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RESTRICTED L1 ESTIMATORS AND THEIR COVARIANCES, (U)

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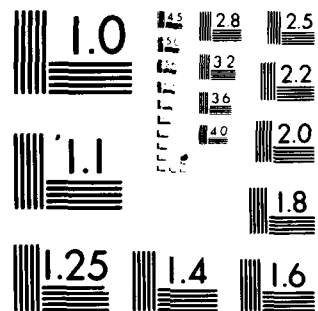
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$$m = \frac{2}{3} \frac{1}{1 + \frac{1}{2} \frac{1}{1 + \frac{1}{2} \frac{1}{1 + \dots}}}$$

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PROJECT THEMIS

Technical Report #65

RESTRICTED L_1 ESTIMATORS
AND THEIR COVARIANCES

by

Book, D., Bocker, J.,
Hartley, H.O., and Sielken, R.L. Jr.

Texas A&M University
Office of Naval Research
Contract N00014-78-C-0426
Project NR047-179

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ATTACHMENT I

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THEMIS OPTIMIZATION RESEARCH PROGRAM
Technical Report No. 65
June 1980

INSTITUTE OF STATISTICS
Texas A&M University

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ATTACHMENT II

ABSTRACT

The parameters in a linear regression model can be estimated by minimizing the sum of the absolute residuals (L_1 estimation) instead of the more classical approach of minimizing the sum of squared residuals (least squares estimation). In addition to other nice properties L_1 estimators are less sensitive to outliers than least squares estimators. This paper describes a linear programming algorithm and computer program for obtaining L_1 estimators and estimates of their covariances when the regression parameters are restricted to satisfy specified linear constraints. These estimated covariances are the new feature in this work and are an extremely important ingredient in hypothesis tests and confidence interval construction. Technical Report 64 describes a similar procedure for obtaining unbiased L_1 estimators when there are no constraints on the parameters.



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Restricted L_1 Estimators and their Covariances

1. An Introduction to MR. A

Consider the linear regression model in the form

$$y = X\beta + \epsilon \quad (1)$$

where y is a vector of n observations, X is an $n \times p$ matrix of rank p of known constants, β is a vector of p unknown parameters and ϵ is a vector of independent random variables (noise) symmetrically distributed with mean zero and variance σ^2 . The estimation of β can be obtained under several different optimality criteria. For β unrestricted, the classical least squares estimator, $\tilde{\beta}$, where

$$\tilde{\beta} = (X^T X)^{-1} X^T Y, \quad (2)$$

has the smallest variance among the class of unbiased linear functions of y . However, the least squares estimator is extremely sensitive to large values of $|\epsilon|$, outliers, particularly when the sample size, n , is small relative to p , say $n \leq 2(p+1)$. In addition, the least squares estimator does not have the flexibility of allowing restrictions to be placed on β . These two drawbacks suggest that an optimality criteria other than least squares be considered. Several authors (Barrodale (1968), Charnes and Cooper (1964), Gentle, Kennedy and Sposito (1977), Harris (1950), Harter (1974), Rice and White (1964), Taylor (1973) have suggested that

$$\sum_{i=1}^n |y_i - X_i \beta| \quad (3)$$

should be minimized with respect to β where y_i is the i -th observation and X_i is the i -th row of X . The estimator, $\hat{\beta}$, which minimizes the sum of the absolute residuals is often called the L_1 estimator.

The L_1 estimate is not necessarily unique since it depends upon the

method of computation. However, conventional linear programming algorithms can calculate the L_1 estimate with m added linear restrictions on β in the forms,

$$\begin{aligned} A\beta &\leq b & \text{or} & & (4) \\ A\beta &\leq b & \text{or} & & \\ A\beta &= b \end{aligned}$$

where A is an $m \times p$ matrix of known coefficients for β and b is a vector of m constants.

The computer program MR. A implements an algorithm for Minimizing the sum of the Absolute Residuals where β is restricted by any of the relationships given in (4). The algorithm uses the least squares estimator $\tilde{\beta}$ as an initial unbiased antisymmetrical estimator of β where

$$\beta - \tilde{\beta}(\epsilon) = -[\beta - \tilde{\beta}(-\epsilon)] \quad (5)$$

in order to provide efficiency in computation as suggested by Barrodale and Roberts (1973).

The estimation of the unrestricted L_1 estimator is only a first step in the completion of the task of estimating β in model (1) subject to restrictions in (4). The second step is the estimation of the covariances of $\hat{\beta}$. This estimation would usually be required for any further developments in hypothesis testing or confidence intervals. MR. A contains a mini-Monte Carlo procedure similar to that used in the algorithm MRS. A (Book, Booker, Hartley, Sielken 1980) for estimating the covariance of the restricted L_1 estimator. This feature sets MR. A apart from other restricted L_1 procedures.

2. Computational Procedure

The problem of minimizing the sum of the absolute residuals can be formulated as follows:

$$\min \sum_{i=1}^n (r_{1i} + r_{2i}) \quad (6)$$

subject to

$$y_i - X_i \beta = (r_{1i} - r_{2i}), \quad i = 1, \dots, n,$$

$$r_{1i}, r_{2i} \geq 0,$$

(7)

$$A_i \beta \begin{cases} < \\ > \\ = \end{cases} b_i,$$

where the i -th residual, r_i , is expressed as the difference between two positive components, r_{1i} and r_{2i} . To promote the unbiasedness of the resulting L_1 estimator and to increase the efficiency of computation in the algorithm, the problem is reformulated following Hartley and Sielken (1973) by introducing the antisymmetrical least squares estimator, $\beta_0 = \hat{\beta}$, and transforming (7) to

$$y_i - X_i \beta_0 - X_i (\beta - \beta_0) = r_{1i} - r_{2i}, \quad i = 1, \dots, n \quad (8)$$

$$A_i (\beta - \beta_0) \begin{cases} < \\ > \\ = \end{cases} b_i - A_i \beta_0$$

Then, using

$$\beta = \beta^{(1)} - \beta^{(2)},$$

$$\beta_0 = \beta_0^{(1)} - \beta_0^{(2)}$$

with $\beta^{(1)}, \beta^{(2)}, \beta_0^{(1)}, \beta_0^{(2)} \geq 0$ in (8) yields

$$X_i (\beta^{(1)} + \beta_0^{(2)}) - X_i (\beta^{(2)} + \beta_0^{(1)}) + r_{1i} - r_{2i} = y_i - X_i \beta_0$$

$$A_i (\beta^{(1)} + \beta_0^{(2)}) - A_i (\beta^{(2)} + \beta_0^{(1)}) \begin{cases} < \\ > \\ = \end{cases} b_i - A_i \beta_0 \quad (9)$$

or equivalently

$$X_i B_1 - X_i B_2 + r_{1i} - r_{2i} = y_i - X_i \beta_0$$

$$A_i B_1 - A_i B_2 \begin{cases} < \\ > \\ = \end{cases} b_i - A_i \beta_0$$

for $i = 1, \dots, n$ where

$$B_1 = \beta^{(1)} + \beta_0^{(2)} \geq 0 \quad (10)$$

$$B_2 = \beta^{(2)} + \beta_0^{(1)} \geq 0.$$

Now, in order to promote unbiasedness of $\hat{\beta}$ and to partially compensate for any idiosyncrasies in the particular linear programming algorithm used to solve the problem in (6), (7), (9), (10), the problem is considered in two equivalent forms P_1 and P_2 with MR. A randomly selecting either P_1 or P_2 with probability $1/2$. In matrix notation, the problems P_1 and P_2 are as follows:

$$P_1: \min \sum_{i=1}^n (r_{1i} + r_{2i})$$

subject to

$$\begin{bmatrix} X & -X & I & -I \\ A & -A & 0 & 0 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ r_1 \\ r_2 \end{bmatrix} \begin{cases} = \\ < \\ > \\ = \end{cases} \begin{bmatrix} y - X\beta_0 \\ b - A\beta_0 \end{bmatrix}$$

$$B_1, B_2, r_1, r_2 \geq 0;$$

$$P_2: \min \sum_{i=1}^n (r_{1i} + r_{2i})$$

subject to

$$\begin{bmatrix} -X & X & I & -I \\ -A & A & 0 & 0 \end{bmatrix} \begin{bmatrix} B_2 \\ B_1 \\ r_1 \\ r_2 \end{bmatrix} \begin{cases} = \\ < \\ > \\ = \end{cases} \begin{bmatrix} y - X\beta_0 \\ b - A\beta_0 \end{bmatrix}$$

$$B_1, B_2, r_1, r_2 \geq 0.$$

After MR. A selects and solves either P_1 or P_2 , the L_1 estimator is $\hat{\beta}$ where

$$\hat{\beta} = B_1 - B_2 + \beta_0. \quad (11)$$

In cases where the sample size is large, say $n \gg (p+1)^2$, the sample could be randomly divided into G groups, then $\hat{\beta}_g$ estimated for each group g separately, and the covariance of $\hat{\beta}$ estimated from the sample covariance of the $\hat{\beta}_g$'s.

MR. A estimates the covariance of $\hat{\beta}$ for any sample size using a mini-Monte Carlo procedure. A Monte-Carlo estimate is obtained by essentially generating several sets of n y 's, finding the L_1 estimate for each set, and computing the sample covariance. There are two difficulties with generating the y 's; namely,

(1) β is unknown, and

(2) σ^2 is unknown.

To overcome the first difficulty, the sum of the absolute residuals is expressed as

$$\begin{aligned} \sum_{i=1}^n |y_i - X_i \hat{\beta}| &= \sum_{i=1}^n |X_i \beta + \epsilon_i - X_i \hat{\beta}| \\ &= \sum_{i=1}^n |\epsilon_i - X_i (\hat{\beta} - \beta)| \\ &= \sum_{i=1}^n |\epsilon_i - X_i \delta\beta| \end{aligned} \quad (12)$$

using (1) and $\delta\beta = \hat{\beta} - \beta$. Note that the covariance of the L_1 estimator of $\delta\beta$ is the same as the covariance of $\hat{\beta}$. Thus, only sets of n ϵ 's need be generated. To deal with the second difficulty (the unknown σ^2), note that

$$\sum_{i=1}^n | \epsilon_i - X_i \delta \beta | = \sigma \sum_{i=1}^n | \epsilon_i^* - X_i \delta \beta^* | \quad (13)$$

and the restrictions on β in (4) become

$$\begin{aligned} A\hat{\beta} - A\beta & \begin{cases} < \\ > \\ = \end{cases} b - A\beta \\ A(\delta\beta) & \begin{cases} < \\ > \\ = \end{cases} b - A\beta \\ A(\delta\beta^*) & \begin{cases} < \\ > \\ = \end{cases} \frac{b - A\beta}{\sigma} \end{aligned} \quad (14)$$

where $\delta\beta^* = (\hat{\beta} - \beta)/\sigma$ and where $\epsilon_i^* = \epsilon_i/\sigma$ and is symmetrically distributed with mean 0 and variance 1. Furthermore,

$$\text{Covariance } (\hat{\beta}) = \text{Covariance } (\delta\beta) = \sigma^2 \text{Covariance } (\delta\beta^*). \quad (15)$$

The mini-Monte Carlo procedure generates K sets of n ϵ_i^* 's; finds the K L_1 estimates $\hat{\delta\beta}_k^*$; and estimates the covariance of $\hat{\beta}$ by $\hat{\sigma}$ times the sample covariance of $\hat{\delta\beta}_k^*$ where $\hat{\sigma}$ is calculated from

$$\hat{\sigma} = \left\{ \frac{\sum_{i=1}^n | y_i - X_i \hat{\beta} | / (n-p)}{\frac{1}{K} \sum_{k=1}^K \sum_{i=1}^n | \epsilon_{ik}^* - X_i \hat{\delta\beta}_k^* | / (n-p)} \right\}^{1/2} \quad (16)$$

This estimator of σ reflects both the variability in $y_i - X_i \beta$ and the variability associated with the linear programming algorithm.

In the estimation of each $\delta\beta_k^*$, an additional problem arises from the right hand side of (14) being a function of β and σ . This problem is overcome by estimating β using (11) and estimating σ using (16) for each sample, $k \geq 2$, as it is generated. For the first sample ($k = 1$), σ is estimated by the numerator of (16). This numerator has been shown to provide a reasonable alternative estimator of σ [Book, Booker, Hartley,

MR. A allows the user to generate the ϵ^* 's from the uniform, normal or double exponential distributions. These distributions were selected in order to represent short, medium and long tailed distributions, respectively. These three distributions are also interesting because maximum likelihood estimation corresponds to minimizing the maximum absolute residual if the ϵ 's are uniform, minimizing the sum of squared residuals if the ϵ 's are normal, and minimizing the sum of the absolute residuals if the ϵ 's are double exponential.

MR. A also allows the user to assign weight coefficients to the residuals, changing (6) to

$$\min \sum_{i=1}^n w_i (r_{1i} + r_{2i})$$

where w_i is the weight coefficient for the i -th residual, r_i . If the w_i 's are not all equal to one, then the objective function in the mini-Monte Carlo study becomes

$$\sum_{i=1}^n w_i | \epsilon_i^* - X_i \delta \beta^* |$$

and the estimate of σ also reflects the w_i , namely,

$$\hat{\sigma} = \frac{\sum_{i=1}^n w_i | y_i - X_i \hat{\beta} |}{(n-p)}$$

$$\frac{1}{K} \sum_{k=1}^K \sum_{i=1}^n w_i | \epsilon_{ik}^* - X_i (\hat{\delta \beta}^*)_k | / (n-p)$$

3. MR. A Users' Guide and Sample Problem

MR. A consists of a main program and subroutines. The functions of these components are given in Table 1 at the end of this section.

The input to the computer program consists of five basic card groups. Input instructions are also briefly documented in the program comment statements.

The first card or card image contains the ancillary statistics required for the specific problem as follows:

First Card:

<u>Card Column</u>	<u>Variable Name</u>	<u>Description</u>
1-3	IP	= p = number of beta parameters (format I3; i.e., a 3 digit integer, right justified)
4-6	NOBS	= n = number of observations (format I3)
7-9	NC	= m = number of constraints (format I3)
11-20	NSEED	Ten digit random number ≤ 2147483647 (format I10)
21-25	ISAM	= K = number of samples for the mini-Monte Carlo study (format I5)
27	IWRIT0	= 1 if the problem input is to be printed = 0 otherwise
29	IWRIT1	= 1 if the main results of the L_1 estimation are to be printed = 0 otherwise
31	IWRIT2	= 1 if the intermediate results of the L_1 estimation are to be printed = 0 otherwise
33	IWRIT3	= 1 if the basis inverse is to be printed = 0 otherwise
35	IWRIT4	= 1 if the main results of the mini-Monte Carlo study are to be printed = 0 otherwise
37	IWRIT5	= 1 if the intermediate results of the mini-Monte Carlo study are to be printed = 0 otherwise

<u>Card Column</u>	<u>Variable Name</u>	<u>Description</u>
39	IWRIT6	= 1 if the intermediate results in the determination of the covariance of $\hat{\beta}$ are to be printed = 0 otherwise
41	IOPTN	= 1 if the ϵ^* 's are to be normally distributed = 2 if the ϵ^* 's are to be double exponentially distributed = 3 if the ϵ^* 's are to be uniformly distributed.
43	IWT	= 1 if the residuals are assigned weights, w_1 = 0 if the residuals are not weighted.

The remaining card input instructions are as follows:

<u>Card Type</u>	<u>Variable Name</u>	<u>Description</u>
Second card group:	$ITYPE_i, i=1, \dots, NC$	The type of the i-th constraint on β = 0 if the constraint is $\leq$ type = 1 if the constraint is $\geq$ type = 2 if the constraint is = type (Format I2)
Third card group:	$y_i, i=1, \dots, NOBS$	The observations [format (10F8.3); i.e., ten eight digit numbers with either a decimal point included or last five digits are assumed to be to the right of a supplied decimal point]

<u>Card Type</u>	<u>Variable Name</u>	<u>Description</u>
	x_{ij} , $i=1, \dots, \text{NOBS}$ $j=1, \dots, \text{IP}$	The matrix of beta coefficients, read in by rows [format (10F8.3)].
Fourth card group:	A_{ij} , $i=1, \dots, \text{NC}$ $j=1, \dots, \text{IP}$	The matrix of beta coefficients in the constraints on beta, read in by rows [format (10F8.3)].
	b_i , $i=1, \dots, \text{NC}$	The right-hand sides of the beta constraints [format (10F8.3)].
Fifth card group	w_i , $i=1, \dots, \text{NOBS}$	The weights assigned to the re- siduals [format (10F8.3)]. (Included only if IWT = 1).

The user then supplies a title card of length 80 spaces or less following the fifth card group.

The size of the problem which MR. A can solve is limited only by the current dimension statements. These restrict the number of observations to 20 ($n \leq 20$), the number of parameters to 10 ($p \leq 10$), the number of constraints to 10 ($m \leq 10$), and the number of samples in the mini-Monte Carlo study to 100 ($K \leq 100$). However, expansion can easily be accomplished by increasing these dimensions in the dimension statements as documented in the program.

MR. A is written in Fortran IV language and is compatible with Fortran G and H and WATFIV language compilers. The program uses double precision arithmetic.

MR. A has been tested on several problems on an AMDAHL 470 V6 and should be compatible with all IBM compilers. MR. A will execute small problems such as $n = 5$, $p = 3$, $K = 6$, $m = 3$ in less than two seconds. Problems of the size $n = 20$, $p = 2$, $K = 30$, $m = 3$ take up to a minute of execution time. A sample problem with sample input and output is given

in Appendix A and B respectively. The program listing of MR. A is given
in Appendix C.

Table 1

Components of MR. A and their Functions

<u>Component</u>	<u>Function</u>
MAIN	Reads data and generates output. Performs L_1 estimation of $\hat{\beta}$. Determines $\hat{\sigma}$ and the estimated covariance of $\hat{\beta}$.
DUAL	Carries out the mini-Monte Carlo study.
MCRHS	Forms the right hand sides of (14) for the mini-Monte Carlo study.
SELECT	Selects P_1 or P_2 for solution.
RAND	Generates random uniform variables ranging from 0 to 1.
RHANDS	Forms the right-hand sides of (9).
BTILDE	Constructs the least squares estimate of β ; i.e., β_0 .
INVERT	Inverts an $n \times n$ matrix.
NORMAL	Generates a vector of normally distributed random variables with mean 0 and variance 1 for the mini-Monte Carlo study.
DOUBLE	Generates a vector of double exponentially distributed random variables with mean 0 and variance 1 for the mini-Monte Carlo study.
UNIFRM	Generates a vector of uniformly distributed random variables with mean 0 and variance 1 for the mini-Monte Carlo study.

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APPENDIX A. SAMPLE INPUTS

2 5 3 1872539680 20 0 0 0 0 0 0 0 1 0
2 1 0
4.257 -1.983 0.024 -3.180 -3.986
1.000 1.120
1.000 1.950
1.000 3.020
1.000 5.430
1.000 6.590
1.000 3.000 -1.000

THIS IS AN EXAMPLE PROBLEM OF MR. A. WITH NO OPTIONAL PRINTOUTS.

2 5 3 1872539680 20 1 1 1 1 1 1 1 1 0
2 1 0
4.257 -1.983 0.024 -3.180 -3.986
1.000 1.120
1.000 1.950
1.000 3.020
1.000 5.430
1.000 6.590
1.000 3.000 -1.000

THIS IS AN EXAMPLE PROBLEM OF MR. A. WITH ALL OPTIONAL PRINTOUTS.

APPENDIX B. SAMPLE OUTPUTS

MR. A. :

MINIMIZES THE ABSOLUTE RESIDUALS.

THIS PROGRAM ESTIMATES A LINEAR REGRESSION BY MINIMIZING THE SUM OF THE ABSOLUTE RESIDUALS - L1 ESTIMATION. IN ADDITION, A MINI-MONTE CARLO SIMULATION GENERATES AN ESTIMATED COVARIANCE MATRIX FOR THE ESTIMATED REGRESSION PARAMETERS.

(Sample output with no optional printouts)

IN ADDITION, LINEAR CONSTRAINTS MAY BE PLACED ON THE PARAMETER VECTOR, BETA. THESE CONSTRAINTS MAY BE EQUALITY OR INEQUALITY CONSTRAINTS. THE FOLLOWING PROCEDURE DEVELOPED BY :

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H.O. HARTLEY
R.L. SIELKEN, JR.
INSTITUTE OF STATISTICS
TEXAS A & M UNIVERSITY
COLLEGE STATION, TEXAS 77843

INQUIRIES AND COMMENTS SHOULD BE ADDRESSED TO:
ROBERT L. SIELKEN, JR.

THE SUPPORT OF THE OFFICE OF NAVAL RESEARCH IS GRATEFULLY ACKNOWLEDGED. THIS IS AN EXAMPLE PROBLEM OF MR. A. WITH NO OPTIONAL PRINTOUTS.

NUMBER OF OBSERVATIONS = 5
NUMBER OF PARAMETERS = 2
USER SUPPLIED RANDOM INTEGER, NSEED = 1872539680
SAMPLE SIZE FOR THE MINI MONTE CARLO STUDY = 20
NUMBER OF BETA CONSTRAINTS = 3
NUMBER OF < OR = BETA CONSTRAINTS = 1
NUMBER OF > OR = BETA CONSTRAINTS = 1
NUMBER OF = BETA CONSTRAINTS = 1

THE AUXILIARY LEAST SQUARES ESTIMATE, BETA0, OF THE REGRESSION PARAMETER VECTOR, BETA

LEAST SQUARES ESTIMATE OF BETA(1) = 3.227002

LEAST SQUARES ESTIMATE OF BETA(2) = -1.159747

MR.A'S ANSWER : THE ESTIMATE OF THE REGRESSION PARAMETER VECTOR WHICH MINIMIZES THE SUM OF THE ABSOLUTE RESIDUALS

L1 ESTIMATE OF BETA(1) = 3.000000

L1 ESTIMATE OF BETA(2) = -2.000000

THE RESIDUALS :

THE 1-TH RESIDUAL = 3.497000

THE 2-TH RESIDUAL = -1.083000

THE 3-TH RESIDUAL = 3.064000

THE 4-TH RESIDUAL = 4.680000

THE 5-TH RESIDUAL = 6.194000

THE SUM OF THE ABSOLUTE RESIDUALS = 18.518000

THE MAXIMUM ABSOLUTE RESIDUAL = 6.194000

MAIN RESULTS OF THE MINI MONTE CARLO STUDY

ESTIMATED VALUE OF SIGMA = 2.817978

ESTIMATED COVARIANCE OF THE REGRESSION PARAMETER VECTOR, BETA

3.565439 -0.970588
-0.970588 0.435623

MINIMIZES THE ABSOLUTE RESIDUALS.

THIS PROGRAM ESTIMATES A LINEAR REGRESSION BY MINIMIZING THE SUM OF THE ABSOLUTE RESIDUALS - L1 ESTIMATION. IN ADDITION, A MINI-MONTE CARLO SIMULATION GENERATES AN ESTIMATED COVARIANCE MATRIX FOR THE ESTIMATED REGRESSION PARAMETERS.

IN ADDITION, LINEAR CONSTRAINTS MAY BE PLACED ON THE PARAMETER VECTOR, BETA. THESE CONSTRAINTS MAY BE EQUALITY OR INEQUALITY CONSTRAINTS.

THE FOLLOWING PROCEDURE DEVELOPED BY :

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H.O. HARTLEY
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COLLEGE STATION, TEXAS 77843

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NUMBER OF OBSERVATIONS = 5
NUMBER OF PARAMETERS = 2
USER SUPPLIED RANDOM INTEGER, NSEED = 1872539680
SAMPLE SIZE FOR THE MINI MONTE CARLO STUDY = 20
NUMBER OF BETA CONSTRAINTS = 3
NUMBER OF < OR = BETA CONSTRAINTS = 1
NUMBER OF > OR = BETA CONSTRAINTS = 1
NUMBER OF = BETA CONSTRAINTS = 1
THE LINEAR REGRESSION IS $Y = X \cdot BETA + EPSILON$

WHERE
Y CONTAINS THE OBSERVATIONS,
BETA IS A VECTOR CONTAINING THE REGRESSION PARAMETERS,
THE I-TH ROW OF X CONTAINS THE COEFFICIENTS OF BETA
CORRESPONDING TO THE I-TH OBSERVATION, AND
EPSILON IS A RANDOM VARIABLE WITH MEAN ZERO AND VARIANCE
SIGMA-SQUARED REPRESENTING RANDOM VARIABILITY FROM
 $X \cdot BETA$.

THE Y VECTOR

4.25700
-1.98300
0.02400
-3.18000
-3.98600

THE X MATRIX

1.0 1.1
1.0 2.0
1.0 3.0
1.0 5.4
1.0 6.6

THE CONSTRAINTS PLACED ON BETA OF THE FORM $A \cdot BETA = B$
WHERE THE CONSTRAINTS MAY BE EQUALITY OR INEQUALITY CONSTRAINTS.
A = THE MATRIX OF COEFFICIENTS WHOSE I-TH ROW CORRESPONDS TO THE I-TH INEQUALITY.
THE RIGHT HAND SIDES OF THE BETA CONSTRAINTS, THE VECTOR B

1.00000
3.00000
-1.00000

THE COEFFICIENTS FOR THE BETA CONSTRAINTS, THE A MATRIX

1.0000 1.0000
1.0000 0.0000
0.0000 1.0000

THE INVERSE OF XTX

0.810298 -0.168497
-0.168497 0.046521

THE VALUE OF BETA0 TO COMPUTE RHS

3.227002 -1.159747

THE RIGHT HAND SIDES, $Y - X \cdot BETA0$

2.32891 -2.94850 0.29943 -0.10958 0.42973

THE RIGHT HAND SIDES, $B - A \cdot BETA0$

-1.06726 -0.22700 0.15975

THE PROBLEM P2 HAS BEEN SELECTED

THE LINEAR PROGRAMMING PROBLEM AS IT WAS CREATED
THE OBJECTIVE FUNCTION COEFFICIENTS

C(1) = 0.00000
C(2) = 0.00000

(Sample output with optional printouts)

C(3) = 0.00000
 C(4) = 0.00000
 C(5) = -1.00000
 C(6) = -1.00000
 C(7) = -1.00000
 C(8) = -1.00000
 C(9) = -1.00000
 C(10) = -1.00000
 C(11) = -1.00000
 C(12) = -1.00000
 C(13) = -1.00000
 C(14) = -1.00000
 C(15) = 0.00000
 C(16) = 0.00000

THE RIGHT-HAND SIDES

BRHS(1) = 2.32891
 BRHS(2) = -2.94850
 BRHS(3) = 0.29943
 BRHS(4) = -0.10958
 BRHS(5) = 0.42973
 BRHS(6) = -1.06726
 BRHS(7) = -0.22700
 BRHS(8) = 0.15975

THE CONSTRAINT MATRIX A

	-0.00000	-0.00000	-0.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	-0.00000
-0.00000				
	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000				
	-1.00000	-1.12000	1.00000	1.12000
0.00000	0.00000	0.00000	0.00000	-1.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000				
	-1.00000	-1.95000	1.00000	1.95000
1.00000	0.00000	0.00000	0.00000	0.00000
-1.00000	0.00000	0.00000	0.00000	0.00000
0.00000				
	-1.00000	-3.02000	1.00000	3.02000
0.00000	1.00000	0.00000	0.00000	0.00000
0.00000	-1.00000	0.00000	0.00000	0.00000
0.00000				
	-1.00000	-5.43000	1.00000	5.43000
0.00000	0.00000	1.00000	0.00000	0.00000
0.00000	0.00000	-1.00000	0.00000	0.00000
0.00000				
	-1.00000	-6.59000	1.00000	6.59000
0.00000	0.00000	0.00000	1.00000	0.00000
0.00000	0.00000	0.00000	-1.00000	0.00000
0.00000				
	-1.00000	-1.00000	1.00000	1.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000				
	-1.00000	-0.00000	1.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	-1.00000
0.00000				
	-0.00000	-1.00000	0.00000	1.00000
0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000
1.00000				

THE INITIAL BASIS INVERSE

THE 1 -TH ROW:	0.10000000D 01	0.00000000D 00	-0.10000000D 01	0.10000000D 01	-0.10000000D 01
THE 2 -TH ROW:	-0.10000000D 01	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00
THE 3 -TH ROW:	0.00000000D 00	0.10000000D 01	0.00000000D 00	0.00000000D 00	0.00000000D 00
THE 4 -TH ROW:	0.00000000D 00	0.00000000D 00	0.10000000D 01	0.00000000D 00	0.00000000D 00
THE 5 -TH ROW:	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.10000000D 01	0.00000000D 00
THE 6 -TH ROW:	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00
THE 7 -TH ROW:	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.10000000D 01
THE 8 -TH ROW:	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00
THE 9 -TH ROW:	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00
THE 10 -TH ROW:	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00
THE 11 -TH ROW:	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00
THE 12 -TH ROW:	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00
THE 13 -TH ROW:	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00
THE 14 -TH ROW:	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00
THE 15 -TH ROW:	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00
THE 16 -TH ROW:	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00

THE 9 -TH ROW: 0.00000000D 00 0.00000000D 00 0.00000000D 00 0.00000000D 00 0.00000000D 00
 0.00000000D 00 0.00000000D 00 0.00000000D 00 -0.10000000D 01 0.00000000D 00
 THE 10 -TH ROW: 0.00000000D 00 0.00000000D 00 0.00000000D 00 0.00000000D 00 0.00000000D 00
 0.00000000D 00 0.00000000D 00 0.00000000D 00 0.00000000D 00 0.10000000D 01

THE VARIABLES INITIALLY IN THE AUGMENTED BASIS MATRIX

0,50, 5,11, 7,13, 9,51,16,15,

THE INITIAL VALUES OF THE BASIC VARIABLES

XB(1) = -0.61161D 01
 XB(2) = -0.10673D 01
 XB(3) = 0.23289D 01
 XB(4) = 0.29485D 01
 XB(5) = 0.29943D 00
 XB(6) = 0.10958D 00
 XB(7) = 0.42973D 00
 XB(8) = 0.10673D 01
 XB(9) = 0.22700D 00
 XB(10) = 0.15975D 00

THE INITIAL VALUE OF THE OBJECTIVE FUNCTION = -0.1067255616D 01

PHASE 1 NOW BEGINS.

PHASE 1 : ITERATION 1

THE REDUCED COSTS

THE 1-TH REDUCED COST = 0.1000000000D 01
 THE 2-TH REDUCED COST = 0.1000000000D 01
 THE 3-TH REDUCED COST = -0.1000000000D 01
 THE 4-TH REDUCED COST = -0.1000000000D 01
 THE 5-TH REDUCED COST = -0.0000000000D 00
 THE 6-TH REDUCED COST = -0.0000000000D 00
 THE 7-TH REDUCED COST = -0.0000000000D 00
 THE 8-TH REDUCED COST = -0.0000000000D 00
 THE 9-TH REDUCED COST = -0.0000000000D 00
 THE 10-TH REDUCED COST = -0.0000000000D 00
 THE 11-TH REDUCED COST = -0.0000000000D 00
 THE 12-TH REDUCED COST = -0.0000000000D 00
 THE 13-TH REDUCED COST = -0.0000000000D 00
 THE 14-TH REDUCED COST = -0.0000000000D 00
 THE 15-TH REDUCED COST = -0.0000000000D 00
 THE 16-TH REDUCED COST = -0.0000000000D 00

X(1) IS THE NON-BASIC VARIABLE WITH THE SMALLEST INDEX WHICH HAS THE LARGEST POSITIVE REDUCED COST.

THEREFORE X(1) WILL ENTER THE BASIS.

THE COMPONENTS OF Y(-, 1)

Y(3, 1) = -0.1000000000D 01
 Y(4, 1) = 0.1000000000D 01
 Y(5, 1) = -0.1000000000D 01
 Y(6, 1) = 0.1000000000D 01
 Y(7, 1) = -0.1000000000D 01
 Y(8, 1) = 0.1000000000D 01
 Y(9, 1) = 0.1000000000D 01
 Y(10, 1) = 0.0000000000D 00

THE VECTOR TO LEAVE THE BASIS IS NOW DETERMINED.

THE XB(I)/Y(I, 1) RATIOS WITH Y(I, 1) > 0, ARE:

0.29485D 01= XB(4)/Y(4, 1)
 0.10958D 00= XB(6)/Y(6, 1)
 0.10673D 01= XB(8)/Y(8, 1)
 0.22700D 00= XB(9)/Y(9, 1)

THE MINIMUM RATIO IS XB(6)/Y(6, 1) = 0.1095781257D 00.

THEREFORE THE 6-TH VARIABLE LEAVES THE BASIS.

THE BASIC VARIABLES ARE NOW

0,50, 5,11, 7, 1, 9,51,16,15,

THE VALUES OF THE BASIC VARIABLES ARE NOW

XB(1) = X(0) = -0.6225727047D 01
 XB(2) = X(50) = -0.9576774908D 00
 XB(3) = X(5) = 0.2438492103D 01
 XB(4) = X(11) = 0.2838918209D 01
 XB(5) = X(7) = 0.4090106653D 00
 XB(6) = X(1) = 0.1095781257D 00
 XB(7) = X(9) = 0.5393060698D 00
 XB(8) = X(51) = 0.9576774908D 00
 XB(9) = X(16) = 0.1174241027D 00
 XB(10) = X(15) = 0.1597466119D 00

THE CURRENT VALUE OF THE OBJECTIVE FUNCTION = -0.95768D 00

THE CURRENT BASIS INVERSE IS AS FOLLOWS:

THE 1-TH ROW: 0.10000000D 01 -0.00000000D 00 -0.10000000D 01 0.10000000D 01 -0.10000000D 01
 0.20000000D 01 -0.10000000D 01 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00
 THE 2-TH ROW: 0.00000000D 00 0.10000000D 01 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00
 -0.10000000D 01 -0.00000000D 00 0.10000000D 01 -0.00000000D 00 -0.00000000D 00
 THE 3-TH ROW: 0.00000000D 00 -0.00000000D 00 0.10000000D 01 -0.00000000D 00 -0.00000000D 00
 -0.10000000D 01 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00
 THE 4-TH ROW: 0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.10000000D 01 -0.00000000D 00
 0.10000000D 01 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00
 THE 5-TH ROW: 0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 0.10000000D 01

```

-0.10000000D 01  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00
THE 6-TH ROW:      0.00000000D 00  0.00000000D 00  0.00000000D 00  0.00000000D 00  0.00000000D 00
-0.10000000D 01  0.00000000D 00  0.00000000D 00  0.00000000D 00  0.00000000D 00  0.00000000D 00
THE 7-TH ROW:      0.00000000D 00  0.00000000D 00  0.00000000D 00  0.00000000D 00  0.00000000D 00
-0.10000000D 01  0.10000000D 01  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00
THE 8-TH ROW:      0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00
0.10000000D 01  -0.00000000D 00  -0.10000000D 01  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00
THE 9-TH ROW:      0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00
0.10000000D 01  -0.00000000D 00  -0.00000000D 00  -0.10000000D 01  -0.00000000D 00  -0.00000000D 00
THE 10-TH ROW:     0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00
-0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00  0.10000000D 01

```

PHASE 1 : ITERATION 2

THE REDUCED COSTS

```

THE 1-TH REDUCED COST = -0.0000000000D 00
THE 2-TH REDUCED COST = -0.4430000000D 01
THE 3-TH REDUCED COST = -0.0000000000D 00
THE 4-TH REDUCED COST = 0.4430000000D 01
THE 5-TH REDUCED COST = -0.0000000000D 00
THE 6-TH REDUCED COST = -0.0000000000D 00
THE 7-TH REDUCED COST = -0.0000000000D 00
THE 8-TH REDUCED COST = 0.1000000000D 01
THE 9-TH REDUCED COST = -0.0000000000D 00
THE 10-TH REDUCED COST = -0.0000000000D 00
THE 11-TH REDUCED COST = -0.0000000000D 00
THE 12-TH REDUCED COST = -0.0000000000D 00
THE 13-TH REDUCED COST = -0.1000000000D 01
THE 14-TH REDUCED COST = -0.0000000000D 00
THE 15-TH REDUCED COST = -0.0000000000D 00
THE 16-TH REDUCED COST = -0.0000000000D 00

```

X(4) IS THE NON-BASIC VARIABLE WITH THE SMALLEST INDEX WHICH HAS THE LARGEST POSITIVE REDUCED COST.

THEREFORE X(4) WILL ENTER THE BASIS.

THE COMPONENTS OF Y(-, 4)

```

Y( 3, 4) = -0.4310000000D 01
Y( 4, 4) = 0.3480000000D 01
Y( 5, 4) = -0.2410000000D 01
Y( 6, 4) = -0.5430000000D 01
Y( 7, 4) = 0.1160000000D 01
Y( 8, 4) = 0.4430000000D 01
Y( 9, 4) = 0.5430000000D 01
Y( 10, 4) = 0.1000000000D 01

```

THE VECTOR TO LEAVE THE BASIS IS NOW DETERMINED. THE XB(I)/Y(I, 4) RATIOS WITH Y(I, 4) > 0. ARE:

```

0.81578D 00= XB( 4)/Y( 4, 4)
0.46492D 00= XB( 7)/Y( 7, 4)
0.21618D 00= XB( 8)/Y( 8, 4)
0.21625D 01= XB( 9)/Y( 9, 4)
0.15975D 00= XB( 10)/Y( 10, 4)

```

THE MINIMUM RATIO IS XB(9)/Y(9, 4) = 0.2162506495D-01.

THEREFORE THE 9-TH VARIABLE LEAVES THE BASIS.

THE BASIC VARIABLES ARE NOW

0,50, 5,11, 7, 1, 9,51, 4,15,

THE VALUES OF THE BASIC VARIABLES ARE NOW

```

XB( 1) = X( 0) = -0.6270707182D 01
XB( 2) = X( 50) = -0.86187E-530D 00
XB( 3) = X( 5) = 0.2531696133D 01
XB( 4) = X( 11) = 0.2763662983D 01
XB( 5) = X( 7) = 0.4611270718D 00
XB( 6) = X( 1) = 0.2270022283D 00
XB( 7) = X( 9) = 0.5142209945D 00
XB( 8) = X( 51) = 0.8618784530D 00
XB( 9) = X( 4) = 0.2162506495D-01
XB( 10) = X( 15) = 0.1381215470D 00

```

THE CURRENT VALUE OF THE OBJECTIVE FUNCTION = -0.86188D 00

THE CURRENT BASIS INVERSE IS AS FOLLOWS:

```

THE 1-TH ROW:      0.10000000D 01  -0.00000000D 00  -0.10000000D 01  0.10000000D 01  -0.10000000D 01
0.16169429D 01  -0.10000000D 01  -0.00000000D 00  0.38305709D 00  -0.00000000D 00
THE 2-TH ROW:      0.00000000D 00  0.10000000D 01  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00
-0.18416206D 00  -0.00000000D 00  0.10000000D 01  -0.81583794D 00  -0.00000000D 00
THE 3-TH ROW:      0.00000000D 00  -0.00000000D 00  0.10000000D 01  -0.00000000D 00  -0.00000000D 00
-0.20626151D 00  -0.00000000D 00  -0.00000000D 00  -0.79373849D 00  -0.00000000D 00
THE 4-TH ROW:      0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.10000000D 01
0.35911602D 00  -0.00000000D 00  -0.00000000D 00  0.64088398D 00  -0.00000000D 00
THE 5-TH ROW:      0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00
-0.55616943D 00  -0.00000000D 00  -0.00000000D 00  -0.44383057D 00  -0.00000000D 00
THE 6-TH ROW:      0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00
-0.69388939D-14  -0.00000000D 00  -0.00000000D 00  -0.10000000D 01  -0.00000000D 00
THE 7-TH ROW:      0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00
-0.12136280D 01  0.10000000D 01  -0.00000000D 00  0.21362799D 00  -0.00000000D 00
THE 8-TH ROW:      0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00  -0.00000000D 00
0.18416206D 00  -0.00000000D 00  -0.10000000D 01  0.81583794D 00  -0.00000000D 00

```

THE 9-TH ROW: 0.00000000D 00 0.00000000D 00 0.00000000D 00 0.00000000D 00 0.00000000D 00
 0.18416206D 00 0.00000000D 00 0.00000000D 00 -0.18416206D 00 0.00000000D 00 0.00000000D 00
 THE 10-TH ROW: 0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00
 -0.18416206D 00 -0.00000000D 00 -0.00000000D 00 0.18416206D 00 0.10000000D 01

PHASE 1 : ITERATION 3

THE REDUCED COSTS

THE 1-TH REDUCED COST = -0.000000000D 00
 THE 2-TH REDUCED COST = -0.2220446049D-15
 THE 3-TH REDUCED COST = -0.000000000D 00
 THE 4-TH REDUCED COST = 0.2220446049D-15
 THE 5-TH REDUCED COST = -0.000000000D 00
 THE 6-TH REDUCED COST = -0.000000000D 00
 THE 7-TH REDUCED COST = -0.000000000D 00
 THE 8-TH REDUCED COST = 0.1841620626D 00
 THE 9-TH REDUCED COST = -0.000000000D 00
 THE 10-TH REDUCED COST = -0.000000000D 00
 THE 11-TH REDUCED COST = -0.000000000D 00
 THE 12-TH REDUCED COST = -0.000000000D 00
 THE 13-TH REDUCED COST = -0.1841620626D 00
 THE 14-TH REDUCED COST = -0.000000000D 00
 THE 15-TH REDUCED COST = -0.8158379374D 00
 THE 16-TH REDUCED COST = -0.000000000D 00

X(8) IS THE NON-BASIC VARIABLE WITH THE SMALLEST INDEX WHICH HAS THE LARGEST POSITIVE REDUCED COST.

THEREFORE X(8) WILL ENTER THE BASIS.

THE COMPONENTS OF Y(-, 8)

Y(3, 8) = -0.2062615101D 00
 Y(4, 8) = 0.3591160221D 00
 Y(5, 8) = -0.5561694291D 00
 Y(6, 8) = -0.6938893904D-16
 Y(7, 8) = -0.1213627993D 01
 Y(8, 8) = 0.1841620626D 00
 Y(9, 8) = 0.1841620626D 00
 Y(10, 8) = -0.1841620626D 00

THE VECTOR TO LEAVE THE BASIS IS NOW DETERMINED.

THE XB(I)/Y(I, 8) RATIOS WITH Y(I, 8) > 0. ARE:

0.76957D 01= XB(4)/Y(4, 8)
 0.46800D 01= XB(8)/Y(8, 8)
 0.11742D 00= XB(9)/Y(9, 8)

THE MINIMUM RATIO IS XB(9)/Y(9, 8) = 0.1174241027D 00.

THEREFORE THE 9-TH VARIABLE LEAVES THE BASIS.

THE BASIC VARIABLES ARE NOW

0,50, 5,11, 7, 1, 9,51, 8,15,

THE VALUES OF THE BASIC VARIABLES ARE NOW

XB(1) = X(0) = -0.6577999355D 01
 XB(2) = X(50) = -0.8402533881D 00
 XB(3) = X(5) = 0.2555916205D 01
 XB(4) = X(11) = 0.2721494107D 01
 XB(5) = X(7) = 0.5264347680D 00
 XB(6) = X(1) = 0.2270022283D 00
 XB(7) = X(9) = 0.6567301725D 00
 XB(8) = X(51) = 0.8402533881D 00
 XB(9) = X(8) = 0.1174241027D 00
 XB(10) = X(15) = 0.1597466119D 00

THE CURRENT VALUE OF THE OBJECTIVE FUNCTION = -0.84025D 00

THE CURRENT BASIS INVERSE IS AS FOLLOWS:

THE 1-TH ROW: 0.10000000D 01 -0.00000000D 00 -0.10000000D 01 0.10000000D 01 -0.10000000D 01
 -0.10000000D 01 -0.10000000D 01 -0.00000000D 00 0.30000000D 01 -0.00000000D 00 -0.00000000D 00
 THE 2-TH ROW: 0.00000000D 00 0.10000000D 01 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00
 -0.00000000D 00 -0.00000000D 00 0.10000000D 01 -0.10000000D 01 -0.00000000D 00 -0.00000000D 00
 THE 3-TH ROW: 0.00000000D 00 -0.00000000D 00 -0.00000000D 00 0.10000000D 01 -0.00000000D 00 -0.00000000D 00
 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.10000000D 01 -0.00000000D 00 -0.00000000D 00
 THE 4-TH ROW: 0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.10000000D 01 -0.00000000D 00
 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 0.10000000D 01 -0.00000000D 00 -0.00000000D 00
 THE 5-TH ROW: 0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 0.10000000D 01
 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.10000000D 01 -0.00000000D 00 -0.00000000D 00
 THE 6-TH ROW: 0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00
 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.10000000D 01 -0.00000000D 00 -0.00000000D 00
 THE 7-TH ROW: 0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00
 -0.00000000D 00 0.10000000D 01 -0.00000000D 00 -0.10000000D 01 -0.00000000D 00 -0.00000000D 00
 THE 8-TH ROW: 0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00
 -0.00000000D 00 -0.00000000D 00 -0.10000000D 01 0.10000000D 01 -0.00000000D 00 -0.00000000D 00
 THE 9-TH ROW: 0.00000000D 00 0.00000000D 00 0.00000000D 00 0.00000000D 00 0.00000000D 00 0.00000000D 00
 0.10000000D 01 0.00000000D 00 0.00000000D 00 -0.10000000D 01 0.00000000D 00 0.00000000D 00
 THE 10-TH ROW: 0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00
 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 -0.00000000D 00 0.10000000D 01 -0.00000000D 00

PHASE 1 : ITERATION 4

THE REDUCED COSTS

THE 1-TH REDUCED COST = -0.000000000D 00
 THE 2-TH REDUCED COST = 0.100000000D 01
 THE 3-TH REDUCED COST = -0.000000000D 00

THE 4-TH REDUCED COST = -0.1000000000 01
 THE 5-TH REDUCED COST = -0.0000000000 00
 THE 6-TH REDUCED COST = -0.0000000000 00
 THE 7-TH REDUCED COST = -0.0000000000 00
 THE 8-TH REDUCED COST = -0.0000000000 00
 THE 9-TH REDUCED COST = -0.0000000000 00
 THE 10-TH REDUCED COST = -0.0000000000 00
 THE 11-TH REDUCED COST = -0.0000000000 00
 THE 12-TH REDUCED COST = -0.0000000000 00
 THE 13-TH REDUCED COST = -0.0000000000 00
 THE 14-TH REDUCED COST = -0.0000000000 00
 THE 15-TH REDUCED COST = -0.1000000000 01
 THE 16-TH REDUCED COST = -0.0000000000 00

X(2) IS THE NON-BASIC VARIABLE WITH THE SMALLEST INDEX
 WHICH HAS THE LARGEST POSITIVE REDUCED COST.
 THEREFORE X(2) WILL ENTER THE BASIS.

THE COMPONENTS OF Y(-, 2)

Y(3, 2) = -0.1120000000 01
 Y(4, 2) = 0.1950000000 01
 Y(5, 2) = -0.3020000000 01
 Y(6, 2) = 0.0000000000 00
 Y(7, 2) = -0.6590000000 01
 Y(8, 2) = 0.1000000000 01
 Y(9, 2) = -0.5430000000 01
 Y(10, 2) = -0.1000000000 01

THE VECTOR TO LEAVE THE BASIS IS NOW DETERMINED.
 THE XB(I)/Y(I, 2) RATIOS WITH Y(I, 2) > 0. ARE:

0.13956D 01= XB(4)/Y(4, 2)
 0.84025D 00= XB(8)/Y(8, 2)

THE MINIMUM RATIO IS XB(8)/Y(8, 2) = 0.8402533981D 00.

THEREFORE THE 8-TH VARIABLE LEAVES THE BASIS.

THE BASIC VARIABLES ARE NOW

0.50, 5, 11, 7, 1, 9, 2, 8, 15,

THE VALUES OF THE BASIC VARIABLES ARE NOW

XB(1) = X(0) = -0.1851800000 02
 XB(2) = X(50) = -0.0000000000 00
 XB(3) = X(5) = 0.3497000000 01
 XB(4) = X(11) = 0.1083000000 01
 XB(5) = X(7) = 0.3064000000 01
 XB(6) = X(1) = 0.2270022283D 00
 XB(7) = X(9) = 0.6194000000 01
 XB(8) = X(2) = 0.8402533981D 00
 XB(9) = X(8) = 0.4680000000 01
 XB(10) = X(15) = 0.1000000000 01

THE CURRENT VALUE OF THE OBJECTIVE FUNCTION = -0.000000 00

THE CURRENT BASIS INVERSE IS AS FOLLOWS:

THE 1-TH ROW:	0.1000000000 01	-0.0000000000 00	-0.1000000000 01	0.1000000000 01	-0.1000000000 01
-0.1000000000 01	-0.1000000000 01	0.1421000000 02	-0.1121000000 02	-0.0000000000 00	-0.0000000000 00
THE 2-TH ROW:	0.0000000000 00	0.1000000000 01	-0.0000000000 00	-0.0000000000 00	-0.0000000000 00
-0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.0000000000 00
THE 3-TH ROW:	0.0000000000 00	-0.0000000000 00	0.1000000000 01	-0.0000000000 00	-0.0000000000 00
-0.0000000000 00	-0.0000000000 00	-0.1120000000 01	0.1200000000 00	-0.0000000000 00	-0.0000000000 00
THE 4-TH ROW:	0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.1000000000 01
-0.0000000000 00	-0.0000000000 00	0.1950000000 01	-0.9500000000 00	-0.0000000000 00	-0.0000000000 00
THE 5-TH ROW:	0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.0000000000 00
-0.0000000000 00	-0.0000000000 00	-0.3020000000 01	0.2020000000 01	-0.0000000000 00	0.1000000000 01
THE 6-TH ROW:	0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.0000000000 00
-0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.1000000000 01	-0.0000000000 00	-0.0000000000 00
THE 7-TH ROW:	0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.0000000000 00
-0.0000000000 00	0.1000000000 01	-0.6590000000 01	0.5590000000 01	-0.0000000000 00	-0.0000000000 00
THE 8-TH ROW:	0.0000000000 00	0.0000000000 00	0.0000000000 00	0.0000000000 00	0.0000000000 00
0.0000000000 00	0.0000000000 00	-0.1000000000 01	0.1000000000 01	0.0000000000 00	0.0000000000 00
THE 9-TH ROW:	0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.0000000000 00
0.1000000000 01	-0.0000000000 00	-0.5430000000 01	0.4430000000 01	-0.0000000000 00	-0.0000000000 00
THE 10-TH ROW:	0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.0000000000 00	-0.0000000000 00
-0.0000000000 00	-0.0000000000 00	-0.1000000000 01	0.1000000000 01	0.1000000000 01	-0.0000000000 00

PHASE 1 : ITERATION 5

THE REDUCED COSTS

THE 1-TH REDUCED COST = -0.0000000000 00
 THE 2-TH REDUCED COST = -0.0000000000 00
 THE 3-TH REDUCED COST = -0.0000000000 00
 THE 4-TH REDUCED COST = -0.0000000000 00
 THE 5-TH REDUCED COST = -0.0000000000 00
 THE 6-TH REDUCED COST = -0.0000000000 00
 THE 7-TH REDUCED COST = -0.0000000000 00
 THE 8-TH REDUCED COST = -0.0000000000 00
 THE 9-TH REDUCED COST = -0.0000000000 00
 THE 10-TH REDUCED COST = -0.0000000000 00
 THE 11-TH REDUCED COST = -0.0000000000 00
 THE 12-TH REDUCED COST = -0.0000000000 00
 THE 13-TH REDUCED COST = -0.0000000000 00

THE 14-TH REDUCED COST = -0.0000000000 00
 THE 15-TH REDUCED COST = -0.0000000000 00
 THE 16-TH REDUCED COST = -0.0000000000 00
 THE VALUE OF THE SUM OF THE ARTIFICIAL VARIABLES AT THE END OF PHASE 1
 IS ZERO. (ACTUALLY IT IS 0.0000000000 00)

PHASE 2 NOW BEGINS.

THE INITIAL FEASIBLE SOLUTION IS AS FOLLOWS:

X(1) = 0.2270022283D 00
 X(2) = 0.8402533881D 00
 X(3) = 0.0000000000D 00
 X(4) = 0.0000000000D 00
 X(5) = 0.3497000000D 01
 X(6) = 0.0000000000D 00
 X(7) = 0.3064000000D 01
 X(8) = 0.4680000000D 01
 X(9) = 0.6194000000D 01
 X(10) = 0.0000000000D 00
 X(11) = 0.1083000000D 01
 X(12) = 0.0000000000D 00
 X(13) = 0.0000000000D 00
 X(14) = 0.0000000000D 00
 X(15) = 0.1000000000D 01
 X(16) = 0.0000000000D 00

THE CURRENT VALUE OF THE OBJECTIVE FUNCTION = -0.18518D 02
 PHASE 2 : ITERATION 1

THE REDUCED COSTS

THE 1-TH REDUCED COST = -0.2220446049D-15
 THE 2-TH REDUCED COST = -0.0000000000D 00
 THE 3-TH REDUCED COST = 0.2220446049D-15
 THE 4-TH REDUCED COST = -0.0000000000D 00
 THE 5-TH REDUCED COST = -0.0000000000D 00
 THE 6-TH REDUCED COST = -0.2000000000D 01
 THE 7-TH REDUCED COST = -0.0000000000D 00
 THE 8-TH REDUCED COST = -0.0000000000D 00
 THE 9-TH REDUCED COST = -0.0000000000D 00
 THE 10-TH REDUCED COST = -0.2000000000D 01
 THE 11-TH REDUCED COST = -0.0000000000D 00
 THE 12-TH REDUCED COST = -0.2000000000D 01
 THE 13-TH REDUCED COST = -0.2000000000D 01
 THE 14-TH REDUCED COST = -0.2000000000D 01
 THE 15-TH REDUCED COST = -0.1121000000D 02
 THE 16-TH REDUCED COST = -0.0000000000D 00

ALL OF THE REDUCED COSTS ARE NON-POSITIVE.
 THEREFORE THE CURRENT BASIC FEASIBLE SOLUTION IS AN OPTIMAL SOLUTION.
 THE OPTIMAL SOLUTION IS AS FOLLOWS:

X(1) = 0.2270022283D 00
 X(2) = 0.8402533881D 00
 X(3) = 0.0000000000D 00
 X(4) = 0.0000000000D 00
 X(5) = 0.3497000000D 01
 X(6) = 0.0000000000D 00
 X(7) = 0.3064000000D 01
 X(8) = 0.4680000000D 01
 X(9) = 0.6194000000D 01
 X(10) = 0.0000000000D 00
 X(11) = 0.1083000000D 01
 X(12) = 0.0000000000D 00
 X(13) = 0.0000000000D 00
 X(14) = 0.0000000000D 00
 X(15) = 0.1000000000D 01
 X(16) = 0.0000000000D 00

THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS -0.18518D 02

THE AUXILIARY LEAST SQUARES ESTIMATE, BETA0, OF THE REGRESSION PARAMETER VECTOR, BETA

LEAST SQUARES ESTIMATE OF BETA(1) = 3.227002
 LEAST SQUARES ESTIMATE OF BETA(2) = -1.159747

MR.A'S ANSWER : THE ESTIMATE OF THE REGRESSION PARAMETER VECTOR WHICH MINIMIZES THE SUM OF THE ABSOLUTE RESIDUALS

L1 ESTIMATE OF BETA(1) = 3.000000
 L1 ESTIMATE OF BETA(2) = -2.000000

THE RESIDUALS :

THE 1-TH RESIDUAL = 3.497000
 THE 2-TH RESIDUAL = -1.083000
 THE 3-TH RESIDUAL = 3.064000
 THE 4-TH RESIDUAL = 4.680000
 THE 5-TH RESIDUAL = 6.194000

THE SUM OF THE ABSOLUTE RESIDUALS = 18.518000
 THE MAXIMUM ABSOLUTE RESIDUAL = 6.194000

THE MONTE CARLO STUDY TO ESTIMATE THE COVARIANCE OF THE L1 ESTIMATE OF BETA NOW BEGINS

RESULTS FOR MONTE CARLO SAMPLE NUMBER 1:

THE PROBLEM P2 HAS BEEN SELECTED

THE INVERSE OF XTX

0.810298 -0.168497
-0.168497 0.046521

THE VALUE OF BETA0 TO COMPUTE RHS

-0.594250 0.136933

THE RIGHT HAND SIDES FOR THIS SAMPLE

BRHS1(1) = 0.000000
BRHS1(2) = 0.000000
BRHS1(3) = 0.067696
BRHS1(4) = -0.244174
BRHS1(5) = 0.226595
BRHS1(6) = -0.039891
BRHS1(7) = -0.010226
BRHS1(8) = 0.236092
BRHS1(9) = 0.582285
BRHS1(10) = -0.184188

THE LEAST SQUARES ESTIMATE OF BETA IN THIS SAMPLE

BETA(1) = -0.594250
BETA(2) = 0.136933

THE EPSILONS GENERATED FOR THIS SAMPLE

EPSILON(1) = -0.373189
EPSILON(2) = -0.571406
EPSILON(3) = 0.045882
EPSILON(4) = 0.109403
EPSILON(5) = 0.297911

THE INITIAL VALUES OF THE BASIC VARIABLES

XB(0) = -0.36609D 01
XB(1) = 0.00000D 00
XB(2) = -0.12685D 00
XB(3) = 0.15138D 00
XB(4) = 0.68981D 00
XB(5) = -0.58228D 00
XB(6) = 0.16889D 01
XB(7) = 0.34619D 00
XB(8) = 0.12577D 01
XB(9) = 0.16200D 00

THE INITIAL VALUE OF THE OBJECTIVE FUNCTION = -0.36609D 01

THE REDUCED COSTS

THE 1-TH REDUCED COST = -0.00000D 00
THE 2-TH REDUCED COST = -0.00000D 00
THE 3-TH REDUCED COST = 0.22204D-15
THE 4-TH REDUCED COST = -0.00000D 00
THE 5-TH REDUCED COST = -0.00000D 00
THE 6-TH REDUCED COST = -0.20000D 01
THE 7-TH REDUCED COST = -0.00000D 00
THE 8-TH REDUCED COST = -0.00000D 00
THE 9-TH REDUCED COST = -0.00000D 00
THE 10-TH REDUCED COST = -0.20000D 01
THE 11-TH REDUCED COST = -0.00000D 00
THE 12-TH REDUCED COST = -0.20000D 01
THE 13-TH REDUCED COST = -0.20000D 01
THE 14-TH REDUCED COST = -0.20000D 01
THE 15-TH REDUCED COST = -0.00000D 00
THE 16-TH REDUCED COST = -0.00000D 00

TENTATIVELY THE 6-TH BASIC VARIABLE IS LEAVING THE BASIS

THE YRJ'S

(YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.

IT CAN BE ZERO OTHERWISE TOO.)

YR(1) = 0.00000D 00
YR(2) = 0.00000D 00
YR(3) = -0.10000D 01
YR(4) = 0.00000D 00
YR(5) = 0.00000D 00
YR(6) = 0.00000D 00
YR(7) = 0.00000D 00
YR(8) = 0.00000D 00
YR(9) = 0.00000D 00
YR(10) = 0.00000D 00
YR(11) = 0.00000D 00
YR(12) = 0.00000D 00
YR(13) = 0.00000D 00
YR(14) = 0.00000D 00
YR(15) = 0.00000D 00
YR(16) = 0.00000D 00

THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S

THE RATIO OF THE 3-TH NET PRICE TO YR(3) = 0.22204D-15

THE 6-TH VARIABLE IS LEAVING THE BASIS.

THE 3-TH VARIABLE IS ENTERING THE BASIS.

THE CURRENT BASIS INVERSE IS AS FOLLOWS:

THE 1-TH ROW:	0.10000000D 01	-0.00000000D 00	-0.10000000D 01	0.10000000D 01	-0.10000000D 01
-0.10000000D 01	-0.10000000D 01	0.14210000D 02	-0.11210000D 02	-0.00000000D 00	-0.00000000D 00
THE 2-TH ROW:	0.00000000D 00	0.10000000D 01	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
THE 3-TH ROW:	0.00000000D 00	-0.00000000D 00	0.10000000D 01	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	-0.11200000D 01	0.12000000D 00	-0.00000000D 00	-0.00000000D 00
THE 4-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.10000000D 01	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	0.19500000D 01	-0.95000000D 00	-0.00000000D 00	-0.00000000D 00
THE 5-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	0.10000000D 01
-0.00000000D 00	-0.00000000D 00	-0.30200000D 01	0.20200000D 01	-0.00000000D 00	-0.00000000D 00
THE 6-TH ROW:	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00	0.00000000D 00
0.00000000D 00	0.00000000D 00	0.00000000D 00	0.10000000D 01	0.00000000D 00	-0.00000000D 00
THE 7-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	0.10000000D 01	-0.65900000D 01	0.55900000D 01	-0.00000000D 00	-0.00000000D 00
THE 8-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	-0.10000000D 01	0.10000000D 01	-0.00000000D 00	-0.00000000D 00
THE 9-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
0.10000000D 01	-0.00000000D 00	-0.54300000D 01	0.44300000D 01	-0.00000000D 00	-0.00000000D 00
THE 10-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	-0.10000000D 01	0.10000000D 01	0.10000000D 01	-0.00000000D 00

THE BASIC VARIABLES ARE NOW

0, 50, 5, 11, 7, 3, 9, 2, 8, 15,

THE VALUES OF THE BASIC VARIABLES ARE NOW

XB(1) = -0.000000D 00
 XB(2) = -0.12685D 00
 XB(3) = 0.15138D 00
 XB(4) = 0.68981D 00
 XB(5) = 0.58228D 00
 XB(6) = 0.16889D 01
 XB(7) = 0.34619D 00
 XB(8) = 0.12577D 01
 XB(9) = 0.16200D 00

THE CURRENT VALUE OF THE OBJECTIVE FUNCTION = -0.36609D 01

THE REDUCED COSTS

THE 1-TH REDUCED COST = -0.000000D 00
 THE 2-TH REDUCED COST = -0.000000D 00
 THE 3-TH REDUCED COST = -0.000000D 00
 THE 4-TH REDUCED COST = -0.000000D 00
 THE 5-TH REDUCED COST = -0.000000D 00
 THE 6-TH REDUCED COST = -0.200000D 01
 THE 7-TH REDUCED COST = -0.000000D 00
 THE 8-TH REDUCED COST = -0.000000D 00
 THE 9-TH REDUCED COST = -0.000000D 00
 THE 10-TH REDUCED COST = -0.200000D 01
 THE 11-TH REDUCED COST = -0.000000D 00
 THE 12-TH REDUCED COST = -0.200000D 01
 THE 13-TH REDUCED COST = -0.200000D 01
 THE 14-TH REDUCED COST = -0.200000D 01
 THE 15-TH REDUCED COST = -0.000000D 00
 THE 16-TH REDUCED COST = -0.000000D 00

TENTATIVELY THE 3-TH BASIC VARIABLE IS LEAVING THE BASIS

THE YR(J)'S

(YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.

IT CAN BE ZERO OTHERWISE TOO.)

YR(1) = 0.000000D 00
 YR(2) = 0.000000D 00
 YR(3) = 0.000000D 00
 YR(4) = 0.000000D 00
 YR(5) = 0.000000D 00
 YR(6) = 0.000000D 00
 YR(7) = 0.000000D 00
 YR(8) = 0.000000D 00
 YR(9) = 0.000000D 00
 YR(10) = -0.100000D 01
 YR(11) = 0.000000D 00
 YR(12) = 0.000000D 00
 YR(13) = 0.000000D 00
 YR(14) = 0.000000D 00
 YR(15) = 0.000000D 00
 YR(16) = 0.000000D 00

THE RATIO OF THE NET PRICES TO THE NEGATIVE YR(J)'S

THE RATIO OF THE 10-TH NET PRICE TO YR(10) = -0.200000D 01

THE 3-TH VARIABLE IS LEAVING THE BASIS.

THE 10-TH VARIABLE IS ENTERING THE BASIS.

THE CURRENT BASIS INVERSE IS AS FOLLOWS:

THE 1-TH ROW:	0.10000000D 01	-0.00000000D 00	0.10000000D 01	0.10000000D 01	-0.10000000D 01
-0.10000000D 01	-0.10000000D 01	0.11970000D 02	-0.10970000D 02	-0.00000000D 00	-0.00000000D 00
THE 2-TH ROW:	0.00000000D 00	0.10000000D 01	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00

THE 3-TH ROW: 0.00000000 00 0.00000000 00 -0.10000000 01 0.00000000 00 0.00000000 00
 0.00000000 00 0.00000000 00 0.11200000 01 -0.12000000 00 0.00000000 00 0.00000000 00
 THE 4-TH ROW: 0.00000000 00 -0.00000000 00 -0.00000000 00 -0.10000000 01 -0.00000000 00
 -0.00000000 00 -0.00000000 00 0.19500000 01 -0.95000000 00 -0.00000000 00 -0.00000000 00
 THE 5-TH ROW: 0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00 0.10000000 01
 -0.00000000 00 -0.00000000 00 -0.30200000 01 0.20200000 01 -0.00000000 00 -0.00000000 00
 THE 6-TH ROW: 0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00
 -0.00000000 00 -0.00000000 00 -0.00000000 00 0.10000000 01 -0.00000000 00 -0.00000000 00
 THE 7-TH ROW: 0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00
 -0.00000000 00 0.10000000 01 -0.65900000 01 0.55900000 01 -0.00000000 00 -0.00000000 00
 THE 8-TH ROW: 0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00
 -0.00000000 00 -0.00000000 00 -0.10000000 01 0.10000000 01 -0.00000000 00 -0.00000000 00
 THE 9-TH ROW: 0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00
 0.10000000 01 -0.00000000 00 -0.54300000 01 0.44300000 01 -0.00000000 00 -0.00000000 00
 THE 10-TH ROW: 0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00 -0.00000000 00
 -0.00000000 00 -0.00000000 00 -0.10000000 01 0.10000000 01 0.10000000 01 0.10000000 01

THE BASIC VARIABLES ARE NOW

0, 50, 10, 11, 7, 3, 9, 2, 8, 15,

THE VALUES OF THE BASIC VARIABLES ARE NOW

XB(1) = -0.000000 00
 XB(2) = 0.126850 00
 XB(3) = 0.151380 00
 XB(4) = 0.689810 00
 XB(5) = 0.582280 00
 XB(6) = 0.168890 01
 XB(7) = 0.346190 00
 XB(8) = 0.125770 01
 XB(9) = 0.162000 00

THE CURRENT VALUE OF THE OBJECTIVE FUNCTION = -0.391460 01

THE REDUCED COSTS

THE 1-TH REDUCED COST = -0.000000 00
 THE 2-TH REDUCED COST = -0.000000 00
 THE 3-TH REDUCED COST = -0.000000 00
 THE 4-TH REDUCED COST = -0.000000 00
 THE 5-TH REDUCED COST = -0.200000 01
 THE 6-TH REDUCED COST = -0.200000 01
 THE 7-TH REDUCED COST = -0.000000 00
 THE 8-TH REDUCED COST = -0.000000 00
 THE 9-TH REDUCED COST = -0.000000 00
 THE 10-TH REDUCED COST = -0.000000 00
 THE 11-TH REDUCED COST = -0.000000 00
 THE 12-TH REDUCED COST = -0.200000 01
 THE 13-TH REDUCED COST = -0.200000 01
 THE 14-TH REDUCED COST = -0.200000 01
 THE 15-TH REDUCED COST = -0.000000 00
 THE 16-TH REDUCED COST = -0.000000 00

THE CURRENT BASIC SOLUTION IS FEASIBLE AND HENCE OPTIMAL.

THE NONZERO VARIABLES ARE AS FOLLOWS:

X(0) = -0.391460 01
 X(50) = -0.000000 00
 X(10) = 0.126850 00
 X(11) = 0.151380 00
 X(7) = 0.689810 00
 X(3) = 0.582280 00
 X(9) = 0.168890 01
 X(2) = 0.346190 00
 X(8) = 0.125770 01
 X(15) = 0.162000 00

THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS -0.391460 01

RESULTS FOR MONTE CARLO SAMPLE NUMBER 2:

THE PROBLEM P1 HAS BEEN SELECTED

THE INVERSE OF XTX

0.810298 -0.168497
 -0.168497 0.046521

THE VALUE OF BETAO TO COMPUTE RMS

0.217174 0.114161

THE RIGHT HAND SIDES FOR THIS SAMPLE

BRHS1(1) = 0.000000
 BRHS1(2) = 0.000000
 BRHS1(3) = 0.534109
 BRHS1(4) = 0.157815
 BRHS1(5) = -0.919343
 BRHS1(6) = -0.320504
 BRHS1(7) = 0.547923
 BRHS1(8) = 0.070676
 BRHS1(9) = 0.294137
 BRHS1(10) = -0.117764

THE LEAST SQUARES ESTIMATE OF BETA IN THIS SAMPLE

BETA(1) = 0.217174
 BETA(2) = 0.114161

(ETC.)

BRHS1(3) = -1.062358
 BRHS1(4) = 1.453735
 BRHS1(5) = -0.206058
 BRHS1(6) = -0.171215
 BRHS1(7) = -0.014105
 BRHS1(8) = 0.358040
 BRHS1(9) = 0.985521
 BRHS1(10) = -0.297676

(RECONTINUED)

THE LEAST SQUARES ESTIMATE OF BETA IN THIS SAMPLE

BETA(1) = 0.011806
 BETA(2) = -0.097417

THE EPSILONS GENERATED FOR THIS SAMPLE

EPSILON(1) = -1.159658
 EPSILON(2) = 1.275578
 EPSILON(3) = -0.488451
 EPSILON(4) = -0.688384
 EPSILON(5) = -0.644277

THE INITIAL VALUES OF THE BASIC VARIABLES

XB(0) = -0.91263D 01
 XB(1) = 0.00000D 00
 XB(2) = 0.13451D 01
 XB(3) = 0.16918D 01
 XB(4) = 0.70341D 00
 XB(5) = 0.98552D 00
 XB(6) = 0.31355D 01
 XB(7) = 0.62748D 00
 XB(8) = 0.22505D 01
 XB(9) = 0.32980D 00

THE INITIAL VALUE OF THE OBJECTIVE FUNCTION = -0.91263D 01
 THE REDUCED COSTS

THE 1-TH REDUCED COST = -0.00000D 00
 THE 2-TH REDUCED COST = 0.28866D-14
 THE 3-TH REDUCED COST = -0.28866D-14
 THE 4-TH REDUCED COST = -0.00000D 00
 THE 5-TH REDUCED COST = -0.20000D 01
 THE 6-TH REDUCED COST = -0.00000D 00
 THE 7-TH REDUCED COST = -0.00000D 00
 THE 8-TH REDUCED COST = -0.00000D 00
 THE 9-TH REDUCED COST = -0.00000D 00
 THE 10-TH REDUCED COST = -0.00000D 00
 THE 11-TH REDUCED COST = -0.20000D 01
 THE 12-TH REDUCED COST = -0.20000D 01
 THE 13-TH REDUCED COST = -0.20000D 01
 THE 14-TH REDUCED COST = -0.20000D 01
 THE 15-TH REDUCED COST = -0.00000D 00
 THE 16-TH REDUCED COST = -0.00000D 00

THE CURRENT BASIC SOLUTION IS FEASIBLE AND HENCE OPTIMAL,
 THE NONZERO VARIABLES ARE AS FOLLOWS:

X(0) = -0.91263D 01
 X(50) = 0.00000D 00
 X(10) = 0.13451D 01
 X(6) = 0.16918D 01
 X(7) = 0.70341D 00
 X(1) = 0.98552D 00
 X(9) = 0.31355D 01
 X(4) = 0.62748D 00
 X(8) = 0.22505D 01
 X(15) = 0.32980D 00

THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS -0.91263D 01

RESULTS FOR MONTE CARLO SAMPLE NUMBER 20:

THE PROBLEM P2 HAS BEEN SELECTED

THE INVERSE OF XTX

0.810298 -0.168497
 -0.168497 0.046521

THE VALUE OF BETA0 TO COMPUTE RMS

0.173768 -0.207514

THE RIGHT HAND SIDES FOR THIS SAMPLE

BRHS1(1) = 0.000000
 BRHS1(2) = 0.000000
 BRHS1(3) = 0.478624
 BRHS1(4) = -0.259669
 BRHS1(5) = -0.702085
 BRHS1(6) = 0.942444
 BRHS1(7) = -0.459314
 BRHS1(8) = 0.349361
 BRHS1(9) = 0.955143
 BRHS1(10) = -0.267826

THE LEAST SQUARES ESTIMATE OF BETA IN THIS SAMPLE

BETA(1) = 0.173768
 BETA(2) = -0.207514

THE EPSILONS GENERATED FOR THIS SAMPLE

EPSILON(1) = 0.419977
 EPSILON(2) = -0.490553
 EPSILON(3) = -1.155009
 EPSILON(4) = -0.010589
 EPSILON(5) = -1.653063

THE INITIAL VALUES OF THE BASIC VARIABLES

XB(0) = -0.57911D 01
 XB(1) = 0.00000D 00
 XB(2) = -0.20196D 00
 XB(3) = -0.33537D-01
 XB(4) = 0.17223D 00
 XB(5) = 0.95514D 00
 XB(6) = 0.25776D 01
 XB(7) = 0.60578D 00
 XB(8) = 0.32767D 01
 XB(9) = 0.33796D 00

THE INITIAL VALUE OF THE OBJECTIVE FUNCTION = -0.57911D 01
 THE REDUCED COSTS

THE 1-TH REDUCED COST = -0.28866D-14
 THE 2-TH REDUCED COST = -0.00000D 00
 THE 3-TH REDUCED COST = -0.00000D 00
 THE 4-TH REDUCED COST = 0.28866D-14
 THE 5-TH REDUCED COST = -0.20000D 01
 THE 6-TH REDUCED COST = -0.00000D 00
 THE 7-TH REDUCED COST = -0.00000D 00
 THE 8-TH REDUCED COST = -0.00000D 00
 THE 9-TH REDUCED COST = -0.00000D 00
 THE 10-TH REDUCED COST = -0.00000D 00
 THE 11-TH REDUCED COST = -0.20000D 01
 THE 12-TH REDUCED COST = -0.20000D 01
 THE 13-TH REDUCED COST = -0.20000D 01
 THE 14-TH REDUCED COST = -0.20000D 01
 THE 15-TH REDUCED COST = -0.00000D 00
 THE 16-TH REDUCED COST = -0.00000D 00

TENTATIVELY THE 3-TH BASIC VARIABLE IS LEAVING THE BASIS
 THE YRJ'S

(YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
 IT CAN BE ZERO OTHERWISE TOO.)

YR(1) = 0.00000D 00
 YR(2) = 0.00000D 00
 YR(3) = 0.00000D 00
 YR(4) = 0.00000D 00
 YR(5) = -0.10000D 01
 YR(6) = 0.00000D 00
 YR(7) = 0.00000D 00
 YR(8) = 0.00000D 00
 YR(9) = 0.00000D 00
 YR(10) = 0.00000D 00
 YR(11) = 0.00000D 00
 YR(12) = 0.00000D 00
 YR(13) = 0.00000D 00
 YR(14) = 0.00000D 00
 YR(15) = 0.00000D 00
 YR(16) = 0.00000D 00

THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S

THE RATIO OF THE 5-TH NET PRICE TO YR(5) = -0.20000D 01

THE 3-TH VARIABLE IS LEAVING THE BASIS.

THE 5-TH VARIABLE IS ENTERING THE BASIS.

THE CURRENT BASIS INVERSE IS AS FOLLOWS:

THE 1-TH ROW:	0.10000000D 01	-0.00000000D 00	-0.10000000D 01	-0.10000000D 01	-0.10000000D 01
-0.10000000D 01	-0.10000000D 01	0.18110000D 02	-0.13110000D 02	-0.00000000D 00	-0.00000000D 00
THE 2-TH ROW:	0.00000000D 00	0.10000000D 01	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
THE 3-TH ROW:	0.00000000D 00	0.00000000D 00	0.10000000D 01	0.00000000D 00	0.00000000D 00
0.00000000D 00	0.00000000D 00	-0.11200000D 01	0.12000000D 00	0.00000000D 00	0.00000000D 00
THE 4-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	0.10000000D 01	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	-0.19500000D 01	0.95000000D 00	-0.00000000D 00	-0.00000000D 00
THE 5-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	0.10000000D 01
-0.00000000D 00	-0.00000000D 00	-0.30200000D 01	0.20200000D 01	-0.00000000D 00	-0.00000000D 00
THE 6-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	0.10000000D 01	-0.00000000D 00	-0.00000000D 00
THE 7-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	0.10000000D 01	-0.65900000D 01	0.55900000D 01	-0.00000000D 00	-0.00000000D 00
THE 8-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	-0.10000000D 01	0.10000000D 01	-0.00000000D 00	-0.00000000D 00
THE 9-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
0.10000000D 01	-0.00000000D 00	-0.54300000D 01	0.44300000D 01	-0.00000000D 00	-0.00000000D 00
THE 10-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	-0.10000000D 01	0.10000000D 01	0.10000000D 01	0.10000000D 01

THE BASIC VARIABLES ARE NOW

0, 50, 5, 6, 7, 3, 9, 2, 8, 15,
THE VALUES OF THE BASIC VARIABLES ARE NOW

XB(1) = -0.00000D 00
XB(2) = 0.20196D 00
XB(3) = -0.33537D-01
XB(4) = 0.17223D 00
XB(5) = 0.95514D 00
XB(6) = 0.25776D 01
XB(7) = 0.60578D 00
XB(8) = 0.32767D 01
XB(9) = 0.33796D 00

THE CURRENT VALUE OF THE OBJECTIVE FUNCTION = -0.61950D 01
THE REDUCED COSTS

THE 1-TH REDUCED COST = -0.28866D-14
THE 2-TH REDUCED COST = -0.00000D 00
THE 3-TH REDUCED COST = -0.00000D 00
THE 4-TH REDUCED COST = -0.00000D 00
THE 5-TH REDUCED COST = -0.00000D 00
THE 6-TH REDUCED COST = -0.00000D 00
THE 7-TH REDUCED COST = -0.00000D 00
THE 8-TH REDUCED COST = -0.00000D 00
THE 9-TH REDUCED COST = -0.00000D 00
THE 10-TH REDUCED COST = -0.20000D 01
THE 11-TH REDUCED COST = -0.20000D 01
THE 12-TH REDUCED COST = -0.20000D 01
THE 13-TH REDUCED COST = -0.20000D 01
THE 14-TH REDUCED COST = -0.20000D 01
THE 15-TH REDUCED COST = -0.00000D 00
THE 16-TH REDUCED COST = -0.00000D 00

TENTATIVELY THE 4-TH BASIC VARIABLE IS LEAVING THE BASIS
THE YR(J)'S

(YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
IT CAN BE ZERO OTHERWISE TOO.)

YR(1) = 0.00000D 00
YR(2) = 0.00000D 00
YR(3) = 0.00000D 00
YR(4) = 0.00000D 00
YR(5) = 0.00000D 00
YR(6) = 0.00000D 00
YR(7) = 0.00000D 00
YR(8) = 0.00000D 00
YR(9) = 0.00000D 00
YR(10) = 0.00000D 00
YR(11) = -0.10000D 01
YR(12) = 0.00000D 00
YR(13) = 0.00000D 00
YR(14) = 0.00000D 00
YR(15) = 0.00000D 00
YR(16) = 0.00000D 00

THE RATIO OF THE NET PRICES TO THE NEGATIVE YR(J)'S
THE RATIO OF THE 11-TH NET PRICE TO YR(11) = -0.20000D 01

THE 4-TH VARIABLE IS LEAVING THE BASIS.

THE 11-TH VARIABLE IS ENTERING THE BASIS.

THE CURRENT BASIS INVERSE IS AS FOLLOWS:

THE 1-TH ROW:	0.10000000D 01	-0.00000000D 00	-0.10000000D 01	0.10000000D 01	-0.10000000D 01
-0.10000000D 01	-0.10000000D 01	0.14210000D 02	-0.11210000D 02	-0.00000000D 00	
THE 2-TH ROW:	0.00000000D 00	0.10000000D 01	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	
THE 3-TH ROW:	0.00000000D 00	-0.00000000D 00	0.10000000D 01	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	-0.11200000D 01	0.12000000D 00	-0.00000000D 00	
THE 4-TH ROW:	0.00000000D 00	0.00000000D 00	0.00000000D 00	-0.10000000D 01	0.00000000D 00
0.00000000D 00	0.00000000D 00	0.19500000D 01	-0.95000000D 00	0.00000000D 00	
THE 5-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	0.10000000D 01
-0.00000000D 00	-0.00000000D 00	-0.30200000D 01	0.20200000D 01	-0.00000000D 00	
THE 6-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	0.10000000D 01	-0.00000000D 00	
THE 7-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	0.10000000D 01	-0.65900000D 01	0.55900000D 01	-0.00000000D 00	
THE 8-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	-0.10000000D 01	0.10000000D 01	-0.00000000D 00	
THE 9-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
0.10000000D 01	-0.00000000D 00	-0.54300000D 01	0.44300000D 01	-0.00000000D 00	
THE 10-TH ROW:	0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00	-0.00000000D 00
-0.00000000D 00	-0.00000000D 00	-0.10000000D 01	0.10000000D 01	0.10000000D 01	

THE BASIC VARIABLES ARE NOW

0, 50, 5, 11, 7, 3, 9, 2, 8, 15,

THE VALUES OF THE BASIC VARIABLES ARE NOW

XB(1) = -0.00000D 00
XB(2) = 0.20196D 00
XB(3) = 0.33537D-01
XB(4) = 0.17223D 00

XB(5) = 0.95514D 00
 XR(6) = 0.25776D 01
 XB(7) = 0.60578D 00
 XB(8) = 0.32767D 01
 XB(9) = 0.33796D 00

THE CURRENT VALUE OF THE OBJECTIVE FUNCTION = -0.62621D 01
 THE REDUCED COSTS

THE 1-TH REDUCED COST = -0.28866D-14
 THE 2-TH REDUCED COST = -0.00000D 00
 THE 3-TH REDUCED COST = -0.00000D 00
 THE 4-TH REDUCED COST = 0.28866D-14
 THE 5-TH REDUCED COST = -0.00000D 00
 THE 6-TH REDUCED COST = -0.20000D 01
 THE 7-TH REDUCED COST = -0.00000D 00
 THE 8-TH REDUCED COST = -0.00000D 00
 THE 9-TH REDUCED COST = -0.00000D 00
 THE 10-TH REDUCED COST = -0.20000D 01
 THE 11-TH REDUCED COST = -0.00000D 00
 THE 12-TH REDUCED COST = -0.20000D 01
 THE 13-TH REDUCED COST = -0.20000D 01
 THE 14-TH REDUCED COST = -0.20000D 01
 THE 15-TH REDUCED COST = -0.00000D 00
 THE 16-TH REDUCED COST = -0.00000D 00

THE CURRENT BASIC SOLUTION IS FEASIBLE AND HENCE OPTIMAL.
 THE NONZERO VARIABLES ARE AS FOLLOWS:

X(0) = -0.62621D 01
 X(50) = -0.00000D 00
 X(5) = 0.20196D 00
 X(11) = 0.33537D-01
 X(7) = 0.17223D 00
 X(3) = 0.95514D 00
 X(9) = 0.25776D 01
 X(2) = 0.60578D 00
 X(8) = 0.32767D 01
 X(15) = 0.33796D 00

THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS -0.62621D 01

AUXILIARY RESULTS OF THE MINI MONTE CARLO STUDY

VALUES OF DELTA BETA STAR

SAMPLE NUMBER = 1	-0.01197	-0.20926
SAMPLE NUMBER = 2	0.51131	-0.10930
SAMPLE NUMBER = 3	1.43557	-0.71028
SAMPLE NUMBER = 4	0.61193	-0.44646
SAMPLE NUMBER = 5	0.61553	-0.51802
SAMPLE NUMBER = 6	-0.28073	-0.21389
SAMPLE NUMBER = 7	-0.18772	-0.34817
SAMPLE NUMBER = 8	-0.36551	-0.05786
SAMPLE NUMBER = 9	0.95646	-0.28508
SAMPLE NUMBER = 10	0.03818	-0.28217
SAMPLE NUMBER = 11	1.59676	-0.62135
SAMPLE NUMBER = 12	0.62508	-0.34275
SAMPLE NUMBER = 13	-0.10425	-0.41664
SAMPLE NUMBER = 14	0.67201	-0.34557
SAMPLE NUMBER = 15	1.18308	-0.61111
SAMPLE NUMBER = 16	1.26476	-0.56381
SAMPLE NUMBER = 17	0.66750	-0.66472
SAMPLE NUMBER = 18	1.85856	-0.79734
SAMPLE NUMBER = 19	0.99733	-0.72490
SAMPLE NUMBER = 20	1.12891	-0.81330

ESTIMATED COVARIANCE OF DELTA BETA STAR

0.448991	-0.122225
-0.122225	0.054857

SUM OF THE OPTIMAL OBJECTIVE FUNCTIONS OVER ALL SAMPLES = 131.427570
 MAIN RESULTS OF THE MINI MONTE CARLO STUDY

ESTIMATED VALUE OF SIGMA = 2.817978
 AUXILIARY RESULT : THE ESTIMATE OF SIGMA S = 7.736290
 ESTIMATED COVARIANCE OF THE REGRESSION PARAMETER VECTOR, BETA

3.565439	-0.970588
-0.970588	0.435623

APPENDIX C. PROGRAM LISTING

```

MAIN0061 DIMENSION C(72),BRHS2(42),A2(42,72),BRHS1(42),DINDX(42)
MAIN0062 DIMENSION X(2,42),Y(2,42),REDCOS(72),E2IN(42,42)
MAIN0063 DIMENSION Y(20),ESTAR(20),RES1(20),RES2(20),RES(20),TITLE(20)
MAIN0064 DIMENSION BETA0(10),YXRB(20),RVAR(20),ITYPE(20),IRHS(20)
MAIN0065 DIMENSION DBI(10),DBR2(10),DELB(10,100),REETA0(10),DBX(135)
MAIN0066 DIMENSION INBASE(42),ISTAT(135),XSOL(135),XB1(42)
MAIN0067 DIMENSION BMT(10),SUM(10),VARDBS(10,10),VARB(10,10),WT(40)
MAIN0068 INTEGER PHASE
MAIN0069
MAIN0070 THE DIMENSIONED ARRAYS HAVE THE FOLLOWING DIMENSIONS:
MAIN0071 C(N),BRHS2(M+2),A2(M+2,N),INBASE(M+2),BRHS1(M+2),
MAIN0072 B2IN(M+2,M+2),XB2(M+2),Y2(M+2),REDCOS(N),DINDX(M+2),
MAIN0073 A(MC,IP),X(NORS,IP),B(NC),Y(NORS),ESTAR(NORS),RES(NORS)
MAIN0074 BETA0(IP),YXRB(NORS),RVAR(NC),ITYPE(NC),IRES(NC)
MAIN0075 DBI(1P),DBR2(IP),DELB(IP),ISAM),RETA(IP),DEB(N)
MAIN0076 INBASE(MP2),ISTAT(N),X(2,MP2),XSOL(N),XB1(MP2)
MAIN0077 BMT(1P),SUM(IP),VARDBS(IP,IP),VARB(IP,IP),WT(2NORS)
MAIN0078
MAIN0079 THE FOLLOWING 'TOLERANCES' ARE USED IN THE ALGORITHM.
MAIN0080 THEY MUST BE NON-NEGATIVE AND WOULD BE ZERO EXCEPT FOR THE
MAIN0081 NUMERICAL INACCURACY OF THE COMPUTER.
MAIN0082 TOLR1 : IF THE MAX REDUCED COST IS LESS THAN OR EQUAL TO
MAIN0083 TOLR1 THEN ALL REDUCED COSTS ARE CONSIDERED TO BE
MAIN0084 NON-POSITIVE.
MAIN0085 TOLR2 : ANY COMPONENT Y(I,J) LESS THAN OR EQUAL TO TOLR2
MAIN0086 IS CONSIDERED NON-POSITIVE.
MAIN0087 TOLR3 : IF AT THE END OF PHASE 1 THE SUM OF THE ARTIFICIAL
MAIN0088 VARIABLES IS GREATER THAN OR EQUAL TO TOLR3, THEN
MAIN0089 THE SUM IS CONSIDERED NON-ZERO AND PHASE 2 IS NOT
MAIN0090 REGUN. OTHERWISE, THE SUM OF THE ARTIFICIAL
MAIN0091 VARIABLES IS CONSIDERED TO HAVE BEEN DRIVEN TO
MAIN0092 ZERO AND PHASE 2 IS REGUN.
MAIN0093
TOLR1=1.00-07
TOLR2=1.00-07
TOLR3=1.00-05
MAIN0094
MAIN0095 THE INPUT - CARD NUMBER ONE OF INPUT CARDS:
MAIN0096 IP = THE LENGTH OF THE VECTOR OF UNKNOWN PARAMETERS.
MAIN0097 NORS = THE NUMBER OF OBSERVATIONS.
MAIN0098 NC = THE NUMBER OF CONSTRAINTS PLACED UPON BETA.
MAIN0099 NSEED = A TEN DIGIT RANDOM NUMBER LESS THAN 2147483647.
MAIN0100 ISAM = THE NUMBER OF SAMPLES TO BE GENERATED FOR THE MINI
MAIN0101 MONTE CARLO STUDY.
MAIN0102
MAIN0103 INWRIT0 = 1 IF THE INPUTTED PROBLEM IS TO BE DESCRIBED;
MAIN0104 0 IF OTHERWISE
MAIN0105
MAIN0106 INWRIT1 = 1 IF THE MAIN RESULTS ARE TO BE DESCRIBED;
MAIN0107 0 OTHERWISE
MAIN0108
MAIN0109 INWRIT2 = 1 IF THE INTERMEDIATE RESULTS ARE TO BE DESCRIBED;
MAIN0110 0 OTHERWISE
MAIN0111
MAIN0112 INWRIT3 = 1 IF THE BASIS INVERSE IS TO BE DESCRIBED
MAIN0113 0 OTHERWISE
MAIN0114
MAIN0115 INWRIT4 = 1 IF THE MAIN RESULTS OF THE MINI MONTE CARLO STUDY
MAIN0116 ARE TO BE PRINTED.
MAIN0117 = 0 OTHERWISE.
MAIN0118
MAIN0119
MAIN0120

```

```

MAIN0001 MR.A: MINIMIZES THE ABSOLUTE RESIDUALS.
MAIN0002
MAIN0003 THIS PROGRAM ESTIMATES A LINEAR REGRESSION BY MINIMIZING THE
MAIN0004 SUM OF THE ABSOLUTE RESIDUALS - L1 ESTIMATION, IN THE MODEL:
MAIN0005 Y = XBETA + EPSILON
MAIN0006 WHERE,
MAIN0007 Y = A VECTOR OF NORS OBSERVATIONS,
MAIN0008 X = AN NORS BY IP MATRIX OF CONSTANTS,
MAIN0009 BETA = AN IP LENGTH VECTOR OF UNKNOWN PARAMETERS
MAIN0010 SUBJECT TO RESTRICTIONS OF THE FORMS
MAIN0011 ARBETA = B OR
MAIN0012 ARBETA < OR = B OR
MAIN0013 ARBETA > OR = B.
MAIN0014
MAIN0015 THE L1 ESTIMATION PROBLEM IS SOLVED USING THE LINEAR PROGRAM
MAIN0016 REISED SIMPLEX ALGORITHM : STANDARD FORM 2
MAIN0017
MAIN0018 THIS ALGORITHM ALWAYS BEGINS WITH AN ALL ARTIFICIAL BASIS
MAIN0019 THE PRODUCT FORM OF THE BASIS INVERSE IS NOT USED
MAIN0020
MAIN0021 THE LINEAR PROGRAMMING PROBLEM IS ASSUMED TO BE IN THE FORM
MAIN0022 MAX CX
MAIN0023 SUBJECT TO
MAIN0024 AX = BRHS
MAIN0025 X GREATER THAN OR EQUAL TO 0
MAIN0026
MAIN0027 WHERE
MAIN0028 BRHS IS A COLUMN OF NON-NEGATIVE CONSTANTS
MAIN0029 A IS AN M-BY-N MATRIX OF CONSTANTS
MAIN0030
MAIN0031 THE AUGMENTED VERSIONS OF A,BRHS,B,XB,AND Y ARE REFERRED TO AS
MAIN0032 A2,BRHS2,B2,XB2,AND Y2 RESPECTIVELY.
MAIN0033
MAIN0034 THE VARIANCE OF THE L1 ESTIMATOR IS ESTIMATED USING A MINI-
MAIN0035 MONTE CARLO APPROACH INCORPORATING THE DUAL SIMPLEX
MAIN0036 ALGORITHM.
MAIN0037
MAIN0038 THE FOLLOWING PROCEDURE WAS DEVELOPED BY :
MAIN0039 D.N. BOOK
MAIN0040 J.R. BOOKER
MAIN0041 H.O. HARTLEY
MAIN0042 R.L. SIELKEN, JR.
MAIN0043 INSTITUTE OF STATISTICS
MAIN0044 TEXAS A&M UNIVERSITY
MAIN0045 COLLEGE STATION, TEXAS 77843
MAIN0046
MAIN0047 INQUIRIES AND COMMENTS SHOULD BY ADDRESSED TO :
MAIN0048 ROBERT SIELKEN, JR.
MAIN0049
MAIN0050 THE SUPPORT OF THE OFFICE OF NAVAL RESEARCH IS GRATEFULLY
MAIN0051 ACKNOWLEDGED.
MAIN0052
MAIN0053 IMPLICIT REAL*(A-H,O-Z)
MAIN0054 COMMON/VALUE/A(20,10),X(20,10),B(20),MP2,N,N,NORS,IP
MAIN0055 COMMON/INWRIT/INWRIT1,INWRIT2,INWRIT3,INWRIT4,INWRIT5,INWRIT6
MAIN0056 COMMON/NFEED/NSEED
MAIN0057
MAIN0058
MAIN0059
MAIN0060

```


PRINTING OF HEADINGS AND INFORMATION

WRIGHT (6,2321)

FORMAT(1H,1X,'MR. A. ',//,16X,
 ' * THIS PROGRAM ESTIMATES A LINEAR REGRESSION BY MINIMIZING',
 ' *',//,6X,' THE SUM OF THE ABSOLUTE RESIDUALS - LI ESTIMATION. ',//,
 ' *',//,6X,' IN ADDITION, A MINI-MONTE CARLO STIMULATION GENERATES AN',
 ' *',//,6X,' ESTIMATED COVARIANCE MATRIX FOR THE ESTIMATED. ',//,6X,
 ' *',//,6X,' REGRESSION PARAMETERS. ',//,6X,' IN ADDITION, LINEAR COVARI',
 ' *',//,6X,' COVARIANTS MAY BE PLACED ON THE PARAMETER VECTOR. REFA.',
 ' *',//,6X,' THESE CONSTRAINTS MAY BE EQUALITY OR INEQUALITY CONSTRAINT',
 ' *',//,6X,'',
 ' *',//,6X,' THE FOLLOWING PROCEDURE DEVELOPED BY ',//,15X,
 ' *',//,6X,' D.N. ROOK',//,15X,' J.B. ROOKER',//,15X,' H.O. HARTLEY',//,15X,
 ' *',//,6X,' R.L. STELKEN, JR.',//,11X,' INSTITUTE OF STATISTICS',//,11X,
 ' *',//,6X,' TEXAS A & M UNIVERSITY',//,11X,' COLLEGE STATION, TEXAS 77943',
 ' *',//,11X,' INQUIRIES AND COMMENTS SHOULD BE ADDRESSED TO:',//,
 ' *',//,15X,' ROBERT L. STELKEN, JR.',//,16X,' THE SUPPORT OF THE',
 ' *',//,6X,' OFFICE OF NAVAL RESEARCH IS GRATEFULLY ACKNOWLEDGED.)

PRINTOUT OF INPUTTED QUANTITIES.

WRITE(6,2332) (TITLE(I),I=1,20)

```

23332 FORMAT(1H1,5X,20A47/)
      WRITE(6,2920) N0B5,IP,NSEED,ISAM,NC
2920  FORMAT(1H0,5X,'NUMBER OF OBSERVATIONS = ',I5/,
      6X,'NUMBER OF PARAMETERS = ',I5/,
      6X,'USER SUPPLIED RANDOM INTEGER, .SEED = ',I11,
      6X,'SAMPLE SIZE FOR THE MINI MONTE CARLO STUDY = ',I6,
      6X,'NUMBER OF BETA CONSTRAINTS = ',I5)
      IF(INHNC.EQ.1) WRITE(6,2921) N10,N11,N12
2921  FORMAT(1H0,9X,'NUMBER OF < OR = BETA CONSTRAINTS = ',I11,
      11H,9X,'NUMBER OF > OR = BETA CONSTRAINTS = ',I11,
      11H,9X,'NUMBER OF = BETA CONSTRAINTS = ',I5)

```

OPTIONAL PRINTOUT OF X, Y, A, AND R WHEN IWRIT=1.

```
IF(IWRITO.EQ.0) GO TO 2305
```

WRITE(6,2324)

```

32324 FORMAT(1H,5X,'THE LINEAR REGRESSION IS Y = X*BETA + EPSILON',/,
      1X6X,'WHERE',/,6X,'Y CONTAINS THE OBSERVATIONS',/,6X,
      1X6X,'BETA IS A VECTOR CONTAINING THE REGRESSION PARAMETERS',
      1X6X,'THE I-TH ROW OF X CONTAINS THE COEFFICIENTS OF BETA',
      1X6X,'CORRESPONDING TO THE I-TH OBSERVATION, AND',/,6X,
      1X6X,'EPSILON IS A RANDOM VARIABLE WITH MEAN ZERO AND VARIANCE',
      1X6X,'SIGMA-SQUARED REPRESENTING RANDOM VARIABILITY FROM',
      1X6X,'X*BETA,')

```

```
WRITE(6,2306)
2306 FORMAT(1H0,10X,'THE Y VECTOR')
```

00 2307 I=1,NOPS

23 200 1 1000
22307 WRITE(6,2308) Y(I)

2308 FORMAT(1H,15X,F16.5)

WRITE(6,2301)

```
22301  FORMAT(1H0,10X,'THE X MATRIX')
```

100 2302 I=1,NORS

2302 WRITE(6,2303) (X(I,J), J=1,IP)

MAIN0241	MAIN0242	MAIN0243	MAIN0244	MAIN0245	MAIN0246	MAIN0247	MAIN0248	MAIN0249	MAIN0250	MAIN0251	MAIN0252	MAIN0253	MAIN0254	MAIN0255	MAIN0256	MAIN0257	MAIN0258	MAIN0259	MAIN0260	MAIN0261	MAIN0262	MAIN0263	MAIN0264	MAIN0265	MAIN0266	MAIN0267	MAIN0268	MAIN0269	MAIN0270	MAIN0271	MAIN0272	MAIN0273	MAIN0274	MAIN0275	MAIN0276	MAIN0277	MAIN0278	MAIN0279	MAIN0280	MAIN0281	MAIN0282	MAIN0283	MAIN0284	MAIN0285	MAIN0286	MAIN0287	MAIN0288	MAIN0289	MAIN0290	MAIN0291	MAIN0292	MAIN0293	MAIN0294	MAIN0295	MAIN0296	MAIN0297	MAIN0298	MAIN0299	MAIN0300
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```

2303 FORMAT(1H, '15X,16F7.1'
      WRITE(6,2309)
2309 FORMAT(1H0,5X,' THE CONSTRAINTS PLACED ON BETA OF THE FORM',
      X' A*BETA = B',9X,'WHERE THE CONSTRAINTS MAY BE EQUALITY OR',
      X' INEQUALITY CONSTRAINTS.',9X,'A = THE MATRIX OF COEFFICIENTS',
      X', WHOSE I-TH ROW CORRESPONDS TO THE I-TH INEQUALITY,')
      WRITE(6,2310)
2310 FORMAT(1H0,10X,'THE RIGHT HAND SIDES OF THE BETA CONSTRAINTS',
      X', THE VECTOR B')
      DO 2311 J=1,NC
2311 WRITE(6,2312) B(J)
2312 FORMAT(1H,9X,F16.5)
      WRITE(6,2313)
2313 FORMAT(1H0,10X,'THE COEFFICIENTS FOR THE BETA ',
      X' CONSTRAINTS, THE A MATRIX')
      DO 2314 I=1,NC
2314 WRITE(6,2315) (A(I,J),J=1,IP)
2315 FORMAT(1H,9X,10F10.4)
      IF(IWT.EQ.0) GO TO 2316
      WRITE(6,2317)
2317 FORMAT(1H0,10X,'THE RESIDUAL WEIGHT COEFFICIENTS')
      DO 2318 I=1,NORS
2318 WRITE(6,2319) WT(I)
2319 FORMAT(1H, '15X,F16.5)
2316 CONTINUE
2305 CONTINUE

      CALL SUBROUTINE BTILDE AND RHANDS TO FORMULATE BETAO AND
      THE PROPER RIGHT HAND SIDES RESPECTIVELY.

      CALL BTILDE(BETAO,Y)
      CALL RHANDS(YHXR,RHAR,BETAO,Y)

      FORM INDICATOR OF RHAR, IRHS,
      IRHS = 1 IF RHAR IS POSITIVE OR ZERO
           2 IF RHAR IS NEGATIVE.

      DO 106 I=1,NC
      IF (RHAR(I).GE.0) IRHS(I) = 1
106 IF (RHAR(I).LT.0) IRHS(I) = 2

      AUGMENT THE RIGHT HAND SIDES FORMING RHHS2.

      RHHS2(1)=0.D0
      RHHS2(2)=0.D0

      FORMULATE THE OBJECTIVE FUNCTION COEFFICIENTS, C(J),J=1,N

      IF(IWT.EQ.0) GO TO 117
      GO TO 116
117 DO 114 J=1,NORS
      WT(J)=1.D0
114 WT(J+NORS)=1.D0
116 DO 13 J=1,N
13 C(J)=0.D0
      DO 14 K=1,NORS
      J=K+2*IP
      C(J)=1.D0*WT(K)
      C(JJ)=-1.D0*WT(K)

```

14 CONTINUE

FORM THE M BY MRV (NUMBER OF REAL VARIABLES) MATRIX A2
OF THE FORM:

X -X 1 -I
A -A 0 0

CALL SURROUTINE SELECT TO SELECT PROBLEMS P1 OR P2
EACH WITH PROBABILITY ONE-HALF.

CALL SELECT(IP1P2,ITERA)

DO 107 I=1,NORS

III=I²

DO 108 J=1,IP

A2(III,J)=X(I,J)

A2(III,J+IP)=X(I,J)

IF(IP1P2.EQ.2) A2(III,J)=X(I,J)

IF(IP1P2.EQ.2) A2(III,J+IP)=X(I,J)

108 CONTINUE

DO 110 K=1,NORS

A2(III,2*IP+K)=0.00

110 A2(III,2*IP+NORSK)=0.00

A2(III,1+2*IP)=1.00

A2(III,1+2*IP+NORS)=1.00

107 CONTINUE

DO 213 I=1,NORS

DO 213 K=1,NC

KK=2*NORSK

A2(KK,1+NORS+2*IP)=0.00

213 A2(KK,1+2*IP)=0.00

AUGMENTATION OF MATRIX A2 BY THE ADDITION OF THE COEFFICIENTS
OF THE BETA CONSTRAINTS, MATRIX A.

DO 109 I=1,NC

DO 118 J=1,IP

A2(NORS+1+2,J)=A(I,J)

A2(NORS+1+2,J+IP)=A(I,J)

IF(IP1P2.EQ.2) A2(NORS+1+2,J)=A(I,J)

IF(IP1P2.EQ.2) A2(NORS+1+2,J+IP)=A(I,J)

118 CONTINUE

109 CONTINUE

AUGMENTATION OF MATRIX A2 BY ADDING SURPLUS/SKACK VARIABLES.

NRV=2*IP+2*NORS

KK=NRV+NT1

IF(KK.EQ.0) GO TO 202

DO 210 I=1,M

DO 210 K=1,KK

II=I+2

J=NRV+K

A2(II,J)=0.00

210 CONTINUE

JJ=0

DO 211 J=1,NC

IF(ITYPE(J).EQ.2) GO TO 211

JJ=JJ+1

I=NRV+JJ

K=NORS+J+2

MAIN0361

MAIN0362

MAIN0363

MAIN0364

MAIN0365

MAIN0366

MAIN0367

MAIN0368

MAIN0369

MAIN0370

MAIN0371

MAIN0372

MAIN0373

MAIN0374

MAIN0375

MAIN0376

MAIN0377

MAIN0378

MAIN0379

MAIN0380

MAIN0381

MAIN0382

MAIN0383

MAIN0384

MAIN0385

MAIN0386

MAIN0387

MAIN0388

MAIN0389

MAIN0390

MAIN0391

MAIN0392

MAIN0393

MAIN0394

MAIN0395

MAIN0396

MAIN0397

MAIN0398

MAIN0399

MAIN0400

MAIN0401

MAIN0402

MAIN0403

MAIN0404

MAIN0405

MAIN0406

MAIN0407

MAIN0408

MAIN0409

MAIN0410

MAIN0411

MAIN0412

MAIN0413

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MAIN0439

MAIN0440

MAIN0441

MAIN0442

MAIN0443

MAIN0444

MAIN0445

MAIN0446

MAIN0447

MAIN0448

MAIN0449

MAIN0450

MAIN0451

MAIN0452

MAIN0453

MAIN0454

MAIN0455

MAIN0456

MAIN0457

MAIN0458

MAIN0459

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MAIN0478

MAIN0479

MAIN0480

A2(K,I)=1.00

IF(ITYPE(J).EQ.1) A2(K,I)=-1.00

211 CONTINUE

202 DO 200 J=1,N

A2(1,J)=C(J)

200 A2(2,J)=0.00

FORM THE BASIS INVERSE, M2 BY M2 MATRIX, AND THE INITIAL
BASIC FEASIBLE SOLUTION.

DO 17 I=1,NORS

DO 16 L=1,NP2

16 R2INV(L,I)=0.00

17 R2INV(L,I)=1.00

CONSTRUCT THE INITIAL BASIS INVERSE, INDEX OF THE INITIAL
BASIC VARIABLES, AND THE INITIAL BASIC VARIABLE VALUES.

DO 240 I=1,NORS

IF(YNXP(I).GE.0) GO TO 241

INBASE(2+I)=2*IP+NORS+1

R2INV(1,2+I)=1.00

R2INV(2+I,2+I)=-1.00

XB2(2+I)=YNXP(I)

BRHS2(2+I)=YNXP(I)

GO TO 243

241 INBASE(2+I)=2*IP+1

R2INV(1,2+I)=-1.00

XB2(2+I)=YNXP(I)

BRHS2(2+I)=YNXP(I)

243 CONTINUE

240 CONTINUE

INBASE(1)=0

INBASE(2)=50

VARIABLE 50 EQUALS MINUS THE SUM OF THE OTHER ARTIFICIAL
VARIABLES WHICH ARE NUMBERED AS 51-52. . .

XB2(1)=0.00

NORSF2=NORS+2

DO 220 I=3,NORSF2

220 XB2(1)=XB2(1)-DBS(BRHS2(I))

NARTS=0

NSLACK=0

NSURF=0

III=2*IP+NORS+1

DO 235 I=1,NC

IJ=NORSF2+1

IBHS2(IJ)=EMAR(I)

IF(ITYPE(I).EQ.2) GO TO 222

IF(ITYPE(I).EQ.1) GO TO 221

TYPE 0 A CONSTRAINT:

IF(EMAR(I).LT.0) GO TO 230

NSLACK=NSLACK+1

INBASE(IJ)=NRV+NSLACK

XB2(IJ)=EMAR(I)

GO TO 235

230 NARTS=NARTS+1

235

```

MAIN0481 INBASE(IJ)=50+NARTS
MAIN0482 XB2(IJ)=RMBAR(I)
MAIN0483 R2INV(IJ,IJ)=1.00
MAIN0484 R2INV(2,IJ)=1.00
MAIN0485 GO TO 235
MAIN0486
MAIN0487 TYPE 1 A CONSTRAINT:
MAIN0488
MAIN0489 221 IF (RMBAR(I).GT.0) GO TO 231
MAIN0490 NSURF=NSURF+1
MAIN0491 INBASE(IJ)=NRVHNT0+NSURF
MAIN0492 XB2(IJ)=RMBAR(I)
MAIN0493 R2INV(IJ,IJ)=1.00
MAIN0494 R2INV(2,IJ)=1.00
MAIN0495 GO TO 235
MAIN0496
MAIN0497 231 NARTS=NARTS+1
MAIN0498 INBASE(IJ)=50+NARTS
MAIN0499 XB2(IJ)=RMBAR(I)
MAIN0500 R2INV(2,IJ)=1.00
MAIN0501 R2INV(IJ,IJ)=1.00
MAIN0502 GO TO 235
MAIN0503
MAIN0504 TYPE 2 A CONSTRAINT:
MAIN0505
MAIN0506 222 NARTS=NARTS+1
MAIN0507 INBASE(IJ)=50+NARTS
MAIN0508 XB2(IJ)=RMBAR(I)
MAIN0509 R2INV(2,IJ)=1.00
MAIN0510 R2INV(IJ,IJ)=1.00
MAIN0511 GO TO 235
MAIN0512
MAIN0513 THE SLACKS ARE NUMBERED AS (2*IP+2*NORS)+1,...
MAIN0514 (2*IP+2*NORS)+NTO.
MAIN0515 THE SURPLUS VARIABLES ARE NUMBERED AS (2*IP+2*NORS)+NTO)+1,
MAIN0516 ....,(2*IP+2*NORS)+NTO)+NNT1.
MAIN0517
MAIN0518 CALCULATION OF XB2(2) = - SUM OF THE ARTIFICIAL VARIABLES.
MAIN0519
MAIN0520 IF (NARTS.EQ.0) GO TO 269
MAIN0521 SUMS=0.00
MAIN0522 DO 267 J=1,NP2
MAIN0523 DINDEX(I)=0.00
MAIN0524 IF (INBASE(I).GT.50) DINDEX(I)=1.00
MAIN0525 DO 266 J=1,N
MAIN0526 JM=J+2
MAIN0527 SUMS=SUMS+XB2(JM)*DINDEX(JM)
MAIN0528 XB2(2)=-SUMS
MAIN0529 GO TO 268
MAIN0530 269 XB2(2)=0.00
MAIN0531 268 CONTINUE
MAIN0532
MAIN0533 OPTIONAL PRINTOUT OF BASIS INVERSE, INBASE, AND INITIAL
MAIN0534 BASIC SOLUTION.
MAIN0535
MAIN0536 IF (IWRIT1.EQ.0) GO TO 2000
MAIN0537 WRITE OUT THE INITIAL PROBLEM AND THE INITIAL BASIC VARIABLES
MAIN0538
MAIN0539 WRITE(6,500)
MAIN0540 FORMAT(1H1, ' THE LINEAR PROGRAMMING PROBLEM AS IT WAS CREATED'
MAIN0541
MAIN0542
MAIN0543
MAIN0544
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MAIN0546
MAIN0547
MAIN0548
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MAIN0597
MAIN0598
MAIN0599
MAIN0600

```

THE OBJECTIVE FUNCTION COEFFICIENTS'

THE RIGHT-HAND SIDES'

THE CONSTRAINT MATRIX A'

THE INITIAL BASIS INVERSE'

THE VARIABLES INITIALLY IN THE AUGMENTED BASIS

THE INITIAL VALUES OF THE BASIC VARIABLES'

THE INITIAL VALUE OF THE OBJECTIVE FUNCTION = '02'

START THE BASIC SIMPLEX ALGORITHM

ITERA=ITERA+1

IF (ITERA.EQ.1) WRITE(6,1051) PHASE, ITERA
FORMAT(1H1,25X, ' PHASE ',11, ' ; ITERATION ',13)

COMPUTE THE REDUCED COSTS: 'C(J) - Z(J)'
THE J-TH REDUCED COST IS DENOTED BY REDUCOS(J)

IF (IWRIT2.EQ.1) WRITE(6,516)
THE REDUCED COSTS'

IF (PHASE.EQ.1) IJLKM=2
IF (PHASE.EQ.2) IJLKM=1
DO 23 J=1,N
S=0.00


```

C      NORMAL (NORMAL), UNIFORM (UNIFRM) OR DOUBLE EXPONENTIAL (DOUBLE) MAIN0841
C      IF (IOPTN.EQ.1) CALL NORMAL(NORS,ESTAR) MAIN0842
C      IF (IOPTN.EQ.2) CALL DOUBLE(NORS,ESTAR) MAIN0843
C      IF (IOPTN.EQ.3) CALL UNIFRM(NORS,ESTAR) MAIN0844
C      ITERA=KKK MAIN0845
C      AK=KKK MAIN0846
C      P=IP MAIN0847
C      AN=NORS MAIN0848
C      IF (ITERA.EQ.1) GO TO 310 MAIN0849
C      SIGNAL=OPTORJ/(SUMORJ/AN) MAIN0850
C      GO TO 311 MAIN0851
C      310 SIGNAL=OPTORJ/(AN-P) MAIN0852
C      311 CONTINUE MAIN0853
C      FORM THE INDICATOR ISTAT TO USE IN DUAL SIMPLEX. MAIN0854
C      DO 281 J=1,N MAIN0855
C      ISTAT(J)=0 MAIN0856
C      DO 282 K=1,NP2 MAIN0857
C      IF (INBASE(K).EQ.50) ISTAT(50)=1 MAIN0858
C      IF (INBASE(K).EQ.J) ISTAT(J)=1 MAIN0859
C      281 CONTINUE MAIN0860
C      CALL THE SUBROUTINE TO SELECT PROBLEM P1 OR P2 TO SOLVE. MAIN0861
C      CALL SELECT(IP1P2,ITERA) MAIN0862
C      IF (IP1P2.EQ.NOMF1) GO TO 343 MAIN0863
C      DO 345 J=1,NP2 MAIN0864
C      DO 346 K=1,IP MAIN0865
C      L=K+IP MAIN0866
C      IF (INBASE(J).EQ.K) GO TO 344 MAIN0867
C      IF (INBASE(J).EQ.L) GO TO 346 MAIN0868
C      GO TO 345 MAIN0869
C      344 INBASE(J)=L MAIN0870
C      ISTAT(K)=0 MAIN0871
C      ISTAT(L)=1 MAIN0872
C      GO TO 345 MAIN0873
C      346 INBASE(J)=L-IP MAIN0874
C      ISTAT(L)=0 MAIN0875
C      ISTAT(L-IP)=1 MAIN0876
C      345 CONTINUE MAIN0877
C      343 CONTINUE MAIN0878
C      IF (IP1P2.EQ.1) GO TO 312 MAIN0879
C      DO 313 I=1,NORS MAIN0880
C      I1=I+2 MAIN0881
C      DO 313 J=1,IP MAIN0882
C      A2(I1,J)=X(I1,J) MAIN0883
C      A2(I1,JIP)=X(I1,J) MAIN0884
C      313 CONTINUE MAIN0885
C      DO 314 I=1,NC MAIN0886
C      DO 314 J=1,IP MAIN0887
C      A2(MORS+I+2,J)=-A(I,J) MAIN0888
C      A2(MORS+I+2,JIP)=-A(I,J) MAIN0889
C      GO TO 111 MAIN0890
C      312 CONTINUE MAIN0891
C      DO 112 I=1,NORS MAIN0892
C      I1=I+2 MAIN0893
C      DO 112 J=1,IP MAIN0894
C      A2(I1,J)=X(I1,J) MAIN0895
C      A2(I1,JIP)=X(I1,J) MAIN0896
C      112 CONTINUE MAIN0897
C      DO 113 I=1,NC MAIN0898
C      DO 113 J=1,IP MAIN0899
C      A2(I1,JIP)=-X(I1,J) MAIN0900
C      113 CONTINUE MAIN0901
C      DO 113 J=1,IP MAIN0902
C      DO 113 I=1,NC MAIN0903
C      A2(MORS+I+2,J)=A(I,J) MAIN0904
C      A2(MORS+I+2,JIP)=-A(I,J) MAIN0905
C      113 CONTINUE MAIN0906
C      CALL RTILDE(DBETA0,ESTAR) MAIN0907
C      CALL SUBROUTINES TO GENERATE BETAO FOR THE MMC STUDY AND TO MAIN0908
C      SOLVE THE CURRENT SAMPLE USING THE DUAL SIMPLEX ALGORITHM. MAIN0909
C      CALL RTILDE(DBETA0,ESTAR) MAIN0910
C      CALL SUBROUTINE MCRRHS TO FORM THE RIGHT HAND SIDES IN THE MAIN0911
C      MINI MONTE CARLO STUDY. MAIN0912
C      CALL MCRRHS(ESTAR,SIGNAL,BHAT,DBETA0,BRRHS1,IP1P2) MAIN0913
C      CALCULATE X81, THE INITIAL BASIC VARIABLES FOR USE IN THE MAIN0914
C      DUAL SIMPLEX ALGORITHM IN THE MINI MONTE CARLO STUDY. MAIN0915
C      DO 283 I=1,NP2 MAIN0916
C      A0=0.00 MAIN0917
C      283 X81(I)=A00 MAIN0918
C      284 A00=ADDH2ZINVO(I,J)*BRRHS1(J) MAIN0919
C      CALL THE DUAL SIMPLEX ALGORITHM TO OBTAIN THE SOLUTION, MAIN0920
C      DELTA BETA STAR, FOR THIS SAMPLE. MAIN0921
C      CALL DUAL5(R2INV,A2,INBASE,BRRHS1,ISTAT,DBX,X81) MAIN0922
C      SUMORJ=SUMORJ-X81(1) MAIN0923
C      IF (IP1P2.EQ.1) GO TO 315 MAIN0924
C      DO 316 L=1,IP MAIN0925
C      DB2(L)=DBX(L) MAIN0926
C      DB1(L)=DBX(L+IP) MAIN0927
C      DELR(L,ITERA)=DB1(L)-DB2(L)+DBETA0(L) MAIN0928
C      GO TO 317 MAIN0929
C      315 DO 318 L=1,IP MAIN0930
C      DB1(L)=DBX(L) MAIN0931
C      DB2(L)=DBX(L+IP) MAIN0932
C      DELR(L,ITERA)=DB1(L)-DB2(L)+DBETA0(L) MAIN0933
C      317 CONTINUE MAIN0934
C      NONF12=IP1P2 MAIN0935
C      2222 CONTINUE MAIN0936
C      CALCULATE THE COVARIANCE OF DELTA BETA STAR BASED ON THE MAIN0937
C      SAMPLES OF EPSILONS FROM THE MINI MONTE CARLO STUDY. MAIN0938
C      SHM=ISAH MAIN0939
C      DO 319 J=1,IP MAIN0940
C      SHM=0.00 MAIN0941
C      DO 320 L=1,ISAH MAIN0942
C      SHM=SHM+DELRE(J,L) MAIN0943
C      319 SUM(J)=SHM MAIN0944
C      DO 321 I=1,IP MAIN0945
C      DO 321 J=1,IP MAIN0946
C      SOSUM=0.00 MAIN0947
C      SOSUM=SOSUM+SHM MAIN0948
C      321 SOSUM=0.00 MAIN0949
C      320 SOSUM=0.00 MAIN0950
C      319 SOSUM=0.00 MAIN0951
C      320 SOSUM=0.00 MAIN0952
C      319 SOSUM=0.00 MAIN0953
C      320 SOSUM=0.00 MAIN0954
C      319 SOSUM=0.00 MAIN0955
C      320 SOSUM=0.00 MAIN0956
C      319 SOSUM=0.00 MAIN0957
C      320 SOSUM=0.00 MAIN0958
C      319 SOSUM=0.00 MAIN0959
C      320 SOSUM=0.00 MAIN0960

```

```

854 FORMAT(1H1)
909 CONTINUE
STOP
END
SUBROUTINE DUALS(B1INV,A1,INBASE,BRHS1,ISTAT,DBX,XB1)
SURROUTINE DUALS - THE DUAL SIMPLEX ALGORITHM . . .
THE LINEAR PROGRAMMING PROBLEM IS PUT INTO THE FORM
MAX CX
SUBJECT TO
AX = BRHS
X GREATER THAN OR EQUAL TO 0
WHERE
BRHS IS A COLUMN OF CONSTANTS
A IS AN M-BY-N MATRIX OF CONSTANTS
THE AUGMENTED VERSIONS OF A,BRHS,B,XB,AND Y ARE REFERRED TO AS
A1,BRHS1,B1,XB1,AND Y1 RESPECTIVELY.
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/VALUE/A(20,10),X(20,10),Z(20),MP2,N,NC,NDES,IP
COMMON/TWRITE/TWRITE1,TWRITE2,TWRITE3,TWRITE4,TWRITE5,TWRITE6
DIMENSION B1INV(42,42),A1(42,72),ISTAT(135),INBASE(42),BRHS1(42)
DIMENSION DBX(135),XB1(42),REDCOS(72),YR(72),Y1(42)
DIMENSIONES ARE: B1INV(MP2,MP2),A1(MP2,N),ISTAT(N),INBASE(MP2)
BRHS1(MP2),DBX(N),XB1(MP2),REDCOS(N),Y1(MP2),YR(N)
MP1=MP2
N=MP2-1
TOLR1=1.0D-07
TOLR2=-1.0D-07
TOLR3=-1.0D-07
IF(TWRITE4.EQ.0) GO TO 3555
WRITE(6,513)
THE INITIAL VALUES OF THE BASIC VARIABLES'
513 FORMAT(1H0,'
DO 515 I=1,MP1
II=I-1
515 WRITE(6,514) II,XB1(II)
514 FORMAT(1H ,15X,XB(' ,13,' ) = ',E15.5)
3555 CONTINUE
855 FORMAT(1H0,10X,THE INITIAL VALUE OF THE OBJECTIVE FUNCTION = ',E15.5)
350 CONTINUE
START THE DUAL SIMPLEX ALGORITHM
COMPUTE THE REDUCED COSTS: C(J)-Z(J)
THE J-TH REDUCED COST IS DENOTED BY REDCOS(J)
DO 23 J=1,N
S=0.00
IF(ISTAT(J).EQ.1) GO TO 1021
DO 22 K=1,MP1
R=REB1INV(1,K)*A1(K,J)
REDCOS(J)=R
1021 CONTINUE
23

```

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SSUM=SUM(J)*SUM(I)
DO 322 L=1,ISAM
SUMO=SUMO+DELTA(J,L)*DELTA(I,L)
322 CONTINUE
323 VARDPS(I,J)=(SUMO-SSUM)/(SAM-P)
324 CONTINUE
IF(TWRITE6.EQ.1) WRITE(6,330)
330 FORMAT(1H1,5X,AUXILIARY RESULTS OF THE MINI MONTE CARLO STUDY',/,MAIN0968
8,X,VALUES OF DELTA BETA STAR')
IF(TWRITE6.EQ.0) GO TO 335
DO 331 I=1,ISAM
331 WRITE(6,332) I,DELTA(I),L=1,IP)
332 FORMAT(1H ,9X,SAMPLE NUMBER =',13,3X,10F11.5)
WRITE(6,333)
333 FORMAT(1H0,8X,ESTIMATED COVARIANCE OF DELTA BETA STAR')
DO 334 I=1,IP
334 WRITE(6,336) (VARDPS(I,J),J=1,IP)
336 FORMAT(1H ,12X,8F14.6)
335 CONTINUE
C
C
C
CALCULATE THE ESTIMATE OF SIGMA, SIGMA HAT.
SIGMAHAT=SQRT(SUMOJ/SAM)
IF(TWRITE6.EQ.1) WRITE(6,337) SUMOJ
337 FORMAT(1H0,'SUM OF THE OPTIMAL OBJECTIVE FUNCTIONS OVER',
8,ALL SAMPLES =',F14.6)
WRITE(6,338) SIGMAHAT
338 FORMAT(1H1,5X,MAIN RESULTS OF THE MINI MONTE CARLO STUDY',/,6X,
8,ESTIMATED VALUE OF SIGMA =',F14.6)
C
C
C
CALCULATION OF SIGMA HAT 5 AND OPTIONAL PRINTOUT WHEN TWRITE6=1,MAIN0991
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IF(LOPTN.EQ.1) CCC=2.50642827500/2.00
IF(LOPTN.EQ.2) CCC=DSORT(2.00)
IF(LOPTN.EQ.3) CCC=1.15470053800
SIGMA5=OPTORJACC/(SN-P)
IF(TWRITE6.EQ.1) WRITE(6,329) SIGMA5
329 FORMAT(1H0,5X,AUXILIARY RESULT : THE ESTIMATE OF SIGMA 5 = ',F14,MAIN1000
26)
C
C
C
CALCULATION OF THE COVARIANCE OF DELTA BETA (= COV. OF BETA)
DO 339 I=1,IP
DO 339 J=1,IP
339 VARD(I,J)=(SIGMAHAT**2)*VARDPS(I,J)
WRITE(6,340)
340 FORMAT(1H0,5X,ESTIMATED COVARIANCE OF THE REGRESSION PARAMETER',
8,VECTOR, BETA')
DO 342 I=1,IP
342 WRITE(6,341) (VARD(I,J),J=1,IP)
341 FORMAT(1H ,11X,8F14.6)
WRITE(6,850)
850 FORMAT(1H1)
GO TO 999
402 WRITE(6,530)
530 FORMAT(1H0,10X,ALL OF THE YS ARE NON-POSITIVE.',/,11X,HENCE THE
* PROBLEM HAS AN UNBOUNDED SOLUTION. ')
WRITE(6,854)

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C C C C C
END
SUBROUTINE UNIFRM(NOBS,UNVAR1)
  SUBROUTINE UNIFRM . . .
    GENERATES THE EPSILON STAR'S (ERRORS DISTRIBUTED AS
    UNIFORM VARIABLES WITH MEAN ZERO, VARIANCE ONE.)
  IMPLICIT REAL*8 (A-H,O-Z)
  COMMON/NFEED/NSEED
  DIMENSION UNVAR1(20)
  DO 10 I=1,NOBS
    CALL RAND(UZO)
    10 UNVAR1(I)=UZO*DSORT(12.D0)
  RETURN
  END
  UNIF0018
  UNIF0001
  UNIF0002
  UNIF0003
  UNIF0004
  UNIF0005
  UNIF0006
  UNIF0007
  UNIF0008
  UNIF0009
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  UNIF0014

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The parameters in a linear regression model can be estimated by minimizing the sum of the absolute residuals (L_1 estimation) instead of the more classical approach of minimizing the sum of squared residuals (least squares estimation). In addition to other nice properties L_1 estimators are less sensitive to outliers than least squares estimators. This paper describes a linear programming algorithm and computer program for obtaining L_1 estimators and estimates of their covariances when the regression parameters are restricted to satisfy specified linear constraints. These estimated		

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19. (cont.)

variances and covariances of L_1 estimators
computer algorithm
least squares
Monte Carlo

20. (cont.)

covariances are the new feature in this work and are an extremely important ingredient in hypothesis tests and confidence interval construction. Technical Report 64 describes a similar procedure for obtaining unbiased L_1 estimators when there are no constraints on the parameters.

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