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ON A CONJECTURE OF C. A. MICCHELLI CONCERNING CUBIC SPLINE INTE--ETC(U)

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ON A CONJECTURE OF C. A. MICCHELLI  
CONCERNING CUBIC SPLINE INTERPOLATION  
AT A BIINFINITE KNOT SEQUENCE

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ON A CONJECTURE OF C. A. MICCHELLI CONCERNING CUBIC SPLINE  
INTERPOLATION AT A BIINFINITE KNOT SEQUENCE

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ABSTRACT

It is shown that if the knot sequence  $\underline{t} = (t_i)_{-\infty}^{\infty}$  satisfies

(i) For some  $m \in \left[1, \frac{3+\sqrt{5}}{2}\right)$ ,

$$m^{-1} < \liminf_r \inf_i \frac{t_{i+r+1} - t_{i+r}}{t_{i+1} - t_i} < \limsup_r \sup_i \frac{t_{i+r+1} - t_{i+r}}{t_{i+1} - t_i} < m$$

and

$$(ii) \quad m_{\underline{t}} := \sup_{|i-j| < 1} \frac{t_{i+1} - t_i}{t_{j+1} - t_j} < \infty,$$

then for any given bounded sequence  $y \in m(\mathbb{Z})$  there exists exactly one cubic spline  $s$  with knots  $t_i$  such that

$$s(t_i) = y_i, \text{ for all } i \in \mathbb{Z}.$$

AMS (MOS) Subject Classification: 41A15

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multiplier

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# SIGNIFICANCE AND EXPLANATION

→ Cubic spline interpolation provides a good and handy method to approximate a given function or to fit a given set of points. However, such an interpolation process does not always converge. It is known that the local mesh ratio (that of the lengths of two consecutive intervals) is less than  $\frac{3+\sqrt{5}}{2}$ , the interpolation process works for any given bounded data.

This paper continues such investigation. It is shown that the above restriction on the knots may be relaxed. Thus, for a wider class of knot sequences, the cubic spline interpolation can be still applied. Hopefully, this would make such interpolation process more feasible in practice.

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ON A CONJECTURE OF C. A. MICCHELLI CONCERNING CUBIC SPLINE  
INTERPOLATION AT A BIINFINITE KNOT SEQUENCE

Rong-qing Jia

1. Introduction. Let  $t := (t_i)_{i=-\infty}^{\infty}$  be a biinfinite, strictly increasing sequence, set

$$t_{+\infty} := \lim_{i \rightarrow +\infty} t_i,$$

let  $k$  be an integer,  $k \geq 2$ , and denote by  $\mathcal{S}_{k,t}$  the collection of spline functions of degree  $< k$  with knot sequence  $t$ . Explicitly,  $\mathcal{S}_{k,t}$  consists of exactly those  $k-2$  times continuously differentiable functions on

$$I := (t_{-\infty}, t_{+\infty})$$

which, on each interval  $(t_i, t_{i+1})$ , coincide with some polynomial of degree  $< k$ . Let

$$m_{\mathcal{S}_{k,t}} := \mathcal{S}_{k,t} \cap m(I),$$

i.e., the normed linear space of splines for which

$$\|s\|_{\infty} := \sup_{t \in I} |s(t)|$$

is finite. We are interested in the

Bounded Interpolation Problem (B.I.P.). To construct, for given  $y \in m(\mathbb{Z})$ , some  $s \in m_{\mathcal{S}_{k,t}}$  for which

$$(1) \quad s|_t = y.$$

We will say that the B.I.P. is correct (for the given knot sequence  $t$ ) if it has exactly one solution for each  $y \in m(\mathbb{Z})$ .

In case  $k = 4$  (cubic spline interpolation), de Boor [2] showed that if the local mesh ratio

$$m_t := \sup_{|i-j| \leq 1} \frac{\Delta t_i}{\Delta t_j} \quad (\text{with } \Delta t_i := t_{i+1} - t_i)$$

is less than  $\frac{3+\sqrt{5}}{2}$ , then the B.I.P. is correct. A similar result was also obtained independently by Zmatrakov [7]. The basic idea of [2] was the exponential decay law, which

could be traced back to [1]. This idea was developed in de Boor [3] and Micchelli [6]. For cubic spline interpolation, Micchelli further raised the following conjecture (see [6], p. 236).

Conjecture. If

$$(2) \quad m^{-1} \leq \liminf_{r \rightarrow \infty} \inf_i (\Delta t_{i+r} / \Delta t_{i-1})^{\frac{1}{r}} \leq \limsup_{r \rightarrow \infty} \sup_i (\Delta t_{i+r} / \Delta t_{i-1})^{\frac{1}{r}} \leq m$$

for some  $m \in [1, \frac{3+\sqrt{5}}{2})$ , then the B.I.P. is correct for  $k = 4$ .

In hindsight, it is easy to see that this conjecture is faulty. This can be clarified by the following

Example. Write  $h_i := t_{i+1} - t_i$ . Let

$$t_0 = 0$$

$$h_{2n} = (\frac{1}{2})^n, \quad h_{2n+1} = (\frac{1}{2})^{2n+1} \quad \text{for integer } n \geq 0,$$

$$h_{-1-i} = h_i \quad \text{for integer } i \geq 0.$$

Then

$$\frac{1}{2} \leq \liminf_{r \rightarrow \infty} \inf_i \left( \frac{h_{i+r}}{h_{i-1}} \right)^{\frac{1}{r}} \leq \limsup_{r \rightarrow \infty} \sup_i \left( \frac{h_{i+r}}{h_{i-1}} \right)^{\frac{1}{r}} \leq 2,$$

but

$$m_{\underline{t}} = \sup_{|i-j| \leq 1} \frac{t_{i+1} - t_i}{t_{j+1} - t_j} = \infty.$$

Let  $m^+$  and  $m^-$  have the same meaning as in [4]. Since the knot sequence  $\underline{t}$  is symmetric with respect to the origin, we must have  $m^+ = m^-$ . If this B.I.P. is correct, then [4] tells us that  $m_{\underline{t}} < \infty$ , which is a contradiction.

The above example suggests to us that the condition  $m_{\underline{t}} < \infty$  should be added to the assumption. Thus we will prove the following

Theorem 1. If a knot sequence  $\underline{t}$  satisfies (2) with  $1 \leq m < (3+\sqrt{5})/2$  and

$$(3) \quad m_{\underline{t}} < \infty,$$

then the B.I.P. is correct for  $k = 4$ .

Remark. This theorem covers de Boor's results for cubic splines with bounded global mesh ratio or with local mesh ratio  $< \frac{3+\sqrt{5}}{2}$  (see [2] and [3]).

2. The basic formulae for cubic spline interpolation. For a given knot sequence  $t$ ,

let

$$(4) \quad h_i := t_i - t_{i-1}, \lambda_i = h_{i+1}/(h_i + h_{i+1}), \mu_i = h_i/(h_i + h_{i+1}), i \in \mathbb{Z}.$$

If  $s \in \mathcal{M}_{4,t}$  satisfies (1) for some  $y \in m(\mathbb{Z})$ , then

$$(5) \quad \lambda_i s'(t_{i-1}) + 2s'(t_i) + \mu_i s'(t_{i+1}) = 3\lambda_i \frac{y_i - y_{i-1}}{h_i} + 3\mu_i \frac{y_{i+1} - y_i}{h_{i+1}}, i \in \mathbb{Z}.$$

Moreover, it is easy to check that

$$(6) \quad s(x) = \frac{(x-t_{i-1})(t_i-x)^2}{h_i^2} s'(t_{i-1}) - \frac{(x-t_{i-1})^2(t_i-x)}{h_i^2} s'(t_i) + \frac{y_{i-1} + y_i}{2} + \frac{1}{h_i^2} \left(x - \frac{t_{i-1} + t_i}{2}\right) [(t_i-x)^2 + 4(t_i-x)(x-t_{i-1}) + (x-t_{i-1})^2] \frac{y_i - y_{i-1}}{h_i}$$

for  $x \in [t_{i-1}, t_i]$ .

Let

$$(7) \quad \forall i \in \mathbb{Z}, A(i, j) := \begin{cases} 2 & \text{for } j = i, \\ \lambda_i & \text{for } j = i-1, \\ \mu_i & \text{for } j = i+1, \\ 0 & \text{for } j \in \mathbb{Z} \setminus \{i-1, i, i+1\}. \end{cases}$$

Then  $A$  is a tri-diagonal  $\mathbb{Z} \times \mathbb{Z}$ -matrix. For any  $\beta \in m(\mathbb{Z})$  we have

$$\forall i \in \mathbb{Z}, |\lambda_i \beta_{i-1} + 2\beta_i + \mu_i \beta_{i+1}| \leq \lambda_i |\beta_{i-1}| + 2|\beta_i| + \mu_i |\beta_{i+1}| = 3|\beta_i|,$$

showing  $\|A\| \leq 3$ . Here, we view  $A$  as a mapping from  $m(\mathbb{Z})$  to  $m(\mathbb{Z})$ . Furthermore,

$$\forall i \in \mathbb{Z}, |\lambda_i \beta_{i-1} + 2\beta_i + \mu_i \beta_{i+1}| \geq 2|\beta_i| - \lambda_i |\beta_{i-1}| - \mu_i |\beta_{i+1}| = 2|\beta_i| - |\beta_{i-1}|.$$

Hence  $\|A\beta\| \geq \sup_{i \in \mathbb{Z}} \{2|\beta_i| - |\beta_{i-1}|\} = \|\beta\|$ . This shows that  $A^{-1}$  exists and  $\|A^{-1}\| \leq 1$ .

3. The exponential decay. The following lemma plays an essential role in this paper.

Lemma 1. For any knot sequence  $t$ ,

$$(8) \quad |A^{-1}(j, i)| \leq 2^{-|j-i|}, \forall i, j \in \mathbb{Z},$$

and

$$(9) \quad A^{-1}(j, i)A^{-1}(j+1, i) < 0, \forall i, j \in \mathbb{Z}.$$

Moreover, if  $\underline{t}$  satisfies the following condition: for some integer  $r > 0$

$$(10) \quad h_1/h_j < \frac{3}{4} m_0^{i-j} \quad \text{whenever } |i-j| > r,$$

then

$$(11) \quad |A^{-1}(j,i)| < (1 + m_0^{-1} + \sqrt{1 + m_0^{-1} + m_0^{-2}})^{-|i-j|} \quad \text{whenever } |i-j| > r.$$

Proof. For simplicity we fix  $i$  and write  $b_j := A^{-1}(j,i)$ . Since  $\|A^{-1}\| < 3$ ,  $b(\cdot) \in m(\mathbb{Z})$ . By  $AA^{-1} = 1$  we have

$$(12) \quad \lambda_j b_{j-1} + 2b_j + \mu_j b_{j+1} = \delta_{ij} = \begin{cases} 1 & \text{for } j = i, \\ 0 & \text{for } j \neq i. \end{cases}$$

We claim that

$$(13) \quad |b_j| < |b_{j+1}| \quad \text{for all } j < i.$$

If not, then there exists  $j_0 < i$  such that  $|b_{j_0}| > |b_{j_0+1}|$ . From (12) we have

$$\begin{aligned} |b_{j_0-1}| &= |-(2b_{j_0} + \mu_{j_0} b_{j_0+1})/\lambda_{j_0}| > (2|b_{j_0}| - \mu_{j_0} |b_{j_0+1}|)/\lambda_{j_0} \\ &> (2 - \mu_{j_0})|b_{j_0}|/\lambda_{j_0} = |b_{j_0}| \cdot (1 + \lambda_{j_0})/\lambda_{j_0} > 2|b_{j_0}|. \end{aligned}$$

Then by induction on  $j$ , we can easily show that  $|b_{j-1}| > 2|b_j|$  for all  $j < j_0$ . Hence  $|b_j| > 2^{j_0-j} |b_{j_0}|$  for  $j < j_0$ . This contradicts the fact  $b \in m(\mathbb{Z})$ . Similar to (13), the following also holds:

$$(13') \quad |b_j| < |b_{j-1}| \quad \text{for all } j > i.$$

Now (12) and (13) yield that

$$\begin{aligned} (14) \quad |b_{j+1}| &= |2b_j + \lambda_j b_{j-1}|/\mu_j > (2|b_j| - \lambda_j |b_{j-1}|)/\mu_j \\ &> (2 - \lambda_j)|b_j|/\mu_j = |b_j| \cdot (1 + \mu_j)/\mu_j > 2|b_j| \quad \text{for } j < i. \end{aligned}$$

Similarly

$$(14') \quad |b_{j-1}| > 2|b_j| \quad \text{for } j > i.$$

In particular,  $|b_{i-1}| < |b_i|$  and  $|b_{i+1}| < |b_i|$ . In connection with (12) we obtain

$$\begin{aligned} (15) \quad 1 &= \lambda_i b_{i-1} + 2b_i + \mu_i b_{i+1} = |\lambda_i b_{i-1} + 2b_i + \mu_i b_{i+1}| > \\ &> 2|b_i| - \lambda_i |b_{i-1}| - \mu_i |b_{i+1}| > (2 - \lambda_i - \mu_i)|b_i| = |b_i|. \end{aligned}$$



This proves (8) for  $j = i$ . For  $j \neq i$ , (8) comes from (14), (14') and (15). For the rest of the proof we may assume  $j < i$  without any loss. To prove (9) we argue indirectly. If  $b_{j_0} b_{j_0+1} > 0$  for some  $j_0 < i$ , then

$$|b_{j_0-1}| > |\lambda_{j_0} b_{j_0-1}| = |2b_{j_0} + \mu_{j_0} b_{j_0+1}| > 2|b_{j_0}|.$$

Comparing the above inequality with (13), we must have  $b_j = 0$  for all  $j < j_0$ . It would cause all  $b_j = 0$ , which is absurd. Now we can write down

$$(16) \quad -b_{j+1}/b_j =: 2 + 2q'_j \quad \text{for } j < i$$

with  $q'_j > 0$ . Let  $q_j := h_{j+1}/h_j$ . We deduce from (12) that, for  $j < i-1$ ,

$$\begin{aligned} -\frac{b_{j+2}}{b_{j+1}} &= \frac{2b_{j+1} + \lambda_{j+1}b_j}{\mu_{j+1}b_{j+1}} = \frac{2}{\mu_{j+1}} + \frac{\lambda_{j+1}}{\mu_{j+1}} \frac{b_j}{b_{j+1}} = 2 + 2q_{j+1} + q_{j+1} \frac{-1}{2+2q'_j} \\ &= 2 + 2q_{j+1} \left(1 - \frac{1}{2(2+2q'_j)}\right) = 2 + 2q_{j+1} \frac{4q'_j + 3}{4q'_j + 4}. \end{aligned}$$

This shows that

$$(17) \quad q'_{j+1} = q_{j+1} \frac{4q'_j + 3}{4q'_j + 4} \quad \text{for } j < i-1.$$

Let

$$(18) \quad p_j := \frac{4q'_j + 4}{4q'_j + 3} q'_j.$$

It is easy to verify that

$$(19) \quad 2 + 2q'_j = 1 + p_j + \sqrt{1 + p_j + p_j^2}.$$

Now (16) and (19) give us

$$(20) \quad |b_i/b_j| = \prod_{k=j}^{i-1} |b_{k+1}/b_k| = \prod_{k=j}^{i-1} (2 + 2q'_k) = \prod_{k=j}^{i-1} (1 + p_k + \sqrt{1 + p_k + p_k^2})$$

It follows from (17) and (18) that

$$\begin{aligned} (21) \quad \prod_{k=j}^{i-1} p_k &= \prod_{k=j}^{i-1} \left( \frac{4q'_k + 4}{4q'_k + 3} q'_k \right) = \prod_{k=j}^{i-1} \left( \frac{4q'_k + 4}{4q'_k + 3} \cdot \frac{4q'_{k-1} + 3}{4q'_{k-1} + 4} q_k \right) \\ &= \frac{4q'_{i-1} + 4}{4q'_{i-1} + 3} \cdot \frac{4q'_{j-1} + 3}{4q'_{j-1} + 4} \cdot \prod_{k=j}^{i-1} q_k > \frac{3}{4} \prod_{k=j}^{i-1} q_k = \frac{3}{4} \cdot \frac{h_i}{h_j}. \end{aligned}$$

If  $t$  satisfies (8) and  $|i-j| > r$ , then

$$(22) \quad \prod_{k=j}^{i-1} p_k > \frac{3}{4} \cdot \frac{h_i}{h_j} > \frac{3}{4} \cdot \frac{4}{3} m_0^{-|i-j|} = m_0^{-|i-j|}.$$

Therefore lemma 1 will be proved, once the following lemma is established:

Lemma 2. Suppose  $p_1, \dots, p_n$  and  $p$  are nonnegative real numbers with  $p^n = p_1 p_2 \dots p_n$ . Then

$$\prod_{i=1}^n (1 + p_i + \sqrt{1 + p_i + p_i^2}) > (1 + p + \sqrt{1 + p + p^2})^n.$$

Proof. Let

$$F(p_1, \dots, p_{n-1}, p_n) := \prod_{i=1}^n (1 + p_i + \sqrt{1 + p_i + p_i^2}).$$

We want to determine the minimum of the function  $F$  under the constraint  $\prod_{i=1}^n p_i = c$ , where  $c$  is a constant,  $c = p^n$ . If some  $p_i > (2+2p)^n$ , then

$$F(p_1, \dots, p_n) > (2+2p)^n > \prod_{i=1}^n (1 + p + \sqrt{1 + p + p^2})^n > \inf_{\prod p_i = c} \{F(p_1, \dots, p_n)\}.$$

Hence

$$\inf_{\prod p_i = c} \{F(p_1, \dots, p_n)\} = \inf_{\substack{\prod p_i = c \\ \forall i, p_i \leq (2+2p)^n}} \{F(p_1, \dots, p_n)\}.$$

Thus there exists a point  $(p_1^0, \dots, p_n^0)$  with  $\prod_{i=1}^n p_i^0 = c$  such that  $F(p_1^0, \dots, p_n^0) = \inf_{\prod p_i = c} \{F(p_1, \dots, p_n)\}$ . To find  $(p_1^0, \dots, p_n^0)$  we shall use the method of Lagrange

multipliers and set

$$\Phi(p_1, \dots, p_n) := F(p_1, \dots, p_n) - \lambda p_1 \dots p_n.$$

Then  $\frac{\partial \Phi}{\partial p_i} (p_1^0, \dots, p_n^0) = 0$ ,  $i = 1, \dots, n$ ; that is

$$\prod_{j \neq i} (1 + p_j^0 + \sqrt{1 + p_j^0 + (p_j^0)^2}) \cdot \left(1 + \frac{2p_i^0 + 1}{2\sqrt{1 + p_i^0 + (p_i^0)^2}}\right) - \lambda \prod_{j \neq i} p_j^0 = 0.$$

It follows that

$$\frac{p_i^0 \left(1 + \frac{2p_i^0 + 1}{2\sqrt{1 + p_i^0 + (p_i^0)^2}}\right)}{1 + p_i^0 + \sqrt{1 + p_i^0 + (p_i^0)^2}} = \lambda \cdot \frac{\prod_{j=1}^n p_j^0}{\prod_{j=1}^n (1 + p_j^0 + \sqrt{1 + p_j^0 + (p_j^0)^2})}.$$

Therefore

$$(23) \quad f(p_i^0) = f(p_k^0) \text{ for all } i, k \in \{1, \dots, n\},$$

where

$$f(x) := \frac{x(1 + \frac{2x+1}{2\sqrt{1+x+x^2}})}{1+x+\sqrt{1+x+x^2}} = \frac{1}{2} + \frac{x-1}{2\sqrt{1+x+x^2}}.$$

An easy calculation yields

$$(24) \quad f'(x) = \frac{3(x+1)}{4(1+x+x^2)^{3/2}} > 0 \text{ for } x > 0.$$

This shows that  $f$  is strictly increasing on  $[0, \infty)$ . Thus (23) and (24) give

$$p_1^0 = \dots = p_n^0 = p.$$

This ends the proof of lemma 2. Also the proof of lemma 1 is complete.

4. The proof of theorem 1. By the hypothesis (2) there exist a positive integer  $r$  and a real number  $m_0$  with  $m < m_0 < \frac{3+\sqrt{5}}{2}$  such that

$$h_i/h_j < \frac{3}{4} m_0^{|i-j|} \text{ whenever } |i-j| > r.$$

Then by lemma 1,

$$(25) \quad \forall i, j \in \mathbb{Z}, |A^{-1}(j, i)| < \begin{cases} 2^{-|i-j|} & \text{if } |i-j| < r \\ (1 + m_0^{-1} + \sqrt{1 + m_0^{-1} + m_0^{-2}})^{-|i-j|} & \text{if } |i-j| > r \end{cases}.$$

Let

$$M := (m_t)^T < \infty$$

and

$$c_i = 3\lambda_i \cdot \frac{y_i - y_{i-1}}{h_i} + 3\mu_i \cdot \frac{y_{i+1} - y_i}{h_{i+1}}, \quad i \in \mathbb{Z}.$$

Then it follows that

$$(26) \quad s'(t_j) = \sum_{i \in \mathbb{Z}} A^{-1}(j, i) c_i = \sum_{|i-j| < r} A^{-1}(j, i) c_i + \sum_{|i-j| > r} A^{-1}(j, i) c_i.$$

By the hypotheses of theorem 1 we have the following estimates for  $c_i$ :

$$(27) \begin{cases} |c_i| \leq 6 \cdot \|y\|_{\infty} \cdot (1+M) \cdot \frac{1}{h_i} \leq 6M(1+M) \cdot \|y\|_{\infty} \cdot \frac{1}{h_j} & \text{if } |j-i| < r; \\ |c_i| \leq 6 \cdot \|y\|_{\infty} \cdot (1+M) \cdot \frac{1}{h_i} \leq 6(1+M) \cdot \|y\|_{\infty} \cdot \frac{1}{h_j} \cdot m_0^{|i-j|}, & \text{if } |j-i| \geq r. \end{cases}$$

Write

$$\theta := m_0 \left( 1 + m_0^{-1} + \sqrt{1 + m_0^{-1} + m_0^{-2}} \right)^{-1}.$$

Then  $\theta < 1$  as long as  $m_0 < \frac{3+\sqrt{5}}{2}$ . Applying (25) and (27) to (26), we obtain

$$(28) \quad \begin{aligned} |s'(t_j)| &\leq 6M(1+M) \cdot \|y\|_{\infty} \cdot \frac{1}{h_j} \cdot \sum_{|j-i| < r} 2^{-|i-j|} + 6(1+M) \|y\|_{\infty} \cdot \frac{1}{h_j} \sum_{|i-j| \geq r} \theta^{|i-j|} \\ &\leq \text{const} \cdot \|y\|_{\infty} \cdot \frac{1}{h_j}. \end{aligned}$$

Furthermore (6) tells us

$$\max_{t_{j-1} \leq x \leq t_j} |s(x)| \leq \text{const}(h_{j-1} |s'(t_{j-1})| + h_j \cdot |s'(t_j)| + \|y\|_{\infty}).$$

which in connection with (28) yields the desired result

$$\|s\|_{\infty} \leq \text{const} \cdot \|y\|_{\infty}.$$

Our proof is complete.

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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>It is shown that if the knot sequence $\underline{t} := (t_i)_{-\infty}^{\infty}$ satisfies<br>(i) For some $m \in [1, \frac{3+\sqrt{5}}{2}]$ ,<br>$m^{-1} \leq \liminf_r \inf_i \frac{t_{i+r+1} - t_{i+r}}{t_{i+1} - t_i} \leq \limsup_r \sup_i \frac{t_{i+r+1} - t_{i+r}}{t_{i+1} - t_i} \leq m$<br>and<br><div style="text-align: right;">(continued)</div> |                                     |                                                                                                                                      |

ABSTRACT (continued)

$$(ii) \quad m_t := \sup_{|i-j| \leq 1} \frac{t_{i+1} - t_i}{t_{j+1} - t_j} < \infty ,$$

then for any given bounded sequence  $y \in m(\mathbb{Z})$  there exists exactly one cubic spline  $s$  with knots  $t_i$  such that

$$s(t_i) = y_i , \text{ for all } i \in \mathbb{Z} .$$

