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Research Report CCS 414

KHINČIN-KULLBACK-LEIBLER ESTIMATION WITH INEQUALITY CONSTRAINTS

by

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ABSTRACT

In this paper we extend our grasp of statistical theory, duality theory, and computational convenience to the general linear inequality situation in K^2L (or minimum discrimination information) estimation by exhibiting it as the limit of a simple one parameter sequence of equality problems. Only the finite discrete distribution case is treated here.

KEY WORDS

Khinčin-Kullback-Leibler Estimation

Inferential Distribution

Minimum Discrimination Information

Inequality Constrained Distributions

INTRODUCTION

Frequently in hypothesis testing or estimation of an "inferential" distribution (in Akaike's terminology) by the Khinčin-Kullback-Leibler (or Minimum Discrimination Information) method one has information about the possible candidate distributions in the form of linear inequalities on the components of the distribution in addition to equality (moment) constraints. In MOFS 1978, Charnes, Cooper, and Seiford [1] developed a convex programming duality theory for the K^2L method with linear inequality constraints. It is especially incisive and convenient for constraints in equality (or moment) form which, further, have connections with established statistical theory as well as the analytic and computational facility of an unconstrained extremal problem in simple smooth functions.

In this paper we extend our grasp of statistical theory, duality theory, and computational convenience to the general linear inequality situation by exhibiting it as the limit of a simple one parameter sequence of equality problems. We treat only the finite discrete distribution case here, reserving the general distribution case to a forthcoming paper which also simplifies the duality theory of the "equality" case for general distributions as developed by Ben-Tal and Charnes in [2].

INEQUALITY AND APPROXIMATING EQUALITY FORMS

The dual programs for the inequality form as presented in Charnes-Cooper-Seiford [1] are

$$\max \quad v(\delta) \overset{\Delta}{=} -\delta^T \ln \left[\frac{\delta}{ec}\right]$$

$$\text{(I)} \qquad \text{s.t.} \qquad \begin{aligned} \delta^T A^1 \qquad &= b^{1T} \\ \delta^T A^2 + \gamma^T &= b^{2T} \\ \delta, \gamma &\geq 0 \end{aligned}$$

and

$$\text{(II)} \qquad \inf \xi(z) \overset{\Delta}{=} c^T e^{A^1 z^1 + A^2 z^2} - b^{1T} z^1 - b^{2T} z^2$$

$$\text{s.t.} \qquad - z^2 \geq 0$$

To obtain the dual programs for our approximating (weighted) equality form, we employ the procedure in [1]. Thus, consider

$$\text{(1)} \qquad K(x,y,\delta,\gamma) \overset{\Delta}{=} \sum_{i \varepsilon I^1} (c_i e^{x_i} - \delta_i x_i) + \sum_{i \varepsilon I^2} (\varepsilon c_i e^{(y_i/\varepsilon)} - \gamma_i y_i)$$

where $c_i > 0$ for all i and $\varepsilon > 0$.

Minimizing with respect to x_i and y_i gives:

$$\text{(2)} \qquad x_i^* = \ln\left(\frac{\delta_i}{c_i}\right), \ i \ \varepsilon \ I^1, \ y_i^* = \varepsilon \ln\left(\frac{\gamma_i}{c_i}\right), \ i \ \varepsilon \ I^2$$

Hence,

$$\text{(3)} \qquad -\sum_{i \varepsilon I^1} \delta_i \ln\left(\frac{\delta_i}{c_i}\right) - \varepsilon \sum_{i \varepsilon I^2} \gamma_i \ln\left(\frac{\gamma_i}{ec_i}\right) \leq K(x,y,\delta,\gamma)$$

To decouple, as in [1], we obtain

$$\text{(4)} \qquad \delta^T x + \gamma^T y = b^{1T} z^1 + b^{2T} z^2$$

and

$$\text{(5)} \qquad (\delta^T, \gamma^T) \begin{bmatrix} A^1 & A^2 \\ 0 & I \end{bmatrix} \begin{bmatrix} z^1 \\ z^2 \end{bmatrix} = (b^{1T}, b^{2T}) \begin{bmatrix} z^1 \\ z^2 \end{bmatrix}$$

when we choose

(6) $$x = A^1z^1 + A^2z^2 \ , \ y = z^2$$

Thereby, we have the dual problems

$$\max \ v(\delta,\gamma,\varepsilon) \triangleq -\sum_{i\varepsilon I^1} \delta_i \ln\left(\frac{\delta_i}{ec_i}\right) - \varepsilon \sum_{i\varepsilon I^2} \gamma_i \ln\left(\frac{\gamma_i}{ec_i}\right)$$

(III) s.t.

$$\delta^T A^1 = b^{1T}$$

$$\delta^T A^2 + \gamma^T = b^{2T}$$

$$\delta \ , \ \gamma \geqslant 0$$

and

(IV) $$\inf \ \xi(z) \triangleq c^{1T} e^{A^1z^1 + A^2z^2} - b^{1T}z^1 - b^{2T}z^2 + \varepsilon \ c^{2T} e^{\left(z^2/\varepsilon\right)}$$
with z unconstrained

(Note that the c_i, $i\varepsilon I_2$, may be chosen arbitrarily.)

The duality theory of (III) and (IV) is precisely that of the equailty case in [1], as may be seen by making the change of variables to

$$\tilde{\gamma}_i \triangleq \varepsilon\gamma_i \ , \ \tilde{c}_i \triangleq \begin{bmatrix} c_i \ , \ i\varepsilon I^1 \\ \varepsilon c_i, \ i\varepsilon I^2 \end{bmatrix} .$$

Our present form is, however, more convenient for our arguments.

We now define

(7.1) $$f(z) \triangleq c^{1T} e^{A^1z^1 + A^2z^2} - b^{1T}z^1 - b^{2T}z^2 \ , \ z^2 \leqslant 0$$

(7.2) $$g(z,\varepsilon) \triangleq c^{1T} e^{A^1z^1 + A^2z^2} - b^{1T}z^1 - b^{2T}z^2 + \varepsilon c^{2T} e^{\left(z^2/\varepsilon\right)}$$

We will then have

Theorem 1:

For some $\varepsilon > 0$, $\inf_{z^2 \leq 0} f(z)$ exists if and only if $\inf_{z} g(z,\varepsilon)$ exists.

Further,

$$\lim_{\varepsilon \to 0} \inf_{z} g(z,\varepsilon) = \inf_{z^2 \leq 0} f(z)$$

Proof:

Consider $\frac{\partial g}{\partial z^2_i}$. It may be written as

$$(8) \qquad \frac{\partial g}{\partial z^2_i} = (c^1k)^T e^{A^1z^1 + A^2z^2} - b^2_i + z^2_i \; c_i e^{\left(z^2_i / \varepsilon\right)} \;, \quad i \varepsilon I^2$$

where $k_r \triangleq \frac{\partial}{\partial z^2_i} (r^{A^2z^2})$ and $(c^1k)^T \triangleq (c^1_1k_1, \ldots, c^1_mk_m)$.

For $z^2_i > 0$ we see that by choosing ε small enough we can make $g(z,\varepsilon)$ an increasing function in that direction. Thus, in seeking a minimum we need only consider $z^2 \leq 0$. For notational simplicity in the following and "inf" with respect to z shall always also entail $z^2 \leq 0$.

For $z^2 \leq 0$ and all $\varepsilon > 0$ we have

$$(9) \qquad f(z) \leq g(z,\varepsilon) \leq f(z) + \varepsilon \sum_{i\varepsilon I^2} c_i$$

If $\inf f(z)$ exists, then, by (9), $\inf(z) \leq g(z,\varepsilon)$ so that $\inf g(z,\varepsilon)$ exists for all $\varepsilon > 0$. Further, $\inf f(z) \leq \inf g(z,\varepsilon)$. On the other hand, if $\inf g(z,\varepsilon)$ exists, then, by (9), $\inf g(z,\varepsilon) \leq f(z) + \varepsilon \sum_{i\varepsilon I^2} c_i$ so that $\inf f(z)$ exists. Further, too, $\inf g(z,\varepsilon) \leq \inf f(z) + \varepsilon \sum_{i\varepsilon I^2} c_i$.

Hence

(10.1) $\inf f(z) \leqslant \inf g(z,\varepsilon) \leqslant \inf f(z) + \varepsilon \sum_{i\varepsilon I^2} c_i$

and

(10.2) $\lim_{\varepsilon\to 0} \inf g(z,\varepsilon) = \inf f(z)$

When f(z) has a minimum at z*, again from (9) we can conclude

(11) $f(z^*) = \lim_{\varepsilon\to 0} g(z^*,\varepsilon)$ Q.E.D.

So much for the minimization side of the duality. For the K^2L side we have

Theorem 2:

The maximum in problem (I) and, as $\varepsilon \to 0$, in problem (III) is the same.

Proof:

For $0 \leqslant \gamma_i \leqslant ec_i$, we have $-c_i \leqslant \gamma_i \ln\left(\frac{\gamma_i}{ec_i}\right) \leqslant 0$, whereas for $\gamma_i > ec_i$, $\gamma_i \ln\left(\frac{\gamma_i}{ec_i}\right) > 0$. Therefore,

(12) $-\varepsilon \sum_{i\varepsilon I^2} \gamma_i \ln\left(\frac{\gamma_i}{ec_i}\right) \leqslant \varepsilon \sum_{i\varepsilon I^2} c_i$ for all $\gamma_i \geqslant 0$.

Thus,

(13) $v(\delta,\gamma,\varepsilon) \leqslant v(\delta) + \varepsilon\sum_{i\varepsilon I^2} c_i \leqslant f(z) + \varepsilon\sum_{i\varepsilon I^2} c_i$

From (13) we conclude

(14) $\sup v(\delta,\gamma,\varepsilon) \leqslant \sup v(\delta) + \varepsilon \sum_{i\varepsilon I^2} c_i \leqslant \inf f(z) + \varepsilon \sum_{i\varepsilon I^2} c_i$

But by the theory of [1],

$$\sup v(\delta,\gamma,\varepsilon) = \max v(\delta,\gamma,\varepsilon) = \inf g(z,\varepsilon) \geqslant \inf f(z)$$ by (10.1). Hence

(15) $\inf f(z) \leqslant \max v(\delta,\gamma,\varepsilon) \leqslant \sup v(\delta) + \varepsilon \sum_{i\varepsilon I^2} c_i \leqslant \inf f(z) + \varepsilon \sum_{i\varepsilon I_2} c_i$

Thus letting $\varepsilon \to 0$, we have

(16) $\inf f(z) \leqslant \lim_{\varepsilon\to 0} \max v(\delta,\gamma,\varepsilon) \leqslant \sup v(\delta) \leqslant \inf f(z)$

and

(17) $\lim_{\varepsilon\to 0} \max v(\delta,\gamma,\varepsilon) = \sup v(\delta) = \inf f(z)$

Q.E.D.

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