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DETECTION NETWORKS.(U)

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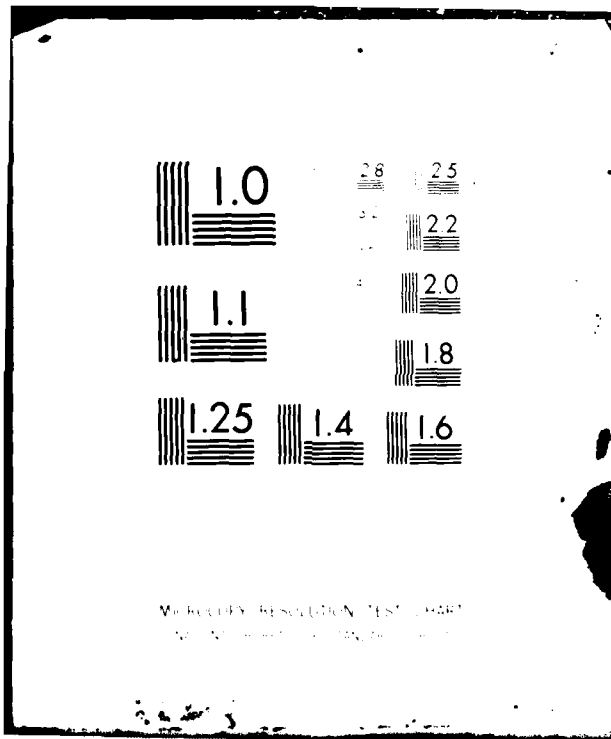
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Detection Networks¹

L.K.Ekchian and R.R.Tenney²

Abstract

Each member of a team of decision agents receives a conditionally independent observation about some underlying discrete hypothesis. Subject to causality constraints, the agents seek to optimize a team cost functional by making discrete decisions which are conveyed to other agents on capacity constrained channels. This paper derives optimal decision rules for a class of problems of this type and discusses their properties.

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I. Introduction

Purpose: Team theory has long sought to unify the joint estimation/ communication/ control problems that arise when several agents that receive different information attempt to cooperate [2]. These problems have proven to be rather difficult in many cases [11], although a surprising number of them can be reduced to an equivalent static framework [3,4] through reformulation.

Building on [8], this paper describes the solution of a number of specific team problems with non-partially nested (non-NP) information structures. The problem is made tractable by assuming that the underlying natural randomness takes the form of a fixed, discrete hypothesis and that the decision agents receive conditionally independent observations. This permits explicit computation of decision rules, even when communication (preassigned flow pattern) is allowed between agents. The decision rules themselves display an interesting structure which aids interpretation of the solution (e.g. in terms of an optimal quantization of the local information). The insight gained from this work will serve as a stepping-stone for the development of a general solution procedure for distributed hypothesis testing with communication constraints.

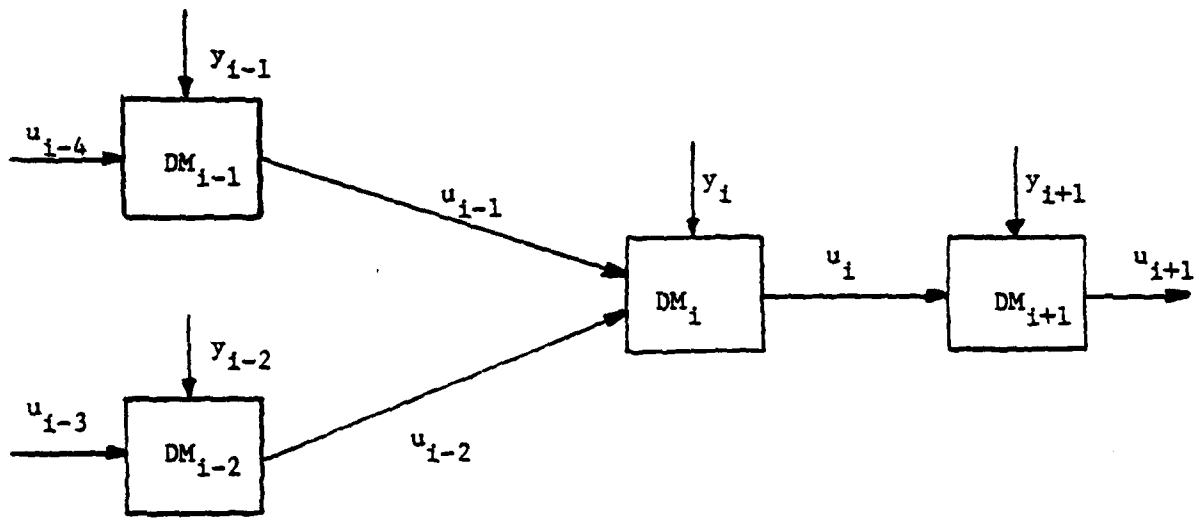
Related Work: As the logical sequel to [8], Sections 2 and 3 present results for two special classes of structures: tandem and hierarchical. The

problems discussed are endowed with digital communication links between the decision-makers (DMs). A predetermined causal precedence ordering is imposed as each DM needs the decisions of his predecessors before it can generate its own decision. We retain however, the assumption that the actions of the DMs do not in any way affect any system dynamics.

However, unlike problems with predetermined communication variables (e.g. one-step delay information sharing [7], [5] and [10]), the communicated variables are limited to m discrete values with their content to be determined. Messages are conveyed error free, i.e. we do not consider noisy channels [1], although generalizations to this case can readily be drawn. While the m discrete value channels provide encoding capability, we are not interested in coding per se [1] but rather seek the overall optimal communication and decision strategy. Note that this allows explicit signalling [9] from a DM to "downstream" DMs through the communication links.

Problem Statement: Find $\gamma_i^* : Z_i \rightarrow U_i, i \in \{0, 1, \dots, N\}$, to minimize $J(\gamma_0, \gamma_1, \dots, \gamma_N) = E\{J(u_0, H)\}$ where u_0 is the output of the detection network; $\{u_1, u_2, \dots, u_N\}$ are internal communication variables; and Z_i is the information available to DM i as specified by its access to: (1) a local observation y_i and (2) other decisions specified by the topology of the communication network (Figure 1). Finally H is the underlying discrete state of nature.

Overview of results: Solutions are available for binary problems where $H \in \{H^0, H^1\}$ and $U_i \in \{0, 1\}$, and a number of network topologies. Extensions to



$$Z_i = \{u_{i-1}, u_{i-2}, y_i\}$$

Figure 1: Topology of Communication Network

M-ary hypotheses and m_i symbol channels are straightforward and discussed in Sections 2 and 3.

II. Tandem Team Configuration

Assumptions: The tandem network topology is illustrated in Figure 2. H^0 and H^1 arise with known a priori probabilities

$$P(H^0) = p^0 \qquad P(H^1) = p^1 \qquad (1)$$

The joint conditional observation distribution is

$$P(y_0, y_1 / H^j) \qquad \text{for } j \in \{0, 1\} \qquad (2)$$

where the subscripts denote the DM. (The y_i 's may be random vectors generated by "preprocessing" of the original measurements). The global cost function is defined by $J(u_0, H)$ where

$$J : \{0, 1\} \times \{H^0, H^1\} \rightarrow \mathbb{R} \qquad (3)$$

The objective of the team is to minimize

$$E \{J(u_0, H)\} \qquad (4)$$

and we will interpret

$$u_0 = \begin{cases} 0 & H^0 \text{ is declared} \\ 1 & H^1 \text{ is declared} \end{cases} \qquad (5)$$

The following assumptions simplify in the derivation of the

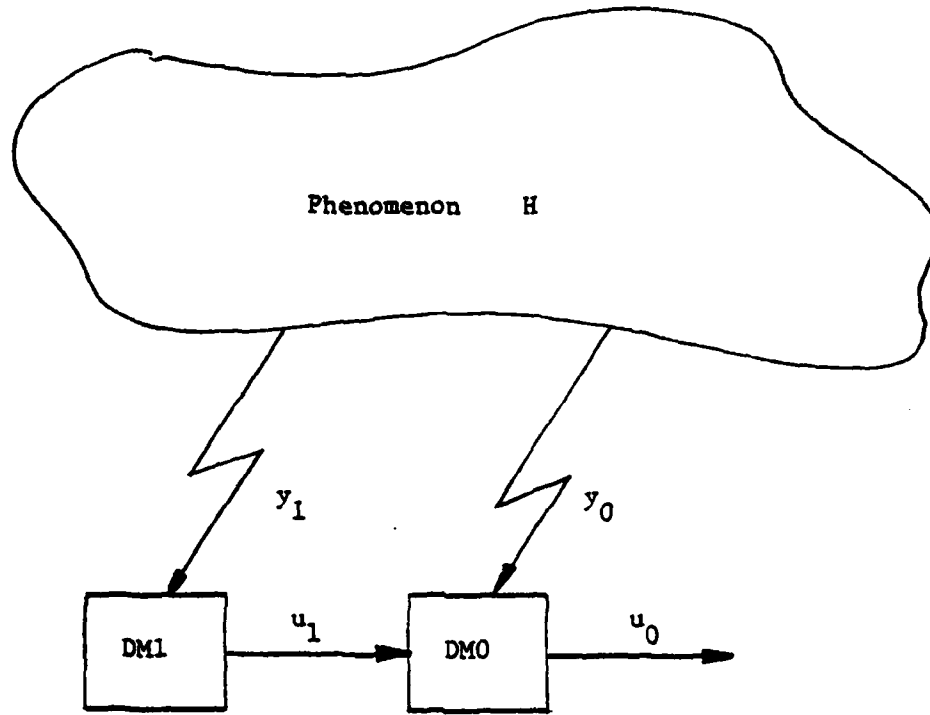


Figure 2: Two DM Tandem Topology

decision rules:

Assumption 1: The observations y_0 and y_1 are statistically independent,
i.e.

$$\begin{aligned} p(y_1/y_0, H) &= p(y_1/H) \\ p(y_0/y_1, H) &= p(y_0/H) \end{aligned} \quad (6)$$

(See [6] for a case of dependent measurements without communication).

Assumption 2:

$$(a) J(1, H^0) > J(0, H^0) \quad | \quad (7)$$

as per the interpretation of J

$$(b) p(u_0=0/u_1=0, y_0) \geq p(u_0=0/u_1=1, y_0) \quad (8)$$

this disambiguates two possible symmetric solutions

Derivation: We begin with

Lemma 1:

$$\sum_{u_0} J(u_0, H^0) [p(u_0/u_1=0, y_1) - p(u_0/u_1=1, y_0)] < 0 \quad (9)$$

Proof: Expanding expression (9)

$$\begin{aligned} &J(0, H^0)p(u_0=0/u_1=0, y_0) + J(1, H^0)p(u_0=1/u_1=0, y_0) \\ &- J(0, H^0)p(u_0=0/u_1=1, y_0) - J(1, H^0)p(u_0=1/u_1=1, y_0) \end{aligned} \quad (10)$$

But

$$\begin{aligned} p(u_0=1/u_1=0, y_0) &= 1 - p(u_0=0/u_1=0, y_0) \\ p(u_0=1/u_1=1, y_0) &= 1 - p(u_0=0/u_1=1, y_0) \end{aligned} \quad (11)$$

Substituting (11) into (10) and simplifying

$$[J(0, H^0) - J(1, H^0)] \{p(u_0=0/u_1=0, y_0) - p(u_0=0/u_1=1, y_0)\} \quad (12)$$

and Assumption 2 gives expression (12) is less than 0.

Q.E.D.

This lemma is required for proving:

Theorem 1: The decision rule for DMO (Figure 2) is given by

$$u_0 = \gamma_0(y_0, u_1) \quad (13)$$

and $\gamma_0(\cdot)$ is the following likelihood ratio test:

$$\Lambda_{0,1}^0 \triangleq \frac{P(y_0|H^0)}{P(y_0|H^1)} \begin{matrix} & u_1=0 \\ & > \\ & < \\ & u_1=1 \end{matrix} \frac{P(H^1)P(u_1|H^1)[J(0,H^1)-J(1,H^1)]}{P(H^0)P(u_1|H^0)[J(1,H^0)-J(0,H^0)]} \quad (14)$$

$$\triangleq \begin{cases} t_0^0 & \text{if } u_1=0 \\ t_0^1 & \text{if } u_1=1 \end{cases} \quad (15)$$

The decision rule for DMI is:

$$u_1 = \gamma_1(y_1) \quad (16)$$

and $\gamma_1(\cdot)$ is the following likelihood ratio test:

$$\Lambda_{0,1}^1 \triangleq \frac{P(y_1|H^0)}{P(y_1|H^1)} \begin{matrix} & u_1=0 \\ & > \\ & < \\ & u_1=1 \end{matrix} \frac{\sum_{u_0} P(H^1)J(u_0,H^1)[P(u_0|u_1=0,H^1)-P(u_0|u_1=1,H^1)]}{\sum_{u_0} P(H^0)J(u_0,H^0)[P(u_0|u_1=1,H^0)-P(u_0|u_1=0,H^0)]} \quad (17)$$

$$\triangleq t_1 \quad (18)$$

Proof: This proof parallels the one presented in [8].

The objective

$$\min E \{ J(u_0, H) \} \quad (19)$$

is expanded explicitly as:

$$\sum_{\substack{u_1, u_0 \\ u_1, \gamma_0, \gamma_1}} \int P(u_0, u_1, \gamma_0, \gamma_1, H) J(u_0, H) dy_0 dy_1 \\ = \sum_{\substack{u_1, u_0 \\ u_1, \gamma_0, \gamma_1}} \int P(H) p(u_0 | u_1, \gamma_0, \gamma_1, H) p(u_1, \gamma_0, \gamma_1 | H) J(u_0, H) dy_0 dy_1 \quad (20)$$

Invoking appropriate independence assumptions yields

$$\sum_{\substack{u_1, u_0 \\ u_1, \gamma_0, \gamma_1}} \int P(H) p(u_0 | u_1, \gamma_0) p(u_1, \gamma_1 | H) p(\gamma_0 | H) J(u_0, H) dy_0 dy_1 \quad (21)$$

Explicitly summing over u_1 and ignoring constant term, the expression to be minimized reduces to

$$\sum_{u_1} \int_{\gamma_0} P(u_0=0 | u_1, \gamma_0) \sum_H \int_{\gamma_1} P(H) p(\gamma_1, u_1 | H) p(\gamma_0 | H) [J(0, H) - J(1, H)] dy_0 dy_1 \quad (22)$$

This expression is minimized by setting

$$p(u_0=0 | u_1, \gamma_0) = \begin{cases} 0 & \text{if } \sum_H P(H) p(u_1 | H) p(\gamma_0 | H) [J(0, H) - J(1, H)] > 0 \\ 1 & \text{otherwise} \end{cases} \quad (23)$$

Expanding over H and invoking Assumption 2(a) yields the decision rule for DMO given in expression (14).

For $\delta_i(\cdot)$, write (21) as

$$\sum_{\substack{u_1, u_0 \\ u_1, \gamma_0, \gamma_1}} \int P(H) p(u_0 | u_1, \gamma_0) p(u_1 | \gamma_1) p(\gamma_1 | H) p(\gamma_0 | H) J(u_0, H) dy_0 dy_1 \quad (24)$$

Explicitly summing over u_1 , substituting $p(u_1=1 | \gamma_1) = 1 - p(u_1=0 | \gamma_1)$ and ignoring the constant term yields

$$\int_{\gamma_1} P(u_1=0 | \gamma_1) \sum_{\substack{u_1, u_0 \\ \gamma_0}} \int P(H) p(\gamma_0 | H) p(\gamma_1 | H) J(u_0, H) [p(u_0 | u_1=0, \gamma_0) - p(u_0 | u_1=1, \gamma_0)] dy_0 dy_1 \quad (25)$$

The expression is minimized for

$$u_1 = \gamma_1(\gamma_1) \tag{30}$$

and $\gamma_1(\cdot)$ is given by the following set of likelihood ratio test:

$$\Lambda_{ij}^1 \triangleq \frac{P(\gamma_1 | H^i)}{P(\gamma_1 | H^j)} > \frac{\sum_{u_0} P(H^j) J(u_0, H^j) \left[\frac{P(u_0 | u_1=j, H^j)}{P(u_0 | u_1=i, H^j)} \right]}{\sum_{u_0} P(H^i) J(u_0, H^i) \left[\frac{P(u_0 | u_1=i, H^i)}{P(u_0 | u_1=j, H^i)} \right]} + \frac{\sum_{u_0} P(H^k) \Delta_{kj} J(u_0, H^k) \left[\frac{P(u_0 | u_1=j, H^k)}{P(u_0 | u_1=i, H^k)} \right]}{\sum_{u_0} P(H^i) J(u_0, H^i) \left[\frac{P(u_0 | u_1=i, H^i)}{P(u_0 | u_1=j, H^i)} \right]} \tag{31}$$

for $(i, j, k) \in \{(0, 1, 2), (0, 2, 1), (1, 2, 0)\}$

Proof: Consider expression (20)

$$\sum_{\substack{H, u_0 \\ u_1, \gamma_0, \gamma_1}} P(H) P(u_0 | u_1, \gamma_0) P(u_1, \gamma_1 | H) P(\gamma_0 | H) J(u_0, H) d\gamma_0 d\gamma_1 \tag{32}$$

$$= \sum_{u_0, u_1, \gamma_0} P(u_0 | u_1, \gamma_0) \sum_H P(H) P(u_1 | H) P(\gamma_0 | H) J(u_0, H) d\gamma_0$$

Hence the decision rule is deterministic and is given by (29).

To show (31) rewrite (32) as

$$\int_{\gamma_1} \sum_{u_1} P(u_1 | \gamma_1) \sum_{H, u_0} P(H) P(u_0 | u_1, H) P(\gamma_1 | H) J(u_0, H) d\gamma_1 \tag{33}$$

and parallel the proof of Theorem 1.

Q.E.D.

Theorem 3: For the Three Tandem DM case (Figure 3) and binary symbol capacity channels, the decision rule for DMO is given by

$$u_0 = \gamma_0(\gamma_0, u_1) \tag{34}$$

and $\gamma_0(\cdot)$ is the following likelihood ratio test:

$$\Lambda_{01}^0 \triangleq \frac{P(\gamma_0 | H^0)}{P(\gamma_0 | H^1)} > \frac{P(H^1) P(u_1 | H^1) [J(0, H^1) - J(1, H^1)]}{P(H^0) P(u_1 | H^0) [J(1, H^0) - J(0, H^0)]} \tag{35}$$

$$\triangleq \begin{cases} t_0 & \text{if } u_1 = 0 \\ t_1 & \text{if } u_1 = 1 \end{cases} \tag{36}$$

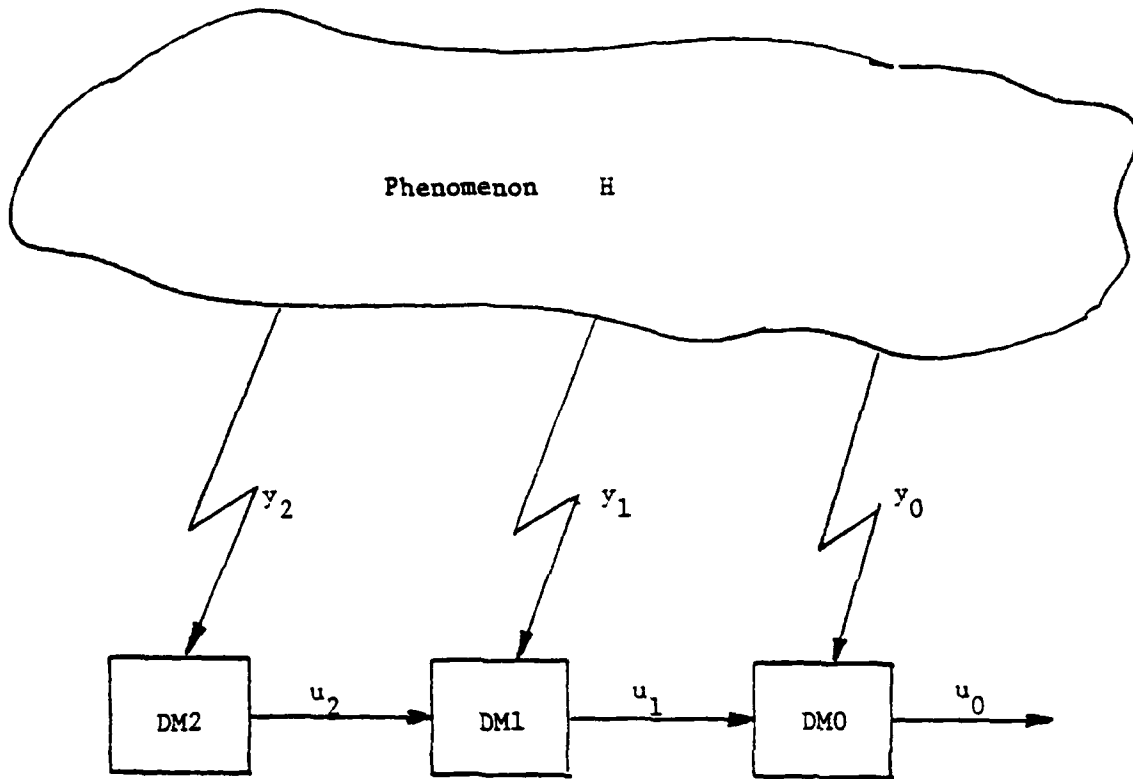


Figure 3: Three DM Tandem Topology.

The decision rule for DM1 is

$$u_1 = \delta_1(\gamma_1, u_2) \quad (37)$$

and $\delta_1(\cdot)$ is the following likelihood ratio test:

$$\Lambda_{01}^1 \triangleq \frac{P(\gamma_1|H^0)}{P(\gamma_1|H^1)} \begin{matrix} >_{u_2=0} \\ <_{u_2=1} \end{matrix} \frac{\sum_{H^1} P(H^1) P(u_2|H^1) J(u_0, H^1) [P(u_0|u_1=0, H^1) - P(u_0|u_1=1, H^1)]}{\sum_{H^0} P(H^0) P(u_2|H^0) J(u_0, H^0) [P(u_0|u_1=1, H^0) - P(u_0|u_1=0, H^0)]} \triangleq \begin{matrix} t_0' & \text{if } u_2=0 \\ t_1' & \text{if } u_2=1 \end{matrix} \quad (38)$$

The decision rule for DM2 is

$$u_2 = \delta_2(\gamma_2) \quad (39)$$

and $\delta_2(\cdot)$ is the following likelihood ratio test:

$$\Lambda_{01}^2 \triangleq \frac{P(\gamma_2|H^0)}{P(\gamma_2|H^1)} \begin{matrix} >_{u_2=0} \\ <_{u_2=1} \end{matrix} \frac{\sum_{H^1} P(H^1) J(u_0, H^1) [P(u_0|u_2=0, H^1) - P(u_0|u_2=1, H^1)]}{\sum_{H^0} P(H^0) J(u_0, H^0) [P(u_0|u_2=1, H^0) - P(u_0|u_2=0, H^0)]} \triangleq t_2. \quad (40)$$

Proof: Invoking similar steps as in Theorem 1.

Q.E.D.

Extensions to M-ary hypothesis, m_i symbol channels and N-tandem DMs are straightforward.

III. Tree Team Configuration

Assumption: Consider same assumptions as in Section II. First consider the simplest tree topology (Data Fusion) illustrated in Figure 4.

Theorem 4: The decision rule for DMO is given by

$$\Lambda_{01}^0 \triangleq \frac{P(\gamma_0|H^0)}{P(\gamma_0|H^1)} \begin{matrix} >_{u_0=0} \\ <_{u_0=1} \end{matrix} \frac{P(H^1) P(u_1|H^1) P(u_2|H^1) [J(0, H^1) - J(1, H^1)]}{P(H^0) P(u_1|H^0) P(u_2|H^0) [J(1, H^0) - J(0, H^0)]} \quad (41)$$

$$\triangleq \begin{cases} t_{00} & \text{if } u_1=0, u_2=0 \\ t_{01} & \text{if } u_1=0, u_2=1 \\ t_{10} & \text{if } u_1=1, u_2=0 \\ t_{11} & \text{if } u_1=1, u_2=1 \end{cases} \quad (42)$$

The decision rule for DM $_i$ ($i \in \{1, 2\}$) is

$$u_i = \delta_i(\gamma_i) \quad \text{for } i \in \{1, 2\}. \quad (43)$$

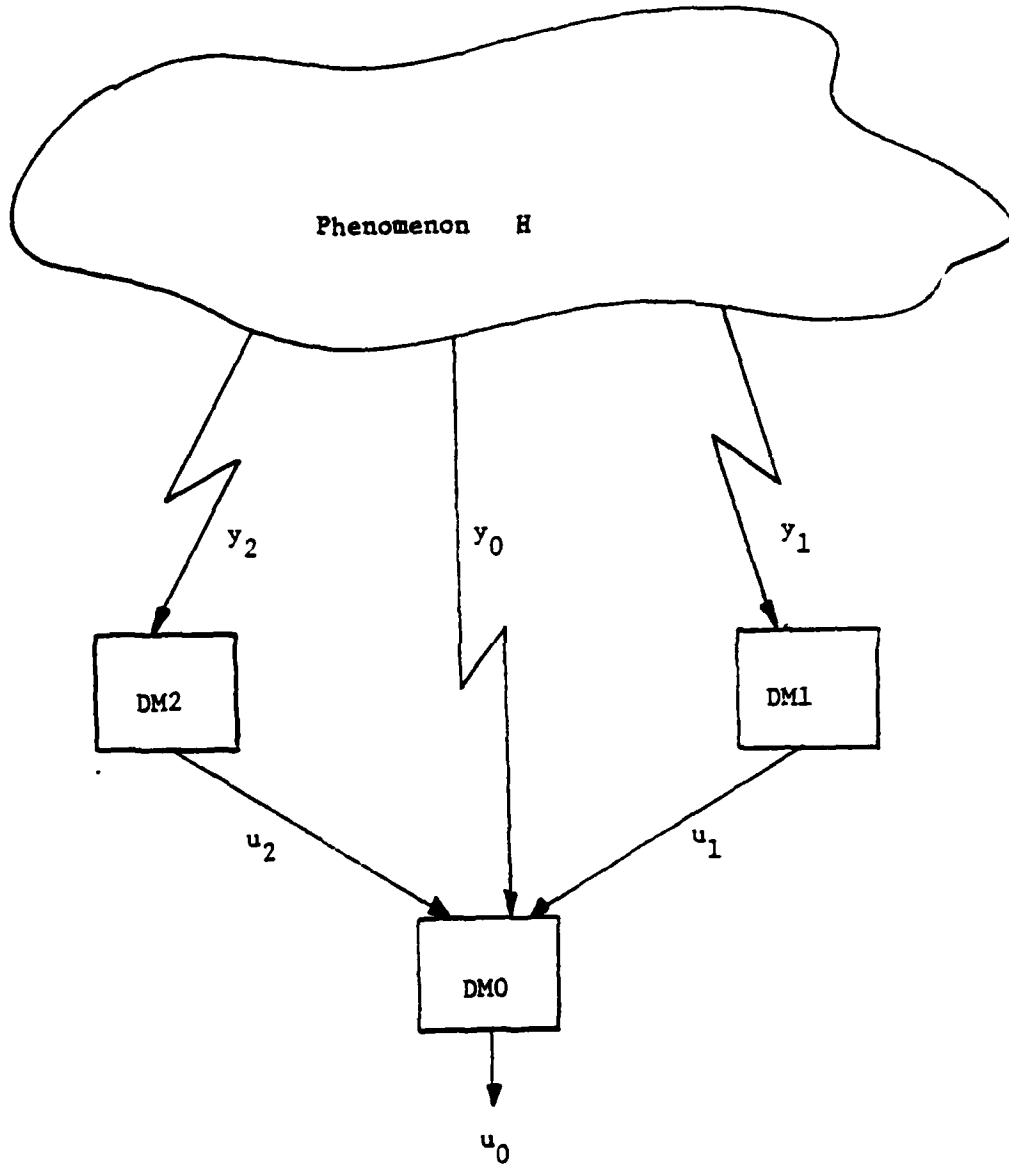


Figure 4: Optimal Data Fusion Topology

and $\gamma_i(\cdot)$ is the following likelihood ratio test:

$$\Lambda_{01}^i \triangleq \frac{P(\gamma_i | H^0)}{P(\gamma_i | H^1)} \begin{matrix} > \\ < \end{matrix} \frac{\sum_{u_0} P(H^1) \tilde{J}(u_0, H^1) [P(u_0 | u_i=0, \gamma_0) - P(u_0 | u_i=1, \gamma_0)]}{\sum_{u_0} P(H^0) \tilde{J}(u_0, H^0) [P(u_0 | u_i=1, \gamma_0) - P(u_0 | u_i=0, \gamma_0)]} \quad (44)$$

$$\triangleq t_i \quad (45)$$

Proof: This proof parallels the one in Theorem 1.

Q.E.D.

Extensions: Consider DML in the tree structure illustrate in Figure 5.

Assume the pair of decision rules in DM4 and DMO are fixed. Then a cost function of u_1 and H at DML, $\tilde{J}(u_1, H)$, can be obtained by taking the expectation over y_0 and y_4 , i.e.

$$\tilde{J}(u_1, H) = E_{\gamma_0, \gamma_4} \left\{ \tilde{J}(u_0, H) \mid \gamma_4(\cdot), \gamma_0(\cdot) \right\}$$

Given this cost, one can solve for decision rules γ_1, γ_2 and γ_3 using Theorem 4. Inserting these decision rules into the the DMs, one can compute $p(u_1/H)$. Then one can solve for the decision rules for DMO and DM4 with u_1 (described by $p(u_1/H)$) by using the results from the tandem case (Theorem 1). Note that u_1 acts as an additional measurement to DMO. The result of this process is an iterative procedure for alternately computing $\{\gamma_4, \gamma_0\}$ and $\{\gamma_1, \gamma_2, \gamma_3\}$, which exploits a decomposition resulting from the independence of observations when conditioned on H (Assumption 1). This suggests a procedure for extending the results for these simple cases to larger networks.

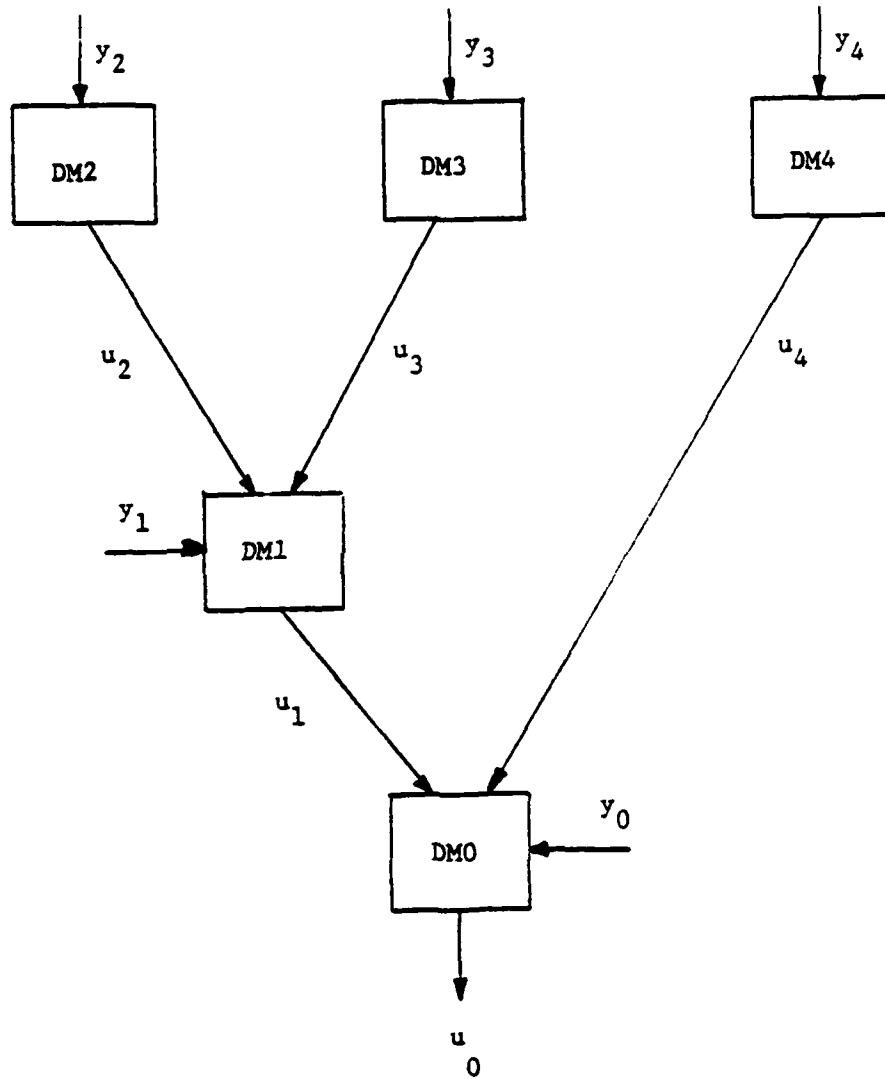


Figure 5: Tree Hierarchical Topology

IV. Summary

Optimal decision rules for a static non-PN detection team have been derived. The decision rules are likelihood ratios in the actual data, with thresholds determined by incoming communicated messages. The number of thresholds at each DM is equal to the number of combinations of these discrete inputs. Moreover, it is apparent that, at least for some tree structured problems, a decomposition principle (along the lines of spatial dynamic programming) can be found for methodically computing decision rules. This is a topic of current research and will be reported in a subsequent paper.

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