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ALTERNATIVE METHODS OF BASE LEVEL DEMAND FORECASTING FOR ECONOM--ETC(U)

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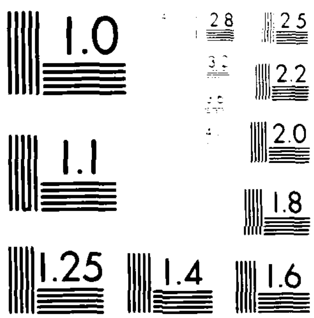

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ALTERNATIVE METHODS

OF

BASE LEVEL DEMAND FORECASTING

FOR

ECONOMIC ORDER QUANTITY ITEMS

A Research Effort Sponsored

by

AIR FORCE INSTITUTE OF TECHNOLOGY,

CIVILIAN INSTITUTIONS DIVISION

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AIR FORCE BUSINESS RESEARCH MANAGEMENT CENTER (HQ USAF)

Wright-Patterson Air Force Base, Ohio

and

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South Central Region, Anchorage, Alaska

by

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DISCLAIMER NOTICE

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Opinions, views or conclusions expressed by the author in this document are his own and are not to be considered or interpreted as official expression, opinion, or policy of the Department of the Air Force.

Acknowledgements

A study of this scope within the given time frame is beyond the capability of any one individual. I therefore wish to express my sincere appreciation to all who have provided assistance in this effort.

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I. Introduction

Problem Statement

The Air Force currently stocks and manages a total of 1.48 million expendable line items in its multi-level inventory system. The dollar investment in inventories of these relatively low cost items is over 1.1 billion.[1]

Efforts have been directed toward developing and applying models as approximations to reality in order to manage these large inventories. These models allow managers to conceptualize the nature of the supply environment, the inventory system, the various subsystems within the inventory system, and the relationships between environment, system and subsystems. An important benefit of this modeling approach is the fact that an appropriate model can provide a means of minimizing total inventory costs, and yet allow the system to remain responsive to the needs of the item users.

The Air Force currently uses a modified version of the classic Wilson lot size economic order quantity (EOQ) model for inventory system management of expendable line items. The objective of this model, designated the D062 model, is to minimize total variable costs of ordering items and maintaining them in inventories. The major constraints are budget restrictions and the requirement for adequate response to the needs of the item users.

Several significant factors of the D062 model have not been adequately refined and have caused problems in the practical application of the model to inventories within the Air Force supply environment. Major problems include inadequate estimates of meaningful cost to order and cost to hold factors, problems of budget constraints, and inaccurate forecasts of future item demand patterns.[2]

Some of these problems, and others not stated, were addressed in the development of the new A022 EOQ model, which the Air Force intends to adopt as a replacement to the D062. Many research efforts have been initiated to make the practical application of this new model more refined and therefore more effective. This paper is the result of one such research effort. The area of concern addressed by this research effort was the problem of inaccurate forecasts of future item demand patterns.

Background

The need for routine forecasting methods exists at all levels in the Air Force multi-level inventory system. Air Force base level forecasting is particularly important since requirements at this level drive total system stock levels. There are two primary objectives at base level for the EOQ management model. The first is to prevent stockouts and subsequent frustration of user needs. The second is to manage base stock levels to minimize the sum of the inventory carrying costs and ordering costs. Accurate forecasts of base level item demand patterns as an input to the EOQ model improves the model's capability to meet these objectives. In turn, base level demand patterns are inputs into higher echelon EOQ management systems, which also must resolve problems of stockouts versus inventory costs. The key then, to improving total system response in the area of stockouts versus inventory costs is more accurate base level forecasting of item demand.

Purpose of Research

The present method of unweighted moving averages to forecast base level demand patterns has a tendency to over-stock inventories during periods of declining demand and under-stock inventories during periods of significant demand increases. This over-stocking or under-stocking of inventories, applied to the world wide scale of the Air Force supply environment, results in large, unnecessary inventory holding costs on the one hand and emergency procurement (ordering) costs on the other.[3]

The purpose of this research effort was to examine and compare five methods of base level demand forecasting which can be considered as alternatives to the present method of unweighted moving averages. All six methods, including unweighted moving averages, were evaluated according to relative accuracy in describing the actual base level demand patterns of a statistically chosen stratified sample of items managed at base level under the Air Force EOQ model.

Scope of Research

This study was exclusively concerned with expendable items which are stocked based upon D062 EOQ procedures. Expendable items are those which are "consumed in use or which lose their original identity during incorporation into, or attachment upon, another assembly".[4] These items, commonly referred to as EOQ items, are designated by expendability, repairability, recoverability category (ERRC) codes "XB2" and "XB3", among others.

This study was strictly limited to a sample of XB2 and XB3 items stocked by the Air Force, and present in a substantial number of base level inventories. The complete definitions of XB2 and XB3 ERRC coded items are as follows:

XB2: Expendable, nonrecoverable (no repair) items with a projected annual requirement of $10,000 or more regardless of unit price.

XB3: Expendable, nonrecoverable (no repair) items with a projected annual requirement of $10,000 or less regardless of unit price.[5]

Five basic forecasting methods were evaluated. These methods were: unweighted moving averages, least squares, and the methods of single, double and triple exponential smoothing. A sixth method was generated by modifying the single exponential smoothing method. The modification consisted of incorporating an adaptive exponential smoothing constant which can be changed according to the affects of exogenous variables. The results obtained with this sixth method were also compared against the results of the five basic methods.

Objective

The validity of a forecasting method rests with its ability to perform in a real world environment. Therefore it was decided to test the six given forecasting methods using actual demand experience. The objective, then, of this study was to evaluate the given forecasting methods using an actual sample of economic order quantity items stocked at a base level consolidated supply activity. The methods were compared to determine if significant differences existed in the accuracies of their respective forecasts.

Brief Summary of Findings

This summary describes the three most accurate of the six evaluated forecasting methods over the total sample set of 316 items and three subsets of the total set. The subsets are: Subset 1, the 71 items with relatively high level, non-erratic demand processes; Subset 2, the 199 items with relatively low level, non-erratic demand processes; and finally, Subset 3, the 46 items with relatively erratic demand processes. An erratic demand process means, in this study, that no discernible constant level, linear trend or curved trend in demand was present over the entire 15 month period for a given item.

Forecasting Method	Mean Squared Error	Mean Forecast Error	Variance
1. Single Exponential Smoothing	16.02	1.74	13.02
2. Moving Average	17.84	1.78	14.72
3. Double Exponential Smoothing	23.89	1.89	20.39

Table 1. Total Set (316 Items)

Forecasting Method	Mean Squared Error	Mean Forecast Error	Variance
1. Single Exponential Smoothing	67.99	5.51	38.16
2. Moving Average	76.18	5.68	44.49
3. Double Exponential Smoothing	103.19	6.14	66.38

Table 2. Subset 1 (71 Items)

Forecasting Method	Mean Squared Error	Mean Forecast Error	Variance
1. Single Exponential Smoothing	.36	.49	.12
2. Moving Average	.37	.49	.12
3. Double Exponential Smoothing	.40	.50	.14

Table 3. Subset 2 (199 Items)

Forecasting Method	Mean Squared Error	Mean Forecast Error	Variance
1. Double Exponential Smoothing	3.56	1.32	1.86
2. Moving Average	3.68	1.31	2.01
3. Single Exponential Smoothing	3.81	1.35	2.04

Table 4. Subset 3 (46 Items)

Single exponential smoothing provided the most accurate forecasts for the total set of 316 items and also for component subsets 1 and 2, based on rankings according to variance, mean forecast error, and mean squared error. Only on the 46 items with relatively erratic demand processes did the order of accuracy change. In this case, double exponential smoothing was the most accurate method. The moving average method consistently ranked second for the total set and subsets 1, 2 and 3.

However, according to the Kruskal-Wallis test which compared the absolute average error distributions, no significant difference was detected in the relative effectiveness of the six forecasting methods. A confidence interval of 95 percent was used for this test.

Organization

The remaining three chapters of this research paper are concerned with the methodology used in the study, an analysis of results, and a detailed summary. Chapter II. Methodology, contains a description of the data, a presentation of the forecasting methods and a discussion of the experimental design. Chapter III. Analysis of Results, compares forecast method performance. Chapter IV. Detailed Summary, presents a detailed summary of the findings, lists the limitations inherent in the research methodology, and provides recommendations for action.

II. Methodology

Description of Data

All six methods of forecasting were evaluated according to relative refinement and accuracy in describing the actual demand patterns of a statistically chosen stratified sample of items managed at base level under the Air Force EOQ model. The methodology used in choosing the sample is important because the results generated by testing the six forecasting methods apply only to the demand processes of which the sample was representative. This section is therefore concerned with the source of data in the sample, the techniques used in sample selection and a description of how the sample was used in the actual testing of the six forecasting methods.

The only source of detailed base level demand data for EOQ managed items is the consolidated and daily transaction registers maintained at base consolidated supply activities. Such registers are detailed records of all transactions on all active items stocked at the base. Registers are maintained for the previous and current calendar years so that historical data is available for one to two years back depending on the current calendar month.[6]

Source of Data. The empirical basis for this research effort was 15 months of demand data (1 January 1973 through 31 March 1974) for selected expendable supply items. The data source for the selected items was the transaction registers maintained by the 21 Supply Squadron, Elmendorf Air Force Base, Alaska.

Time Periods for Demand Data. The actual demand data was aggregated into 15 monthly totals for each item. The first 10 months of demand history (1 January 1973 through 31 October 1973) were used as the experience base from which to make monthly forecasts for the following 5 months (1 November 1973 through 31 March 1974). The 10 month experience base was designated as the base period. The following 5 months, then, was designated the forecast period. The purpose of the actual demand data in the forecast period was to use it for error comparison in evaluating the relative accuracy of the given six forecasting methods.

Sampling Technique. The initial sample included all XB2 and XB3 items for three Air Force weapons/support systems. Those items which had at least one demand during the two year period 1 July 1972 through 31 June 1974 were included in the sample. The selection process for the three systems was essentially random, with the criteria that the systems be commonly deployed at many bases, and not unique to the Alaskan theater.

The sample items for the three chosen systems were stratified according the "ABC" technique. System identity was not maintained in this stratification process. The final sample (selected from the initial sample) was chosen to be reasonably representative of the stratified "B" cost group of all XB2 and XB3 expendable items used in support of common Air Force weapons/support systems. A brief explanation of the ABC technique is pertinent at this point.

The ABC Technique. Inventory management involves the management of individual items. Unless each item stocked is under a reasonable degree of control, the aggregate will not be under adequate control. A technique is needed that will isolate those items that have a high total dollar inventory turnover per year from those that have a relatively low total dollar turnover.

The high dollar turnover items should be of primary concern to the inventory management system and should be controlled with more precision compared to low dollar turnover items. This is simply an application of Pareto's Principle of Maldistribution, which has been expressed as follows: "Very often a small number of important items dominate the results while at the other end of the line are a large number of items whose volume is so small that they have little effect on the results".[7]

Normally, input data for each item undergoing ABC analysis should include the following:

(1) Stock Number

(2) Unit Cost

(3) Usage per time period

These data are manipulated in the following manner:

(1) Multiply usage per time period by unit cost to obtain the dollar demand for each item.

(2) Sort the items according to dollar demand in a descending sequence.

(3) List this sequence. Obtain a cumulative dollar demand figure as each item is added to the list.[8]

The listing will normally show that a relatively few items have a high impact on usage value. The "A" group is the classification of highest dollar demand items. It is usually chosen as those items that make up 85 to 88 percent of the total cumulative dollar demand figure, going down the list from the highest dollar demand item toward the lowest. Usually the A group includes only about 20 percent of the total number of items.[9]

The "B" group classification is usually chosen to be 95 to 98 percent of the total cumulative dollar demand figure. The B group includes those items starting at the top of the list and going down until 95 to 98 percent of the cumulative dollar demand is reached. The B group adds another 20 percent of the total items to those selected by the A group, for a total of about 40 percent of the total items. The remaining, low usage items are classified as the "C" group, which contains about 60 percent of the total items.[10]

The ABC Technique Applied. The initial sample of items chosen for this analysis included, as previously stated, all XB2 and XB3 items required for three weapons/support systems in common Air Force use. For inclusion in the initial sample, each item must have had at least one demand during the two year period 1 July 1972 through 31 June 1974. The three randomly chosen weapons/support systems were: the F4E aircraft, the C-130 aircraft and the Digital Subscriber Terminal Equipment (DSTE).

Following the ABC technique, the required input data for each item of the initial sample were manipulated in the following manner:

(1) The dollar demands of the initial sample of XB2 and XB3 items were calculated according to:

D_i = Dollar demand for item i (i = 1,2,3,..,n).

$D_i = V_i \times P_i$, where

$V_i = \frac{\text{demand volume}}{\text{year}}$,

P_i = unit price and

n = total number of items.

(2) All items were then sorted from highest to lowest value of D_i.

(3) After listing the sorted items, the D_i for each item was added, in sequence, cumulatively.

$$\sum_{i=1}^{n} D_i = \text{Total cumulative dollar demand per year.}$$

(4) Items selected for the final sample included those starting with the highest value of D_i, moving continuously down the list until 95 percent of the total cumulative dollar demand figure was reached.

The final sample was therefore the B cost group of the initial sample. This application of the ABC technique selected the most active

XB2 and XB3 items, dollar wise, in the Elmendorf AFB inventory for the three randomly chosen systems during the period 1 July 1972 through 31 June 1974. Continuous demand data for the final sample was available for the period 1 January 1973 through 31 March 1974. These demand data for the final sample of items were used to evaluate the six forecasting methods considered in this research effort. There were 977 items in the initial sample. The B group reduced the final sample size to 316 items, approximately 32 percent of the total.

Forecasting Methods

The objective of this research effort was to evaluate the six given forecasting methods using an actual sample of EOQ items stocked at a base level consolidated supply activity. The selection of these particular six forecasting methods was based on choosing methods which could be used routinely and inexpensively to forecast demand for a large number of relatively inexpensive items based entirely on historical demand data. The exponential smoothing method with an adaptive smoothing constant was the only one of the six methods evaluated which explicitly accounted for the effects of exogenous variable (variablesother than historical demand), yet the data requirements are the same for this method as for the other five methods. Other more sophisticated methods which employ such techniques as spectral analysis, iterative dynamic programming, Bayesian analysis, etc., were rejected as being impractical for base level use because of the higher levels of computer storage and computational requirements.

Moving Averages Method. The process of computing a moving average is quite straightforward. The mathematical formula for calculating the expected demand in a period simply averages the demand experienced in the previous n periods. The average demand forecast minimizes the sum of the squares of the differences between the most recent n observations and the expected value of the demand in the period being forecasted.[11] It is simple and easily adapted to automatic data processing equipment. The formula is as follows.

$$v_t = \frac{1}{n} \sum_{i=t-n}^{t-1} d_i \qquad (1)$$

where: v_t is the forecast of demand in period t.

d_i is the actual demand in period i.

n is the number of periods of actual demand which are used to develop the forecast.

For example, let the period be one month and n = 10. Then the forecast for the first month after time t = 0 is v_i, and $\frac{1}{10}$ the sum of the demands in months -9 through 0, i.e.,

$$v_1 = \frac{1}{10} \sum_{i=-9}^{0} d_i = \frac{1}{10}(d_{-9} + d_{-8} + \ldots. + d_0) \qquad (2)$$

The effect of moving average forecasting depends on the number of periods (n) used to make the forecast. When a large number of periods are used, the weight given to each period is relatively small, and random fluctuations have little effect on the forecasts. The moving average method is more sensitive to changes in demand if n is small. The moving average method works best when the demand generating process is stable. If the demand generating process has a constant mean and variance, and successive observations are uncorrelated, then the sample mean is an unbiased estimate of the process mean. If these assumptions are met, then the demand generating process can be considered stable.[12]

Least Squares Method. The least squares method of forecasting assumes that the demand d_t in period t can be predicted using $w_t = a + bt$, where a, b are determined from historical data by minimizing

$$F = \sum_t (d_t - w_t)^2 = \sum_t (d_t - a - bt)^2 \qquad (3)$$

In the above equation, w_t is the forecast demand for period t and d_t is the actual demand for period t. Assume that a, b are to be determined by using the demands in the previous n periods. The current time t* is a review time, and that the time period from t* to t* - T, T being the time between reviews, will be referred to as period 0, the period from t* - T to t^* - 2T as period - 1, etc. Thus the number of the n periods to be used in determining a and b will be -(n-1), ..., -1, 0. To determine a and b, set $\partial F/\partial a = \partial F/\partial b = 0$ and solve the resulting equations for a and b.

$$\frac{\partial F}{\partial a} = -2 \sum_{t=0}^{-(n-1)} t(d_t - a - bt) = 0 \tag{4}$$

$$\frac{\partial F}{\partial b} = -2 \sum_{t=0}^{-(n-1)} (d_t - a - bt) = 0 \tag{5}$$

where: $\sum_{t=0}^{-(n-1)} t = \frac{-n(n-1)}{2}$; $\sum_{t=0}^{-(n-1)} t^2 = \frac{1}{6} n(n-1)(2n-1)$

and $$U = \sum_{t=0}^{-(n-1)} d_t \; ; \; P = \sum_{t=0}^{-(n-1)} td_t \tag{6}$$

Then the solution to the equations (4) and (5) are:

$$b = \frac{12}{n(n-1)(n+1)} \left[P + \frac{n-1}{2} U\right] \; ; \; a = \frac{U}{n} + b\frac{(n-1)}{2}$$

so that $w_t = \frac{U}{n} + b \left[t + \frac{n-1}{2}\right]$

$$w_t = \frac{U}{n} + 12 \left[\frac{P + \frac{n-1}{2}U}{n(n-1)(n+1)}\right] \left[t + \frac{n-1}{2}\right], \; t = 1,2,3,.. \tag{7}$$

Equation (7) allows computation of the forecasted demand for period t, $t = 1,2,3,...$, using data for the demands in periods $-(n-1), ..., -1,0$.[13]

For example, let the period be one month and $n=10$. Then the forecast for the first month after time $t^* = 0$ is w_1, and is:

$$w_1 = \frac{U}{10} + 12 \left[\frac{P + \frac{9}{2}U}{990}\right] \left[t + \frac{9}{2}\right]$$

where: $U = \sum_{t=0}^{-(n-1)} d_t$; $P = \sum_{t=0}^{-(n-1)} td_t$

The least squares method, sometimes called simple linear regression, incorporates the ability to follow a demand pattern which has a linear trend. The demand process, as taken from the above derivation is assumed to be:

$$w_t = a + bt$$

The demand in period t is a function of a, the intercept value of the trend line; b, the slope of the trend line and t the number of periods between the ordinate and time t^*.[14]

It is possible to have least squares method forecasts that are less than zero using the given formulas. It was therefore reasonable to impose a requirement that demand forecast must always be greater than or equal to zero. If the forecast demand (using the formula) was less than zero for a particular period, the forecast demand was set to zero. This procedure also had important consequences for subsequent forecast demands, since the forecast demand that was set to zero was used for forecasting demands in the following n periods.

The moving average and least squares methods for forecasting demand both require demand data be available for n periods back. This disadvantage is not inherent in the exponential smoothing methods to follow.

Single Exponential Smoothing Method. Single exponential smoothing is a forecasting method similar to moving averages in the sense that it provides accurate forecasts for a stable demand pattern. The forecasting formula is:

$$X_t = \alpha d_{t-1} + (1 - \alpha)X_{t-1} \tag{8}$$

where: X_t = the single exponential smoothing forecast for demand in period t,

d_{t-1} = the actual demand experienced in period t-1,

X_{t-1} = the single exponential smoothing forecast for demand made for period t-1.

α = the smoothing constant, $0 \leq \alpha \leq 1$.[15]

The effect of single exponential smoothing depends on the size of α, the smoothing constant. If α is large, more weight is given to the most recent demand experience and the forecast is more sensitive to fluctuations. Instead of weighting each past observation equally as does the moving average method, the weight given to previous observations decreases geometrically with age. If the demand generating process has a constant mean and variance, and successive observations are independent, then the expectation of the single exponential forecast is equal to the expectation of the mean of the demand generating process. If the assumptions about the demand generating process are met, single exponential smoothing can be used to produce an accurate estimate of demand. Therefore, single exponential smoothing is as accurate as the moving averages technique, while computation is simpler and storage requirements are reduced.[16]

Double Exponential Smoothing Method. Double exponential smoothing is a forecasting method which uses a first order polynomial. Thus it incorporates the ability to follow a demand pattern which has a linear trend. The demand generating process is assumed to be:

$$d_t = a + bt.$$

The demand in period t is a function of a, b and t. The coefficient a is the intercept value of the trend line; b, the first derivative of d_t with respect to t, is the slope of the trend line; and t is the number of periods between the ordinate and period t*.[17]

In order to develop the forecast Y_t for this process using exponential smoothing, coefficients a and b can be estimated using two smoothed statistics.

$$a_t = d_{t-1} + (1 - \alpha)^2 (Y_{t-1} - d_{t-1})$$

$$b_t = b_{t-1} + \alpha^2 (Y_{t-1} - d_{t-1})$$

The one period double exponential smoothing forecast Y_t is then:

$$Y_t = a_t + b_t \tag{9}$$

The forecast for τ periods in the future is:

$$Y_t = a_t + b_t \tau. \tag{10}$$

Written in terms of single exponential smoothing values and α, the double exponential smoothing forecast for period t is:

$$Y_t = \alpha X_t + (1 - \alpha) Y_{t-1} \tag{11}$$ [18]

Use of the double exponential smoothing model should be based on certain assumptions about the demand pattern. If the time series has a trending average demand rate, either increasing or decreasing, the double exponential smoothing method produces an accurate estimate of demand. For such a demand pattern, the moving averages and single exponential smoothing methods lag behind a trend, while the double exponential smoothing method is more responsive in that it accounts for a trend factor.[19]

Triple Exponential Smoothing Method. Triple exponential smoothing is based upon a second order polynomial time series process which accounts for trend and the rate of change of trend. The assumed demand generating process is:

$$d_t = a + bt = \tfrac{1}{2}\, ct^2$$

where b is the first derivative of d_t with respect to t evaluated at t=0; and c is the second derivative of d_t with respect to t, which makes it the coefficient for the rate of change of trend. Coefficients a, b and c can be estimated by computing three smoothed statistics:

$$a_t = d_{t-1} + (1 - \alpha)^3(Z_{t-1} - d_{t-1}),$$

$$b_t = b_{t-1} + c_{t-1} - \tfrac{3}{2}\alpha^2(2 - \alpha)(Z_{t-1} - d_{t-1}),$$

$$c_t = c_{t-1} - \sigma^3(Z_{t-1} - d_{t-1});$$

where Z_{t-1} is the triple exponential smoothing forecast made for period t-1. The forecast for period t=1 is then:

$$Z_t = a_t + b_t + \tfrac{1}{2}\, c_t. \tag{12}$$

The forecast made for τ periods in the future is:

$$Z_\tau = a_t + b_t\tau + \tfrac{1}{2}\, c_t\tau^2 \tag{13}$$

Written in terms of double exponential smoothing values and α, the triple exponential smoothing forecast for period t is:

$$Z_t = \alpha Y_t + (1 - \alpha)Z_{t-1} \tag{14}$$

Triple exponential smoothing is accurate when the process to be forecasted can be adequately represented as a quadratic function of time. Thus, if there is reasonable justification of assuming that the time series can be represented by a polynomial of the form

$$d_t = a + bt + \frac{1}{2} ct^2,$$

then triple exponential smoothing provides the most accurate and stable forecast methods.[20]

NOTE: For the given three and higher order exponential smoothing methods, Brown, in his proof of the fundamental theorem of exponential smoothing, has demonstrated that it is possible to estimate the (n+1) coefficients in an nth order polynomial for the corresponding order of exponential smoothing by using linear combinations of the first (n+1) orders of exponential smoothing. It should be noted at this point that Brown's proof can be used to derive formulas (8), (11), and (14).[21]

Adaptive Single Exponential Smoothing. The single exponential smoothing forecasting equation is:

$$X_t = \alpha d_{t-1} + (1 - \alpha) X_{t-1} \tag{15}$$

Let X_t^* equal the adaptive single exponential smoothing forecast for period t, and let α^* be an adaptive smoothing constant. The adaptive single exponential smoothing formula is therefore:

$$X_t^* = \alpha^* d_{t-1} + (1 - \alpha^*) X_{t-1} \tag{16}$$

The single exponential smoothing method provides good forecasts when the demand process has a constant mean and variance. However, actual demand is often subject to sudden incremental changes due to variables exogenous to the model. For example, the number of systems for which an item is used may be suddenly increased or decreased. Under these conditions, as the level of demand changes, the single exponential smoothing formula will change the forecasts over time, moving towards the new demand curve at a rate dependent on the value of α, the smoothing constant. The adaptive aspects of this adaptive single exponential smoothing method were built upon this feature.[22]

The basic philosophy was to change α^* to a value of one when sudden shifts in the actual demand level, d, were detected. This has the advantage of changing the estimates of the mean demand, X_t^*, almost immediately to the range of the new demand mean. The procedure to accomplish this was based on the observation of actual demand values outside an acceptable range around the current forecast.

It is assumed for the purposes of this model that the random fluctuations about the basic demand level, d, were normally distributed with a known, constant standard deviation. In such a case, it is natural to use a set of controls on X_t^* which permit a rapid response to abrupt changes in demand patterns. There is, however, a difference between the problem here and the apparent analogy with the classical quality control case of controlling a process by detecting points outside control limits set around the desired process mean. The difference is brought about because the initial demand forecast is likely to be different from the true mean demand by an amount dependent on the smoothing constant and the demand sequence. This possible error means that a single actual demand, d_t, may differ sufficiently from the current forecast to lead one to suspect that the basic demand has shifted when, in fact, it has not.

Because of this complication, two criteria were developed for concluding that a new demand level had been detected and changing the smoothing constant is necessary. The first of these criteria is applied in the case where a single demand is discovered outside a range of four standard deviations around the current mean of actual demand. The second criteria is applied when two successive demands differ from the current estimate by more than 1.2 standard deviations and both of these points are either above or below the mean actual demand.

The second consideration in the development of the adaptive aspects of the model was the choice of the values of the smoothing constant after the detection of a shift in the basic demand level. The model uses a 3-stage change of the smoothing constant. The constant is set equal to 1 for the first period after a change in the basic level of demand has been detected, and to .8 for the second period and .5 for the third period. In the fourth period the value of α is returned to its "normal level.

<u>Choosing the Smoothing Constant</u>

Approach. The smoothing constant, α, is used in each of the exponential smoothing forecasting methods. According to Brown, the value of α is usually chosen between .01 and 0.3. Values between 0.3 and 1 are normally not used, since these values cause the formula to be too sensitive to random, non-representative fluctuations in the demand generating process. For the exponential smoothing methods evaluated in this study, values of α between .05 and .5, in increments of .05, were considered. The value of α which minimized the standard deviation of forecasting errors for the single exponential method was chosen as the optimum α.

The purpose of the smoothing constant is consistent in all of the exponential smoothing methods. This purpose is to weight the past demands used in the forecasting formula in a single, prescribed manner. The α chosen should strike a balance between responsiveness to demand process changes (value of α approaching 0.5) and damping of random fluctuations in the demand process (value of α approaching .05). It should also be noted that in this study, the same set of demand data was operated on by all forecasting methods. Considering the purpose of α and the use of the same demand data, it was reasonably assumed that the optimum value of α for single exponential smoothing approximates the optimum value of α for double and triple exponential smoothing. The α that minimized the standard deviation of forecast errors for the single exponential smoothing method was therefore also used as the optimum α for the double and triple exponential smoothing methods.

This optimum α was also used as the "normal" value of α for the adaptive triple exponential smoothing method.

<u>Method</u>. Specifically, the optimum α from the set $.05 \leq \alpha \leq 0.5$ in increments of .05, was selected in the following manner:

$$(1) \quad X_t' = \alpha d_{t-1} + (1 - \alpha) X_{t-1}, \tag{17}$$

where X_t = the single exponential smoothing forecast for period t. X_{t-1} = single exponential smoothing forecast for period t-1, except $X_{t-1} = X_0$, where $X_0 = \frac{1}{10} \sum_{i=-9}^{0} d_i$.

(2) Find X_t for each item for each value of α, $.05 \leq \alpha \leq 0.5$. There are 10 values of X_t, one for each α, for each item per month. This was repeated for all 316 items for each month of the forecast period, 1 November 1973 through 31 March 1974.

(3) The differences between actual demands d_i and forecast demands X_t for each month of the forecast period were found for each item, for each α.

Let α_j for j = 1,2,3,...,10 and

$$e_{\alpha j} = \frac{1}{5} \sum_{i=t=1}^{5} |X_t - d_i|, \quad (j = 1,2,3,\ldots,10) \tag{18}$$

Repeat for each item.

(4) The mean for each $e_{\alpha j}$ was then found.

$$\mu_j = \frac{1}{n} \sum_{i=1}^{n} (e_{\alpha_j})_i \text{ , n = 312 items. Repeat for each } \alpha_j. \tag{19}$$

(5) The standard deviation for each e_{α_j} was then calculated.

$$s = \text{standard deviation} = \sqrt{\frac{\sum_{i=1}^{n} (e_{\alpha j} - \mu_j)^2}{n - 1}} \qquad (20)$$

Repeat for each α_j.

The value of α_j which produced the smallest standard deviation, s, was chosen as the optimum α. That value of α was then used in the calculations for the given exponential smoothing methods.

Initial Estimates

The first 10 months of demand history were used as the experience base from which to make monthly forecasts for the following five months. The smoothing constant α was selected based on the five month forecasting period. The exponential smoothing methods also require initial estimates for the coefficients X_{t-1}, Y_{t-1}, and Z_{t-1} when $t-1 = 0$. These initial estimates can severly bias method performance over a small number of periods.[23] Since the weight assigned to an estimate decreases geometrically with age, the effect of a biased initial estimate eventually becomes insignificant. However, in this study, the forecasting period included only five months. Reasonable care then, had to be taken in determining the values of the initial coefficients.

Initial Estimate: Single Exponential Smoothing. A common and normally sufficient approach to providing the initial estimate of coefficient X_0 for single exponential smoothing is to take a simple average of a selected number of data points.[24] In this study therefore, the initial coefficient X_0 was determined by taking an average of the actual demand over the 10 month base period.

$$X_0 = \frac{1}{10} \sum_{t=-9}^{0} d_t \tag{21}$$

Initial Estimate: Double Exponential Smoothing. The double exponential smoothing method requires initial estimates for both the slope and the intercept coefficients a and b:

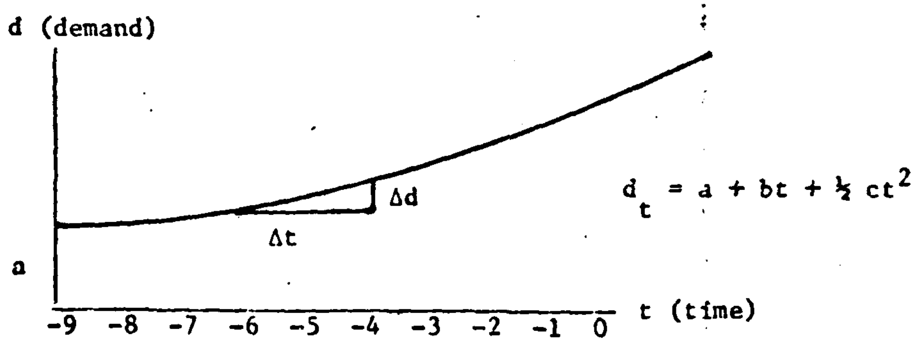
$$Y_t = a + bt \tag{22}$$

The method used to compute Y_0 was a simple linear least squares regression based on the 10 data points of the base period. The least squares estimates of a and b served as the components of Y_0 for the forecast Y_1. The remainder of the 5 months of the forecasting period were forecast using the standard double exponential smoothing formula.

Initial Estimate: Triple Exponential Smoothing. The triple exponential smoothing method requires initial estimates for the intercept, slope and rate of change of slop of the demand process equation.[25] Consider the assumed demand process:

$$d_t = a + bt + \tfrac{1}{2} ct^2 \tag{23}$$

Over the 10 data points of the base period,



Demand Versus Time Distribution

The value of a is the d axis intercept of d_t, and can be estimated by:

$$a = \frac{1}{10} \sum_{t=-9}^{0} d_t. \qquad (24)$$

The value of b is the limit as Δt approaches zero of $\frac{\Delta d}{\Delta t}$. Using data points at $t = -9,-7,-6,-4,=3$ and -1;

$$b = \frac{1}{3}\left[\frac{(d_{-9}-d_{-7})}{2} + \frac{(d_{-6}-d_{-4})}{2} + \frac{(d_{-3}-d_{-1})}{2}\right]$$

which reduces to:

$$b = \frac{1}{6}\,(d_{-9}-d_{-7}+d_{-6}-d_{-4}+d_{-3}-d_{-1}) \qquad (25)$$

The value of c is equal to the rate of change of b with respect to t. *Consider the rate of change of b using two data points of b:*

$$c = \frac{1}{2}\left\{\frac{1}{3}\left[\frac{(d_{-9}-d_{-7})}{2} \; \frac{-(d_{-6}-d_{-4})}{2}\right] + \frac{1}{3}\left[\frac{(d_{-6}-d_{-4})}{2} \; \frac{-(d_{-3}-d_{-1})}{2}\right]\right\}$$

which reduces to:

$$c = \frac{1}{12}\,(d_{-9}-d_{-7}-d_{-3}+d_{-1}) \qquad (26)$$

The following equation was taken from Brown's book, Smoothing, Forecasting and Prediction of Discrete Time Series, and is the triple exponential smoothing demand process quadratic formula in terms of a, b, c and α.[26] In this form, the demand process equation can be used to estimate Z_t. For Z_0 then;

$$Z_0 = a - \frac{3(1-\alpha)}{\alpha} b + \frac{3(1-\alpha)(4-3\alpha)}{2\alpha^2} c. \qquad (27)$$

The range of values taken on by Z_0 must be limited to positive demands. It is also reasonable to impose an upper limit of 2a, so that:

$$0 \leq Z_0 \leq 2a$$

When Z_0 calculated by formulas (24), (25), (26) and (27) is negative, let $Z_0 = 0$. When Z_0 calculated by (24), (25), (26) and (27) is $\geq 2a$, let $Z_0 = 2a$.

The value of Z_{t-1} at t-1 = 0 was calculated in the manner prescribed above. The values of Z_t for subsequent periods of course, were calculated using the standard triple exponential smoothing formula.

Evaluation Criteria For Forecasting Methods

A comparative evaluation of the accuracy of the six forecasting techniques was performed in terms of the forecast error each produced. Three statistical measures of error were selected: the mean square error, mean forecast error and variance of error.

The Mean Square Error. The mean square error (MSE) is a measure of forecast accuracy which emphasizes large errors.[27] The mean square error for each item and forecasting technique was computed by summing the squared forecast error for each of the 5 months and dividing by the number of months forecast. The equation for the MSE is developed from the definition of forecast error:

$$e_j = \frac{|R_t - d_t|}{5}$$

where e_j = the absolute average forecast error over the 5 month forecasting period

$R = V_t, W_t, X_t, Y_t, Z_t$, or X_t^*, depending on the forecast method used.

d_t = the actual demand in period t.

Then the mean square error is:

$$MSE = \frac{1}{n} \sum_{t=1}^{n} (e_t)^2 = \frac{1}{n} \sum_{t=1}^{n} (R_t - d_t)^2.$$

The mean square error for each item was summed and divided by the sample size to obtain an average mean square error. The forecasting technique with the smallest average mean square error was termed the most accurate in that it minimized the number of large errors.[28]

The Mean Forecast Error. This statistic was computed by averaging the absolute value of the forecast errors for each item by forecast model. The equation for the mean forecast error is:

$$MFE = \frac{1}{n} \sum_{t=1}^{n} e_j = \frac{1}{n} \sum_{t=1}^{n} |R_t - d_t|.$$

The average mean forecast error was then computed for each forecasting technique by summing the MFE for all items and dividing by the number of items. The preferred model was that for which the average MFE was closest to zero.

The Variance. The third comparative statistic was the variance of forecast error (VAR). A high variance indicates an unstable or erratic estimator, while a small variance indicates a consistent estimator.[29]

The variance of forecast error was computed for each item for each forecast model. The equation for the variance of forecast error is:

$$[(R_t - d_t) - \Sigma | R_T' - d_T |]$$

$$\text{VAR} = \frac{1}{n-1} \sum_{t=1}^{n} (e_t - \text{MFE})^2$$

The variances for the sample were then averaged for each model. The model with the smallest average variance was considered the most stable estimator.

Statistical Inference. At this point, it is pertinent to discuss statistical inferences concerning the error statistics of the six evaluated forecasting methods. A nonparametric test was necessary to evaluate the error statistics since no specific assumptions could be made about the distribution of the error population. Nonparametric tests are not concerned with parameters of a specific type of distribution, and no assumptions are made about the populations sampled.

The nonparametric Kruskal-Wallis test (H test) was used to determine if there was a significant difference in the effectiveness of the six forecasting methods evaluated in this study. The H test evaluates the null hypothesis that k independent random sample distributions come from identical populations against the alternative that the means of the populations are not all equal.[30]

In this case, the absolute average error distributions generated by each forecasting method were considered the independent sample distributions. Within each subset, the error distributions were compared to determine if they did or did not come from identical populations. If the pertinent error distributions were from identical populations, then there were no significant differences in the effectiveness of the respective forecasting methods. On the other hand, if they were not from identical polulations, then there were significant differences in effectiveness.

The combined absolute average errors from the forecasting methods being compared were ranked according to size. R_i was the sum of the ranks assigned to the n_i observations in the ith method and $n = n_1 + n_2 + \ldots\ldots + n_k$. The test was based on the statistic

$$H = \frac{12}{n(n+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3\ (n+1)$$

If the null hypothesis was true, the sampling distribution of this statistic could be approximated closely with a chi-square distribution with k-1 degrees of freedom. Consequently, the null hypothesis at the level of significance Z could be rejected if H exceeds χ^2_Z for k-1 degrees of freedom. An $\chi^2_{.05}$ was used to give a confidence interval of 95 percent.

<u>Final Sample Item Classifications</u>. From an examination of the toal demand data and the error statistics for each forecasting method, a relationship was noted between the demand characteristics of an item and the magnitude of its absolute average forecast error. Items with high demand rates tended to have larger error statistics than did items with low demand rates. Consequently, the former played a more significant role in the determination of the best forecast model. In order to equalize the effect that items had on this determination, total sample of items was divided into three subsets based on the demand process characteristics of each item. The six demand forecasting methods evaluated were therefore

applied to the following set and subsets of the 316 item final sample.

(1) The total set of 316 items.

(2) The subset which included the 71 items with relatively high level, non-erratic demand processes. Non-erratic means, in this study, a relatively constant demand process, or a demand with a discernible trend (linear or curved).

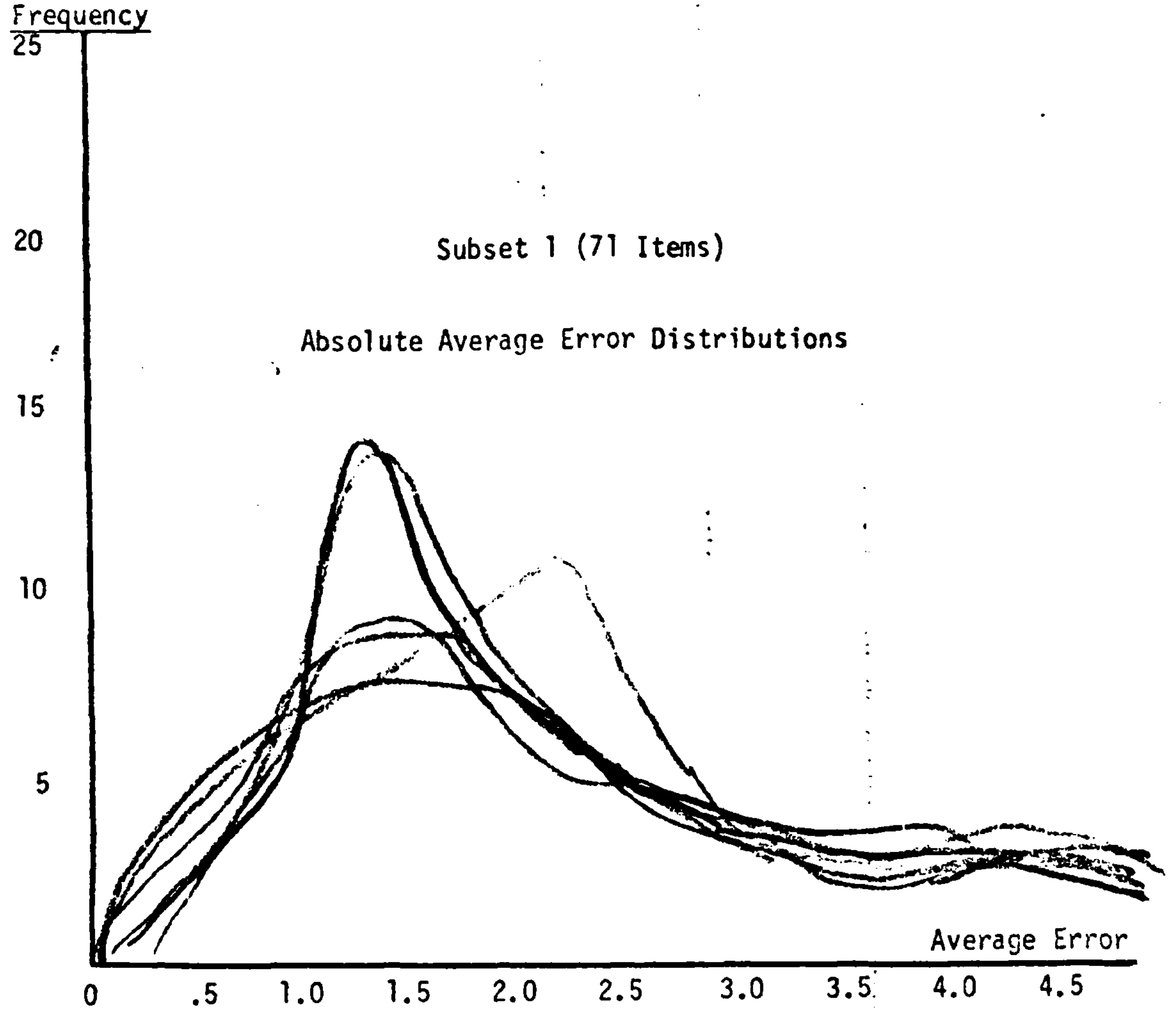
(3) The subset which included the 199 items with relatively low-level, non-erratic demand processes.

(4) The subset which included the 46 items with erratic demand processes (no discernible constant level or trend, linear or curved).

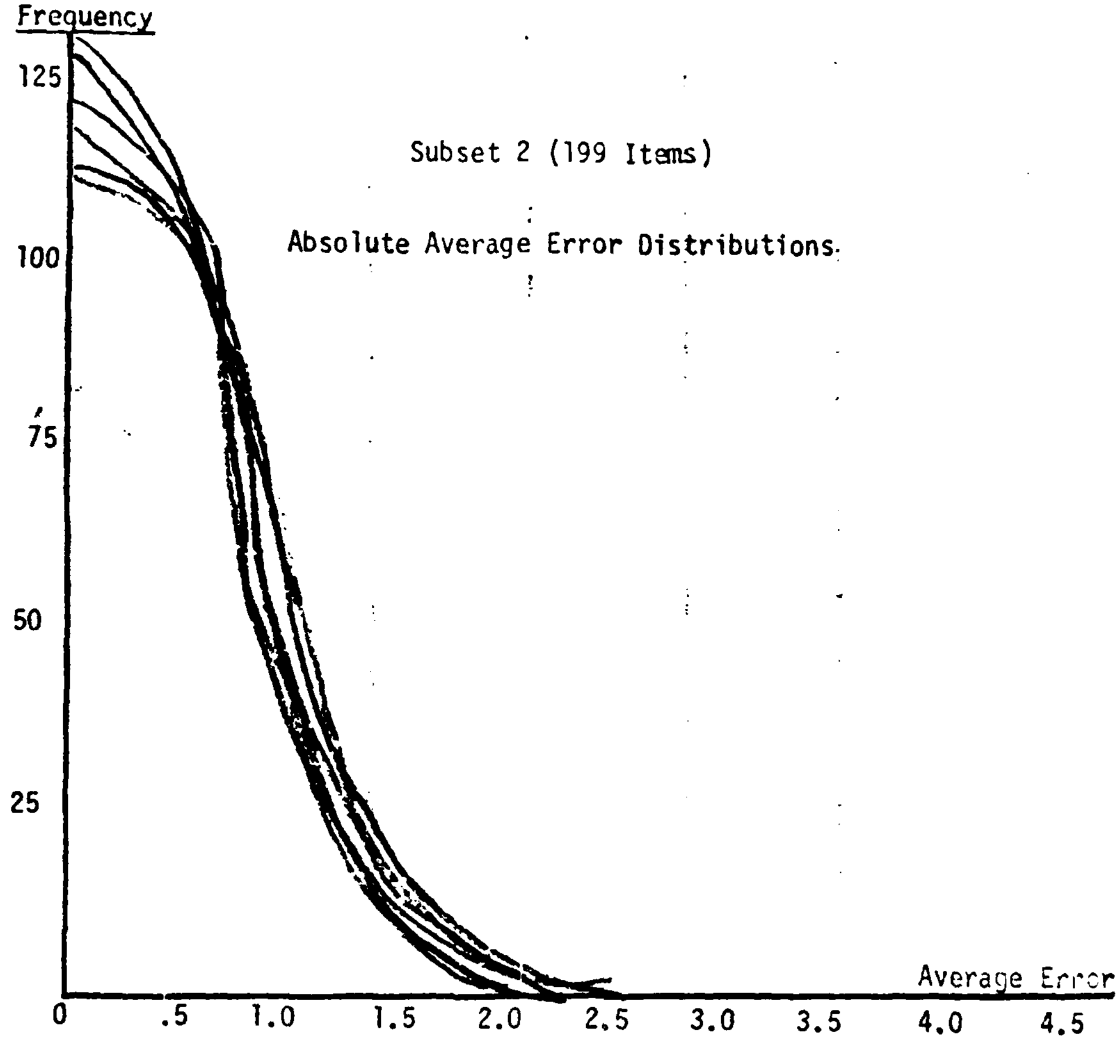
Standardization of Error Statistics. Consideration was given to standardizing the absolute average error statistics used as a basis for evaluating the six forecasting methods. For example, the absolute average error for each item could have been divided by the average monthly demand rate for that item to produce a standardized statistic independent of demand level. However, the absolute average error for each item would have been divided by the same average monthly demand rate for each forecasting method. The standardization process would not change the relative ranking of the forecasting methods. Standardization of the absolute average error statistics was therefore not used.

III. Analysis of Results

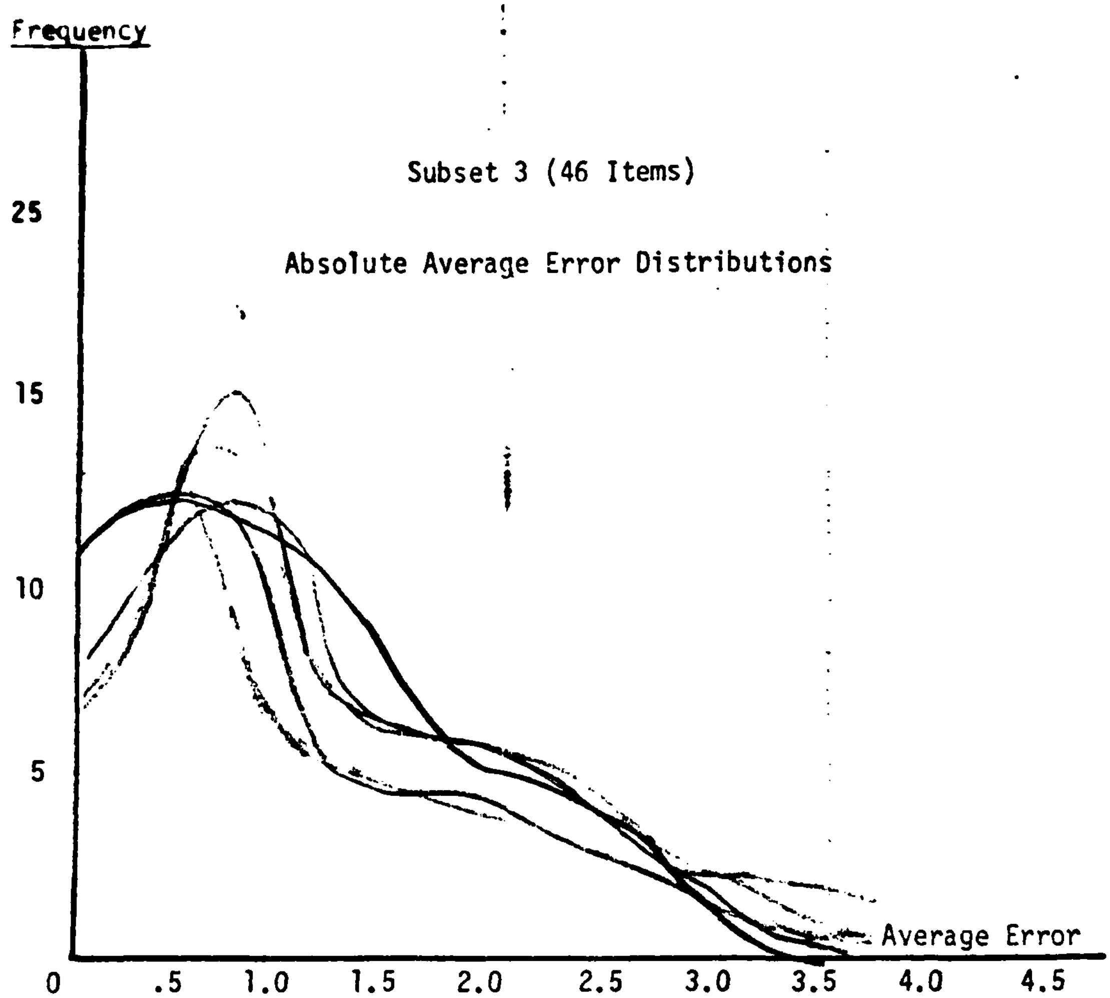
General Comparison of Forecast Method Performance



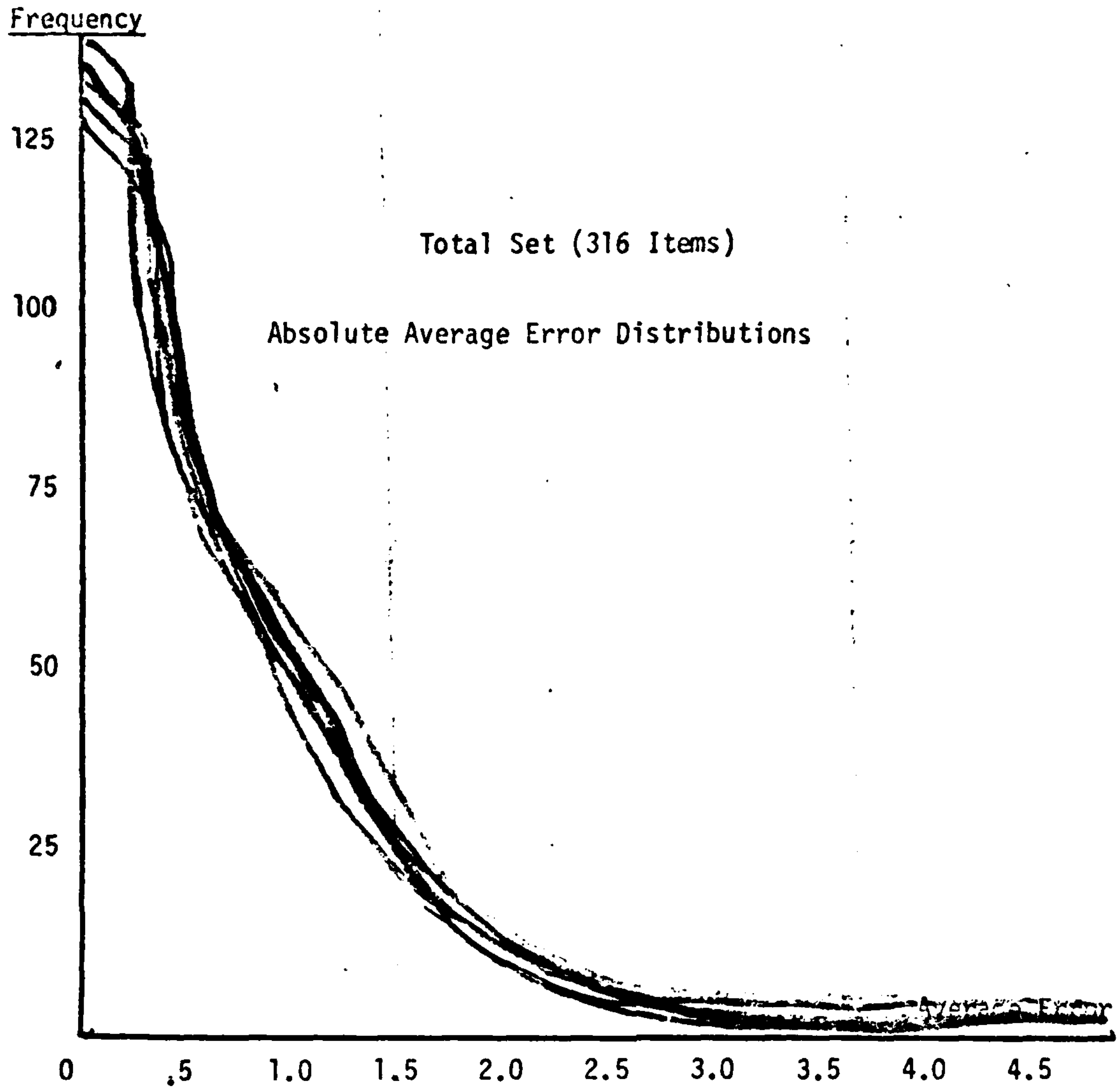
Method	Mean Squared Error	Mean Forecast Error	Variance
– Single Exponential Smoothing	67.99	5.51	38.15
– Moving Average	76.18	4.68	44.49
– Double Exponential Smoothing	103.19	6.14	66.38
– Least Squares	108.92	6.24	71.02
– Triple Exponential Smoothing	124.67	7.18	74.21
– Adaptive Single Exponential	120.78	6.22	83.28



Method	Mean Squared Error	Mean Forecast Error	Variance
Single Exponential Smoothing	0.36	0.49	0.12
Moving Average	0.37	0.49	0.12
Double Exponential Smoothing	0.40	0.50	0.14
Triple Exponential Smoothing	0.41	0.49	0.17
Least Squares	0.47	0.53	0.20
Adaptive Single Exponential	0.52	0.56	0.20



Method	Mean Squared Error	Mean Forecast Error	Variance
— Double Exponential Smoothing	3.56	1.32	1.86
— Moving Average	3.68	1.31	2.01
— Single Exponential Smoothing	3.81	1.35	2.04
— Adaptive Single Exponential	4.51	1.51	2.28
— Least Squares	4.46	1.32	2.79
— Triple Exponential Smoothing	5.52	1.46	3.47



Method	Mean Squares Error	Mean Forecast Error	Variance
Single Exponential Smoothing	16.02	1.74	13.02
Moving Average	17.84	1.78	14.72
Double Exponential Smoothing	23.89	1.89	20.39
Least Squares	25.36	1.92	21.72
Adaptive Single Exponential	28.05	1.97	24.24
Triple Exponential Smoothing	29.00	2.13	24.55

Analysis of Forecast Methoc

Effectiveness

Method	H Test Subset 1	Significant Difference In Effectiveness	H Test Subset 2	Significant Difference In Effectiveness	H Test Subset 3	Significant Difference In Effectiveness
Single Exponential Smoothing	0	-	1.020	-	0.360	-
Moving Average	-	No	-	No	-	No
Double Exponential Smoothing	-	No	-	No	-	No

Table 9. Results of Kruskal-Wallis Test on Forecast Error Distributions for the Three Top Rated Forecasting Methods. (Confidence Interval is 95 Percent).

Method	H Test Subset 1	Significant Difference In Effectiveness	H Test Subset 2	Significant Difference In Effectiveness	H Test Subset 3	Significant Difference In Effectiveness
Single Exponential Smoothing	0.280	-	0.014	-	-	-
Double Exponential Smoothing	-	-	-	-	0.070	-
Least Squares	-	No	-	No	-	No

Table 10 Results of Kruskal-Wallis Test on Forecast Error Distributions for Least Squares. (Confidence Interval is 95 Percent)

Method	H Test Subset 1	Significant Difference In Effectiveness	H Test Subset 2	Significant Difference In Effectiveness	H Test Subset 3	Significant Difference In Effectiveness
Single Exponential Smoothing	1.000	-	0.375	-	-	-
Double Exponential Smoothing	-	-	-	-	1.94	-
Triple Exponential Smoothing	-	No		No	-	No

Table 11. Results of Kruskal-Wallis Test on Forecast Error Distributions for Triple Exponential Smoothing. (Confidence Interval is 95 Percent).

Method	H Test Subset 1	Significant Difference In Effectiveness	H Test Subset 2	Significant Difference In Effectiveness	H Test Subset 3	Significant Difference In Effectiveness
Single Exponential Smoothing	0.163	-	0.402	-	-	-
Double Exponential Smoothing	-	-	-	-	0.191	-
Adaptive Single Exponential	-	No	-	No	-	No

Table 12. Results of Kruskal-Wallis Test on Forecast Error Distributions for Adaptive Single Exponential Smoothing (Confidence Interval is 95 Percent).

IV. Detailed Summary

Experiment Summary

Sampling Technique. There are many possible ways of selecting a sample of EOQ items. One method is by stock number classification. Other methods are demand level and cost. In this study, the stratified base level XB2 and XB3 item sample (B cost group) was selected from three randomly chosen systems. Within the toal sample set, items were grouped according to demand level (subsets 1, 2 and 3). These important facts should be kept in mind when drawing conclusions from the results of the analysis.

Evaluation Criteria. The forecasting methods were ranked according to error statistics. Variance, mean forecast error and mean squared error were the error statistics used. The smaller the error statistic, the more accurate the forecasting method. The Kruskal-Wallis test was used to determine whether there was significant differences in the relative effectiveness of the six forecasting methods.

Findings

Error Statistic Ranking. Single exponential smoothing proved to be the most accurate alternative to the present method of moving average forecasting. Double exponential smoothing also ranked in the top three for the total sample set and for subsets 1, 2 and 3. Least squares, triple exponential smoothing and adaptive single exponential smoothing ranked fourth, fifth and sixth, respectively. The following paragraphs discuss the reasons for their relatively poor showings.

The least squares method tended to respond to sudden, large, non-representative demands by excessive changes in the slope of the trend line. In some cases, this change of slope resulted in forecasts of negative demands for the following period or periods. This type of error was midigated to some extent by setting all negative least squares forecasts to zero. On the other hand, double exponential smoothing, which also assumes a linear trend in demand, gave relatively more weight to demand history with subsequently less drastic changes in trend. The non-negative demand constraint was not necessary for double exponential smoothing.

The two primary causes for the relatively poor showing of triple exponential smoothing were the absence of demand processes which could be accurately approximated by a second order polynomial, and the poor results obtained with the formula for initial conditions (Z_0). None of the 316 items in the sample had a demand pattern with a consistent rate of change of slope. The formula used to calculate initial conditions, though mathematically correct, was therefore not representative of the actual demand processes. Starting with relatively inaccurate initial conditions, the errors tended to compound during the forecast period.

The results obtained with the adaptive single exponential smoothing were also disappointing. The major cause of error relative to standard single exponential smoothing was the first criteria for detecting a change in demand level; i.e., a single demand detected outside four standard deviations of the mean demand. When such a demand was detected, the smoothing constant was changed to one. This criteria and the subsequent smoothing constant change caused the forecast to change excessively in response to abrupt, non-representative demands. The second criteria; i.e., two successive demands differing from the mean demand by more than 1.2 standard deviations, both demands being above or below the mean, appeared to change the smoothing constant at appropriate times for the given sample.

Three obvious approaches to improving the accuracy of adaptive single exponential smoothing are open. The first would be to drop the first criteria for changing the smoothing constant, retaining only the second criteria. The second approach would be to change the maximum smoothing constant to some value less than one. The third approach would be a combination of the first two approaches. However, these possible improvements were not attempted in this study and are left as subjects for further research.

Finally, a few words concerning the choice of smoothing constants for double and triple exponential smoothing are in order. It was assumed that the optimum smoothing constant for single exponential smoothing would also be optimum for double and triple exponential smoothing. No proof was offered for this assumption. It would be of interest to compare the accuracy of double and triple exponential smoothing forecasts using smoothing constant (α) values in increments of .05 between $.05 \leq \alpha \leq .5$. This comparison was not attempted for this study, and is also left as a subject for pertinent future research.

The Kruskal-Wallis Test. This test indicated that the populations of absolute average forecast errors are nearly identical for the six forecast methods. Based on this test, there were no significant differences in the relative effectiveness of the six methods. However, as previously no ·d, there is substantial evidence that improvement is required in the formulation of least squares, triple exponential smoothing and adaptive single exponential smoothing forecasts. It would seem appropriate then, to withhold final judgement on these three forecasting methods until the suggested improvements are made and tested.

Recommendations

The purpose of this research effort was to examine and compare five methods of base level demand forecasting which can be considered as alternatives to the present method of unweighted moving averages. All six methods, including unweighted moving averages, were evaluated according to relative accuracy in describing the actual base level demand patterns of a statistically chosen, stratified sample of XB2 and XB3 EOQ items.

There is little doubt that the single exponential smoothing method of forecasting item demand is a viable alternative to the present method of unweighted moving averages for items represented by the sample used in this study. Double exponential smoothing also appears viable.

Research using the given forecasting methods should certainly continue. Recommend that subsequent research efforts use item samples gathered by different methodologies at base level. In this way, a broad base of item samples, wholly representative of all EOQ items Air Force wide, can be established. Given that the results of future research efforts bear out the findings of this study, serious consideration ought to be given to single and double exponential smoothing methods as alternatives to the present method of forecasting demands. Single and double exponential smoothing have the well documented, inherent advantages of smaller data storage requirements, greater computational ease, and greater flexibility in adapting to demand pattern trends compared to unweighted moving averages.

Further research, as indicated under the Findings Section of this chapter, is necessary to properly evaluate all pertinent improvements concerning least squares, triple exponential smoothing and adaptive single exponential smoothing. Relatively simple adaptations to the FORTRAN program used in this research effort can be made to analyze the suggested improvements. Recommend these adaptions be made and the program used to evaluate the given sample and other base level samples selected by different methodologies.

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Appendixes

Appendix A

FORTRAN Computer Program

```
IDENT MAIN
FILE 5 = INPUT , UNIT = READER
FILE 6  = OUTPUT , UNIT = PRINTER
FILE 10 = DSKFIL, UNIT = DISK,WORKFILE,RANDOM,RECORD = 196
FILE 11 = DSKFL2,UNIT=DISK,WORKFILE,RANDOM,RECORD=300
      DIMENSION NSN(4),OBSVD(15),CALC(20),PERIOD(10),SMOOTH(50),
     1CONSNT(10),STORE(10),HOLD(10),STV(3),STW(3),STX(3),STY(3),STZ(3),
     2STZA(3)
      ALPHA NSN
      DATA ZERO/0.0/,PLOAD/-10.0/,ALP/0.0/,KOUNT/0/,MARK/1/
      INN = 5
      IOUT =6
C
C     ZEROING ARRAY CALC IN PREPARATION FOR CALCULATED VALUES
C
      DO 2 N=1,20
      CALC(N) = ZERO
    2 CONTINUE
      DO 3 M = 1,400
      READ(INN,51,END = 4) (NSN(I),I=1,4),(OBSVD(J),J=1,15)
   51 FORMAT(2X,4A4,15F4.0)
      KOUNT = KOUNT + 1
      WRITE(10≥KOUNT,52) (NSN(I),I=1,4),(OBSVD(J),J=1,15),(CALC(K),K=1,
     120)
   52 FORMAT(4A4,15F4.0,20F6.2)
    3 CONTINUE
    4 WRITE(IOUT,60)
   60 FORMAT(1H1,50X,31HSMOOTHING CONSTANT CALCULATIONS,//,2X,21HNATIONA
     1L STOCK NUMBER,7X,5HA=.05,5X,5HA=.10,5X,5HA=.15,5X,5HA=.20,5X,5HA=
     2.25,5X,5HA=.30,5X,5HA=.35,5X,5HA=.40,5X,5HA=.45,5X,5HA=.50,//)
      DO 5 K=1,10
      ALP=ALP+.05
      CONSNT(K)=ALP
      PERIOD(K)=ZERO
    5 CONTINUE
   21 READ(10≥MARK,52)(NSN(I),I=1,4),(OBSVD(J),J=1,15),(CALC(K),K=1,20)
      N=1
      V=0.0
      DO 6 I=1,10
    6 V=V+OBSVD(I)
```

```
      V1=V/10.0
      V2=V1
      DO 8 K=1,10
      V1=V2
      DIFF=0.0
      DO 7 J=1,5
      N=J+9
      L=J+10
      X1=(CONSNT(K)*OBSVD(N))+((1.0-CONSNT(K))*V1)
      SMOOTH(N)=X1
      DIFF=DIFF+ABS(X1-OBSVD(L))
      V1=X1
      N=N+1
    7 CONTINUE
      CALC(K)=DIFF/5.0
      PERIOD(K)=PERIOD(K)+CALC(K)
    8 CONTINUE
      DO 47 I=1,3
      STV(I)=0.0
      STW(I)=0.0
      STX(I)=0.0
      STY(I)=0.0
      STZ(I)=0.0
      STZA(I)=0.0
   47 CONTINUE
      WRITE(IOUT,61)(NSN(I),I=1,4),(CALC(K),K=1,10)
   61 FORMAT(1H ,3X,A4,3H 00 ,3A4,6X,F6.2,9(F10.2))
      WRITE(10≥MARK,52)(NSN(I),I=1,4),(OBSVD(J),J=1,15),(CALC(K),K=1,20)
      WRITE(11≥MARK,53)(SMOOTH(J),J=1,50)
   53 FORMAT(50F6.2)
      MARK=MARK+1
      IF(MARK.LE.KOUNT) GO TO 21
      COUNT=KOUNT
      DO 9 I=1,10
      HOLD(I)=ZERO
    9 STORE(I)=PERIOD(I)/COUNT
      WRITE(IOUT,62)(STORE(M),M=1,10)
   62 FORMAT(3(/),20X,7HMEAN = ,2X,F6.2,9(4X,F6.2),//)
      MARK=1
      COUNT1=COUNT-1.0
```

```
80 READ(10≥MARK,52)(NSN(I),I=1,4),(OBSVD(J),J=1,15),(CALC(K),K=1,20)
   DO 81 J=1,10
   DIFF=CALC(J)-STORE(J)
   PLACE=DIFF*DIFF
   HOLD(J)=HOLD(J)+PLACE
81 CONTINUE
   MARK=MARK+1
   IF(MARK.LE.KOUNT) GO TO 80
   MARK=1
   DO 82 K=1,10
   PLACE=SQRT(HOLD(K)/COUNT1)
   HOLD(K)=PLACE
82 CONTINUE
   WRITE(IOUT,63)(HOLD(M),M=1,10)
63 FORMAT(1H0,4X,20HSTANDARD DEVIATION =,10(4X,F6.2),//)
   PLACE=HOLD(1)
   IPOINT=1
   DO 83 I=2,10
   IF(HOLD(I).GE.PLACE) GO TO 83
   PLACE=HOLD(I)
   IPOINT=I
83 CONTINUE
   ALP=CONSNT(IPOINT)
   WRITE(IOUT,64) ALP
64 FORMAT(1H0,44X,23H*****  ALPHA SELECTED = ,F3.2,7H  ***** )
   KPOINT=0
   DO 84 K=1,IPOINT
84 KPOINT=KPOINT+5
   LPOINT=KPOINT-4
   WRITE(IOUT,65)
65 FORMAT(1H1,49X,35HSINGLE EXPONENTIAL SMOOTHING METHOD,/)
   WRITE(IOUT,66)
66 FORMAT(1H0,42X,14HFORCAST DEMAND,31X,13HACTUAL DEMAND,19X,
  18HABSOLUTE)
   WRITE(IOUT,67)
67 FORMAT(1H0,2X,21HNATIONAL STOCK NUMBER,    2(10X,7HNOV    ,
  17HDEC    ,7HJAN    ,7HFEB    ,7HMAR    ),2X,13HAVERAGE ERROR,//)
   V=0.0
   DIFF=0.0
   DO 85 MARK=1,KOUNT
```

```
      READ(10≥MARK,52)(NSN(I),I=1,4),(OBSVD(J),J=1,15),(CALC(K),K=1,20)
      READ(11≥MARK,53)(SMOOTH(J),J=1,50)
      JPOINT=LPOINT
      DO 86 N=1,5
      L=N+10
      DIFF=DIFF+ABS(SMOOTH(JPOINT)-OBSVD(L))
      JPOINT=JPOINT+1
   86 CONTINUE
      DIFF=DIFF/5.0
      WRITE(IOUT,68)(NSN(I),I=1,4),(SMOOTH(K),K=LPOINT,KPOINT),
     1(OBSVD(J),J=11,15),DIFF
   68 FORMAT(1H ,3X,A4,3H 00 ,3A4, 8X,5(F7.2),10X,5(F7.2),6X,F8.4)
      CALC(16)=DIFF
      STX(1)=STX(1)+(DIFF*DIFF)
      STX(2)=STX(2)+DIFF
      DIFF=0.0
      V=0.0
      DO 10 I=1,10
C     ZEROING ARRAY CALC IN PREPARATION FOR CALCULATED VALUES
      V=V+OBSVD(I)
      COUNT=I
      PERIOD(I)=PLOAD*COUNT
   10 CONTINUE
      COUNT=KOUNT
      V1=V/10.0
      CALC(1)=V1
      DO 11 J=2,5
      K=J-1
      L=J+9
      V=(V-OBSVD(K))+OBSVD(L)
      V1=V/10.0
      CALC(J)=V1
   11 CONTINUE
      CNNST=(9.0/2.0)
C
C     THIS LOOP CALCULATES DEMANDS BY THE LEAST SQUARES METHOD
C
      U=0.0
      P=0.0
      P1=0.0
```

```
      P2=0.0
      P3=0.0
      P4=0.0
      DO 12 K= 1,10
      U= U+OBSVD(K)
      P= P+(PERIOD(K)*OBSVD(K))
      P1=P1+(PERIOD(K)*OBSVD(K+1))
      P2=P2+(PERIOD(K)*OBSVD(K+2))
      P3=P3+(PERIOD(K)*OBSVD(K+3))
      P4=P4+(PERIOD(K)*OBSVD(K+4))
   12 CONTINUE
C
C        CARDS NUMBERED 1685, 1725, 1765, 1805, 1845 PREVENT LEAST
C        SQUARES FROM GOING NEGATIVE. A NEGATIVE IS SET TO 0.
C
      Q=U/10.0
      W1=((P+(CNNST*U))/990.0)*(1.0+CNNST)*12.0+Q
      IF(W1.LT.0.0) W1=0.0
      CALC(6)=W1
      U=U-OBSVD(1)+OBSVD(11)
      Q=U/10.0
      W2=((P1+(CNNST*U))/990.0)*(1.0+CNNST)*12.0+Q
      IF(W2.LT.0.0) W2=0.0
      CALC(7)=W2
      U=U-OBSVD(2)+OBSVD(12)
      Q=U/10.0
      W3=((P2+(CNNST*U))/990.0)*(1.0+CNNST)*12.0+Q
      IF(W3.LT.0.0) W3=0.0
      CALC(8)=W3
      U=U-OBSVD(3)+OBSVD(13)
      Q=U/10.0
      W4=((P3+(CNNST*U))/990.0)*(1.0+CNNST)*12.0+Q
      IF(W4.LT.0.0) W4=0.0
      CALC(9)=W4
      U=U-OBSVD(4)+OBSVD(14)
      Q=U/10.0
      W5=((P4+(CNNST*U))/990.0)*(1.0+CNNST)*12.0+Q
      IF(W5.LT.0.0) W5=0.0
      CALC(10)=W5
      WRITE(102MARK,52)(NSM(I),I=1,4),(OBSVD(J),J=1,15),(CALC(K),K=1,20)
```

```
85 CONTINUE
   STX(2)=STX(2)/COUNT
   DIFF=0.0
   WRITE(IOUT,69)
69 FORMAT(1H1,55X,21HMOVING AVERAGE METHOD,/)
   WRITE(IOUT,66)
   WRITE(IOUT,67)
   DO 87 MARK=1,KOUNT
   READ(10≥MARK,52)(NSN(I),I=1,4),(OBSVD(J),J=1,15),(CALC(K),K=1,20)
   READ(11≥MARK,53)(SMOOTH(J),J=1,50)
   DO 88 N=1,5
   L=N+10
   DIFF=DIFF+ABS(CALC(N)-OBSVD(L))
88 CONTINUE
   DIFF=DIFF/5.0
   STV(1)=STV(1)+(DIFF*DIFF)
   WRITE(IOUT,68)(NSN(I),I=1,4),(CALC(K),K=1,5),
  1(OBSVD(J),J=11,15),DIFF
   STV(2)=STV(2)+DIFF
   CALC(17)=DIFF
   IND=1
   DIFF=0.0
   X=1.0-ALP
   YO=CALC(6)
   DO 89 L=LPOINT,KPOINT
   Y1=ALP*(SMOOTH(L))+(X*YO)
   CALC(IND)=Y1
   YO=Y1
   IND=IND+1
89 CONTINUE
   WRITE(10≥MARK,52)(NSN(I),I=1,4),(OBSVD(J),J=1,15),(CALC(K),K=1,20)
87 CONTINUE
   STV(2)=STV(2)/COUNT
   DIFF=0.0
   WRITE(IOUT,70)
70 FORMAT(1H1,56X,20HLEAST SQUARES METHOD,/)
   WRITE(IOUT,66)
   WRITE(IOUT,67)
   DO 90 MARK=1,KOUNT
   READ(10≥MARK,52)(NSN(I),I=1,4),(OBSVD(J),J=1,15),(CALC(K),K=1,20)
```

```
      DO 91 N=6,10
      L=N+5
      DIFF=DIFF+ABS(CALC(N)-OBSVD(L))
   91 CONTINUE
      DIFF=DIFF/5.0
      WRITE(IOUT,68)(NSN(I),I=1,4),(CALC(K),K=6,10),
     1(OBSVD(J),J=11,15),DIFF
      STW(2)=STW(2)+DIFF
      CALC(18)=DIFF
      STW(1)=STW(1)+(DIFF*DIFF)
      DIFF=0.0
      V=0.0
      DO 92 I=1,10
      V=V+OBSVD(I)
   92 CONTINUE
      V1=V/10.0
      A=V1
      B=OBSVD(1)-OBSVD(3)+OBSVD(4)-OBSVD(6)+OBSVD(7)-OBSVD(9)
      B=B/6.0
      C=OBSVD(1)-OBSVD(3)-OBSVD(7)+OBSVD(9)
      C=C/12.0
      ALPH=ALP
      ALPH2=ALPH*ALPH
      B1=((3.*X)/ALPH)*B
      C1=((3.*X)*(4.-3.*ALPH)/(2.*ALPH2))*C
      Z0=A-B1+C1
      IF(Z0.LE.0.0)Z0=0.0
      IF(Z0.GT.(2.0*A)) Z0=2.0*A
      Z=ALPH*CALC(1)+(X*Z0)
      CALC(11)=Z
      DO 93 J=2,5
      L=J+10
      ZT=(ALPH*CALC(J))+(X*Z)
      CALC(L)=ZT
      Z=ZT
   93 CONTINUE
      WRITE(10≥MARK,52)(NSN(I),I=1,4),(OBSVD(J),J=1,15),(CALC(K),K=1,20)
   90 CONTINUE
      STW(2)=STW(2)/COUNT
      DIFF=0.0
```

```
      WRITE(IOUT,71)
   71 FORMAT(1H1,49X,35HDOUBLE EXPONENTIAL SMOOTHING METHOD , / )
C
      WRITE(IOUT,66)
      WRITE(IOUT,67)
      DO 94 MARK=1,KOUNT
      READ(10≥MARK,52)(NSN(I),I=1,4),(OBSVD(J),J=1,15),(CALC(K),K=1,20)
      DO 95 N=1,5
      L=N+10
      DIFF=DIFF+ABS(CALC(N)-OBSVD(L))
   95 CONTINUE
      DIFF=DIFF/5.0
      WRITE(IOUT,68)(NSN(I),I=1,4),(CALC(K),K=1,5),
     1(OBSVD(J),J=11,15),DIFF
      CALC(19)=DIFF
      STY(1)=STY(1)+(DIFF*DIFF)
      STY(2)=STY(2)+DIFF
      DIFF=0.0
      WRITE(10≥MARK,52)(NSN(I),I=1,4),(OBSVD(J),J=1,15),(CALC(K),K=1,20)
   94 CONTINUE
      STY(2)=STY(2)/COUNT
      WRITE(IOUT,72)
   72 FORMAT(1H1,49X,35HTRIPLE EXPONENTIAL SMOOTHING METHOD)
      WRITE(IOUT,66)
      WRITE(IOUT,67)
      DO 96 MARK=1,KOUNT
      READ(10≥MARK,52)(NSN(I),I=1,4),(OBSVD(J),J=1,15),(CALC(K),K=1,20)
      DO 97 N=11,15
      DIFF=DIFF+ABS(CALC(N)-OBSVD(N))
   97 CONTINUE
      DIFF=DIFF/5.0
      WRITE(IOUT,68)(NSN(I),I=1,4),(CALC(K),K=11,15),
     1(OBSVD(J),J=11,15),DIFF
      CALC(20)=DIFF
      STZ(1)=STZ(1)+(DIFF*DIFF)
      STZ(2)=STZ(2)+DIFF
      WRITE(10≥MARK,52)(NSN(I),I=1,4),(OBSVD(J),J=1,15),(CALC(K),K=1,20)
      DIFF=0.0
   96 CONTINUE
      STZ(2)=STZ(2)/COUNT
```

```
      WRITE(IOUT,73)
   73 FORMAT(1H1,45X,44HSINGLE EXPONENTIAL SMOOTHING ADAPTIVE METHOD ,/)
C
      WRITE(IOUT,66)
      WRITE(IOUT,74)
   74 FORMAT(1H0,21HNATIONAL STOCK NUMBER,3X,7HA    NOV,5X,7HA    DEC,5X,
     17HA    JAN,5X,7HA    FEB,5X,7HA    MAR,5X,3HNOV,4X,3HDEC,4X,3HJAN,4X,
     23HFEB,4X,3HMAR,4X,13HAVERAGE ERROR,//)
      DO 98 MARK=1,KOUNT
      READ(10≥MARK,52)(NSN(I),I=1,4),(OBSVD(J),J=1,15),(CALC(K),K=1,20)
      READ(11≥MARK,53)(SMOOTH(J),J=1,50)
      V=0.0
      DO 99 I=1,10
   99 V=V+OBSVD(I)
      V1=V/10.0
      X0=V1
      A=ALPH
      K=1
      L=10
   59 PLACE=0.0
      DO 100 J=K,L
      DIFF=OBSVD(J)-V1
  100 PLACE=PLACE+(DIFF*DIFF)
      STDEV=SQRT(PLACE/9.0)
      TEST=4.0*STDEV
      V2=V1+TEST
      V3=V1-TEST
      IF(OBSVD(L+1).GE.V2) A=1.0
      IF(OBSVD(L+1).LE.V3) A=1.0
      IF(A.EQ.1.0) GO TO 103
      IF(L.GE.14) GO TO 103
      TEST2=1.2*STDEV
      V2=V1+TEST2
      V3=V1-TEST2
      IF((OBSVD(L+1).GE.V2).AND.(OBSVD(L+2).GE.V2)) A=1.0
      IF(A.EQ.1.0) GO TO 103
      IF((OBSVD(L+1).LE.V3).AND.(OBSVD(L+2).LE.V3)) A=1.0
  103 X=1.0-A
      XT=(A*OBSVD(L))+(X*X0)
      STORE(2*K-1)=A
```

```
      STORE(2*K)=XT
      XO=XT
      V=(V-OBSVD(K))+OBSVD(L+1)
      V1=V/10.0
      IF(STORE(2*K-1).EQ.1.0) A=.8
      IF(STORE(2*K-1).EQ..8) A=.5
      IF(STORE(2*K-1).EQ..5) A=ALPH
C
C        IF YOU WANT ALPHA TO SKIP .8 VALUE, CHANGE THE .8 VALUE ON
C        CARD NUMBERED 3551 TO A .5 AND REMOVE CARD NUMBER 3555.
C
      K=K+1
      L=L+1
      IF(L.LE.14) GO TO 59
      DIFF=0.0
      M1=11
      DO 104 M=2,10,2
      DIFF=DIFF+ABS(STORE(M)-OBSVD(M1))
      M1=M1+1
  104 CONTINUE
      DIFF=DIFF/5.0
      WRITE(IOUT,75)(NSN(I),I=1,4),(STORE(K),K=1,10) ,
     1(OBSVD(J),J=11,15),DIFF
   75 FORMAT(2X,A4,3H 00,3A4,10(F6.2),1X,5(F7.2),5X,F8.4)
      STZA(1)=STZA(1)+(DIFF*DIFF)
      STZA(2)=STZA(2)+DIFF
      SMOOTH(1)=DIFF
      WRITE(11≥MARK,53)(SMOOTH(J),J=1,50)
      STV(3)=STV(3)+((CALC(17)-STV(2))*(CALC(17)-STV(2)))
      STW(3)=STW(3)+((CALC(18)-STW(2))*(CALC(18)-STW(2)))
      STX(3)=STX(3)+((CALC(16)-STX(2))*(CALC(16)-STX(2)))
      STY(3)=STY(3)+((CALC(19)-STY(2))*(CALC(19)-STY(2)))
      STZ(3)=STZ(3)+((CALC(20)-STZ(2))*(CALC(20)-STZ(2)))
   98 CONTINUE
      STZA(2)=STZA(2)/COUNT
      DO 105 MARK=1,KOUNT
      READ(11≥MARK,53)(SMOOTH(J),J=1,50)
      STZA(3)=STZA(3)+((SMOOTH(1)-STZA(2))*(SMOOTH(1)-STZA(2)))
  105 CONTINUE
      COUNT1=KOUNT-1
   :
```

```
      WRITE(IOUT,76)
   76 FORMAT(1H1,61X,10HEVALUATION,//,15X, 6HMETHOD,19X,18HMEAN SQUARED
     1ERROR,10X,18HMEAN FORCAST ERROR,12X,8HVARIANCE,///)
      STV(3)=STV(3)/COUNT1
      STV(1)=STV(1)/COUNT
      WRITE(IOUT,77)(STV(J),J=1,3)
   77 FORMAT(1H0,10X,14HMOVING AVERAGE,18X,3(F8.2,20X),///)
      STW(3)=STW(3)/COUNT1
      STW(1)=STW(1)/COUNT
      WRITE(IOUT,78)(STW(J),J=1,3)
   78 FORMAT(1H0,10X,13HLEAST SQUARES,19X,3(F8.2,20X),///)
      STX(3)=STX(3)/COUNT1
      STX(1)=STX(1)/COUNT
      WRITE(IOUT,79)(STX(J),J=1,3)
   79 FORMAT(1H0,5X,28HSINGLE EXPONENTIAL SMOOTHING,9X,3(F8.2,20X),///)
      STY(3)=STY(3)/COUNT1
      STY(1)=STY(1)/COUNT
      WRITE(IOUT,30)(STY(J),J=1,3)
   30 FORMAT(1H0,5X,28HDOUBLE EXPONENTIAL SMOOTHING,9X,3(F8.2,20X),///)
      STZ(3)=STZ(3)/COUNT1
      STZ(1)=STZ(1)/COUNT
      WRITE(IOUT,31) (STZ(J),J=1,3)
   31 FORMAT(1H0,5X,28HTRIPLE EXPONENTIAL SMOOTHING ,9X,3(F8.2,20X),///)
      STZA(3) = STZA(3)/COUNT1
      STZA(1) = STZA(1)/COUNT
      WRITE(IOUT,32) (STZA(J),J=1,3)
   32 FORMAT(1H0,1X,37HADAPTIVE SINGLE EXPONENTIAL SMOOTHING , 4X,
     13(F8.2,20X),///)
      WRITE(IOUT,33)
   33 FORMAT(1H1)
      STOP
      END

5   0:14 P.M.    ASR  5.4 USING 73/345   XFORTN COMPILER
 TIME   196 SECS        432 CARDS AT  132 C.P.M.         0 FLAGS     0 ERRORS
=      0  DATA =    6174  TEMPORARIES =      72  CODE =  27900      DIGITS
```

DATE
FILMED

7-8