# DAVID W. TAYLOR NAVAL SHIP
# RESEARCH AND DEVELOPMENT CENTER

Bethesda, Maryland 20084

PERFORMANCE PREDICTIONS FOR A ROOM
TEMPERATURE, ERICSSON CYCLE MAGNETIC HEAT PUMP

by

John G. Purnell

PROPULSION AND AUXILIARY SYSTEM DEPARTMENT
RESEARCH AND DEVELOPMENT

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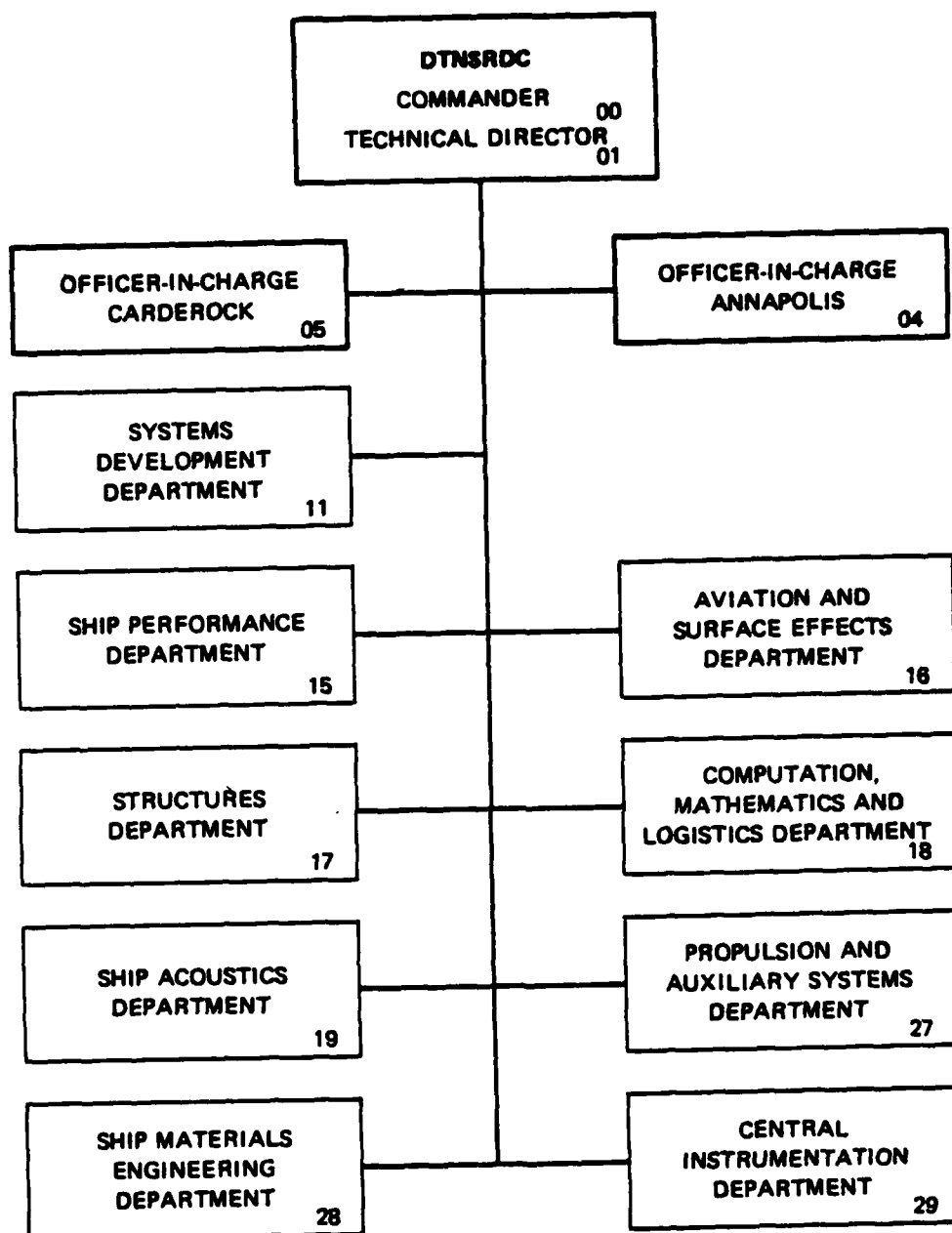
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The performance potential of a room temperature magnetic heat pump utilizing Gadolinium and operating on an Ericsson Cycle was investigated at magnetic flux densities of 2 and 7-Tesla which represent the upper limits of conventional and superconducting electromagnetics, respectively. At a coefficient of performance of 5, a 7-Tesla system would provide a cooling capacity of at best 1200 BTU per hour per pound of Gadolinium while a 2-Tesla system would operate at approximately 130 BTU per hour per pound of

Gadolinium. Magnetic circuit efficiency was not determined but must be high (95-percent or better) in order for the magnetic heat pump performance to compete with conventional cooling systems. It is unlikely the magnetic heat pump investigated could approach the performance and compactness of the conventional cooling systems unless field strengths much greater than 7-Tesla are possible.

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## Nomenclature

| Symbol | Description | Units |
|---|---|---|
| $A_{CL}$ | Column Cross Sectional Area | $Ft^2$ ($m^2$) |
| $A_{FF}$ | Free Flow Area Between Plates | $Ft^2$ ($m^2$) |
| $A_S$ | Surface Area Per Plate | $Ft^2$ ($m^2$) |
| $c_p$ | Specific Heat | $BTU/LB_m °F$ ($CAL/gm °C$) |
| $c_v$ | Specific Heat | $BTU/LB_m °F$ ($CAL/gm °C$) |
| COP | Coefficient of Performance | -- |
| $D_H$ | Hydraulic Diameter | inches (CM) |
| f | Friction Factor | -- |
| F | Heat Flow Correction | - |
| g | Gravitational Constant | $FT/sec^2$ ($m/sec^2$) |
| $h_D$ | Duct Heat Transfer Coefficient | $BTU/Hr Ft^2$ ($CAL/Hr CM^2 °C$) |
| $H_{CH}$ | Channel Height | Inches (CM) |
| $H_l$ | Friction Head Loss | Ft (m) |
| k | Thermal Conductivity | $BTU/Hr ft °F$ ($CAL/Hr CM °C$) |
| $L_p$ | Plate Length | Inches (CM) |
| $L_S$ | System Length | Ft (M) |
| $\dot{m}$ | Mass Flow Rate | $LB_m/Hr$ ($Kg/Hr$) |
| M | Molecular Weight | $LB/Mole$ ($Kg/Mole$) |
| $N_p$ | Number of Plate | ---- |
| $N_{RPM}$ | Cycle Rate Per Minute | $1/Min.$ ($1/Min.$) |
| Pr | Prandtl Number | ---- |
| Q | Heat Flow | $BTU/Hr LB_m$, ($CAL/Hr gm$) |
| $R_A$ | Free Flow Area Ratio | ---- |
| Re | Reynolds Number | ---- |
| S | Entropy | $BTU/Mole °F$ ($CAL/Mole °C$) |
| $S_p$ | Plate Spacing | Inches (CM) |
| t | Time | Hours (Hours) |

## Nomenclature

| Symbol | Description | Units |
|--------|-------------|-------|
| $t_p$ | Plate Thickness | Inches (CM) |
| $T$ | Temperature | °F (°C) |
| $U$ | Overall Transmittance | $BTU/Hr\ Ft^2\ °F$ ($CAL/Hr\ CM^2\ °C$) |
| $V_{CL}$ | Column Velocity | $FT/sec$ ($m/sec$) |
| $V_p$ | Plate Channel Velocity | $Ft/sec$ ($m/sec$) |
| $W_{CL}$ | Column Width | Inches (CM) |
| $\eta_m$ | Magnetic Circuit Efficiency | ---- |
| $\rho$ | Density | $LB_m/Ft^3$ ($Kg/m^3$) |
| $\Delta T$ | Temperature Difference | °F (°C) |

## Nomenclature/Subscripts

| Symbol | Description |
|--------|-------------|
| a | Refers to State Point a |
| b | Refers to State Point b |
| c | Refers to State Point c |
| cold or C | Refers to Cold Column End |
| d | Refers to State Point d |
| e | Refers to State Point e |
| f | Refers to State Point f |
| F | Refers to Frictional Effect |
| GD | Refers to Gadolinium |
| HE | Refers to the External Heat Exchange Loop |
| hot or H | Refers to Hot Column End |
| I | Refers to Ideal Condition |
| mix | Refers to Mixing |
| REG | Refers to the Regenerative Column |
| TOT | Refers to Total |
| W | Refers to the Working Fluid |
| 1 | Refers to State Point 1 |
| 2 | Refers to State Point 2 |
| 3 | Refers to State Point 3 |

## Nomenclature/Subscripts

| Symbol | Description |
|--------|-------------|
| 4 | Refers to State Point 4 |
| 5 | Refers to State Point 5 |
| 6 | Refers to State Point 6 |

## List of Abbreviations

COP    Coefficient of Performance

CPM    Cycles Per Minute

GD     Gadolinium

KJ     Kilojoules

## Conversion Constants

(Work was done in English units and the following
provides conversion to metric units)

1 Foot   =   0.3048 Meters

1 Inch   =   2.54 Centimeters

1 BTU    =   252 Gram-Calories = 1.0548 Kilojoules

1 Pound  =   453.5924 Grams = 0.4536 Kilograms

$^\circ F$       =   $9/5 \, C^\circ + 32$

## ABSTRACT

The performance potential of a room
temperature magnetic heat pump utilizing
Gadolinium and operating on an Ericsson Cycle was
investigated at magnetic flux densities of 2 and
7-Tesla which represent the upper limits of
conventional and superconducting
electromagnetics, respectively. At a coefficient
of performance of 5, a 7-Tesla system would
provide a cooling capacity of at best 1200 BTU
per hour per pound of Gadolinium while a 2-Tesla
system would operate at approximately 130 BTU per
hour per pound of Gadolinium. Magnetic circuit
efficiency was not determined but must be high
(95 - percent or better) in order for the
magnetic heat pump performance to compete with
conventional cooling systems. It is unlikely the
magnetic heat pump investigated could approach
the performance and compactness of the
conventional cooling systems unless field
strengths much greater than 7-Tesla are possible.

## ADMINISTRATIVE INFORMATION

## INTRODUCTION

Magnetic heat pumps are based on the magnetocaloric effect by which certain
materials exhibit an increase (or decrease) in temperature with the application (or
removal) of a magnetic field. Magnetic heat pumps originally found applications only
at temperature near absolute zero using paramagnetic materials [1]. Using a
ferromagnetic material having a Curie point in the ambient temperature range, a
magnetic heat pump for air conditioning is possible by utilizing a suitable
regenerative thermodynamic cycle [2], [3]. Rare earth materials are effective
in this application and Gadolinium (GD) with a Curie point of 68 °F (293 °K)
is a good working material. At its Curie point, the application of a 7-Tesla
field to GD causes a temperature rise of 25°F (14 °K) or the release of 1.7
BTU/lb (4 KJ/Kg) of heat under isothermal conditions.

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Magnetic heat pumps are of interest for several reasons. The number of moving parts in the system should be small and thus high reliability is expected. For a ferromagnetic material like Gadolinium, an Ericsson or Stirling cycle are similar and cycle efficiency can approach the Carnot cycle efficiency. This should provide considerable improvement over conventional room temperature refrigeration systems which generally operate on a Rankine type cycle.

In this study, a magnetic heat pump operating on an Ericsson cycle was evaluated parametrically to determine expected thermodynamic performance. Losses associated with the generation of the proper magnetic field are not determined in this report however the effect of the magnet circuit efficiency on the cycle performance is shown. A 2 and 7 - Tesla field strength were examined with 2-Tesla representing the limit of an electromagnet and 7-Tesla the limit of a superconducting magnet.

## BACKGROUND

A typical magnetic heat pump consists of a porous mass of ferromagnetic material, Gadolinium plates in this case, located in a vertical column of fluid with an energizing magnet surrounding the column at the location of the Gadolinium plates. At least one of the working substances must move. One case is to allow the column of fluid to remain stationary and to move the Gadolinium located in the column up and down. This also requires moving the magnet which surrounds the column with the Gadolinium, resembling test setups by Brown [3]. One of the drawbacks of this setup is that the magnet and Gadolinium are separated by the column wall plus gaps to allow for movement, all resulting in a degradition of magnet performance.

The arrangement investigated keeps the Gadolinium (and thus the magnet) stationary and moves the column of fluid back and forth through the Gadolinium. This allows the Gadolinium to be in as close a proximity to the magnet as possible to minimize magnetic losses. Moving the liquid column requires the system to be about twice as long as the above stationary fluid case using the same Gadolinium geometry.

Figure 1 shows a schematic of the magnetic Ericsson cycle investigated.

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Figure 2 and 3 show the magnetic Ericsson cycle entropy-temperature diagram and the fluid column and Gadolinium temperature profiles, respectively. The working fluid was assumed to be water and all points in the cycle are numbered similarly in the figures. The column would be mounted vertically with the cold end at the bottom and the hot end at the top to avoid fluid mixing caused by density differences in the fluid column. Heat exchangers are located at the top of the column to remove the heat of magnetization from the fluid and



HOT END HEAT EXCHANGER

MAGNET AND GADOLINIUM PLATES

COLUMN LENGTH

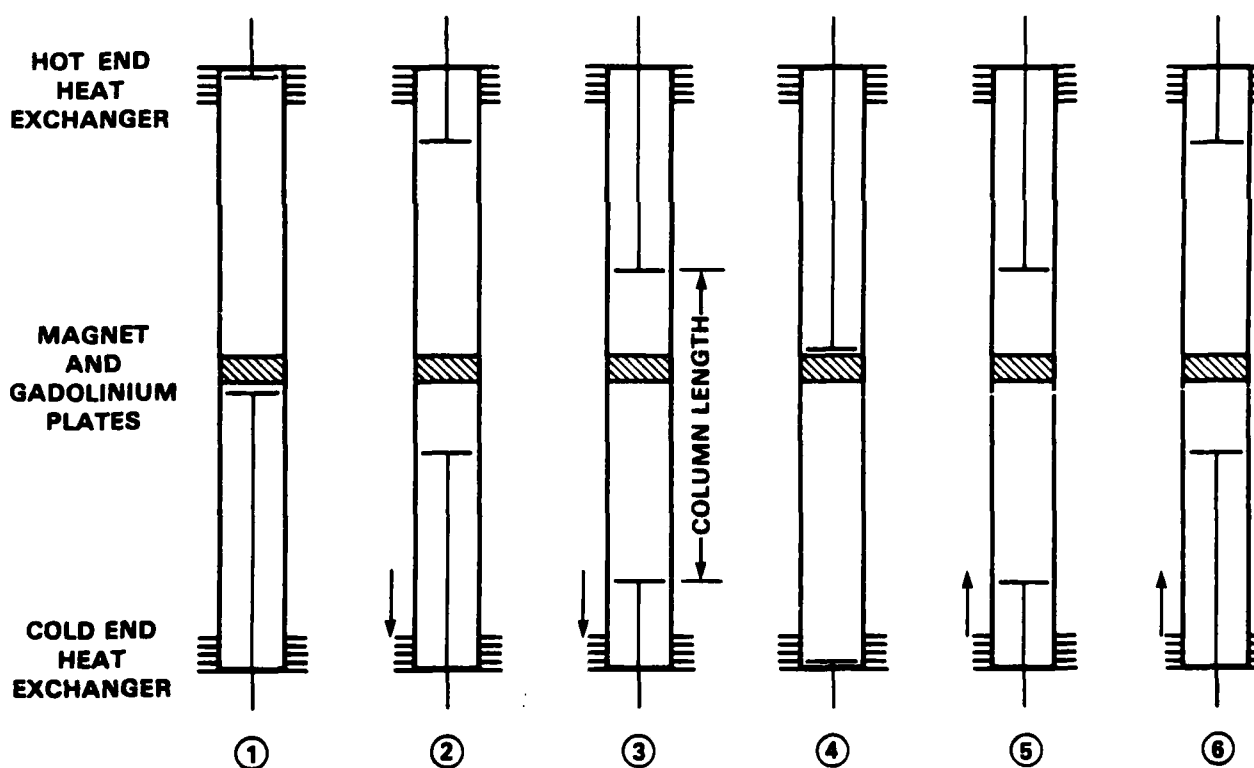COLD END HEAT EXCHANGER

① ② ③ ④ ⑤ ⑥

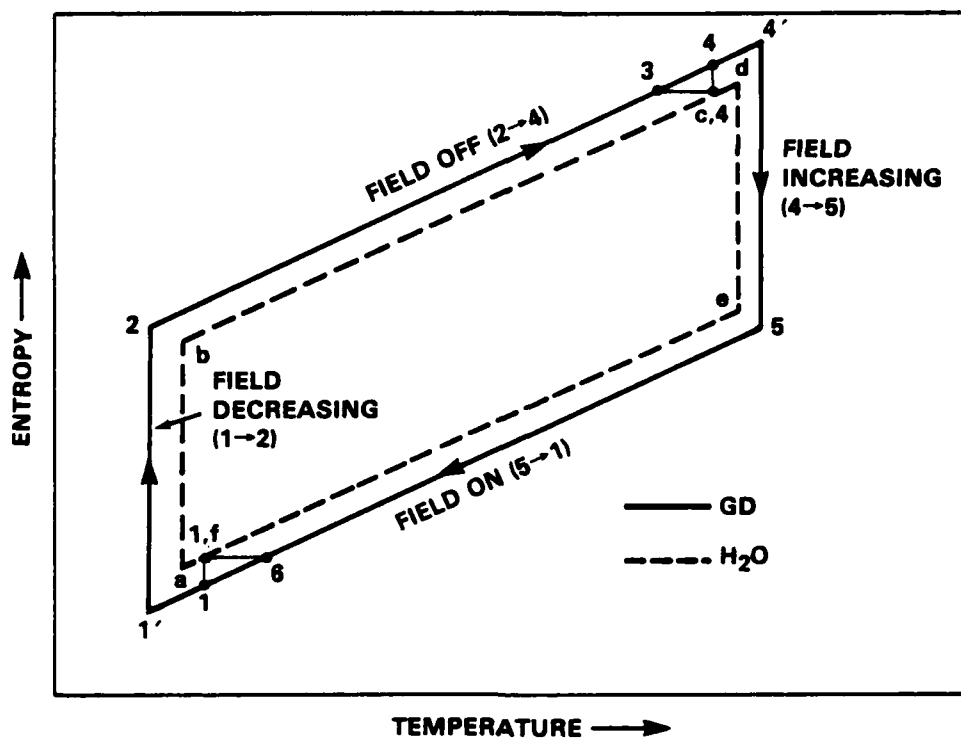Figure 1: Magnetic Ericsson Cycle Heat Pump Schematic

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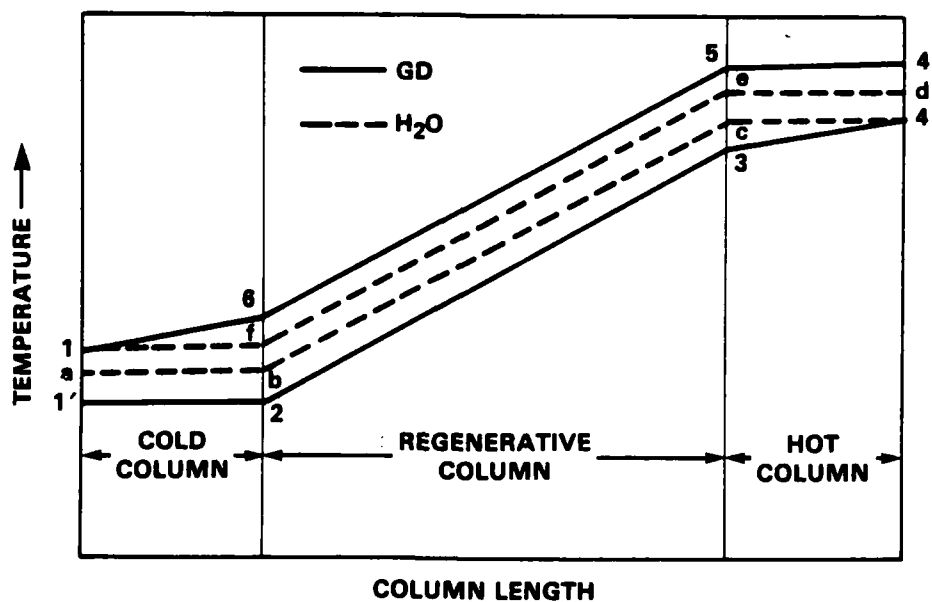Figure 2: Magnetic Ericsson Cycle Entropy-Temperature Diagram



Figure 3: Fluid Column and Gadolinium Temperature Profile

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at the bottom to interface with the refrigeration load. Centrally located in the column is the stationary Gadolinium surrounded by its magnet. The fluid column is composed of essentially three sections. The cold column, the regenerative column, and the hot column as shown in figure 1 and 3.

The Ericsson cycle ideally consists of two isothermal processes and two constant pressure processes. Referring to figure 1 to 3, the cycle operation is as described below:

1-2: Ideally, this is an isothermal process during which the magnetic field is decreased from some maximum value to some minimum value. The demagnization produce a cooling effect in the Gadolinium plates and heat is transferred from the water flowing thru the Gadolinium plates to maintain the plates at constant temperature while chilling the water. Because temperature differences are required between the water and the plates for heat transfer to actually occur, isothermal operation is possible from state points 1' to 2 only with 1 to 1' representing a loss due to the finite temperature differences required for heat transfer.

2-3: This is a regenerative process during which heat is transferred from the water column to heat the plates. Because entropy change in the Gadolinium at low or no field shows a definite peak centered on the Curie temperature, an appropriate variation of the applied magnetic field during this regeneration process is necessary so that the entropy change from statepoints 2 to 3 can be balanced with an equal but opposite entropy change during regenerative process 5 - 6.

3-4: The hot water column passes through the Gadolinium plates and acts as a continuation of the regeneration process 2 - 3. Heat is transferred from the fluid to the plates.

4-5: Ideally this is an isothermal process during which the magnetic field is increased to its maximum value. The magnization of the Gadolinium produces a heating effect and this heat is transferred to

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the water flowing through the plates to maintain the plate at constant temperature. As in process 1 - 2, temperature differences are required between the water and the plates for heat transfer to actually occur. Thus isothermal operation is only possible from state points 4' to 5 with 4 to 4' representing a loss due to the establishment of the finite temperature differences.

5-6: This is a regenerative process during which heat is transferred from the Gadolinium plates to the water flowing through them. The magnetic field is at its maximum and held constant while the plates are cooled in the regeration.

6-1: The cold column passes through the Gadolinium plates and acts as a continuation of the regeneration of process 5 - 6. Heat is transferred from the plates to the fluid.

The fluid is pumped toward the cold end heat exchanger for cycle points 1-2-3-4. Then the direction of pumping is reversed toward the hot end heat exchanger for cycle points 4-5-6-1. When the cold column reaches the cold end heat exchanger or the hot column reaches the hot end exchanger, external loops exchange the cooling load and the heat of magnetization, respectively.

## CYCLE DETAILS

To simplify the analysis it was assumed that the Gadolinium geometry would be parallel flat plates as shown in figure 4. Water flowing between the plates either adds heat to or removes heat from the plates. Since the plates act as a restriction to the flow in the assumed constant cross-sectional area fluid column channel, the water velocity between the plates, $V_p$, is greater than the fluid column velocity, $V_{CL}$, and is given by,

$$V_p = \frac{t_p + S_p}{S_p} * V_{CL} \tag{1}$$

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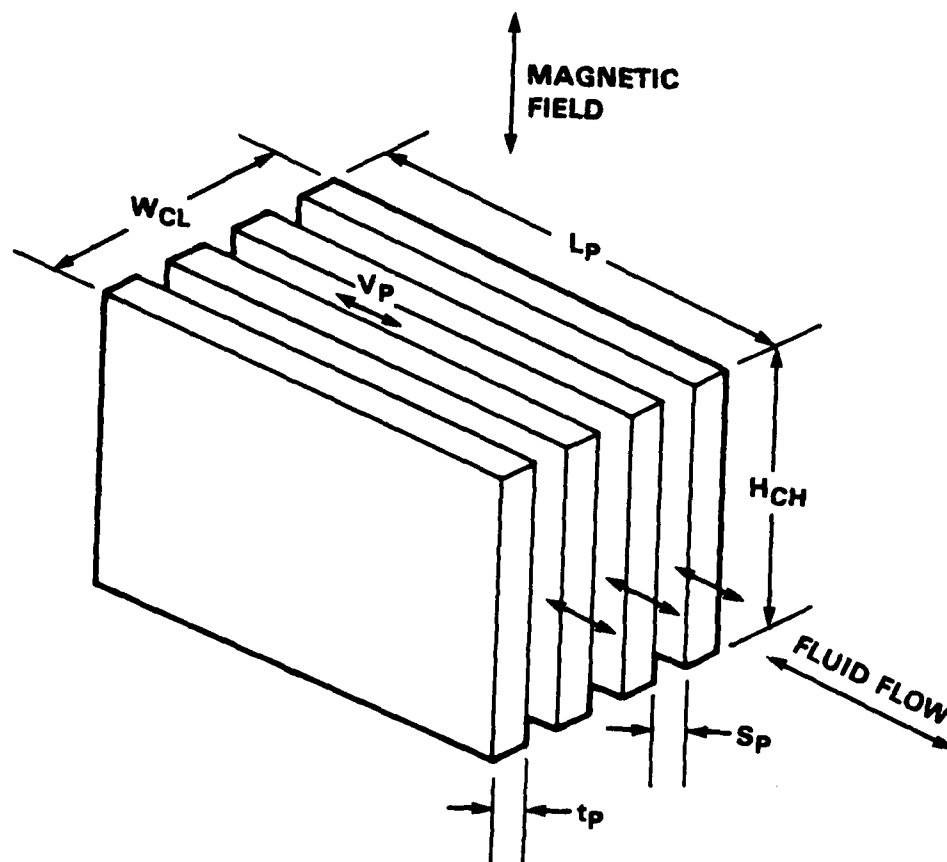
W<sub>CL</sub> = COLUMN WIDTH
L<sub>P</sub>  = PLATE LENGTH
H<sub>CH</sub> = CHANNEL HEIGHT
t<sub>P</sub>  = PLATE THICKNESS
S<sub>P</sub>  = PLATE SPACING
V<sub>P</sub>  = PLATE CHANNEL VELOCITY

Figure 4:  Gadolinium Plate Geometry

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Velocity through the plates is important since it helps determine whether the water side heat transfer coefficient is based on a laminar or turbulent heat transfer correlation. Laminar flow heat-transfer coefficients were based on average conditions from tabulated data in reference 4 for flat ducts. Turbulent heat transfer coefficients were obtained using the following correlation for turbulent flow[5]:

$$h_D = 0.023 \frac{K_W}{D_H} Re^{0.8} Pr^{0.33} \tag{2}$$

The overall transmittance between the Gadolinium plates and the water was calculated by

$$U = \cfrac{1}{\cfrac{1}{h_D} + \cfrac{t_p}{24 * k_{GD}}} \tag{3}$$

Table 1 presents physical properties of Gadolinium that were used in the calculations in this report. For all calculations in this report average conditions are used. Thermodynamic properties of Gadolinium in the presence of a magnetic field were obtained from reference 6 and are plotted in figure 5 as a function of entropy versus temperature for magnetic fields from 0 to 7 Tesla. This data was tabulated for use in the computer program (See Appendix A) that was written to perform the cycle calculations. Figure 5 shows the nonlinearity of the entropy change at low field strength around the Curie temperature ( 293 °K). This nonlinearity affects the cycle regeneration processes and necessitates the variation of the field strength discussed earlier in order to balance the entropy change during the regeneration process between the low field strength state and high field strength state.
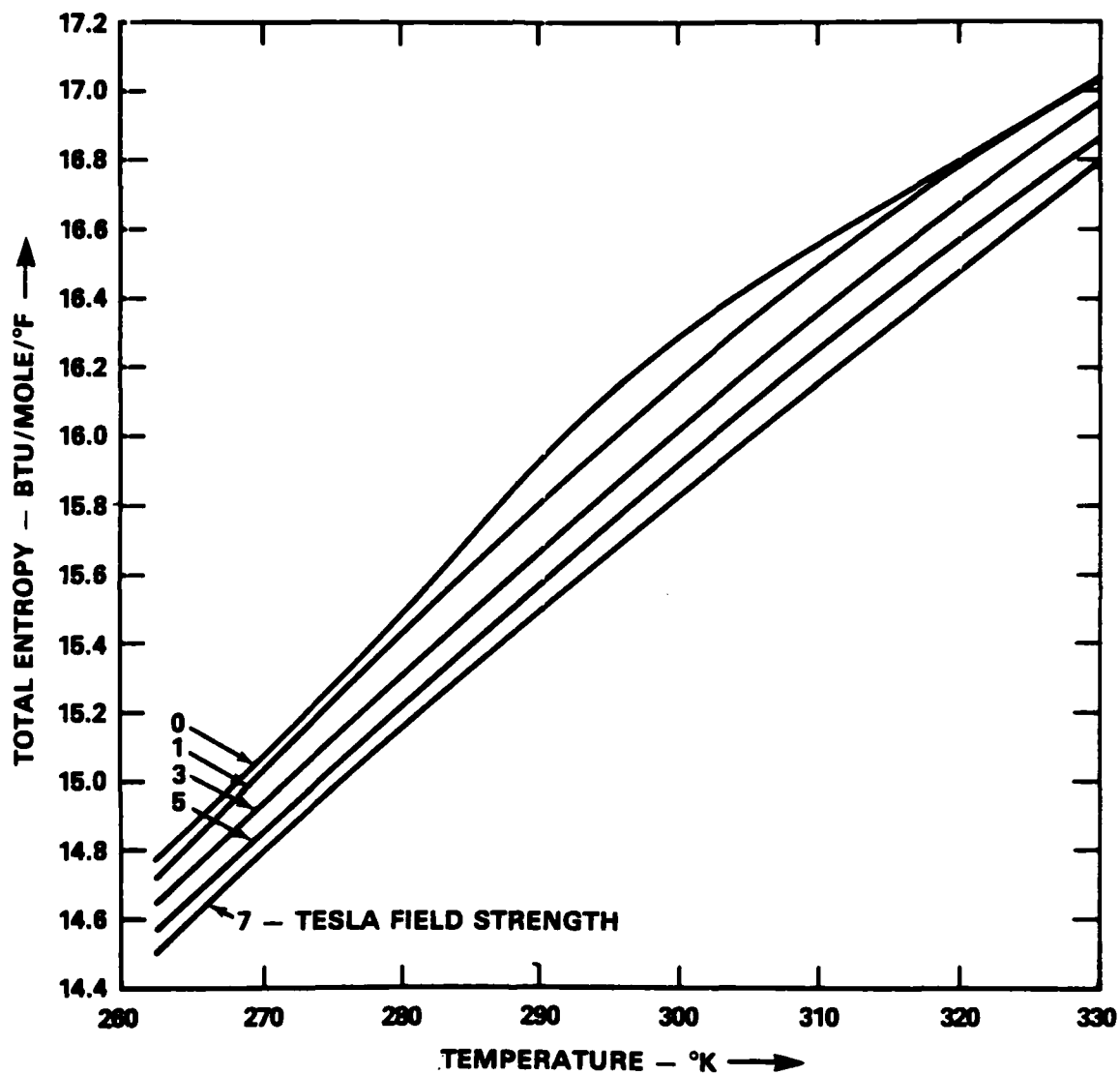
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Figure 5: Total Entropy of Gadolinium As A Function of Temperature And Field Strength

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Table 1 - Physical Properties of Gadolinuim at 68°F (20°C)

Molecular Weight ($M_{GD}$): 157.25 lbm/mole (346.74 Kg/mole)
Density ($\rho_{GD}$): 490.7 lbm/ft$^3$ (7.86 gm/cm$^3$)
Speicifc Heat ($C_{V_{GD}}$) 0.071 BTU/lbm °F (.71 CAL/gm °C)
Thermal Conductivity ($k_{GD}$): 5.8 BTU/Hr ft °F (.1 Watt/cm °C)

In evaluating conditions around the cycle, one of the losses that affects performance is frictional flow losses between the plates. It was assumed that frictional losses were dissipated as heat which heated the working fluid. The amount of heat generated by friction is found by

$$Q_F = 4.6263 \ H_1 \quad _W A_S \cdot V_P \cdot t \cdot N_P \tag{4}$$

where $\quad H_1$ = Head Loss $= \dfrac{V_P^2}{2} \cdot \dfrac{L_P}{D_H} \cdot \dfrac{f}{g}$ $\tag{5a}$

and $\quad A_S$ = Plate Surface Area $= 2 \cdot L_P \cdot H_{CH}$ $\tag{5b}$

In the laminar flow region, friction factor can be found by assuming the plate geometries satisfy conditions for two-dimensional, steady flow between infinite parallel plates. Then solving the Navier-Stokes equation to obtain heat loss and equating to equation 5 yields

$$f = \dfrac{96}{Re * (1 + \dfrac{S_P}{H_{CH}})^2} \tag{6}$$

In the turbulent flow region, friction factor between the plates was assumed to be comparable to the friction factor for a smooth pipe in turbulent flow and was estimated from the following curve fit of the smooth pipe friction factor curve,

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$$f = \frac{0.184}{Re^{0.2}} \qquad\qquad (7)$$

No friction factor was determined in the transitional flow region and the flow was assumed to go from laminar to turbulent at a Reynolds number (Re) of 3000.

Mixing losses can occur in the fluid column and at the plate inlets and exits. Ideally, a horizontal cut through the fluid column would show the entire layer of fluid to be at uniform temperature and velocity. However, moving the fluid column up and down can result in velocity profile being established across the column due to the presence of the stationary walls. Developing turbulent flow has a nonuniform velocity profile which essentially becomes uniform when fully developed, except close to the walls. Laminar flow has a parabolic velocity profile when fully developed. The velocity profile causes fluid of one temperature to be introduced into a region of fluid at another temperature causing mixing losses due to the temperature differential. In the analysis, mixing losses were considered to cause a temperature differential across the hot and cold column ( $\Delta T_{mix}$ ).

Other loses can occur in the system but were not specifically considered. Other losses can include:

o  Inlet and exit losses at the Gadolinium plates
o  Friction between the water column and the container walls
o  Pumping power to overcome losses

Although these losses are not directly considered, the first two could be considered to be grouped in with the mixing loss. Pumping power will reduce the system overall performance but would not change the cycle state points provided the losses requiring pumping power are considered. Any losses over and above those considered in this analysis would further reduce system overall performance. The loses considered in this analysis should give an overall indication of the system performance.

## CYCLE ANALYSIS

In the analysis, a cold end, external sink temperature, $T_{cold}$, that is to be maintained by the heat pump is specified, as is the hot end, external sink

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temperature, $T_{hot}$. In order for heat transfer to occur between the heat pump
fluid and the external heat exchanger fluids, some heat exchanger temperature
difference, $T_{HE}$, is required and is specified in this analysis and assumed to
be the same for both the hot and cold end heat exchangers. Thus cold and hot
columns must be operated at some lower and higher temperatures, respectively,
tu effect the heat transfer to the external heat exchangers and are define by,

$$T(C) = T_{cold} - \Delta T_{HE} \tag{8}$$

$$T(H) = T_{hot} + \Delta T_{HE} \tag{9}$$

T(C) is the temperature of the cold fluid column between state points f and 1
and T(H) is the hot fluid column temperature between state points c and 4 for
the ideal cycle of figures 2 and 3. However, due to friction, mixing, and
heat transfer temperature differences, process f   1 and process c   4 are not
isothermal and T(C) and T(H) were specified to be the average cold and hot
column temperatures between state points f and 1 & c and 4, respectively.

Figure 6 shows the actual cycle considering losses and the heat transfer
tempeature differences. Figure 7 shows the actual fluid column and Gadolinium
temperature profiles.

The system was evaluated on a per pound of Gadolinium basis. Cooling
load was evaluated by,

$$Q_{1-2} = (\frac{T_1 + T_2}{2} + 459.7) \cdot \frac{(S_2 - S_1)}{M_{GD}} \cdot (60 \, N_{RPM}) - Q_{F_{1-2}} \tag{10}$$

Where $S_2$ and $S_1$ are the total entropy of Gadolinium at $T_2$ and $T_1$
respectively. During this process (1→2) the magnet field is decreased from
a maximum value at $T_1$ to near zero at $T_2$ and values of entropy are obtained
from figure 5. Since this is a cooling process, fricton generated by the
water flowing between the plates reduces the cooling load by $Q_{F_{1-2}}$ which is
evaluated using equation 4. Not all values for equation 10 and subsequent
equations are known at the start but initial assumptions were made and
subsequent iterations in the computer program to evaluate the cycle provided
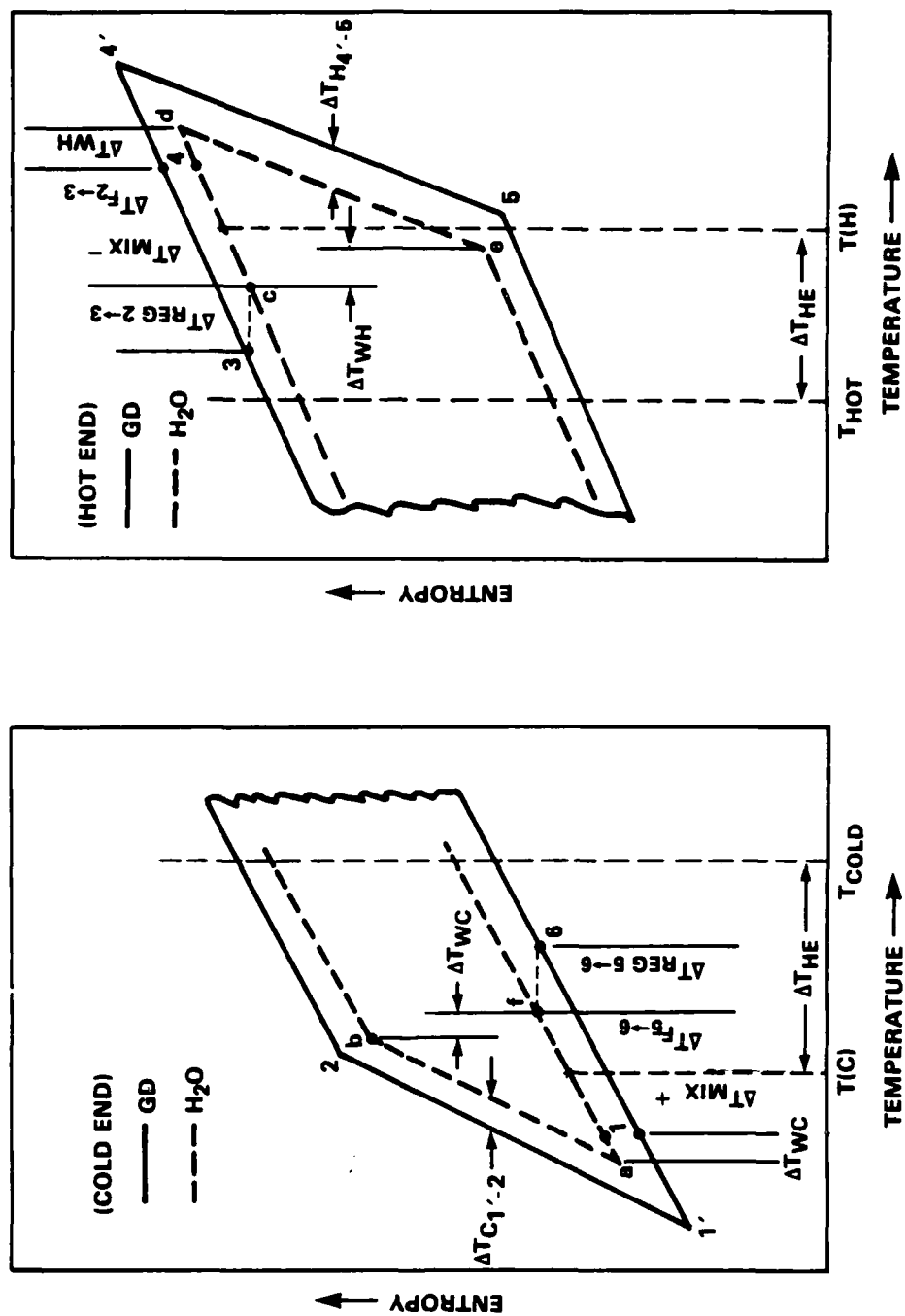rapid convergence to the correct values.

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Figure 6: Actual Cycle Considering Losses and Heat Transfer Temperature Differences
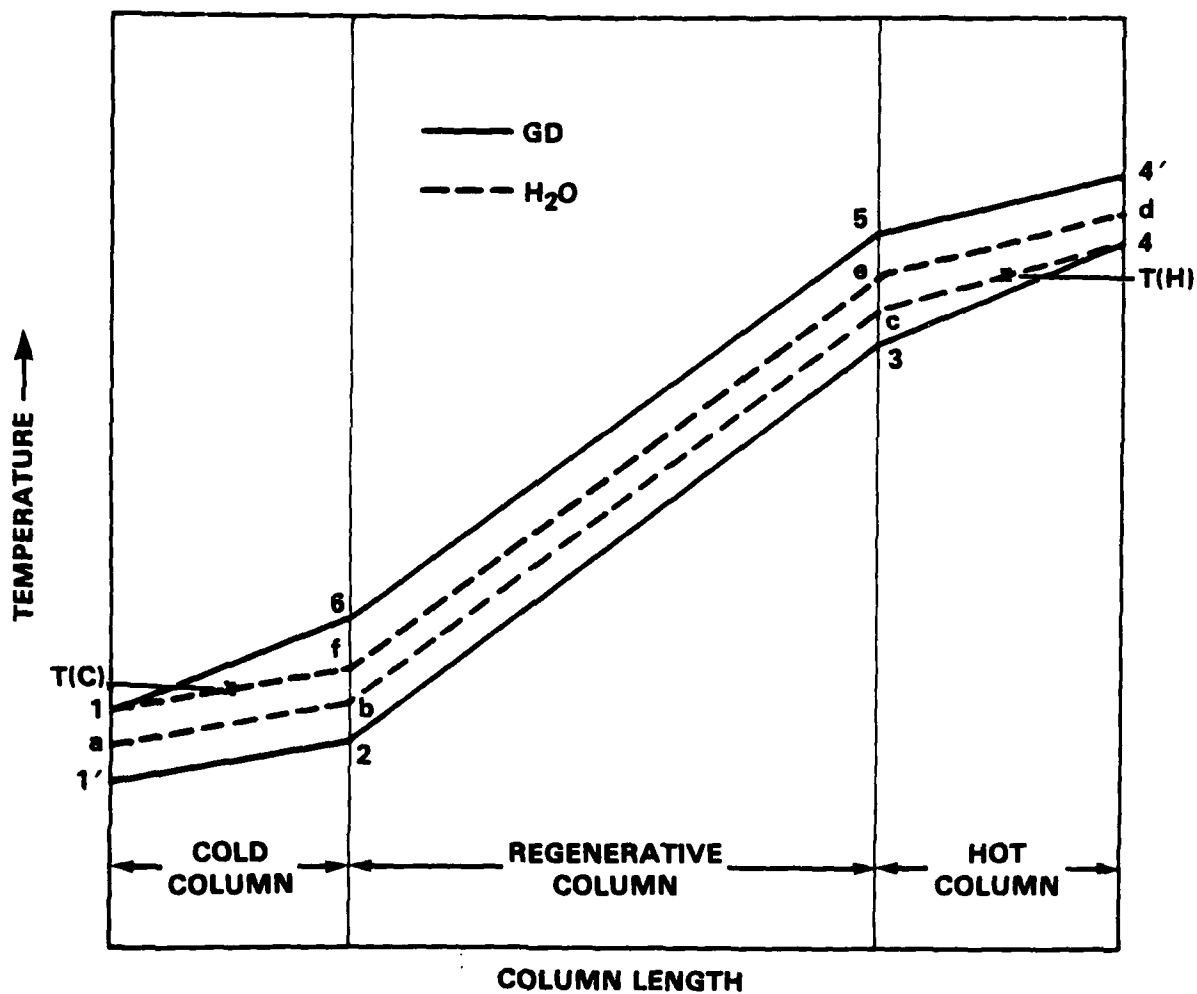
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Figure 7: Actual Column and Gadolinium Temperature Profiles

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The entropy change at the hot and cold end will be equal in magnitude but opposite in sign for the ideal isothermal case. Thus for the Gadolinium,

$$(S_{2I} - S_{1I})\, T(C) = (S_{4I} - S_{5I})\, T(H) \tag{11}$$

However due to losses and temperature difference, the entropy change at the hot and cold end are no longer necessarily equal in magnitude. It was assumed that the temperature - entropy line from state points 2 to 4 was parallel to the maximum, constant field temperature - entropy line from state points 5 to 1 in order to balance the loads on the regenerative portion of the cycle. This requires some magnetic field variation from state points 2 to 4 and assuming the lines are linear, entropy at point 4 for Gadolinium is found by

$$S_4 = S_{4I} + (S_{4I} - S_{2I}) * \frac{T_4 - T(H)}{T(H) - T(C)} \tag{12}$$

Thus the heat load to the hot column is found from,

$$Q_{4-5} = (\frac{T_4 + T_5}{2} + 459.7) \cdot \frac{(S_4 - S_5)}{M_{GD}} \cdot (60\ N_{RPM}) + Q_{F_{4-5}} \tag{13}$$

with $S_5$ being at the maximum field strength and $Q_{F\ 4-5}$ being heat added due to friction between the plates, which now adds to the hot end heat rejection load. The regenerative heat loads are found from

$$Q_{2-3} = 60\ c_{v_{GD}}\ N_{RPM}\ (T_3 - T_2) \tag{14}$$

and

$$Q_{5-6} = 60\ c_{v_{GD}}\ N_{RPM}\ (T_5 - T_6) \tag{15}$$

Likewise the cold and hot columns pass through the Gadolinium prior to the change in magnetic field and exchange heat with each other where,

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$$Q_{6-1} = 60 \, c_{v_{GD}} \, N_{RPM} \, (T_6 - T_1) \tag{16}$$

and

$$Q_{3-4} = 60 \, c_{v_{GD}} \, N_{RPM} \, (T_4 - T_3) \tag{17}$$

Heat transfer to and from the water is related to the mass flow rate of water and the channel geometry. Based on a one pound weight of Gadolinium, the number of plates, $N_p$, in the system is,

$$N_p = \frac{1728}{H_{CH} t_p L_p \, \rho_{GD}} \tag{18}$$

Thus the column width, $W_{CH}$, is

$$W_{CH} = (t_p + S_p) \, N_p \tag{19}$$

The free flow area between the plates, $A_{FF}$, is

$$A_{FF} = \frac{H_{CH} S_p N_p}{144} \tag{20}$$

Thus the mass flow rate of water, $\dot{m}_W$, through the plates is,

$$\dot{m}_W = 3600 \, \rho_W A_{FF} \, V_p \tag{21}$$

In calculating temperature changes in the water, an estimate of the time for each of the three column sections (cold, regenerative, and hot) to pass through the plates was required. the column passes through the plates twice to complete a cycle. The overall tansmittance and temperature differential in

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the regenerative column and the hot and cold columns during the change in magnetic field, were close and thus the ratio of the heat transfers in these areas, provided a close estimate of time for each column section to pass through the plate.

$$Q_{TOT} = Q_{2-3} + Q_{5-6} + 2* (Q_{1-2} + Q_{F_{1-2}} + Q_{4-5} - Q_{F_{4-5}}) \quad (22)$$

then for the cold column

$$t_{cold} = \frac{Q_{1-2} + Q_{F_{1-2}}}{Q_{TOT}} \ 60 \ N_{RPM} \quad (23)$$

for the regenerative column

$$t_{REG} = \frac{Q_{2-3} + Q_{5-6}}{2* \ Q_{TOT}} \ 60 \ N_{RPM} \quad (24)$$

and for the hot column

$$t_{hot} = \frac{Q_{4-5} - Q_{F_{4-5}}}{Q_{TOT}} \ 60 \ N_{RPM} \quad (25)$$

Determining the temperatures at the state points is accomplished based on a few assumptions. It is assumed at state points 1 and 4, that the temperature of the fluid and Gadolinium are the same prior to the changing of the magnetic field (see figs 6 & 7). Also the mean temperature of the cold and hot fluid columns prior to changing the magnetic field was equal to the values defined by equations (8) and (9) respectively (see figs. 6 & 7). Thus, for the cold column

$$T(C) = \frac{T_f + T_1}{2} \quad (26a)$$

and for the hot column

$$T(H) = \frac{T_4 + T_C}{2} \tag{26b}$$

If no losses or temperature difference were present, lines f-1 and c-4 would be isothermal as previously discussed. However the two temperatures are not equal but are different by

$$T_f - T_1 = \Delta T_{Mix} + \Delta T_{F_{5-6}} \tag{27a}$$

for the cold end and for the hot end

$$T_4 - T_c = \Delta T_{Mix} - \Delta T_{F_{2-3}} \tag{27b}$$

The mixing temperature change, $T_{mix}$, was difficult to evaluate and was assumed to be some fixed temperature difference across the hot and cold column. The fluid flowing through the plates generates friction which in effect adds heat to the fluid. The temperature increase of the fluid caused by friction is obtained in the different column sections by use of ether laminar or turbulent equations (4 to 7) and

$$\Delta T_{F_{6-1}} = \frac{Q_{F_{6-1}}}{\dot{m}_W \, c_{p_W} \, t_{cold}} \tag{28a}$$

$$\Delta T_{F_{5-6}} = \frac{Q_{F_{5-6}}}{\dot{m}_W \, c_{p_W} \, t_{REG}} \tag{28b}$$

$$\Delta T_{F_{4-5}} = \frac{Q_{F_{4-5}}}{\dot{m}_W \, c_{p_W} \, t_{hot}} \tag{28c}$$

$$\Delta T_{F_{3-4}} = \frac{Q_{F_{3-4}}}{\dot{m}_W \, c_{p_W} \, t_{hot}} \tag{28d}$$

$$\Delta T_{F_{2-3}} = \frac{Q_{F_{2-3}}}{\dot{m}_W \, c_{p_W} \, t_{REG}} \tag{28e}$$

$$\Delta T_{F_{1-2}} = \frac{Q_{F_{1-2}}}{\dot{m}_W \, c_{p_W} \, t_{REG}} \qquad (28f)$$

In order for the Gadolinium and the fluid to achieve the same temperature at points 1 and 4 heat must be transfered between the two.  Thus the heat gain or loss of the Gadolinium was reflectd in the fluid as a heat loss or gain respectively.  The change in temperature of the fluid in exchanging heat with the Gadolinium was at the cold end,

$$\Delta T_{C_{W-GD}} = \frac{60 \, N_{RPM} \, c_{v_{GD}} \, (T_6 - T_1)}{\dot{m}_W \, c_{p_W} \, t_{cold}} \qquad (29a)$$

and at the hot end

$$\Delta T_{H_{W-GD}} = \frac{60 \, N_{RPM} \, c_{v_{GD}} \, (T_4 - T_3)}{\dot{m}_W \, c_{p_W} \, t_{hot}} \qquad (29b)$$

The temperature changes are shown in figure 8 which will be discussed shortly.

Additional information is still needed to obtain $T_6$ and $T_3$ for equation 29.  This involves finding the average temperature difference in the regenerator between the plates and the fluid.  Thus,

$$\Delta T_{REG_{5-6}} = \frac{Q_{5-6}}{U_{5-6} \, A_S \, t_{REG}} \qquad (30a)$$

and

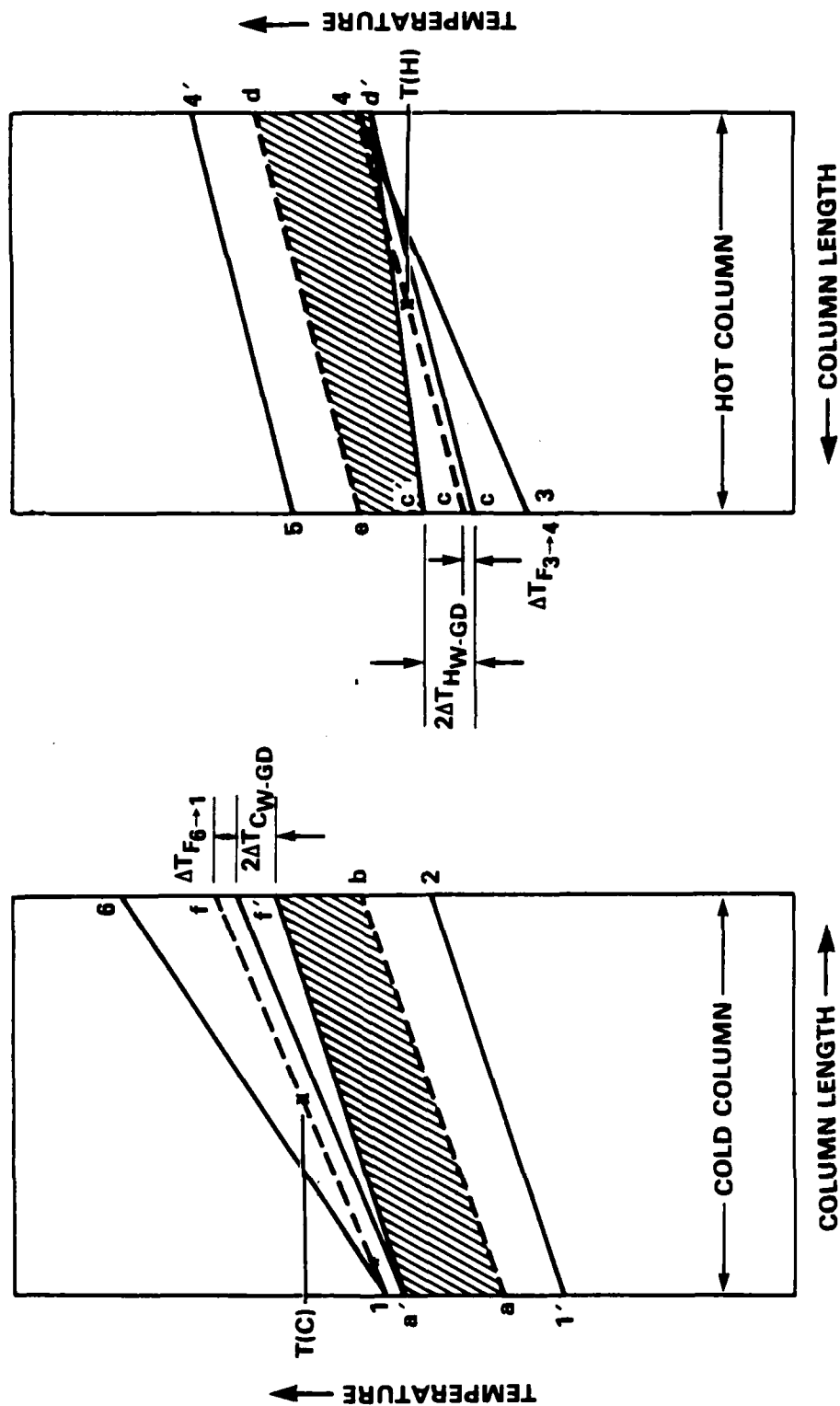$$\Delta T_{REG_{2-3}} = \frac{Q_{2-3}}{U_{2-3} \, A_S \, t_{REG}} \qquad (30b)$$

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Figure 8: Temperature Changes in Cold and Hot Column

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with the assumption that,

$$T_6 - T_f = \Delta T_{REG_{5-6}} \tag{31a}$$

and

$$T_c - T_3 = \Delta T_{REG_{2-3}} \tag{31b}$$

The average temperature change of the cold and hot fluid column due to the change in magnetic field is found by

$$\Delta T_{W_c} = \frac{Q_{1-2}}{\dot{m}_W c_{P_W} t_{cold}} \tag{32a}$$

and for the hot end

$$\Delta T_{W_H} = \frac{Q_{4-5}}{\dot{m}_W c_{P_W} t_{hot}} \tag{32b}$$

The temperature difference between the Gadolinium and the water to effect the heat transfer in the cold and hot column during field change is found by

$$\Delta T_{c_{1'-2}} = \frac{Q_{1-2} + Q_{F_{1-2}}}{U_{1-2} A_S t_{cold}} \tag{33a}$$

and for the hot column

$$\Delta T_{H_{4'-5}} = \frac{Q_{4-5} - Q_{F_{4-5}}}{U_{4-5} A_S t_{hot}} \tag{33b}$$

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All the above temperatures are illustrated in Figure 6.

All the heat loads and temperatures have been described up to this point. Repeated iteration with the equations presented led to a rapid solution of the cycle in the computer program that was written to evaluate the cycle (see Appendix A). All that remains is to predict performance. The cooling load that can be transferred at the cold end to the external load is less than the change of magnet energy in the Gadolinium due to friction and heat transfer. Likewise, friction and heat transfer between the Gadolinium and fluid make the heat rejected at the hot end differ from the magnetic energy change. Figure 8 shows in more detail the variation of Gadolinium and fluid temperature in the cold and hot column. Line 1f represents the temperature distribution in the cold column when the magnetic field starts to decrease at point 1. Line ab is the temperature distribution in the cold column fluid after the drop in magnetic field is completed and the fluid has been chilled by transferring heat to the Gadolinium. Thus area 1abf represents the amount that the fluid is chilled and is equal to the value obtained by equation (10). However only the area a'abf' can be used to chill the external loop. Part of the cooling load, area 1a'f'f must be kept in the loop to offset frictional heating of the fluid and heat transfer from the Gadolinium to the fluid during process 6-1. Line a'f' represents the temperature profile in the cold column after heat transfer is completed with the cold end external loop.

In the hot end, line c4 is the temperature distribution in the hot column at the start of the increase in magnetic field at point 4. Line de is the temperature distribution in the fluid after the magnetic field is brought to its maximum value and heat generated in the Gadolinium has been transferred to the fluid. Area ec4d is the heat generated in the Gadolinium from equation (13) that is transferred to the fluid. However area ec'd'd is the heat that will be removed from the fluid by the external loop and may be greater than or less than that of equation (13) depending on the magnitude of various temperature differences and changes. Line c'd' represents the temperature profile in the hot column after heat is removed by the hot end external loop.

In order to calculate the actual cooling output, $Q_{cold}$, to the external

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loop and the system coefficient of performance (COP) corrections must be made to heat flow calculated for the Gadolinium alone in equations (10). Based on figure 8, a correction for the cooling output was made where:

$$Q_{cold} = Q_{1-2} * F_{cold} \tag{34}$$

where

$$F_{cold} = 1 - \frac{\Delta T_{F_{6-1}} + \Delta T_{GD_{cold}}}{\Delta T_{W_c}} \tag{35}$$

for the hot end, the heat load to be removed by the external loop, $Q_{hot}$, was found by correcting the magnetic heat output of equation (13) by,

$$Q_{hot} = Q_{4-5} * F_{hot} \tag{36}$$

where

$$F_{hot} = 1 + \frac{\Delta T_{F_{3-4}} - \Delta T_{GD_{hot}}}{\Delta T_{W_h}} \tag{37}$$

Thus the heat load to the external hot end loop can be greater than the magnetic heat load of equation (13) if the frictional term is larger than the heat lost from the fluid to heating the Gadolinium.

In calculating COP, the work input to the loop is that obtained from equation (13) without the frictional term. This value does not include any losses in generating the magnetic field which would increase the actual work input to the system necessary to generate the amount of cooling given by equation (34). Thus

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$$COP = \left(\dfrac{Q_{cold}}{Q_{4-5} - Q_{F_{4-5}}}\right) \dfrac{1}{\eta_m} \quad -Q_{cold} \qquad (38)$$

where $\eta_m$ is the efficiency of generating and transmitting the magnetic field.

## CYCLE RESULTS

In the analysis, it was desired to maintain an external chilled water loop at 45°F (7.2°C) cold end with an external hot end sink temperature of 85°F (29.4°C). No loss effects were assumed in the magnetic cicuit at this time or in the external cooling loops. Thus only the performance of the Ericsson cycle was analyzed. The effects of the following parameters on the cycle performance were evaluated:

o Plate Spacing, $S_p$

o Channel Length, $L_p$

o Plate Thickness, $t_p$

o Mixing Temperature Change, $\Delta T_{mix}$

o External heat exchanger temperature differential, $\Delta T_{HE}$

The effects of channel height, $H_{CH}$, made no noticeable change in cycle performance for the thermal/hydraulic analysis performed and was not considered. However, channel height is important in considering the magnetic circuit since it represents the magnetic flow path length which affects magnet performance.

Figures 9 to 13 show the effects of varing the above parameter for a magnetic field strength of 7-Tesla while figures 14 to 18 show similar results for a 2-Tesla field strength. Figure 9 through 18 are plotted for cooling capacity to the external loop and coefficient of performance versus the Gadolinium plate channel velocity, $V_p$. The plate channel velocity is of importance since it determines whether laminar or turbulent flow and heat transfer models are used. The figures were computer plotted using a spline fit of the data points. Some irregularities in the curves occurred at higher velocities where the laminar to turbulent change occurs between the plates and the prediction models change.

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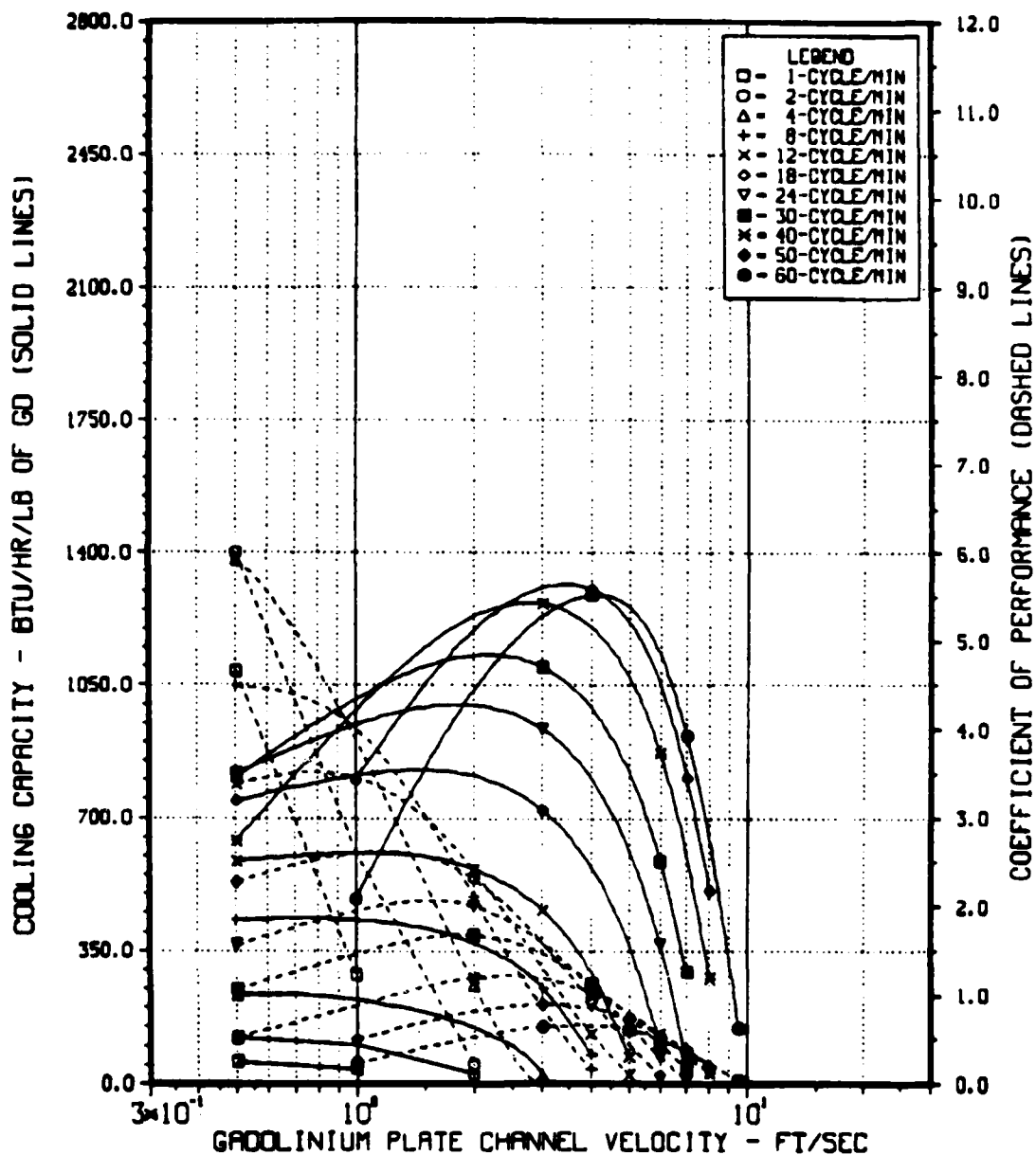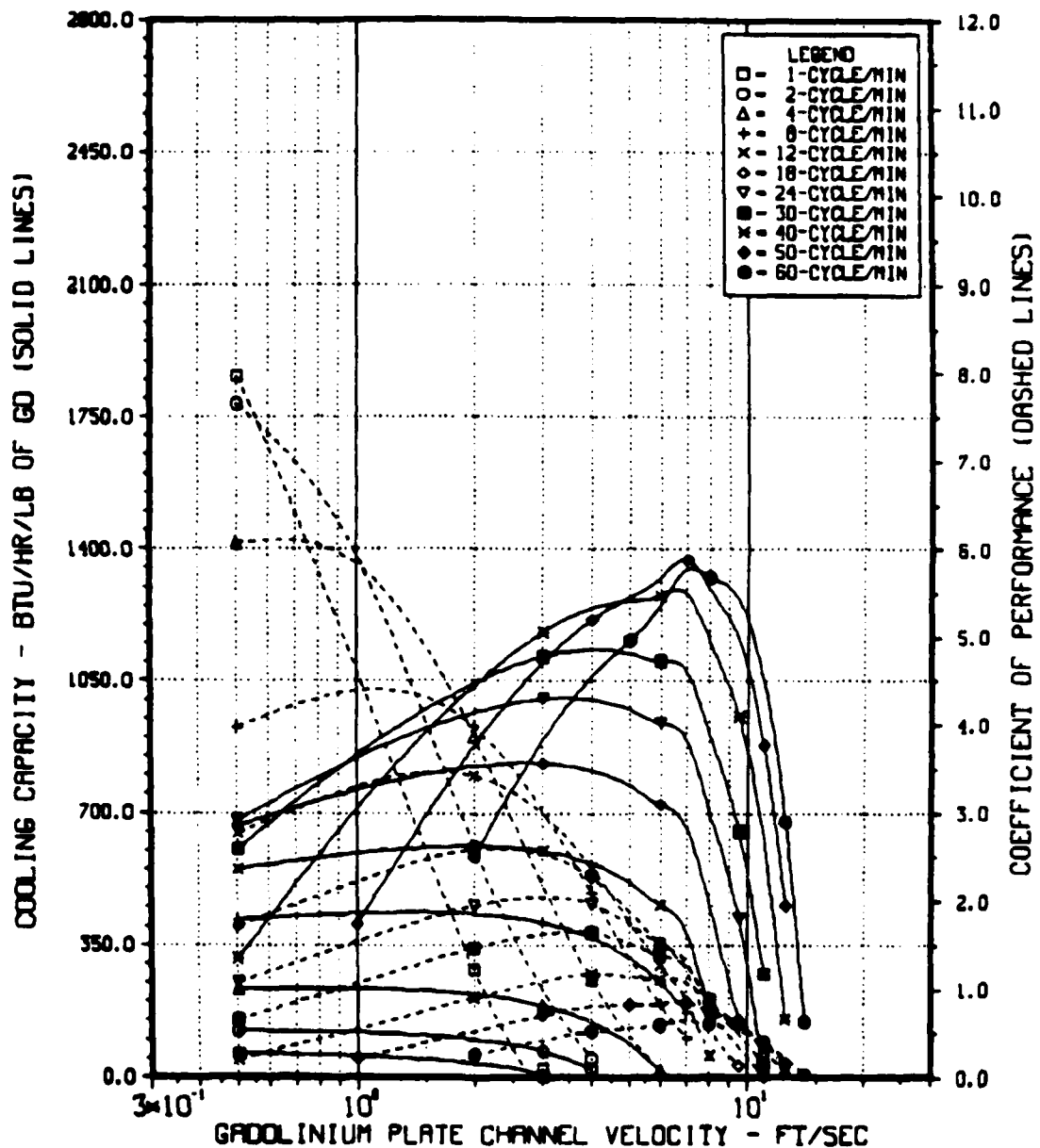MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

Figure 9a:   Effects of Gadolinium Plate Spacing on Performance at 7-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

CHANNEL HEIGHT, IN.- 2.00    PLATE SPACING, IN. - 0.040    TEMP.HOT END,F - 85.    MIXING TEMP,F- 0.0

CHANNEL LENGTH, IN.- 0.50    PLATE THICKNESS, IN.- 0.010    TEMP COLD END,F- 45.    H.E.DELTA-T,F- 0.0



Figure 9b:    Effects of Gadolinium Plate Spacing on Performance at 7-Tesla

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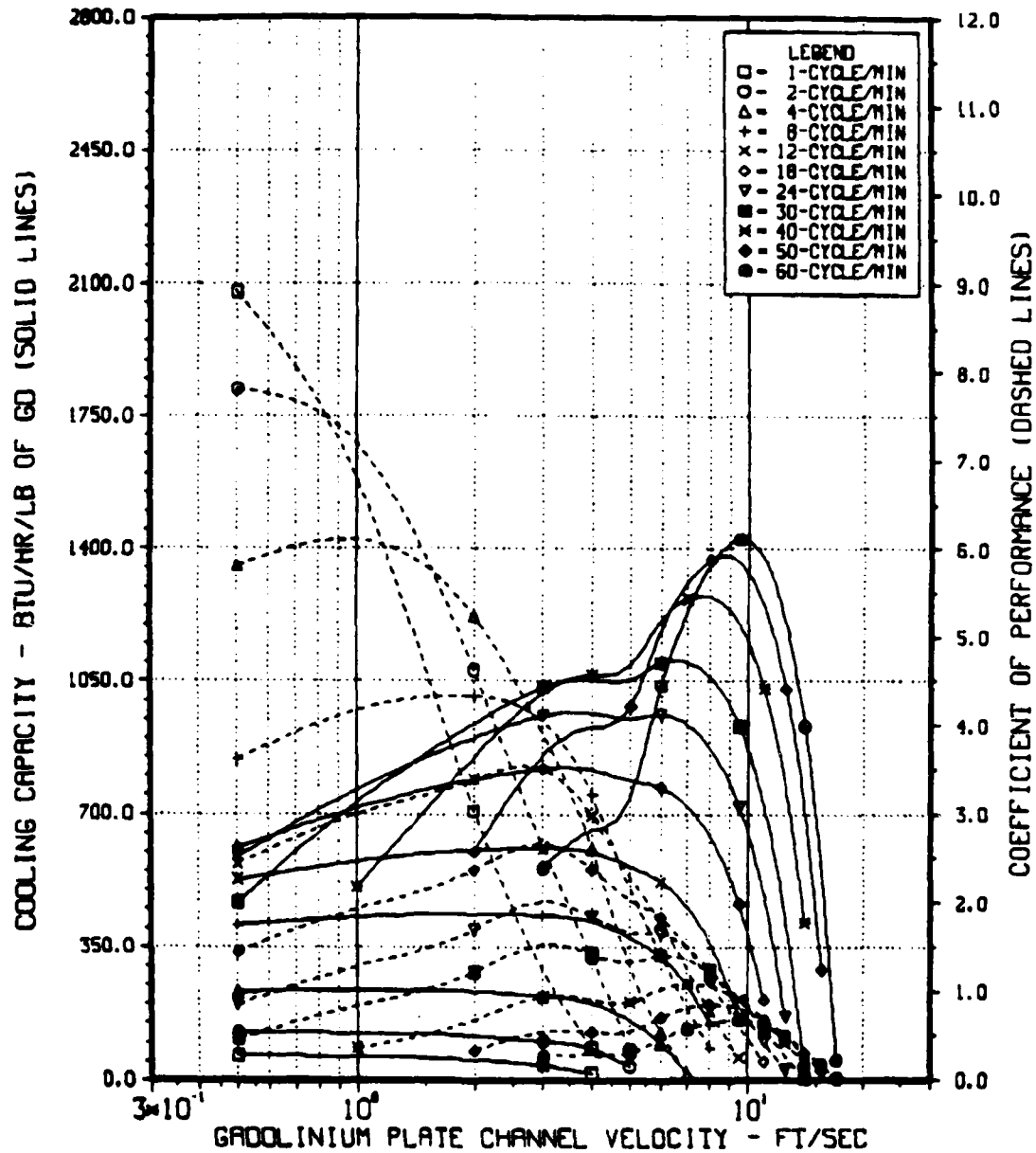MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

Figure 9c: Effects of Gadolinium Plate Spacing on Performance at 7-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

CHANNEL HEIGHT, IN.- 2.00    PLATE SPACING, IN. - 0.040    TEMP.HOT END,F - 85.    MIXING TEMP,F- 0.0

CHANNEL LENGTH, IN.- 0.25    PLATE THICKNESS, IN.- 0.010    TEMP COLD END,F- 45.    H.E.DELTA-T,F- 0.0



Figure 10a:   Effects of Gadolinium Channel Length on Performance at 7-Tesla

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# MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

CHANNEL HEIGHT, IN.- 2.00    PLATE SPACING, IN. - 0.040    TEMP.HOT END,F - 85.    MIXING TEMP,F- 0.0

CHANNEL LENGTH, IN.- 0.50    PLATE THICKNESS, IN.- 0.010    TEMP COLD END,F- 45.    H.E.DELTA-T,F- 0.0
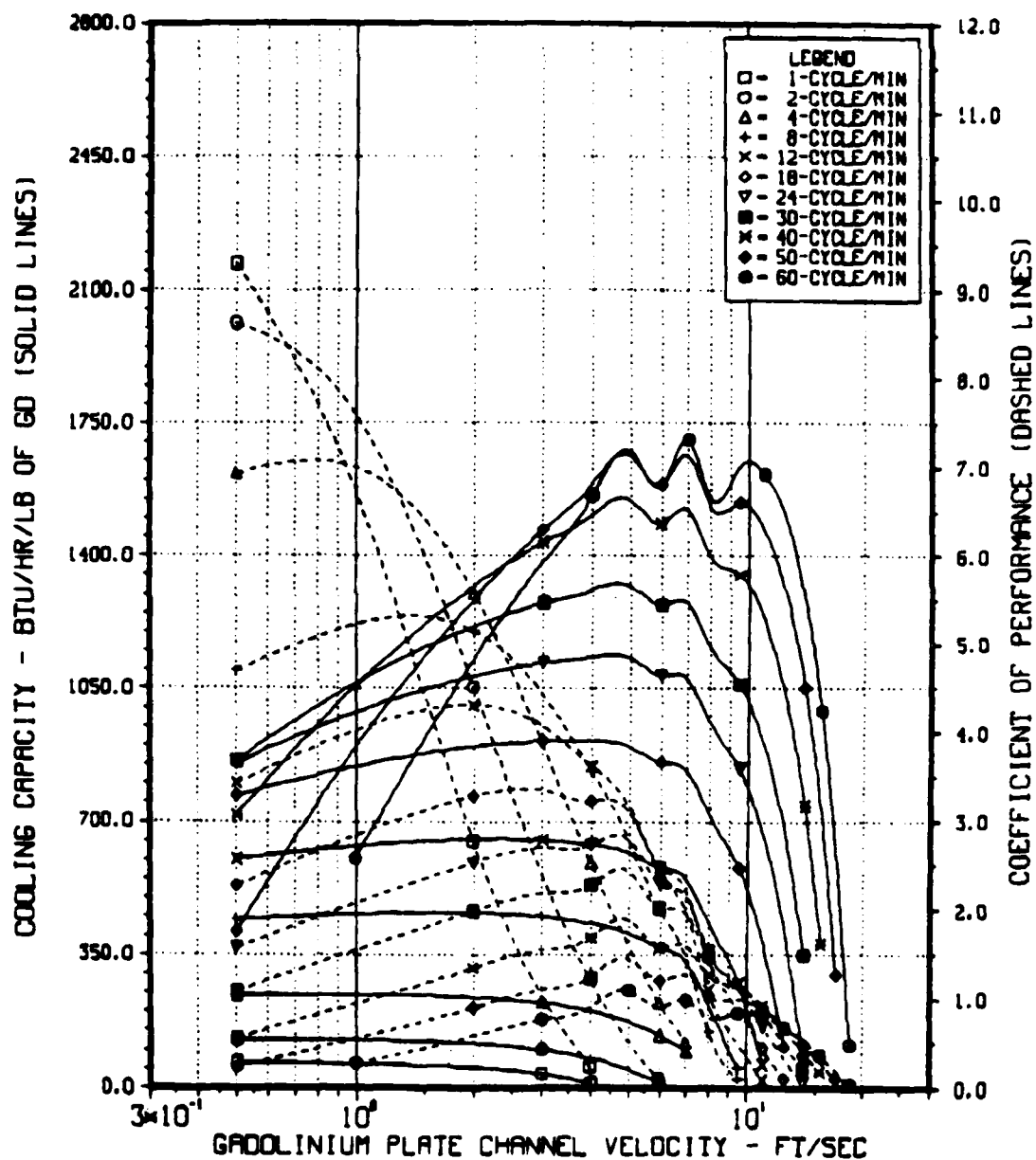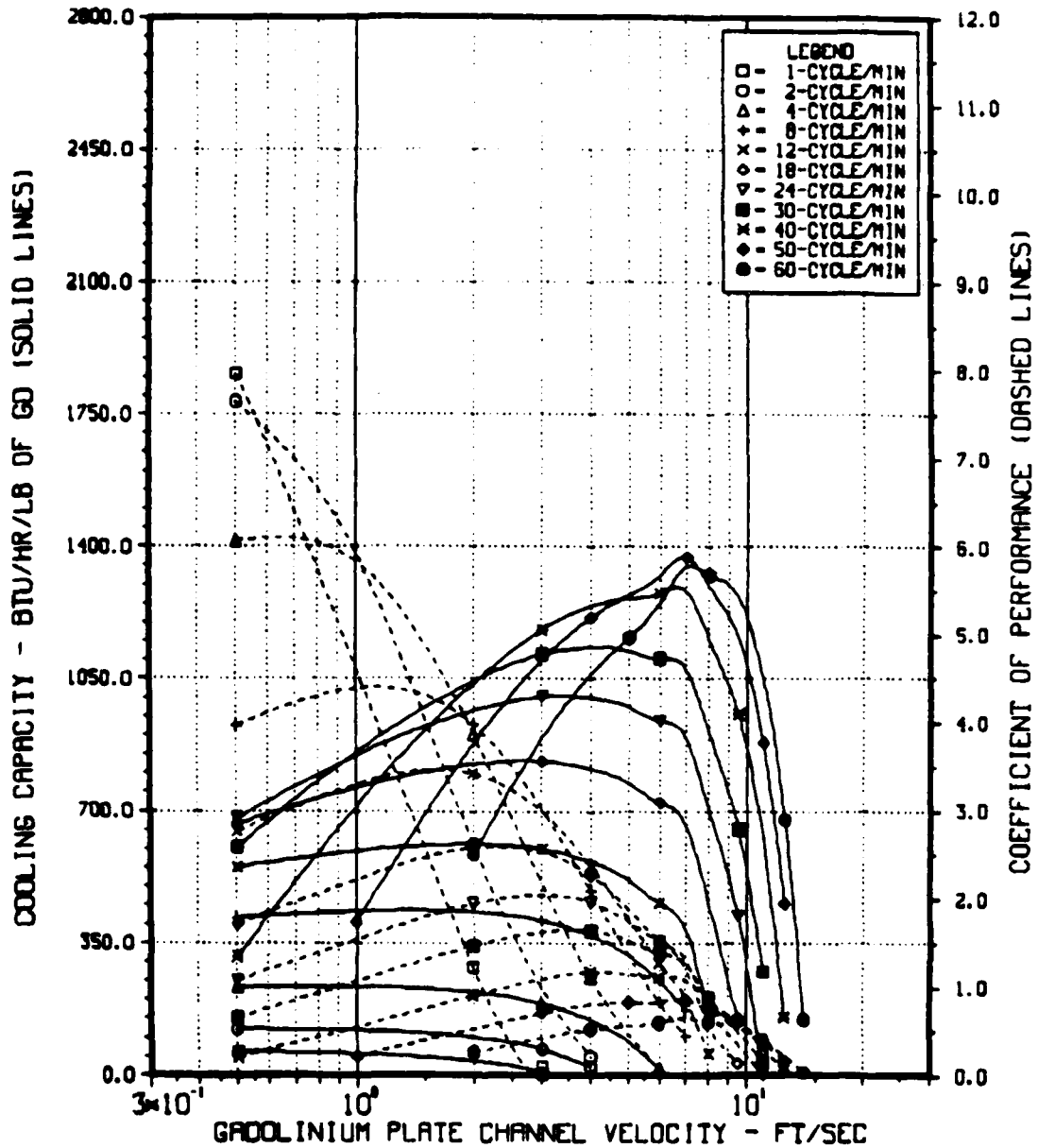


Figure 10b: Effects of Gadolinium Channel Length on Performance at 7-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

CHANNEL HEIGHT,IN.- 2.00  PLATE SPACING,IN. - 0.040  TEMP.HOT END,F - 85.  MIXING TEMP,F- 0.0

CHANNEL LENGTH,IN.- 1.00  PLATE THICKNESS,IN.- 0.010  TEMP COLD END,F- 45.  H.E.DELTA-T,F- 0.0

LEGEND
□ - 1-CYCLE/MIN
○ - 2-CYCLE/MIN
△ - 4-CYCLE/MIN
+ - 8-CYCLE/MIN
× - 12-CYCLE/MIN
◇ - 18-CYCLE/MIN
▽ - 24-CYCLE/MIN
■ - 30-CYCLE/MIN
✕ - 40-CYCLE/MIN
◆ - 50-CYCLE/MIN
● - 60-CYCLE/MIN

COOLING CAPACITY - BTU/HR/LB OF GD (SOLID LINES)

COEFFICIENT OF PERFORMANCE (DASHED LINES)

GADOLINIUM PLATE CHANNEL VELOCITY - FT/SEC

Figure 10c:  Effects of Gadolinium Channel Length on Performance at 7-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

CHANNEL HEIGHT, IN.- 2.00    PLATE SPACING, IN.  - 0.040    TEMP.HOT END,F - 85.    MIXING TEMP,F- 0.0

CHANNEL LENGTH, IN.- 2.00    PLATE THICKNESS, IN.- 0.010    TEMP COLD END,F- 45.    M.E.DELTA-T,F- 0.0
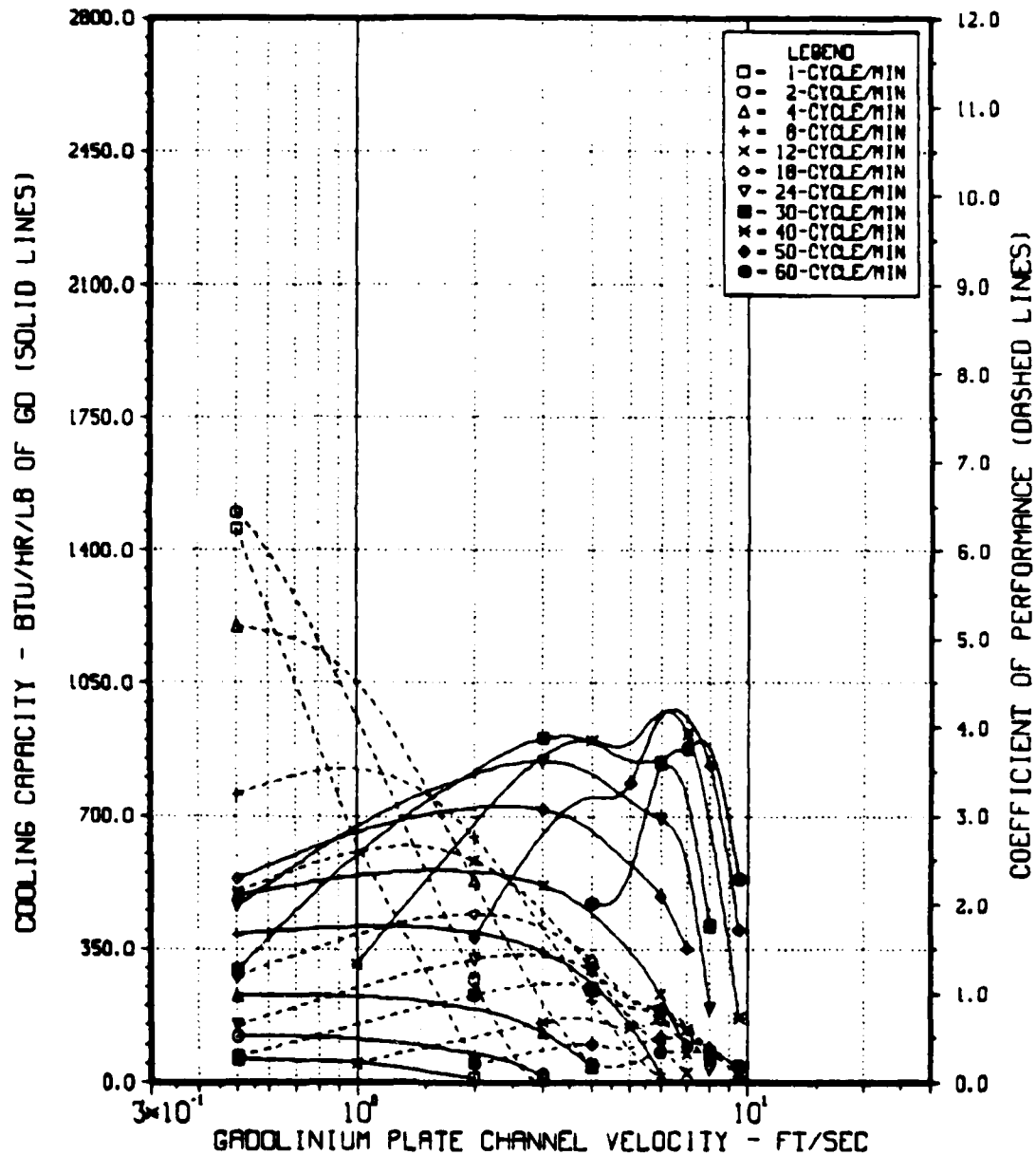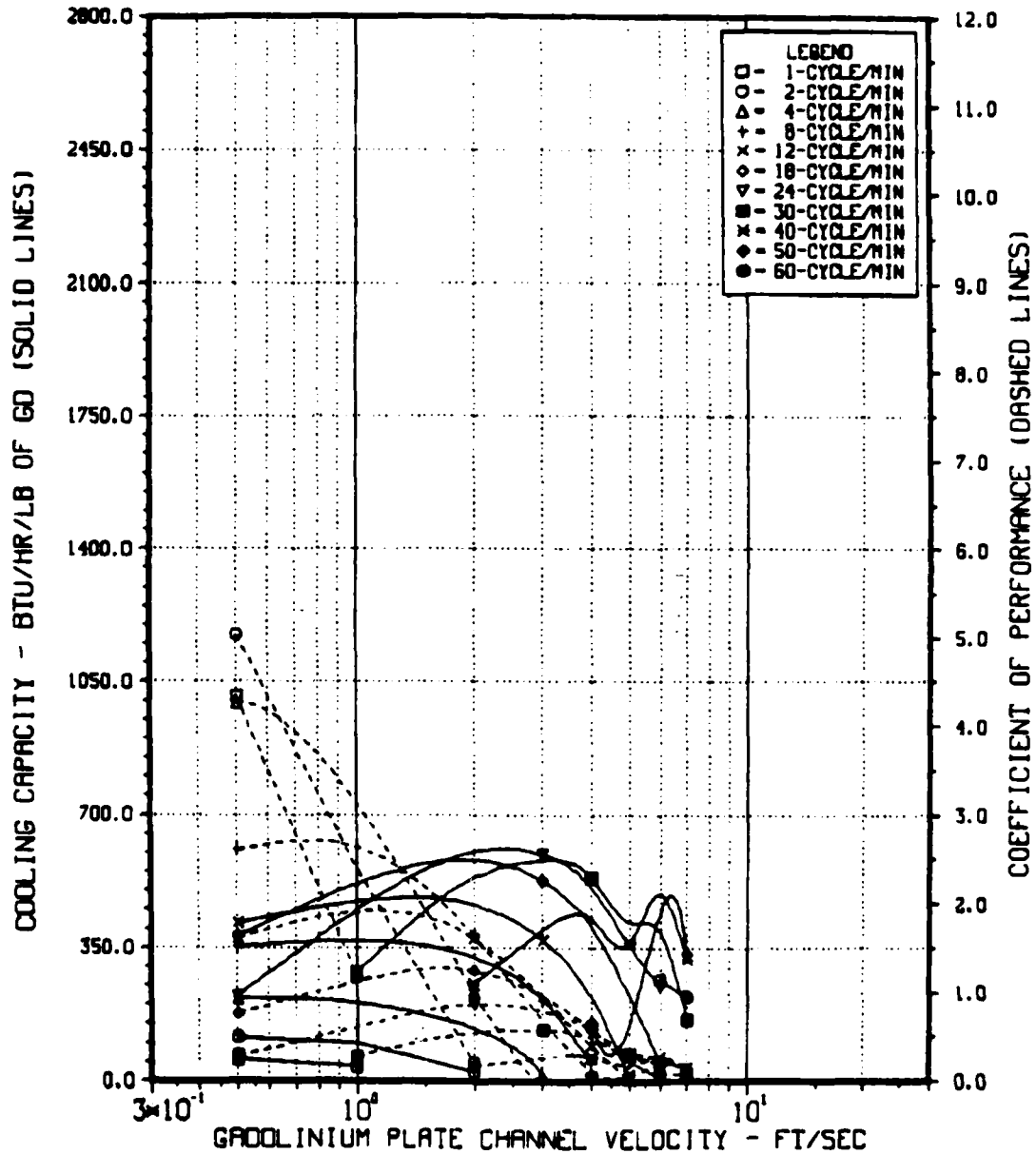


Figure 10d:   Effects of Gadolinium Channel Length on Performance at 7-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

CHANNEL HEIGHT, IN. - 2.00   PLATE SPACING, IN. - 0.040   TEMP HOT END, F - 85.   MIXING TEMP, F - 0.0

CHANNEL LENGTH, IN. - 0.50   PLATE THICKNESS, IN. - 0.005   TEMP COLD END, F - 45.   H.E. DELTA-T, F - 0.0



Figure 11a:   Effects of Gadolinium Plate Thickness on Performance at 7-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

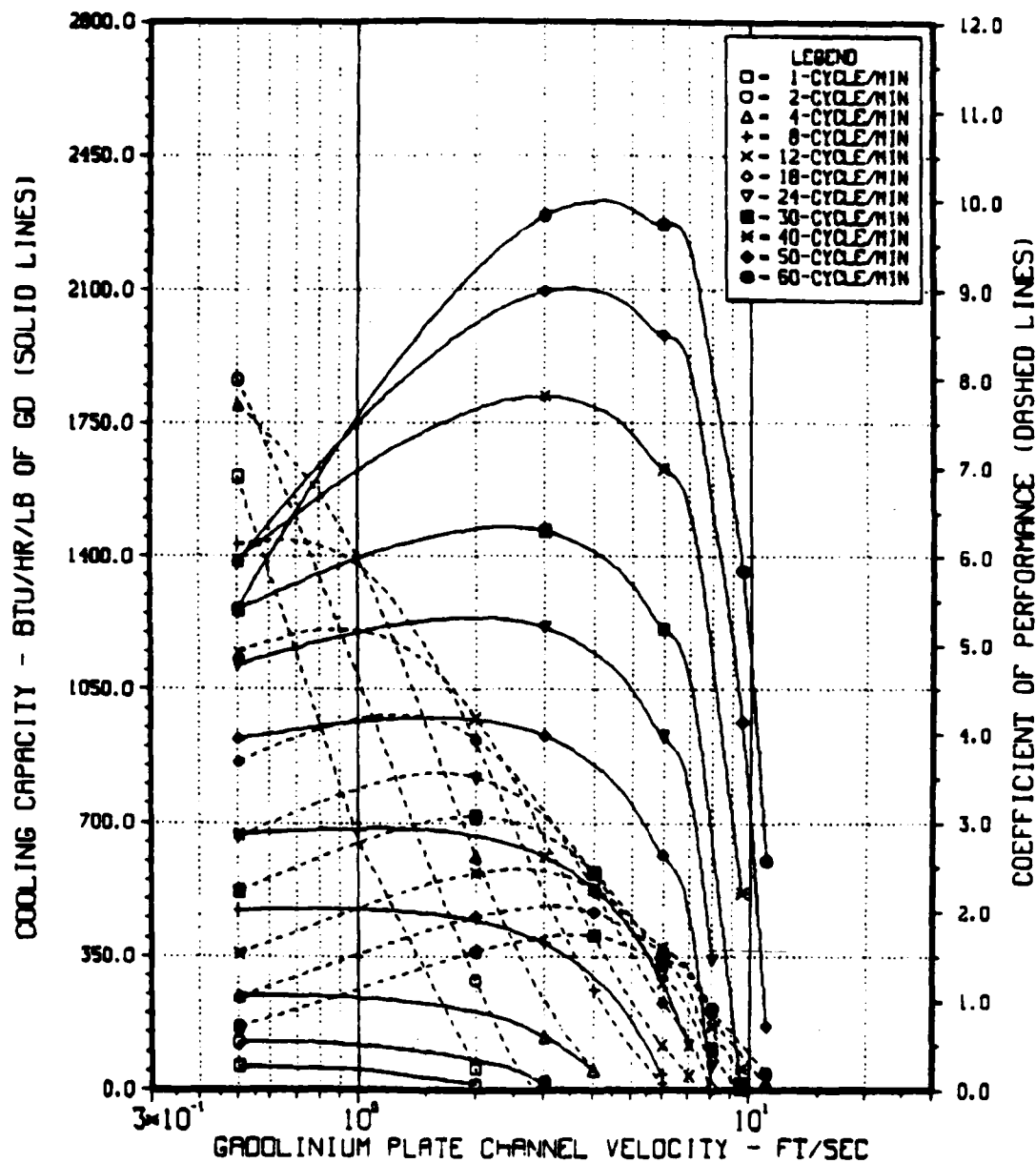Figure 11b: Effects of Gadolinium Plate Thickness on Performance at 7-Tesla

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Figure 11c: Effects of Gadolinium Plate Thickness on Performance at 7-Tesla

MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

CHANNEL HEIGHT, IN.- 2.00    PLATE SPACING, IN.  - 0.040    TEMP.HOT END,F - 85.    MIXING TEMP,F- 0.0

CHANNEL LENGTH, IN.- 0.50    PLATE THICKNESS,IN.- 0.010    TEMP COLD END,F- 45.    H.E.DELTA-T,F- 0.0

Figure 12a:  Effects of Thermal Mixing on Performance at 7-Tesla

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DTNSRDC PAS-82-11

MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

CHANNEL HEIGHT, IN.- 2.00    PLATE SPACING, IN.  - 0.040    TEMP.HOT END,F - 85.    MIXING TEMP,F- 1.0

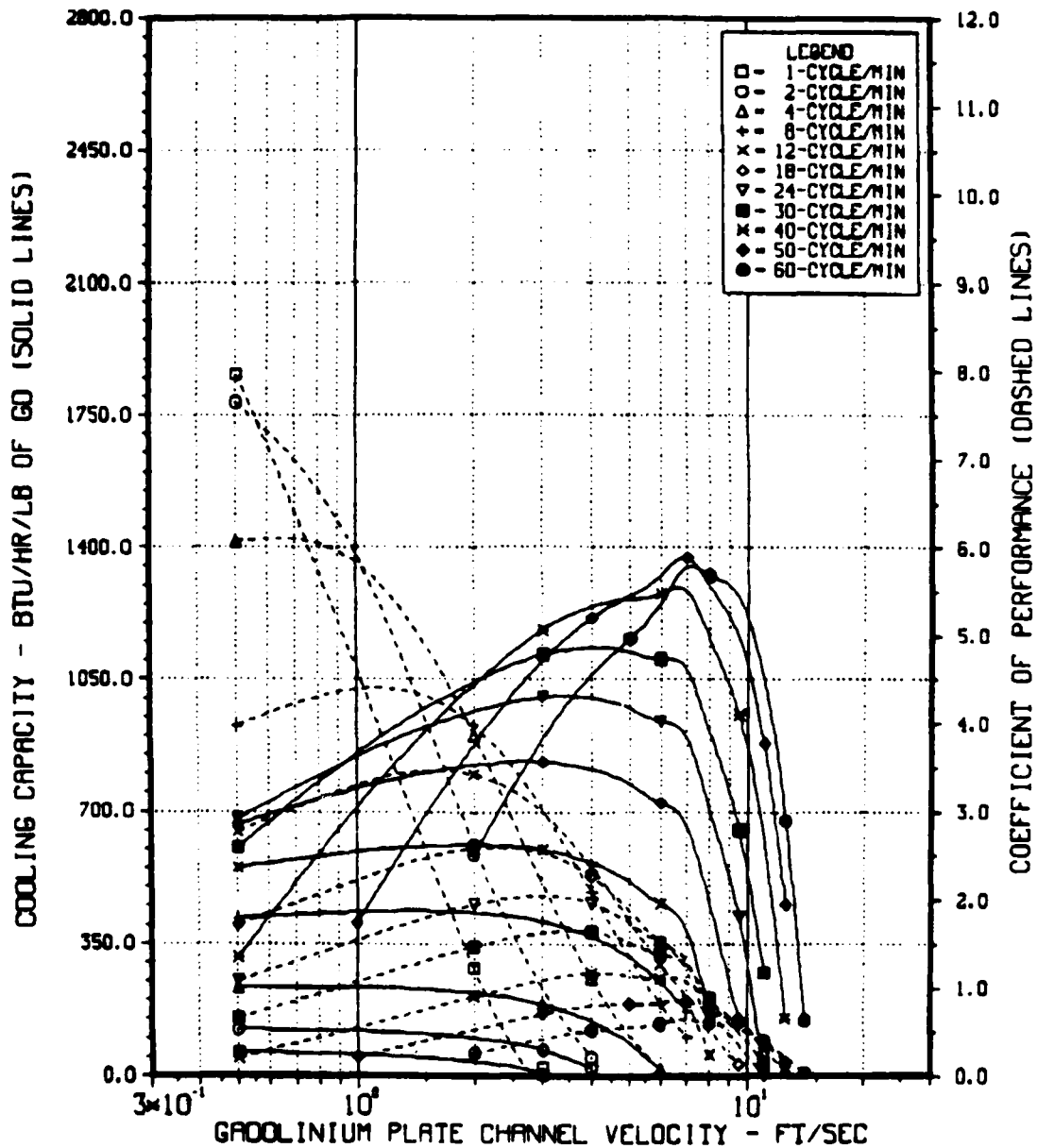CHANNEL LENGTH, IN.- 0.50    PLATE THICKNESS, IN.- 0.010    TEMP COLD END,F- 45.    H.E.DELTA-T,F- 0.0



Figure 12b:   Effects of Thermal Mixing on Performance at 7-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

CHANNEL HEIGHT, IN.- 2.00    PLATE SPACING, IN.  - 0.040    TEMP. HOT END, F - 85.    MIXING TEMP, F- 2.0

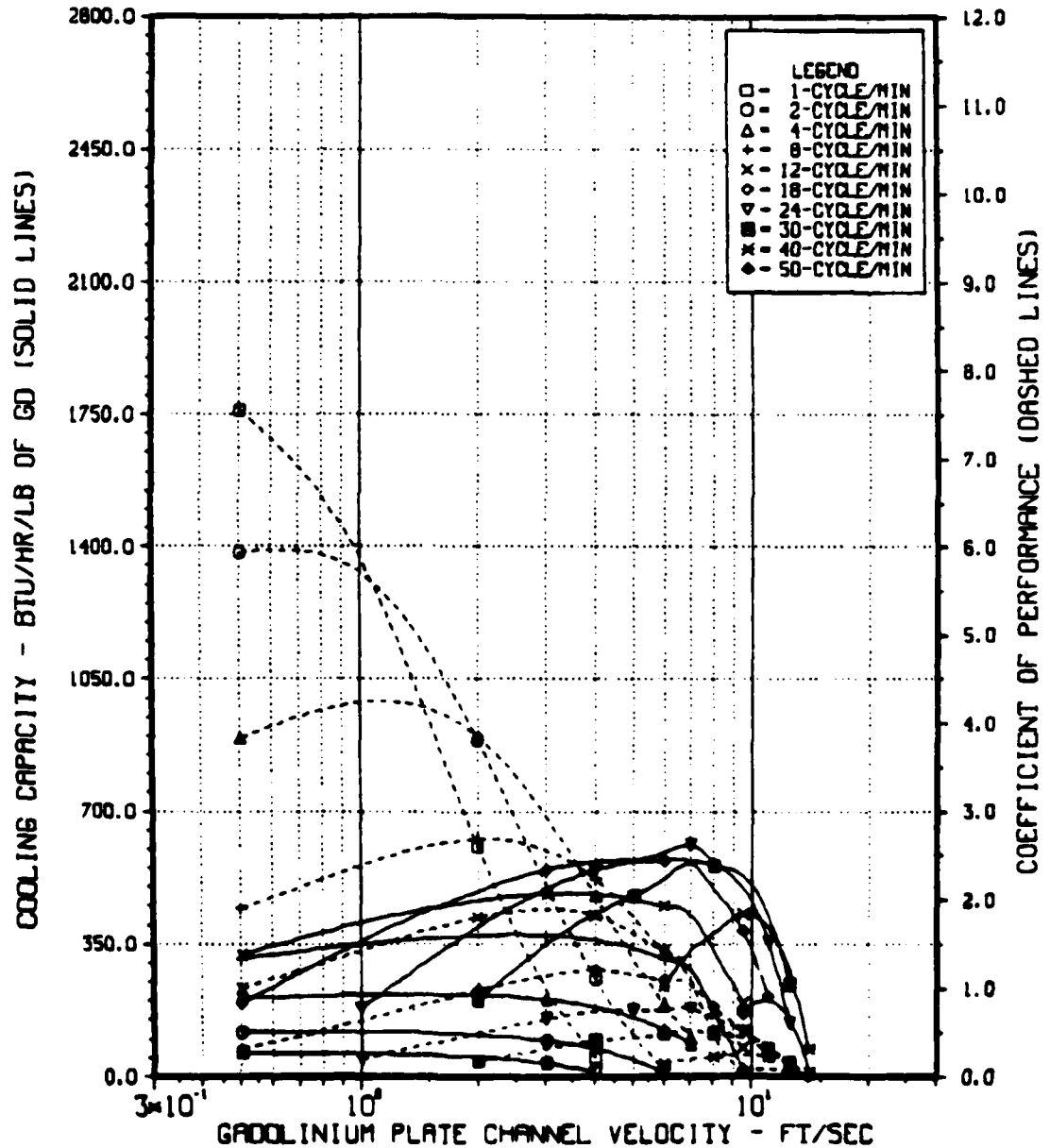CHANNEL LENGTH, IN.- 0.50    PLATE THICKNESS, IN.- 0.010    TEMP COLD END, F- 45.    H.E. DELTA-T, F- 0.0



Figure 12c:   Effects of Thermal Mixing on Performance at 7-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

Figure 13a: Effects of External Heat Exchanger Temperature Difference on Performance at 7-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

CHANNEL HEIGHT, IN.- 2.00   PLATE SPACING, IN. - 0.040   TEMP.HOT END,F - 85.   MIXING TEMP,F- 0.0

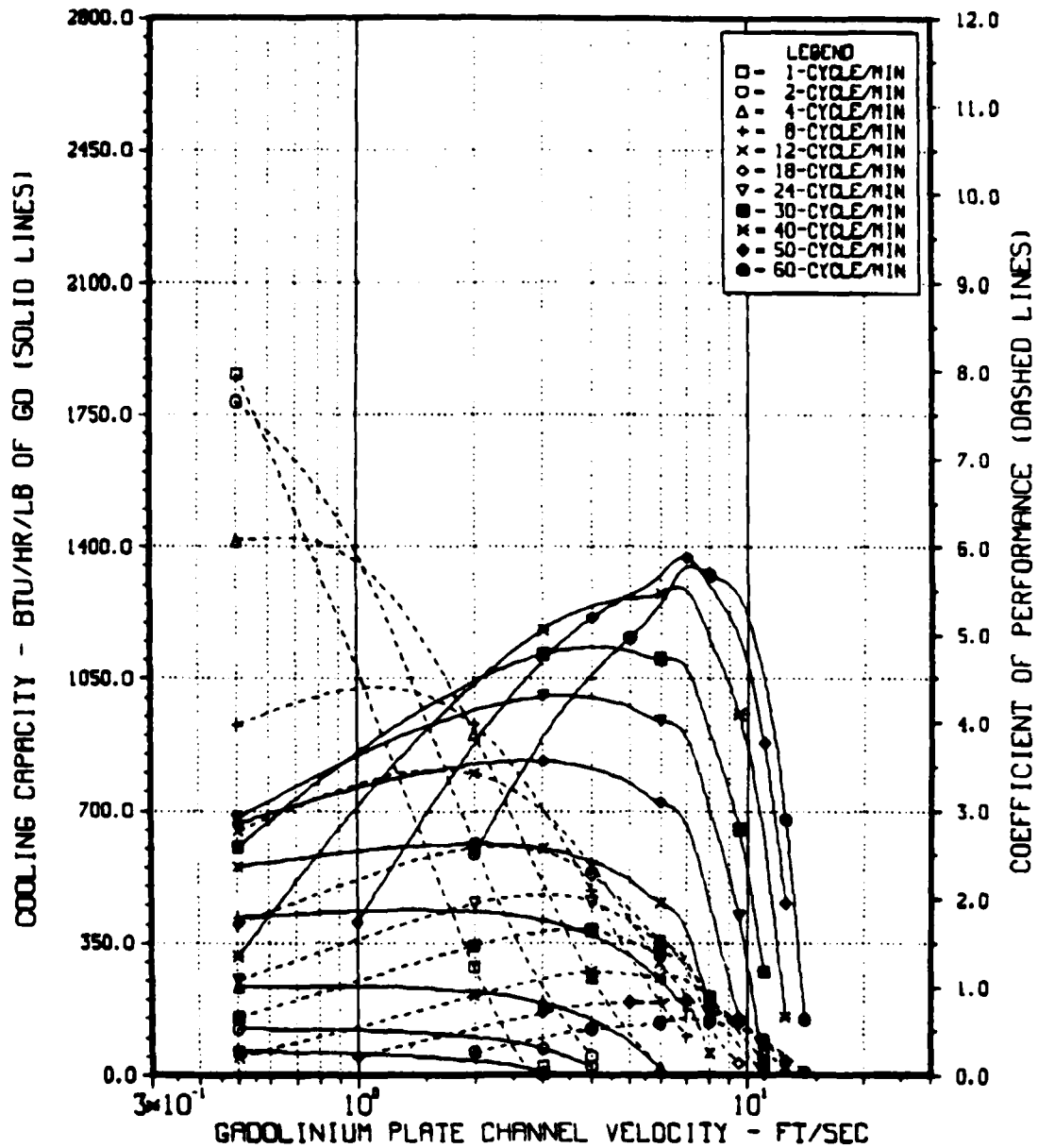CHANNEL LENGTH, IN.- 0.50   PLATE THICKNESS, IN.- 0.010   TEMP COLD END,F- 45.   H.E.DELTA-T,F- 2.0

Figure 13b:   Effects of External Heat Exchanger Temperature Difference
On Performance at 7 Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

CHANNEL HEIGHT, IN.- 2.00    PLATE SPACING, IN.  - 0.040    TEMP.HOT END,F - 85.    MIXING TEMP,F- 0.0

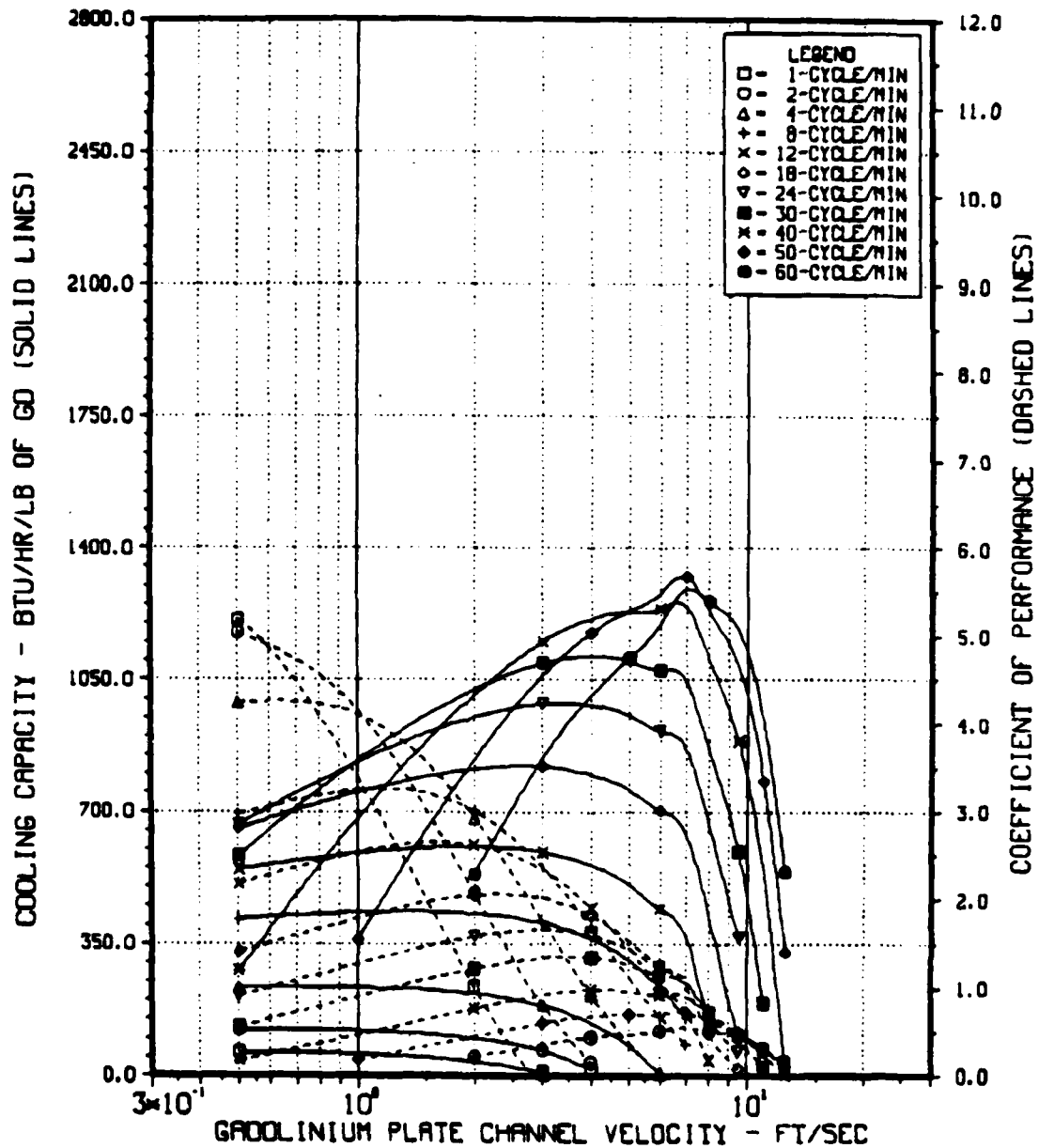CHANNEL LENGTH, IN.- 0.50    PLATE THICKNESS, IN.- 0.010    TEMP COLD END,F - 45.   H.E.DELTA-T,F- 5.0



Figure 13c:   Effects of External Heat Exchanger Temperature
Difference on Performance at 7-Tesla

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CHANNEL HEIGHT, IN.- 2.00   PLATE SPACING, IN.  - 0.020   TEMP.HOT END,F - 85.   MIXING TEMP,F- 0.0

CHANNEL LENGTH, IN.- 0.50   PLATE THICKNESS,IN.- 0.010   TEMP COLD END,F- 45.   H.E.DELTA-T,F- 0.0



Figure 14a:   Effects of Gadolinium Plate Spacing on Performance at 2-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 2.0 TESLA

CHANNEL HEIGHT, IN.- 2.00    PLATE SPACING, IN. - 0.040    TEMP.HOT END,F - 85.    MIXING TEMP,F- 0.0

CHANNEL LENGTH, IN.- 0.50    PLATE THICKNESS, IN.- 0.010    TEMP COLD END,F- 45.    H.E.DELTA-T,F- 0.0
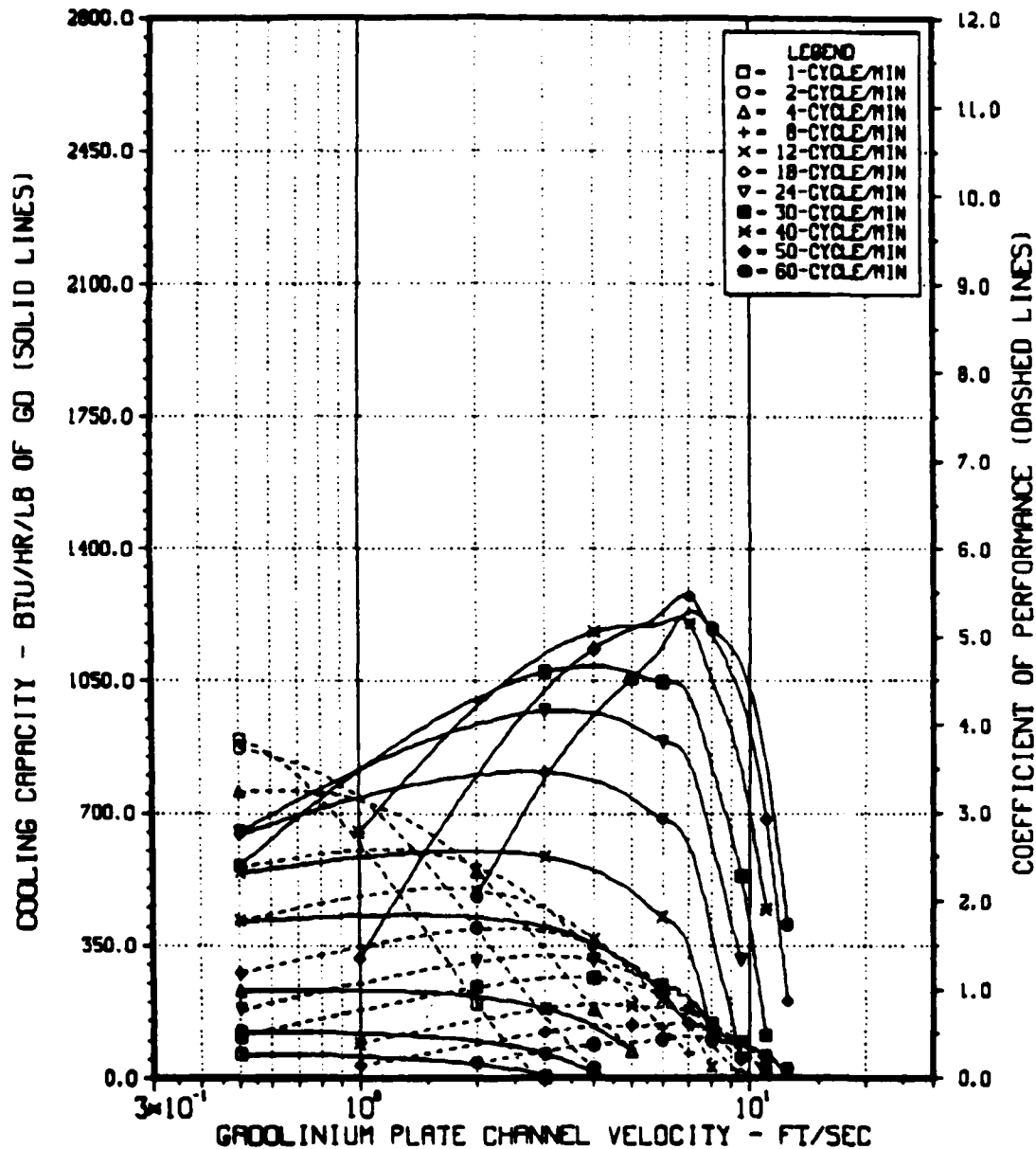


Figure 14b:    Effects of Gadolinium Plate Spacing on Performance at 2-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 2.0 TESLA

CHANNEL HEIGHT, IN.- 2.00   PLATE SPACING, IN.  - 0.060   TEMP.HOT END,F - 85.   MIXING TEMP,F- 0.0

CHANNEL LENGTH, IN.-`0.50   PLATE THICKNESS, IN.- 0.010   TEMP COLD END,F- 45.   H.E.DELTA-T,F- 0.0
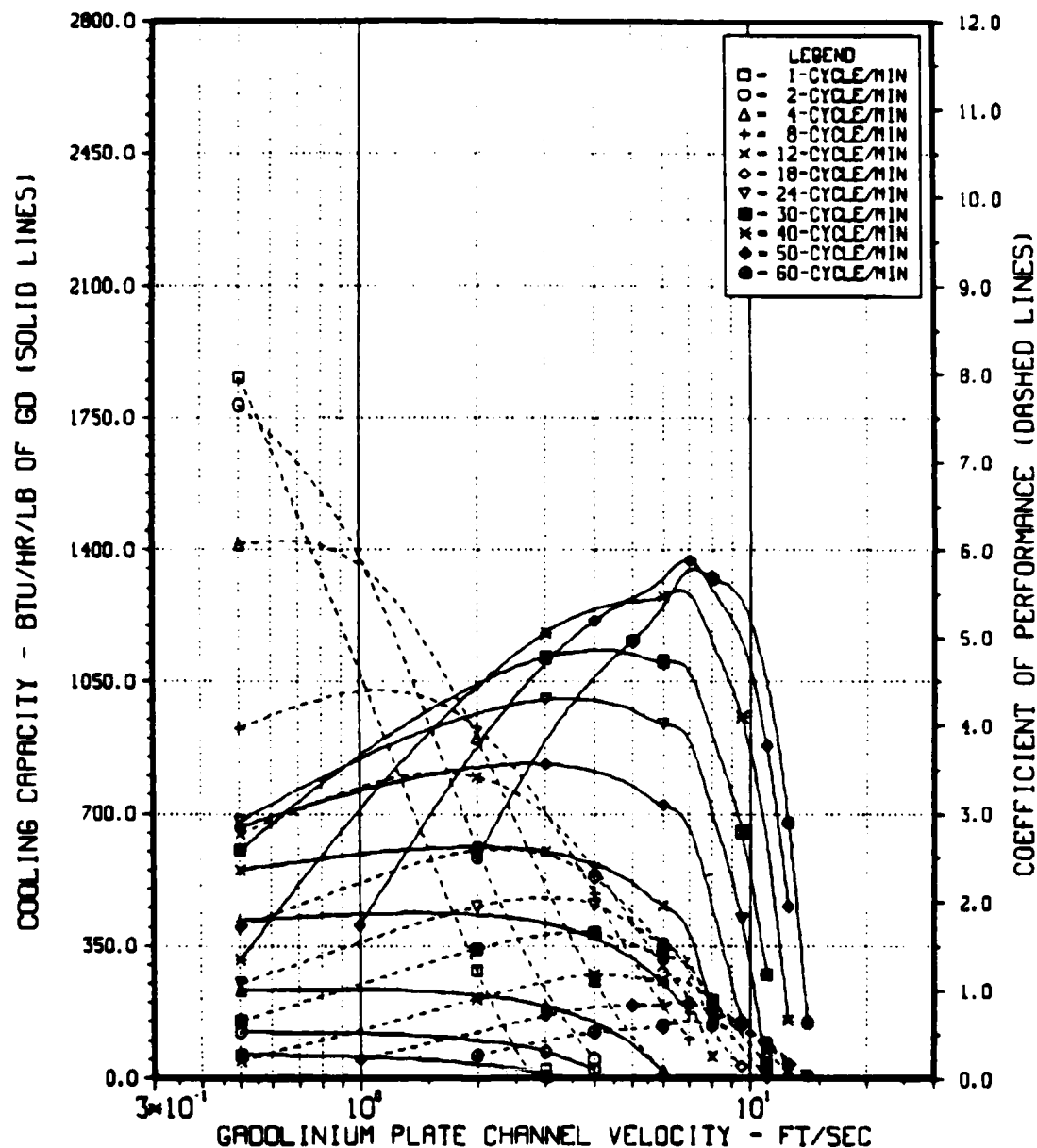


Figure 14c:   Effects of Gadolinium Plate Spacing on Performance at 2-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 2.0 TESLA

Figure 15a:  Effects of Gadolinium Channel Length on Performance at 2-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 2.0 TESLA

CHANNEL HEIGHT, IN.- 2.00   PLATE SPACING, IN. - 0.040   TEMP.HOT END,F - 85.   MIXING TEMP,F- 0.0

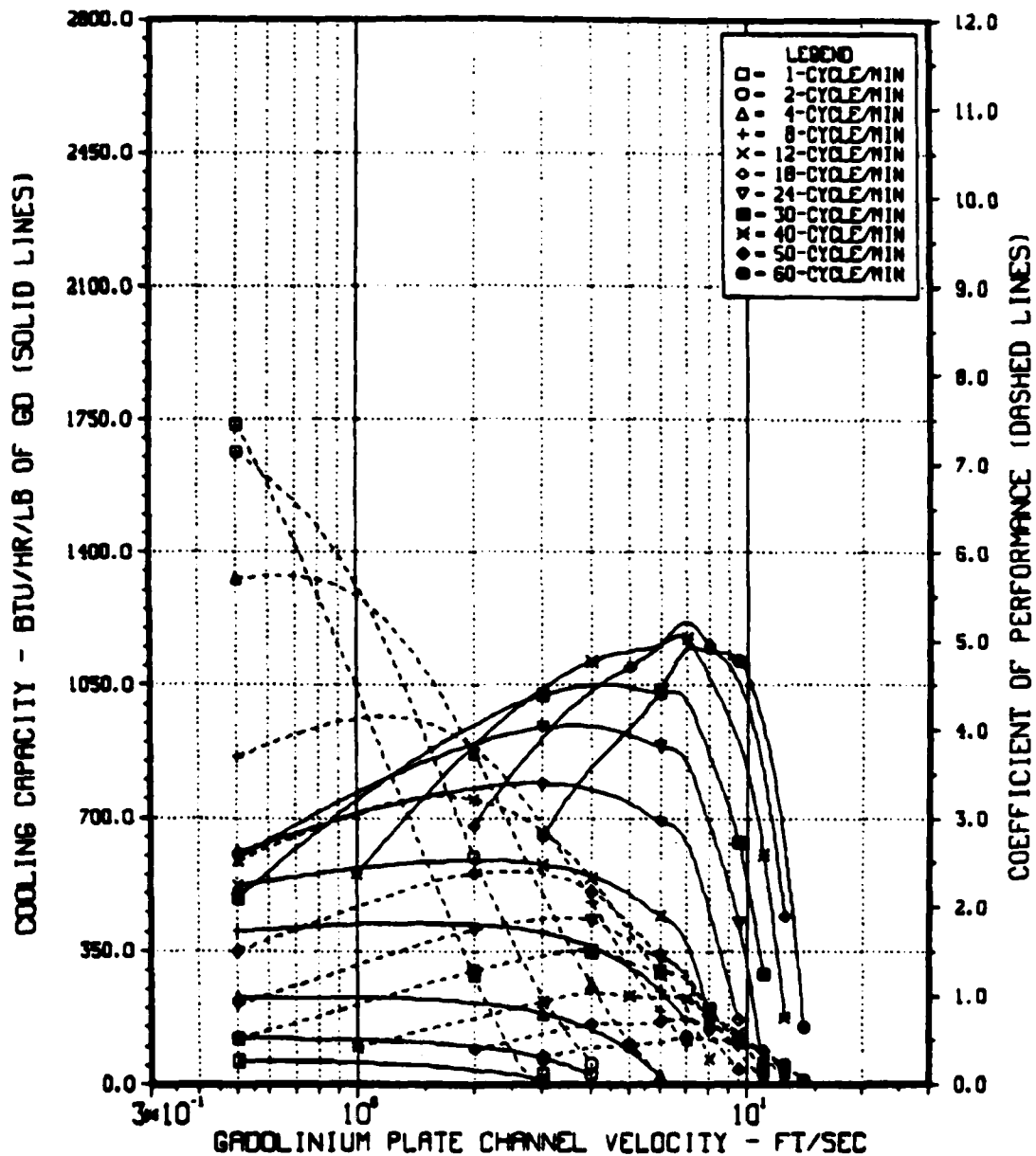CHANNEL LENGTH, IN.- 0.50   PLATE THICKNESS, IN.- 0.010   TEMP COLD END,F- 45.   H.E.DELTA-T,F- 0.0



Figure 15b:  Effects of Gadolinium Channel Length on Performance at 2-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 2.0 TESLA

Figure 15c:   Effects of Gadolinium Channel Length on Performance at 2-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 2.0 TESLA

CHANNEL HEIGHT,IN.- 2.00    PLATE SPACING,IN. - 0.040    TEMP.HOT END,F - 85.    MIXING TEMP,F- 0.0

CHANNEL LENGTH,IN.- 2.00    PLATE THICKNESS,IN.- 0.010    TEMP COLD END,F- 45.    H.E.DELTA-T,F- 0.0
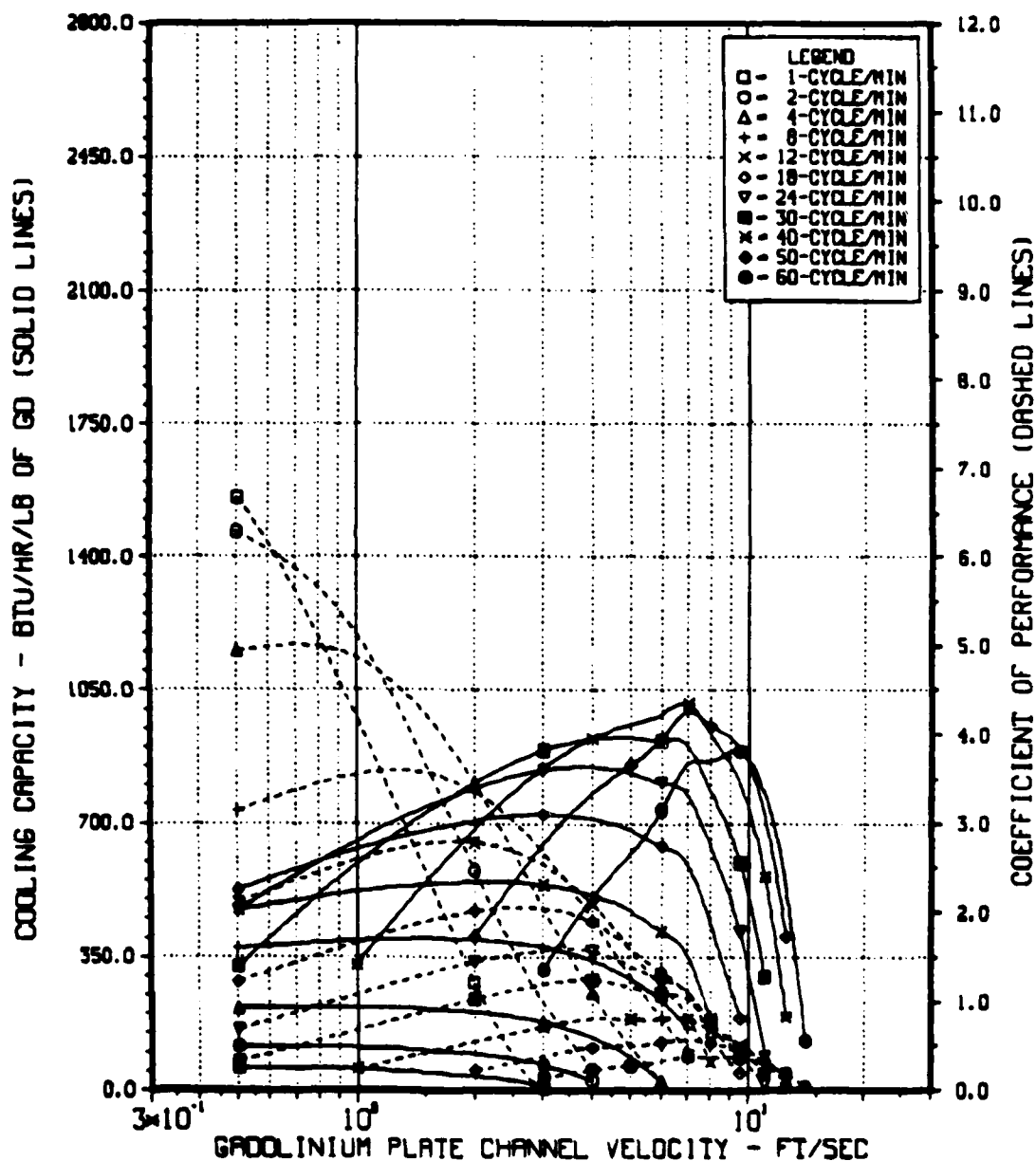


Figure 15d:   Effects of Gadolinium Channel Length on Performance at 2-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 2.0 TESLA

CHANNEL HEIGHT, IN.- 2.00    PLATE SPACING, IN.  - 0.040    TEMP.HOT END,F - 85.    MIXING TEMP,F- 0.0

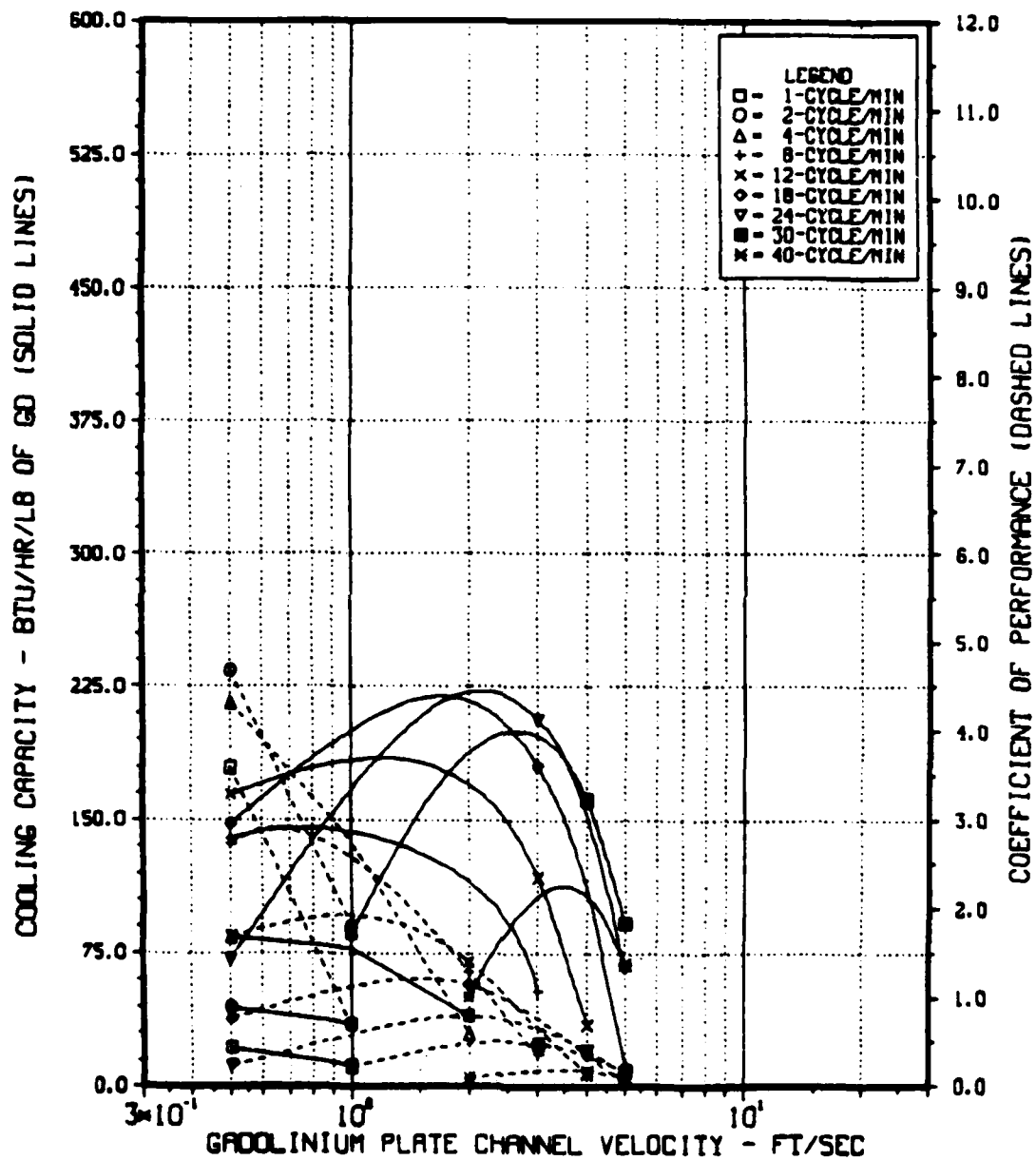CHANNEL LENGTH, IN.- 0.50    PLATE THICKNESS, IN.- 0.005    TEMP COLD END,F- 45.    H.E.DELTA-T,F- 0.0



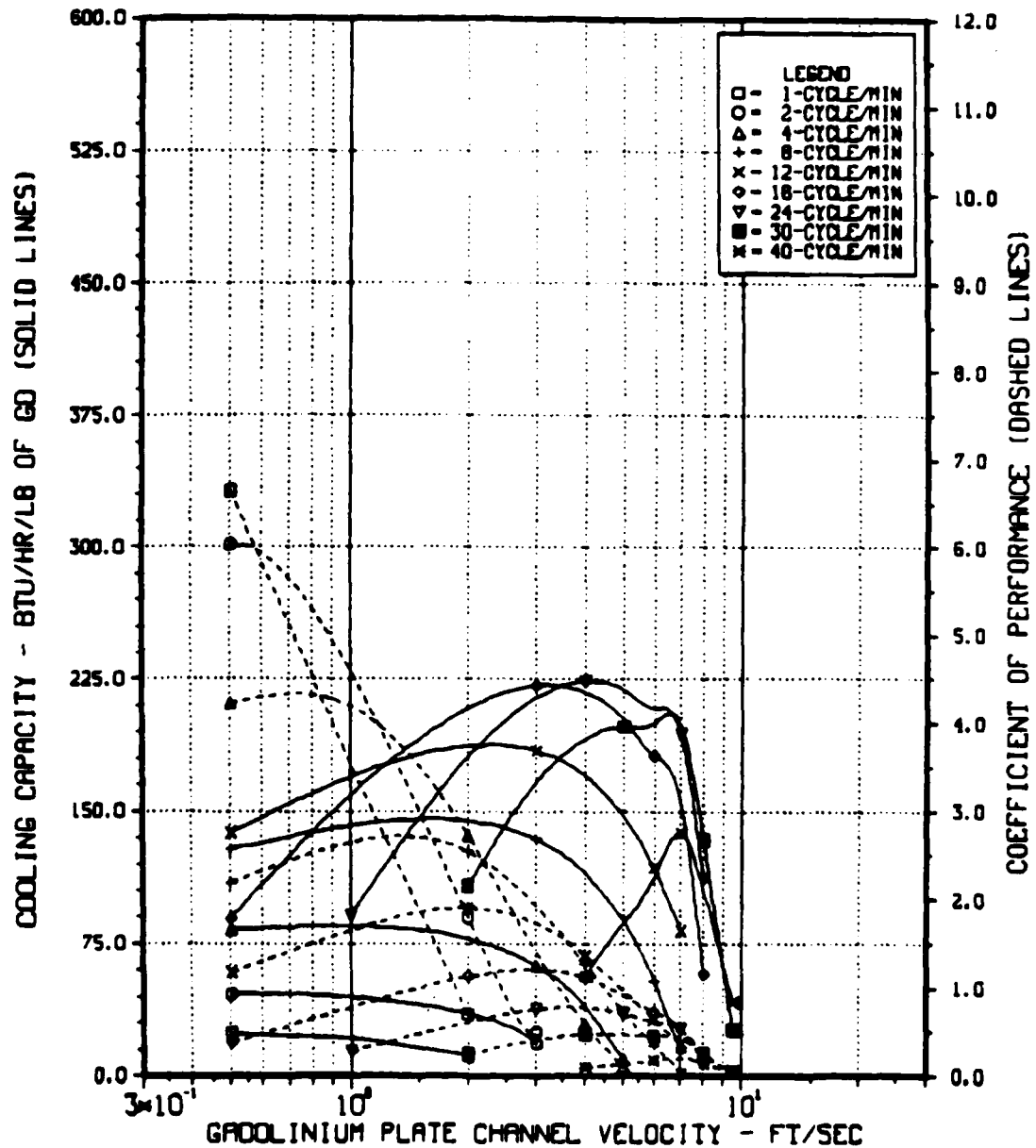Figure 16a:    Effects of Gadolinium Plate Thickness on Performance at 2-Tesla

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Figure 16b: Effects of Gadolinium Plate Thickness on Performance at 2-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 2.0 TESLA

CHANNEL HEIGHT, IN.- 2.00    PLATE SPACING, IN. - 0.040    TEMP.HOT END,F - 85.    MIXING TEMP,F- 0.0

CHANNEL LENGTH, IN.- 0.50    PLATE THICKNESS, IN.- 0.020    TEMP COLD END,F- 45.    H.E.DELTA-T,F- 0.0



Figure 16c:    Effects of Gadolinium Plate Thickness on Performance at 2-Tesla

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# MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 2.0 TESLA



Figure 17a:  Effects of Thermal Mixing on Performance at 2-Tesla

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Figure 17b:  Effects of Thermal Mixing on Performance at 2-Tesla

MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 2.0 TESLA

Figure 17c:  Effects of Thermal Mixing on Performance at 2-Tesla

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## MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 2.0 TESLA

CHANNEL HEIGHT, IN.- 2.00    PLATE SPACING, IN.  - 0.040    TEMP.HOT END,F - 85.    MIXING TEMP,F- 0.0

CHANNEL LENGTH, IN.- 0.50    PLATE THICKNESS, IN.- 0.010    TEMP COLD END,F- 45.    H.E.DELTA-T,F- 0.0
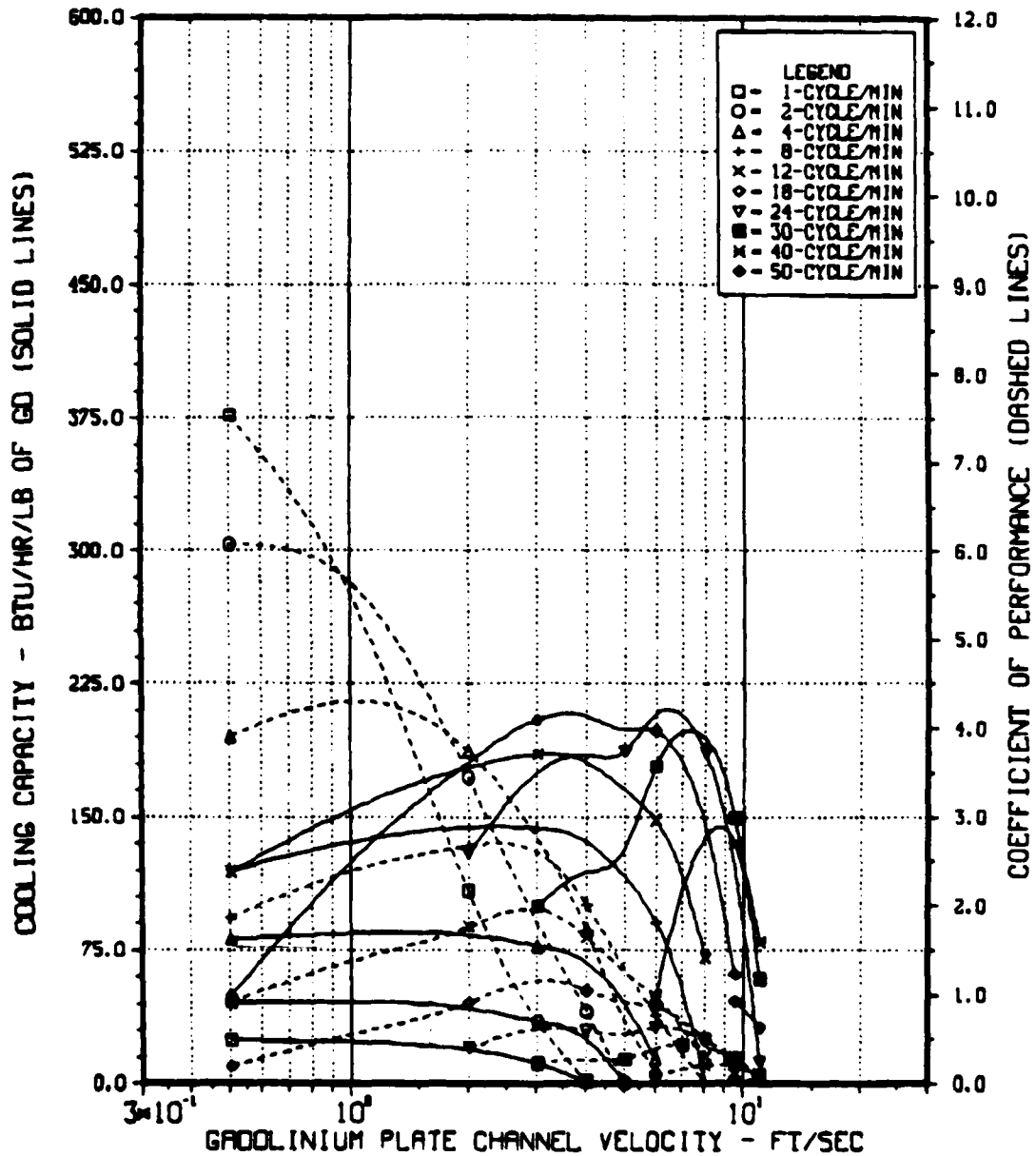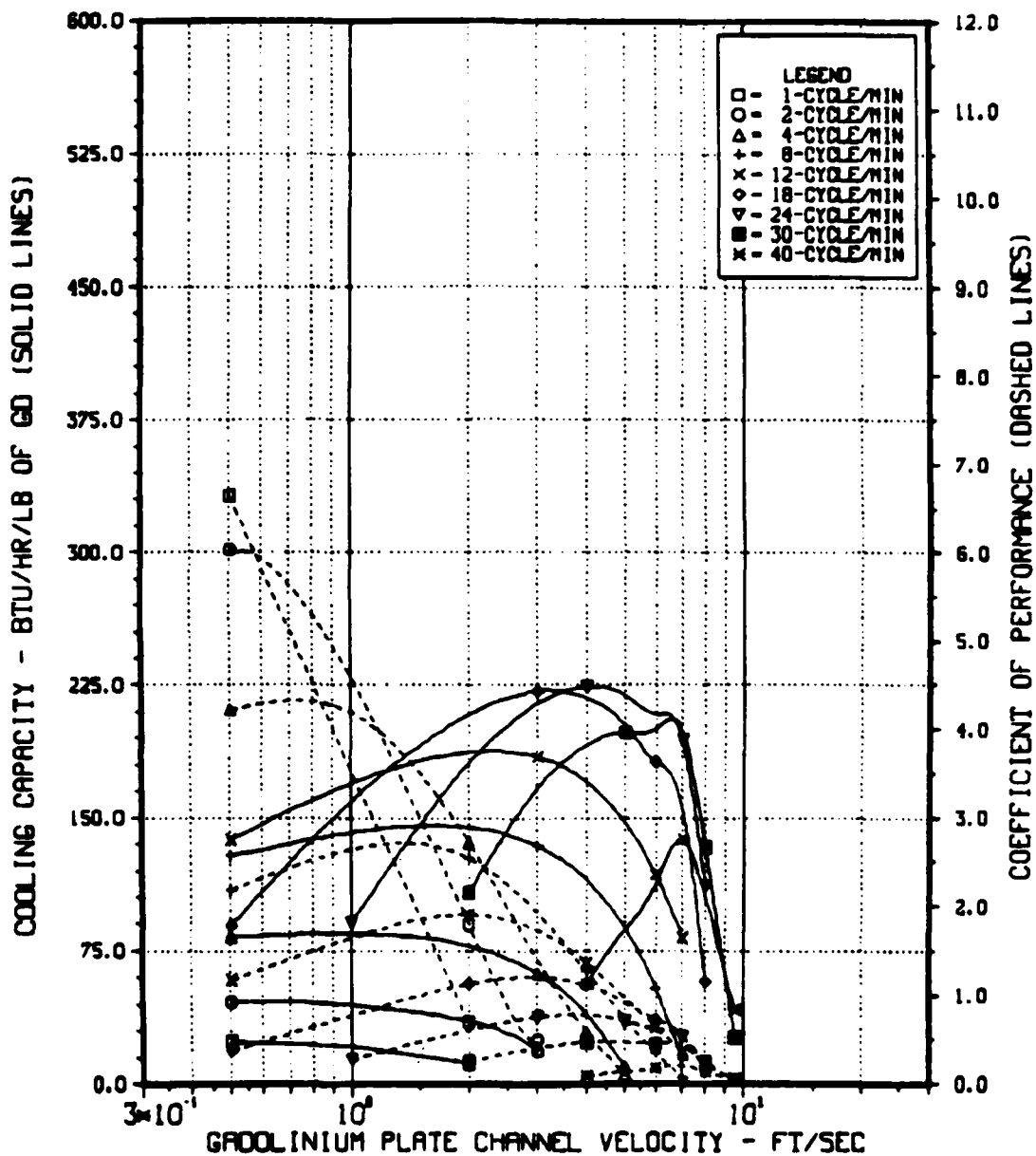


Figure 18a:  Effects of External Heat Exchanger Temperature
Difference on Performance at 2-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 2.0 TESLA

CHANNEL HEIGHT, IN.= 2.00   PLATE SPACING, IN.  - 0.040   TEMP.HOT END,F - 85.   MIXING TEMP,F- 0.0

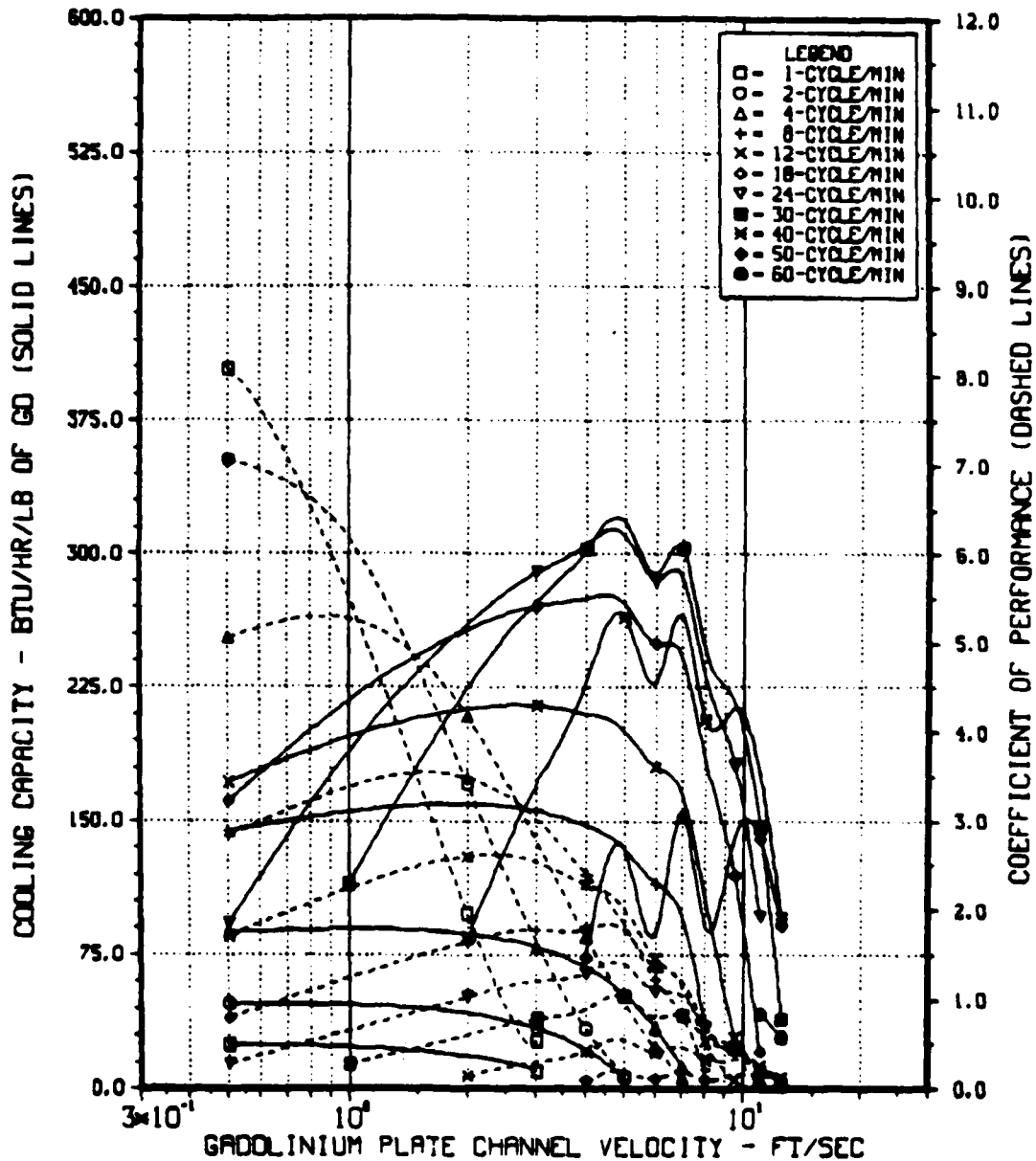CHANNEL LENGTH, IN.= 0.50   PLATE THICKNESS, IN.- 0.010   TEMP COLD END,F- 45.   H.E.DELTA-T,F- 2.0
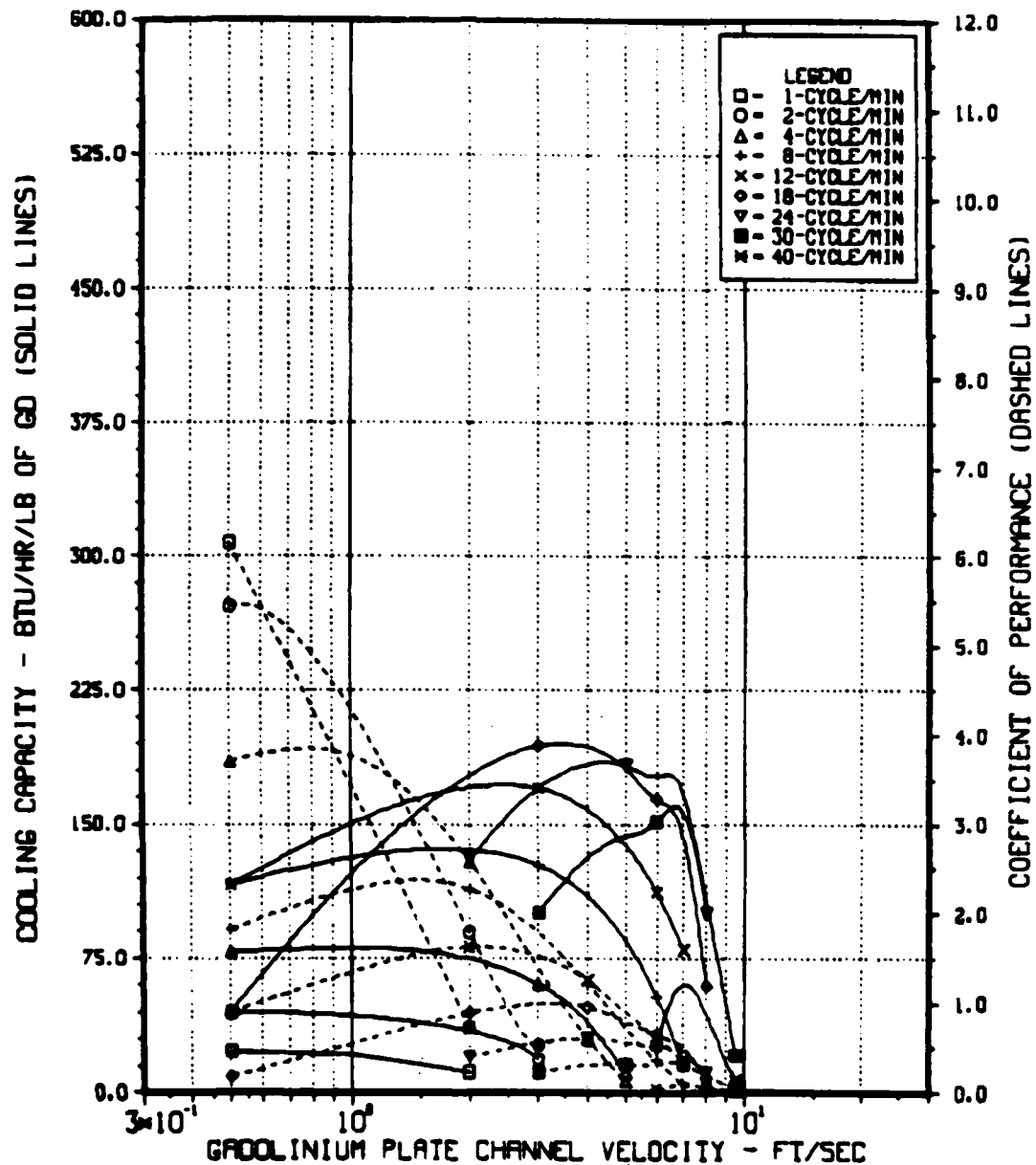


Figure 18b:   Effects of External Heat Exchanger Temperature
Difference on Performance at 2-Tesla

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 2.0 TESLA

CHANNEL HEIGHT, IN.- 2.00    PLATE SPACING, IN.  - 0.040    TEMP.HOT END,F - 85.    MIXING TEMP,F- 0.0

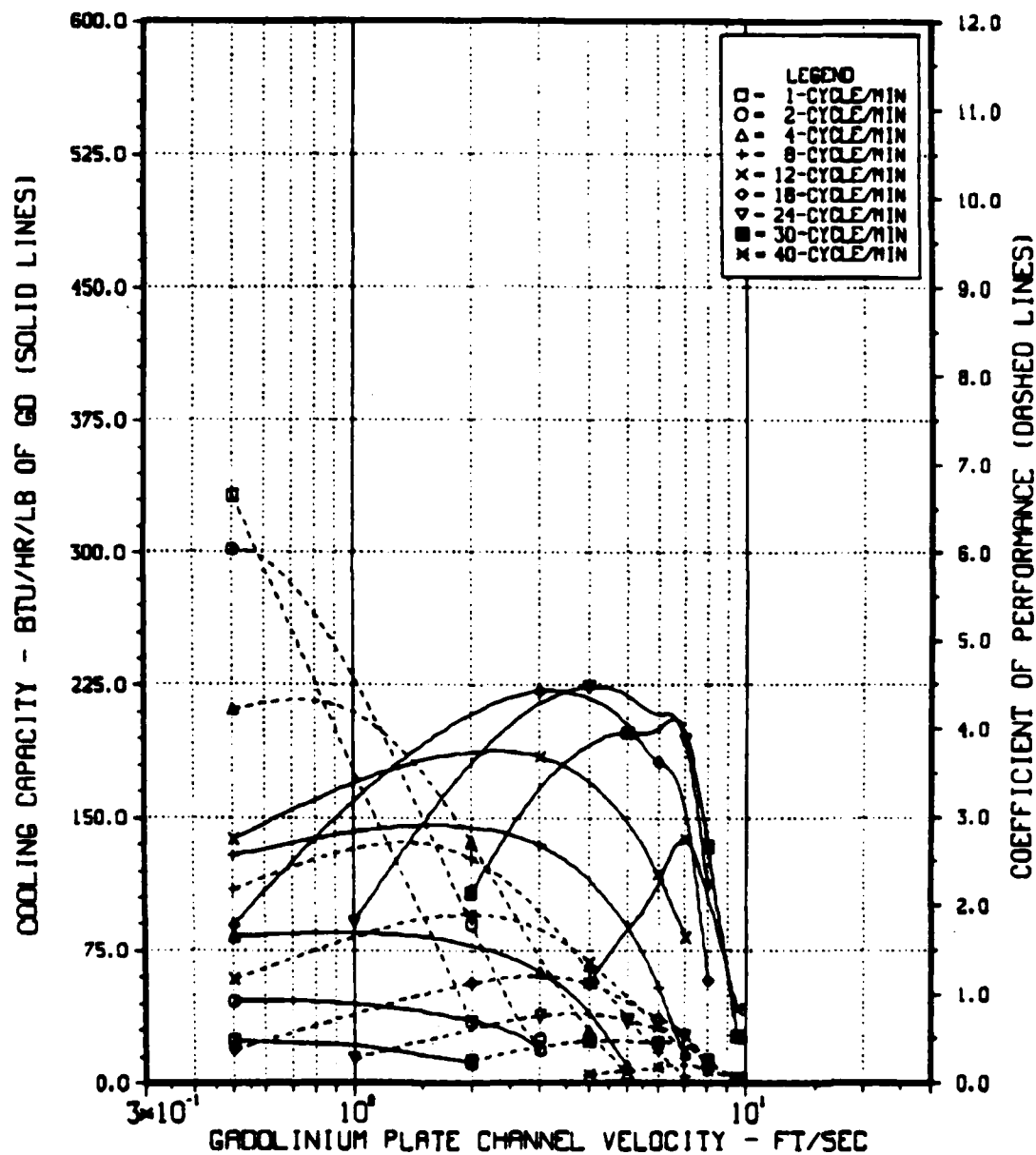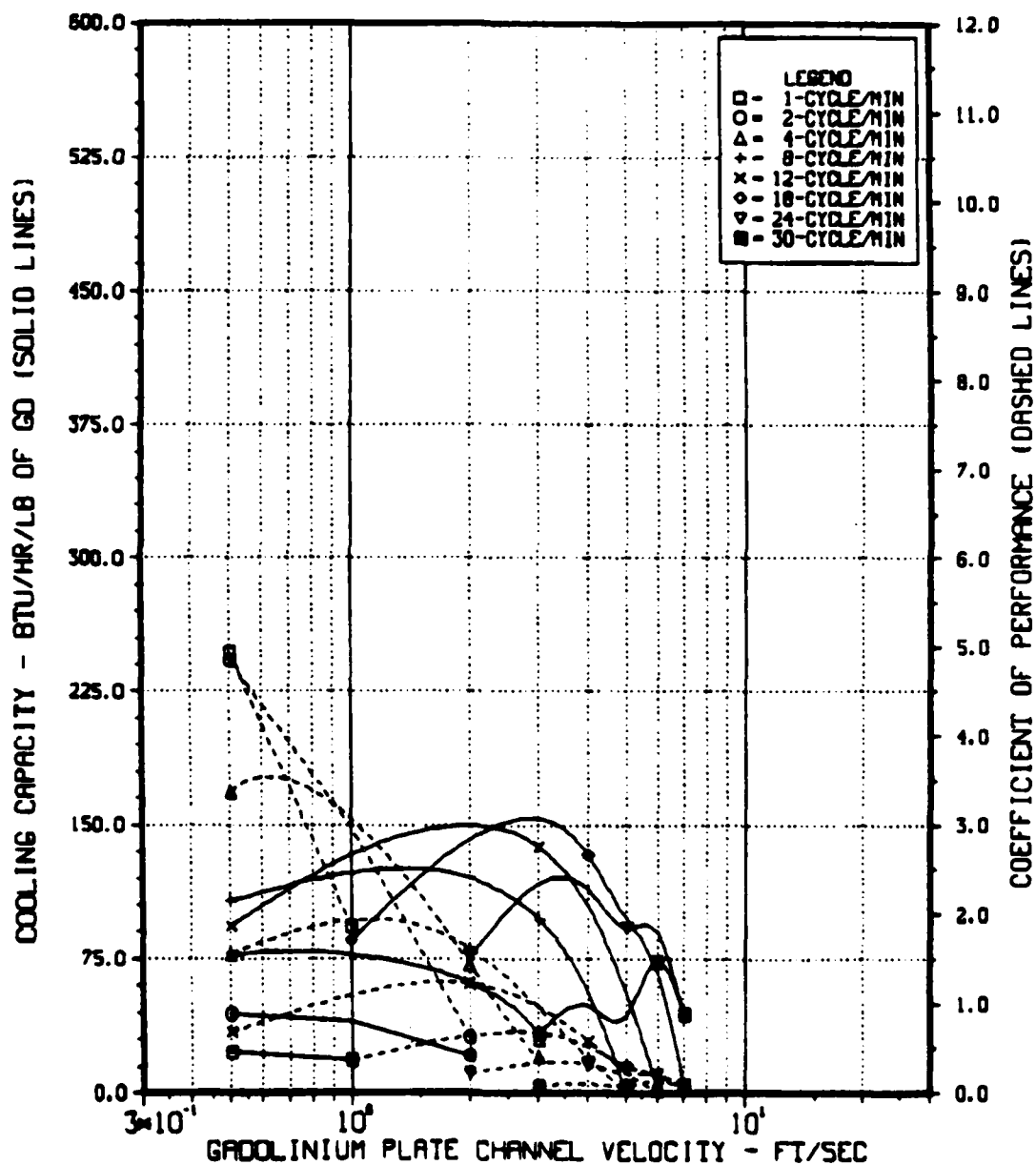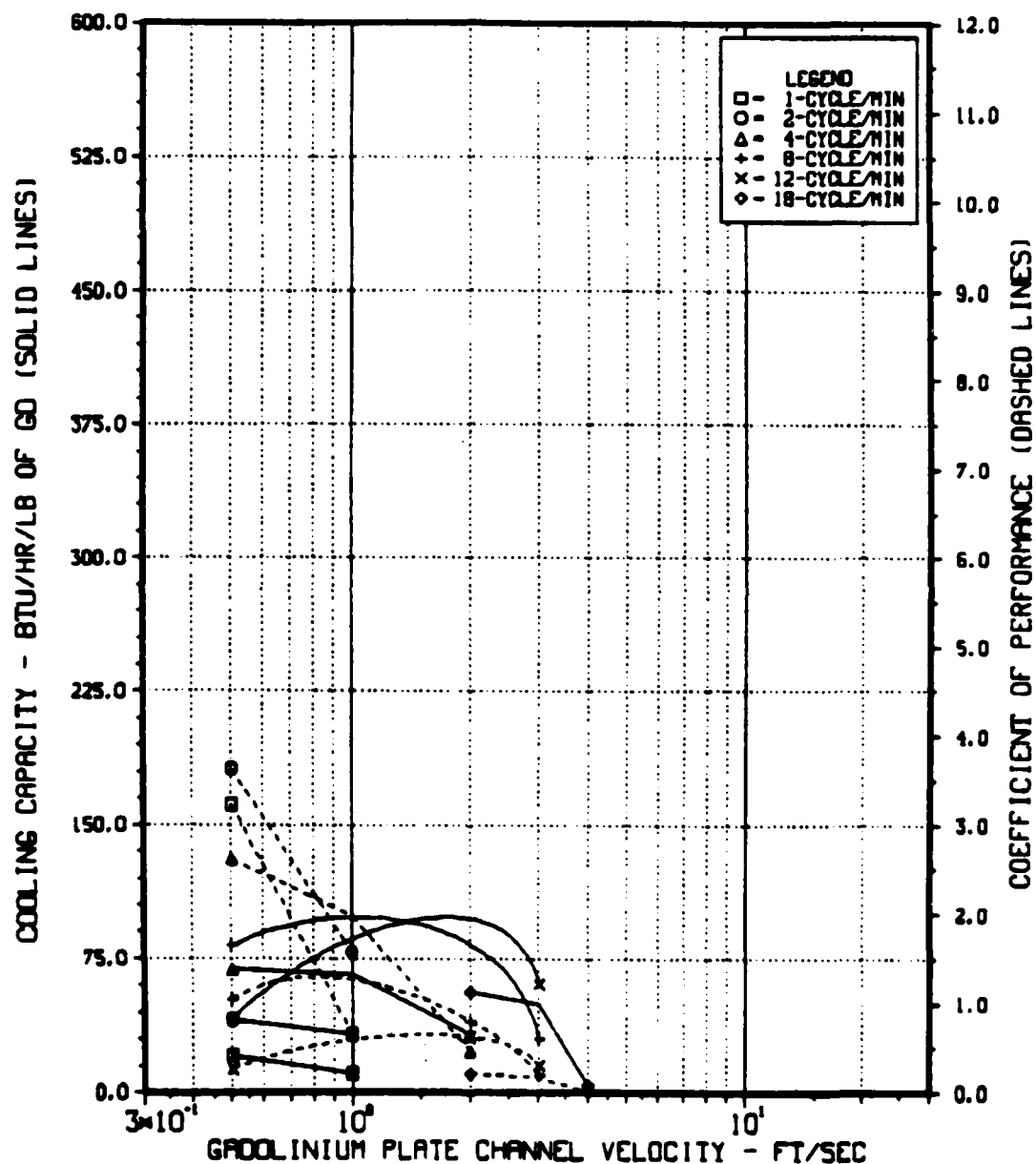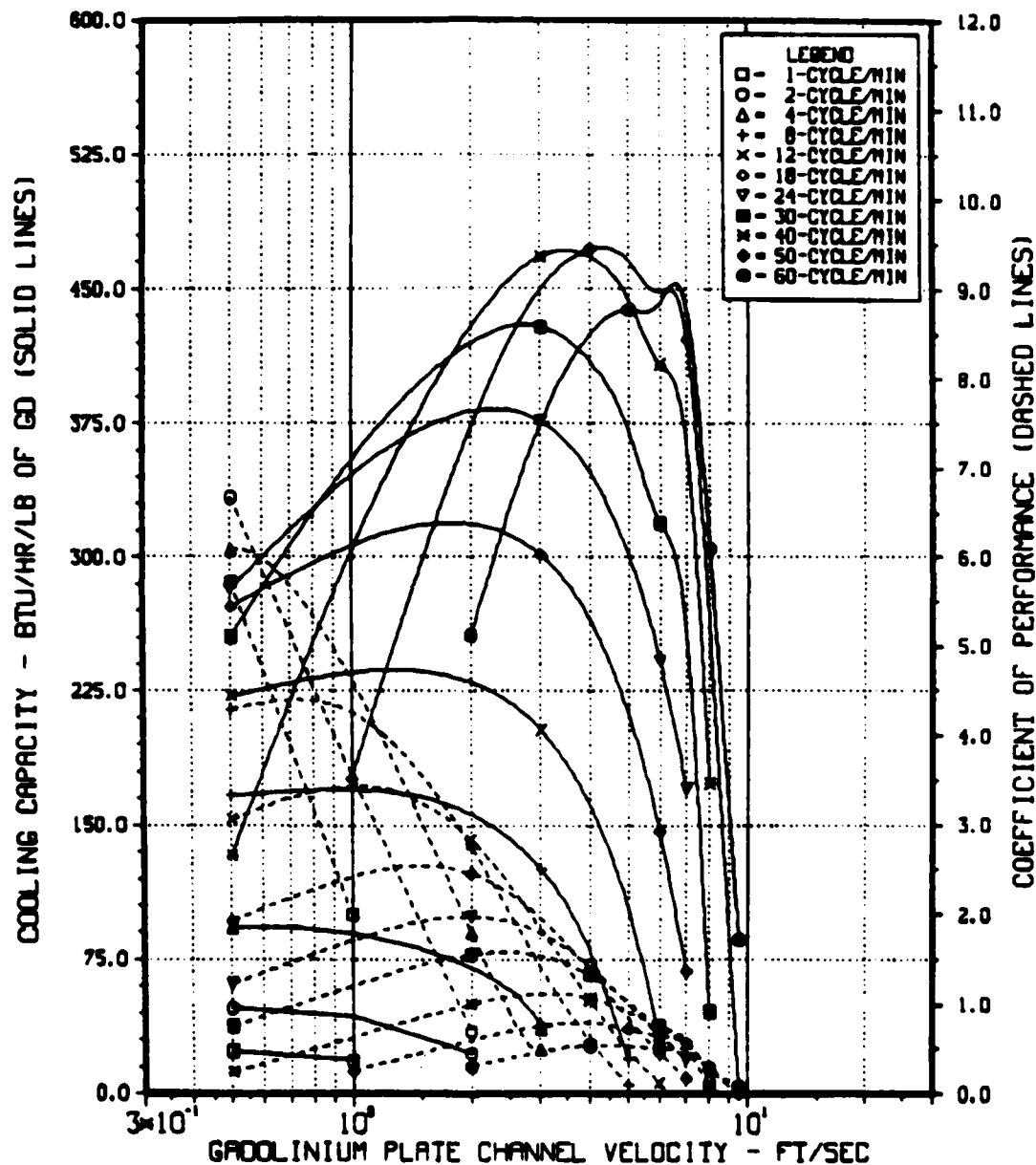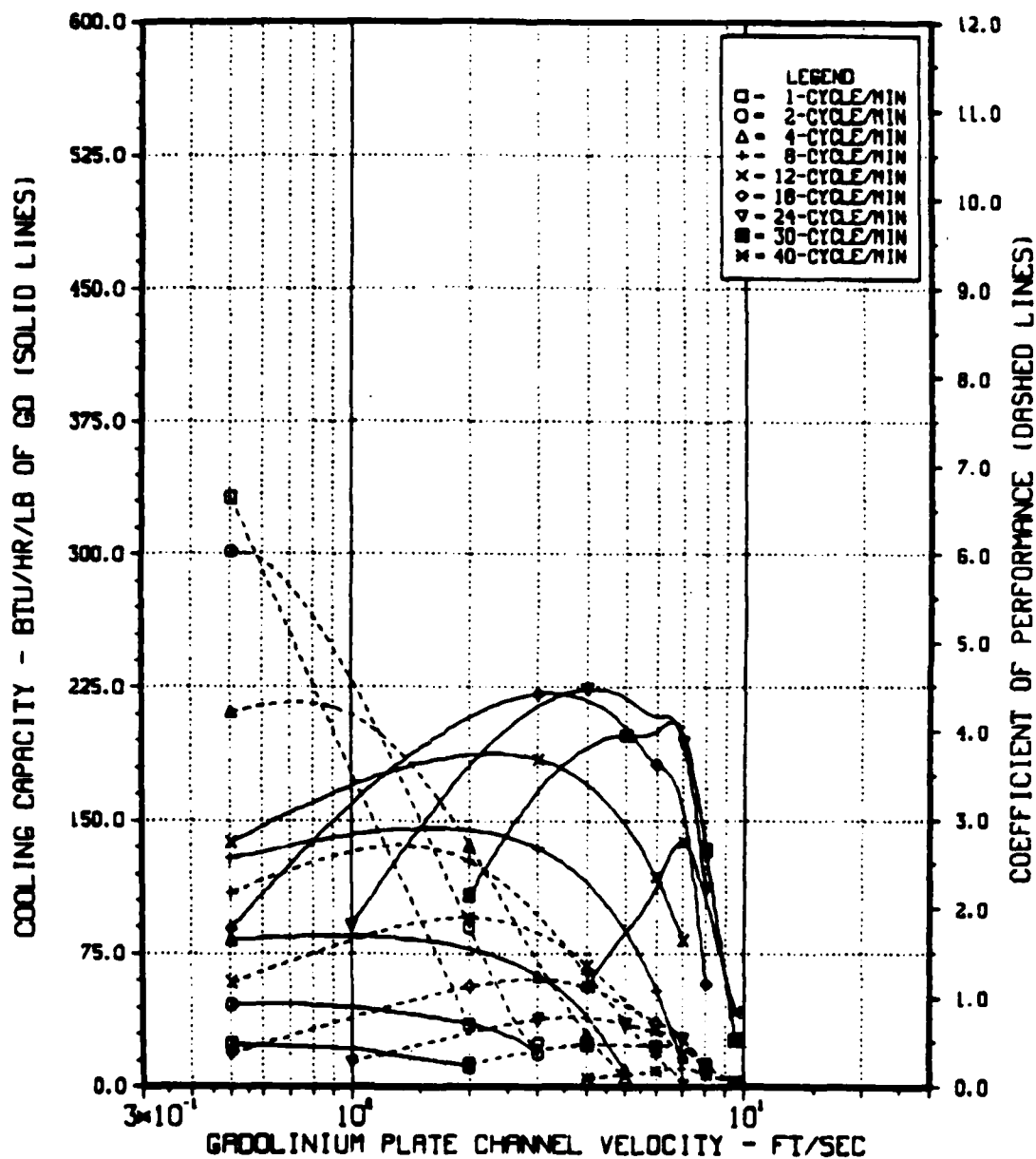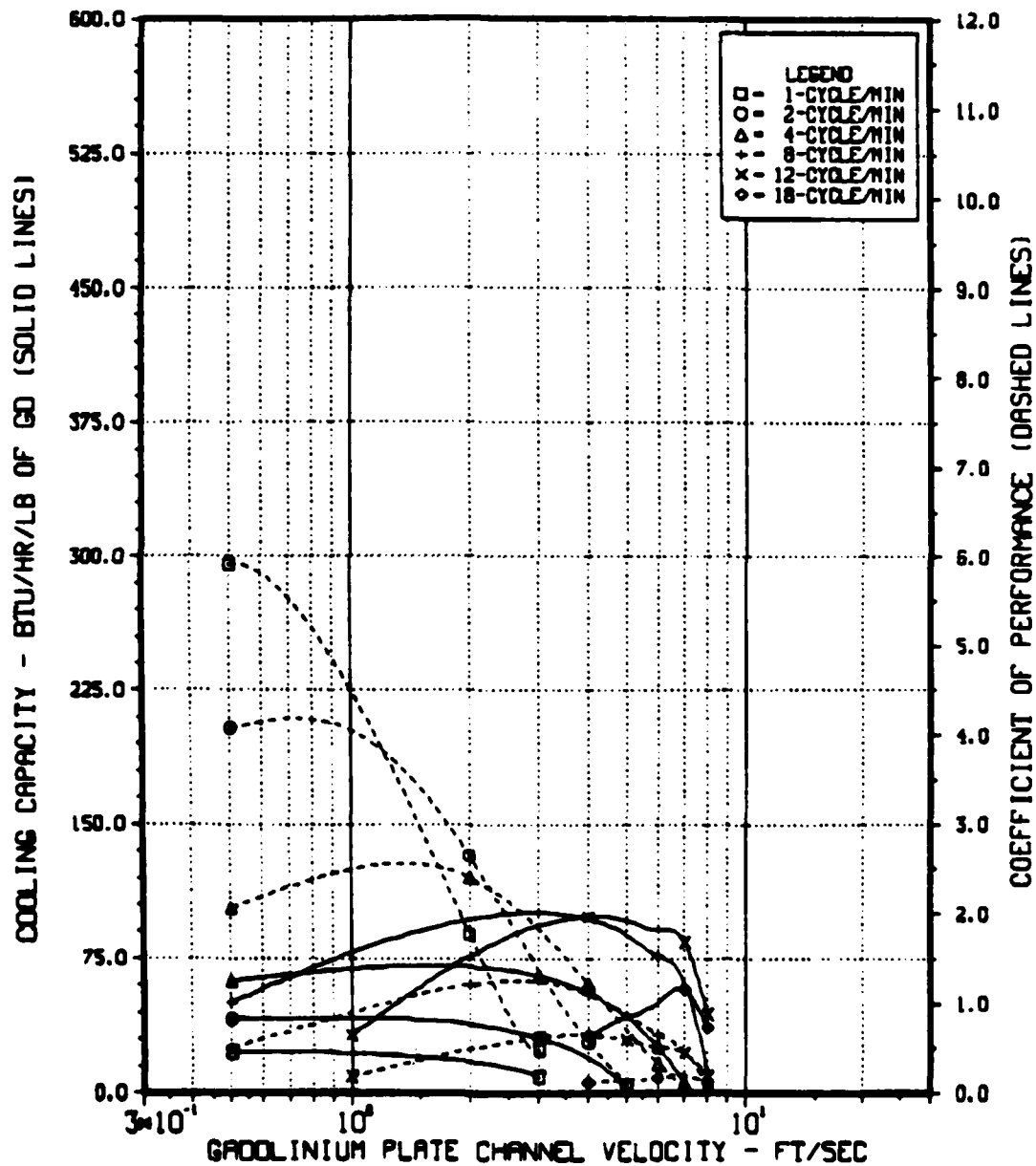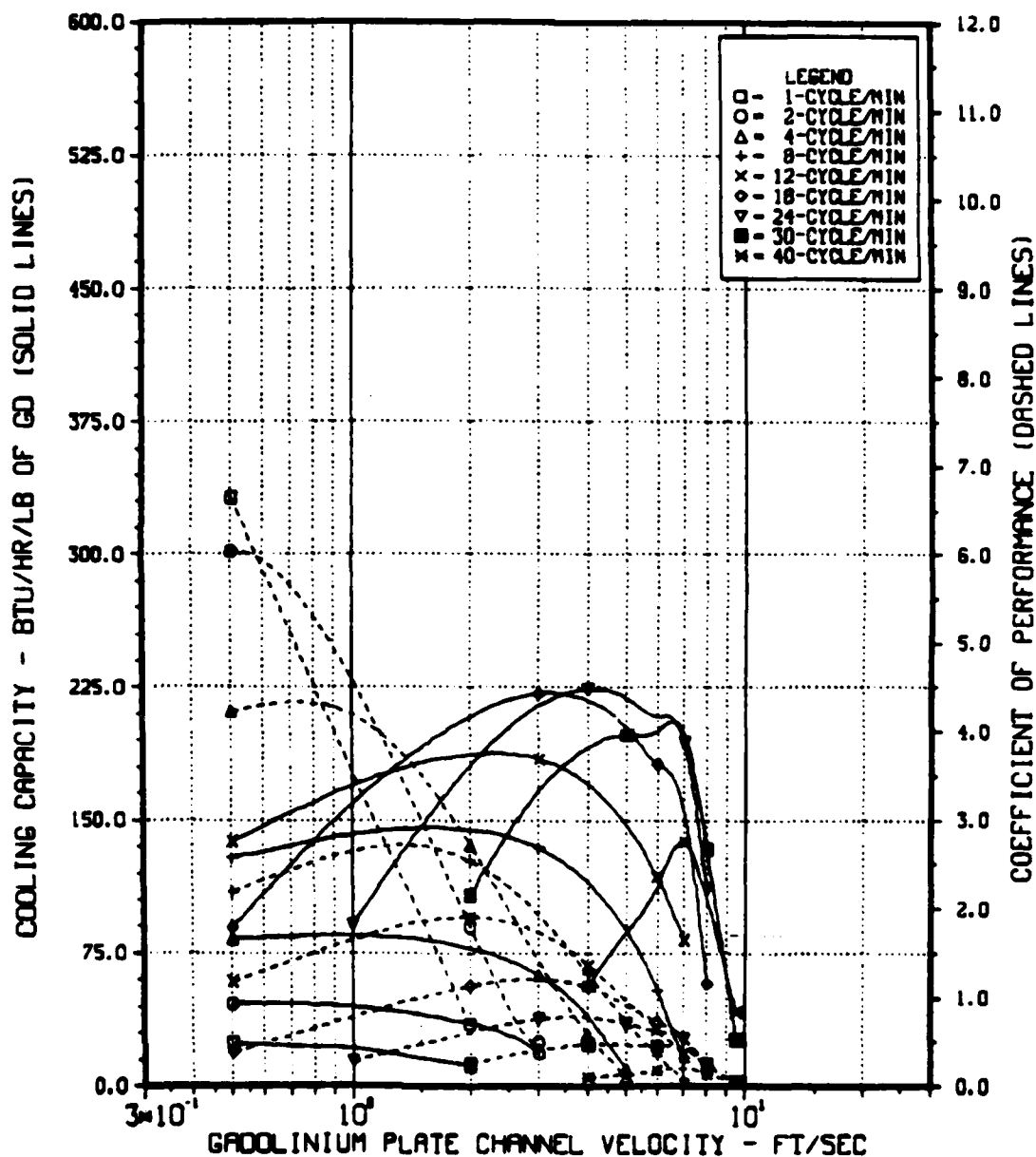CHANNEL LENGTH, IN.- 0.50   PLATE THICKNESS, IN.- 0.010    TEMP COLD END,F- 45.    H.E.DELTA-T,F- 5.0



Figure 18c:   Effects of External Heat Exchanger Temperature
Difference on Performance at 2-Tesla

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The results presented in figures 9 through 18 all show the highest COP's to be occurring at the low velocities while maximum cooling capacity is occurring at higher velocities. In general, cooling capacity increased with channel velocity until frictional effects exceeded cooling gains and the cooling capacity dropped off. Increasing the cycle rate increases the cooling capacity until losses associated with establishing larger finite temperature differences between the Gadolinium and fluid to effect heat transfer at the higher cycle rates offsets any further gains. These losses result in a decrease in maximum COP with increasing cycle rate. The maximum cooling capacity for a given cycle rate essentially coincides with the maximum COP for that cycle rate.

Comparing the 7 and 2-Tesla systems for similar geometries and conditions, the 7-Tesla system provide approximately 4 to 6 times the cooling capacity of the 2-Tesla system. The 7-Tesla system yields higher COP's than the similar 2-Tesla system operating at the same plate channel velocity. The 7-Tesla system can operate at higher plate channel velocities and shows increasing cooling capacity up to approximately twice the cycle rate of the similar 2-Tesla system.

Figures 9 and 14 show the effects of plate spacing, $S_p$, on the system performace at 7 and 2-Tesla, respectively. Increasing plate spacing increases the COP and permits operation over a wider range of plate channel velocities with maximum cooling capacity being shifted to higher velocities. Maximum cooling capacity increased slightly for the 7-Tesla case but started to decline some for the 2-Tesla case. Thus increasing plate spacing improves performance of the cycle. However spacing the plates further apart can have an adverse affect on the magnet performance because of the increased void facing the magnet and its effect on magnet performance which is not considered in the figures.

Figures 10 and 15 show the effects of the Gadolinium channel length, $L_p$, on the performance for a 7 and 2-Tesla system, respectively. Increasing channel length causes a significant reduction in both cooling capacity and in COP due to the added frictional losses in the longer channel. Cycle rate at the maximum cooling capacity also decreased noticeably with increased channel

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length. Thus the shorter the Gadolinium channel length the better the cycle performance.

Figures 11 and 16 show the effect of the Gadolinium plate thickness on performance for a 7 and 2-Tesla system, respectively. Reducing plate thickness, significantly increased cooling capacity and enabled maximum cooling capacity to occur at higher cycle rates. Maximum COP's remained about the same for any given plate channel velocity but occurred for slightly higher cycle rates at reduced plate thicknesses. The thinner plates improve heat transfer because of less thermal resistance and provide increased surface area per pound of Gadolinium which lowers finite temperature differences between the fluid and plate necessary for heat transfer, but increases frictional losses due to the increased area. However, thinner plate thicknesses are desirable for improving cycle performance neglecting the magnet performance which could be adversely affected by facing a larger void with the thinner plates.

Figure 12 and 17 show the effect of thermal mixing on the perforance for a 7 and 2-Tesla system respectively. Thermal mixing results from such things as boundary layer formation in the column causing warmer fluid to be moved into a region of cooler fluid and vise versa. Thermal mixing was expressed as a temperature difference along the cold and the hot column caused by the thermal mixing and was assumed to be the same in both end columns. Increased mixing temperature decreases cooling capacity slightly and reduces COP noticeably. Increased mixing temperature results in more of the cooling load being required to be used in regeneration to cool the Gadolinium plates as the cold column passes through the plates (process 6 to 1) prior to the reverse trip (process 1 to 2) where the magnetic field is dropped to obtain the cooling load. Thus, less of the cooling load is available for the external cooling loop.

Figure 13 and 18 show the effect of the heat transfer temperature difference between the external loop and the cycle fluid for a 7 and 2-Tesla system, respectively. Increasing heat transfer temperature difference reduces both cooling capacity and COP. This results from the cycle operating over a larger temperature range, thus more heat must be transferred in the same amount of time. Thus, larger heat transfer temperature differences are

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required between the fluid and the Gadolinium which result in the performance decrease.

## SYSTEM SIZING

An important consideration in selecting a system is its physical size. To avoid gravitational mixing due to differences in fluid density, the column would operate vertically with the hot column on top and the more dense fluid of the cold column on the bottom. The system size is primarily a function of plate geometry and plate channel velocity. It is assumed that the frontal area of the plates is the same as the cross-sectional area of the column. Thus the flow through the plates is at a higher velocity than the column velocity due to the reduced free flow area caused by plate blockage. The ratio of free flow area through the plates to the column cross-sectional area is expressed by

$$R_A = \frac{S_p}{t_p + S_p} \tag{39}$$

and the cross sectional area of the column, $A_{CL}$, is found by

$$A_{CL} = \frac{12}{\rho_{GD} (1 - R_A) L_p} \tag{40}$$

figure 19 shows the effects of the free flow area ratio, $R_A$, and plate channel length on the column cross-sectional area. Very short plate length and high free flow area, which show good performance trends, are high in cross-sectional area and thus volume.

The fluid column length represents about half the system length since the column must be moved completely through the plates and back again to complete one cycle. System length was calculated from
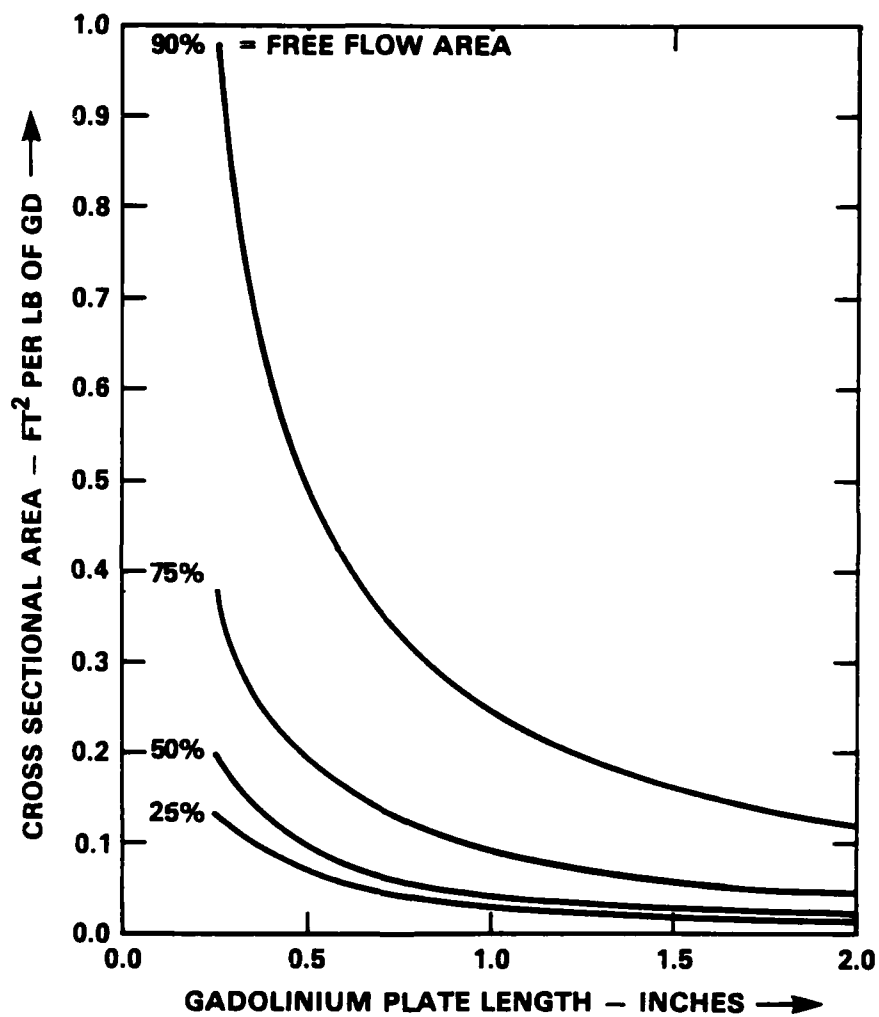
Figure 19:   Column Cross Sectional Area

$$L_s = \frac{60\ R_A\ V_p}{N_{RPM}} + \frac{L_p}{12} \tag{41}$$

Figure 20 shows a plot of system length as a function of plate channel velocity, cycle rate, and free flow area ratio assuming plate length equals zero. Increasing cycle rate decreasing free flow area, and reducing plate channel velocity will reduce system length. Multiplying equations (40) and (41) would give an idea of system volume per pound of Gadolinium not including the magnetic portion of the system. System volumes will be larger than conventional systems to improve performance.

## DISCUSSION OF RESULTS

Present room temperature refrigeration systems operate with best COP's on the order of 3 to 4. For Gadolinium magnetic heat pumps to be of interest COP's of at least 5 or greater would be necessary to provide any improvement over present state-of-the-art systems. Reviewing figures 9 to 18 show that COP's of 5 or greater occur at plate channel velocities of 2.5 FPS or less for a 7-Tesla system and at 1.5 FPS or less for a 2-Tesla system. At these conditions the 7-Tesla system showed a maximum cycle rate of 12 cycles per minute and a maximum cooling capacity of about 690 BTU per hour per pound of Gadolinium. For the 2-Tesla system, maximum cycle rate was 4 cycles per minute with a maximum cooling capacity of about 95 BTU per hour per pound of Gadolinium. At higher plate channel velocities, maximum cooling capacities of as much as 475 and 2300 BTU per hour per pound of Gadolinium were shown for a 2 and 7-Tesla system, respectively, but COP was less than two.

Figures 9 to 18 presented the effects of the various parameters on the system. Based on this information a more optimum system was evalauted using what were felt to be reasonable geometries and losses. The system shown in Table 2 was evaluated and presented in figures 21 and 22 for a 7 and 2-Tesla system respectively.

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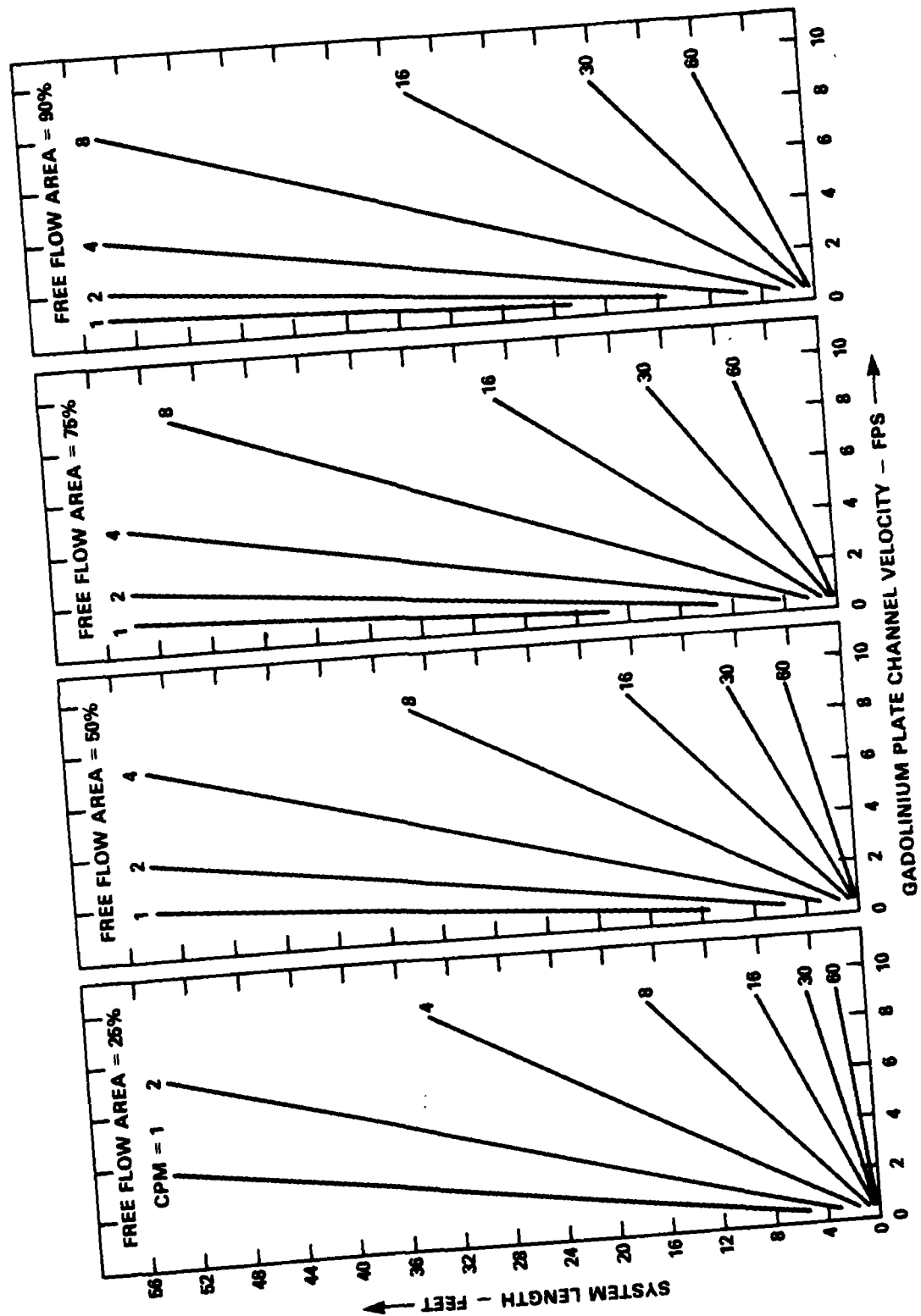
Figure 20: System Length

62                                         DTNSRDC PAS-82-11

### Table 2 - Optimum System Parameters

| Field Strength | 7 Tesla | 2 Tesla |
|---|---|---|
| Plate Spacing, $S_p$ | .060 Inches | .060 Inches |
| Plate Thickness, $t_p$ | .005 Inches | .005 Inches |
| Plate Length, $L_p$ | .250 Inches | .250 Inches |
| Mixing Temperature, $\Delta T_{mix}$ | 0.1 °F | 0.1 °F |
| Heat Exchanger, $\Delta T_{HE}$ | 1.0 °F | 0.5 °F |

Using the more optimum plate geometry even with some mixing temperature effects, system performance was improved. External heat exchanger temperature difference was assumed higher for a 7-Tesla system since it can operate at higher cycle rate which requires the larger value. Figures 21 and 22 show the performance of the 7 and 2-Tesla systems respectively of Table 2. Table 3 tabulates the optimum performance as well as its column size for the system based on a COP of 5. Maximum cooling capacities at the COP of 5 turned out to be 1200 BTU/HR/LB of GD at 7-Tesla and 130 BTU/HR/LB of GD at 2-Tesla.

### Table 3 - Optimum System Performance & Size

| Field Strength | 7 Tesla | 2 Tesla |
|---|---|---|
| COP | 5 | 5 |
| Plate Channel Velocity, Vp | 3 FPS | 2 FPS |
| Cycle Rate, $N_{RPM}$ | 22 cpm | 6 cpm |
| Cooling Capacity, $Q_{Cold}$ | 1200 BTU/HR/LB of GD | 130 BTU/HR/LB of GD |
| System Length, $L_s$ | 7.6 ft | 18.5 ft |
| Column Cross-Sectional Area ($A_{CL}$) per ton of refrigeration | 12.7 ft$^2$/ton | 117.4 ft$^2$/ton |
| Column Volume per ton of refrigeration | 96.5 ft$^3$/ton | 2172 ft$^3$/ton |

Higher cooling capacities are possible but only at COP's less than or comparable to present state-of-the-art systems and thus no performance incentive to consider magnetic heat pump application.

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 7.0 TESLA

CHANNEL HEIGHT,IN.- 2.00    PLATE SPACING,IN.  - 0.060    TEMP.HOT END,F - 85.    MIXING TEMP,F- 0.1

CHANNEL LENGTH,IN.- 0.25    PLATE THICKNESS,IN.- 0.005    TEMP COLD END,F- 45.    H.E.DELTA-T,F- 1.0
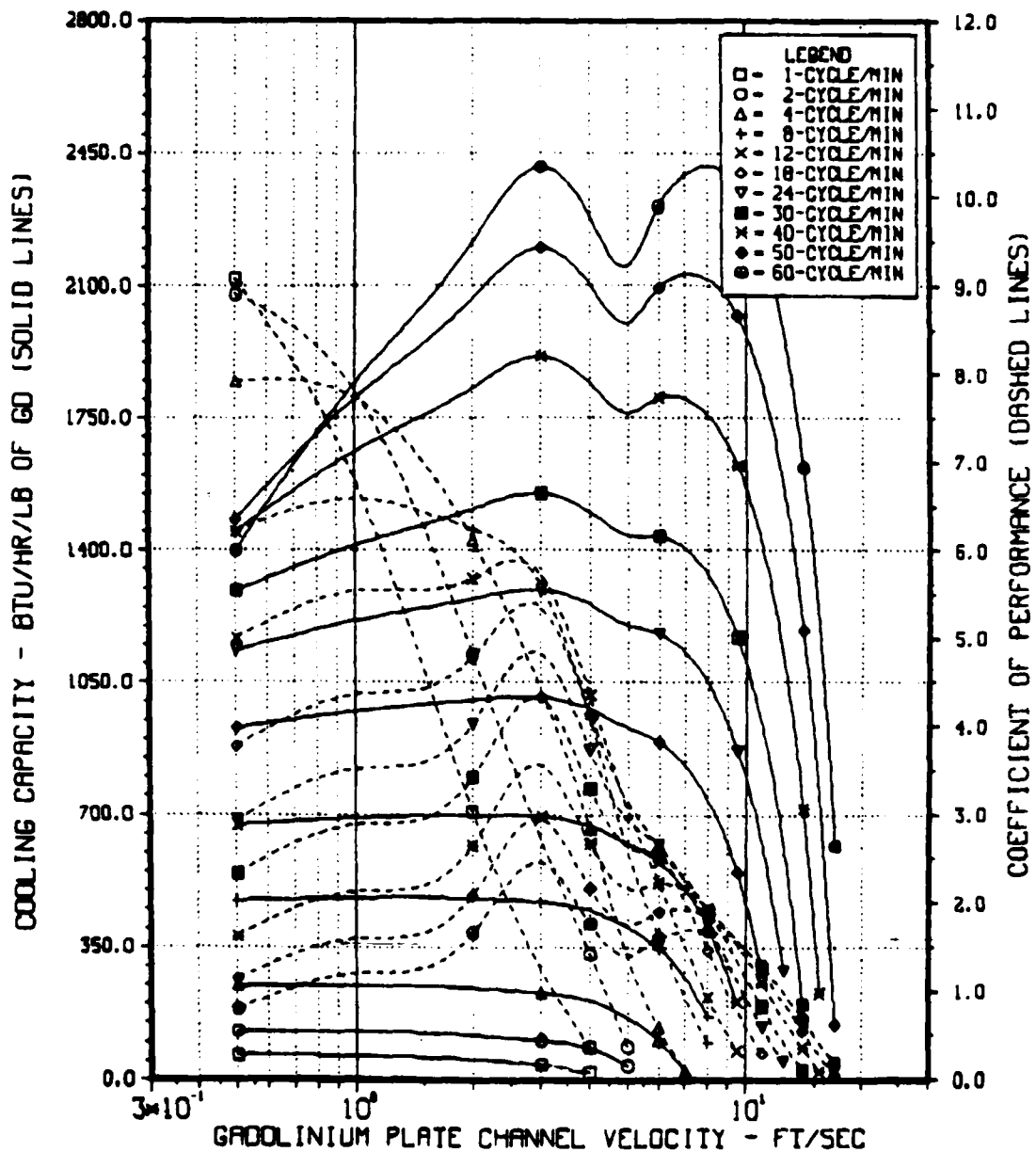


Figure 21:   Improved Performance of a 7-Tesla System

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MAGNETO THERMAL HEAT PUMP PERFORMANCE AT 2.0 TESLA

CHANNEL HEIGHT, IN. - 2.00    PLATE SPACING, IN.  - 0.060    TEMP.HOT END,F - 85.    MIXING TEMP,F- 0.1

CHANNEL LENGTH, IN. - 0.25    PLATE THICKNESS, IN.- 0.005    TEMP COLD END,F- 45.    H.E.DELTA-T,F- 0.5
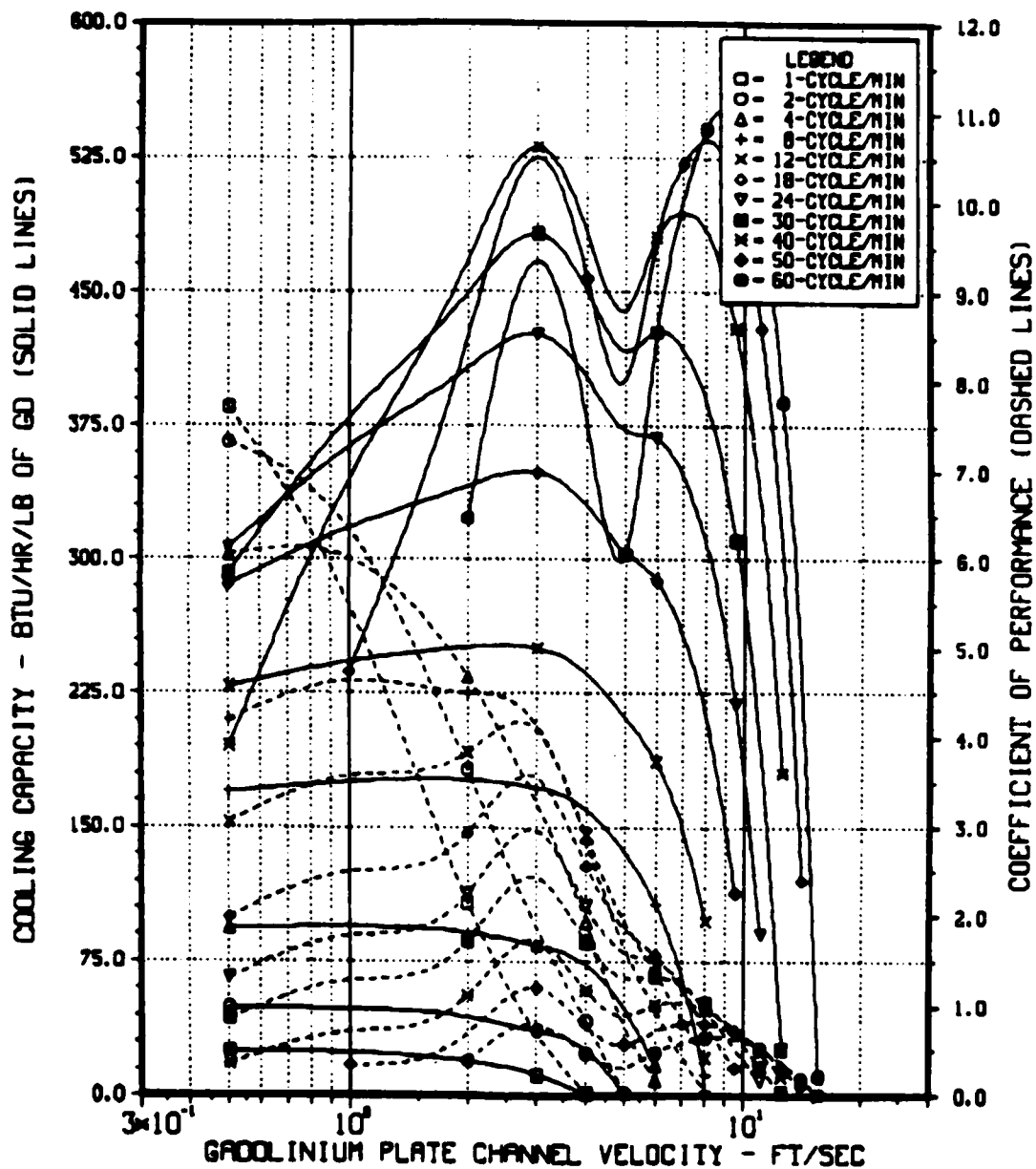


Figure 22:   Improved Performance of a 2-Tesla System

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The 1200 and 130 BTU/HR/LB of GD for the above system (COP = 5) is the maximum to be expected with any significant improvement in COP. Mixing temperature loss was assumed to be 0.1°F in the above system. Low mixing temperature loss may only be possible if velocity profile formation in the column is retarded. This could require some arrangement such as walls that move with the column to avoid velocity profile formation across the column. The above system will require 10-pounds of Gadolinium to obtain one-ton of refrigeration (12,000 BTU/HR) at 7-Tesla and 92.3 pounds of Gadolinium at 2-Tesla.

Table 3 also includes an idea of the column size obtained from use of equations 39 to 41. Column volumes of 96.5 and 2,171 $ft^3$/ton of refrigeration were obtained for a 7 and 2-Tesla system respectively. Column lengths were 7.6 and 18.5 ft for the 7 and 2-Tesla system respectively. These systems were sized for optimum performance rather than minimum size. Size could be reduced some at the expense of performance but would be considerably higher than the estimated 4-8 $ft^3$/ton for a freon type air conditioning system. The above column volumes do not include the volume of the magnetic system. The magnetic heat pump system will be considerably more voluminous than the more conventional state-of-the-art systems. Unless the field strengths could be pushed considerably beyond the 7 Tesla limit considered, magnetic heat pump column size can not be significantly reduced to compete with conventional systems.

In this analysis, the magnetic circuit efficiency, $\eta_m$, was taken as unity. Any losses in the magnetic circuit are not reflected in the system COP's that are presented. Generating the magnetic field represents the work input to the system which is part of the basis on which COP is determined. In equation (38) for COP, $\left(Q_{4-5}-Q_{F4-5}\right)$ represents the energy drawn from the magnet by the Gadolinium plates. Any losses in generating and transmitting this energy to the plates is included in the magnetic circuit efficiency and makes the work input to the magnet equal $\left(Q_{4-5}-Q_{F4-5}\right)/\eta_m$. Figure 23 shows the effect of magnet circuit efficiency on COP calculated from equation (38) compared to an $\eta_m = 1$ ideal case. For the COP of 5 case, a magnetic circuit efficiency of 0.9 would result in an actual COP of only 3.0 which barely approaches state-of-the-art systems. Magnetic circuit efficiencies of greater

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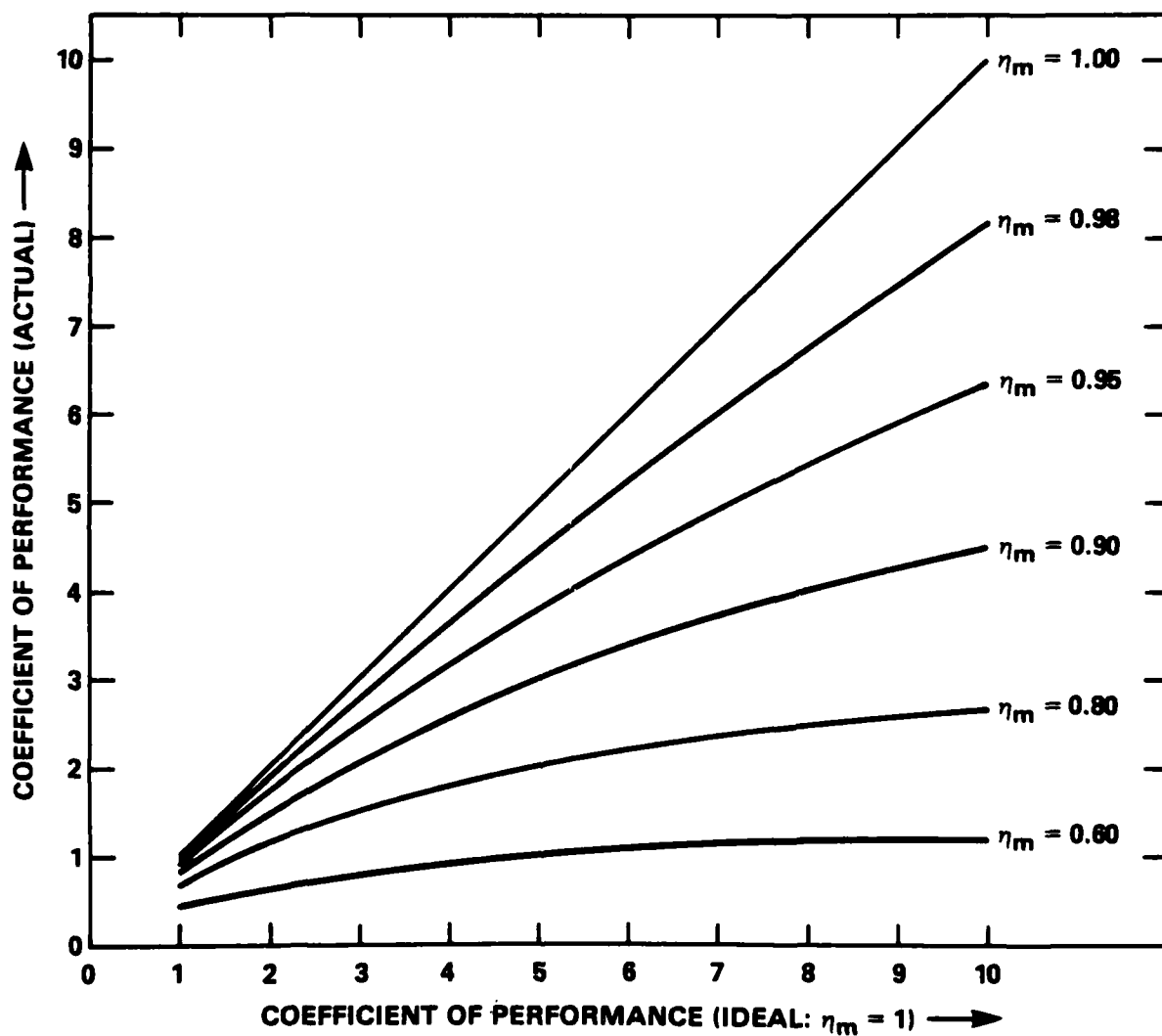
Figure 23:  Effects of Magnetic Circuit Efficiency on COP

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than .95 appear to be necessary to assure high COP performance.

The magnetic heat pump has been analyzed starting with a series of ideal processes and modifying these for the major losses. Friction losses could be calculated with a reasonable degree of certainty. However, mixing losses in the column were difficult to evaluate and were estimated over a reasonable range to show its effects. Actual calculated estimates for mixing losses would be required for any further serious consideration of the cycle. The cycle was evaluated with magnetic field strengths up to 7-Tesla which is the present limit of superconducting magnets. Improvements in field strength above this limit should impact favorably on this magnetic heat pump cycle by improved performance per pound of Gadolinium and reduced column size. The external heat exchanger loop which removes the cycle cooling load would ideally operate over a very small heat transfer temperature difference to improve cycle performance and the impact of the actual heat exchanger design on the system must be considered in any further consideration of cycle.

## CONCLUSIONS

Based upon the idealized thermal analysis of the magnetic heat pump the overall conclusion is that the particular system studied is not attractive for shipboard use due to its large size and unspectacular perfomance. An improved analysis and consideration of the magnetic and electrical portion of the system will most likely show that the performance, weight and volume are inferior to conventional freon heat pumps.

The following specific conclusions should be noted:

    o  A 7-Tesla system would provide approximately 4 to 6 times the cooling capacity of a 2-Tesla system for similar conditions and with COP's as much as 30 to 50 percent higher.

    o  The ideal plate geometry would minimize plate thickness and plate length and increase plate spacing to provide best cycle performance.

    o  At a cycle COP of 5, a 7-Tesla system could provide a cooling capacity of 1200 BTU per hour per pound of Gadolinium while a 2

Tesla system would operate around 130 BTU per hour per pound of
Gadolinium.  Higher cooling capacities are possible but at
unfavorable COP's while higher COP's are possible but at reduced
cooling capacities.

o  Magnet circuit efficiency must be high to obtain high COP's.
Magnet circuit efficiency of 80 percent or less would result in
actual operating COP's that are lower than best conventional systems
that range around COP equal 3 to 4.  Magnet circuit efficiencies of
95 percent or better are needed to consider magnet heat pumps for
improved performance.

o  The magnetic heat pump investigated would be considerably larger
than a conventional state-of-the-art cooling system by as much as an
order of magnitude.

o  Increasing magnetic field strength can decrease the system size
significantly by increasing the amount of cooling capacity, possible
per pound of Gadolinium.  However magnetic field strengths much
greater than the present superconducting state-of-the-art limit of
7-Tesla must be possible before the magnetic heat pump investigated
could compete in size with conventional systems.

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Appendix A:  Magnetic Heat Pump Performance Computer Program

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      PROGRAM MAGHEAT (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C      THIS PROGRAM EVALUATES THE PERFORMANCE OF AN ERICSSON CYCLE
C      MAGNETIC HEAT PUMP FOR ROOM TEMPERATURE APPLICATIONS USING
C      GADOLINIUM.
       COMMON QCX(50,50),VELX(50,50),NRX,NV(50),COPX(50,50)
       COMMON WC,LC,TC,TP,TFH,TFC,DTM,DTW,BCU
       COMMON N,NN
       REAL LC,KW,KF,MUF,NUF,LFN,PR
       INTEGER PHI
       DIMENSION PHI(32),NUF(32),TFX(10),RHOX(10),CFX(10),XKF(10),XMU(10)
      *,T(3),FQ(11),V(30)
       DATA((NUF(I),I=1,32)=.5,1.,1.5,2.,2.48,2.95,3.42,3.85,4.23,4.58,
      *5.19,5.89,6.67,7.16,7.50,8.04,8.67,9.54,10.4,11.6,13.1,14.4,16.2,
      *19.5,23.9,26.9,29.3,33.5,39.9,50.2,57.2,62.8)
       DATA((PHI(J),J=1,32)=1,2,3,4,5,6,7,8,9,10,12,15,20,25,30,40,60,90,
      *130,200,300,400,600,1000,2000,3000,4000,6000,10000,20000,30000,
      *40000)
       DATA((TFX(I),I=1,9)=32.,40.,50.,60.,70.,80.,90.,100.,150.)
       DATA((RHOX(I),I=1,9)=62.42,62.42,62.42,62.33,62.31,62.23,62.11,61.
      *88,61.20)
       DATA((CFX(I),I=1,9)=1.01,1.00,1.00,.999,.998,.998,.997,.998,1.00)
       DATA((XKF(I),I=1,9)=.319,.325,.332,.340,.347,.353,.359,.364,.384)
       DATA((XMU(I),I=1,9)=1.2,1.04,.88,.75,.658,.579,.514,.458,.292)
       DATA((FQ(J),J=1,11)=1.,2.,4.,8.,12.,18.,24.,30.,40.,50.,60.)
       DATA((V(K),K=1,20)=.5,1.,2.,3.,4.,5.,6.,7.,8.,9.5,11.,12.5,14.,15.
      15,17.,18.5,20.,21.5,22.5,23.5)
       N=0
       NN=1
    1 READ(5,100)WC,TC,LC,TP,DTM,BCU,DTW
  100 FORMAT(8F10.4)
       NPX=0
       IF(WC.EQ.0.0) GO TO 999
       DO 60 J=1,11
       FREQ=FQ(J)
       WRITE(6,101) WC,LC,TC,TP
  101 FORMAT(1H1/2X,"CHANNEL WIDTH =",F6.3," IN",5X,"CHANNEL LENGTH =",F
      *7.3," IN",5X,"PLATE SPACING =",F6.3," IN",5X,"PLATE THICKNESS =",F
      *6.3," IN"/)
       TFH=85.
       TFC=45.
       TFI=(TFH+TFC)*.5
       T(1)=TFC-DTW
       T(2)=TFH+DTW
       T(3)=TFI
       TIRP=1.
       BHU=BCU
       BCL=0.
       CALL GDPROP(1,BCU,T(1),DTCU,STCUI,BTCU)
       CALL GDPROP(1,BCL,T(1),DTCL,STCLI,BTCL)
       CALL GDPROP(1,BHU,T(2),DTHU,STHUI,BTHU)
       STHLI=STHUI+STCLI-STCUI
       STOT=(STHUI-STCUI)/(T(2)-T(1))
       AF=WC*TC/144.
       AS=WC*LC/144.*2.
       DL=4.*LC/12.*AF/AS
       DH=2.*WC*TC/(TC+WC)/12.
       RHOW=490.7
```

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```
      KW=5.8
      CPGD=0.071
      NPASS=1728./RHOW/(WC*LC*TP) + .1
      GDMOLE=157.26
      TO=459.7
      WRITE(6,102) TFH,TFC,FREQ,NPASS,DTW,DTM,BCU
102 FORMAT(2X,"TEMPERATURE HOT END =",F6.1," F      TEMPERATURE COLD EN
     *D =",F6.1," F      CYCLES/MINUTE =",F5.1,5X,"NO.OF CHANNELS/LB =",I
     *5//2X,"HEAT EXCHANGER TEMPERATURE DIFFERENCE =",F5.1," F",5X,"MIXI
     *NG TEMPERATURE CHANGE =",F5.1," F",3X," MAX.FIELD =",F4.1," TESLA"
     *//)
      WRITE(6,198)
      WRITE(6,199)
198 FORMAT(1X," VEL   REY.NO.    HAF   TIME    DEL-T   PHI        F
     *HL        U      UA      QR       QF  DT-CHR   TIME  LENGTH")
199 FORMAT(1X," (FPS)                      (SEC)                     (IN
     *.H2O) (B/HF2R)(B/H-R)  (B/H)  (B/H)   (F)    (SEC)    (FT)")
      NVX=0
      DO 50 K=1,20
      CCX=0.
      CHX=0.
      DCF=0.
      DHF=0.
      DFC=0.
      DFH=0.
      DFFC=0.
      DFFH=0.
      DGDC=0.
      DGDH=0.
      NCT=0
      DTPEG=0.
      TIRP=1.
   3  VEL=V(K)
      VELCH=VEL*TC/(TC+TP)
      DO 40 L=1,3
      DO 5 I=2,9
      IF(T(L).LT.TFX(I)) GO TO 30
   5  CONTINUE
  30  FBC=(T(L)-TFX(I-1))/(TFX(I)-TFX(I-1))
      RHOF=RHOX(I-1)*(1.-FBC)+RHOX(I)*FBC
      CF=CFX(I-1)*(1.-FBC)+CFX(I)*FBC
      KF=XKF(I-1)*(1.-FBC)+XKF(I)*FBC
      MUF=XMU(I-1)*(1.-FBC)+XMU(I)*FBC
      MUF=MUF*0.001
      PP=CF*MUF/KF*3600.
      GF=RHOF*VEL*3600.
      REY=RHOF*VEL*DH/MUF
      IF(REY.GT.3000.) GO TO 7
      PH=CF*GF*DE*DE/(KF*LC/12.)
      F=96./(REY*(1.+TC/WC)**2)
      GO TO 8
   7  F=0.184/REY**0.2
   8  HL=LC/DH*VEL*VEL/2.*F/32.174
      IF(REY.LT.3000.) GO TO 21
      HAF =0.023*KF/DH*REY**0.8*PR**0.33
      GO TO 22
  21  CONTINUE
```

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```
          DO 10 I=2,32
          IF(PH-PHI(I)) 15,20,10
   10 CONTINUE
   15 FAC=(PH-PHI(I-1))/(PHI(I)-PHI(I-1))
          FNU=NUF(I-1)+FAC*(NUF(I)-NUF(I-1))
          GO TO 25
   20 FNU=NUF(I)
   25 HAF=FNU*KF/DE
   22 TIMS=LC/(12.*VEL)
          TIMH=TIMS/3600.
          U=1./(1./HAF+TP/24./KW)
          UAS=U*AS
          IF(TIRR.EQ.1.) GO TO 23
          IF(L-2) 26,27,28
   23 DTWC=(DTM+DCF)/2.
          TCU=T(1)-DTWC
          TCUW=T(1)+DTWC
          TCLW=TCUW-DFC
          TCL=TCLW-DCX
          IF(DGDC.GT.DFC) GO TO 50
          CALL GDPROP(1,BCU,TCU,DTCU,STCU,BTCU)
          CALL GDPROP(1,BCL,TCL,DTCL,STCL,BTCL)
          IF(STCL.LT.STCU) GO TO 50
          DTC=DTCU-DTCL
          STC=STCU-STCL
          DTWH=(DTM-DHF)/2.
          THL=T(2)+DTWH
          THLW=T(2)-DTWH
          THUW=THLW+DFH
          THU=THUW+DHX
          CALL GDPROP(1,BHU,THU,DTHU,STHU,BTHU)
          STHL=STHLI+STDT*(THL-T(2))
          IF(STHL.LT.STHU) GO TO 50
          CALL GDPROP(2,STHL,THL,DTHL,STHL,BTHL)
          DTH=DTHU-DTHL
          STH=STHL-STHU
          QPG=((THU-TCUW-DTREG)+(THLW-DTREG-TCL))*CPGC*FREQ*60.
          QR=QPG
          QC=(STCL-STCU)/GDMOLE*((TCL+TCU)/2. +459.7)*FREQ*60.
          QH=(STHL-STHU)/GDMOLE*((THL+THU)/2. +459.7)*FREQ*60.
          QTOT=QR+(QC+QH)*2.
          TICC=QC/QTOT
          TIRR=QR/QTOT
          TIHH=QH/QTOT
          GO TO 3
   26 QP=QC
          TIM=TICC/FREQ*60.
          DTREG=QC/UAS/TICC/NPASS
          DCX=DTREG
          LEN=VFLCH*TIM
          CLEN=LEN
          CW=(STCL-STCU)/GDMOLE*(T(1)+459.7)/DTC
          DFLT=EXP(-HAF*TIMH/(CW*RHOW*TP/24.))
          QF=HL*RHOF*AS*VEL*4.6293*TICC/12.*NPASS
          QCF=QF
          HCNT=GF*AF*TICC*CF*NPASS
          DFFC=QF/HCNT
```

```
      DFC=(QC-QF)/HCNT
      IF(NCT.LT.3) GO TO 40
      WRITE(6,103)
  103 FORMAT(1H )
      APER=(DFC-DFFC-DGDC)/DFC
      IF(APER.LE.0.) GO TO 50
      QP=(QC-QF)*APER
      GO TO 29
   27 QR=QH
      TIM=TIHH/FREQ*60.
      DTREG=QH/UAS/TIHH/NPASS
      DHX=DTREG
      LEN=VELCH*TIM
      HLEN=LEN
      CW=(STHL-STHU)/GDMOLE*(T(2)+459.7)/DTH
      DELT=EXP(-HAF*TIMH/(CW*RHOW*TP/24.))
      QF=HL*RHOF*AS*VEL*4.6293*TIHH/12.*NPASS
      QHF=QF
      HCNT=GF*AF*TIHH*CF*NPASS
      DFFH=QF/HCNT
      DFH=(QH+QF)/HCNT
      IF(NCT.LT.3) GO TO 40
      APEH=(DFH+DFFH-DGDH)/DFH
      GO TO 29
   28 QR=QRG
      TIM=TIRR/FREQ*60.
      LEN=VELCH*TIM/2.
      RLEN=LEN
      DTREG=QR/UAS/TIRR/NPASS
      CW=(STHU-STCU)/GDMOLE*(TFI+459.7)/(TFH-TFC)
      DELT=EXP(-HAF*TIMH/(CW*RHOW*TP/24.))
      QRF=HL*RHOF*AS*VEL*4.6293*TIRR/12.*NPASS
      QF=QRF
      HCNT=GF*AF*TIRR*CF*NPASS
      DCF=QRF/HCNT
      DHF=DCF
      DGDC=CPGD*FREQ*60.*(2.*DTWC+DTREG)/(GF*AF*TICC*CF*NPASS)
      DGDH=CPGD*FREQ*60.*(2.*DTWH+DTREG)/(GF*AF*TIHH*CF*NPASS)
      IF(NCT.LT.3) GO TO 40
   29 WRITE(6,200) VEL,REY,HAF,TIMS,DELT,PH,F,HL,U,UAS,QR,QF,
     *DTREG,TIM,LEN
  200 FORMAT(F6.2,2F3.2,F6.3,F7.4,F8.2,F8.4,F7.3,F9.2,F8.3,F7.1,F7.2,F7.
     *3,F7.2,F8.3)
   40 CONTINUE
      IF(NCT.LT.3) GO TO 48
      BETA=(TFC+459.7)/(TFH-TFC)
      COP=(QC-QCF)*APER/(QH-(QC-QCF)*APER)
      RATIO=COP/BETA
      TLEN=CLEN+HLEN+RLEN+LC/12.*(1.-TC/(TC+TP))
      TILEN=TLEN*12.
      TLPAT=TILEN/LC
      GO TO 49
   48 NCT=NCT+1
      TIRR=1.
      GO TO 3
   49 WRITE(6,300) BTHL,DTC,DTH,RCU,BETA,COP,RATIO
  300 FORMAT( /5X,"BHL =",F6.3,5X,"DTC =",F7.2,5X,"DTH =",F7.2,5X,"BCU =
```

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```
     *",F6.3,5X,"BETA =",F7.2,5X,"COP =",F7.3,5X,"RATIO(COP/BETA)=",F7.4
     *)
     WRITE(6,301) STCU,STCL,STC,STHU,STHL,STH,TLRAT
301 FORMAT( 5X,"STCU =",F7.3,4X,"STCL =",F7.3,4X,"STC =",F7.3,4X,"STHU
     * =",F7.3,4X,"STHL =",F7.3,4X,"STH =",F7.3,5X,"LENGTH RATIO=",F8.2)
     WRITE(6,302) TCL,TCU,THL,THU,TLEN,TILEN
302 FORMAT( 5X,"TC(LB)=",F7.2,5X,"TC(UB)=",F7.2,5X,"TH(LB)=",F7.2,5X,
     *"TH(UB)=",F7.2,5X,"TOTAL LENGTH =",F7.3," FT (",F7.2," IN)")
     WRITE(6,104) TCU,TCLW,TCL,CCF,DFC,DFFC,DGOC,DTWC,APER
     WRITE(6,104) THU,THUW,THL,DHF,DFH,DFFH,DGOH,DTWH,APEH
104 FORMAT(9F14.4)
     IF(APER.LT.0.) GO TO 60
     IF(QCF.GT.QC) GO TO 60
     IF(NVX.EQ.0) NRX=NRX+1
     NVX=NVX+1
     VELX(NRX,NVX)=VEL
     QCX(NRX,NVX)=(QC-QCF)*APER
     COPX(NRX,NVX)=COP
     NV(NRX)=NVX
 50 CONTINUE
 60 CONTINUE
     N=N+1
     CALL GRNBRD
     GO TO 1
999 STOP
     END
```

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```
      SUBROUTINE GDPROP(NCASE,X,Y,CT,ST,DT)
C     THIS SUBROUTINE CONTAINS PHYSICAL PROPERTIES OF GADOLINIUM.
      DIMENSION TK(14),B(5),DTK(5,14),S(5,14),DTL(14),SIL(14)
C     CASE 1= X(TESLA)    Y(TEMP)        CASE 2= X(ENTROPY)    Y(TEMP)
      DATA((TK(I),I=1,14)=263.,269.,273.,279.,283.,288.,293.,298.,303.,
     *308.,313.,318.,323.,328.)
      DATA((B(I),I=1,5)=7.,5.,3.,1.,0.)
      DATA(((DTK(J,I),I=1,14),J=1,5)=
     7 7.40, 8.30, 8.70, 9.65,10.80,12.31,13.95,13.72,13.28,12.51,11.63,
     *10.95,10.20, 9.31,
     5 5.44, 6.00, 6.62, 7.32, 8.20, 9.75,11.02,10.70,10.13, 9.45, 8.70,
     * 7.92, 7.11, 6.40,
     3 3.45, 3.76, 4.20, 4.72, 5.50, 6.70, 7.82, 7.35, 6.70, 5.85, 5.12,
     * 4.40, 3.74, 3.18,
     1 1.26, 1.41, 1.56, 1.79, 2.14, 2.72, 3.54, 2.92, 2.31, 1.67, 1.26,
     * 0.92, 0.65, 0.53,
     Z0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.)
      DATA(((S(J,I),I=1,14),J=1,5)=
     714.51,14.70,14.89,15.07,15.24,15.42,15.59,15.77,15.93,16.10,16.26,
     *16.41,16.58,16.73,
     514.58,14.77,14.96,15.13,15.31,15.49,15.68,15.85,16.02,16.20,16.36,
     *16.51,16.67,16.81,
     314.66,14.85,15.04,15.22,15.40,15.59,15.78,15.96,16.13,16.30,16.47,
     *16.61,16.77,16.90,
     114.74,14.94,15.14,15.33,15.53,15.72,15.92,16.11,16.29,16.44,16.60,
     *16.73,16.87,16.98,
     Z14.79,14.99,15.19,15.40,15.60,15.83,16.06,16.22,16.37,16.50,16.63,
     *16.76,16.89,16.99)
      IF(NCASE.EQ.2) GO TO 20
10    T=(Y-32.)*5./9.+273.
      DO 11 J=3,5
      IF(X.GE.B(J)) GO TO 12
11    CONTINUE
12    DO 13 I=3,14
      IF(T.LE.TK(I)) GO TO 14
13    CONTINUE
14    L2=I
      L1=I-2
      DO 15 L=L1,L2
      CALL TRIQUA(B(J-2),B(J-1),B(J),DTK(J-2,L),DTK(J-1,L),DTK(J,L),A1,
     *A2,A3)
      CALL TRIQUA(B(J-2),B(J-1),B(J),S(J-2,L),S(J-1,L),S(J,L),B1,B2,B3)
      DTL(L)=A1*X*X+A2*X+A3
      SIL(L)=B1*X*X+B2*X+B3
15    CONTINUE
      CALL TRIQUA(TK(I-2),TK(I-1),TK(I),DTL(I-2),DTL(I-1),DTL(I),C1,C2,
     *C3)
      CALL TRIQUA(TK(I-2),TK(I-1),TK(I),SIL(I-2),SIL(I-1),SIL(I),D1,D2,
     *D3)
      CT=C1*T*T+C2*T+C3
      DT=DT*9./5.
      ST=D1*T*T+D2*T+D3
      RETURN
20    T=(Y-32.)*5./9.+273.
      DO 21 I=3,14
      IF(T.LE.TK(I)) GO TO 22
21    CONTINUE
```

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```
22 DO 23 J=1,5
   CALL TRIQUA(TK(I-2),TK(I-1),TK(I),DTK(J,I-2),DTK(J,I-1),DTK(J,I),
  *E1,E2,E3)
   CALL TRIQUA(TK(I-2),TK(I-1),TK(I),S(J,I-2),S(J,I-1),S(J,I),F1,F2,
  *F3)
   DTL(J)=E1*T*T+E2*T+E3
   SIL(J)=F1*T*T+F2*T+F3
23 CONTINUE
   DO 24 J=3,5
   IF(X.LE.SIL(J)) GO TO 25
24 CONTINUE
25 CALL TRIQUA(SIL(J-2),SIL(J-1),SIL(J),DTL(J-2),DTL(J-1),DTL(J),
  *G1,G2,G3)
   CALL TRIQUA(SIL(J-2),SIL(J-1),SIL(J),B(J-2),B(J-1),B(J),H1,H2,H3)
   DT=G1*X*X+G2*X+G3
   DT=DT*9./5.
   BT=H1*X*X+H2*X+H3
   RETURN
   END




      SUBROUTINE TRIQUA(X1,X2,X3,Y1,Y2,Y3,A,B,C)
C     THIS SUBROUTINE FINDS COEFFICENTS FOR A QUADRATIC EQUATION
      A=(1./(X2-X3))*((Y1-Y2)/(X1-X2)-(Y1-Y3)/(X1-X3))
      B=(Y1-Y3)/(X1-X3)-A*(X1+X3)
      C= Y2-A*X2*X2-B*X2
      RETURN
      END
```

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```
      SUBROUTINE GRNBRD
C     THIS SUBROUTINE PLOTS THE OUTPUT OF THE MAIN PROGRAM.
      COMMON QCX(50,50),VELX(50,50),NRX,NV(50),COPX(50,50)
      COMMON WC,LC,TC,TP,TFH,TFC,DTM,DTW,BCU
      COMMON N,NN
      DIMENSION VELXX(50),QCXX(50),COPXX(50),IPAK(120)
      REAL LC
      CALL COMPOS
      CALL BGNPL(N)
      CALL PAGE(11.,8.5)
      CALL PHYSOR(9.8,1.5)
      CALL NOBRDR
      CALL BANGLE(90.)
      CALL TITLE(" ",0,"GADOLINIUM PLATE CHANNEL VELOCITY - FT/SEC$",100
     1,"COOLING CAPACITY - BTU/HR/LB OF GD (SOLID LINES)$",100,6.,8.)
      CALL HEIGHT(.13)
      CALL MESSAG("MAGNETO THERMAL HEAT PUMP PERFORMANCE AT$",100,.2,9.)
      CALL REALNO(BCU,1,"ABUT","ABUT")
      CALL MESSAG(" TESLA$",100,"ABUT","ABUT")
      CALL HEIGHT(.09)
      CALL MESSAG("CHANNEL HEIGHT,IN.=$",100,-.7,9.6)
      CALL REALNO(WC,2,"ABUT","ABUT")
      CALL MESSAG("    PLATE SPACING,IN.  =$",100,"ABUT","ABUT")
      CALL REALNO(TC,3,"ABUT","ABUT")
      CALL MESSAG("    TEMP,HOT END,F =$",100,"ABUT","ABUT")
      CALL REALNO(TFH,0,"ABUT","ABUT")
      CALL MESSAG("   MIXING TEMP,F=$",100,"ABUT","ABUT")
      CALL REALNO(DTM,1,"ABUT","ABUT")
      CALL MESSAG("CHANNEL LENGTH,IN.=$",100,-.7,9.3)
      CALL REALNO(LC,2,"ABUT","ABUT")
      CALL MESSAG("    PLATE THICKNESS,IN.=$",100,"ABUT","ABUT")
      CALL REALNO(TP,3,"ABUT","ABUT")
      CALL MESSAG("   TEMP COLD END,F=$",100,"ABUT","ABUT")
      CALL REALNO(TFC,0,"ABUT","ABUT")
      CALL MESSAG("    H.E.DELTA-T,F=$",100,"ABUT","ABUT")
      CALL REALNO(DTW,1,"ABUT","ABUT")
      HPL=0.1
      CALL HEIGHT(HPL)
      IDUMMY=LINEST(IPAK,120,40)
      CALL LINES(13H 1-CYCLE/MIN$,IPAK,1)
      CALL LINES(13H 2-CYCLE/MIN$,IPAK,2)
      CALL LINES(13H 4-CYCLE/MIN$,IPAK,3)
      CALL LINES(13H 8-CYCLE/MIN$,IPAK,4)
      CALL LINES(13H12-CYCLE/MIN$,IPAK,5)
      CALL LINES(13H18-CYCLE/MIN$,IPAK,6)
      CALL LINES(13H24-CYCLE/MIN$,IPAK,7)
      CALL LINES(13H30-CYCLE/MIN$,IPAK,8)
      CALL LINES(13H40-CYCLE/MIN$,IPAK,9)
      CALL LINES(13H50-CYCLE/MIN$,IPAK,10)
      CALL LINES(13H60-CYCLE/MIN$,IPAK,11)
      XW=XLEGND(IPAK,NRX+1)
      YH=YLEGND(IPAK,NRX+1)
      XWA=5.8-XW
      YHA=7.8-YH
      CALL BLNK1(-7.9,-7.7+YH,5.7-XW,5.9,1)
      CALL FRAME
      CALL RESET("HEIGHT")
```

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```
      CALL XTICKS(2)
      CALL YTICKS(5)
      CALL XLOG(.3,3.,0.,75.0)
      CALL DOT
      CALL GRID(1,1)
      CALL RESET("DOT")
      CALL SPLINE
      CALL NOCHEK
      DO 10 NC=1,NRX
      NPX=NV(NC)
      DO 8 I=1,NPX
      VELXX(I)=VF_X(NC,I)
      QCXX(I)=QCX(NC,I)
    8 CONTINUE
      CALL CURVE(VELXX,QCXX,NPX,3)
   10 CONTINUE
      CALL YTICKS(2)
      CALL YGRAXS(0.,1.,12.,8.,"COEFFICIENT OF PERFORMANCE (DASHED LINES
     1)$",-100,6.,0.)
      CALL DASH
      DO 20 NC=1,NRX
      NPX=NV(NC)
      CALL MARKER(NC-1)
      DO 18 I=1,NPX
      VELXX(I)=VF_X(NC,I)
      COPXX(I)=COPX(NC,I)
   18 CONTINUE
      CALL CURVE(VELXX,COPXX,NPX,2)
   20 CONTINUE
      CALL RESET("BLNK1")
      CALL HEIGHT(HP_)
      CALL LEGEND(IPAK,NRX,XWA,YHA)
      CALL ENDPL(N)
      IF(N.EQ.NN) GO TO 25
      RETURN
   25 CALL DONEPL
      RETURN
      END
```

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