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A GENERALIZED CURVE APPROACH TO ELEMENTARY PARTICLES.(U)
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A GENERALIZED CURVE APPROACH TO ELEMENTARY PARTICLES

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L. C. Young** and P. Nowosad***

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ABSTRACT

The theory of MRC Technical Summary Report 2067 is applied in a simplified form to a Relativistic One-Particle Universe, and boundary and symmetry conditions are imposed. The existence, for the particle, of a series of resonance states and a series of gauge states is derived. For the proton and the electron, a selection of such states account for the elementary particles observed to date.

AMS (MOS) Subject Classifications: 81G25, 81G20, 81N05, 35D05, 49A10

Key Words: generalized curves, wave-algebras, dislocation line, tunnel condition, symmetry, group representations, unimodular matrices, Laplacian, eigenfunctions, harmonic functions, analytic functions, relativistic metric, energy integral, resonance, gauge, proton, electron, photon, neutrino, cosmological constant, elementary particles

Work Unit Number 2 (Physical Mathematics)

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*This work was carried out at IMPA while the first author was visiting there in November 1981.

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A GENERALIZED CURVE APPROACH TO ELEMENTARY PARTICLES*

L. C. Young** and P. Nowosad***

(Dedicated to Lamberto Cesari.)

§1. Introduction. It is often stressed that the Calculus of Variations, or its modern form, Optimal Control, originated concepts of importance in other fields. This was illustrated, in the case of the concept of generalized curve, in the MRC report [8]. As regards Part I of that report, we take this opportunity of drawing attention to some recent researches, in fields not always considered close to variational theory. In differential and partial differential equations (where Cesari pioneered methods from Optimal Control), problems of electro-magnetism and the like have now led Tartar and Murat to "compensated compactness", a topic intimately related to generalized curves. Probability distributions, of the same general nature as in generalized curves, are also beginning to be considered, for instance by Sinai, in dynamical systems. In engineering, lines of stress, that approximate generalized curves, have been found by Sneddon, and an appropriate theory of the limiting behavior of porous or re-inforced material has been developed by the school of Lions. However the present note is intended to continue Part II of the report, and to provide adjustments and developments for what was then, to an even larger extent than now, a preview. The full details will be given in a book by Nowosad, nearing completion.

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§2. The, ρ,t metric and its transformation to a Poincaré half-plane. We need to fit in material about wave-functions and eigenfunctions: it is available from a few pages of Titchmarsh [7, p. 93 sequ]. Also as regards symmetries, we need material given in a different metric in the book of Gelfand and others [3, p. 33 sequ]. We therefore require a $C \times C \to C \times C$ transformation of the metric

$$(1) \qquad d\sigma^2 = \frac{dx_1^2 - dx_4^2}{\cosh^2 \mu x_1} \quad \text{into (1*)} \quad d\sigma^2 = \frac{dx^2 + dy^2}{y^2} .$$

Let β be a complex constant (eventually chosen to be $-i$), and let x_1, x_4, x, y be complex variables. An appropriate transformation, ignoring in (1*) a constant factor $-1/\mu^2$ of no importance, is

$$(T) \qquad x + iy = \beta - ie^{\mu(x_1+x_4)}, \quad x - iy = \beta + ie^{\mu(-x_1+x_4)} .$$

For reference to Gelfand, we shall use the metric (1*) in the real half-plane $y > 0$, the so-called Poincaré half-plane. This we defer to the next section, but we shall here quote some of the results affecting the metric (1) for real x_1, x_4. These two real variables we shall often write ρ,t: the metric (1) is then viewed as an abstract form of one with more familiar coordinates, and for this, ρ becomes the function

$$(2) \qquad \rho(r) = \pm \frac{1}{2\mu} \log \coth\left(\frac{1}{2} \mu|r - r_0|\right)$$

of a radius r, so that in particular $r = \infty$ corresponds to $\rho = 0$. Eventually our results must be translated into those in familiar coordinates.

In the metric (1) we require eigenfunctions $U = V(\rho)T(t)$ for the associated Laplacian $\Delta_1 = \cosh^2\mu\rho(\partial^2/\partial\rho^2 - \partial^2/\partial t^2)$. They satisfy $\Delta_1 U = kU$, so that we can write

$$(3) \qquad T(t) = Ae^{t\sqrt{p}} + Be^{-t\sqrt{p}}, \qquad Y'' + \left(\Lambda - \frac{\alpha(1-\alpha)}{4\cosh^2 \frac{X}{2}}\right)Y = 0 ,$$

if (to agree with Titchmarsh) we set $X = 2\mu\rho$, $Y(X) = V(\rho)$, $-p/(4\mu^2) = \Lambda$, $k/\mu^2 = \alpha(1-\alpha)$. Here $\Lambda > 0$, as we must exclude, from (3), solutions which die or blow up as functions of t.

For $k = 0$, the harmonic case in the metric, we have no need of Titchmarsh and we retain the notation $U = VT$. Then in (3) T is written as usual as the combination of $e^{\pm i\omega t}$, and $V(\rho)$ is $\sin(\omega\rho + \gamma_\omega)$. However $V = 0$ at $r = \infty$ (i.e. at $\rho = 0$) and $V_r = 0$ at $r = 0$ (i.e. $V_\rho = 0$ at $\rho = \rho_0 = \rho(r_0)$) are here boundary conditions. We can thus set $\gamma_\omega = 0$, and the conditions at $r = 0$, together with the symmetry condition of the next section, lead to

(4) $$\omega = (n + \tfrac{1}{2})\pi/\rho_0, \quad \omega/\mu = s + 1, \quad \text{where } n,s = 0,1,2,\ldots .$$

For $k \neq 0$, Titchmarsh gives the solutions Y. Those for $\Lambda \leq 0$ hold only for particular α, and if we exclude $\Lambda < 0$, they are Legendre polynomials in $\tanh \frac{1}{2} X$ and α has to be an integer > 1. For $\Lambda > 0$ we get on α a restriction from a boundary condition on V, imposed at $X = \pm\infty$. We thus find

(5) $$\text{either } \Lambda = 0, \quad \alpha = N + 1, \quad \text{or} \quad \Lambda > 0, \quad \alpha = \tfrac{1}{2} \pm i\gamma_N$$

where $\gamma_N = \sqrt{N(N+1) - 1/4}$, $N = 1,2,\ldots$. For $\Lambda > 0$ the boundary condition is referred to as a tunnel condition (Landau [6], p. 102). Near $X = \pm\infty$, we have to a high approximation $Y'' + \Lambda Y = 0$, so that Y is a virtual combination of $\exp(\pm iX\sqrt{\Lambda})$. The condition requires that if Y at $+\infty$ is $\exp(+iX\sqrt{\Lambda})$, then the term $\exp(-iX\sqrt{\Lambda})$ must drop out also at $-\infty$. Landau gives as the coefficient that must vanish

$$\Gamma(i\Lambda)\Lambda(1 - i\Lambda)/\{\Gamma(-a)\Gamma(1 + a)\} \quad \text{where} \quad a = \tfrac{1}{2}\sqrt{4\alpha(1-\alpha) + 1} - \tfrac{1}{2} .$$

As this only vanishes when $\Gamma(-a) = \infty$, we see that a is a positive integer N, and this gives $\alpha = \frac{1}{2} \pm i\gamma_N$.

On the continuous spectrum $\Lambda > 0$, Titchmarsh gives an integral representation of the Fourier type for square integrable functions of X. It involves two densities $\xi'(\Lambda), \zeta'(\Lambda)$ where ζ' increases, while ξ' has a maximum depending on the parameter α, and so in our case on N. The approximate values of $\sqrt{\Lambda}$ turn out to be

(6) $$\tfrac{3}{4} \text{ for } N = 1, \quad \tfrac{1}{10} + \tfrac{1}{2}\gamma_N \text{ for } N \geq 2 \text{ where } \gamma_N = \sqrt{N(N+1) - 1/4} .$$

These high density positions are important in that their neighborhood contributes most highly to the Fourier type integral.

§3. Symmetries. In the Poincaré half-plane of $z = x + iy$, $y > 0$, the symmetries we need correspond to the subgroup K of rotations

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

in the unimodular group G of 2×2 matrices. The rest of G is of no particular interest. K matters: anything in the metric (1) whose image in (1*) lacks the proper invariance relative to K will be unstable. Our symmetry condition is so formulated because Gelfand is available. Gelfand defines, for appropriate functions, transformations $T_g = T_{s,g}$, where $g \in G$ and where s is a parameter. There is a principal series, with $s = i\tau$, τ real, a supplementary series with $-1 < s < 1$, and there are two analytic series, corresponding to positive or negative integers s. The first two series can be defined on the C^∞ bounded functions $\varphi(z)$ subject to

$$-\Delta^*\varphi = \frac{1}{4}(1 - s^2)\varphi ,$$

where Δ^* is the Laplacian in (1*), i.e. the usual one times y^2. These φ are eigenfunctions of Δ^* invariant under the rotations in K. They are also images of eigenfunctions in (1), and the boundedness of such an image reduces, in view of the explicit hypergeometric form given by Titchmarsh, to the condition $\Lambda > 0$ already imposed. Taking account of a change of scale by μ, our symmetry condition is thus satisfied if $k/\mu^2 = \frac{1}{4}(1 - s^2)$, i.e. if this last quantity is $\alpha(1 - \alpha)$, i.e. if

$$\alpha = \frac{1}{2} \pm \frac{s}{2} .$$

Evidently this is satisfied, with $s = i\tau$ (principal series), for $\Lambda > 0$, on account of (5). Our tunnel condition thus already implies the desirable symmetry for this case.

We next come to the first of the analytic series of $T_{s,g}$. Now the transformations are on functions φ analytic for $y > 0$ and such that both φ and the function $\hat{\varphi}(z) = \varphi(z^{-1})/z^{s+1}$ are C^∞ in $y > 0$. The common eigenfunctions of the matrices of K under $T_{s,g}$, where now

$$T_{s,g}\varphi(z) = \varphi\left(\frac{az + c}{bz + d}\right)(bz + d)^{-s-1} ,$$

are the functions

$$\varphi_\ell(z) = (z - i)^\ell (z + i)^{-\ell-s-1}, \qquad \ell = 0,1,2,\ldots .$$

The symmetry for $k = 0$ amounts to $U = VT$ having as its (1*) image one of these φ_ℓ. We clearly must then have $\ell = 0$, $\beta = -i$. The condition moreover involves z and so the first equation (T), not $\bar{z}$, but to make real $x_1, x_4 \to$ real x,y we should write $\pm ix_1, \pm ix_4$ in place of x_1, x_4. We find that $(z + i)^{-s-1}$ becomes a constant multiple of $\exp(\pm i\omega(x_1 \pm x_4))$ only if $\omega = (s + 1)\mu$.

In the case $\Lambda = 0$, which still remains, the form of Y as a Legendre polynomial in $\tanh \frac{1}{2} X$ is enough to convince us of its invariance under rotations of the appropriate type. However the passage to the metric (1*) will suggest an important physical idea. We now need the second analytic series, which Gelfand gives in his other book [4, p. 468 sequ]. We write F_{-s} for the space of analytic functions φ of the first series $s = 0,1,2,\ldots$. We define further the space $\mathcal{D}_s$ of analytic functions φ on $y > 0$, such that $\varphi, \hat{\varphi}$ are C^∞ on $y \geq 0$, where $\hat{\varphi}(z) = z^{s-1}\varphi(-z^{-1})$. We define our new transformations $T_{s,g}$ on $\mathcal{D}_s$ by setting

$$T_{s,g}\varphi(z) = \left(\frac{az + c}{bz + d}\right)(bz + d)^{s-1} \quad \text{when} \quad g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in G .$$

Evidently these have in $\mathcal{D}_s$ an invariant subspace E_s consisting of the polynomials of degree $\leq s - 1$. Moreover, it turns out that we can map $\mathcal{D}_s/E_s$ on F_{-s} by differentiating the $\varphi(z)$ s times, and so wiping out the above polynomials. Conversely, from F_{-s} we get back to $\mathcal{D}_s$ by s integrations, which then restore the arbitrary additive term from E_s. Clearly it is in E_s that we must look for the images of our Legendre polynomials Y in $\tanh \frac{1}{2} X$, under a map suitably related to (T). This suggests that these polynomials Y play like their images in x,y, the same part of arbitrary additive terms, tacked on to an s-th "integral", which in the invariant harmonic case corresponds to the s-th integral of $(z + i)^{-s-1}$, i.e. apart from a constant factor to the lowest of the latter, namely $(z + i)^{-1}$. We shall loosely speak of our Y, for $\Lambda = 0$, as giving to a harmonic U a "twist" of order s, where U is itself associated with the value $s = 0$ in (4). The degree of the polynomial corresponds to N in the first part of (5), so that we must set $N \leq s - 1$.

§4. Adjustments to the theory. The things physicists speak of as quanta associated with elementary particles, arises from jumps of the energy, or Dirichlet integral, in the relevant metric, which we now take to be that of a one-particle universe, namely

(7) $$ds^2 = (\operatorname{sech}^2 \mu\rho)(d\rho^2 - dt^2) + dy^2 + dz^2 .$$

By limiting ourselves to this, we eliminate interaction, and so questions about quarks or the Pauli exclusion. But we may also be excluding the instabilities that might eliminate from our theoretical states some so far not observed. Here as before, ρ is related by (2) to a radius r, while t,y,z are usual coordinates.

We recall from the report [8], that when exponentials are used to ensure a multiplicative superposition of states, the energy integrand may become $f^{-1}\Delta_7 f$, where Δ_7 is the Laplacian in the metric (7). Jumps then come from zeros of particular f in an algebra M of wave-functions. One such algebra suffices for a photon, but we need two related M, M' for our particle, which then describes a generalized curve. The apparent velocity is $v = p_1 v_1 + p_2 v_2$ where p_1, p_2 are probabilities and $p_1 + p_2 = 1$; p_1, p_2 are associated also with an $f \in M$ and an $f' \in M'$, where the zeros of f, f' give the particle's position. In the report, v_1, v_2 were velocities of emitted and absorbed photons. We took for granted that a photon has a position, and for better localization the waves f had a real part f_1 and a complex part $f_2 + if_3$. M and M' were generated by the same complex parts, but different real parts. Here we do not localize photons: we allow real or complex f as the occasion arises. Nevertheless in effect a similar separation is obtained by the classical process of separation of variables in the solutions to Laplace's equation in the metric (7).

These solutions, the harmonic functions in the metric, are the main intrinsic objects that this theory is built on. From them, other objects are derived, just as from straight lines in the plane we derive curves as envelopes, and generalized curves as limiting zigzags.

§5. The radiation field of an electron or a proton. Nature has few principles, but an extraordinary variety of structures. The few preceding pages are the key to results of great diversity, and whose agreement with observation can hardly be fortuitous. But some cherished notions may have to be abandoned - particularly those Fourier tells us Nature has no room for: "les notions confuses".

The Laplacian for the metric (7) is the sum of the Laplacian Δ_1 for the metric 1 in ρ, t, and the ordinary Laplacian Δ in y, z. The algebras M, M' we naturally come to, will be derived from the harmonic functions for $\Delta_1 + \Delta$, and will be generated by products $U(\rho,t)W(y,z)$. The most basic turn out to be those for which U, W are themselves harmonic in their metrics. To retain some analogy with the separation of an f into f_1 and $f_2 + if_3$ in the preceding section, it is natural to identify U with f_1 and W with $f_2 + if_3$. The two algebras are then generated by (real) functions of $\rho \pm t$, multiplied by (complex) functions of $y + iz$, and the latter - if we subject them to the monochromatic conditions of the previous report - are analytic, or else anti-analytic (i.e. with the sign of i reversed). We leave out of our discussion the anti-analytic case, it applies to anti-particles. The pair of real functions of $\rho \pm t$ now balances in a sense the real and imaginary parts of the complex function of $y + iz$. (A more complete interpretation, in terms of Penrose twistors, will be given in Nowosad's book.)

For the desirable zeros of suitable members of M, M', we do not have to look far. As a standing singularity, there will be the line $y = z = 0$ in $\mathbb{R}_3$, on which $y + iz = 0$. It is of the same nature as the dislocation lines in crystals, found by Hirschfelder, Nye and Berry, and described by Dirac in 1948. (See for instance Jackson [5], p. 258 sequ.) But the really important singularity is the wave-front $\rho = \pm t$ of an emitted or absorbed spherical wave of light, as conceived by Huygens. The point, the Newtonian picture, what we observe when we speak of a photon hitting a photographic plate, is merely the intersection of the wave-front with the standing dislocation line. The spherical wave itself, as function of ρ, t is harmonic in the metric (1) and satisfies the boundary conditions of section 2, when we replace it by its constituent parts, which are products VT in the separate variables. We therefore get first of all the first relation (4); it expresses Planck's law (but with $n + \frac{1}{2}$ in place of n). In this formula π/ρ_0 is the quantum of energy except for units; μ is $\sqrt{(2\lambda)}$ where λ is the cosmological constant, and ρ_0 is $\frac{1}{2}\mu^{-1}\log\coth\frac{1}{2}R$, where $R = \mu r_0$. Of course ρ_0 here depends on the particle whose field determines the metric (7), and whose algebras are the pair M, M'. The superposition of out and in going waves from M, M' describes periodic ones with the above Planck frequencies.

We shall need some constants for the particle. They can now be calculated. The charge Q becomes an integral in $\mathbf{R}_3$ of μ times a Dirac function (0 except for $r = r_0$). The value found is $r_0^2\sqrt{8\pi\lambda}$, or using familiar cgs units

$$Q = \pm c^2 r_0^2 \sqrt{\lambda K/G}\ ,$$

where K is a coupling factor (from well-known equations $KT_{ij} = G_{ij} + \lambda g_{ij}$) and G and c are known. The energy E is similarly an integral outside and inside $r = r_0$, and we find that the mass

$$m_0 = E/c^2 = \left(\frac{\pi}{16} Kc^2/(G\mu)\right)\varphi(R)$$

where

$$\varphi(R) = 2R^2 + \frac{\pi^2}{12} + (\log 2)^2 + \frac{4}{K}\sum_n \frac{1\cdot 3\cdot 5\cdots\cdots(2n+1)}{2\cdot 4\cdots\cdots(2n)}\ \frac{1 - 2e^{-(n+1)R}}{(2n+1)^2}\ .$$

However we also had, from symmetry, the second equation (4). Thus R depends on $n, s = 0, 1, 2, \ldots$ by the equation

$$s + 1 = \frac{(2n+1)\pi}{\log\coth\frac{R}{2}}\ . \tag{8}$$

Experimentally, $\log\coth R/2$ is found in quite a different way. For the electron, for $n = 0$, the right hand side of (8) is found experimentally to be 3.02806. For the proton, for $n = 0, 1, 2, 3$ the experimental values are

1.143076, 3.429227, 5.715378, 8.00153 .

From this we should infer (taking account of experimental errors, admittedly large for the electron, where the whole procedure is very indirect) that the true values of s are 2 for the electron with $n = 0$, and 7 for the proton with $n = 3$. We are then able to get from (8) theoretical values for log coth $R/2$, more reliable than the experimental ones. The proton has moreover indeed what physicists refer to as a "strange" property: its most stable excited state is not $n = 1$, but $n = 3$. Finally we get, for the radius of the proton $.4283 \times 10^{-13}$cm, and for that of the electron 8.253×10^{-13}. For the electron, this is the first reliable estimate, as we cannot bombard it with smaller particles.

A related question concerns the intrinsic magnetic m ment. The formula for this is $\frac{1}{2} g_S \cdot (e\hbar/2m)$ and can now be proved with the above theory. Here $\pm\hbar/2$ is the "spin". The coefficient $\frac{1}{2} g_S$ has so far been an experimental one, with the values* 1.0012 for the electron, 2.7928 for the proton. However the theoretical coefficient is simply log coth $R/2$, and its corresponding values are 1.047198 and 2.748936, assuming as before $n = 0$, $s = 2$ for the electron, and $n = 3$, $s = 7$ for the proton. These are the values ($\pi/3, 7\pi/8$) experiments should give for $\frac{1}{2} g_S$, under conditions which do away with the disturbing magnetic field of the earth (about $\pm$ .045).

We come to the neutrino: like the photon it is not here a particle at all. Instead of a product, its algebra involves a sum. A typical function is $\varepsilon \exp i(\rho - t) - (y + iz)$. Thus the zeros are on the locus $y = \varepsilon \cos(\rho - t)$, $z = \varepsilon \sin(\rho - t)$, i.e. they lie on a cylinder of radius ε, on which the motion is a right or left handed screw. To detect a neutrino at all, we need a long portion of its screw path.

*Alonso-Finn [1, p. 135, p. 289].

§6. Resonance states and gauge states. Now, in the metric (7), we look for harmonic functions of the form $U(\rho,t)W(y,z)$, where U,W are not harmonic in their metrics, but subject to

$$\Delta W = -kW, \qquad \Delta_1 U = kU .$$

Circular symmetry in y,z leads to a separation of variables in polar coordinates there, and W becomes the product of a Bessel function and a trigonometric one. With U we proceed as in section 2, so that $k = \mu^2\alpha(1-\alpha)$ and we have the further parameter $\Lambda > 0$ and the relations (5).

The solutions corresponding to $\Lambda > 0$ give rise to what we shall call resonance states of our particle. They depend on the integer $N = 1,2,3,\ldots$. We have $\alpha = \frac{1}{2} \pm i\gamma_N$ where $\gamma_N = \sqrt{N(N+1) - 1/4}$; and we have a value Λ_N whose vicinity contains the important part of the range of Λ, and therefore contributes a major part to the energy. The approximate square root of Λ_N is given by (6), and we can now get a good theoretical estimate of the energy E_N arising from our "resonance". This increases, as is easy to see, the energy E of our particle to $(E^2 + E_N^2)^{1/2}$, so that it becomes

$$\left(n' + \sqrt{1 + \left(\frac{n+\frac{1}{2}}{s+1}\right)^2 \Lambda_N}\right)E_0' + E ,$$

where E_0'/E is .143 for the proton and 197.205 for the electron! (Quite a difference!) n' is the energy level of the internal excited state of the particle. n' for the electron is 0, for the proton it is 1 or 2. In the case of the proton, this resonance state has been reached experimentally for $n' = 2$ and for a number of corresponding N. The resulting energies are given for instance in Frazer [2], but the figures soon become unreliable as N increases. Moreover for $N = 2$ there is no experimental figure, and for $N = 1$ the value of n' is 1 instead of 2. However all these figures are, under the circumstances, in excellent agreement with our theoretical ones, even for instance for $N = 30$, when the experimental energy is 3030 and the theoretical one 3026.

The same theoretical formula applies to the resonance states of the electron, but as pointed out with very different numerical constants. (Incidentally E_0' for the electron would have been about 206, if we had not made the theoretical correction, indicated in the preceding section, to log coth R/2.) Besides, there is no known experimental way of inducing resonance states in the electron Instead, in the "experimental" line belo", we have noted the energies of the "particles" $\mu^{\pm}, \pi^{\pm}, K^{\pm}, K^{*\pm}, X_0$.

Electron (with $n = n' = 0$, $s = 2$) $N = 1, 5, 28, 52, 56$

Theoretical resonance energies:	104, 138, 492, 891, 958
"Experimental"	105, 139, 493, 891, 959

Can anyone doubt, after this, that these supposed particles, which decay ultimately into the electron, are the electron's observed resonance states? What about other values of N? Should we look for them, in our laboratories? In the stars? Are some of them unstable when there is interaction with other particle?

We pass on to the case $\Lambda = 0$, $\alpha = N + 1$. There is the further complication that $N \leq s - 1$, however this last condition becomes irrelevant, because the crucial ratio $(n + \frac{1}{2})/(s + 1)$ can be m d` to take the same value, 1/3 or 7/8, for pairs n,s in which s is as large as we please. These ratios 1/3 for the electron, 7/8 for the proton, we must stick to. They determine the radius and so forth, which does not change.

Mathematically, as we saw, a harmonic two-dimensional state was first reduced to $s = 0$ by s integrations, and then calibrated by specifying the additional polynomial of degree $\leq s - 1$. We may speak of a similar calibration or gauge in the ρ, t variables, arising from adding the appropriate Legendre polynomial in tanh X/2. Physically, one thinks rather of a rotation or "twist", of order s, applied only to the inside (or to the outside) of the spherical shell $r = r_0$. In Jackson [5, p. 259], such a twist is attributed to a shift of a dislocation line. At any rate, as in the case of resonance, the twist increases the energy of the original particle, by a factor which turns out to be

$$1 + \frac{n + \frac{1}{2}}{s + 1} (E_0'/E)\sqrt{N(N + 1)} ,$$

where E_0' is as before. In the tables below, $(n + \frac{1}{2})/(s + 1)$ is of course 7/8 for the proton, 1/3 for the electron.

Proton $N = 2, 4, 6, 12$

Theoretical gauge energies factor 1.154, 1.281, 1.407, 1.785

Experimental 1.187, 1.274, 1.405, 1.783

Electron $N = 32, 45, 46, 60, 73$

Theoretical g.e.f. 546, 765, 781, 1017, 1235

Experimental 548, 769 ± 3, 783, 1019, 1253 ± 20

Here again, in the "experimental" rows, the figures are ratios, to the energy of our particle, of the energies of supposed "particles": in the first list $\Lambda, \Sigma^-, \Xi_0, \Omega^-$ (for Σ^+, Σ^0 the figures would be 1.265, 1.270); in the second list η, $\rho^{\pm}$ and ρ_0, ω, ϕ, f. Again, therefore, these "particles" are really states (gauge states) of the proton or the electron or of their antiparticles.

Main References

1. Alonso-Finn, Universal Physics III Quantum and Statistical Physics (Addison-Wesley).
2. Frazer, Elementary Particles (Prentice-Hall).
3. Gelfand and others, Representation theory and automorphic functions (Saunders).
4. Gelfand, Les distributions (Tome 5) translated from Russian by Vasilach-Gussi (Dunod).
5. Jackson, Classical electrodynamics (Wiley).
6. Landau-Lifchitz, Théorie du Cnamp (Mir).
7. Titchmarsh, Eigenfunction expansions associated with second order differential equations (Oxford).
8. Young, L. C., Mathematics Research Center Technical Summary Report #2067 (May 1980). To appear in Proc. Ioffe Conf. Imperial College London (1980) (Springer).

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The theory of MRC Technical Summary Report 2067 is applied in a simplified form to a Relativistic One-Particle Universe, and boundary and symmetry conditions are imposed. The existence, for the particle, of a series of resonance states and a series of gauge states is derived. For the proton and the electron, a selection of such states account for the elementary particles observed to date.

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