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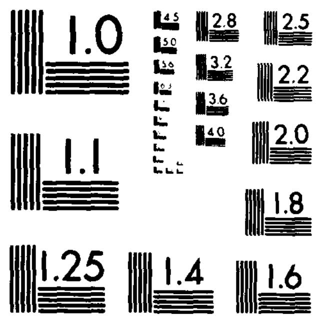

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IN A SPATIALLY PERIODIC TRANSVERSE MAGNETIC FIELD

by

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RADIATION SPECTRUM OF RELATIVISTIC ELECTRONS IN A SPATIALLY PERIODIC TRANSVERSE MAGNETIC FIELD

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Abstract

The motion track of relativistic electrons in a spatially periodic transverse magnetic field has been analyzed. Under some conditions, the electron motion is harmonic in the direction perpendivular to the original incident one. Radiation spectrum and spectral width have been obtained using harmonic mode. In the last section, nonharmonic mode is also discussed. The results show that the distribution of the radiation spectrum depends on the initial electron velocity and the intensity of the magnetic field.

Forward

In he early fifties, H. Motz [1] analyzed the case of radiation generated by a beam of relativistic electrons moving through a periodic electromagnetic field. According to the results of this study, in the early sixties physicists developed the laser technology of free electrons [2]. In 1974 results were obtained in the emission of the millimeter wave band [3]. In 1975 similar results were obtained concerning the wave band next to the red [4]. This type of laser equipment offers excellent harmonic characteristics, and it is therefore possible to generate excited radiations of extremely small amplitude. In addition one can expect a high efficiency. For these reasons, scientists concentrated on free electron laser equipment. In this study we discuss the emission spectrum of radiations generated by relativistic electrons moving in a spatially periodic transverse magnetic field.

HARMONIC MODE

Suppose a magnetic field distribution as in Chart 1. The field is oriented along x, the electrons along y, perpendicular to the field, and enter the region of the field.

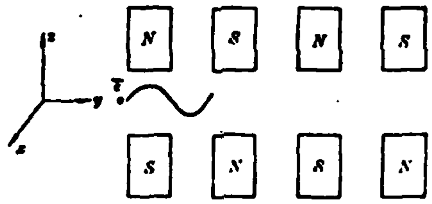

Chart 1.

Considering the magnetic field is oriented along x, along the transverse period y, then $\boldsymbol{B} = B_x \boldsymbol{e}_x$ is satisfied if $\nabla \cdot \boldsymbol{B} = 0$ with $\boldsymbol{B}$ as in the following form of the distribution:

$$\boldsymbol{B} = \left(\frac{4B_x}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \sin \frac{2n+1}{\lambda} \pi y \right) \boldsymbol{e}_x \qquad (1)$$

in which B_x is the density of the magnetic field, it is a constant, $B = B_x$, λ is the length of the spatial period. This is the integral of the movement of the electrons in such a field:

$$\frac{d\boldsymbol{P}}{dt} = \frac{-|e|}{c} \boldsymbol{V} \times \boldsymbol{B} \qquad (2)$$

in which equality $\boldsymbol{P}$ is the momentum of the electron, $|e|$ is the electronic charge, $\boldsymbol{V}$ is the speed of the electrons, c is the speed of light. From (2) we write the movement oriented along z:

$$\frac{dP_z}{dt} = \frac{4e}{c} V_y \frac{B}{\pi} \sum_{n}^{\infty} \frac{1}{2n+1} \times \sin \frac{2n+1}{\lambda} \pi y \tag{3}$$

From this integral we obtain:

$$P_z = \frac{4eB\lambda}{c\pi^2} \sum_{n}^{\infty} \frac{1}{(2n+1)^2} \cos \frac{2n+1}{\lambda} \pi y \tag{4}$$

Let us assume that the energy of the radiation of the electrons takes only a small portion of its entire energy, then P_y the momentum aligned on y is determined by the following relation:

$$P_y = \sqrt{P_0^2 - P_z^2} \tag{5}$$

in which P_o is the initial momentum at the time the electrons enter the magnetic field. The momentum and the speed of the electrons are related:

$$V = \frac{c^2}{\varepsilon} P \tag{6}$$

in which ε is the kinetic energy of the electrons.

$$\varepsilon = \frac{m_0 c^2}{\sqrt{1 - \frac{v_e^2}{c^2}}}.$$

with m_o being the static character of the electrons, v_e is the speed of the electrons. From equality (6) we get:

$$t = \frac{\varepsilon}{c^2} \times \int_0^y \frac{dy}{P_0^2 \sqrt{1 - K^2 f^2(y)}} \tag{7}$$

in which

$$f(y) = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} - \cos \frac{2n+1}{\lambda} \pi y$$

$$K = \frac{4Be\lambda}{\pi^2 P_0}.$$

Equation (7) is resolved using a digital microcomputer of type TQ-16. The results are exposed on chart 2. From chart 2 we observe the tendency of the motion track to become asymptotic, when K < 1. We then have the following approximation:

$$y = at \tag{8}$$

in which the coefficient a tends toward the initial velocity v_e. In fact we know from (7) that the highest value of the denominator in brackets is $(\pi^2/8)^2 \approx 1$, when K < 1.

$$K^2\left[\sum \frac{1}{(2n+1)^2} \cos \frac{2n+1}{\lambda} \pi y\right]^2 < 1$$

Consequently, the equation (6) can be approximated:

$$\frac{eP_0}{c^2} t + \int_0^y \left\{1 + \frac{K^2}{2} \times \left[\sum \frac{1}{(2n+1)^2} \cos \frac{2n+1}{\lambda} \pi y\right]^2\right\} dy \approx y + \frac{3K^2}{1180} y$$

and $$\frac{eP_0}{c^2} \approx v_e$$

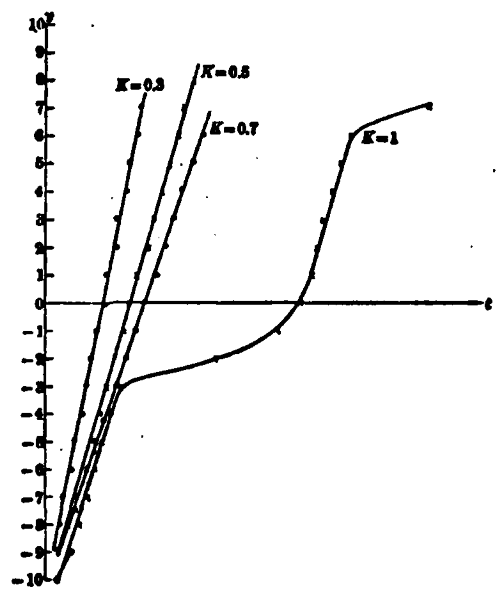

Chart 2.

We will now discuss the case of the electrons movement oriented with z.

$$\frac{dz}{dt} = \frac{c^2}{e} P_z - \frac{c^2}{\pi^2 e} \cdot \frac{4eB\lambda}{c} \sum \frac{1}{(2n+1)^2} \cos \frac{2n+1}{\lambda} \pi y$$

Substituting with (8) and integration gives us:

$$z = \frac{4eB\lambda c}{\pi^2 s}\frac{\lambda}{\pi a} \times \left(\sin\frac{\pi a}{\lambda}t + \frac{1}{27}\sin\frac{3\pi a t}{\lambda}\cdots\right) \quad (9)$$

In order to observe the physical pattern of the radiation produced by the free electrons moving in a periodic magnetic field, we will start with the simplest case, and when we consider only the first term between brackets of the equality (9), we have:

$$z = \frac{4eB\lambda c}{s\pi^2}\frac{\lambda}{\pi a}\sin\frac{\pi a}{\lambda}t \quad (10)$$

Obviously this is the simple harmonic oscillation's equation. From equality (10) we get:

$$\ddot{z} = -\left(\frac{\pi a}{\lambda}\right)^2 z$$

The potential energy U of this harmonic oscillation is:

$$U = m_0\int_0^z \left(\frac{\pi a}{\lambda}\right)^2 z\,dz = \frac{m_0}{2}\left(\frac{\pi a}{\lambda}\right)^2 z^2 = 2\pi^2\nu_0^2 m_0 z^2 \quad (11)$$

in which v_o is:

$$\nu_0 = a/2\lambda \quad (12)$$

This is the equation of Xuẹ Ding-E of this harmonic oscillation:

$$\frac{d^2\psi}{dz^2} + \frac{8\pi^2 m_0}{h^2}(E - 2\pi^2\nu_0^2 m_0 z^2)\psi = 0 \quad (13)$$

in which ψ is the wave function of the oscillator, E its general energy. For each wave function in the completed equation (13), the oscillator's energy E accepts only the following values:

$$E_n = h\nu_0\left(n+\frac{1}{2}\right)$$

$$n = 1, 2, 3\cdots$$

In this case only boundary conditions are fulfilled. According to the above discussion, it is possible to look at the movement of the free electron within a spatially periodic magnetic field, as an oscillator in harmonic oscillation outside the field. Then the following frequency can be considered as the electromagnetic radiation of the basic frequency:

$$\nu_0 = \frac{a}{2\lambda} \qquad (14)$$

The electron being effected by the movement, according to the principle of Doppler, we have obtained in laboratory the following radiation frequency:

$$\nu' = \nu_0(1-\beta^2)^{\frac{1}{2}}/(1-\beta) \qquad (15)$$

with $\beta = v_e/c_0$

And because the electrons are evolving in a magnetic field with limited length, the radiation they emit is not strictly monochromatic, and offers a definite width. In our approximation, we assume that the photoelectric field density vector ε showing radiations is comparable to the oscillation band of the oscillator.*
Then:

$$\varepsilon = \varepsilon_0 \sin\frac{\pi a}{\lambda}t = \varepsilon_0 \sin 2\pi\nu_0 t \qquad (16)$$

applying the transformation of (16) we get:

$$\varepsilon(\nu) = \varepsilon_0\int_0^{\frac{T}{2}} \sin 2\pi\nu_0 t e^{i2\pi\nu} dt \qquad (17)$$

in which T the limit of the integral is:

$$T = N\cdot\frac{\lambda}{a}$$

with N being a spatially periodical value of the magnetic field.

*Strictly speaking $\varepsilon\sim E$.

After systematization of the equality (17), we get:

$$\varepsilon(\nu) = \frac{\varepsilon_0}{2\pi}\left[\frac{\sin(\nu+\nu_0)\pi T}{\nu_0+\nu} + \frac{\sin(\nu-\nu_0)\pi T}{\nu-\nu_0}\right]$$

Because $\nu \sim \nu_0$, the second term is in fact much more important and we have

$$\varepsilon(\nu) \approx \frac{\varepsilon_0}{2\pi}\frac{\sin(\nu-\nu_0)\pi T}{\nu-\nu_0} \qquad (18)$$

when $v = v_0$, $\varepsilon(v)$ grows extremely, and when $(v-v_0)xT = nx$, $\varepsilon(v) = 0$, and therefore the radiation spectrum total width is:

$$2(\nu-\nu_0) = \frac{2}{T} = \frac{2}{N}\frac{a}{\lambda} \qquad (19)$$

and the emission spectrum frequency v_0 will be: $\nu_0 = \frac{a}{2\lambda}$

Therefore, the total width of the spectrum, Δv, is:

$$\Delta\nu = \frac{4}{N}\nu_0 \qquad (20)$$

And this is to say that larger is the spatially periodic number N of the magnetic field, better are the monochromatic qualities of the emission.

NON HARMONIC MODE

From the harmonic oscillation mode described above, we obtained a radiation with equivalent emission frequency intervals, when electrons move in a spatially periodic magnetic field. In fact, the movement of the electrons is not absolutely simply harmonic , especially in the case of a magnetic field of great strength. According to equality (9), the movement of the electron will further be considered under the form:

$$z = \frac{4\lambda Bce}{\varepsilon\pi^2}\cdot\frac{\lambda}{\pi a} \times\left[\sin\frac{\pi at}{\lambda} + \frac{1}{27}\sin\frac{3\pi a}{\lambda}t\right] \qquad (10')$$

$$\dot{z} = \frac{4\lambda Bce}{\varepsilon\pi^2}\left[\cos\frac{\pi at}{\lambda} + \frac{1}{9}\cos\frac{3\pi a}{\lambda}t\right].$$

$$\ddot{z} = -\left(\frac{\pi a}{\lambda}\right)^2 z - \frac{32}{27} \times \frac{4\lambda^2 Bec}{\pi^2 ae}\left(\frac{\pi a}{\lambda}\right)^2 \sin\frac{3\pi a}{\lambda} t$$

z is the derivative at time t. In the same manner, the potential energy U of the oscillator is:

$$\begin{aligned} U &= \frac{1}{2}\left(\frac{\pi a}{\lambda}\right)^2 m_0 z^2 + \frac{32}{27}\frac{4eB\lambda^2 mc}{e\pi^2 a}\left(\frac{\pi a}{\lambda}\right)^2 \\ &\times \int_0^z \sin\frac{3\pi y}{\lambda}\left(\frac{dz}{dy}\right)dy \\ &\approx 2\pi^2 m_0 \nu_0^2 z^2 + 3Q - 2Q\cos\frac{2\pi}{\lambda}z \\ &- Q\cos\frac{4\pi}{\lambda}z \end{aligned} \qquad (21)$$

in which $Q = \frac{4c}{27e}\left(\frac{4eB\lambda}{\pi^3}\right)^2$

From (21) we realize that the movement in the vicinity of $z \approx n \cdot \frac{\lambda}{4\pi}$ is simply harmonic.

In the case of non harmonic mode, the equation of Xue Ding-E for the oscillation is:

$$\begin{aligned} \frac{d^2\psi}{dz^2} + \frac{8\pi^2 m_0}{h^2}\Big\{E - 3Q - 2\pi^2\nu_0^2 m_0 z^2 \\ + 2Q\cos\frac{2\pi}{\lambda}z + Q\cos\frac{4\pi}{\lambda}z\Big\}\psi = 0 \end{aligned} \qquad (22)$$

The equation (22) can be resolved by the development of series. In order to meet the need to satisfy equation (22) and the boundary conditions, the value of the energy E is:

$$E_n = h\nu_0\left(n + \frac{1}{2}\right) - Qf(a, n) \qquad (23)$$

in which f(a,n) is function of the parameter a, and of the value of the radiation spectrum n. Because the parameter Q is positive in relation to the strength of the magnetic field, the energy of the free electrons moving in the spatial magnetic field depends on the velocity of the electrons, and on the strength of the field. The frequency intervals follow the high frequency developments and then diminish. The transformation quantity also depends on the strength of the magnetic field. Consequently, the frequency spectrum of the free electrons emission is not homogeneous. Its intervals are not equivalent. The distribution of the radiation spectrum also depends on the initial velocity of the electrons and on the strength of the magnetic field.

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