phase and gas phase, while the second involves the modification of the particle trajectories by the motion of the gas phase.

The gas phase may be modelled as a continuum using an Eulerian approach while the liquid phase, using a Lagrangian approach is modelled by considering a finite set of particles of varying size/initial trajectory.

2.2 The gas phase model

2.2.1 Gas phase equations

Since the gas phase occupies the most significant portion of the flow domain, it may be treated as a continuum. Making the standard assumptions of incompressibility and Newtonian fluid the following form of the Navier Stokes equations may be used. Here, due to the axisymmetric nature of the problem, the equations are written in cylindrical coordinates :

Continuity

$$\frac{1}{R}\frac{\partial}{\partial R}(RV_r) + \frac{\partial V_z}{\partial z} = 0 \qquad (2.1)$$

Momentum

r-component

$$V_r \frac{\partial V_r}{\partial R} + V_z \frac{\partial V_r}{\partial z} = - \frac{\partial \frac{P}{\rho}}{\partial R} + \nu \left[\frac{\partial}{\partial R} \left\{ \frac{1}{P} \frac{\partial}{\partial R} (RV_r) \right\} + \frac{\partial^2 V_r}{\partial z^2} \right] + F_r \quad (2.2)$$

z-component

$$V_r \frac{\partial V_z}{\partial R} + V_z \frac{\partial V_z}{\partial z} = - \frac{\partial \frac{P}{\rho}}{\partial z} + \nu \left[\frac{1}{R} \frac{\partial}{\partial R} \left\{ R \frac{\partial V_z}{\partial R} \right\} + \frac{\partial^2 V_z}{\partial z^2} \right] + F_z \quad (2.3)$$

where V_r, V_z, P are the radial and axial component of velocity, and the pressure respectively; R and z are the radial and axial coordinate and ρ and ν represent the fluid density and kinematic viscosity. The terms F_r and F_z are the volume force terms which account for the contribution of momentum per unit volume from the liquid phase.

2.2.2 Boundary conditions for the gas phase

Considered here are three types of boundaries; a single axis of symmetry boundary, free boundaries and wall boundaries. These are shown in figure 1.

Velocity boundary conditions

Axis of symmetry

At the axis of symmetry the radial component of velocity must be equal to zero because of mass conservation and the symmetry assumption. Symmetry also requires that there be no shear stress in the axial direction, requiring the derivative of the axial velocity with respect to the radial coordinate to be zero.

$$V_r = 0 \tag{2.4}$$

$$\frac{\partial V_z}{\partial R} = 0 \tag{2.5}$$

Wall boundary

Along the wall boundaries the usual conditions for an impermeable, non-slip wall are used :

$$V_r = V_z = 0 \tag{2.6}$$

Free boundary

In the flow domain considered the free boundary is taken to be far from the spray domain. With this assumption the following approximate conditions may be applied. The velocity tangential to the boundary is small, and may be assumed equal to zero. Applying continuity, the boundary condition for the component of velocity normal to the boundary may be obtained.

Vertical free boundary

$$V_z = 0 \tag{2.7}$$

$$\frac{\partial (RV_r)}{\partial R} = 0 \tag{2.8}$$

Horizontal free boundary

$$V_r = 0 \tag{2.9}$$

$$\frac{\partial V_z}{\partial R} = 0 \tag{2.10}$$

Pressure boundary conditions

The pressure boundary conditions may be derived from the momentum equations 2.2 and 2.3. Since most gases have small viscosity it is permissible here to ignore the viscous terms of the momentum equations in deriving the pressure boundary conditions.

The resulting derivation shows that for the wall and axis of symmetry boundaries, the normal derivative of the pressure is equal to zero.

The resulting boundary condition on the free boundary is a bit more complicated using this approach. However, making use of experimental observation which has shown that variation of the pressure field is small throughout the flow domain, the pressure at the free boundary is assumed to be constant.

Wall or axis of symmetry

$$\frac{\partial P}{\partial N} = 0 \qquad (2.11)$$

Free boundary

$$P = P_{ref} \qquad (2.12)$$

2.3 The particle phase model

2.3.1 Particle equations

The liquid phase is modelled as a distribution of droplets of distinct sizes and trajectories. Using a Lagrangian approach individual particles are followed from injection until hitting the floor. By considering the number of particles of similar sizes/trajectories, the magnitude of droplet-gas momentum exchange throughout the flow field may be determined. This approach assumes that the particles do not interact with each other, i.e., no collisions or particle break up. Making the additional assumptions that the particles are spherical and non-evaporating, the following equations of motion may be written.

$$m \frac{du_r}{dt} = - f_r \qquad (2.13)$$

$$m \frac{du_z}{dt} = - f_z \pm mg \qquad (2.14)$$

$$\frac{dr_0(t)}{dt} = u_r \qquad (2.15)$$

$$\frac{dz_0(t)}{dt} = u_z \qquad (2.16)$$

where u_r, u_z are the particle radial and axial velocities and r_0 and z_0 are the radial and axial position of the particles. "m" is the mass of the particle calculated as follows :

$$m = \rho_L \frac{\pi}{6} D^3 \qquad (2.17)$$

where ρ_L is the density of the liquid and D is the particle diameter. f_r and f_z are the radial and axial component of drag force, related to F_r and F_z in equations 2.2 and 2.3 and are calculated as follows :

$$f_r = C_D \; Re \; \frac{\pi D \nu \;\; \rho}{8} \; (u_r - V_r) \qquad (2.18)$$

$$f_z = C_D \; Re \; \frac{\pi D \nu \;\; \rho}{8} \; (u_z - V_z) \qquad (2.19)$$

Re is the Reynolds number defined as

$$Re = \frac{\sqrt{(u_z - V_z)^2 + (u_r - V_r)^2}\ D}{\nu} \tag{2.20}$$

The drag coefficient C_D is calculated using a standard form fit for the drag coefficient of a sphere :

$$C_D = \frac{24}{Re} + \frac{6.}{1+\sqrt{Re}} + .4 \tag{2.21}$$

2.3.2 Initial conditions for the particle equations

The initial conditions are derived from the properties of the spray nozzle. By definition the coordinate system is fixed at the nozzle so the initial conditions for equations 2.15 and 2.16 are simply :

$$r_0(0) = z_0(0) = 0 \tag{2.22}$$

The initial conditions for the force equations are derived by considering the particle ejection velocity from the nozzle and the initial angle of ejection (see figure 2) :

$$u_r(0) = U_0 \sin\theta$$

$$u_z(0) = U_0 \cos\theta \tag{2.23}$$

U_0 is calculated from the volume flow of the nozzle, Q_W :

$$U_0 = \frac{Q_W}{\frac{\pi}{4}\, d_0^2} \tag{2.24}$$

where d_0 is the inside diameter of the nozzle. The initial angle of ejection of the particle must be between 0 and θ_{max}. θ_{max} and d_0 are available from the manufacturers data.

2.3.3 Particle size

Particle size i´ calculated by using the following expression

$$D = C_N \frac{d_0^{2/3}}{\Delta P_N^{1/3}} \tag{2.25}$$

where C_N is a nozzle design parameter and ΔP_N is the water delivery pressure. These values may be determined from experiment or from manufacturers data.

2.4 Nondimensionalization of the model equations

To allow the resulting program to be used easily with any system of units, the model equations are nondimensionalized.

Velocities are nondimensionalized by the particle ejection velocity

$$\hat{V}_r = \frac{V_r}{U_0}, \quad \hat{V}_z = \frac{V_z}{U_0}, \quad \hat{u}_r = \frac{u_r}{U_0}, \quad \hat{u}_z = \frac{u_z}{U_0} \tag{2.26}$$

Pressure is non dimensionalized by the gas density, and the particle ejection velocity squared

$$\hat{P} = \frac{P}{\rho U_0^2} \tag{2.27}$$

Lengths are nondimensionalized by the spray height, H_ℓ, (see figure 1)

$$\hat{R} = \frac{R}{H_\ell}, \quad \hat{z} = \frac{z}{H_\ell}, \quad \hat{r}_0 = \frac{r_0}{H_\ell}, \quad \hat{z}_0 = \frac{z_0}{H_\ell} \tag{2.28}$$

Time is nondimensionalized by the spray height divided by the ejection velocity

$$\hat{t} = \frac{tU_0}{H_\ell} \tag{2.29}$$

The momentum source terms are nondimensionalized by the ejection velocity squared divided by the spray height

$$\hat{F}_r = F_r \frac{H_\ell}{U_0^2}, \qquad \hat{F}_z = F_z \frac{H_\ell}{U_0^2} \tag{2.30}$$

Thus in the momentum equations (2.2) and (2.3), the viscosity is replaced by the reciprocal of the "Reynolds number of nondimensionalization"

$$\frac{1}{RE} = \frac{\nu}{U_0 H_\ell} \tag{2.31}$$

The nondimensionalization of the other model equations is straightforward. From this point on, the model equations are assumed nondimensional, and the hat (^) is neglected.

3. THE NUMERICAL METHOD

3.1 Introduction

It is difficult indeed to conceptualize a numerical method for the simultaneous solution of both sets of equations. A more natural method of solution involves iterative solution of these equations. Such an iterative solution is outlined by the flow chart in figure 3.

The specific numerical method to solve each set of equations differs due to their nature (ODE, PDE) and are considered in appropriate sections.

3.2 Numerical solution of the particle equations

The solution of the set of ordinary differential equations (ODE) governing the particle phase is made by a simultaneous fourth order Runge-Kutta technique which is well documented in the literature (Ref. 9).

3.3 Numerical solution of the gas equations

The numerical scheme for the solution of the gas phase equations is based on the MAC method (Refs. 8,10,11).

The gas phase equations are recast into the following form

$$\frac{\partial V_r}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}(V_r^2 R) + \frac{\partial (V_r V_z)}{\partial z} = -\frac{\partial P}{\partial R} + \frac{1}{RE}\left[\frac{\partial^2 V_r}{\partial R^2} + \frac{\partial}{\partial R}\left(\frac{V_r}{R}\right) + \frac{\partial^2 V_r}{\partial z^2}\right] + F_r \tag{3.1}$$

$$\frac{\partial V_z}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}(V_r V_z R) + \frac{\partial V_z^2}{\partial z} = -\frac{\partial P}{\partial z} +$$

$$+ \frac{1}{RE}\left[\frac{1}{R}\frac{\partial V_z}{\partial R} + \frac{\partial^2 V_z}{\partial R^2} + \frac{\partial^2 V_z}{\partial z^2}\right] + F_z \qquad (3.2)$$

$$\frac{1}{R}\frac{\partial P}{\partial R} + \frac{\partial^2 P}{\partial R^2} + \frac{\partial^2 P}{\partial z^2} = -\frac{\partial D}{\partial t} - \frac{\partial^2 (V_r)^2}{\partial R^2} - \frac{\partial^2 (V_z)^2}{\partial z^2}$$

$$-\frac{2}{R}\frac{\partial}{\partial R}(V_r)^2 - \frac{2}{R}\frac{\partial}{\partial z}(V_z V_r) - 2\frac{\partial^2 (V_r V_z)}{\partial R \partial z} +$$

$$+ \frac{1}{RE}\left[\frac{1}{R}\frac{\partial D}{\partial R} + \frac{\partial^2 D}{\partial R^2} + \frac{\partial^2 D}{\partial z^2}\right] + \frac{1}{R}\frac{\partial (RF_r)}{\partial R} + \frac{\partial F_z}{\partial z} \qquad (3.3)$$

These equations are written in unsteady form as the MAC solution procedure is an iterative one in which the steady state solution is the desired solution. To be noted here is the exchange of the continuity equation for a Poisson equation for the pressure. Contained in this Poisson equation is the variable D which is a dilation term representing the amount of continuity existing.

$$D = \frac{1}{R}\frac{\partial (V_r R)}{\partial R} + \frac{\partial V_z}{\partial z} \qquad (3.4)$$

This term is used to reduce the nonlinear instabilities existing in the numerical solution of the Navier-Stokes equations (Ref. 11). Continuity is solved implicitly using this term.

3.3.1 Discretization of the MAC method equations

An appropriate discretization of the MAC method equation on a staggered grid (see Fig. 4) is given below :

$$\tilde{V}_{r_{i+1/2,j}} = V_{r_{i+1/2,j}} + \Delta t \Big\langle - \frac{1}{R_{i+1/2}\Delta R} \left(V^2_{r_{i+1,j}} R_{i+1} - V^2_{r_{i,j}} R_i \right)$$

$$- \frac{1}{\Delta z} \left(V_r V_{z_{i+1/2,j+1/2}} - V_r V_{z_{i+1/2,j-1/2}} \right)$$

$$- \frac{1}{\Delta R} \left(P_{i+1,j} - P_{i,j} \right) + \frac{1}{RE} \left[\frac{1}{2\Delta R} \left(\left(\frac{V_r}{R} \right)_{i+3/2,j} \right. \right.$$

$$\left. - \left(\frac{V_r}{R} \right)_{i-1/2,j} \right) + \frac{V_{r_{i+3/2,j}} - 2V_{r_{i+1/2,j}} + V_{r_{i-1/2,j}}}{\Delta R^2}$$

$$\left. + \frac{V_{r_{i+1/2,j+1}} - 2V_{r_{i+1/2,j}} + V_{r_{i+1/2,j-1}}}{\Delta z^2} \right] + F_r \Big\rangle \qquad (3.5)$$

$$\tilde{V}_{z_{i,j+1/2}} = V_{z_{i,j+1/2}} + \Delta t \left\{ \frac{-1}{R_i \Delta R} \left(V_r V_{z_{i+1/2,j+1/2}} R_{i+1/2} - V_r V_{z_{i-1/2,j+1/2}} R_{i-1/2} \right) - \frac{1}{\Delta z} \left(V^2_{z_{i,j+1}} - V^2_{z_{i,j}} \right) \right.$$

$$- \frac{1}{\Delta z} \left(P_{i,j+1} - P_{i,j} \right) +$$

$$+ \frac{1}{RE} \left[\frac{1}{R_i} \left(\frac{V_{z_{i+1,j+1/2}} - V_{z_{i-1,j+1/2}}}{2\Delta R} \right) \right.$$

$$+ \frac{V_{z_{i+1,j+1/2}} - 2V_{z_{i,j+1/2}} + V_{z_{i-1,j+1/2}}}{\Delta R^2}$$

$$\left. \left. + \frac{V_{z_{i,j+1/2}} - 2V_{z_{i,j+1/2}} + V_{z_{i,j-1/2}}}{\Delta z^2} \right] + F_z \right\} \tag{3.6}$$

$$\frac{1}{R_i} \frac{\tilde{P}_{i+1,j} - \tilde{P}_{i-1,j}}{2\Delta R} + \frac{\tilde{P}_{i+1,j} - 2\tilde{P}_{i,j} + \tilde{P}_{i-1,j}}{\Delta R^2} + \frac{\tilde{P}_{i,j+1} - 2\tilde{P}_{i,j} + \tilde{P}_{i,j-1}}{\Delta z^2} =$$

$$= - \frac{\tilde{V}^2_{r_{i+1,j}} - 2\tilde{V}^2_{r_{i,j}} + \tilde{V}^2_{r_{i-1,j}}}{\Delta R^2} - \frac{\tilde{V}^2_{z_{i,j+1}} - 2\tilde{V}^2_{z_{i,j}} - \tilde{V}^2_{z_{i,j-1}}}{\Delta z^2}$$

$$- \frac{2}{R_i} \frac{V^2_{r_{i+1,j}} - V^2_{r_{i-i,j}}}{2\Delta R} - \frac{2}{R_i} \frac{V_z V_{r_{i,j+1}} - V_z V_{r_{i,j-1}}}{2\Delta z}$$

$$- 2 \frac{V_r V_{z_{i+1/2,j+1/2}} + V_r V_{z_{i-1/2,j-1/2}} - V_r V_{z_{i+1/2,j-1/2}} - V_r V_{z_{i-1/2,j+1/2}}}{\Delta R \Delta z}$$

$$+ \frac{1}{RE} \left[\frac{1}{R_i} \frac{D_{i+1,j} - D_{i-1,j}}{2\Delta R} + \frac{D_{i+1,j} - 2D_{i,j} + D_{i-1,j}}{\Delta R^2} + \frac{D_{i,j+1} - 2D_{i,j} + D_{i,j-1}}{\Delta z^2} \right]$$

$$+ \frac{D_{i,j}}{\Delta t} + \frac{1}{R_i} \left(\frac{R_{i+1/2} F_{r_{i+1/2,j}} - R_{i-1/2} F_{r_{i-1/2,j}}}{\Delta R} \right) + \frac{F_{z_{i,j+1/2}} - F_{z_{i,j-1/2}}}{\Delta z} \tag{3.7}$$

Here the tilde (~) indicates the updated quantities (N+1 iteration).

As can be seen, the MAC method is a two level scheme involving explicit solution of equations 3.5 and 3.6 for the velocities of the N+1 iteration and an iterative solution of equations 3.7 for the updated pressure field.

Note the discretization of the $\frac{\partial D}{\partial t}$ term. D^{N+1} is set equal to zero in an attempt to force continuity to exist at the N+1 time step. At steady state the time derivative term disappears and continuity will be satisfied, thereby solving the original problem.

This method was applied to the solution of laminar flow in a pipe as a test case. Solutions were obtained for low Reynolds numbers (Re < 100) but instabilities were observed for higher Reynolds numbers.

Because the solution of the gas equations involves a low gas viscosity ($\nu = 1.5 \times 10^{-5}$ m^2/s) and therefore high Reynolds numbers, the numerical method has to be modified.

3.3.2 Modification to the MAC method

To remove the instabilities upwind differencing was employed. This is not so straightforward for the staggered grid used and involves some additional approximations. The resulting discretization of the momentum equations is made below. Here only the convective terms are affected and therefore are the only ones considered.

$$\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + V_z \frac{\partial V_r}{\partial z}$$

$$\cong \frac{\tilde{V}_{r_{i+1/2,j}} - V_{r_{i+1/2,j}}}{\Delta t}$$

$$+ V_{r_{i+1/2,j}} \begin{cases} \dfrac{V_{r_{i+1/2,j}} - V_{r_{i-1/2,j}}}{\Delta r}, & V_{r_{i+1/2,j}} > 0 \\[2ex] \dfrac{V_{r_{i+3/2,j}} - V_{r_{i+1/2,j}}}{\Delta r}, & V_{r_{i+1/2,j}} < 0 \end{cases}$$

$$\text{VZAUE}\begin{cases}\dfrac{V_{r_{i+1/2,j+1}}-V_{r_{i+1/2,j}}}{\Delta z}, & \text{VZAUE} > 0\\[2ex] \dfrac{V_{r_{i+1/2,j}}-V_{r_{i+1/2,j-1}}}{\Delta z}, & \text{VZAUE} < 0\end{cases}$$

$$\text{VZAUE} = \frac{1}{4}\left\{V_{z_{i,j+1/2}}+V_{z_{i+1,j+1/2}}+V_{z_{i,j-1/2}}+V_{z_{i+1,j-1/2}}\right\} \quad (3.8)$$

$$\frac{\partial V_z}{\partial t} + V_r\frac{\partial V_z}{\partial r} + V_z\frac{\partial V_z}{\partial z}$$

$$\cong \frac{\tilde{V}_{z_{i,j+1/2}}-V_{z_{i,j+1/2}}}{\Delta t} +$$

$$+\ \text{VRAUE}\begin{cases}\dfrac{V_{z_{i+1,j+1/2}}-V_{z_{i,j+1/2}}}{\Delta R}, & \text{VRAUE} > 0\\[2ex] \dfrac{V_{z_{i,j+1/2}}-V_{z_{i-1,j+1/2}}}{\Delta R}, & \text{VRAUE} < 0\end{cases}$$

$$+V_{z_{i,j+1/2}}\begin{cases}\dfrac{V_{z_{i,j+3/2}}-V_{z_{i,j+1/2}}}{\Delta z}, & V_{z_{i,j+1/2}} > 0\\[2ex] \dfrac{V_{z_{i,j+1/2}}-V_{z_{i,j-1/2}}}{\Delta z}, & V_{z_{i,j+1/2}} < 0\end{cases} \quad (3.9)$$

The pressure equation remains the same.

This modification allowed a stable solution to be obtained, though the scheme did not allow the lower wall to be sensed, and no recirculation occurred (see Fig. 5). There another modification was employed. This involved changing the unsteady term in the momentum equations along the lower and axis of symmetry boundaries.

$$\frac{\partial V_z}{\partial t} \cong \frac{\tilde{V}_{z_{i,j+1/2}} - \frac{1}{2}\left(V_{z_{i,j+3/2}} + V_{z_{i,j-1/2}}\right)}{\Delta t} \tag{3.10}$$

on lower boundary,

$$\frac{\partial V_r}{\partial t} \cong \frac{\tilde{V}_{r_{i+1/2,j}} - \frac{1}{2}\left(V_{r_{i+3/2,j}} + V_{r_{i-1/2,j}}\right)}{\Delta t} \tag{3.11}$$

on axis of symmetry boundary.

This finally yielded plausible results.

3.3.3 Solution procedure for the MAC method

The solution procedure for the MAC method, as mentioned previously, involves an explicit solution of equations (3.5) and (3.6) with the modifications for the velocity field at the N+1 iteration and an iterative solution of (3.7) for the pressure field at the N+1 iteration. This iterative solution was made using a point by point SOR method. The overrelaxation factor used was $\omega = 1.5$, though no optimization has been done on this parameter.

3.4 Calculation of the source terms in the Navier-Stokes equations

For the calculation of the source term, the particle trajectories are superimposed over the mesh used for the solution of the gas equations, (see figure 6). To calculate the source terms the arc length of each particle trajectory in a given cell is determined. Because the velocity along this trajectory is known, the time that a particle spends in this cell can be determined :

$$\delta t = \int \frac{ds}{\sqrt{V_r^2+V_z^2}} \cong \frac{\delta s}{\sqrt{V_r^2+V_z^2}} \qquad (3.12)$$

for small δs and gradually varying V_r and V_z.

From this time the average number of particles in the cell is determined by considering the fraction of the mass flow through the nozzle assigned to this particular trajectory, f_p. The average force along the arc length is determined and then the contribution to the momentum source term along this trajectory in this particular cell is found as follows.

Number of particles on given trajectory in control volume :

$$N_p = \frac{Q_w . f_p . \delta t}{\frac{\pi}{6} D^3} \qquad (3.13)$$

$$F_r = \frac{F_{rav} \times N_p}{\text{Volume of cell}} \qquad (3.14)$$

similarly for F_z.
It should be noted that F_r and F_z are not calculated in the same cell because of the staggered grid (see figure 4).

4. RESULTS

Results were obtained for the consideration of the effect of increase of mass flow through the nozzle and the effect of a ceiling on entrainment properties of the spray, and finally, to examine the pressure field. The results were obtained by using the properties of a Lechler SZ1 spray nozzle as input to the program (see Table 1).

4.1 Comparison with experiment

First to validate the program a comparison of the entrainment properties of the calculated spray was made with the experimental results of an unconfined spray. These results are shown in figure 7. Here the entrainment efficiency is plotted versus the inverse of the non dimensional spray envelope diameter squared. The entrainment efficiency is defined as the volume flow of air entrained into the spray divided by the volume flow of water through the nozzle. The amount of air entrained into the spray is defined as follows :

$$Q_a = 2\pi \int_0^{D_t/2} r\, V_z(r)\, dr \tag{4.1}$$

Results were obtained by considering a spray nozzle mounted on a ceiling two meters above the floor, for all three mass flows considered. Although this is not an unconfined spray, previous results have shown little effect of the presence of the ceiling (Ref. 6). The axial positions where these results were obtained ranged from 20 percent of the spray height from the nozzle to 55 percent of this distance. It was impossible to calculate these values any closer to the nozzle because of the lack of resolution with 21 × 21 pressure nodes. At axial positions greater than this the floor seems to have a significant effect on the entrainment, so no comparison is made in this area.

The results displayed in figure 7 show good agreement with the McQuaid correlation (solid line) and the ± 20% scatter of experimental data about this line. This gives confidence that the bulk of the flow field is well predicted.

4.2 Effect of increasing mass flow through nozzle

The effect of increasing mass flow is displayed in figures 8, 9 and 10. Here the velocity vectors are non dimensionalized by the ejection velocity of the particle. For the determination of the entrained air flow equation 4.1 is rearranged

$$Q_a = 2\pi\ U_0 \int_0^{D_t/2} r\ \frac{V_z(r)}{U_0}\ dr \propto 2\pi\ Q_w \int_0^{D_t/2} r\ \frac{V_z(r)}{U_0}\ dr \qquad (4.2)$$

As can be seen from the results, the flow field non dimensionalized by the ejection velocity does not change significantly. Thus the integral in 4.2 is of the same order of magnitude for all three mass flows considered. It seems that the air entrainment in the spray is increased strongly by increasing the mass flow through the nozzle. This is a physically observable result.

Finally the efficiency of the spray is examined.

$$\eta = \frac{Q_a}{Q_w} = 2\pi \int_0^{D_t} r\,\frac{V_z(r)}{U_0}\ dr \qquad (4.3)$$

Close examination of figures 8, 9 and 10 shows a very slight increase in the efficiency of the spray with increased mass flow, reflected by a slight increase in the above integral. This result is found to be true for some cases, though the inverse can also be true. This is because higher mass flow results in small droplets which exchange momentum with air more rapidly resulting in higher entrainment, but because of the small droplet size the spray envelope contracts quickly slowing entrainment. Thus, for small spray lengths an increase in efficiency is observed relative to larger droplet sizes, but this is reversed as the spray length is increased.

4.3 Effect of ceiling on entrainment

Results were obtained for the case where the ceiling is removed. These results are shown in figures 11 and 12. Though not shown a free boundary was placed at a height of two meters above the spray nozzle. Comparison with the previous results of corresponding mass flow with ceiling included shows little change in the entrainment properties with or without a ceiling. This result is supported by experimental results (Ref. 3).

4.4 Examination of the pressure field

Along with the velocity field each calculation also gives the corresponding pressure field. The isobars of a typical pressure field are shown in figure 13. This figure validates the previous assumption in the derivation of the boundary condition that at the free boundary the pressure field varies little and the pressure can be set to a reference value.

The area of steepest pressure gradients is near the floor inside the spray envelope. However, the pressure gradient in this region is small compared to the momentum source term. The other area of high pressure gradient seems to be at the nozzle. In this region, again the pressure gradient is small compared to momentum source term inside the spray envelope, though outside the spray envelope the pressure gradient may be significant. Better resolution is necessary to draw a firm conclusion.

It can be concluded that for the bulk of the flow field ignoring the pressure gradient may be a good approximation. This is a particularly important result since the pressure calculations are the most time consuming and require the most number of statements in program. In the future the program may be modified to include only a closed form expression for a pressure-like variable which forces continuity to be satisfied. This should result in large savings in computer time and allow better resolution in the mesh.

5. CONCLUSIONS

It has been shown that the axisymmetric spray model implemented gives good results for the entrainment of air inside the spray envelope for the portion at the spray envelope which contains most of the entrained air. The bulk of the surrounding flow field is thus considered to be more or less correctly predicted.

The physically observable result that increasing the outflow from the nozzle increases entrainment of air is also found.

The effect of the ceiling on entrainment is seen to be negligible.

Finally, the examination of the pressure field indicates that neglecting the pressure gradient is a good approximation for the bulk of the flow field. This result leads to the recommendation that work continues on the model to incorporate a closed form expression for a pressure-like variable to force continuity to hold, while eliminating the need for the Poisson equation for the pressure. This will allow better results since the mesh may be made smaller due to the saving in computer time.

Application of this model to a spray in the upward facing mode is straightforward.

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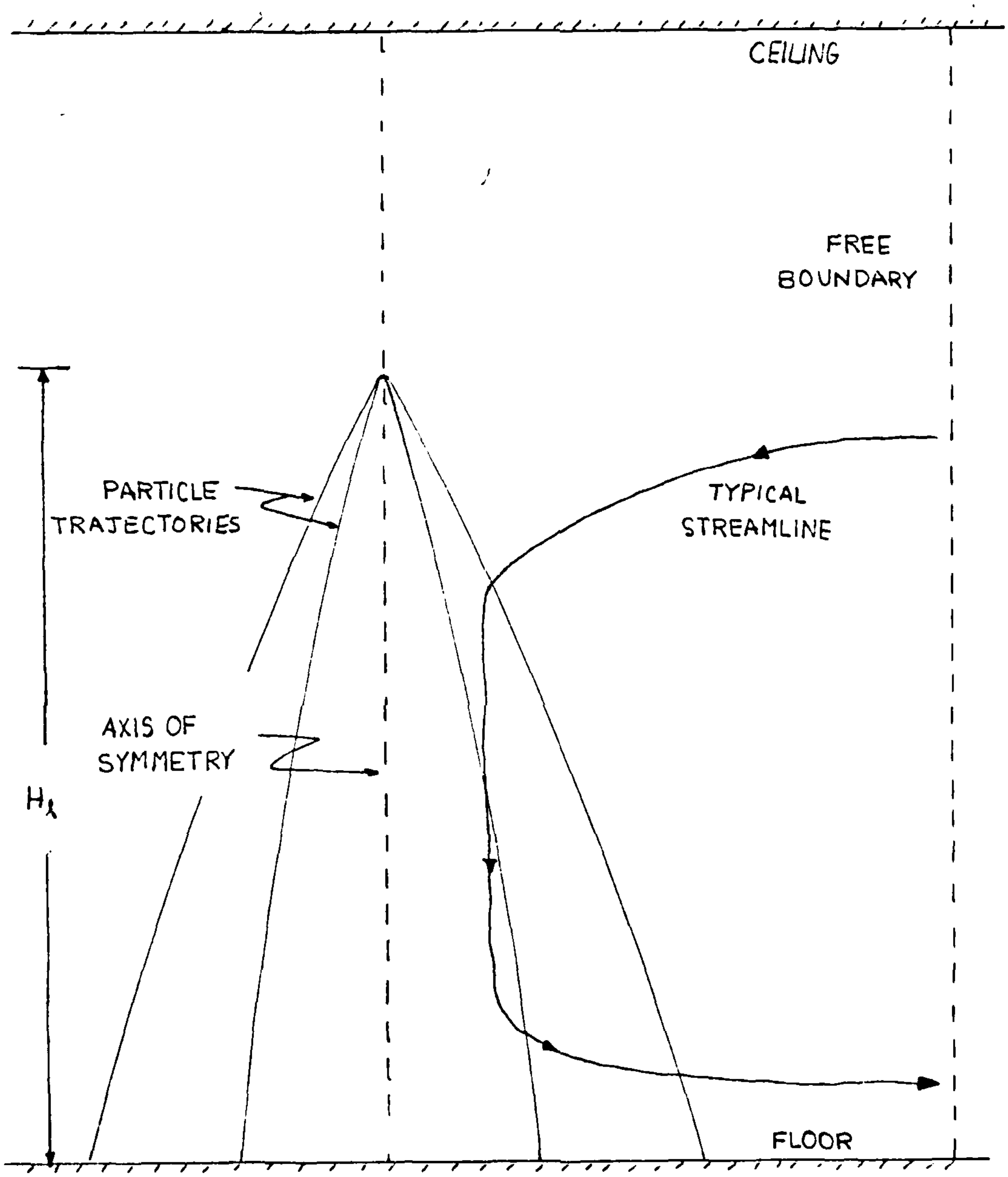


Figure 1 Spray and flow domain in downward facing mode

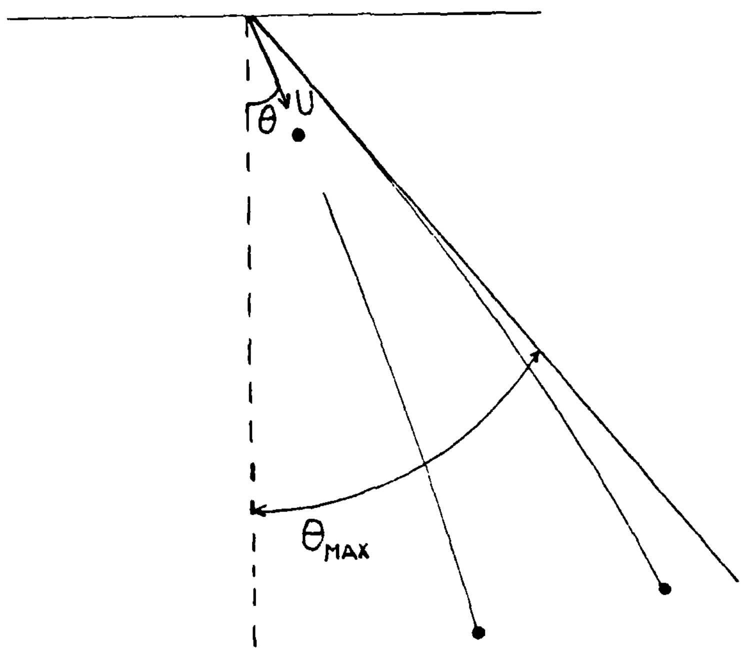


Figure 2 Nozzle Configuration

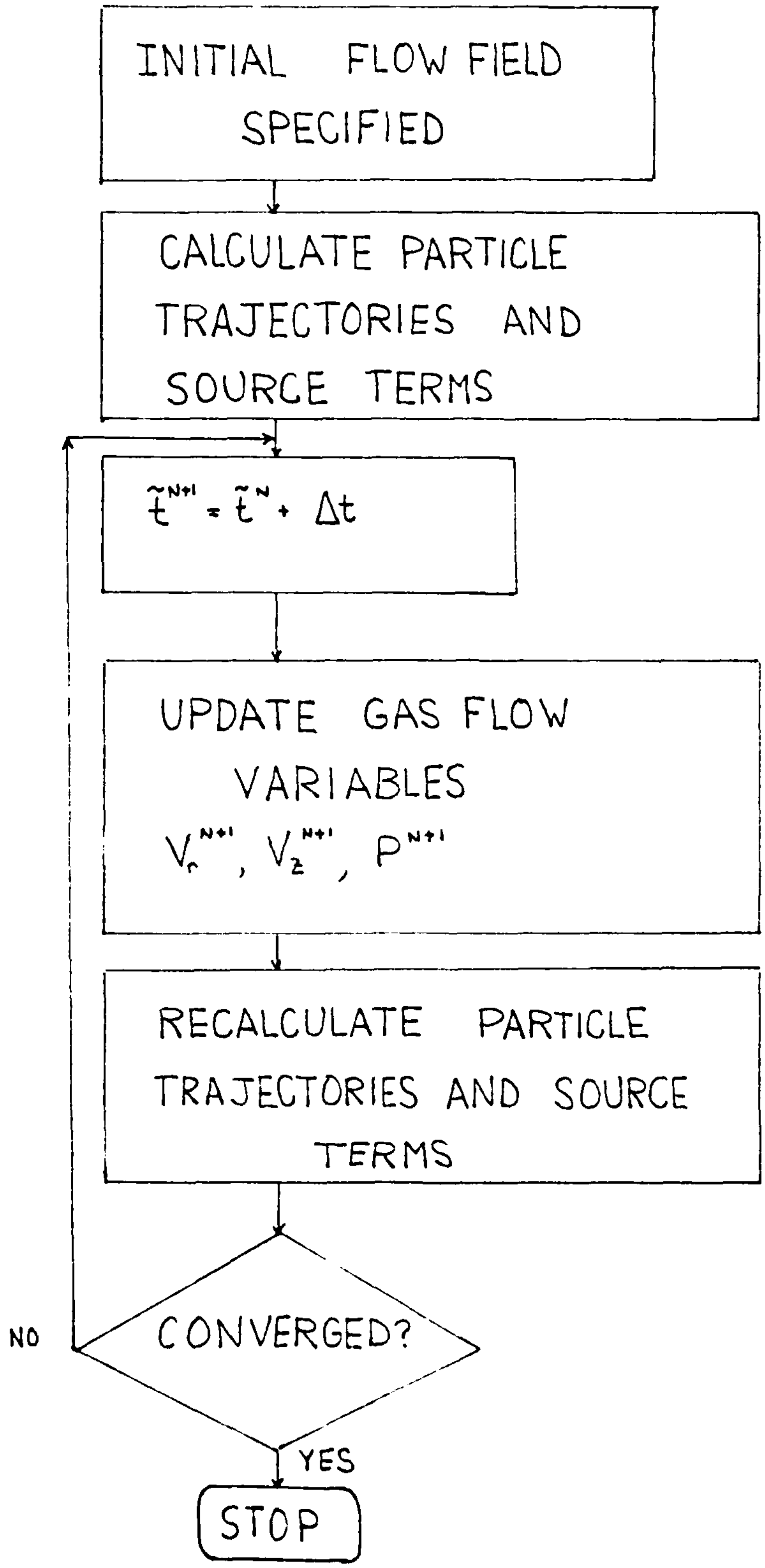


Figure 3 Scheme for numerical solution

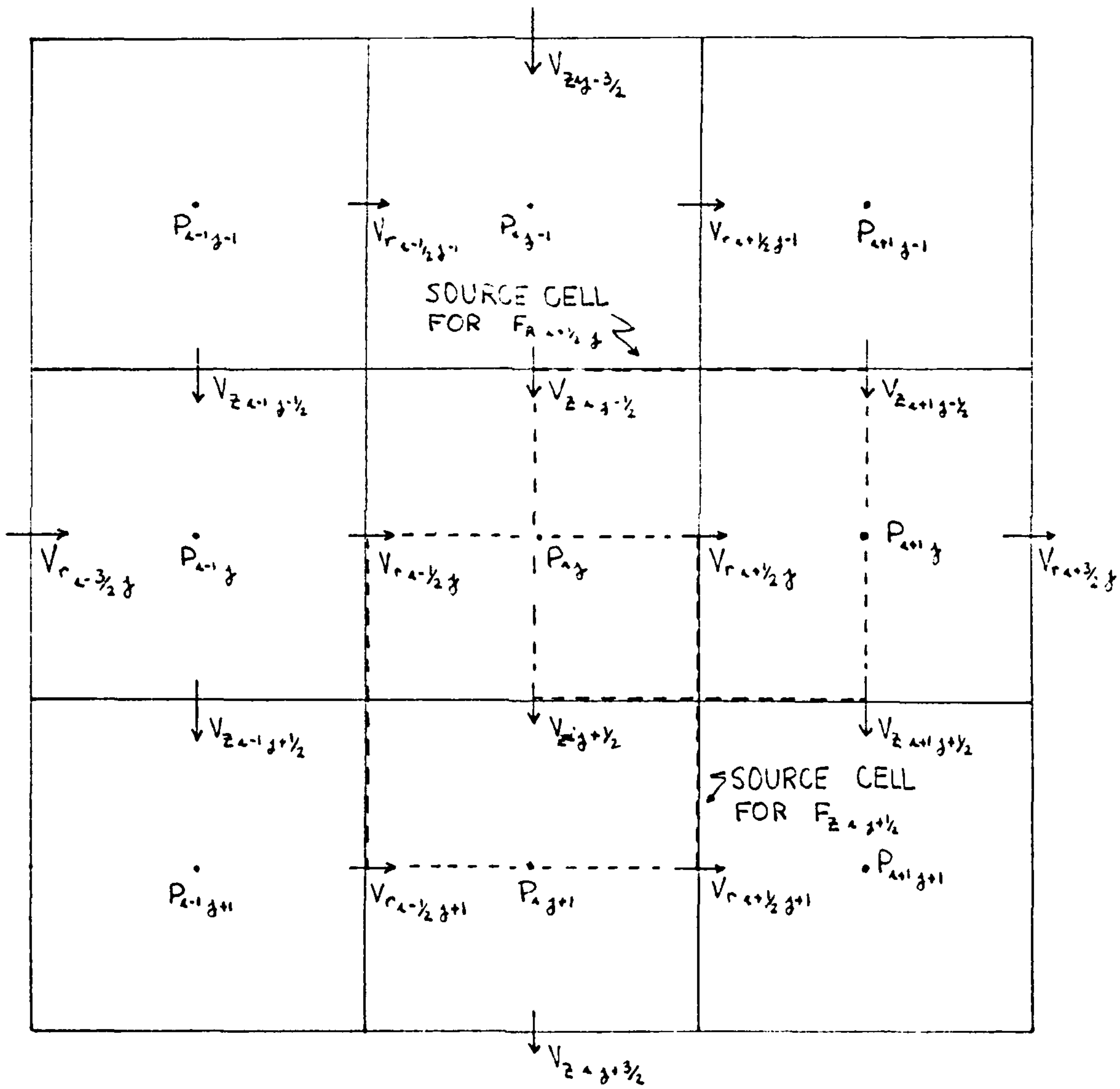


Figure 4 Staggered MAC grid with source cells

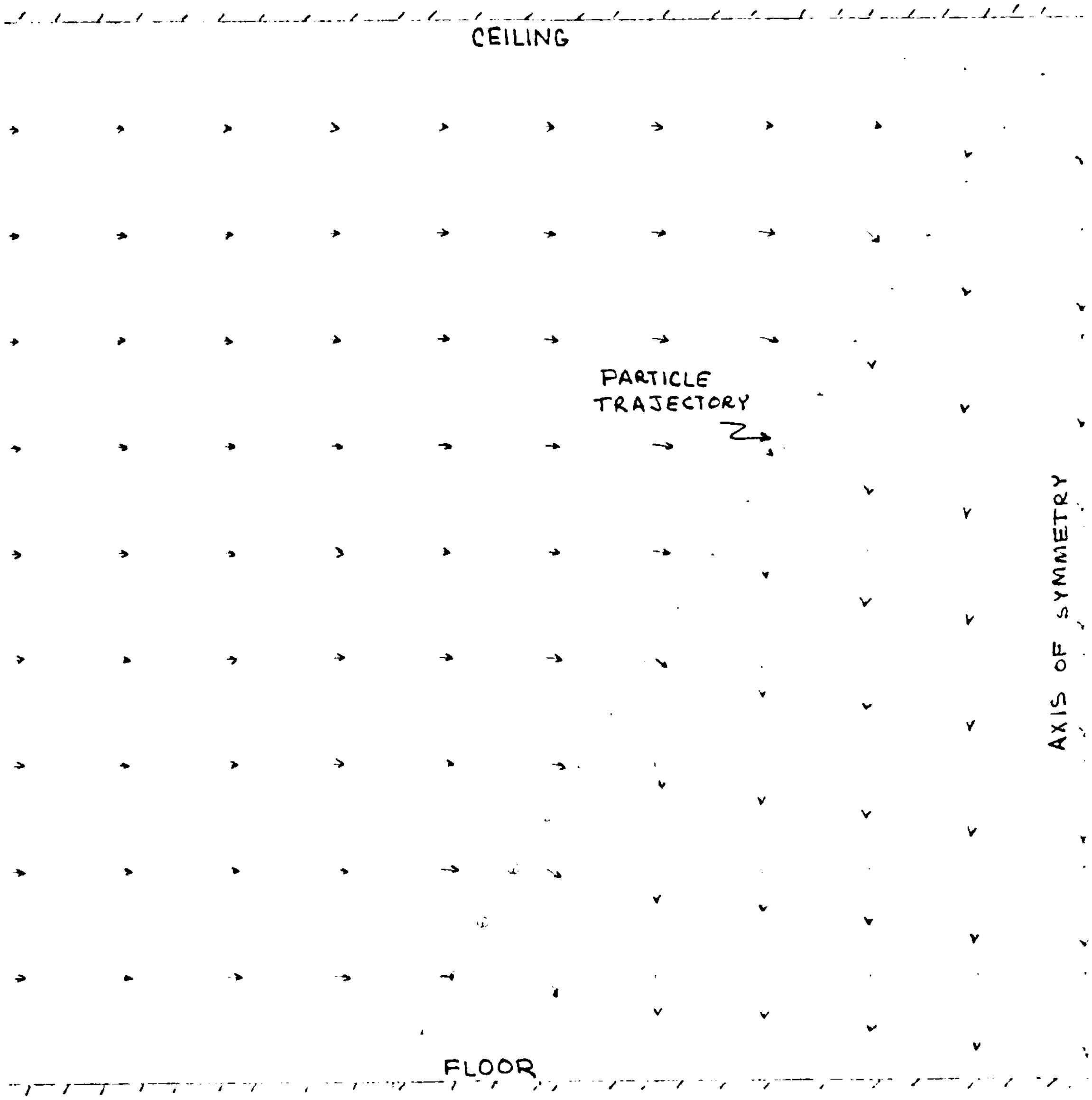


Figure 5 Non-Recirculating Spray

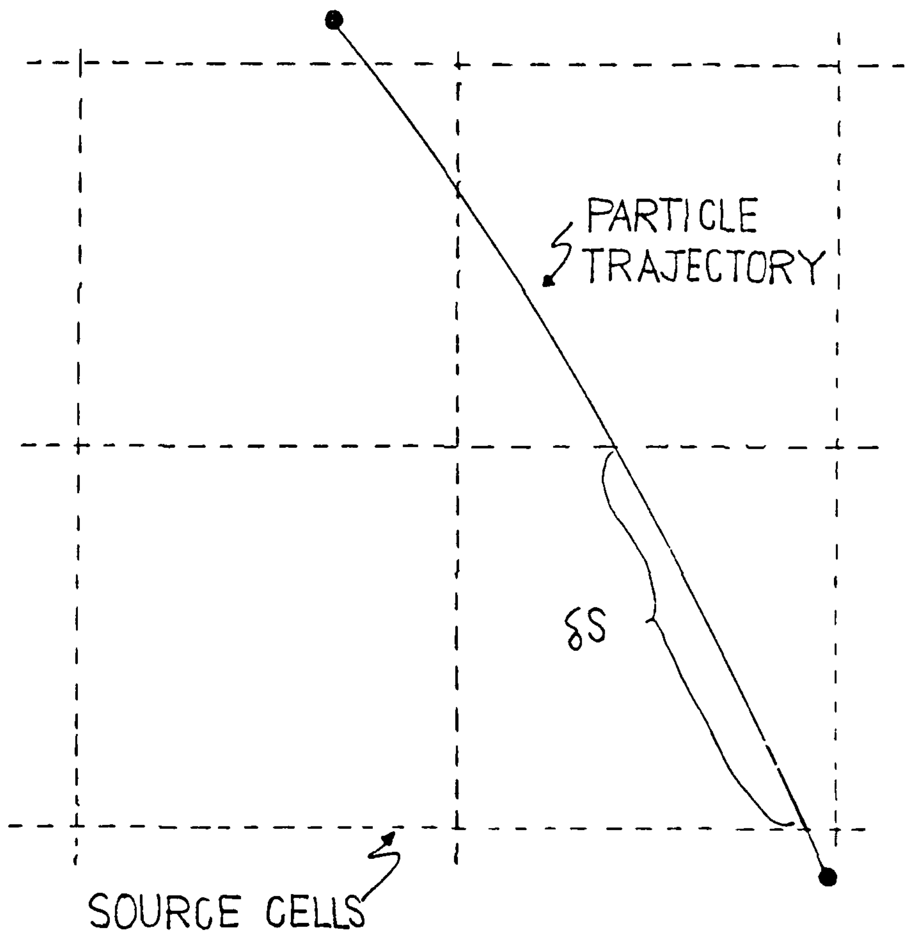


Figure 8 Particle trajectory superimposed on source cells

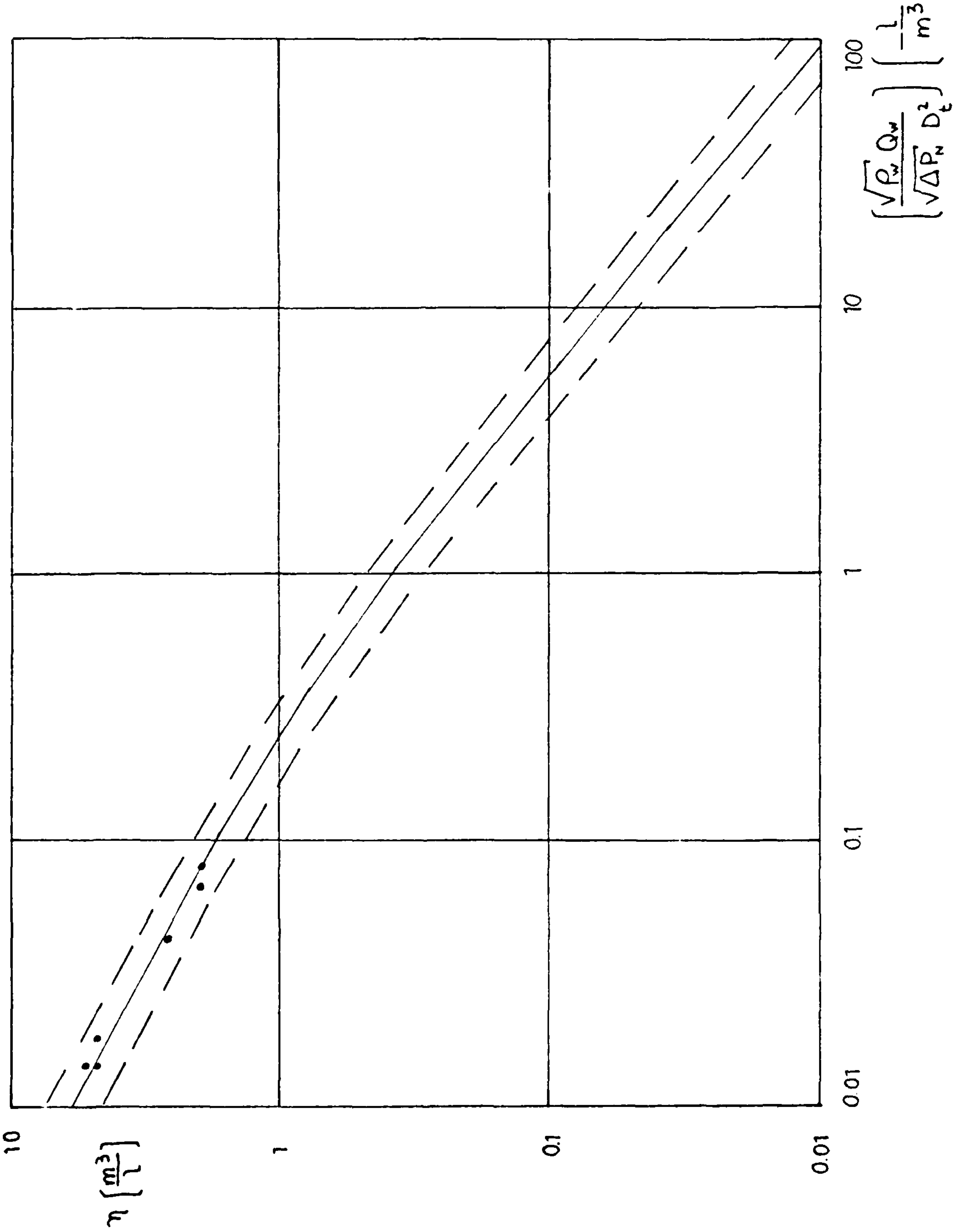
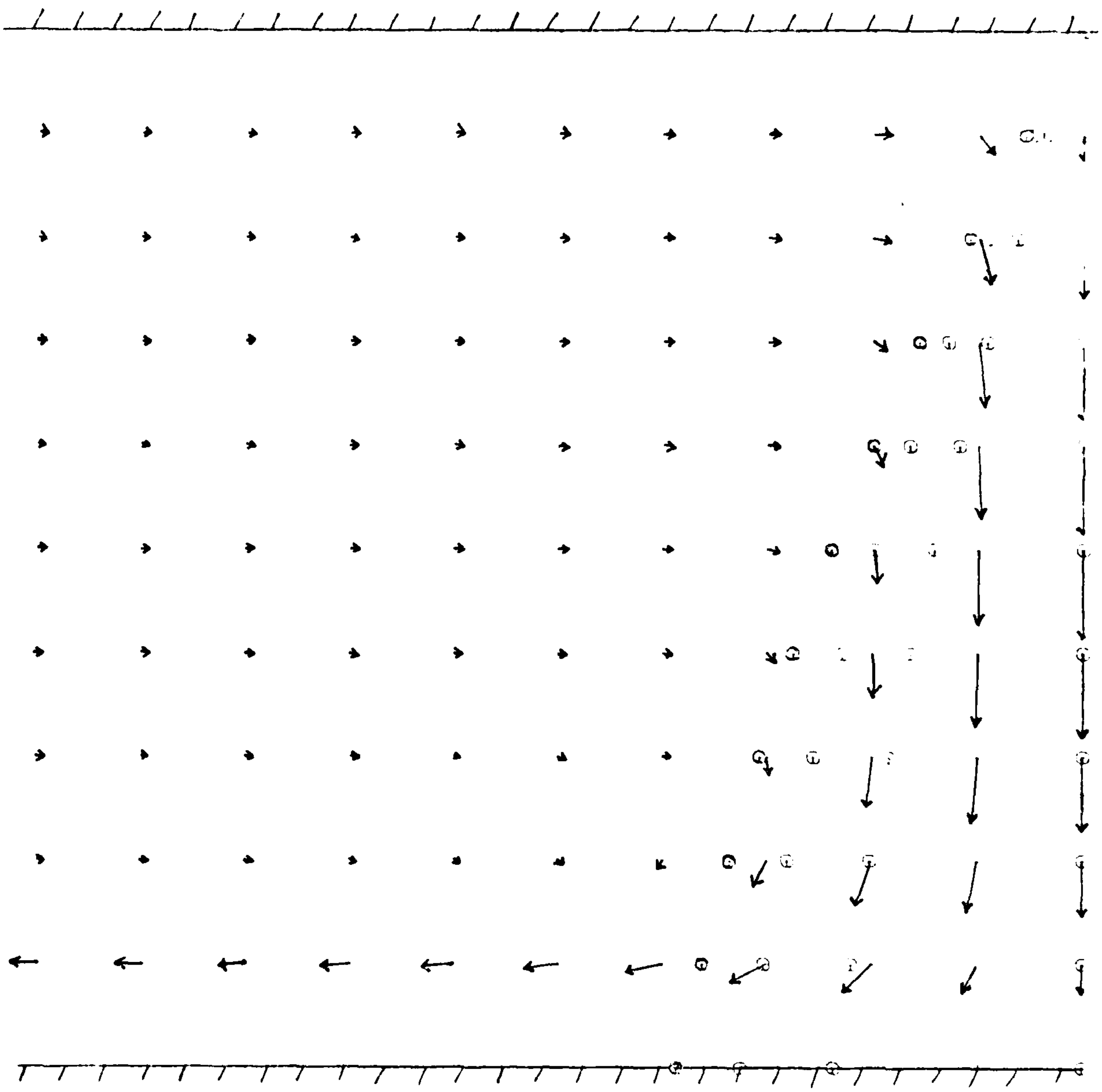


Figure 7 Comparison of air entrainment with experimental results

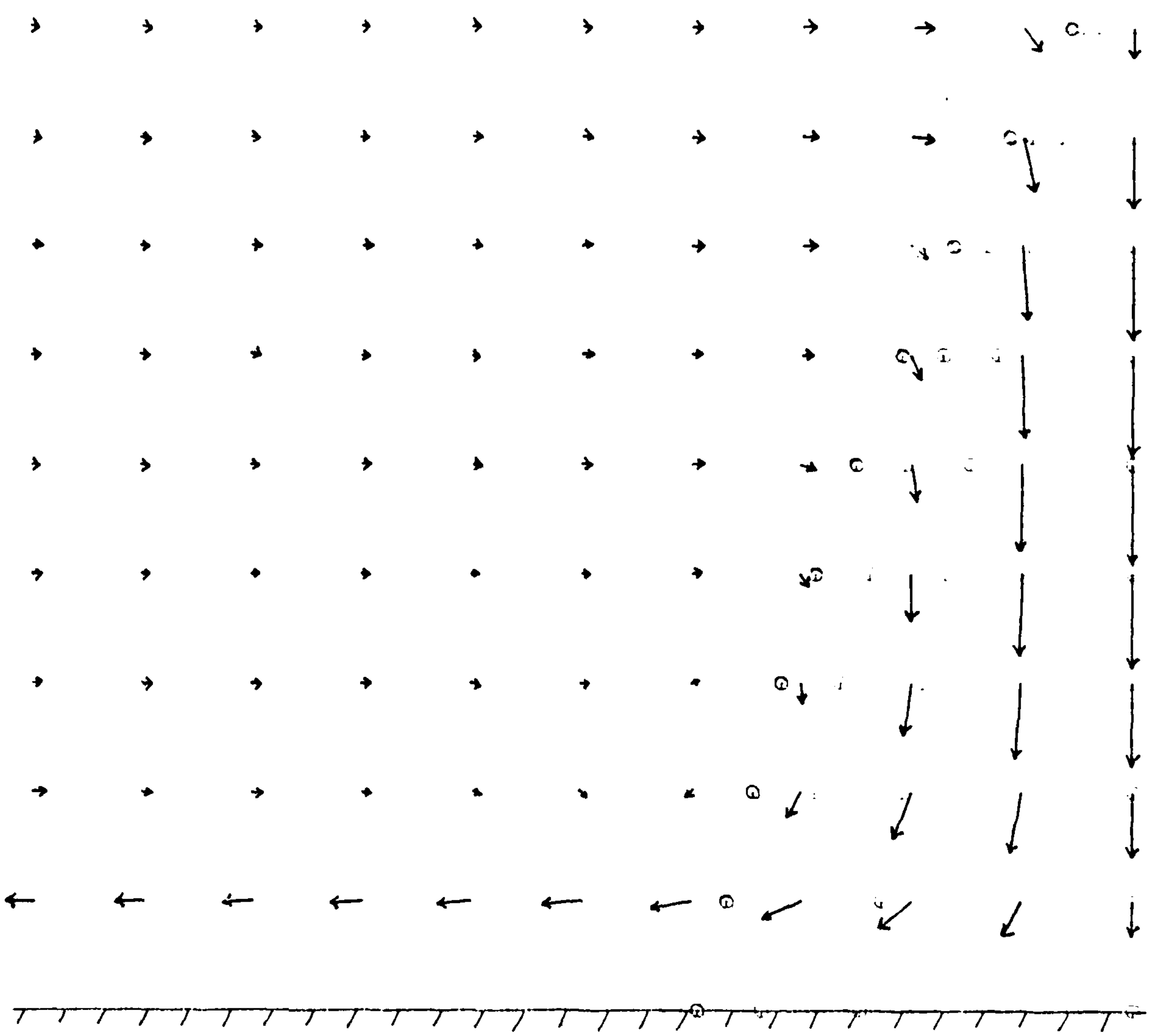


←——— CORRESPOND TO A GAS VELOCITY TO INJECTION VELOCITY RATIO OF [illegible] PERCENT

o o o o REPRESENT PARTICLE TRAJECTORIES

LE[illegible]LER [illegible]Z1 N[illegible]ZZLE
NOZZLE DIAMETER [illegible] MM
HALF AN[illegible]LE OF SPRAY 30[illegible] DEGREE
LIQUID VOLUME FLOW [illegible] LT PER MIN
DROPLET SIZE 1.0[illegible] MM
SPRAY HEIGHT [illegible].0 M

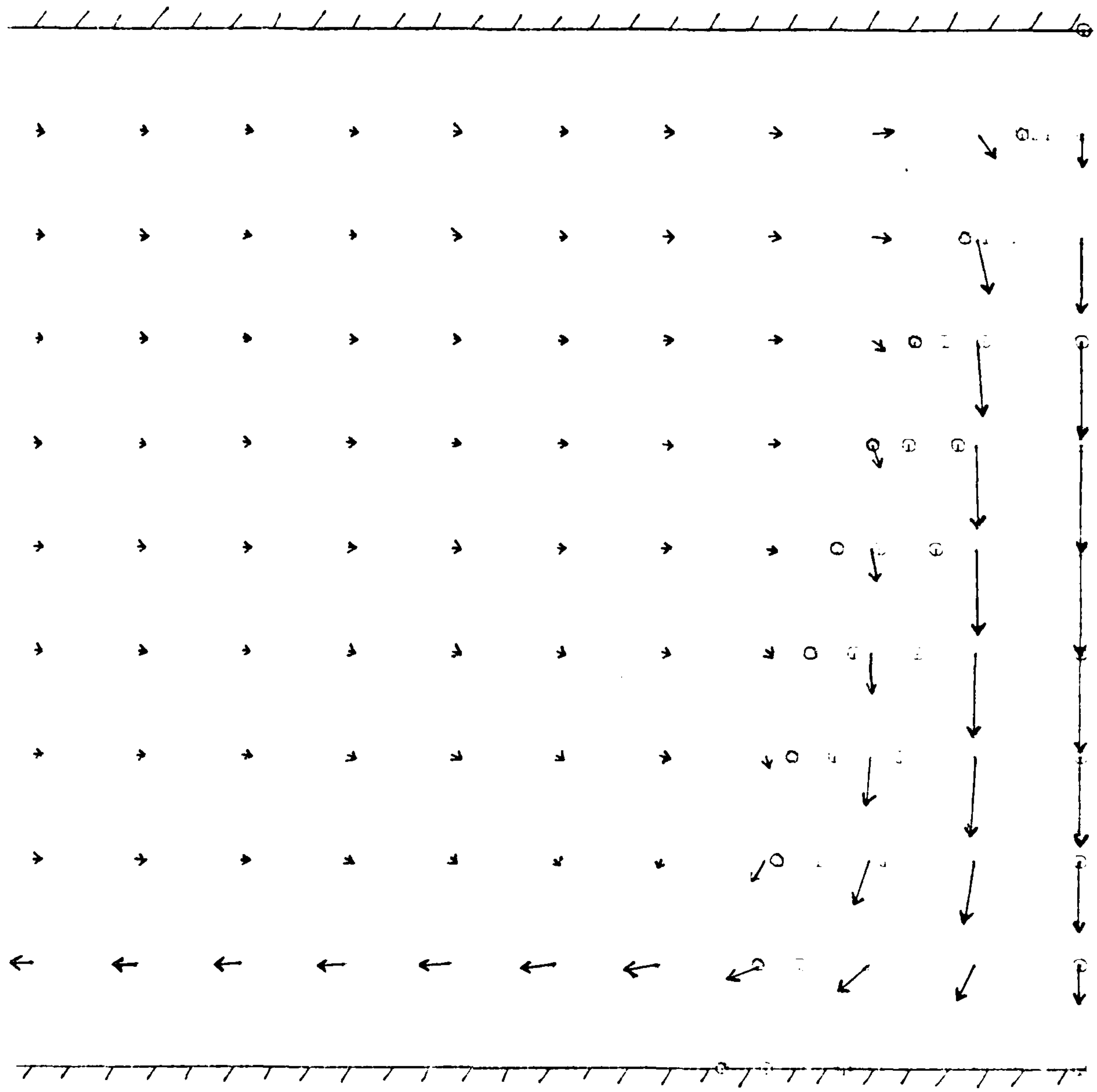
FIGURE 8 FLOW FIELD [illegible] PARTICLE TRAJECTORIES



← CORRESPONDS TO A GAS VELOCITY TO INJECTION VELOCITY RATIO OF 20 PERCENT
o o o o REPRESENT PARTICLE TRAJECTORIES

[illegible] NOZZLE
NOZZLE DIAMETER [illegible] MM
HALF ANGLE OF SPRAY [illegible]
LIQUID VOLUME FLOW [illegible].3 LT PER MIN
DROPLET SIZE 0.01 MM
SPRAY HEIGHT [illegible]

FIGURE 9 [illegible]

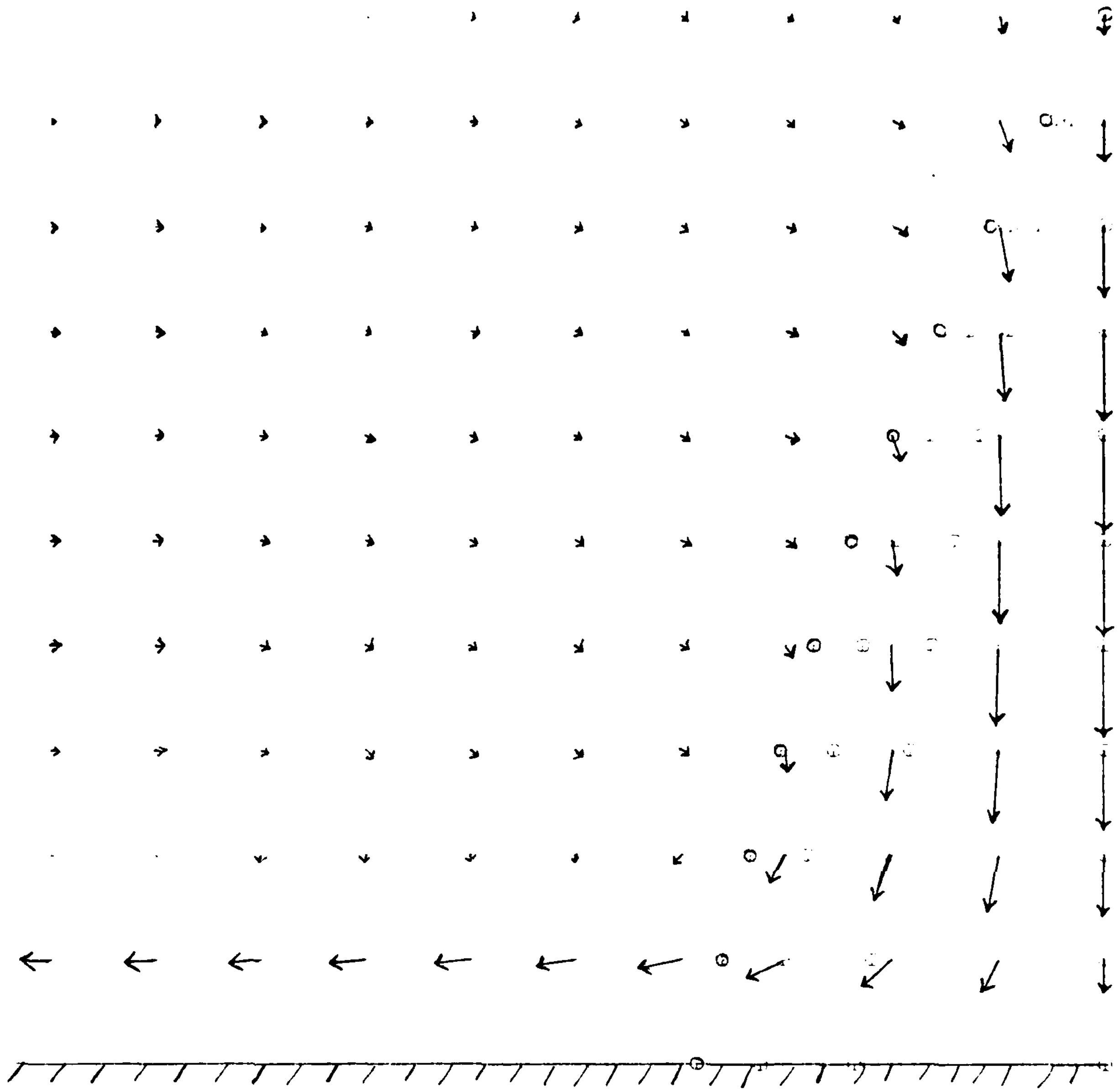


——— CORRESPONDS TO A GAS VELOCITY TO INJECTION VELOCITY RATIO OF 10 PERCENT

o o o o REPRESENT PARTICLE TRAJECTORIES

[illegible]
NOZZLE DIAMETER [illegible] MM
HALF ANGLE OF SPRAY [illegible]
LIQUID VOLUME FLOW [illegible] LT PER MIN
DROPLET SIZE [illegible] MM
SPRAY HEIGHT [illegible]

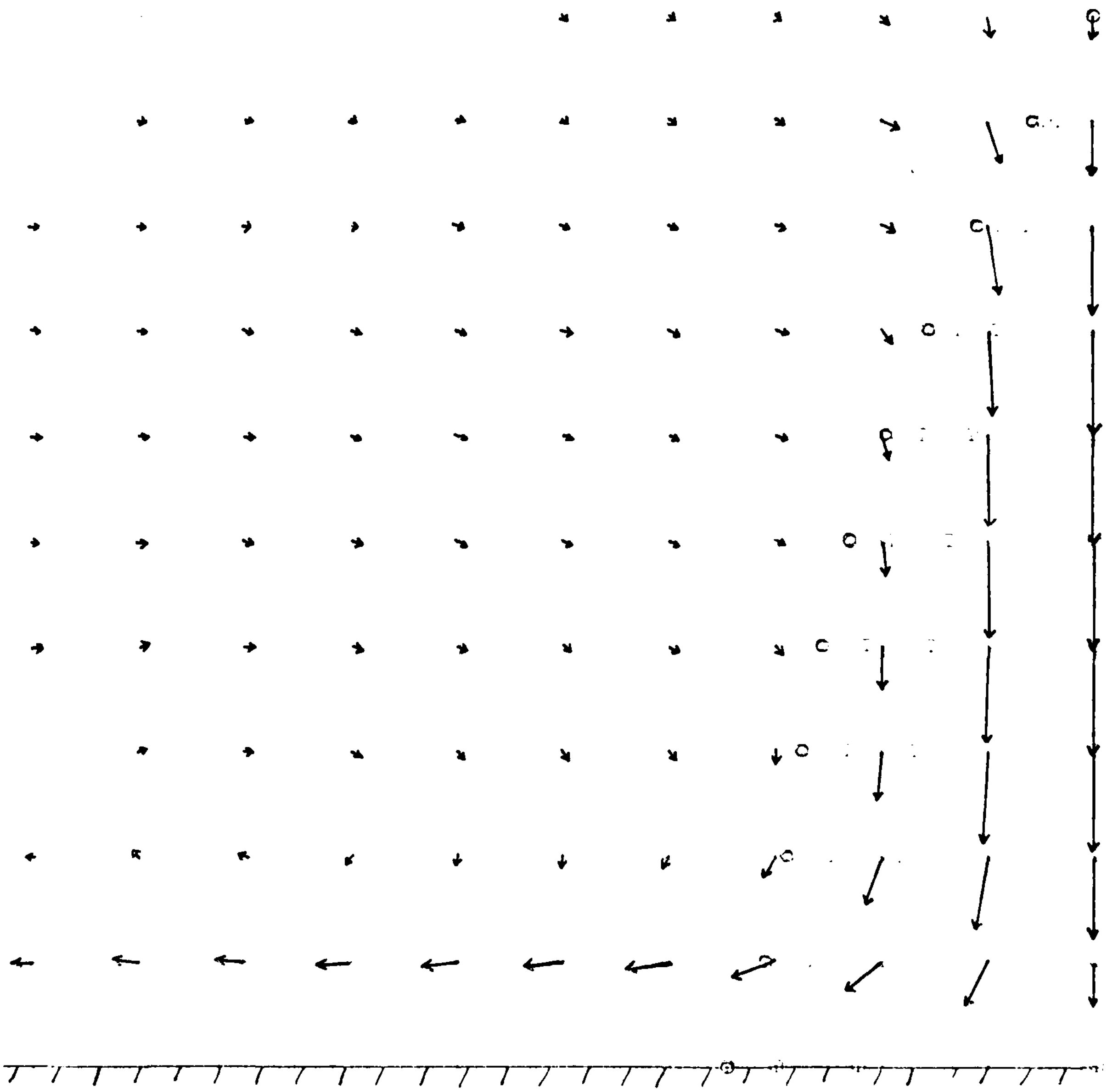
FIGURE 10 FLOW FIELD AND PARTICLE TRAJECTORIES



CORRESPONDS TO A GAS VELOCITY TO
INJECTION VELOCITY RATIO OF 2 [illegible]
0 0 0 0 REPRESENTS PARTICLE TRAJECTORIES

LECHLER [illegible] NOZZLE
NOZZLE DIAMETER 4.4 MM
HALF ANGLE OF SPRAY [illegible] DEG
LIQUID VOLUME FLOW [illegible] LT PER MIN
DROPLET SIZE [illegible] MM
SPRAY HEIGHT 1.0 M

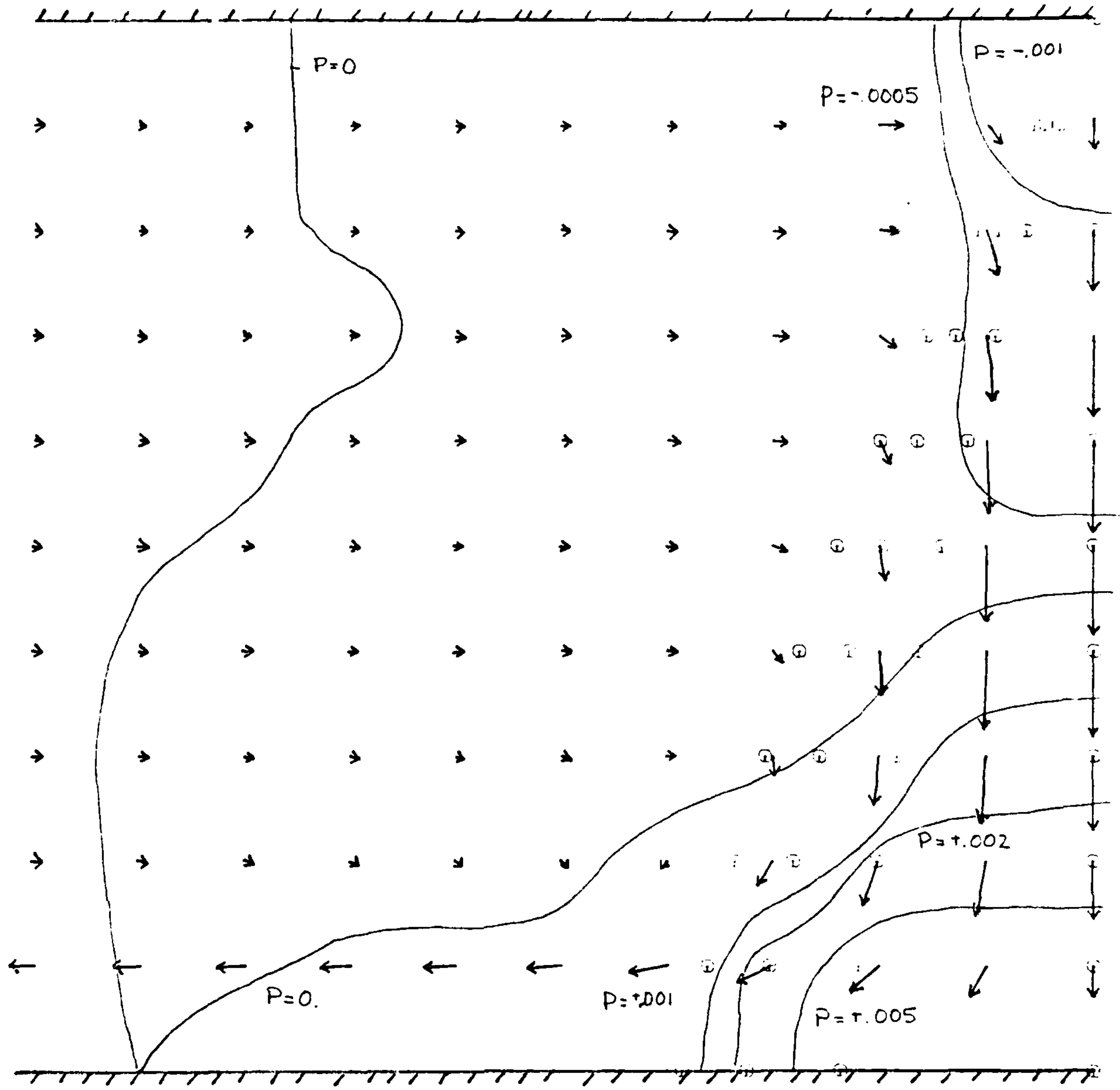
FIG [illegible] 11 [illegible]



⟵ CORRESPONDS TO A GAS VELOCITY [illegible]
INJECTION VELOCITY RATIO OF [illegible] PER [illegible]
o o o o REPRESENTS PARTICLE TRAJECTORIES

[illegible]
[illegible] DIAMETER [illegible] MM
[illegible] SPRAY [illegible]
[illegible] FLOW [illegible] PER MIN
[illegible] MM
[illegible]

12 [illegible]



← CORRE[illegible] VEL[illegible]ITY TO
INJE[illegible] RATIO [illegible]
o o o o REPRES[illegible] PART[illegible]
P = PRESSURE / ρU_0^2

[illegible]
[illegible]
HA[illegible] OF [illegible]
[illegible]
DROPLET [illegible]
[illegible]

F[illegible] 13 [illegible]

d_o mm	Q_w lt/min	ΔP_N kPa	D mm	θ_{MAX}
4.4	6.6	34.3	1.05	30°
	13.3	176.5	.61	30°
	21.1	617.8	.40	30°

Table 1 Properties of Lechler S21 Nozzle

APPENDIX - LISTING OF SPRAY COMPUTER PROGRAM

MAIN PROGRAM

```
      DIMENSION VR(50,50),VZ(50,50),X(50),Y(50)
      DIMENSION VRN(50,50),VZN(50,50)
      DIMENSION PRESSR(50,50),GUESS(50,50)
      DIMENSION ABC(50),BBC(50),CBC(50),DBC(50)
      DIMENSION CNE(50,50),CNW(50,50),CSW(50,50),CSE(50,50)
      DIMENSION CN(50,50),CS(50,50),CW(50,50),CE(50,50)
      DIMENSION F(50,50),CP(50,50)
      DIMENSION Z(50),R(50),T(50),UR(50),UZ(50)
      DIMENSION FR(50,50),FZ(50,50)
      DIMENSION DP(10),THETA0(10),FRACO(10)
      COMMON CNE,CNW,CSE,CSW,CN,CS,CE,CW,CP,F
      COMMON /BOUNDARY/ ABC,BBC,CBC,DBC,ACROSS,BCROSS,CCROSS,DCROSS
      COMMON /BOUNDARY2/ ILEFT,IRIGHT,ITOP,IBOTTM
      COMMON /AIR V/ VZ,VR
      COMMON /COORD/ X,Y
      COMMON /DEL/ DELX,DELY,DELT
      COMMON /REYNOLDS/ RE
      COMMON /SPRAY DIS/ ZL,R,RL,RP,G,RO
      COMMON /SOURCE/ FR,FZ
      READ N,NP
C****************************************************************
C
C    THIS PROGRAM SOLVES ITERATIVELY BOTH THE NAVIER STOKES
C    EQUATIONS WITH MOMENTUM SOURCE TERMS INCLUDED, USING THE
C    MAC METHOD, AND THE EQUATIONS OF MOTION OF A PARTICLE
C    USING A FOURTH ORDER RUNGE-KUTTA METHOD TO DETERMINE THE
C    SOURCE TERMS
C
C    THIS MAIN PROGRAM SETS UP THE INPUT FOR THE FIVE SUBROUTINES
C    COEFF, PSOURC, SOR, GASVEL, AND PARTCL, AND THEN ITERATATES
C    FOR THE SOLUTION.
C
C    COEFF CALCULATES THE COEFFICIENTS OF THE LEFT HAND SIDE OF
C    THE POISSON EQUATION FOR THE PRESSURE.  THIS CALCULATION
C    IS ONLY REQUIRED ONCE SINCE THESE COEFFICIENTS ARE ONLY
C    DEPENDENT ON SPACIAL POSITION
C
C    PSOURC CALCULATES THE SOURCE TERM (RIGHT HAND SIDE) OF THE
C    POISSON EQUATION.  THE SOURCE TERM IS DEPENDENT ON THE
C    LOCAL VELOCITY AND MUST BE UPDATED AT EACH ITERATION.
C
C    SOR COMPUTES THE SOLUTION OF THE DISCRETIZED POISSON
```

```
C     EQUATION BY THE SOR METHOD USING THE COEFFICIENTS AND
C     SOURCE TERM CALCULATED BY COEFF AND PSOURC.
C
C     GASVEL CALCULATES EXPLICITLY THE UPDATED VELOCITY FIELD
C     USING THE UPDATED PRESSURE FIELD AND THE OLD VELOCITY
C     FIELD.
C
C     PARTCL CALCULATES THE PARTICLE TRAJECTORIES UNDER THE
C     INFLUENCE FROM THE AIR FLOW AND THEN CALCULATES THE MOMENTUM
C     SOURCE TERMS.
C
C     THIS PROGRAM IS DIVIDED INTO FIVE PARTS;
C
C     THE FIRST PART SETS UP THE GRID FOR THE SOLUTION,
C     THE SECOND PART SETS UP THE SUBROUTINE INPUTS FOR THE
C     SOLUTION OF THE POISSON EQUATION FOR THE PRESSURE,
C     THE THIRD PART SETS UP THE SUBROUTINE INPUTS FOR THE
C     SOLUTION OF THE UPDATED VELOCITY FIELD,
C     THE FOURTH PART INVOLVES THE ITERATION FOR THE
C     SOLUTION,
C     AND THE FIFTH PART OUTPUTS THE SOLUTION.
C
C*****************************************************************
C
C     INPUTS FOR PROGRAM FROM DATA FILE 20
C
C*****************************************************************
C     SET DOMAIN DIMENSIONS
C     ENTER RADIAL DOMAIN LENGTH, RL, AND AXIAL DOMAIN LENGTH, ZL
      READ(20,201)RL,ZL
  201 FORMAT(/////1X,2(F10.5,2X))
      WRITE(30,202)RL,ZL
  202 FORMAT(1X,'RADIAL LENGTH= ',F10.5,2X,'AXIAL LENGTH= ',F10.5)
C     ENTER SPRAY HEIGHT H, AND SPRAY HEIGHT TO NOZZLE DIAMETER RATIO
      READ(20,203)H,HF
  203 FORMAT(//1X,2(F10.5,2X))
      WRITE(30,208)H,HF
  208 FORMAT(1X,'SPRAY HEIGHT= ',F10.5,2X,'SPRAY LENGTH TO NOZZLE
     1 DIAMETER RATIO= ',F10.5)
C     ENTER NUMBER OF DROPLET SIZES OF TRAJECTORIES
```

```
      READ(20,220)NUMPAR
  220 FORMAT(//1X,I2)
      WRITE(30,226)NUMPAR
  226 FORMAT(1X,'NUMBER OF PARTICLE SIZES/TRAJECTORIES= ',I2)
      READ(20,221)
  221 FORMAT(//)
C     ENTER DROPLET DIAMETER/NOZZLE DIAMETER, INITIAL HALF ANGLE OF
C     SPRAY, AND FRACTION OF VOLUME FLOW FOR EACH PARTICULAR
C     DROPLET SIZE/TRAJECTORY
      TOTFRQ=0.
      DO 222 I=1,NUMPAR
      READ(20,223)DP(I),THETA0(I),FRACQ(I)
      WRITE(30,212)DP(I),THETA0(I),FRACQ(I)
  212 FORMAT(1X,'DROPLET DIAMETER/NOZZLE DIAMETER= ',F10.5,2X,
     1'HALF ANGLE OF SPRAY= ',F10.5,2X,'VOLUME FLOW FRACTION= ',
     2F10.5)
      TOTFRQ=TOTFRQ+FRACQ(I)
  222 CONTINUE
  223 FORMAT(1X,3(F10.5,2X))
      IF(ABS(1.-TOTFRQ).LT.0.02)GO TO 225
      WRITE(30,224)
  224 FORMAT(1X,'SUM OF NOZZLE VOLUME FLOW FRACTIONS IS NOT
     1EQUAL TO ONE')
      STOP
  225 CONTINUE
C     ENTER INVERSE FROUDE NUMBER SQUARED, G, AND REYNOLDS NUMBER
C     OF NONDIMENSIONALIZATION
      READ(20,213)G,RE
  213 FORMAT(//1X,2(F10.5,2X))
      WRITE(30,214)G,RE
  214 FORMAT(1X,'INVERSE FROUDE NUMBER SQUARED= ',F10.5,2X,
     1'REYNOLDS NUMBER OF NONDIMENSIONALIZATION= ',E12.5)
C     ENTER DENSITY OF GAS/DENSITY OF LIQUID
      READ(20,215)RO
  215 FORMAT(//1X,2(F10.5,2X))
      WRITE(30,216)RO
  216 FORMAT(1X,'DENSITY OF GAS/DENSITY OF LIQUID= ',F10.5)
C     SET THE TIME STEP-(STEADY STATE EQUALS THE CONVERGED SOLUTION)
C     THE SIZE OF THE TIME STEP IS SUBJECT TO STABILITY RESTRICTIONS
      READ(20,206)DELT
  206 FORMAT(//1X,F10.7)
      WRITE(30,205)DELT
  205 FORMAT(1X,'TIME STEP = ',F10.7)
C     SET THE NUMBER OF ITERATIONS FOR OUTER LOOP OF MAC METHOD
      READ(20,217)NMIT
  217 FORMAT(//1X,I3)
      WRITE(30,218)NMIT
  218 FORMAT(1X,'NUMBER OF ITERATIONS= ',I3)
```

```
C     SET SWITCH FOR INITIAL SOLUTION (0=START SOLN FROM AIR AT REST)
C        (1=INITIAL SOLUTION PROVIDED IN FILE 19)
         READ(20,219)INTSOL
  219 FORMAT(//1X,I1)
C**********************************************************************
C
C     SET UP GRID
C
C**********************************************************************
C
C     SET NUMBER OF PRESSURE NODES IN RADIAL DIRECTION
C     SET NUMBER OF PRESSURE NODES IN AXIAL DIRECTION
         READ(20,209)NX,NY
  209 FORMAT(///1X,2(I2,2X))
         WRITE(6,210)NX,NY
  210 FORMAT(1X,'NX= ',I2,2X,'NY= ',I2)
C     CALCULATE PRESSURE NODE LOCATIONS
         DELX=RL/FLOAT(NX-1)
         DELY=ZL/FLOAT(NY-1)
         DO 5 I=1,NX
         X(I)=FLOAT(I-1)*DELX
    5    CONTINUE
         DO 6 J=1,NY
         Y(J)=FLOAT(J-1)*DELY
    6    CONTINUE
C**********************************************************************
C
C     SET UP SUBROUTINE INPUTS FOR SOLUTION OF POISSON EQUATION
C
C**********************************************************************
C     SPECIFY TYPE OF BOUNDARY CONDITION ON EACH BOUNDARY
C     0=DIRICHLET BC
C     1=NEUMANN BC
C
C     I=NX BOUNDARY OR X=RADIUS BOUNDARY
          IRIGHT=0
C     I=1 BOUNDARY OR X=0 BOUNDARY
          ILEFT=1
C     J=NY BOUNDARY OR Y=XLENG BOUNDARY
          ITOP=1
```

```
C     J=1 BOUNDARY OF Y=0 BOUNDARY
         IBOTTM=1
C     PRESSURE SOLUTION AT BOUNDARIES
C     X=0 AND X=L BOUNDARIES
C     NORMAL DERVITIVE OF PRESSURE EQUAL ZERO
         DO 20 J=1,NY
         BBC(J)=0.
         DBC(J)=0.
   20    CONTINUE
C     NORMAL DERVITIVE OF PRESSURE EQUALS MINUS THE
C     PRESSURE GRADIENT AT Y=XLENG BOUNDARY
         DO 21 I=1,NX
         ABC(I)=0.
         CBC(I)=0.
   21    CONTINUE
C     Y=0 AND Y=1 BOUNDARIES
         DO 30 I=1,NX
         PRESSR(I,1)=0.
         PRESSR(I,NY)=0.
   30    CONTINUE
C
C     CALCULATE INITIAL GUESSED PRESSURE AT INTERIOR POINTS
C
         DO 10 I=1,NX-1
         DO 10 J=1,NY
         PRESSR(I,J)=0.
   10    CONTINUE
C     SET MAXIMUM NUMBER OF ITERATIONS FOR SOR ROUTINE
         ITRMAX=400
C     SET INITIAL OVERRELAXATION FACTOR
         WOMEGA=1.5
C*****************************************************************
C
C     SET UP SUBROUTINE INPUTS FOR SOLUTION OF VELOCITY FIELD
C
C*****************************************************************
C     GUESSED SOLUTION FOR VELOCITY
C     IF INTSOL=0 INITIAL FLOW FIELD SET TO ZERO
C     IF INTSOL=1 INITIAL FLOW FIELD FROM FILE 19
         IF(INTSOL.EQ.1)GO TO 91
         DO 9 J=1,NY+1
         DO 9 I=1,NX
         VZ(I,J)=0.
    9    CONTINUE
```

```
      DO 19 I=1,NX+1
      DO 19 J=1,NY
      VR(I,J)=0.
   19 CONTINUE
C     CALCULATE BOUNDARY VALUES OF VELOCITY
      DO 7 I=1,NX
      VZ(I,NY+1)=0.
      VZ(I,1)=VZ(I,NY+1)
    7 CONTINUE
      DO 8 I=1,NX+1
      VR(I,1)=0.
      VR(I,NY)=0.
    8 CONTINUE
      GO TO 92
C     INITIAL SOLUTION READ FROM FILE 19 IF INFSOL=1
   91 READ(19,93)
   93 FORMAT(/////////)
      DO 401 I=1,NUMPAR
  401 READ(19,407)
  407 FORMAT(1X)
      DO 45 J=1,NY
      DO 46 I=1,NX
      READ(19,304)X1,Y1,PRESSF(I,J),X2,Y2,VZ(I,J),
     1X3,Y3,VR(I,J)
   46 CONTINUE
      READ(19,305)X1,Y1,VR(NX+1,J)
   45 CONTINUE
      DO 47 I=1,NX
      READ(19,306)X1,Y1,VZ(I,NY+1)
   47 CONTINUE
   92 CONTINUE
C*******************************************************************
C
C     ITERATE FOR THE SOLUTION OF THE PRESSURE AND VELOCITY FIELDS
C
C*******************************************************************
C
C     CALCULATE COEFFICIENTS OF LEFT HAND SIDE OF PDE
C     SAME FOR ALL TIME
C
      CALL COEFF(NX,NY,PRESSF)
C
C     SET SOURCE TERMS EQUAL TO ZERO
      DO 82 I=1,NX
```

```
      DO 82 J=1,NY
      FR(I,J)=0.
      FZ(I,J)=0.
   82 CONTINUE
C     CALCULATE INITIAL SOURCE TERMS
      DO 81 I=1,NUMPAR
      WRITE(28,307)
      CALL PARTCL(R,Z,T,UR,UZ,DP(I),THETAO(I),FRACO(I))
      DO 84 J=1,IY+2
      WRITE(28,308)R(J),Z(J),T(J),UR(J),UZ(J)
   84 CONTINUE
   81 CONTINUE
      DO 138 I=1,NY
      WRITE(28,351)FR(1,I),FR(2,I),FR(3,I),FR(4,I),FR(5,I)
      WRITE(28,351)FZ(1,I),FZ(2,I),FZ(3,I),FZ(4,I),FZ(5,I)
  351 FORMAT(1X,5(F12.5,2X))
  138 CONTINUE
      DO 52 ITER1=1,NOMIT
C
C     UPDATE SOURCE TERM FOR PRESSURE EQUATION USING NEW
C     VELOCITY
      CALL PSOURC(NX,NY,PRESSR)
C     UPDATE PARTICLE FIELD AND PARTICLE SOURCE TERMS
C     EVERY TENTH ITERATION
      L=L+1
      IF(L.LT.10)GO TO 234
      L=0
C     SET SOURCE TERMS EQUAL TO ZERO
      DO 83 I=1,NX
      DO 83 J=1,NY
      FR(I,J)=0.
      FZ(I,J)=0.
   83 CONTINUE
C     RECALCULATE SOURCE TERMS
      DO 85 I=1,NUMPAR
      CALL PARTCL(R,Z,T,UR,UZ,DP(I),THETAO(I),FRACO(I))
   85 CONTINUE
  234 CONTINUE
C
```

```
C     ITERATE FOR NEW PRESSURE FIELD USING SOR ROUTINE
         CALL SOR(PRESSR,ITER,OMEGA,NX,NY,ITRMAX)
C
C     DETERMINE NEW VELOCITY FIELD
         CALL GASVEL(NX,NY,VZN,VRN,PRESSR)
C
C     STORE NEW VELOCITY FIELD AND CALCULATE CAUCHY RESIDUAL
         VNOR=0.
         SOLNOR=0.
         DO 29 I=1,NX+1
         DO 29 J=1,NY
         VNOR=VNOR+(VRN(I,J)-VR(I,J))**2
         SOLNOR=SOLNOR+VRN(I,J)**2
         VR(I,J)=VRN(I,J)
   29    CONTINUE
         DO 28 J=1,NY+1
         DO 28 I=1,NX
         VNOR=VNOR+(VZN(I,J)-VZ(I,J))**2
         SOLNOR=SOLNOR+VZN(I,J)**2
         VZ(I,J)=VZN(I,J)
   28    CONTINUE
         VNOR=SQRT(VNOR/SOLNOR)
C     IF SOLUTION STARTS TO BLOW UP STOP PROGRAM
C        IF(SOLNOR/FLOAT(NX*NY).GT.100.)STOP
         NXH=NX/2+1
         NYH=NY/2+1
         IF(L.EQ.0)WRITE(6,53)ITER1,VNOR,VR(NXH,NYH),VZ(NXH,NYH),
        1PRESSR(NXH,NYH),VR(2,NYH),VZ(2,NYH),PRESSR(2,NYH)
   53    FORMAT(1X,I4,2X,E12.6,6(2X,F10.5))
C        DO 51 I=1,NX+1
C        DO 51 J=1,NY+1
C        WRITE(30,202)X(I),Y(J),PRESSR(I,J),VZN(I,J),VRN(I,J)
C  51    CONTINUE
   52    CONTINUE
C**********************************************************************
C
C     OUTPUT SOLUTION-BOTH PRESSURE AND VELOCITY FIELDS
C     AND PARTICLE TRAJECTORIES
C
C**********************************************************************
         WRITE(30,301)
  301  FORMAT(1X,7X,'NODE LOCATION',6X,'PRESSURE',9X,
        1'NODE LOCATION',7X,'AXIAL',10X,'NODE LOCATION',8X,
        2'RADIAL')
         WRITE(30,302)
  302  FORMAT(1X,3X,'RADIAL',6X,'AXIAL',19X,'RADIAL',
        16X,'AXIAL',6X,'VELOCITY',4X,'RADIAL',6X,'AXIAL',7X,
        2'VELOCITY')
         WRITE(30,303)
```

```
303 FORMAT(1X,2X,'POSITION',4X,'POSITION',16X,
   1'POSITION',4X,'POSITION',15X,'POSITION',4X,'POSTTION')
    DO 35 J=1,NY
    DO 36 I=1,NX
    XIH=X(I)-DELX/2.
    YJH=Y(J)-DELY/2.
    WRITE(30,304)X(I),Y(J),PRESSR(I,J),X(I),YJH,VZ(I,J)
   1,XIH,Y(J),VR(I,J)
36  CONTINUE
304 FORMAT(1X,9(F10.5,2X))
    XIH=X(NX)+DELX/2.
    WRITE(30,305)XIH,Y(J),VR(NX+1,J)
35  CONTINUE
305 FORMAT(1X,72X,3(F10.5,2X))
    YJH=Y(NY)+DELY/2.
    DO 37 I=1,NX
    WRITE(30,306)X(I),YJH,VZ(I,J)
37  CONTINUE
306 FORMAT(1X,36X,3(F10.5,2X))
    WRITE(30,309)
309 FORMAT(//1X,'PARTICLE TRAJECTORIES'//)
    DO 310 J=1,NUMPAR
    WRITE(30,311)DP(J),THETA0(J),FRACV(J)
311 FORMAT(1X,'PARTICLE TRAJECTORY FOR DP= ',F10.5,2X,'THETA0= ',
   1F10.5,2X,'VOLUME FLOW FRACTION= ',F10.5)
    WRITE(30,307)
307 FORMAT(1X,'    RADIUS    ',2X,'AXIAL DIST',2X,'     TIME     ',
   11X,'RADIAL VEL',2X,'AXIAL VEL')
    CALL PARTCL(R,Z,T,UR,UZ,DP(J),THETA0(J),FRACO(J))
    DO 38 I=1,NT*2-1
    WRITE(30,308)R(I),Z(I),T(I),UR(I),UZ(I)
38  CONTINUE
310 CONTINUE
308 FORMAT(1X,5(F10.5,2X))
    STOP
    END
```

COEFF SUBROUTINE

```
      SUBROUTINE COEFF(NX,NY,GUESS)
      DIMENSION CNE(50,50),CNW(50,50),CSE(50,50),CSW(50,50)
      DIMENSION CW(50,50),CN(50,50),CS(50,50)
      DIMENSION CE(50,50),CP(50,50),F(50,50)
      DIMENSION X(50),Y(50),GUESS(50,50)
      DIMENSION ABC(50),BBC(50),CBC(50),DBC(50)
      DIMENSION FBOUND(50,50)
      COMMON CNE,CNW,CSE,CSW,CN,CS,CE,CW,CP,F
      COMMON /BOUNDARY/ABC,BBC,CBC,DBC,ACROSS,BCROSS,CROSS,DCROSS
      COMMON /BOUNDARY2/ ILEFT,IRIGHT,ITOP,IBOTTM
      COMMON /BOUNDARY3/ FBOUND
      COMMON /DEL/ DELX,DELY,DELT
      COMMON /COORD/ X,Y
C*************************************************************
C    THIS SUBROUTINE CALCULATES THE THE LEFT HAND SIDE OF THE
C    DISCRETIZED PDE FOR THE POISSSON TYPE EQUATION FOR
C    THE PRESSURE
C
C***INPUT TO THIS SUBPROGRAM IS AS FOLLOWS:
C
C    "NX" AND "NY" ARE THE NUMBER OF PRESSURE NODES IN THE
C    X AND Y DIRECTIONS
C
C    X=RADIAL DIRECTION, Y=AXIAL DIRECTION
C
C    "GUESS" IS THE GUESSED SOLUTION VECTOR FOR USE WITH
C    NONLINEAR COEFFICIENTS (NOT USED HERE)
C
C***INPUT THRU COMMON BLOCK
C
C    THE "BOUNDARY" COMMON BLOCK CONTAINS THE INFORMATION FOR THE
C    NEUMANN BOUNDARY CONDITIONS AT THE FOUR BOUNDARIES.  THE
C    FIRST FOUR VECTORS CONTAIN THE COEFFICIENTS OF THE NORMAL
C    DERIVATIVES.  THE LAST FOUR ENTRIES DEAL WITH THE CROSS
C    DERIVATIVES AT THE FOUR CORNERS.  SEE WRITE-UP FOR MORE
C    INFORMATION
C
C    THE "BOUNDARY2" COMMON BLOCK CONTAINS THE SWITCHES FOR THE
C    TYPE OF BOUNDARY CONDITION AT THE FOUR BOUNDARIES.
C    =1 FOR NEUMANN BOUNDARY CONDITIONS
C    =0 FOR DIRICHLET BOUNDARY CONDITIONS
```

```
C
C     THE "BOUNDARY3" COMMON BLOCK CONTAINS THE CONTRIBUTIONS TO
C     THE RIGHT HAND SIDE OF THE PDE DUE TO NEUMANN BOUNDARY
C     CONDITIONS
C
C     THE "DEL" COMMON BLOCK CONTAINS THE RADIAL AND AXIAL
C     INCREMENTAL LENGTHS "DELX" AND "DELY", AND THE
C     INCREMENTAL TIME "DELT"
C
C     THE "COORD" COMMON BLOCK CONTAINS THE RADIAL AND AXIAL
C     COORDINATES X AND Y OF THE PRESSURE NODES
C
C
C***OUTPUT THRU THE COMMON BLOCK
C
C     THE COMMON BLOCK NOT NAMED CONTAINS THE COEFFICIENTS OF THE
C     DISCRETIZED PDE AT EACH POINT IN THE COMPUTATIONAL MOLECULE
C     AS A FUNCTION OF THE MOLECULE'S POSITION IN THE DOMAIN. MOST
C     OF THESE COEFFICIENTS ARE CALCULATED IN THIS SUBROUTINE AND
C     ARE DEFINED BELOW:
C     CNE-COEFF OF NORTHEAST POINT
C     CN -COEFF OF NORTH POINT
C     CNW-COEFF OF NORTHWEST POINT
C     CW -COEFF OF WEST POINT
C     CSW-COEFF OF SOUTHWEST POINT
C     CS -COEFF OF SOUTH POINT
C     CSE-COEFF OF SOUTHEAST POINT
C     CE -COEFF OF EAST POINT
C     CP -COEFF OF CENTRAL POINT
C     R  -RIGHT HAND SIDE OF DISCRETIZED PDE (CALCULATED IN
C        SUBROUTINE PSOURC)
C***********************************************************************
C
C*********************************************************
C     CALCULATE COEFFICIENTS OF INTERIOR POINTS
C*********************************************************
         DO 50 I=2,NX-1
         DO 50 J=2,NY-1
         CNE(I,J)=0.
         CNW(I,J)=-CNE(I,J)
         CSE(I,J)=-CNE(I,J)
```

```
      CSW(1,J)=CNE(1,J)
      CC1=1./DELY**2
      CA=1./DELX**2
      CDY=0.
      CDX=(1./X(1))/(2.*DELX)
      CN(1,J)=CC1+CDY
      CS(1,J)=CC1-CDY
      CE(1,J)=CA+CDX
      CW(1,J)=CA-CDX
      CP(1,J)=-2.*CA-2.*CC1
      FBOUND(1,J)=0.
   50 CONTINUE
C
C*****************************************************************
C    MODIFY COEFFICIENTS OF INTERIOR POINTS NEXT TO BOUNDARY
C    FOR NEUMANN CONDITIONS
C*****************************************************************
C
C
C    MODIFY COEFFICIENTS AT LEFT BOTTOM CORNER FOR NEUMANN
C    CONDITIONS ON BOTTOM AND/OR LEFT SIDES
C
      CP(2,2)=CP(2,2)+CW(2,2)*FLOAT(ILEFT)+CS(2,2)
     1*FLOAT(IBOTTM)+CSW(2,2)*FLOAT(ILEFT*IBOTTM)
      CN(2,2)=CN(2,2)+CNW(2,2)*FLOAT(ILEFT)
      CE(2,2)=CE(2,2)+CSE(2,2)*FLOAT(IBOTTM)
      FBOUND(2,2)=-((-CW(2,2)*DELX*BBC(2)-CNW(2,2)*DELX*BBC(3))
     1*FLOAT(ILEFT)+(-CS(2,2)*DELY*CBC(2)-CSE(2,2)*DELY*CBC(3))*
     2FLOAT(IBOTTM)+(CSW(2,2)*(-DELX*BBC(2)-DELY*CBC(2)+DELX*DELY*
     3*CCROSS))*FLOAT(ILEFT*IBOTTM))
      CSW(2,2)=CSW(2,2)*FLOAT(1-(ILEFT+IBOTTM)/2)
      CW(2,2)=CW(2,2)*FLOAT(1-ILEFT)
      CNW(2,2)=CNW(2,2)*FLOAT(1-ILEFT)
      CS(2,2)=CS(2,2)*FLOAT(1-IBOTTM)
      CSE(2,2)=CSE(2,2)*FLOAT(1-IBOTTM)
C
C    MODIFY COEFFICIENTS NEXT TO LEFT BOUNDARY FOR NEUMANN
C    CONDITIONS ON LEFT SIDE
C
      DO 10 J=3,NY-2
      CN(2,J)=CN(2,J)+CNW(2,J)*FLOAT(ILEFT)
      CP(2,J)=CP(2,J)+CW(2,J)*FLOAT(ILEFT)
      CS(2,J)=CS(2,J)+CSW(2,J)*FLOAT(ILEFT)
      FBOUND(2,J)=-(-CNW(2,J)*DELX*BBC(J+1)-CW(2,J)*DELX*BBC(J)
     1-CSW(2,J)*DELX*BBC(J-1))*FLOAT(ILEFT)
      CNW(2,J)=CNW(2,J)*FLOAT(1-ILEFT)
```

```
      CW(2,J)=CW(2,J)*FLOAT(1-ILEFT)
      CSW(2,J)=CSW(2,J)*FLOAT(1-ILEFT)
   10 CONTINUE
C
C   MODIFY COEFFICIENTS AT LEFT TOP CORNER FOR NEUMANN
C   CONDITIONS ON TOP AND/OR LEFT SIDES
C
      CS(2,NY-1)=CS(2,NY-1)+CSW(2,NY-1)*FLOAT(ILEFT)
      CE(2,NY-1)=CE(2,NY-1)+CW(2,NY-1)*FLOAT(ILEFT)+
     1CN(2,NY-1)*FLOAT(ITOP)+CNW(2,NY-1)*FLOAT(ILEFT*ITOP)
      CE(2,NY-1)=CE(2,NY-1)+CNE(2,NY-1)*FLOAT(ITOP)
      ICROSS=ILEFT*ITOP
      FBOUND(2,NY-1)=-((-CW(2,NY-1)*DELX*BBC(NY-1)-CSW(2,NY-1)
     1*DELX*BBC(NY-2))*FLOAT(ILEFT)+(-CN(2,NY-1)*DELY*ABC(2)-
     2CNE(2,NY-1)*DELY*ABC(3))*FLOAT(ITOP)+(CNW(2,NY-1)*(-DELX
     3*BBC(NY-1)-DELY*ABC(2)-DELX*DELY*BCROSS))*FLOAT(ICROSS))
      CSW(2,NY-1)=CSW(2,NY-1)*FLOAT(1-ILEFT)
      CW(2,NY-1)=CW(2,NY-1)*FLOAT(1-ILEFT)
      CNW(2,NY-1)=CNW(2,NY-1)*FLOAT(1-(ILEFT+ITOP)/2)
      CN(2,NY-1)=CN(2,NY-1)*FLOAT(1-ITOP)
      CNE(2,NY-1)=CNE(2,NY-1)*FLOAT(1-ITOP)
C
C   MODIFY COEFFICIENTS NEXT TO TOP BOUNDARY FOR NEUMANN
C   CONDITIONS ON TOP SIDE
C
      DO 20 I=3,NX-2
      CW(I,NY-1)=CW(I,NY-1)+CNW(I,NY-1)*FLOAT(ITOP)
      CP(I,NY-1)=CP(I,NY-1)+CN(I,NY-1)*FLOAT(ITOP)
      CE(I,NY-1)=CE(I,NY-1)+CNE(I,NY-1)*FLOAT(ITOP)
      FBOUND(I,NY-1)=-(-CNW(I,NY-1)*DELY*ABC(I-1)-
     1CN(I,NY-1)*DELY*ABC(I)-CNE(I,NY-1)*DELY*ABC(I+1))*FLOAT(ITOP)
      CNW(I,NY-1)=CNW(I,NY-1)*FLOAT(1-ITOP)
      CN(I,NY-1)=CN(I,NY-1)*FLOAT(1-ITOP)
      CNE(I,NY-1)=CNE(I,NY-1)*FLOAT(1-ITOP)
   20 CONTINUE
C
C   MODIFY COEFFICIENTS AT RIGHT TOP CORNER FOR NEUMANN
C   CONDITIONS ON TOP AND/OR RIGHT SIDES
C
      NX1=NX-1
      NY1=NY-1
      CS(NX1,NY1)=CS(NX1,NY1)+CSE(NX-1,NY-1)*FLOAT(IRIGHT)
```

```
      CP(NX1,NY1)=CP(NX1,NY1)+CE(NX1,NY1)*FLOAT(IRIGHT)
     1+CN(NX1,NY1)*FLOAT(ITOP)+CNE(NX1,NY1)*FLOAT(ITOP*IRIGHT)

      CW(NX1,NY1)=CW(NX1,NY1)+CNW(NX1,NY1)*FLOAT(ITOP)
      ICROSS=IRIGHT*ITOP
      FBOUND(NX1,NY1)=-((-CE(NX1,NY1)*DELX*DBC(NY-1)
     1-CSE(NX1,NY1)*DELY*DBC(NX-2))*FLOAT(IRIGHT)+(-CN(NX1,NY1)*
     2DELY*ABC(NY-1)-CNW(NX1,NY1)*DELY*ABC(NX-2))*FLOAT(ITOP)+
     3(CNE(NX1,NY1)*(-DELX*DBC(NY1)-DELY*ABC(NY1)+DELY*DELX*
     4ACROSS))*FLOAT(ICROSS))
      CNW(NX1,NY1)=CNW(NX1,NY1)*FLOAT(1-ITOP)
      CN(NX1,NY1)=CN(NX1,NY1)*FLOAT(1-ITOP)
      CNE(NX1,NY1)=CNE(NX1,NY1)*FLOAT(1-(IRIGHT+ITOP)/2)
      CE(NX1,NY1)=CE(NX1,NY1)*FLOAT(1-IRIGHT)
      CSE(NX1,NY1)=CSE(NX1,NY1)*FLOAT(1-IRIGHT)
C
C   MODIFY COEFFICIENTS NEXT TO RIGHT BOUNDARY FOR NEUMANN
C   CONDITIONS ON RIGHT SIDE
C
      DO 30 J=3,NY-2
      CN(NX-1,J)=CN(NX-1,J)+CNE(NX-1,J)*FLOAT(IRIGHT)
      CP(NX-1,J)=CP(NX-1,J)+CE(NX-1,J)*FLOAT(IRIGHT)
      CS(NX-1,J)=CS(NX-1,J)+CSE(NX-1,J)*FLOAT(IRIGHT)
      FBOUND(NX-1,J)=-(-CNE(NX-1,J)*DELX*DBC(J+1)-
     1CE(NX-1,J)*DELX*DBC(J)-CSE(NX-1,J)*DELX*DBC(J-1))*
     2FLOAT(IRIGHT)
      CNE(NX-1,J)=CNE(NX-1,J)*FLOAT(1-IRIGHT)
      CE(NX-1,J)=CE(NX-1,J)*FLOAT(1-IRIGHT)
      CSE(NX-1,J)=CSE(NX-1,J)*FLOAT(1-IRIGHT)
   30 CONTINUE
C
C   MODIFY COEFFICIENTS AT RIGHT BOTTOM CORNER FOR NEUMANN
C   CONDITIONS ON BOTTOM AND/OR RIGHT SIDES
C
```

```
      CN(NX-1,2)=CN(NX-1,2)+CNE(NX-1,2)*FLOAT(IRIGHT)
      CP(NX-1,2)=CP(NX-1,2)+CE(NX-1,2)*FLOAT(IRIGHT)+CS(NX-1,2)*
     1FLOAT(IBOTTM)+CSE(NX-1,2)*FLOAT(IBOTTM*IRIGHT)
      CW(NX-1,2)=CW(NX-1,2)+CSW(NX-1,2)*FLOAT(IBOTTM)
      FBOUND(NX-1,2)=-((-CE(NX-1,2)*DELX*DBC(2)-CNE(NX-1,2)*
     1DELX*DBC(3))*FLOAT(IRIGHT)+(-CS(NX-1,2)*DELY*CBC(NX-1)
     2-CSW(NX-1,2)*DELY*CBC(NX-2))*FLOAT(IBOTTM)+(CSE(NX-1,2)*
     3(-DELX*DBC(2)-DELY*CBC(NX-1)-DELX*DELY*DCROSS))*FLOAT(
     4IRIGHT*IBOTTM))
      CNE(NX-1,2)=CNE(NX-1,2)*FLOAT(1-IRIGHT)
      CE(NX-1,2)=CE(NX-1,2)*FLOAT(1-IRIGHT)
      CSE(NX-1,2)=CSE(NX-1,2)*FLOAT(1-(IRIGHT+IBOTTM)/2)
      CS(NX-1,2)=CS(NX-1,2)*FLOAT(1-IBOTTM)
      CSW(NX-1,2)=CSW(NX-1,2)*FLOAT(1-IBOTTM)
C
C     MODIFY COEFFICIENTS NEXT TO BOTTOM BOUNDARY FOR NEUMANN
C     CONDITIONS ON BOTTOM SIDE
C
      DO 40 I=3,NX-2
      CW(I,2)=CW(I,2)+CSW(I,2)*FLOAT(IBOTTM)
      CP(I,2)=CP(I,2)+CS(I,2)*FLOAT(IBOTTM)
      CE(I,2)=CE(I,2)+CSE(I,2)*FLOAT(IBOTTM)
      FBOUND(I,2)=-(-CSW(I,2)*DELY*CBC(I-1)-CS(I,2)*DELY*CBC(I)-
     1CSE(I,2)*DELY*CBC(I+1))*FLOAT(IBOTTM)
      CSW(I,2)=CSW(I,2)*FLOAT(1-IBOTTM)
      CS(I,2)=CS(I,2)*FLOAT(1-IBOTTM)
      CSE(I,2)=CSE(I,2)*FLOAT(1-IBOTTM)
   40 CONTINUE
      RETURN
      END
```

PSOURC SUBROUTINE

```
      SUBROUTINE PSOURC(NX,NY,GUESS)
      DIMENSION CNE(50,50),CNW(50,50),CSE(50,50),CSW(50,50)
      DIMENSION CW(50,50),CN(50,50),CS(50,50)
      DIMENSION CE(50,50),CP(50,50),F(50,50),GUESS(50,50)
      DIMENSION VR(50,50),VZ(50,50),X(50),Y(50),FBOUND(50,50)
      DIMENSION ABC(50),BBC(50),CBC(50),DBC(50)
      DIMENSION FR(50,50),FZ(50,50)
      COMMON CNE,CNW,CSE,CSW,CN,CS,CE,CW,CP,F
      COMMON /DEL/ DELX,DELY,DELT
      COMMON /AIR V/ VZ,VR
      COMMON /BOUNDARY3/ FBOUND
      COMMON /COORD/ X,Y
      COMMON /REYNOLDS/ RE
      COMMON /SOURCE/ FR,FZ
C*************************************************************
C   THIS SUBROUTINE CALCULATES THE RIGHT HAND SIDE OF THE
C   DISCRETIZED PDE FOR THE POISSON TYPE EQUATION FOR
```

```
C     THE PRESSURE
C
C***INPUT TO THIS SUBPROGRAM IS AS FOLLOWS:
C
C     "NX" AND "NY" ARE THE NUMBER OF PRESSURE NODES IN THE
C     X AND Y DIRECTIONS
C
C     X=RADIAL DIRECTION, Y=AXIAL DIRECTION
C
C     "GUESS" IS THE GUESSED SOLUTION VECTOR FOR USE WITH
C     NONLINEAR COEFFICIENTS
C
C     THE "DEL" COMMON BLOCK CONTAINS THE RADIAL AND AXIAL
C     INCREMENTIAL LENGTHS AND "DELX" AND "DELY", AND THE
C     INCREMENTIAL TIME "DELT"
C
C     THE "AIR V" COMMON BLOCK CONTAINS THE RADIAL AND AXIAL
C     COORDINATES X AND Y OF THE PRESSURE NODES
C
C     THE "BOUNDARY3" COMMON BLOCK CONTAINS THE CONTRIBUTIONS
C     TO THE RIGHT HAND SIDE OF THE PDE DUE TO NEUMANN
C     BOUNDARY CONDITIONS
C
C     THE "REYNOLDS" COMMON BLOCK CONTAINS THE REYNOLD'S NUMBER
C     OF NONDIMENSIONALIZATION
C
C     THE "SOURCE" COMMON BLOCK CONTAINS THE SOURCE TERMS OF THE
C     MOMENTUM EQUATIONS
C
C***ALSO REQUIRED AS INPUT FOR THIS SUBPROGRAM ARE THE SET OF
C     FUNCTION SUBPROGRAMS WHICH CALCULATE THE SQUARE OF THE
C     VELOCITIES, THE CROSS MULTIPLICATION OF THE VELOCITIES,
C     AND THE DILATION TERM.  THESE ARE GIVEN AFTER THIS
C     THIS PROGRAM.
C
C***OUTPUT THRU THE COMMON BLOCK
C
C     THE COMMON BLOCK NOT NAMED CONTAINS THE COEFFICIENTS OF THE
C     DISCRETIZED PDE AT EACH POINT IN THE COMPUTATIONAL MOLECULE
C     AS A FUNCTION OF THE MOLECULE'S POSITION IN THE DOMAIN. ONE
C     OF THESE COEFFICIENTS IS CALCULATED IN THIS SUBROUTINE.
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C     THESE COEFFICIENTS ARE DEFINED BELOW:
C     CNE-COEFF OF NORTHEAST POINT (CALCULATED IN SUB COEFF)
C     CN -COEFF OF NORTH POINT     (CALCULATED IN SUB COEFF)
C     CNW-COEFF OF NORTHWEST POINT (CALCULATED IN SUB COEFF)
C     CW -COEFF OF WEST POINT      (CALCULATED IN SUB COEFF)
C     CSW-COEFF OF SOUTHWEST POINT (CALCULATED IN SUB COEFF)
C     CS -COEFF OF SOUTH POINT     (CALCULATED IN SUB COEFF)
C     CSE-COEFF OF SOUTHEAST POINT (CALCULATED IN SUB COEFF)
C     CE -COEFF OF EAST POINT      (CALCULATED IN SUB COEFF)
C     CP -COEFF OF CENTRAL POINT   (CALCULATED IN SUB COEFF)
C     F  -RIGHT HAND SIDE OF DISCRETIZED PDE (CALCULATED HERE)
C**********************************************************************
C
C*********************************************************
C     CALCULATE RIGHT HAND SIDE OF INTERIOR POINTS
C*********************************************************
        DO 50 I=2,NX-1
        DO 50 J=2,NY-1
        F(I,J)=-(VRSQR(I+1,J)-2.*VRSQR(I,J)+VRSQR(I-1,J))/DELX**2
       1-(VZSQR(I,J+1)-2.*VZSQR(I,J)+VZSQR(I,J-1))/DELY**2
       2-(VRSQR(I+1,J)-VRSQR(I-1,J))/(X(I)*DELX)
       3-(VZVR(I,J+1)-VZVR(I,J-1))/(X(I)*DELY)
       4-2.*(VRVZ(I+1,J+1)+VRVZ(I,J)-VRVZ(I+1,J)-VRVZ(I,J+1)
       5)/(DELX*DELY)+D(I,J,X(I))/DELT
       6+(1./RE)*((1./X(I))*(D(I+1,J,X(I+1))-D(I-1,J,X(I-1)))/(2.*DELX)
       7+(D(I+1,J,X(I+1))-2.*D(I,J,X(I))+D(I-1,J,X(I-1)))/DELX**2
       8+(D(I,J+1,X(I))-2.*D(I,J,X(I))+D(I,J-1,X(I)))/DELY**2)
       9+(FZ(I,J)-FZ(I,J-1))/DELY+(FR(I,J)*(X(I)+DELX/2.)-
       1FR(I-1,J)*(X(I)-DELX/2.))/(DELX*X(I))
   50   CONTINUE
C
C************************************************************
C     MODIFY RIGHT HAND SIDE OF INTERIOR POINTS NEXT TO BOUNDARY
C     FOR NEUMANN CONDITIONS
C************************************************************
C
C
C     MODIFY RIGHT HAND SIDE AT LEFT BOTTOM CORNER FOR NEUMANN
C     CONDITIONS ON BOTTOM AND/OR LEFT SIDES
C
        F(2,2)=F(2,2)+FBOUND(2,2)
C
C     MODIFY RIGHT HAND SIDE NEXT TO LEFT BOUNDARY FOR NEUMANN
C     CONDITIONS ON LEFT SIDE
C
        DO 10 J=3,NY-2
        F(2,J)=F(2,J)+FBOUND(2,J)
```

```
   10 CONTINUE
C
C     MODIFY RIGHT HAND SIDE AT LEFT TOP CORNER FOR NEUMANN
C     CONDITIONS ON TOP AND/OR LEFT SIDES
C
      F(2,NY-1)=F(2,NY-1)+FBOUND(2,NY-1)
C
C     MODIFY RIGHT HAND SIDE NEXT TO TOP BOUNDARY FOR NEUMANN
C     CONDITIONS ON TOP SIDE
C
      DO 20 I=3,NX-2
      F(I,NY-1)=F(I,NY-1)+FBOUND(I,NY-1)
   20 CONTINUE
C
C     MODIFY RIGHT HAND SIDE AT RIGHT TOP CORNER FOR NEUMANN
C     CONDITIONS ON TOP AND/OR RIGHT SIDES
C
      NX1=NX-1
      NY1=NY-1
      F(NX1,NY1)=F(NX1,NY1)+FBOUND(NX1,NY1)
C
C     MODIFY RIGHT HAND SIDE NEXT TO RIGHT BOUNDARY FOR NEUMANN
C     CONDITIONS ON RIGHT SIDE
C
      DO 30 J=3,NY-2
      F(NX-1,J)=F(NX-1,J)+FBOUND(NX-1,J)
   30 CONTINUE
C
C     MODIFY RIGHT HAND SIDE AT RIGHT BOTTOM CORNER FOR NEUMANN
C     CONDITIONS ON BOTTOM AND/OR RIGHT SIDES
C
      F(NX-1,2)=F(NX-1,2)+FBOUND(NX-1,2)
C
C     MODIFY RIGHT HAND SIDE NEXT TO BOTTOM BOUNDARY FOR NEUMANN
C     CONDITIONS ON BOTTOM SIDE
C
      DO 40 I=3,NX-2
      F(I,2)=F(I,2)+FBOUND(I,2)
   40 CONTINUE
      RETURN
      END
C
C
C
```

```
C*****************************************************************
C     FUNCTION SUBPROGRAMS FOR SUBROUTINES PSOURC AND GASVEL
C
C
C     COMPUTES THE SQUARE OF THE RADIAL VELOCITY
         FUNCTION VRSQR(I,J)
         DIMENSION VR(50,50),VZ(50,50)
         COMMON /AIR V/ VZ,VR
         VRSQR=.25*(VR(I+1,J)+VR(I,J))**2
         RETURN
         END
C     COMPUTES THE SQUARE OF THE AXIAL VELOCITY
         FUNCTION VZSQR(I,J)
         DIMENSION VZ(50,50),VR(50,50)
         COMMON /AIR V/ VZ,VR
         VZSQR=.25*(VZ(I,J+1)+VZ(I,J))**2
         RETURN
         END
C     COMPUTES THE QUANTITY VR*VZ AT PRESSURE NODE I,J
         FUNCTION VZVR(I,J)
         DIMENSION VZ(50,50),VR(50,50)
         COMMON /AIR V/ VZ,VR
         VZVR=.25*(VR(I+1,J)+VR(I,J))*(VZ(I,J+1)+VZ(I,J))
         RETURN
         END
C     COMPUTES THE QUANTITY VR*VZ AT THE MIDPOINT OF A RECTANGLE
C     FORMED BY FOUR PRESSURE NODES (AT NODE (I-.5,J-.5) IN MAC
C     NOTATION)
         FUNCTION VRVZ(I,J)
         DIMENSION VZ(50,50),VR(50,50)
         COMMON /AIR V/ VZ,VR
         VRVZ=.25*(VR(I,J)+VR(I,J-1))*(VZ(I,J)+VZ(I-1,J))
         RETURN
         END
C     COMPUTES THE DILATION TERM
         FUNCTION D(I,J,R)
         DIMENSION VR(50,50),VZ(50,50)
         COMMON /AIR V/ VZ,VR
         COMMON /DEL/ DELX,DELY,DELT
         CENT=(VR(I+1,J)-VR(I,J))/DELX
         IF(R.LT.0.0001)GO TO 1
         CENT=(VR(I+1,J)*(R+DELX/2.)-VR(I,J)*(R-DELX/2.)
        1)/(R*DELX)
    1    D=(VZ(I,J+1)-VZ(I,J))/DELY+CENT
         RETURN
         END
```

SOR SUBROUTINE

```
      SUBROUTINE SOR(SOLN,ITER,WOMEGA,NX,NY,ITRMAX)
      DIMENSION SOLN(50,50),GUESS(50,50)
      DIMENSION CNE(50,50),CNW(50,50),CSE(50,50),CSW(50,50)
      DIMENSION CN(50,50),CS(50,50),CE(50,50),CW(50,50)
      DIMENSION CP(50,50),F(50,50)
      DIMENSION ABC(50),BBC(50),CBC(50),DBC(50)
      COMMON CNE,CNW,CSE,CSW,CN,CS,CE,CW,CP,F
      COMMON /BOUNDARY/ ABC,BBC,CBC,DBC,ACROSS,BCROSS,CCROSS,DCROSS
      COMMON /BOUNDARY2/ ILEFT,IRIGHT,ITOP,IBOTTM
      COMMON /DEL/ DELX,DELY,DELT
C*****************************************************************
C   THIS PROGRAM USES THE SUCCESSIVE-OVER-RELAXATION SCHEME TO
C   ITERATE FOR THE ELLIPTIC EQUATION SOLUTION
C
C***INPUT TO THIS SUBPROGRAM IS AS FOLLOWS:
C
C   "SOLN" HOLDS THE GUESSED SOLUTION VECTOR INITIALLY
C   AFTER EACH ITERATION "SOLN" HOLDS THE UPDATED SOLUTION
C   AND "GUESS" HOLDS THE OLD SOLUTION
C
C   DETAILS ON THE GUESSED SOLUTION
C   THE GUESSED SOLUTION CAN BE CHOSEN AT INTERIOR POINTS AS THE
C   USER SEES FIT.  AT THE BOUNDARIES THE GUESSED SOLUTION MUST
C   CONTAIN THE VALUES OF THE SOLUTION AT THE BOUNDARIES WHERE THE
C   DIRICHLET CONDITIONS EXIST, INCLUDING CORNER POINTS.  ALONG THE
C   BOUNDARIES WHERE THE NEUMANN CONDITIONS EXIST THE USER MAY
C   SUPPLY GUESSED VALUES THOUGH THESE ARE NOT USED IN THE
C   CALCULATION
C
C   "WOMEGA" CONTAINS THE FIRST GUESSED VALUE OF THE RELAXATION
C   FACTOR, UPDATED AT SUCCESSIVE ITERATIONS BY USING THE RATIO
C   OF THE RESIDUALS AT SUCCESSIVE ITERATIONS.
C
C   "NX" AND "NY" ARE THE NUMBER OF DISCRETIZED POINTS IN
C   THE X AND Y DIRECTIONS
C
C   "ITRMAX" IS THE MAXIMUM NUMBER OF ITERATIONS TO BE ALLOWED.
C   IF THE SOLUTION DOES NOT CONVERGE BY ITRMAX ITERATIONS A
C   MESSAGE IS PRINTED OUT ON THE SCREEN
C
```

```
C***INPUT THRU THE COMMON BLOCKS ARE AS FOLLOWS:
C
C    THE COMMON BLOCK BUF NAMED CONTAINS THE COEFFICIENTS OF THE
C    DISCRETIZED PDE AT EACH POINT IN THE COMPUTATIONAL MOLECULE
C    FOR EACH COMPUTATIONAL MOLECULE.  THIS INFORMATION IS
C    CALCULATED IN THE SUBROUTINE COEFF.
C
C    THE "BOUNDARY" COMMON BLOCK CONTAINS THE INFORMATION FOR THE
C    NEUMANN BOUNDARY CONDITIONS AT THE FOUR BOUNDARIES.   THE
C    FIRST FOUR VECTORS CONTAIN THE COEFFICIENTS OF THE NORMAL
C    DERIVATIVES.  THE LAST FOUR ENTRIES DEAL WITH THE CROSS
C    DERIVATIVES AT THE FOUR CORNERS. SEE WRITE-UP FOR MORE
C    INFORMATION
C
C    THE "BOUNDARY2" COMMON BLOCK CONTAINS THE SWITCHES FOR THE
C    TYPE OF BOUNDARY CONDITIONS AT THE FOUR BOUNDARIES
C    =1 FOR NEUMANN BOUNDARY CONDITIONS
C    =0 FOR DIRICHLET BOUNDARY CONDITIONS
C    TOP BOUNDARY IS DEFINED AS BOUNDARY WITH J=NY
C    BOTTOM BOUNDARY IS DEFINED AS BOUNDARY WITH J=1
C    RIGHT BOUNDARY IS DEFINED AS BOUNDARY WITH I=NX
C    LEFT BOUNDARY IS DEFINED AS BOUNDARY WITH I=1
C
C    THE "DEL" COMMON BLOCK CONTAINS THE RADIAL AND AXIAL
C    INCREMENTIAL LENGTHS "DELX" AND "DELT", AND THE
C    INCREMENTIAL TIME "DELT"
C
C***OUTPUT OF THE SUBPROGRAM IS AS FOLLOWS:
C
C    "SOLN" IS THE SOLUTION VECTOR SOLVED FOR DURING THE ITERATIONS
C    IT REPRESENTS THE UPDATED SOLUTION VECTOR DURING THE ITERATIONS
C    AND THE FINAL SOLUTION AT CONVERGENCE
C
C    "ITER" IS THE NUMBER OF ITERATIONS REQUIRED FOR CONVERGENCE
C
C***OUTPUT INTO DATA FILES
C
C    THE VALUES OF THE RESIDUAL ,THE RESIDUAL NORMALIZED BY THE
C    SOLUTION, AND THE RELAXATION FACTOR AT EACH ITERATION ARE
C    OUTPUTTED IN FILE FOR031.DAT
C
C************************************************************************
```

```
C
C     PUT GUESSED SOLUTION INTO SOLUTION VECTOR
        DO 5 I=1,NX
        DO 5 J=1,NY
        GUESS(I,J)=SOLN(I,J)
    5   CONTINUE
C
C     START ITERATION FOR SOLUTION
C
        DO 100 ITER=1,ITRMAX
C
C     DETERMINE SOLN AT INTERIOR POINTS FOR CURRENT SWEEP
C
        IODEVE=(ITER+1)/2-ITER/2
C     FOR ITER ODD SWEEP DIRECTION IS FORWARD, ITER EVEN SWEEP
C     DIRECTION IS BACKWARD
C     IODEVE=0 FOR ITER=EVEN, IODEVE=1 FOR ITER=ODD
        IFACT=0
        JFACT=0
        IF(IODEVE.EQ.1) GO TO 1
        IFACT=NX+1
        JFACT=NY+1
    1   LPLMIN=(-1.)**(1+IODEVE)
        DO 10 J=2,NY-1
        JC=JFACT+LPLMIN*J
        DO 10 I=2,NX-1
        IC=IFACT+LPLMIN*I
C     PUT OLD SOLUTION INTO GUESS SOLUTION VECTOR AT POINT IC,JC
        GUESS(IC,JC)=SOLN(IC,JC)
C     CALCULATE NEW SOLUTION AT IC,JC
        COCROS=CE(IC,JC)*SOLN(IC+1,JC)+CW(IC,JC)*SOLN(IC-1,JC)
       1+CN(IC,JC)*SOLN(IC,JC+1)+CS(IC,JC)*SOLN(IC,JC-1)
        COCORN=CNE(IC,JC)*SOLN(IC+1,JC+1)+CNW(IC,JC)*SOLN(IC-1,JC+1)
       1+CSE(IC,JC)*SOLN(IC+1,JC-1)+CSW(IC,JC)*SOLN(IC-1,JC-1)
        SOLN(IC,JC)=(1.-WOMEGA)*SOLN(IC,JC)-(WOMEGA/CP(IC,JC))*
       1(COCORN+COCROS-F(IC,JC))
   10   CONTINUE
C
C     CALCULATE RESIDUAL - QUADRATIC NORM
C     NORM OF RESIDUAL NORMALIZED BY NORM OF SOLUTION
C     TO DETERMINE CONVERGENCE
C
        RESOLD=RESNEW
        RESNEW=0.
        SOLNOP=0.
```

```
      DO 20 J=2,NY-1
      DO 20 I=2,NX-1
      COCROS=CE(I,J)*SOLN(I+1,J)+CW(I,J)*SOLN(I-1,J)+
     1CN(I,J)*SOLN(I,J+1)+CS(I,J)*SOLN(I,J-1)
      COCORN=CNE(I,J)*SOLN(I+1,J+1)+CNW(I,J)*SOLN(I-1,J+1)
     1+CSE(I,J)*SOLN(I+1,J-1)+CSW(I,J)*SOLN(I-1,J-1)
      SOLRES=COCROS+COCORN+CP(I,J)*SOLN(I,J)-F(I,J)
      RESNEW=RESNEW+SOLRES**2
      SOLNOR=SOLNOR+SOLN(I,J)**2
   20 CONTINUE
      RESNEW=SQRT(RESNEW)
      RESNOR=RESNEW/SQRT(SOLNOR)
C    ITERATION FINISHED IF RESIDUAL IS SMALL ENOUGH
      IF(RESNOR.LT.0.01)GO TO 2
      IF(ITER.LT.2)GO TO 100
C
C    FOR ITERATIONS AFTER THE FIRST, RECALCULATE OMEGA BASED ON
C    THE CURRENT RESIDUAL AND THE RESIDUAL AT THE PREVIOUS ITERATION
C
      XLAMDA=RESNEW/RESOLD
C     WRITE(6,301)ITER,WOMEGA,RESNEW,RESNOR
C     WRITE(31,301)ITER,WOMEGA,RESNEW,RESNOR
C 301 FORMAT(1X,'ITERATION ',I4,1X,'OMEGA=',F7.5,1X,'RESIDUAL=',
C    1F10.5,1X,'NORMALIZED RESIDUAL=',F10.5)
      IF(XLAMDA.GT.1.)GO TO 100
      WOMEGA=2./(1.+SQRT(1.-XLAMDA))
  100 CONTINUE
      WRITE(6,201)ITMAX
  201 FORMAT(1X,'SOLUTION NOT CONVERGED AFTER ',I4,' ITERATIONS')
C
C    DETERMINE SOLUTION AT BOUNDARY FOR NEUMANN BOUNDARY CONDITIONS
C
C    DETERMINE SOLUTION AT BOUNDARY FOR NEUMANN BC AT LEFT
    2 IF(ILEFT.EQ.0)GO TO 31
      DO 41 J=2,NY-1
      SOLN(1,J)=SOLN(2,J)-DELX*BNC(J)
   41 CONTINUE
C    DETERMINE SOLUTION AT BOUNDARY FOR NEUMANN BC AT RIGHT
   31 IF(IRIGHT.EQ.0)GO TO 32
      DO 42 J=2,NY-1
      SOLN(NX,J)=SOLN(NX-1,J)-DELX*DNC(J)
   42 CONTINUE
C    DETERMINE SOLUTION AT BOUNDARY FOR NEUMANN BC AT TOP
```

```
   32 IF(ITOP.EQ.0)GO TO 33
      DO 43 I=2,NX-1
      SOLN(I,NY)=SOLN(I,NY-1)-DELY*ABC(I)
   43 CONTINUE
C     DETERMINE SOLUTION AT BOUNDARY FOR NEUMANN BC AT BOTTOM
   33 IF(IBOTTM.EQ.0)GO TO 34
      DO 44 I=2,NX-1
      SOLN(I,1)=SOLN(I,2)-DELY*CBC(I)
   44 CONTINUE
C     DETERMINE SOLUTION AT TOP LEFT CORNER FOR NEUMANN CONDITION
C     AT TOP AND LEFT SIDES
   34 IF(ILEFT*ITOP.EQ.0)GO TO 35
      SOLN(1,NY)=SOLN(1,NY-1)+SOLN(2,NY)-SOLN(2,NY-1)-
     1DELX*DELY*BCROSS
C     DETERMINE SOLUTION AT BOTTOM LEFT CORNER FOR NEUMANN CONDITION
C     AT BOTTOM AND LEFT SIDES
   35 IF(ILEFT*IBOTTM.EQ.0)GO TO 36
      SOLN(1,1)=SOLN(2,1)+SOLN(1,2)-SOLN(2,2)+DELX*DELY*CCROSS
C     DETERMINE SOLUTION AT TOP RIGHT CORNER FOR NEUMANN CONDITION
C     AT TOP AND RIGHT SIDES
   36 IF(IRIGHT*ITOP.EQ.0)GO TO 37
      SOLN(NX,NY)=SOLN(NX,NY-1)+SOLN(NX-1,NY)-SOLN(NX-1,NY-1)+
     1DELX*DELY*ACROSS
C     DETERMINE SOLUTION AT BOTTOM RIGHT CORNER FOR NEUMANN CONDITION
C     AT BOTTOM AND RIGHT SIDES
   37 IF(IRIGHT*IBOTTM.EQ.0)GO TO 38
      SOLN(NX,1)=SOLN(NX,2)+SOLN(NX-1,1)-SOLN(NX-1,2)-
     1DELX*DELY*DCROSS
   38 RETURN
      END
```

GASVEL SUBROUTINE

```
      SUBROUTINE GASVEL(NX,NY,VZN,VRN,P)
      DIMENSION VZ(50,50),VZN(50,50),VR(50,50),VRN(50,50)
      DIMENSION X(50),Y(50),P(50,50)
      DIMENSION FR(50,50),FZ(50,50)
      COMMON /DEL/ DELX,DELY,DELT
      COMMON /COORD/ X,Y
      COMMON /AIR V/ VZ,VR
      COMMON /REYNOLDS/ RE
      COMMON /SOURCE/ FR,FZ
C*****************************************************************
C
C     THIS SUBROUTINE CALCULATES THE UPDATED VELOCITIES
C     (AXIAL VELOCITY = VZ; RADIAL VELOCITY = VR)
C     EXPLICITLY FROM THE VELOCITY AND PRESSURE AT THE
C     PREVIOUS ITERATION.  THESE EQUATIONS ARE DISCRETIZED
C     FORMS OF THE MOMENTUM EQUATIONS IN THE AXIAL AND
C     RADIAL DIRECTIONS.  HERE UPWIND DIFFERENCING IS USED
C     AND AVERAGING OF THE POINTS NEAR THE BOUNDARY IS DONE
C     (SEE REPORT)
C
C*****************************************************************
C
C***INPUT TO THIS SUBROUTINE IS AS FOLLOWS
C
C     "NX" AND "NY" ARE THE NUMBER OF PRESSURE NODES IN
C     THE X (RADIAL) AND Y (AXIAL) DIRECTIONS
C
C     "P" IS THE NX BY NY ARRAY OF PRESSURES IN THE FLOW
C     FIELD
C
C***INPUT THRU COMMON BLOCKS
C
C     THE "DEL" COMMON BLOCK CONTAINS THE RADIAL AND AXIAL
C     INCREMENTAL LENGTHS "DELX" AND "DELY", AND THE
C     INCREMENTAL TIME "DELT"
C
C     THE "COORD" COMMON BLOCK CONTAINS THE RADIAL AND AXIAL
C     COORDINATES X AND Y OF THE PRESSURE NODES
C
C     THE "AIR V" COMMON BLOCK CONTAINS THE ARRAY OF AXIAL
C     AND RADIAL VELOCITIES AT THE PREVIOUS ITERATION
C     THE RADIAL VELOCITY ARRAY VR IS NX+1 BY NY
C     THE AXIAL VELOCITY ARRAY VZ IS NX BY NY+1
C     THE "REYNOLDS" COMMON BLOCK CONTAINS THE REYNOLD'S
C     NUMBER OF NONDIMENSIONALIZATION
C
C     THE "SOURCE" COMMON BLOCK CONTAINS THE SOURCE TERMS OF THE
C     MOMENTUM EQUATIONS
C
C
```

```
C***ALSO REQUIRED AS INPUT FOR THIS SUBROUTINE ARE THE SET
C    OF FUNCTION SUBPROGRAMS WHICH CALCULATE THE SQUARE OF THE
C    VELOCITIES, AND THE CROSS MULTIPLICATION OF THE VELOCITIES.
C    THESE ARE GIVEN AFTER THE SUBROUTINE PSOURC
C
C***OUTPUT
C
C    "VRN" AND "VZN" ARE THE UPDATED ARRAYS OF RADIAL AND
C    AXIAL VELOCITY
C******************************************************
C
C    CALCULATE VELOCITIES AT INTERIOR POINTS
C    CALCULATE THE AXIAL COMPONENT OF VELOCITY
        DO 10 I=2,NX-1
        DO 10 J=2,NY
C    UPWIND DIFFERENCING
        VRAVIJ=.25*(VR(I,J-1)+VR(I,J)+VR(I+1,J)+VR(I+1,J-1))
        DERVR=(VZ(I,J)-VZ(I-1,J))/DELX
        IF(VRAVIJ.LT.0.)DERVR=(VZ(I+1,J)-VZ(I,J))/DELX
        DERVZ=(VZ(I,J)-VZ(I,J-1))/DELY
        IF(VZ(I,J).LT.0.)DERVZ=(VZ(I,J+1)-VZ(I,J))/DELY
        VZMOD=VZ(I,J)
        IF(J.EQ.NY)VZMOD=.5*(VZ(I,J+1)+VZ(I,J-1))
        VZN(I,J)=VZMOD
       1+DELT*(-VRAVIJ*DERVR-VZ(I,J)*DERVZ
       2-(P(I,J)-P(I,J-1))/DELY+(1./RE)*((VZ(I+1,J)-
       3VZ(I-1,J))/(X(I)*2.*DELX)+(VZ(I+1,J)-2.*VZ(I,J)+
       4VZ(I-1,J))/DELX**2+(VZ(I,J+1)-2.*VZ(I,J)+VZ(I,J-1))
       5/DELY**2)+FZ(I,J-1))
   10  CONTINUE
C    CALCULATE THE RADIAL COMPONENT OF VELOCITY
        DO 20 I=2,NX
        DO 20 J=2,NY-1
C    UPWIND DIFFERENCING
        VZAVIJ=.25*(VZ(I,J)+VZ(I,J+1)+VZ(I-1,J)+VZ(I-1,J+1))
        DERVR=(VR(I,J)-VR(I-1,J))/DELX
        IF(VR(I,J).LT.0.)DERVR=(VR(I+1,J)-VR(I,J))/DELX
        DERVZ=(VR(I,J)-VR(I,J-1))/DELY
        IF(VZAVIJ.LT.0.)DERVZ=(VR(I,J+1)-VR(I,J))/DELY
        RHALF=(X(I+1)+X(I))/2.
        VRMOD=VR(I,J)
        IF(I.EQ.2)VRMOD=.5*(VR(I+1,J)+VR(I-1,J))
        VRN(I,J)=VRMOD
       1+DELT*(-VR(I,J)*DERVR-VZAVIJ*DERVZ
       2-(P(I,J)-P(I-1,J))/DELX+(1./RE)*((1./(2.*DELX))*
       3((VR(I+1,J)/(RHALF+DELX))-(VR(I-1,J)/(RHALF-DELX)))+
       4(VR(I+1,J)-2.*VR(I,J)+VR(I-1,J))/DELX**2+
       5(VR(I,J+1)-2.*VR(I,J)+VR(I,J-1))/DELY**2)+FR(I-1,J))
   20  CONTINUE
C
```

```
C
C    CALCULATE VELOCITIES AT BOUNDARIES
C
C***CALCULATE SOLUTION ON LEFT BOUNDARY
C    ZERO SHEAR STRESS ON AXIS OF SYMMETRY
C    DVZ/DZ=0.

         DO 30 J=2,NY
         VZN(1,J)=(4.*VZN(2,J)-VZN(3,J))/3.
   30    CONTINUE
C      NORMAL DERIVATIVE OF VELOCITY IS ZERO
         DO 40 J=2,NY-1
         VRN(1,J)=-VRN(2,J)
   40    CONTINUE
C***CALCULATE SOLUTION AT BOTTOM BOUNDARY
C    AXIAL VELOCITY =0. SOLID BOUNDARY
         DO 50 I=1,NX
         VZN(I,NY+1)=-VZN(I,NY)
C        VZN(I,NY+1)=0.
   50    CONTINUE
C    RADIAL VELOCITY =0. NO SLIP CONDITION
         DO 60 I=1,NX+1
         VRN(I,NY)=0.
   60    CONTINUE
C***CALCULATE SOLUTION AT TOP BOUNDARY
C    AXIAL VELOCITY =0. SOLID BOUNDARY
         DO 90 I=1,NX
         VZN(I,1)=-VZN(I,2)
   90    CONTINUE
C    RADIAL VELOCITY =0. NO SLIP CONDITION
         DO 100 I=1,NX+1
         VRN(I,1)=0.
  100    CONTINUE
C***CALCULATE SOLUTION AT RIGHT BOUNDARY
C    MASS CONSERVATION
         DO 70 J=2,NY
         VRN(NX+1,J)=VRN(NX,J)*(X(NX-1)+DELX/2.)/(X(NX)+DELX/2.)
   70    CONTINUE
C    NO VORTICITY
         DO 80 J=2,NY-1
C        VZN(NX,J)=VZN(NX-1,J)+(VRN(NX,J)-VRN(NX,J-1))*DELX/DELY
C    NO AXIAL VELOCITY
         VZN(NX,J)=0.
   80    CONTINUE
         RETURN
         END
```

PARTCL SUBROUTINE

```
      SUBROUTINE PARTCL(F,Z,T,UR,UZ,DF,THETA0,FRACQ)
      DIMENSION UR(50),UZ(50),T(50),R(50),Z(50)
      DIMENSION FORCER(50),FORCEZ(50)
      DIMENSION FR(50,50),FZ(50,50)
      COMMON /DEL/ DELX,DELY,DELT
      COMMON /SPRAY DIM/ ZD,N,RL,DP,G,RO
      COMMON /REYNOLDS/ RE
      COMMON /SOURCE/ FR,FZ
      REAL N,NF
      PI=3.14159
C*******************************************************************
C     THIS SUBROUTINE CALCULATES THE PARAMETERS OF THE LIQUID
C     PHASE, NAMELY THE PARTICLE TRAJECTORIES, THE PARTICLE
C     VELOCITY ALONG THE TRAJECTORY, AND THE EXCHANGE OF MOMENTUM
C     TO THE GAS PHASE (THE SOURCE TERMS).
C
C     THESE CALCULATIONS ARE MADE BY INTEGRATING THE EQUATIONS OF
C     MOTION OF THE PARTICLE, AND DETERMINING THE NET DRAG FORCE
C     ACTING ON THE PARTICLES IN A GIVEN VOLUME.
C     A FOURTH ORDER INTEGRATION IS USED FOR THE INTEGRATION OF
C     THE EQUATIONS OF MOTION.
C
C     NOTE:THIS PROGRAM IS SET UP ONLY FOR THE CASE WHERE THE NOZZLE
C          POSITION CORRESPONDS WITH A PRESSURE NODE LOCATION
C
C***INPUT TO THIS PROGRAM IS AS FOLLOWS:
C
C     "DP" - PARTICLE DIAMETER / NOZZLE DIAMETER
C     "THETA0" - THE INITIAL HALF ANGLE OF THE SPRAY AT THE
C                INJECTION POINT
C     "FRACQ"  - IS THE FRACTION OF MASS FLOW OF EACH PARTICULAR
C                TRAJECTORY OR DROPLET SIZE
C
C
C***INPUT THRU COMMON BLOCK
```

```
C
C    THE "SPRAY DIM" COMMON BLOCK CONTAINS THE PARAMETERS OF THE
C    SPRAY
C
C    "ZL"-THE AXIAL LENGTH OF THE DOMAIN DIVIDED BY THE HEIGHT OF
C    SPRAY
C    "H"-EQUALS ONE FOR A DOWNWARD FACING SPRAY
C    "RL"-THE RADIAL LENGTH OF THE DOMAIN DIVIDED BY THE HEIGHT OF
C    THE SPRAY
C    "HP"-HEIGHT OF THE SPRAY FROM GROUND / NOZZLE DIAMETER
C    "G"-INVERSE FROUDE NUMBER SQUARED
C        ACCERATION OF GRAVITY*HEIGHT OF SPRAY/EJECTION VELOCITY**2
C    "RO"-DENSITY OF GAS/DENSITY OF LIQUID
C
C    THE "DEL" COMMON BLOCK CONTAINS THE RADIAL AND AXIAL INCREMENTAL
C    DISTANCE AND (THOUGH NOT USED IN THIS SUBROUTINE) THE INCREMENT/
C    TIME
C
C    THE "REYNOLDS" COMMON BLOCK CONTAINS THE REYNOLDS NUMBER OF
C    NONDIMENSIONALIZATION - EJECTION VELOCITY*SPRAY HEIGHT/GAS
C    VISCOSITY
C
C***OUTPUT
C
C    "Z" AND "R" ARE THE ARRAYS CONTAINING THE AXIAL AND RADIAL
C    COMPONENTS OF THE PARTICLE TRAJECTORIES
C
C    "UZ" AND "UR" ARE THE ARRAYS CONTAINING THE AXIAL AND RADIAL
C    COMPONENTS OF THE SPRAY VELOCITY ALONG THE SPRAY TRAJECTORY
C***OUTPUT THRU COMMON BLOCK
C
C    THE "SOURCE" COMMON BLOCK CONTAINS THE TWO DIMENSIONAL
C    ARRAY OF THE RADIAL AND AXIAL SOURCE TERMS "FR" AND "FZ"
C
C*******************************************************************
         DELZ=DELY/2.
         NUMY=IFIX(ZL/DELZ)
         NX=IFIX(RL/DELX)+1
         NY=IFIX(ZL/DELY)+1
C*********************************************************
C    DETERMINATION OF PARTICLE TRAJECTORY AND VELOCITY
C    ALONG THE TRAJECTORY
C*********************************************************
C    INITIAL CONDITIONS FOR PARTICLE EQUATIONS
        UR(1)=SIN(THETA0)
        UZ(1)=COS(THETA0)
        R(1)=0.
```

```
      R(1)=0.
      Z(1)=0.
      CONST=.75*RO*HP**2/(DP**2*RL)
      CONST2=1.5/(DP**3*HP**2)
      DO 10 I=1,NUMT
C     START FIRST STEP OF RUNGE-KUTTA INTEGRATION
      RK0=DELZ*(UR(I)/UZ(I))
      TK0=DELZ/UZ(I)
C     CALCULATE AIRFIELD VELOCITY
      CALL AIRVEL(R(I),Z(I),VR,VZ)
      REYNLD=RE*DP*SQRT((UR(I)-VR)**2+(UZ(I)-VZ)**2)/HP
      C2=-CD(REYNLD)*REYNLD*CONST
      URK0=DELZ*(C2*(UR(I)-VR))/UZ(I)
      UZK0=DELZ*(C2*(UZ(I)-VZ)+G)/UZ(I)
C     START SECOND STEP OF RUNGE-KUTTA INTEGRATION
      UR1=UR(I)+URK0/2.
      UZ1=UZ(I)+UZK0/2.
      RK1=DELZ*(UR1/UZ1)
      TK1=DELZ/UZ1
      R1=R(I)+RK1/2.
      Z1=Z(I)+DELZ/2.
C     CALCULATE AIRFIELD VELOCITY
      CALL AIRVEL(R1,Z1,VR,VZ)
      REYNLD=RE*DP*SQRT((UR1-VR)**2+(UZ1-VZ)**2)/HP
      C2=-CD(REYNLD)*REYNLD*CONST
      URK1=DELZ*(C2*(UR1-VR))/UZ1
      UZK1=DELZ*(C2*(UZ1-VZ)+G)/UZ1
C     START THIRD STEP OF RUNGE-KUTTA INTEGRATION
      UR1=UR(I)+URK1/2.
      UZ1=UZ(I)+UZK1/2.
      RK2=DELZ*(UR1/UZ1)
      TK2=DELZ/UZ1
      R1=R(I)+RK2/2.
C     CALCULATE AIR FIELD VELOCITY
      CALL AIRVEL(R1,Z1,VR,VZ)
      REYNLD=RE*DP*SQRT((UR1-VR)**2+(UZ1-VZ)**2)/HP
      C2=-CD(REYNLD)*REYNLD*CONST
      URK2=DELZ*(C2*(UR1-VR))/UZ1
      UZK2=DELZ*(C2*(UZ1-VZ)+G)/UZ1
C     START FOURTH STEP OF RUNGE-KUTTA INTEGRATION
      UR1=UR(I)+URK2
      UZ1=UZ(I)+UZK2
      RK3=DELZ*(UR1/UZ1)
      TK3=DELZ/UZ1
      R1=R(I)+RK3
      Z1=Z(I)+DELZ
```

```
C    CALCULATE AIR FIELD VELOCITY
        CALL AIRVEL(R1,Z1,VR,VZ)
        REYNLD=RE*DP*SQRT((UR1-VR)**2+(UZ1-VZ)**2)/HP
        C2=-CD(REYNLD)*REYNLD*CONST
        URK3=DELZ*(C2*(UR1-VR))/UZ1
        UZK3=DELZ*(C2*(UZ1-VZ)+G)/UZ1
C    CALCULATE VALUE AT END OF INTEGRATION STEP
        T(I+1)=T(I)+(1./6.)*(TK0+2.*TK1+2.*TK2+TK3)
        R(I+1)=R(I)+(1./6.)*(RK0+2.*RK1+2.*RK2+RK3)
        UR(I+1)=UR(I)+(1./6.)*(URK0+2.*URK1+2.*URK2+URK3)
        UZ(I+1)=UZ(I)+(1./6.)*(UZK0+2.*UZK1+2.*UZK2+UZK3)
        Z(I+1)=Z(I)+DELZ
   10   CONTINUE
C    CALCULATION OF TRAJECTORY AND VELOCITIES ALONG TRAJECTORY
C    COMPLETED
C*******************************************************************
C    CALCULATE NONDIMENSIONAL FORCE COMPONENTS ALONG THE
C    TRAJECTORY
C*******************************************************************
        CALL AIRVEL(R(1),Z(1),VR,VZ)
        VTOT=SQRT((UR(1)-VR)**2+(UZ(1)-VZ)**2)
        REYNLD=RE*DP*VTOT/HP
        FORCER(1)=-CD(REYNLD)*VTOT*(UR(1)-VR)*HP*PI*DP**2/8.
        FORCEZ(1)=-CD(REYNLD)*VTOT*(UZ(1)-VZ)*HP*PI*DP**2/8.
        CALL AIRVEL(R(NUMY+1),Z(NUMY+1),VR,VZ)
        VTOT=SQRT((UR(NUMY+1)-VR)**2+(UZ(NUMY+1)-VZ)**2)
        REYNLD=RE*DP*VTOT/HP
        FORCER(NUMY+1)=-CD(REYNLD)*VTOT*(UR(NUMY+1)-VR)*HP*PI*DP**2/8
        FORCEZ(NUMY+1)=-CD(REYNLD)*VTOT*(UZ(NUMY+1)-VZ)*HP*PI*DP**2/8
        C1=PI*DP**3/(6.*RO)
        DO 30 J=2,NUMY
        FORCEZ(J)=C1*(UZ(J)*(UZ(J+1)-UZ(J-1))/(2.*DELZ)-G)
        FORCER(J)=C1*(UZ(J)*(UR(J+1)-UR(J-1))/(2.*DELZ))
   30   CONTINUE
C*******************************************************************
C    CALCULATE SOURCE TERMS IN EACH CONTROL VOLUME
C*******************************************************************
        IFR1=1
        JFR1=IFIX((ZL-H)/DELY+.5)+1
        IFZ1=1
        JFZ1=IFIX((ZL-H)/DELY)+1
        DO 50 J=1,NUMY
```

```
      JFR2=IFIX((ZB-H+Z(J+1))/DELY+.5)+1
      IFR2=IFIX(R(J+1)/DELX)+1
      FRAVE=(FORCER(J)+FORCER(J+1))/2.
      JFZ2=IFIX((ZB-H+Z(J+1))/DELY)+1
      IFZ2=IFIX(R(J+1)/DELX+.5)+1
      FZAVE=(FORCEZ(J)+FORCEZ(J+1))/2.
      TOTL=SQRT((R(J+1)-R(J))**2+(Z(J+1)-Z(J))**2)
      ICROSR=IFR2-IFR1+1
      ICROSZ=IFZ2-IFZ1+1
      DO 60 I=1,ICROSR
      DELTR=R(J+1)-R(J)
      DELTB=FLOAT(IFR1)*DELX-R(J)+DELX*FLOAT(I-1)
      DELTE=R(J+1)-FLOAT(IFR2-1)*DELX+DELX*FLOAT(ICROSR-I)
      DELTM=DELX
      IF(DELTR.GT.DELTM)DELTR=DELTM
      IF(DELTR.GT.DELTB)DELTR=DELTB
      IF(DELTR.GT.DELTE)DELTR=DELTE
      SCALE=1.
      IF(ABS(R(J+1)-R(J)).GT.0.)SCALE=DELTR/(R(J+1)-R(J))
      DELTZ=SCALE*DELZ
      DT=DELTZ/((UZ(J)+UZ(J+1))/2.)
      FR(IFR1+I-1,JFR1)=FR(IFR1+I-1,JFR1)-FRAVE*DT*FRACU*CONST2/
     1((2.*DELX*FLOAT(IFR1+I-2)+DELX)*PI*DELX*DELZ)
   60 CONTINUE
      JFR1=JFR2
      IFR1=IFR2
      DO 70 I=1,ICROSZ
      DELTR=R(J+1)-R(J)
      DELTB=(FLOAT(IFZ1)-.5)*DELX-R(J)+DELX*FLOAT(I-1)
      DELTE=R(J+1)-(FLOAT(IFZ2-1)-.5)*DELX+DELX*FLOAT(ICROSZ-I)
      DELTM=DELZ
      IF(DELTR.GT.DELTM)DELTR=DELTM
      IF(DELTR.GT.DELTB)DELTR=DELTB
      IF(DELTR.GT.DELTE)DELTR=DELTE
      SCALE=1.
      IF(ABS(R(J+1)-R(J)).GT.0.)SCALE=DELTR/(R(J+1)-R(J))
      DELTZ=SCALE*DELZ
      DT=DELTZ/((UZ(J)+UZ(J+1))/2.)
      R1=.5+FLOAT(IFZ1+I-3)
      R1=AMAX(0.,R1)*DELX
      R2=(.5+FLOAT(IFZ1+I-2))*DELX
      FZ(IFZ1+I-1,JFZ1)=FZ(IFZ1+I-1,JFZ1)-FZAVE*DT*FRACU*CONST2/
     1(PI*(R2**2-R1**2)*DELZ)
   70 CONTINUE
```

```
      JFZ1=JFZ2
      IFZ1=IFZ2
   50 CONTINUE
      RETURN
      END
C   THIS FUNCTION SUBPROGRAM GIVES THE DRAG COEFFICIENT
C   OF A SPHERE AS A FUNCTION OF REYNOLDS NUMBER
      FUNCTION CD(RE)
      CD=(24./RE)+6./(1.+SQRT(RE))+.4
      RETURN
      END
C   THIS SUBROUTINE CALCULATES THE AXIAL AND RADIAL COMPONENTS
C   OF AIR VELOCITY AT POINTS INTERIOR TO THE MAC CELL BY
C   INTERPOLATING THE DISCRETIZED MAC CELL VELOCITIES
      SUBROUTINE AIRVEL(R,Z,VROFRZ,VZOFRZ)
      DIMENSION VR(50,50),VZ(50,50)
      COMMON /DEL/ DELX,DELY,DELT
      COMMON /AIR V/ VZ,VR
      COMMON /SPRAY DIM/ ZL,H,RL,HP,G,RO
      REAL H,HP
      IVR=IFIX(R/DELX+.5)+1
      JVR=IFIX((ZL-H+Z)/DELY)+1
      IVZ=IFIX(R/DELX)+1
      JVZ=IFIX((ZL-H+Z)/DELY+.5)+1
      DX=(R/DELX+.5-FLOAT(IVR)+1.)*DELX
      DY=((ZL-H+Z)/DELY-FLOAT(JVR)+1.)*DELY
      VROFRZ=VR(IVR,JVR)+(VR(IVR+1,JVR)-VR(IVR,JVR))*DX/DELX
     1+(VR(IVR,JVR+1)-VR(IVR,JVR))*DY/DELY
      DX=(R/DELX-FLOAT(IVZ)+1.)*DELX
      DY=((ZL-H+Z)/DELY+.5-FLOAT(JVZ)+1.)*DELY
      VZOFRZ=VZ(IVZ,JVZ)+(VZ(IVZ+1,JVZ)-VZ(IVZ,JVZ))*DX/DELX
     1+(VZ(IVZ,JVZ+1)-VZ(IVZ,JVZ))*DY/DELY
      RETURN
      END
```

INPUT FILE

```
INPUT FOR SPRAY PROGRAM
DOMAIN DIMENSIONS
ENTER RADIAL AND AXIAL DOMAIN DIMENSIONS; RL AND ZL;
NONDIMENSIONALIZED BY THE SPRAY HEIGHT
1XXXXXXXXXX22XXXXXXXXXX
     1.          1.
ENTER SPRAY HEIGHT/SPRAY LENGTH,H, AND SPRAY LENGTH/NOZZLE DIAMETER,HP
1XXXXX/XXXX22XXXXXXXXXX
   1.           454.55
ENTER NUMBER OF PARTICLE SIZES OR TRAJECTORIES
1XX
  4
ENTER DROPLET DIAMETER/NOZZLE DIAMETER, DP, INITIAL ANGLE OF SPRAY,
THETAO,AND VOLUME FLOW FRACTION OF EACH PARTICULAR TRAJECTORY, FRACO
1XXXXXXXXXX22XXXXXXXXXX22XXXXXXXXXX
    .09091        .5236         .1667
    .09091        .4405         .3333
    .09091        .3217         .3333
    .09091        .0            .1667
ENTER INVERSE FROUDE NUMBER**2 ,G,AND REYNOLDS NUM OF NONDIMEN, RE
1XXXXXXXXXX22XXXXXXXXXX
    .03636      3097300.
ENTER DENSITY OF GAS/DENSITY OF LIQUID, RO
1XXXXXXXXXX
  .0012024
ENTER TIME STEP, DELT, SUBJECT TO STABILITY RESTRICTIONS
1XXXXXXXXXX
   0.02
ENTER NUMBER OF ITERATIONS OF MAC METHOD
1XXXXXXXX
 1000
ENTER SWITCH FOR INITIAL SOLUTION (0=NONE,1=IN FILE 19)
1X
 0
ENTER NUMBER OF PRESSURE NODES IN RADIAL AND AXIAL DIRECTIONS
 NX  NY
1XX22XX
 11  11
```

OUTPUT FILE

RADIAL LENGTH= 1.00000 AXIAL LENGTH= 1.00000
SPRAY HEIGHT= 1.00000 SPRAY LENGTH TO NOZZLE DIAMETER RATIO= 154.54999
NUMBER OF PARTICLE SIZES/TRAJECTORIES= 4
DROPLET DIAMETER/NOZZLE DIAMETER= 0.09091 HALF ANGLE OF SPRAY= 0.52360 VOLUME FLOW FRACTION= 0.16670
DROPLET DIAMETER/NOZZLE DIAMETER= 0.09091 HALF ANGLE OF SPRAY= 0.34050 VOLUME FLOW FRACTION= 0.33330
DROPLET DIAMETER/NOZZLE DIAMETER= 0.09091 HALF ANGLE OF SPRAY= 0.32170 VOLUME FLOW FRACTION= 0.33330
DROPLET DIAMETER/NOZZLE DIAMETER= 0.09091 HALF ANGLE OF SPRAY= 0.00000 VOLUME FLOW FRACTION= 0.16670
INVERSE FROUDE NUMBER SQUARED= 0.03636 REYNOLDS NUMBER OF NONDIMENSIONALIZATION= 0.30973E+07
DENSITY OF GAS/DENSITY OF LIQUID= 0.00120
TIME STEP = 0.0200000
NUMBER OF ITERATIONS= 1000

NODE LOCATION RADIAL POSITION	NODE LOCATION AXIAL POSITION	PRESSURE
0.00000	0.00000	-0.00264
0.10000	0.00000	-0.00284
0.20000	0.00000	-0.00046
0.30000	0.00000	-0.00018
0.40000	0.00000	-0.00008
0.50000	0.00000	-0.00004
0.60000	0.00000	-0.00002
0.70000	0.00000	-0.00001
0.80000	0.00000	0.00000
0.90000	0.00000	0.00000
1.00000	0.00000	0.00000
0.00000	0.10000	-0.00264
0.10000	0.10000	-0.00284
0.20000	0.10000	-0.00046
0.30000	0.10000	-0.00018
0.40000	0.10000	-0.00008
0.50000	0.10000	-0.00004
0.60000	0.10000	-0.00002
0.70000	0.10000	-0.00001
0.80000	0.10000	0.00000
0.90000	0.10000	0.00000
1.00000	0.10000	0.00000
0.00000	0.20000	-0.00108
0.10000	0.20000	-0.00108
0.20000	0.20000	-0.00040
0.30000	0.20000	-0.00017
0.40000	0.20000	-0.00008
0.50000	0.20000	-0.00004
0.60000	0.20000	-0.00002
0.70000	0.20000	-0.00001
0.80000	0.20000	0.00000
0.90000	0.20000	0.00000
1.00000	0.20000	0.00000

NODE LOCATION RADIAL POSITION	NODE LOCATION AXIAL POSITION	AXIAL VELOCITY
0.00000	-0.05000	-0.01625
0.10000	-0.05000	-0.01219
0.20000	-0.05000	0.00000
0.30000	-0.05000	0.00000
0.40000	-0.05000	0.00000
0.50000	-0.05000	0.00000
0.60000	-0.05000	0.00000
0.70000	-0.05000	0.00000
0.80000	-0.05000	0.00000
0.90000	-0.05000	0.00000
1.00000	-0.05000	0.00000
0.00000	0.05000	0.01625
0.10000	0.05000	0.01219
0.20000	0.05000	0.00000
0.30000	0.05000	0.00000
0.40000	0.05000	0.00000
0.50000	0.05000	0.00000
0.60000	0.05000	0.00000
0.70000	0.05000	0.00000
0.80000	0.05000	0.00000
0.90000	0.05000	0.00000
1.00000	0.05000	0.00000
0.00000	0.15000	0.11301
0.10000	0.15000	0.08427
0.20000	0.15000	-0.00196
0.30000	0.15000	-0.00086
0.40000	0.15000	-0.00036
0.50000	0.15000	-0.00011
0.60000	0.15000	0.00001
0.70000	0.15000	0.00005
0.80000	0.15000	0.00003
0.90000	0.15000	0.00001
1.00000	0.15000	0.00000

NODE LOCATION RADIAL POSITION	NODE LOCATION AXIAL POSITION	RADIAL VELOCITY
-0.05000	0.00000	0.00000
0.05000	0.00000	0.00000
0.15000	0.00000	0.00000
0.25000	0.00000	0.00000
0.35000	0.00000	0.00000
0.45000	0.00000	0.00000
0.55000	0.00000	0.00000
0.65000	0.00000	0.00000
0.75000	0.00000	0.00000
0.85000	0.00000	0.00000
0.95000	0.00000	0.00000
1.05000	0.00000	0.00000
-0.05000	0.10000	0.01505
0.05000	0.10000	-0.01505
0.15000	0.10000	-0.05192
0.25000	0.10000	-0.02955
0.35000	0.10000	-0.02035
0.45000	0.10000	-0.01550
0.55000	0.10000	-0.01259
0.65000	0.10000	-0.01066
0.75000	0.10000	-0.00929
0.85000	0.10000	-0.00822
0.95000	0.10000	-0.00736
1.05000	0.10000	-0.00666
-0.05000	0.20000	0.01106
0.05000	0.20000	-0.01106
0.15000	0.20000	-0.03635
0.25000	0.20000	-0.02836
0.35000	0.20000	-0.02022
0.45000	0.20000	-0.01567
0.55000	0.20000	-0.01288
0.65000	0.20000	-0.01101
0.75000	0.20000	-0.00963
0.85000	0.20000	-0.00853
0.95000	0.20000	-0.00764
1.05000	0.20000	-0.00692

0.00000	0.30000	-0.00067	0.00000	0.25000	0.17662
0.10000	0.30000	-0.00087	0.10000	0.25000	0.13399
0.20000	0.30000	-0.00045	0.20000	0.25000	0.00612
0.30000	0.30000	-0.00016	0.30000	0.25000	-0.00088
0.40000	0.30000	-0.00007	0.40000	0.25000	-0.00043
0.50000	0.30000	-0.00003	0.50000	0.25000	-0.00003
0.60000	0.30000	-0.00002	0.60000	0.25000	0.00013
0.70000	0.30000	-0.00001	0.70000	0.25000	0.00014
0.80000	0.30000	0.00000	0.80000	0.25000	0.00007
0.90000	0.30000	.00000	0.90000	0.25000	0.00002
1.00000	0.30000	0.00000	1.00000	0.25000	0.00000
0.00000	0.40000	-0.00094	0.00000	0.35000	0.19849
0.10000	0.40000	-0.00094	0.10000	0.35000	0.15642
0.20000	0.40000	-0.00024	0.20000	0.35000	0.03022
0.30000	0.40000	-0.00011	0.30000	0.35000	-0.00228
0.40000	0.40000	-0.00005	0.40000	0.35000	-0.00045
0.50000	0.40000	-0.00003	0.50000	0.35000	0.00027
0.60000	0.40000	-0.00001	0.60000	0.35000	0.00046
0.70000	0.40000	-0.00001	0.70000	0.35000	0.00038
0.80000	0.40000	0.00000	0.80000	0.35000	0.00021
0.90000	0.40000	0.00000	0.90000	0.35000	0.00007
1.00000	0.40000	0.00000	1.00000	0.35000	0.00000
0.00000	0.50000	-0.00049	0.00000	0.45000	0.20236
0.10000	0.50000	-0.00049	0.10000	0.45000	0.16522
0.20000	0.50000	-0.00013	0.20000	0.45000	0.05321
0.30000	0.50000	-0.00007	0.30000	0.45000	-0.00069
0.40000	0.50000	-0.00003	0.40000	0.45000	0.00046
0.50000	0.50000	-0.00002	0.50000	0.45000	0.00106
0.60000	0.50000	-0.00001	0.60000	0.45000	0.00114
0.70000	0.50000	-0.00001	0.70000	0.45000	0.00089
0.80000	0.50000	0.00000	0.80000	0.45000	0.00051
0.90000	0.50000	0.00000	0.90000	0.45000	0.00018
1.00000	0.50000	0.00000	1.00000	0.45000	0.00000
0.00000	0.60000	0.00039	0.00000	0.55000	0.19907
0.10000	0.60000	0.00039	0.10000	0.55000	0.16736
0.20000	0.60000	-0.00007	0.20000	0.55000	0.07222
0.30000	0.60000	-0.00003	0.30000	0.55000	0.00531
0.40000	0.60000	-0.00002	0.40000	0.55000	0.00161
0.50000	0.60000	-0.00001	0.50000	0.55000	0.00230
0.60000	0.60000	-0.00001	0.60000	0.55000	0.00225
0.70000	0.60000	-0.00001	0.70000	0.55000	0.00172
0.80000	0.60000	-0.00001	0.80000	0.55000	0.00102
0.90000	0.60000	-0.00001	0.90000	0.55000	0.00039
1.00000	0.60000	0.00000	1.00000	0.55000	0.00000

-0.05000	0.30000	0.00538
0.05000	0.36000	-0.00538
0.15000	0.30000	-0.01657
0.25000	0.30000	-0.02934
0.35000	0.30000	-0.01975
0.45000	0.30000	-0.01533
0.55000	0.30000	-0.01282
0.65000	0.30000	-0.01115
0.75000	0.30000	-0.00989
0.85000	0.30000	-0.00886
0.95000	0.30000	-0.00797
1.05000	0.30000	-0.00721
-0.05000	0.40000	0.00227
0.05000	0.40000	-0.00227
0.15000	0.40000	-0.00658
0.25000	0.40000	-0.02288
0.35000	0.40000	-0.01772
0.45000	0.40000	-0.01459
0.55000	0.40000	-0.01265
0.65000	0.40000	-0.01133
0.75000	0.40000	-0.01029
0.85000	0.40000	-0.00936
0.95000	0.40000	-0.00848
1.05000	0.40000	-0.00767
-0.05000	0.50000	0.00058
0.05000	0.50000	-0.00058
0.15000	0.50000	-0.00162
0.25000	0.50000	-0.01573
0.35000	0.50000	-0.01631
0.45000	0.50000	-0.01378
0.55000	0.50000	-0.01240
0.65000	0.50000	-0.01151
0.75000	0.50000	-0.01075
0.85000	0.50000	-0.00997
0.95000	0.50000	-0.00912
1.05000	0.50000	-0.00825
-0.05000	0.60000	-0.00146
0.05000	0.60000	0.00146
0.15000	0.60000	0.00451
0.25000	0.60000	-0.00920
0.35000	0.60000	-0.01459
0.45000	0.60000	-0.01254
0.55000	0.60000	-0.01193
0.65000	0.60000	-0.01162
0.75000	0.60000	-0.01125
0.85000	0.60000	-0.01071
0.95000	0.60000	-0.00997
1.05000	0.60000	[illegible]

0.00000	0.70000	0.00152
0.10000	0.70000	0.00152
0.20000	0.70000	0.00047
0.30000	0.70000	0.00003
0.40000	0.70000	0.00001
0.50000	0.70000	-0.00001
0.60000	0.70000	-0.00002
0.70000	0.70000	-0.00002
0.80000	0.70000	-0.00002
0.90000	0.70000	-0.00001
1.00000	0.70000	0.00000
0.00000	0.80000	0.00436
0.10000	0.80000	0.00436
0.20000	0.80000	0.00217
0.30000	0.80000	0.00035
0.40000	0.80000	0.00003
0.50000	0.80000	-0.00003
0.60000	0.80000	-0.00004
0.70000	0.80000	-0.00004
0.80000	0.80000	-0.00003
0.90000	0.80000	-0.00003
1.00000	0.80000	0.00000
0.00000	0.90000	0.00865
0.10000	0.90000	0.00865
0.20000	0.90000	0.00551
0.30000	0.90000	0.00222
0.40000	0.90000	0.00059
0.50000	0.90000	0.00016
0.60000	0.90000	0.00004
0.70000	0.90000	0.00000
0.80000	0.90000	-0.00001
0.90000	0.90000	-0.00004
1.00000	0.90000	0.00000
0.00000	1.00000	0.00865
0.10000	1.00000	0.00865
0.20000	1.00000	0.00551
0.30000	1.00000	0.00222
0.40000	1.00000	0.00053
0.50000	1.00000	0.00016
0.60000	1.00000	0.00004
0.70000	1.00000	0.00000
0.80000	1.00000	-0.00001
0.90000	1.00000	-0.00004
1.00000	1.00000	0.00000

0.00000	0.65000	0.16599
0.10000	0.65000	0.16126
0.20000	0.65000	0.08707
0.30000	0.65000	0.01460
0.40000	0.65000	0.00295
0.50000	0.65000	0.00413
0.60000	0.65000	0.00391
0.70000	0.65000	0.00298
0.80000	0.65000	0.00184
0.90000	0.65000	0.00080
1.00000	0.65000	0.00000
0.00000	0.75000	0.15927
0.10000	0.75000	0.14302
0.20000	0.75000	0.09427
0.30000	0.75000	0.02988
0.40000	0.75000	0.00308
0.50000	0.75000	0.00723
0.60000	0.75000	0.00639
0.70000	0.75000	0.00490
0.80000	0.75000	0.00327
0.90000	0.75000	0.00176
1.00000	0.75000	0.00000
0.00000	0.85000	0.11267
0.10000	0.85000	0.10401
0.20000	0.85000	0.07801
0.30000	0.85000	0.04244
0.40000	0.85000	0.01955
0.50000	0.85000	0.01168
0.60000	0.85000	0.00820
0.70000	0.85000	0.00629
0.80000	0.85000	0.00511
0.90000	0.85000	0.00468
1.00000	0.85000	0.00000
0.00000	0.95000	0.03787
0.10000	0.95000	0.03500
0.20000	0.95000	0.02637
0.30000	0.95000	0.01440
0.40000	0.95000	0.00657
0.50000	0.95000	0.00391
0.60000	0.95000	0.00274
0.70000	0.95000	0.00210
0.80000	0.95000	0.00170
0.90000	0.95000	0.00156
1.00000	0.95000	0.00000
0.00000	1.05000	-0.03787
0.10000	1.05000	-0.03500
0.20000	1.05000	-0.02637
0.30000	1.05000	-0.01440
0.40000	1.05000	-0.00657
0.50000	1.05000	-0.00391
0.60000	1.05000	-0.00274
0.70000	1.05000	-0.00210
0.80000	1.05000	-0.00170
0.90000	1.05000	-0.00156
1.00000	1.05000	0.00000

-0.05000	0.70000	-0.00445
0.05000	0.70000	0.00445
0.15000	0.70000	0.01356
0.25000	0.70000	0.00232
0.35000	0.70000	-0.01153
0.45000	0.70000	-0.01090
0.55000	0.70000	-0.01100
0.65000	0.70000	-0.01159
0.75000	0.70000	-0.01184
0.85000	0.70000	-0.01179
0.95000	0.70000	-0.01147
1.05000	0.70000	-0.01038
-0.05000	0.80000	-0.00959
0.05000	0.80000	0.00959
0.15000	0.80000	0.02909
0.25000	0.80000	0.03046
0.35000	0.80000	0.01097
0.45000	0.80000	-0.00528
0.55000	0.80000	-0.00838
0.65000	0.80000	-0.00679
0.75000	0.80000	-0.00892
0.85000	0.80000	-0.00961
0.95000	0.80000	-0.01137
1.05000	0.80000	-0.01029
-0.05000	0.90000	-0.01687
0.05000	0.90000	0.01687
0.15000	0.90000	0.05109
0.25000	0.90000	0.07159
0.35000	0.90000	0.07511
0.45000	0.90000	0.07065
0.55000	0.90000	0.06440
0.65000	0.90000	0.05954
0.75000	0.90000	0.05552
0.85000	0.90000	0.05220
0.95000	0.90000	0.04966
1.05000	0.90000	0.04493
-0.05000	1.00000	0.00000
0.05000	1.00000	0.00000
0.15000	1.00000	0.00000
0.25000	1.00000	0.00000
0.35000	1.00000	0.00000
0.45000	1.00000	0.00000
0.55000	1.00000	0.00000
0.65000	1.00000	0.00000
0.75000	1.00000	0.00000
0.85000	1.00000	0.00000
0.95000	1.00000	0.00000
1.05000	1.00000	0.00000

PARTICLE TRAJECTORIES

PARTICLE TRAJECTORY FOR DP= 0.09091 THETA0= 0.52360 VOLUME FLOW FRACTION= 0.16670

RADIUS	AXIAL DIST	TIME	RADIAL VEL	AXIAL VEL
0.00000	0.00000	0.00000	0.50000	0.86602
0.02882	0.05000	0.06299	0.41970	0.72934
0.05749	0.10000	0.13758	0.35197	0.61905
0.08535	0.15000	0.22471	0.29072	0.53413
0.11177	0.20000	0.32512	0.23617	0.46584
0.13643	0.25000	0.43981	0.19386	0.40855
0.15918	0.30000	0.57101	0.15539	0.35585
0.18021	0.35000	0.72201	0.12473	0.30839
0.19961	0.40000	0.89701	0.09857	0.26461
0.21736	0.45000	1.10095	0.07656	0.22840
0.23307	0.50000	1.33558	0.05845	0.19965
0.24661	0.55000	1.60183	0.04409	0.17712
0.25798	0.60000	1.90082	0.03266	0.15857
0.26730	0.65000	2.23161	0.02436	0.14454
0.27500	0.70000	2.59267	0.01866	0.13308
0.28158	0.75000	2.98208	0.01584	0.12467
0.28813	0.80000	3.39359	0.01641	0.11806
0.29611	0.85000	3.83341	0.02033	0.11055
0.30696	0.90000	4.29096	0.02845	0.10711
0.32292	0.95000	4.77512	0.03532	0.09925
0.34092	1.00000	5.30573	0.03057	0.08680

PARTICLE TRAJECTORY FOR DP= 0.09091 THETA0= 0.44050 VOLUME FLOW FRACTION= 0.33330

RADIUS	AXIAL DIST	TIME	RADIAL VEL	AXIAL VEL
0.00000	0.00000	0.00000	0.42639	0.90454
0.02354	0.05000	0.06009	0.36072	0.76728
0.04698	0.10000	0.13074	0.30519	0.65572
0.06984	0.15000	0.21273	0.25495	0.56939
0.09164	0.20000	0.30658	0.21191	0.50067
0.11210	0.25000	0.41255	0.17579	0.44559
0.13105	0.30000	0.53145	0.14472	0.39739
0.14858	0.35000	0.66475	0.11933	0.35446
0.16475	0.40000	0.81329	0.09804	0.31557
0.17965	0.45000	0.98270	0.07981	0.28000
0.19323	0.50000	1.17191	0.06439	0.24950
0.20548	0.55000	1.38487	0.05141	0.22199
0.21636	0.60000	1.62119	0.04124	0.20242
0.22569	0.65000	1.87611	0.03334	0.18705
0.23425	0.70000	2.15542	0.02746	0.17453
0.24177	0.75000	2.45091	0.02367	0.16319
0.24892	0.80000	2.76331	0.02260	0.15546
0.25694	0.85000	3.09623	0.02614	0.14558
0.26736	0.90000	3.45099	0.03340	0.13506
0.28190	0.95000	3.84617	0.03829	0.11862
0.29837	1.00000	4.29944	0.03262	0.10097

PARTICLE TRAJECTORY FOR DP= 0.09091 THETA0= 0.32170 VOLUME FLOW FRACTION= 0.33330

RADIUS	AXIAL DIST	TIME	RADIAL VEL	AXIAL VEL
0.00000	0.00000	0.00000	0.31616	0.94870
0.01665	0.05000	0.05707	0.26975	0.81063
0.03325	0.10000	0.12368	0.23048	0.69793
0.04950	0.15000	0.20045	0.19448	0.61017
0.06596	0.20000	0.28766	0.16400	0.54076
0.07977	0.25000	0.38525	0.13657	0.48698
0.09349	0.30000	0.49398	0.11659	0.44259
0.10622	0.35000	0.61114	0.09935	0.40597
0.11796	0.40000	0.73959	0.08417	0.37356
0.12879	0.45000	0.87909	0.07153	0.34433
0.13874	0.50000	1.03015	0.06068	0.31840
0.14786	0.55000	1.19345	0.05168	0.29494
0.15639	0.60000	1.36917	0.04414	0.27462
0.16410	0.65000	1.55801	0.03858	0.25591
0.17141	0.70000	1.75962	0.03414	0.24017
0.17841	0.75000	1.97516	0.03111	0.22478
0.18533	0.80000	2.20404	0.02978	0.21169
0.19273	0.85000	2.44955	0.03072	0.19632
0.20120	0.90000	2.71426	0.03394	0.18079
0.21172	0.95000	3.00656	0.03705	0.16098
0.22387	1.00000	3.31239	0.03367	0.13643

PARTICLE TRAJECTORY FOR DP= 0.09091 THETA0= 0.00000 VOLUME FLOW FRACTION= 0.16670

RADIUS	AXIAL DIST	TIME	RADIAL VEL	AXIAL VEL
0.00000	0.00000	0.00000	0.00000	1.00000
0.00001	0.05000	0.05393	0.00055	0.85139
0.00009	0.10000	0.11638	0.00170	0.74710
0.00020	0.15000	0.18762	0.00132	0.65855
0.00028	0.20000	0.26815	0.00083	0.59051
0.00034	0.25000	0.35699	0.00053	0.53835
0.00037	0.30000	0.45370	0.00005	0.49752
0.00037	0.35000	0.55784	-0.00006	0.46433
0.00035	0.40000	0.66893	-0.00027	0.43685
0.00032	0.45000	0.78665	-0.00029	0.41346
0.00027	0.50000	0.91077	-0.00037	0.39328
0.00023	0.55000	1.04085	-0.00039	0.37559
0.00016	0.60000	1.17692	-0.00046	0.35962
0.00010	0.65000	1.31896	-0.00051	0.34472
0.00001	0.70000	1.46715	-0.00066	0.33013
-0.00010	0.75000	1.62213	-0.00074	0.31517
-0.00025	0.80000	1.78443	-0.00103	0.29882
-0.00043	0.85000	1.95760	-0.00116	0.28012
-0.00070	0.90000	2.14341	-0.00164	0.25734
-0.00096	0.95000	2.34903	-0.00062	0.22861
-0.00031	1.00000	2.58589	0.00139	0.19310

DATE
FILMED

05-8