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A MODIFIED CRAMER-VON MISES AND ANDERSON-DARLING TEST FOR THE WEIBULL DISTRIBUTION WITH UNKNOWN LOCATION AND SCALE PARAMETERS

THESIS

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A MODIFIED CRAMER-VON MISES AND ANDERSON-DARLING TEST FOR THE WEIBULL DISTRIBUTION WITH UNKNOWN LOCATION AND SCALE PARAMETERS

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
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by
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Preface

The purpose of this thesis is to generate critical values for the modified Anderson-Darling and Cramer-von Mises statistics. These critical values are used for testing whether a set of observations follows a Weibull distribution when the scale and location parameters are unspecified and are estimated from the sample. An extensive power study is made to compare the power of the Anderson-Darling, Cramer-von Mises, Kolmogorov-Smirnov, and the Chi-Square goodness-of-fit tests.

I wish to express my sincere appreciation to my advisor, Capt. Brian Woodruff, for his guidance throughout this study. I also wish to thank my readers, Lt. Col. James Dunne and Dr. Albert H. Moore, for their encouragement throughout my thesis endeavor.

Finally, I wish to acknowledge my gratitude to my wife, Gloria, for her support when my spirits were low, and to my daughter, Kirsten Ashley, born 7 March 1980.

John G. Bush

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Abstract

The Anderson-Darling and Cramer-von Mises critical values are generated for the three-parameter Weibull distribution. The critical values are used for testing whether a set of observations follows a Weibull distribution when the scale and location parameters are unspecified and are estimated from the sample. A Monte Carlo simulation, with 5000 repetitions, is used to generate critical values for sample sizes 5(5)30 and Weibull shape parameters .5(.5)4.0.

A Monte Carlo power investigation of the Anderson-Darling and Cramer-von Mises tests is made using 5, 15, and 25 observations from ten alternate distributions. The power of the two tests are compared to the Kolmogorov-Smirnov and the Chi-Square tests. The power of all the tests are low with a sample size of five. When the hypothesized distribution is the Weibull with shape equal 1.0, the power of the tests in decreasing order are: Cramer-von Mises, Anderson-Darling, Kolmogorov-Smirnov, and Chi-Square. When the hypothesized distribution is the Weibull with shape equal 3.5, the power of the tests are the Anderson-Darling, followed by the Cramer-von Mises, Kolmogorov-Smirnov, and the Chi-Square test.

A relationship between the Anderson-Darling and Cramer-von Mises critical values with the Weibull shape

parameters is investigated. The critical values of both of the tests are found to be a function of the inverse of the shape parameters.

A MODIFIED CRAMER-VON MISES AND ANDERSON-DARLING TEST FOR THE WEIBULL DISTRIBUTION WITH UNKNOWN LOCATION AND SCALE PARAMETERS

I. Introduction

Reliability theory and life testing of equipment is of great importance to the U.S. Air Force. The mean time to failure and the failure rate ot equipment and systems are an important input into the decision making process.

Through experimentation and observation, time to failure data can be collected. The data can be compared to a theoretical probability distribution. The test to determine if the hypothesized distribution "fits" the data in the sample is known as the "goodness-of-fit test." There are several classical goodness-of-fit tests, such as: the Chi-Square test (χ^2), the Kolmogorov-Smirnov test (K-S), the Anderson-Darling test (A^2), and the Cramer-von Mises test (W^2). If such tests show a good "fit," the hypothesized distribution can then be used in simulation modeling techniques to predict the failure rates of Air Force systems or system components.

Background

The general goodness-of-fit tests are valid to test whether a set of observations come from a completely specified distribution. The tests have been modified when the parameters of the distribution are unknown and are estimated from the data sample. The K-S test has been modified by H.W. Lilliefors so the test can be used with the normal (16) and the exponential (17) distributions when the parameters are unknown and are estimated from the data. R. Cortes further extended the K-S test so it can be used with the Gamma and Weibull distributions when the scale and location parameters are unknown (4). In 1969, Green and Hegezy (10) used the K-S, W^2 and the A^2 tests to generate rejection tables for the goodness-of-fit for the uniform, normal, Laplace, exponential, and the Cauchy distribution with unknown parameters. Mann, Scheuer, and Fertig developed in 1973 a new goodness-of-fit test for the two-parameter Weibull distribution with unknown parameters. They called the new statistic the L and S statistic (19). In 1979, Littell, McClave, and Offen (18) used the K-S, W^2, and the A^2 tests to generate the rejection regions for the two-parameter Weibull distribution when the shape and scale parameters are unspecified.

Empirical Distribution Function

The empirical distribution function (EDF) statistics are a class of statistics based on a comparison between the cumulative distribution function F(x) and the empirical distribution function Sn(x). The EDF of a random sample of size n is the proportion of the sample values which do not exceed the number x:

$$Sn(x) = \frac{\text{number of sample values} \leq x}{n} \tag{1}$$

Thus, Sn(x) is a step function with 1/n jumps at each order statistic of the sample (9:73). Letting $x_{(1)}, x_{(2)}, \ldots x_{(n)}$ be order statistics, Sn(x) is defined by Eq (2).

$$Sn(x) = \begin{cases} 0 & , x < x_{(1)} \\ i/n & , x_{(i)} \leq x < x_{(i+1)} \ , \ i=1,2,\ldots,(n-1) \\ 1 & , x > x_{(n)} \end{cases} \tag{2}$$

Since Sn(x) gives the proportion of a random sample less than x, it is reasonable to expect it to give a good estimate of the hypothesized cumulative distribution function F(x). In fact, goodness-of-fit tests based on EDF statistics measure the discrepancy between Sn(x) and F(x).

The EDF statistics used in this thesis are the Anderson-Darling statistic and the Cramer-von Mises statistic. They are compared with perhaps the best known EDF statistic, the Kolmogorov-Smirnov goodness-of-fit test. The K-S statistic measures the maximum vertical discrepancy between the cumulative distribution function and the empirical distribution function (23:2).

Cramer-von Mises Goodness-of-Fit Test Statistic. The Cramer-von Mises statistic is based on the squared integral of the difference between the cumulative distribution tested and the EDF (23:2):

$$W^* = \int_{-\infty}^{\infty} [Sn(x) - F(x)]^2 \psi(x)dx \qquad (3)$$

where $\psi(x)$ gives a weighting to the squared difference. The Cramer-von Mises statistic is figured with $\psi(x) = 1$. The computational formula to calculate W^2 is given by Eq (4).

Letting $x_{(1)}, x_{(2)}, \ldots x_{(n)}$ be the n order statistics and letting $U_i = F(x_i)$, the cumulative distribution function then,

$$W^2 = \frac{1}{12n} + \sum_{i=1}^{n} [U_i - \frac{2i-1}{2n}]^2 \qquad (4)$$

If W^2 is too large, the hypothesized distribution is rejected (1:765).

Anderson-Darling Goodness-of-Fit Test Statistic. The Anderson-Darling goodness-of-fit statistic is also based on the squared integral of the difference between F(x) and Sn(x). The A^2 statistic is given by Eq (3) with

$$\psi(x) = [(F(x))(1-F(x))]^{-1} \qquad (5)$$

This weight function has the effect of giving greater importance to observations in the tails (23:2). The computational formula to calculate A^2 is given by the next equation (16):

$$A^2 = -n - \frac{1}{n}\{\sum_{i=1}^{n} (2i-1)[\ln(U_i)+\ln(1-U_{n+1-i})]\} \qquad (6)$$

If A^2 is too large, then the hypothesis is rejected (23:4).

Unknown Parameters. Because of the probability integral transformation, the values of a completely specified cumulative distribution are ordered values from a uniform distribution over the interval from zero to one. EDF goodness-of-fit tests test the hypothesis that a sample has been drawn from a fully specified continuous cumulative distribution. Thus, EDF statistics are a function of ordered uniform random variables (23:4). In general, if the cumulative distribution function is not completely specified, the distribution of the EDF statistics will depend on the sample size n and the values of the unknown parameters (23:4). However, David and Johnson showed, in 1948, that the distribution of any EDF statistic is simplified when the unknown parameters are the location and scale. They showed the distribution of any test statistic based on the cumulative distribution function using invariant estimators for location and scale will depend on the distribution tested, but not on the specific values of the unspecified scale and location parameters (5:182). Thus, the distribution of the A^2, W^2, and K-S statistic for the Weibull distribution with unknown scale and location parameters will depend on the sample size n, but will be independent of the scale and location parameters.

Problem

It is well known that the EDF goodness-of-fit tests are, in general, more powerful tests of the null hypothesis than is the Chi-Square test (24:730). The most powerful goodness-of-fit tests among the Anderson-Darling, Cramer-von Mises, and the modified Kolmogorov-Smirnov tests for the three-parameter Weibull distribution when the scale and location parameters are not specified is not known.

Objectives

This thesis has the following objectives:

1. To generate the Anderson-Darling rejection tables for the three-parameter Weibull distribution when the scale and location parameters are not specified.

2. To generate the Cramer-von Mises rejection tables for the three-parameter Weibull distribution when the scale and location parameters are not specified.

3. To conduct a power comparison between the K-S, A^2, W^2, and the χ^2 goodness-of-fit tests for the three-parameter Weibull distribution when the scale and location parameters are not specified.

4. To investigate a relationship between the shape parameter of the Weibull distribution and the Anderson-Darling and Cramer-von Mises critical values.

II. The Weibull Distribution

History and Application

The Weibull probability density function (pdf) was developed in 1939 by a Swedish scientist, Waloddi Weibull. In his study, he examined the distribution of the phenomenon of rupture in solids. In a paper published in 1951, Weibull demonstrated the application of the distribution in the investigation of the yield strength and fatigue of steel, the size distribution of fly ash, and the fiber strength of cotton (26:293). Although the Weibull distribution was first applied to the fatigue of materials, the distribution is very flexible and has proved very useful in other fields of study as well. Peto and Lee used the Weibull distribution in their experiments with continuous carcinogenesis experiments with laboratory animals (21:457). Today, the Weibull distribution is best known for its application in the field of reliability.

The Weibull distribution is related to the exponential distribution. However, the Weibull distribution with the three parameters--shape, location, and scale--can model a variety of hazard situations. Where the exponential has a constant failure rate, $1/\theta$, the Weibull distribution can deal with an increasing, constant, or decreasing failure rate. Since many systems often experience a burn-in period (decreasing failure rate) followed by a useful life (constant failure rate) followed by a wearout period (increasing failure rate), the Weibull distribution can be very useful in describing these phenomena.

The Three-Parameter Weibull

The Weibull pdf represents the distribution of x, such as the time of failure of a piece of equipment. Let K denote the shape parameter, C the location parameter, and θ the scale parameter. The Weibull pdf is:

$$F(x:K,C,\theta) = \begin{cases} \dfrac{K(x-C)^{K-1}}{\theta^{K}} \exp\{-(\frac{x-C}{\theta})^{K}\} \ , & K,\theta>0, C\leq x \\ 0 \ , & \text{otherwise} \end{cases} \quad (7)$$

The Weibull cumulative distribution function (CDF), F(x) is given by:

$$F(x) = \int_{0}^{x} f(x:K,C,\theta)\,dx \quad (8)$$

$$= 1 - \exp\{-(\frac{x-C}{\theta})^{K}\} \quad K,\theta>0,\ C\leq x \quad (9)$$

The mean and variance of the Weibull distribution are:

$$\text{Mean} = C + \theta\Gamma(\frac{K+1}{K}) \quad (10)$$

$$\text{Variance} = C^{2}[\Gamma(\frac{K+2}{K}) - \Gamma^{2}(\frac{K+1}{K})] \quad (11)$$

If there are n identical equipment components with the same failure distribution F(x), then the hazard function is the percentage of the n components that will fail in the next time interval x. The hazard function is defined as:

$$h(x) = \frac{f(x)}{(1-F(x))} \tag{12}$$

The hazard function for the three-parameter Weibull distribution is:

$$h(x) = \frac{K(x-C)^{K-1}}{\theta^K} \tag{13}$$

The shape parameter K determines whether the hazard function is decreasing, constant, or increasing. A shape parameter K less than unity has a decreasing hazard rate and describes the burn-in period. A shape parameter K equal to unity has a constant failure rate, $1/\theta$, and describes the useful life period. A shape parameter K greater than unity has an increasing hazard rate and describes the wearout period. This description is commonly known as the bath tub curve as shown in Fig. 1.

The Weibull distribution reduces to the exponential distribution when K is set to one and C is set to zero. The Weibull distribution approximates the normal distribution with K = 3.4 (3:45). Figure 2 depicts the Weibull with different values of K while C and θ are held constant. This flexibility in describing different failure rates makes the Weibull distribution most appealing.

The location parameter C indicates the value x at which failures begin occurring. With C equal zero, failures can occur immediately after the component or device is put into operation. A location parameter greater than zero indicates that there is a period of time which is failure free.

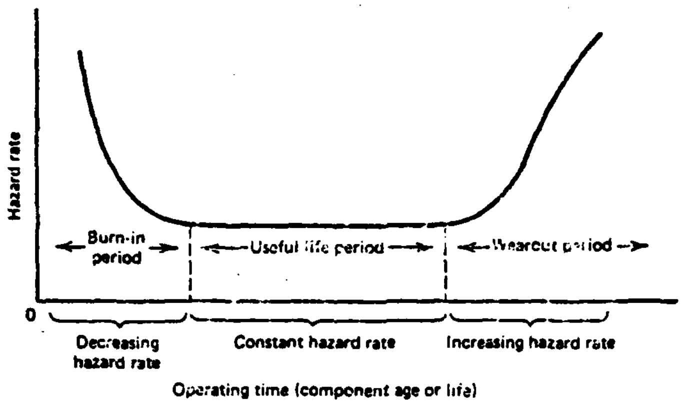


Fig. 1. Bath Tub Curve (7:28)

A location parameter less than zero indicates that the components are defective at the time of initial operation. Failure during storage of the component would be modeled using a negative location parameter. In this thesis, it is assumed C is non-negative. In Fig. 3, C is varied while the shape parameter and scale parameters are held constant.

The scale parameter θ affects the dispersion of the random variable, x, about its mean and is sometimes called the characteristic life (3:48). Figure 4 depicts various values of θ while the shape and location parameters are held constant.

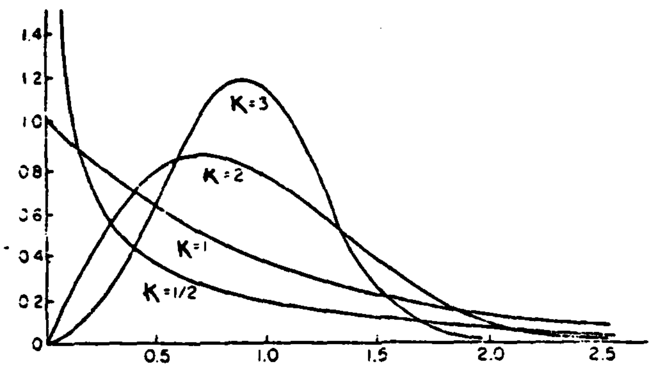


Fig. 2. Weibull Distribution with Different Values of K; θ=1; C=0 (2:47)

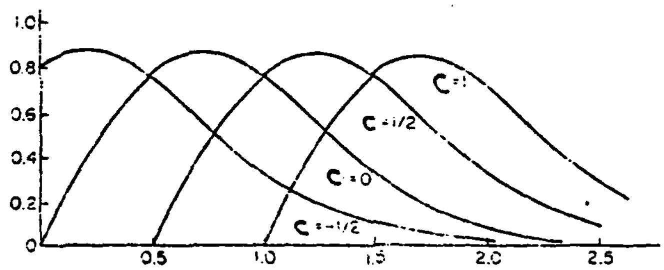


Fig. 3. Weibull Distribution with Different Values of C: θ=1; K=2 (2:48)

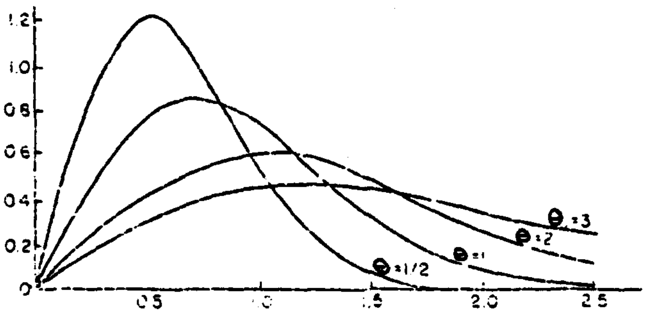


Fig. 4. Weibull Distribution with Different Values of θ; K=2; C=0 (2:46)

III. Methodology

This thesis presents a Monte Carlo method for obtaining the critical values of the modified Anderson-Darling and the modified Cramer-von Mises goodness-of-fit for the three-parameter Weibull distribution when the scale and location parameters are not specified. This chapter discusses the procedures used in this thesis. First, an outline and flow chart of the Monte Carlo method is presented. This is followed by a detailed description of the steps taken in the Monte Carlo procedure. The chapter concludes with a description of the power study and of the analysis of the critical values for each goodness-of-fit test versus the Weibull shape parameters.

Steps in the Monte Carlo Method

The following nine steps outline the Monte Carlo method used to calculate the critical values for the A^2 and W^2 goodness-of-fit tests. The flow chart in Fig. 5 illustrates the method.

1. For a fixed sample size n and fixed shape parameter K, n random Weibull deviates are generated using a computer subroutine. All Weibull deviates are generated with location parameter C=2 and scale parameter θ=1.

2. The n random deviates are ordered, $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$.

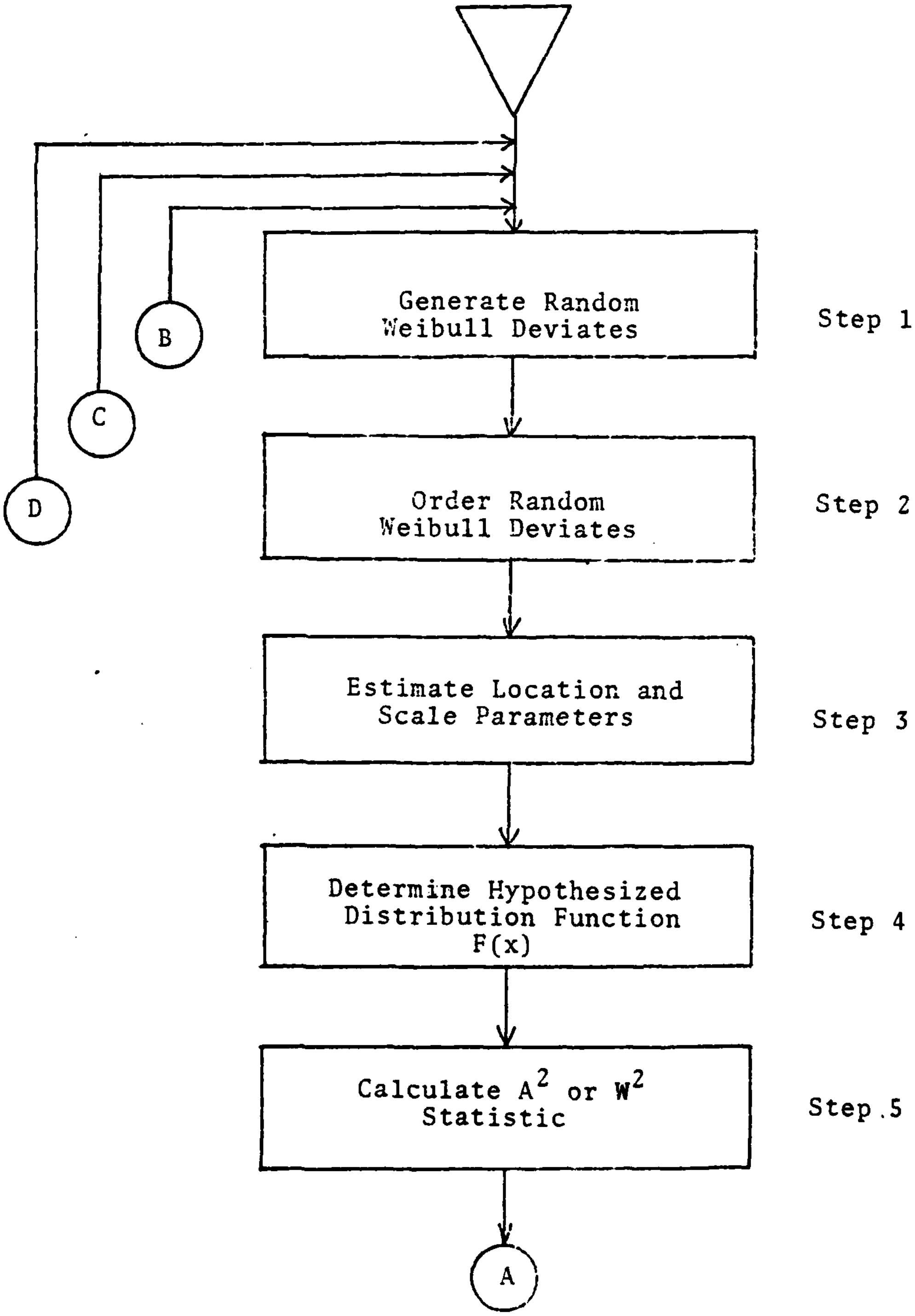
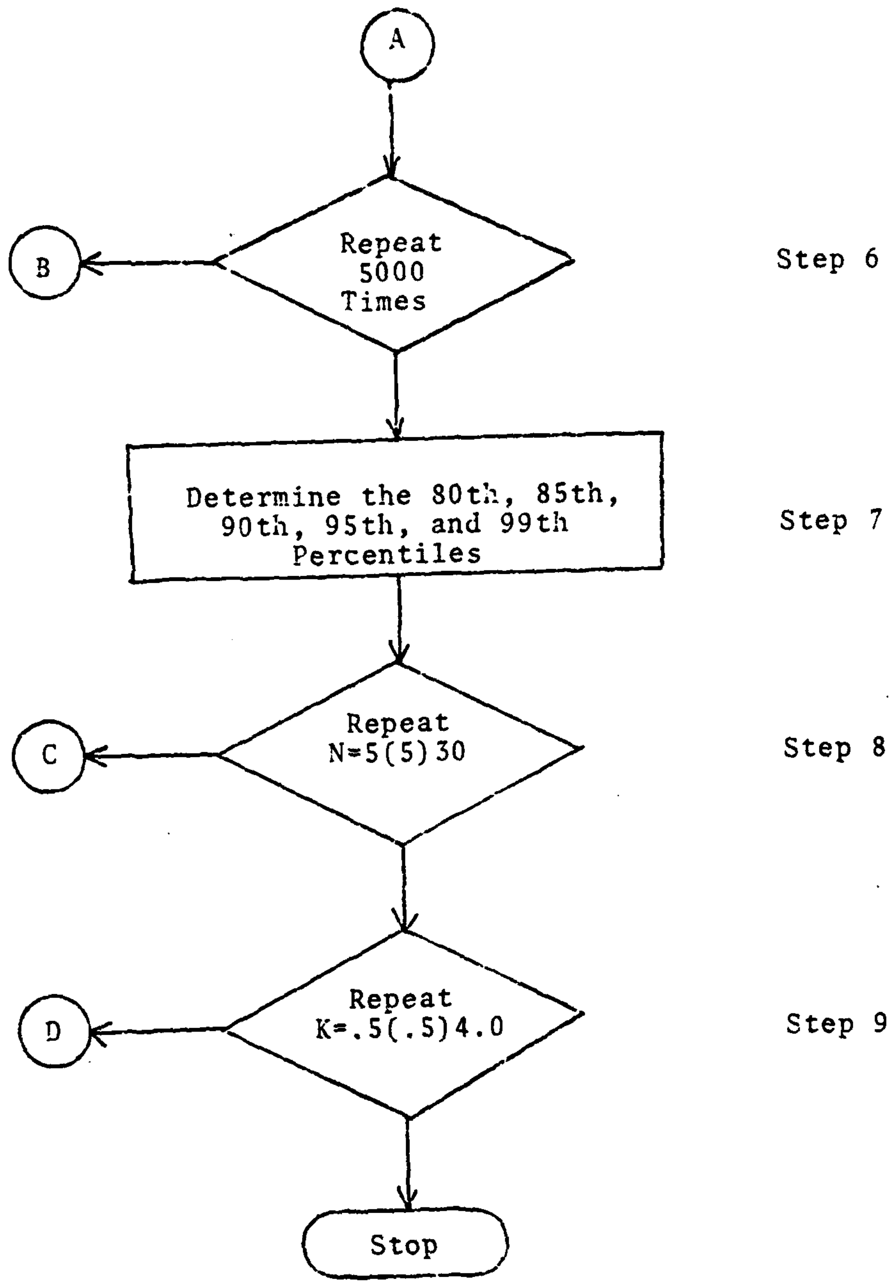


Fig 5. Flow Chart

Fig 5, Flow Chart continued



3. The ordered random Weibull deviates are used to estimate the scale and location parameters by the method of maximum likelihood.

4. The estimated scale and location parameter and the fixed shape parameter are used to determine the hypothesized distribution function F(x).

5. The Anderson-Darling statistic is calculated using Eq (6).

6. Steps 1 to 5 are repeated 5000 times, thus generating 5000 independent A^2 statistics.

7. The 5000 statistics are ordered. Using a plotting position, described later, the 80th, 85th, 90th, 95th, and the 99th percentiles are calculated by linear interpolation.

8. Repeat steps 1 to 7 for sample size n equal to 5, 10, 15, 20, 25, and 30.

9. Repeat steps 1 to 8 for shape parameter K equal to .5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0.

The critical values for the Cramer-von Mises test is calculated using the same procedure by calculating the W^2 statistic in step 5 using Eq (4).

Generation of the Random Weibull Deviates. The random Weibull deviates are obtained on the Control Data System (CDC) 6600 computer using the International Mathematical and Statistics Library (IMSL) subroutine GGWIB (12: Ch. G).

This subroutine generates pseudo-random deviates by

$$x = [-\ln(u)]^{1/K} \tag{14}$$

where u is a pseudo-random deviate from a uniform (0,1) distribution. Thus, the deviates obtained by this subroutine are from the Weibull distribution with location parameter $C = 0$, scale parameter $\theta = 1$, and the shape K set as an input parameter. Deviates from the more general Weibull pdf are obtained using the transformation

$$x^* = \theta \cdot x + C \tag{15}$$

In this thesis $\theta = 1$ and $C = 2$. The reason for this step will be explained after the parameter estimation routine is discussed.

Ordering of the Deviates. The random Weibull deviates are ordered, smallest to largest, using the CDC 6600 IMSL subroutine VSRTA (12:Ch. V).

Method of Maximum Likelihood Estimation of the Weibull Parameters. The procedure to derive the maximum likelihood estimators $\hat{K}$, $\hat{C}$, $\hat{\theta}$ of the Weibull parameters K, C, θ was developed by Harter and Moore (11:639). This is a general purpose iterative method for censored or uncensored samples. The Harter and Moore parameter estimation routine maps negative estimates onto zero. If Weibull deviates are generated with location $C = 0$ and scale $\theta = 1$, the estimators $\hat{C}$ and $\hat{\theta}$ will not retain the invariant property because within the iterative estimation procedure negative location estimators are mapped onto zero. Thus, Weibull deviates are generated with $C = 2$ and $\theta = 1$, preventing the generation of negative location estimators. Because of the invariant property of the EDF statistics, this transformation will not adversely affect the results.

The method of maximum likelihood selects as estimates those values of the parameters that maximize the probability or joint density of the observed sample (20:302). The likelihood function is defined as follows. Let $x_1, x_2, \ldots, x_n$ be sample observations taken on corresponding random variables: $X_1, X_2, \ldots, X_n$. Then if $X_1, X_2, \ldots X_n$ are continuous random variables, the likelihood, L, is defined to be the joint density evaluated at $x_1, x_2, \ldots, x_n$ (20:303). For the Weibull pdf, the likelihood function can be represented by:

$$L = f(x_1, x_2, \ldots, x_n : K, \theta, C) \tag{16}$$

Since the deviates, x_i, are derived randomly, $X_1, X_2, \ldots, X_n$ are mutually independent random variables. Hence,

$$L = \prod_{i=1}^{n} f(x_i : K, \theta, C) \tag{17}$$

The method of maximum likelihood chooses as estimates those values of the parameters that maximize the likelihood L. The procedure is:

1. Take the partial derivatives of the natural logarithm of L with respect to each parameter.

2. Set these equations to zero.

3. Solve simultaneously the equations for the values of the parameters.

The Weibull pdf is

$$f(x:K,\theta,C) = [\frac{K(x-C)^{K-1}}{\theta^K}] \exp [-(\frac{x-C}{\theta})^K],$$

$$\theta,\ K > 0,\ x \geq C \geq 0 \tag{18}$$

then

$$L = K^n\theta^{-Kn}[\prod_{i=1}^{n}(x_i-C)^{K-1}] \exp [-\theta^{-k}\sum_{i=1}^{n}(x_i-C)^K] \tag{19}$$

and

$$\ln(L) = n\ln K - Kn\ln\theta + (K-1)\sum_{i=1}^{n}\ln(x_i-C) - \theta^{-K}\sum_{i=1}^{n}(x_i-C)^K \tag{20}$$

Taking the partial derivatives of ln(L) with respect to K, θ, and C yields:

$$\frac{\partial\ln(L)}{\partial K} = \frac{n}{K} - n\ln\theta + \sum_{i=1}^{n}\ln(x_i-C) - \theta^{-K}\sum_{i=1}^{n}(x_i-C)^k\ln(x_i-C) = 0 \tag{21}$$

$$\frac{\partial\ln(L)}{\partial\theta} = -\frac{Kn}{\theta} + K\theta^{-K-1}\sum_{i=1}^{n}(x_i-C)^K = 0 \tag{22}$$

$$\frac{\partial\ln(L)}{\partial C} = (1-K)\sum_{i=1}^{n}(x_i-C)^{-1} + K\theta^{-K}\sum_{i=1}^{n}(x_i-C)^{K-1} = 0 \tag{23}$$

In this thesis K is an input parameter so the last two equations (22 and 23) are solved simultaneously to determine θ and C. Solving Eq (22) for θ yields

$$\hat{\theta} = [\frac{\sum_{i=1}^{n}(x_i-\hat{C})^K}{n}]^{1/K} \tag{24}$$

The iterative procedure of solving for $\hat{\theta}$ and $\hat{C}$ using Eqs (23) and (24) yields the maximum likelihood estimates.

For the special case when the shape parameter $K = 1$, the likelihood function L is maximized when

$$\hat{C} = x_{(1)} \tag{25}$$

and

$$\hat{\theta} = \sum_{i=2}^{n} (x_{(i)} - x_{(1)}) \tag{26}$$

where $x_{(i)}$ is the i^{th} order statistic of the sample.

The Hypothesized Distribution Function F(x). The maximum likelihood estimates for the location and scale parameters, the known shape parameter K, and the n ordered Weibull deviates, $x_{(i)}$, are used to calculate the hypothesized Weibull distribution function by

$$F(x_i) = 1 - \exp\{[(x_i - \hat{C})/\hat{\theta}]^K\} \tag{27}$$

Determining the Critical Values of the Anderson-Darling and the Cramer-von Mises Goodness-of-Fit Tests. The A^2 or W^2 statistic is calculated using Eq (6) or Eq (5). This procedure is repeated 5000 times. The 5000 A^2 (W^2) statistics are ordered. Using a plotting position and linear interpolation, the 80th, 85th, 90th, 95th, and 99th percentiles are found. The percentiles are the critical values for the goodness-of-fit test.

The use of plotting positions to determine the critical values is based on the "bootstrap" method (8).

Three plotting positions are considered. The first,

$$\frac{(i - .5)}{n} \tag{28}$$

is the middle of the interval from (i-1)/n to i/n. The second plotting position considered,

$$\frac{i - .3}{n + .4} \tag{29}$$

is the median ranks plotting position. The last one considered,

$$\frac{\frac{i}{n+1} + \frac{i-1}{n-1}}{2} \tag{30}$$

is the average of the mean plotting position, i/(n+1) and the mode plotting position, (i-1)/(n-1). One important property shared by all three of the plotting positions is the property of symmetry. For each plotting position over the interval 0 to 1,

$$\frac{i - \frac{1}{2}}{n} = 1 - \frac{(n-1) - \frac{1}{2}}{n} \tag{31}$$

for $1 \leq i \leq n$ for any sample size n. Johnston, in 1980, used the median ranks plotting position in a method similar to the one used in this thesis (14:22). At 5000 repetitions (n = 5000) there is no difference in the third significant digit in calculating the A^2 and W^2 statistics for the three plotting positions. For simplicity, plotting position described by Eq (28) is used.

The critical values are calculated using linear interpolation of the ordered A^2 (W^2) statistics as the abscissa axis and the associated ordinate axis of ordered plotting positions. Figure 6 depicts this procedure. The arrays making up the abscissa and ordinate axes are composed of 5002 entries. The 5000 values of the plotting position, Eq (28) with $n = 5000$ and $i = 1,2,\ldots,5000$, are entered in the ordinate array from position 2 to 5001. The interval [0,1] is completed by entering zero and one into position 1 and 5002, respectively. The 5000 ordered A^2 (W^2) statistics are entered in the abscissa array from position 2 to 5001. The first and last entries of the array are calculated by linear extrapolation. The first entry is calculated by linearly extrapolating from the second and third entry (first and second order statistic) subject to a non-negativity restriction. The 5002nd entry in the abscissa array is calculated by linearly extrapolating from the 5000th and 5001st entry (second to last and last order statistic), not subject to a maximum value. The 80th, 85th, 90th, 95th, and 99th percentiles are calculated by linearly interpolating between the order statistics and the respective plotting positions. For example, the 80th percentile is calculated as shown in Fig. 6. The plotting position just greater than .80, say the i^{th} entry in the array, and the plotting position just smaller than .80, the (i-1)st entry in the array, are found. Thus, the 80th percentile is found by interpolating between the i^{th} entry in the ordinate and abscissa array and the (i-1)st entry.

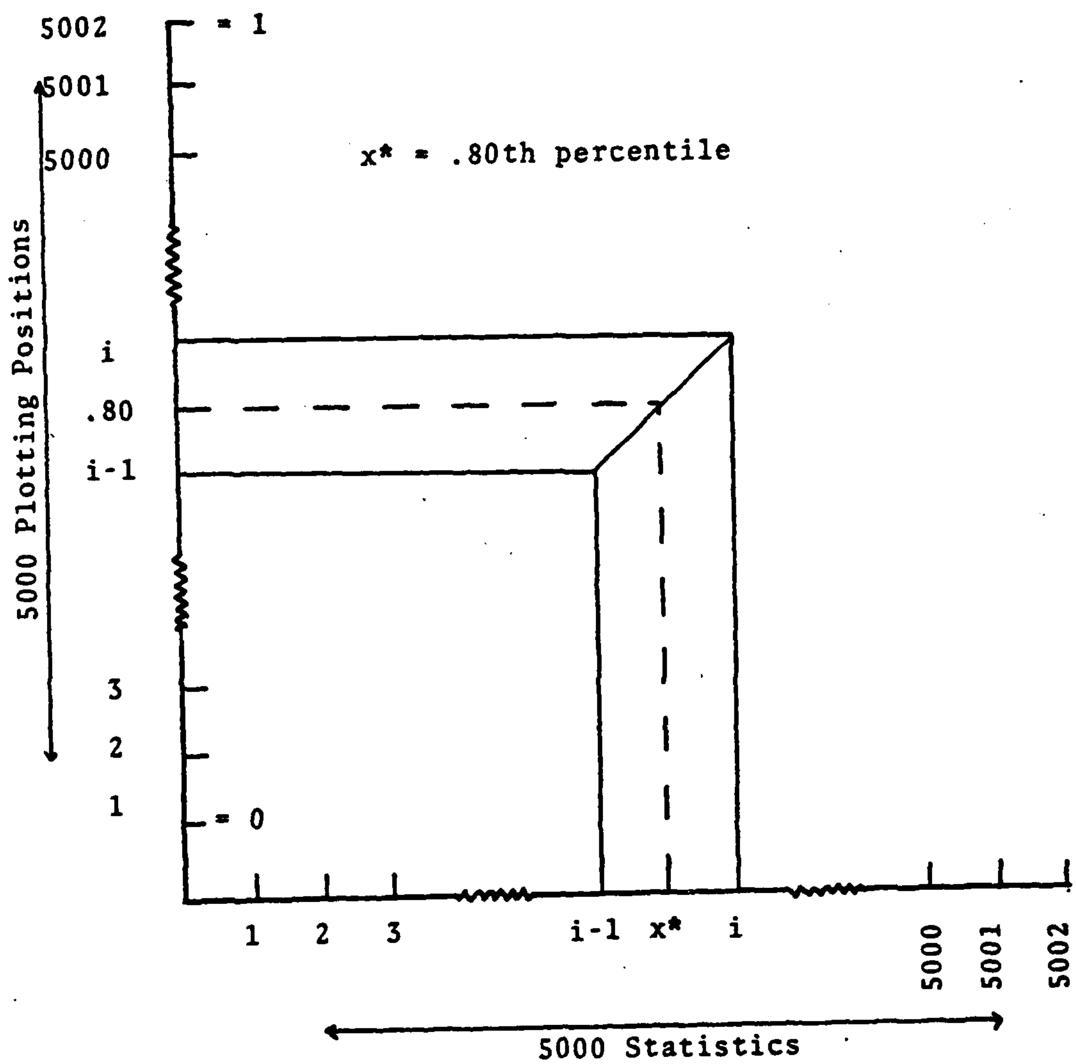


Fig 6. Plotting Positions Versus A^2 or W^2 Statistics

Power Comparison

In this study the powers of the Anderson-Darling test and the Cramer-von Mises test are compared with the modified Kolmogorov-Smirnov test and the Chi-Square test. The power of each test is compared for several alternative distributions. The null hypothesis is

H_o: Sample deviates follow a Weibull distribution, shape parameter K versus

H_a: Sample deviates follow some other distribution

In the power study, two null hypothesis K values are used, K = 1.0 and 3.5. The random deviates of the alternative distributions are generated using CDC 6600 IMSL subroutines. The following distributions are used in the study:

1. Weibull, shape equal 1.0
2. Weibull, shape equal 2.0
3. Weibull, shape equal 3.5
4. Gamma, shape equal 1.0
5. Gamma, shape equal 2.0
6. Uniform (1,2)
7. Normal (10,1)
8. Beta (p=1, q=1)
9. Beta (p=2, q=2)
10. Beta (p=2, q=3)

The Gamma distribution with shape parameter equal one is equivalent to the Weibull distribution with shape equal one since both distributions reduce to the exponential. The Gamma with shape parameter equal two is presented in Fig. 7.

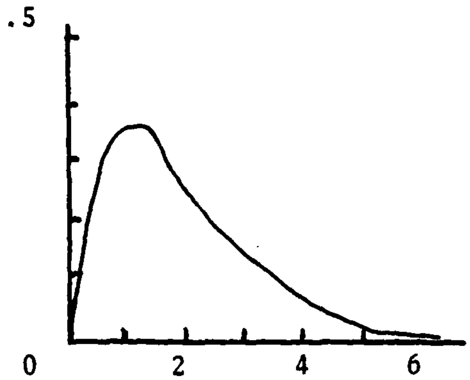


Fig. 7. Gamma Shape = 2

The Beta pdf

$$f(x) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} x^{p-1}(1-x)^{q-1} \quad , \; 0 < x < 1 \qquad (32)$$

for the above values of p and q are presented in Figs. 8a, 8b, and 8c. The other alternate distributions are well known and will not be discussed here.

Five thousand random samples of size n are generated for each of the ten alternate distributions. The respective test statistic, A^2, W^2, K-S, and χ^2 are calculated under the null hypothesis that the random deviates follow the Weibull distribution with shape parameter K. The calculated statistics are then compared to the respective critical value for each goodness-of-fit test. For the A^2 and W^2 tests, the critical values generated in this thesis are used. The K-S critical values presented by Cortes (4:48-57) were generated with C = 0 and θ = 1. Thus, the critical values are in error since the

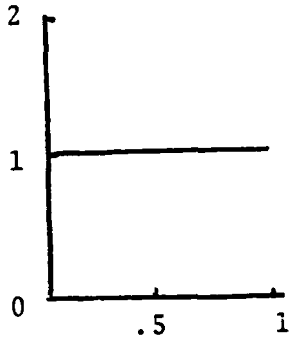


Fig. 8a. Beta,
p=1, q=1

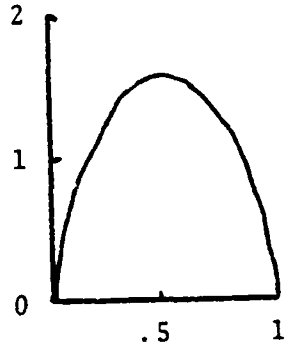


Fig. 8b. Beta
p=2, q=2

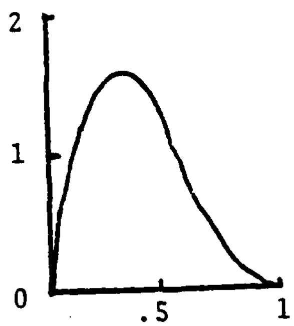


Fig. 8c. Beta
p=2, q=3

the maximum likelihood estimators for C and θ are not invariant. The K-S critical values used in this power study are regenerated using invariant estimators of C and θ and a simulation sample size of 5000. The Chi-Square critical values used in this thesis are generated using a Monte Carlo simulation with five observations per cell, sample size n = 25, and 5000 repetitions. The number of times each test statistic exceeds the respective critical value is counted. This is the number of times the null hypothesis is rejected. The power of the test is the number of times the null hypothesis is rejected divided by the total number of tests, 5000. The two power studies, one for each null hypothesis, are performed with sample size n = 25. The entire power study for the two null hypotheses and the 10 alternate distributions is repeated for n equal to 5 and 15. However, for n equal to 5 and 15, only the power comparisons between the A^2, W^2, and K-S goodness-of-fit tests are considered.

Analysis of Critical Values vs Shape Parameters

The tabled critical values for the Anderson-Darling and the Cramer-von Mises tests are generated for each value of shape parameter K equal to .5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0. The apparent relationship between the shape parameters and the critical values is investigated using regression analysis. A graphical relationship between the shape parameters and the critical values is also presented.

Computer Programs

The computer programs used in this thesis are presented in Appendix E.

IV. Use of the Tables

This chapter discusses the use of the tabled critical values for the A^2 and W^2 goodness-of-fit tests generated in this thesis for the three-parameter Weibull distribution. An example follows an explanation of the basic procedures in the use of the tables.

For both the Anderson-Darling and Cramer-von Mises goodness-of-fit tests, a theoretical distribution F(x) is compared to an observed distribution Sn(x). The W^2 or A^2 statistic is calculated using the appropriate equation (Eq 4 or 6). If the value of the statistic exceeds a certain level (critical value), the theoretical distribution is rejected. The steps in applying this procedure are:

1. Determine the sample size n and the level of significance, α. The α-level is the probability of rejecting the null hypothesis that the sample follows the theoretical distribution when, in fact, the null hypothesis is true.

2. Specify the shape parameter for the hypothesized theoretical Weibull distribution. The invariant property of the A^2 and W^2 test allows the scale and location parameters to be unspecified.

3. Select, in a random manner, the n observations from the population to be tested. The n observations are ordered from smallest to largest.

4. Estimate the unknown location and scale parameters from the observed sample using the method of maximum likelihood. If the parameter estimation method developed by Harter and Moore is used, care must be taken to prevent the location estimators from being mapped onto zero. Any number can be added to all the sample values so the location estimator remains positive. This transformation will not adversely affect the test because of the invariant property of the EDF statistics.

5. Completely specify the hypothesized Weibull distribution F(x) using the estimated scale and location parameters and the set shape parameter.

6. Determine the value of the test statistic, W^2 or A^2 by using Eq (4) or (6).

7. With the sample size n, the significance level α , and the shape parameter K, extract the critical value from the tables in Appendix A or B.

8. Reject the null hypothesis if the value of the test statistic exceeds the critical value. If the test statistic does not exceed the critical value, we fail to reject the null hypothesis and conclude there is insufficient evidence to say the observed sample does not follow the hypothesized Weibull distribution with shape parameter K.

Example

The following example illustrates this procedure for the Cramer-von Mises test.

The following ten failure times are tested to determine if they follow a Weibull distribution with $K = 3.5$ at α equal .05: 2.22, 2.26, 2.26, 2.44, 2.47, 2.49, 2.61, 2.67, 3.14, and 3.44 months. The hypothesis tested is:

H_o: Sample data follows a Weibull distribution, $K = 3.5$ versus

H_a: Sample data follows some other distribution.

The Harter and Moore subroutine yields the estimated parameter values: location $\hat{C} = 1.427$ and scale $\hat{\theta} = 1.328$. Using these two estimators and the shape $K = 3.5$ in Eq (9), the value of the hypothesized distribution is calculated for each sample value. These calculations are shown in Table I.

TABLE I

Example: x and F(x)

i	x	F(x)
1	2.22	.148
2	2.26	.174
3	2.26	.177
4	2.44	.322
5	2.47	.347
6	2.49	.370
7	2.61	.481
8	2.67	.543
9	3.14	.911
10	3.44	.986

Using Eq (4), the W statistic equals .144. The critical value extracted from Table XVI with α equal .05, $n = 10$, and $K = 3.5$ is .142. Since $.144 > .142$, the null hypothesis is rejected.

Thus, the ten failure times follow a distribution other than the Weibull distribution with shape parameter 3.5.

V. Discussion of the Results

This chapter discusses the results obtained in this thesis. The results as they pertain to the four objectives set forth in Chapter I are presented.

Presentation of the Cramer-von Mises and the Anderson-Darling Tables of Critical Values

The tabled critical values for the W^2 and A^2 goodness-of-fit tests for sample sizes n equal to 5, 10, 15, 20, 25, and 30 and shape parameter K equal to .5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0 are presented in Appendices A and B, respectively. Each table presented is valid for a specific shape parameter.

In general, the critical values for the Cramer-von Mises test tend to monotonically increase for each level of significance as the sample size increases from 5 to 20. When the sample size is increased from 20 to 30, this increasing trend breaks down. The trend for the critical values for the Anderson-Darling test is interesting. With shape parameter K equal .5 and 1.0, the A^2 critical values are monotonically decreasing for each level of significance as the sample size increases from 5 to 25. However, with the shape parameter equal to 1.5 and greater, the opposite trend develops; the critical values are monotonically increasing. Similar to the W^2 critical values, the trends for the A^2 critical values breaks down as the sample size becomes large. For both goodness-of-fit tests, the critical values may, in fact, be approaching a limit as the sample size becomes larger than 25. The Monte Carlo variability of the experiment may also cause the critical values to fluctuate. The critical values for sample size greater than 25 can be investigated in two ways. First, the simulation sample size can be increased from 5000 to 10000. It is known that the error associated with a Monte Carlo simulation is proportional to $1/\sqrt{N}$, where N is the number of repetitions of the simulation (22:259). Thus, increasing N would have the effect of reducing the variability. Second, the critical values for both the A^2 and W^2 goodness-of-fit tests can be investigated for sample sizes n equal 40, 50, and 60. The monotonic decreasing or increasing trend, thus, may be easier to detect. However, either increasing the simulation repetitions N or increasing the sample size n would require much more computer time and, therefore, is beyond the scope of this thesis.

Validation of Computer Programs

The computer programs used are validated by comparing a modification of the A^2 and W^2 critical values generated in this thesis to the approximate critical values calculated by M.A. Stephens for the exponential distribution (25:359). This comparison is possible since the Weibull distribution with shape parameter equal to one and location parameter equal to zero reduces to the exponential distribution.

Setting $K = 1$ and $C = 0$, Weibull deviates are generated. The scale parameter θ is estimated and the critical values for the Anderson-Darling and the Cramer-von Mises tests are calculated for sample sizes n equal to 5, 10, 15, 20, and 30. The A^2 critical values are modified using Eq (33) and the W^2 critical values are modified using Eq (34):

$$A^2\left(1 + \frac{1.5}{n} - \frac{5}{n^2}\right) \tag{33}$$

$$W^2\left(1 + \frac{.16}{n}\right) \tag{34}$$

where the A^2 and W^2 are the Anderson-Darling and Cramer-von Mises critical values generated in this thesis and n is the sample size. The modified critical values are compared to the critical values derived by Stephens for a level of significance equal to .15, .10, .05, and .01. The comparisons for the W^2 and A^2 critical values are presented in Tables II and III, respectively. The comparisons are good.

Power Investigation

A power comparison is made between the Chi-Square, Kolmogorov-Smirnov, Cramer-von Mises, and Anderson-Darling goodness-of-fit tests for the three-parameter Weibull distribution with the scale and location parameters unspecified. The power comparisons are made using random deviates generated from the ten alternate distributions discussed in Chapter III. The Weibull distribution with shape parameter $K = 1$ and shape parameter $K = 3.5$ are both considered as the null hypothesis.

TABLE II

Cramer-von Mises W^2

1 - α	$W^2(1+.16/n)$				Stephen's Critical Values
	n				
	5	10	20	30	
.85	.155	.150	.149	.146	.149
.90	.180	.178	.173	.171	.171
.95	.226	.227	.220	.214	.224
.99	.324	.323	.358	.326	.337

TABLE III

Anderson-Darling A^2

1 - α	$A^2(1+1.5/n-5/n^2)$				Stephen's Critical Values
	n				
	5	10	20	30	
.85	.928	.999	.998	.940	.922
.90	1.071	1.173	1.153	1.083	1.078
.95	1.345	1.492	1.438	1.333	1.341
.99	1.996	2.220	2.053	2.000	1.957

The power of the Chi-Square test is only included at sample size n = 25. As expected, the power of the Chi-Square test is lower than the other three goodness-of-fit tests. The power comparisons between the K-S, W^2, and the A^2 tests are discussed by first considering the null hypothesis with K = 1 and then for K = 3.5.

<u>Shape Parameter K Equal 1.0</u>. The hypothesis tested is

H_o: The sample data follow a Weibull distribution with shape equal one versus

H_a: The sample data follow some other distribution.

The power comparisons between the goodness-of-fit tests are presented in Table IV for α-level equal .05, and Table V for α-level equal .01. The power of the goodness-of-fit tests when the null hypothesis is true should achieve the claimed level of significance. There are differences for the power of the tests in the third decimal place from the exact significance level. These discrepancies can possibly be explained by the Monte Carlo variability of the experiment.

At the α equal to the .05 level of significance, the power of the Cramer-von Mises test is generally greater than the power of the Anderson-Darling or the Kolmogorov-Smirnov tests. The trend is evident for sample sizes n equal to 25 and 15 and for the alternate distributions with the exception of the Gamma, shape equal two when n = 15. For the deviates generated from the alternate distributions at α equal .05 and sample size n equal to 25 and 15, the power study indicates the power of the three tests in decreasing order are: W^2, A^2, and K-S. For sample size n = 5, the power of all the goodness-of-fit tests are quite low; they are all very close to the level of significance of the test. This indicates the three goodness-of-fit tests are not very good when only five observations are used in the test. With five

TABLE IV

Power Test for the Weibull Distribution

H_0: Weibull Distribution, K = 1.0 -- H_a: Another Distribution

Level of Significance = .05

Sample Size n	Test Statistics	Alternate Distributions				
		Weibull Shape=1.0	Weibull Shape=2.0	Weibull Shape=3.5	Gamma Shape=1.0	Gamma Shape=2.0
25	K-S	.057	.575	.885	.048	.196
	W^2	.056	.700	.947	.055	.245
	A^2	.047	.655	.930	.044	.231
	X^2	.047	.371	.740	.042	.121
15	K-S	.049	.277	.568	.050	.099
	W^2	.053	.346	.667	.053	.116
	A^2	.043	.321	.624	.050	.122
5	K-S	.045	.051	.079	.047	.040
	W^2	.045	.055	.097	.047	.043
	A^2	.049	.049	.052	.045	.114

TABLE IV, continued

Sample Size n	Test Statistics	Alternate Distributions				
		Uniform (1,2)	Normal (10,1)	Beta P=2 Q=3	Beta P=2 Q=2	Beta P=1 Q=1
25	K-S	.575	.904	.628	.771	.563
	W^2	.746	.950	.773	.899	.737
	A^2	.703	.938	.715	.867	.694
	X^2	.318	.785	.389	.554	.320
15	K-S	.328	.606	.340	.454	.323
	W^2	.446	.699	.428	.578	.434
	A^2	.384	.685	.359	.508	.363
5	K-S	.072	.079	.057	.069	.069
	W^2	.084	.099	.066	.082	.085
	A^2	.050	.202	.014	.031	.048

TABLE V

Power Test for Weibull Distribution

H_0: Weibull Distribution, K = 1.0 -- H_a: Another Distribution

Level of Significance = .01

Sample Size n	Test Statistics	Alternate Distributions				
		Weibull Shape=1.0	Weibull Shape=2.0	Weibull Shape=3.5	Gamma Shape=1.0	Gamma Shape=2.0
25	K-S	.012	.324	.717	.010	.069
	W^2	.011	.424	.828	.011	.083
	A^2	.009	.351	.781	.009	.062
	X^2	.009	.190	.587	.008	.037
15	K-S	.012	.105	.315	.012	.029
	W^2	.012	.139	.422	.012	.027
	A^2	.009	.107	.347	.009	.021
5	K-S	.009	.003	.007	.011	.004
	W^2	.008	.002	.006	.011	.003
	A^2	.007	.004	.007	.007	.017

TABLE V, continued

Sample Size n	Test Statistics	Alternate Distributions Uniform (1,2)	Normal (10,1)	Beta P=2 Q=3	Beta P=2 Q=2	Beta P=1 Q=1
25	K-S	.303	.756	.351	.524	.292
	W^2	.449	.857	.506	.695	.442
	A^2	.361	.819	.406	.618	.151
	χ^2	.187	.636	.215	.371	.180
15	K-S	.125	.362	.134	.199	.117
	W^2	.191	.465	.177	.291	.178
	A^2	.138	.434	.122	.216	.122
5	K-S	.007	.003	.003	.005	.005
	W^2	.005	.004	.002	.005	.005
	A^2	.006	.061	.001	.004	.006

observations, the three tests are not able to differentiate the alternate distributions from the Weibull distribution with shape parameter equal one.

Shape Parameter K Equal to 3.5. The hypothesis tested is:

H_o: The sample data follow a Weibull distribution with shape parameter equal 3.5 versus

H_a: The sample data follow some other distribution.

The power comparisons between the goodness-of-fit tests are presented in Tables VI for α-level equal .05 and VII for α-level equal .01. When the null hypothesis is true, both tests do achieve the claimed level of significance. The differences, again, may be due to the Monte-Carlo variability. Generally, the A^2 test is more powerful than the W^2 and K-S tests. This study showed for the alternate distributions tested at α equal .05 and sample size n equal to 25, 15, and 5 the power of the three tests in decreasing order are: A^2, W^2, and K-S. In contrast with the first hypothesis (Weibull, shape equal one), the three goodness-of-fit tests are better when the sample size is five. However, the value of the three goodness-of-fit tests for small sample sizes is questionable. The problem with small sample sizes could be further investigated by expanding the power study to include other hypothesized Weibull distributions, including sample size n = 10, investigate additional alternative distributions, or investigate other goodness-of-fit tests. However, all these alternatives take additional computer time and are beyond the scope of

TABLE VI

Power Test for the Weibull Distribution

H_0: Weibull Distribution, K = 3.5 -- H_a: Another Distribution

Level of Significance = .05

Sample Size n	Test Statistics	Alternate Distributions				
		Weibull Shape=1.0	Weibull Shape=2.0	Weibull Shape=3.5	Gamma Shape=1.0	Gamma Shape=2.0
25	K-S	.801	.172	.051	.801	.602
	W^2	.881	.176	.045	.878	.652
	A^2	.907	.191	.046	.913	.679
	X^2	.414	.066	.042	.428	.274
15	K-S	.558	.102	.057	.559	.423
	W^2	.651	.117	.058	.646	.465
	A^2	.667	.117	.056	.667	.476
5	K-S	.219	.064	.052	.212	.180
	W^2	.238	.063	.053	.232	.195
	A^2	.241	.060	.052	.237	.207

TABLE VI, continued

Sample Size n	Test Statistics	Alternate Distributions Uniform (1,2)	Normal (10,1)	Beta P=2 Q=3	Beta P=2 Q=2	Beta P=1 Q=1
25	K-S	.129	.076	.093	.067	.132
	W^2	.172	.071	.094	.061	.172
	A^2	.232	.073	.103	.073	.231
	X^2	.094	.045	.048	.044	.095
15	K-S	.086	.063	.061	.044	.079
	W^2	.117	.066	.069	.048	.106
	A^2	.131	.061	.065	.050	.122
5	K-S	.063	.053	.053	.050	.056
	W^2	.069	.057	.056	.054	.062
	A^2	.069	.054	.053	.052	.064

TABLE VII

Power Test for the Weibull Distribution

H_0: Weibull Distribution, K = 3.5 -- H_a: Another Distribution

Level of Significance = .01

Sample Size n	Test Statistics	Alternate Distributions				
		Weibull Shape=1.0	Weibull Shape=2.0	Weibull Shape=3.5	Gamma Shape=1.0	Gamma Shape=2.0
25	K-S	.600	.054	.010	.607	.434
	W^2	.726	.056	.008	.728	.491
	A^2	.782	.071	.009	.780	.520
	X^2	.271	.022	.011	.270	.151
15	K-S	.372	.026	.013	.373	.294
	W^2	.467	.033	.012	.475	.343
	A^2	.467	.031	.010	.469	.339
5	K-S	.106	.013	.008	.107	.085
	W^2	.133	.016	.011	.128	.109
	A^2	.152	.018	.013	.146	.131

TABLE VII, continued

Sample Size n	Test Statistics	Alternate Distributions				
		Uniform (1,2)	Normal (10,1)	Beta P=2 Q=3	Beta P=2 Q=2	Beta P=1 Q=1
25	K-S	.030	.020	.022	.013	.028
	W^2	.042	.020	.020	.014	.057
	A^2	.067	.023	.025	.017	.069
	χ^2	.022	.014	.011	.010	.020
15	K-S	.019	.017	.016	.008	.015
	W^2	.023	.016	.015	.010	.021
	A^2	.024	.013	.011	.007	.022
5	K-S	.010	.010	.007	.008	.010
	W^2	.014	.011	.011	.010	.012
	A^2	.017	.017	.012	.011	.015

this thesis.

Relationship Between the Critical Values and the Shape Parameters

The relationship between the Anderson-Darling and the Cramer-von Mises critical values and the Weibull shape parameters is investigated using both graphical and regression techniques. The graphs of the W^2 critical values versus the Weibull shape parameters are presented in Appendix C. The graphs for the A^2 critical values versus the shape parameters are presented in Appendix D.

For the Anderson-Darling goodness-of-fit test, the relationship between critical values and shape parameters one and greater is investigated using regression analysis. The best model found to represent the relationship is:

$$Y = a_0 + a_1(1/x) \tag{35}$$

where Y is the critical value and x is the shape parameter greater than or equal to one. For the figures in Appendix D, the A^2 critical values increase from shape .5 to 1.0, then decrease as the shape parameter is increased to 4.0. Therefore, the relationship between the A^2 critical values and the shape parameter is only investigated for shape parameters 1.0 and greater. In Eq (35), a_0 is a constant and a_1 is the coefficient of the independent variable. The coefficients, a_0 and a_1, are presented in Table VIII. Separate coefficients are presented for sample sizes 5, 10, 15, 20, 25, and 30 and for levels of significance .20, .15, .10, .05, and .01.

TABLE VIII

Coefficients and R^2 Values for the Relationship Between the Anderson-Darling Critical Values and the Weibull Shape Parameters, 1.0(.5)4.0

Level of Significance	.20	.15	.10	.05	.01
n=5	1.7370 .80 -.0984	1.7545 .81 -.0578	1.7764 .82 -.0015	1.8127 .85 .8970	1.9038 .89 .2809
n=10	.7718 .91 .2829	.8003 .93 .3255	.8293 .95 .3930	.9195 .97 .4859	1.1441 .97 .7052
n=15	.5577 .98 .3680	.6011 .99 .4109	.6593 .99 .4678	.7683 .99 .5686	1.0877 .99 .7680
Legend: a_1 R^2 a_0					

TABLE VIII, continued

Level of Significance	.20	.15	.10	.05	.01
n=20	.4555 .99 .4158	.5072 .99 .4553	.5774 .99 .5182	.7245 .99 .6093	1.0767 .99 .7986
n=25	.4331 .99 .4284	.4897 .99 .4652	.5616 .99 .5253	.6935 .98 .6210	1.1022 .99 .8059
n=30	.4044 .99 .4417	.4495 .99 .4855	.5303 .99 .5374	.6127 .97 .6530	.9969 .92 .8595

Legend:

a_1 R^2

a_0

The R^2 value on the tables is a measurement of the percent of total variation explained by the regression line. All R^2 values are above .80. Twenty-five regressions have an R^2 above .90, and 16 have an R^2 of .99. Thus, Eq (35) is a good approximation of the relationship between the A^2 critical values and the Weibull shape parameters.

Eq (35) is also a very good approximation of the relationship between the Cramer-von Mises critical values and the Weibull shape parameters equal to 1.5 and greater. For most of the figures in Appendix C, the critical values decrease (shape .5 to 1.0), then increase (shape 1.0 to 1.5), and then decrease again (shape 1.5 to 4.0). Therefore, the relationship between the W^2 critical values and the shape parameters is investigated for shape parameters equal to 1.5 to 4.0. All 30 regressions have an R^2 greater than .90, and 25 are above .98. The coefficients and R^2 values are presented in Table IX.

An example will demonstrate the use of these tables. Suppose a W^2 critical value is desired for shape parameter $K = 2.8$, sample size $n = 20$, and an α-level of .10. The value of the a_0 and a_1 coefficients are extracted from Table IX. The coefficients and the inverse of 2.8 are substituted into Eq (35). Thus,

$$Y = .09 + .099(1/2.8) \tag{37}$$

yields a W^2 critical value of .125.

TABLE IX

Coefficients and R^2 Values for the Relationship Between the Cramer-von Mises Critical Values and the Weibull Shape Parameters, 1.5(.5)4.0

Level of Significance	.20	.15	.10	.05	.01
n=5	.0552 .99 .0727	.0689 .99 .0744	.0815 .99 .0856	.1069 .98 .0970	.1643 .91 .1279
n=10	.0643 .99 .0724	.0799 .99 .0779	.0906 .99 .0891	.1217 .99 .1044	.2122 .98 .1331
n=15	.0666 .99 .0715	.0829 .99 .0771	.0970 .99 .0878	.1292 .98 .1032	.2201 .91 .1334

Legend: a_1 R^2 a_0

TABLE IX, continued

Significance	.20	.15	.10	.05	.01
n=20	.0735 .99 .0724	.0800 .99 .0805	.0990 .99 .0900	.1487 .98 .1005	.2497 .96 .1288
n=25	.0629 .99 .0737	.0779 .99 .0796	.0977 .99 .0893	.1163 .99 .1110	.2255 .98 .1394
n=30	.0760 .99 .0714	.0876 .99 .0795	.1005 .99 .0919	.1504 .97 .1034	.2243 .94 .1440
Legend: a_1 R^2 a_0					

VI. Conclusions and Recommendations

The following conclusions are made based on the results obtained in this thesis.

1. The tabled Anderson-Darling and Cramer-von Mises critical values for the three-parameter Weibull distribution are valid. In the Monte Carlo simulation, both tests achieved the claimed level of significance when the null hypothesis is true.

2. The conclusions based on the power comparison study is applicable for at least the ten alternate distributions discussed in Chapter III. The Kolmogorov-Smirnov, Anderson-Darling, and the Cramer-von Mises goodness-of-fit tests are not very powerful when the sample size is five. When the hypothesized distribution is the Weibull with shape parameter equal 1.0, the power of the tests in descending order are: W^2, A^2, and K-S. When the hypothesized distribution is the Weibull with shape parameter equal 3.5, the power of the tests in descending order are: A^2, W^2, and K-S.

3. The Anderson-Darling critical values are related to the inverse of the shape parameter for the Weibull distribution when the shape parameter is one and greater. The same inverse relationship exists for the Cramer-von Mises critical values for the Weibull distribution when the shape parameter is 1.5 and greater.

Recommendations

Based on observations made during the investigation, the following recommendations are proposed for further study.

1. Calculate the critical values for the Anderson-Darling and Cramer-von Mises tests for the three-parameter Gamma distribution when the scale and location parameters are unspecified.

2. Modify Mann, Scheuer, and Fertig's S statistic to calculate the critical values for the three-parameter Weibull distribution. The power of the S test can be compared to the power of the A^2, W^2, and K-S goodness-of-fit tests to determine which is most powerful.

3. Investigate the A^2 and W^2 and possibly other goodness-of-fit tests for small sample sizes. The simulation sample size can be increased, other alternative distributions investigated, or additional shape parameters of the Weibull distribution can be considered as the hypothesized distribution.

4. Investigate the A^2 and W^2 goodness-of-fit tests for large sample sizes. The critical values for n equal to 40, 50, and 60 can be investigated and the simulation sample size can be increased.

5. Investigate the possibility of developing a more efficient (in terms of computer execution time) parameter estimation routine for the Weibull distribution. The new routine can then be compared to the Harter and Moore routine used in this thesis to determine the most efficient.

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APPENDIX A

Tables of the Cramer-von Mises Critical Values for the Weibull Distribution

TABLE X

Cramer-von Mises W^2
Shape Parameter Equals .5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.109	.121	.138	.169	.257
10	.118	.133	.157	.195	.293
15	.120	.138	.160	.204	.310
20	.127	.144	.169	.210	.318
25	.128	.146	.171	.219	.332
30	.125	.142	.166	.211	.334

TABLE XI

Cramer-von Mises W^2
Shape Parameter Equals 1.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.096	.106	.118	.141	.209
10	.111	.125	.146	.181	.271
15	.114	.130	.150	.186	.279
20	.116	.133	.158	.198	.304
25	.118	.136	.160	.196	.299
30	.122	.138	.163	.206	.315

TABLE XII

Cramer-von Mises W^2
Shape Parameter Equals 1.5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.110	.124	.141	.171	.245
10	.116	.131	.150	.186	.279
15	.117	.133	.153	.193	.289
20	.119	.133	.157	.203	.301
25	.117	.132	.155	.190	.292
30	.123	.138	.159	.205	.301

TABLE XIII

Cramer-von Mises W^2
Shape Parameter Equals 2.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.101	.111	.125	.148	.198
10	.104	.118	.134	.165	.232
15	.104	.117	.136	.163	.237
20	.108	.122	.139	.172	.251
25	.104	.118	.137	.167	.245
30	.107	.122	.141	.178	.248

TABLE XIV

Cramer-von Mises W^2
Shape Parameter Equals 2.5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.094	.104	.117	.138	.196
10	.099	.109	.125	.153	.214
15	.098	.110	.127	.153	.203
20	.100	.112	.128	.156	.219
25	.098	.110	.127	.157	.234
30	.102	.115	.132	.163	.229

TABLE XV

Cramer-von Mises W^2
Shape Parameter Equals 3.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.091	.100	.111	.132	.177
10	.093	.103	.117	.144	.207
15	.094	.104	.119	.147	.216
20	.095	.106	.124	.150	.212
25	.095	.106	.123	.149	.220
30	.096	.107	.124	.148	.208

TABLE XVI

Cramer-von Mises W^2
Shape Parameter Equals 3.5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.088	.097	.110	.126	.173
10	.091	.102	.117	.142	.198
15	.090	.100	.115	.141	.197
20	.092	.102	.117	.142	.195
25	.092	.103	.117	.146	.207
30	.094	.107	.124	.153	.218

TABLE XVII

Cramer-von Mises W^2
Shape Parameter Equals 4.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.087	.096	.107	.128	.178
10	.089	.099	.113	.134	.165
15	.089	.099	.113	.138	.196
20	.092	.102	.117	.143	.203
25	.090	.099	.114	.140	.189
30	.091	.101	.116	.140	.206

APPENDIX B

Tables of the Anderson-Darling Critical Values for the Weibull Distribution

TABLE XVIII

Anderson-Darling A^2
Shape Parameter Equals .5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	1.057	1.144	1.309	1.617	2.427
10	.875	.972	1.104	1.365	2.090
15	.834	.937	1.075	1.292	1.946
20	.812	.903	1.051	1.290	1.974
25	.807	.888	1.022	1.312	1.951
30	.802	.903	1.043	1.304	1.888

TABLE XIX

Anderson-Darling A^2
Shape Parameter Equals 1.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	1.918	1.973	2.040	1.147	2.400
10	1.131	1.194	1.284	1.451	1.855
15	.954	1.035	1.147	1.355	1.879
20	.874	.961	1.096	1.322	1.885
25	.850	.941	1.069	1.287	1.875
30	.838	.927	1.055	1.240	1.806

TABLE XX

Anderson-Darling A^2
Shape Parameter Equals 1.5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.670	.728	.814	.966	1.267
10	.698	.775	.873	1.062	1.529
15	.700	.779	.886	1.059	1.482
20	.722	.804	.914	1.131	1.525
25	.741	.822	.939	1.136	1.605
30	.731	.807	.923	1.111	1.672

TABLE XXI

Anderson-Darling A^2
Shape Parameter Equals 2.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.581	.631	.707	.819	1.067
10	.606	.664	.750	.885	1.195
15	.630	.696	.777	.938	1.279
20	.635	.698	.791	.957	1.292
25	.641	.706	.801	.964	1.375
30	.642	.707	.797	.964	1.314

TABLE XXII

Anderson-Darling A^2
Shape Parameter Equals 2.5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.542	.588	.647	.760	.982
10	.573	.623	.698	.819	1.089
15	.581	.645	.719	.867	1.155
20	.594	.656	.742	.885	1.227
25	.601	.657	.743	.899	1.224
30	.590	.655	.734	.881	1.193

TABLE XXIII

Anderson-Darling A^2
Shape Parameter Equals 3.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.534	.580	.647	.739	.963
10	.558	.612	.683	.804	1.101
15	.558	.614	.696	.824	1.114
20	.561	.621	.713	.841	1.156
25	.573	.625	.712	.844	1.148
30	.573	.634	.708	.837	1.127

TABLE XXIV

Anderson-Darling A^2
Shape Parameter Equals 3.5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.515	.561	.621	.714	.904
10	.538	.586	.660	.775	1.044
15	.542	.595	.672	.809	1.126
20	.554	.602	.683	.815	1.118
25	.547	.601	.684	.817	1.104
30	.560	.615	.696	.847	1.207

TABLE XXV

Anderson-Darling A^2
Shape Parameter Equals 4.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.520	.563	.616	.712	.925
10	.528	.574	.652	.765	1.054
15	.527	.577	.644	.768	1.048
20	.536	.588	.672	.805	1.086
25	.535	.587	.659	.782	1.099
30	.546	.600	.670	.792	1.122

APPENDIX C

Graphs of the Cramer-von Mises Critical Values Versus the Weibull Shape Parameters

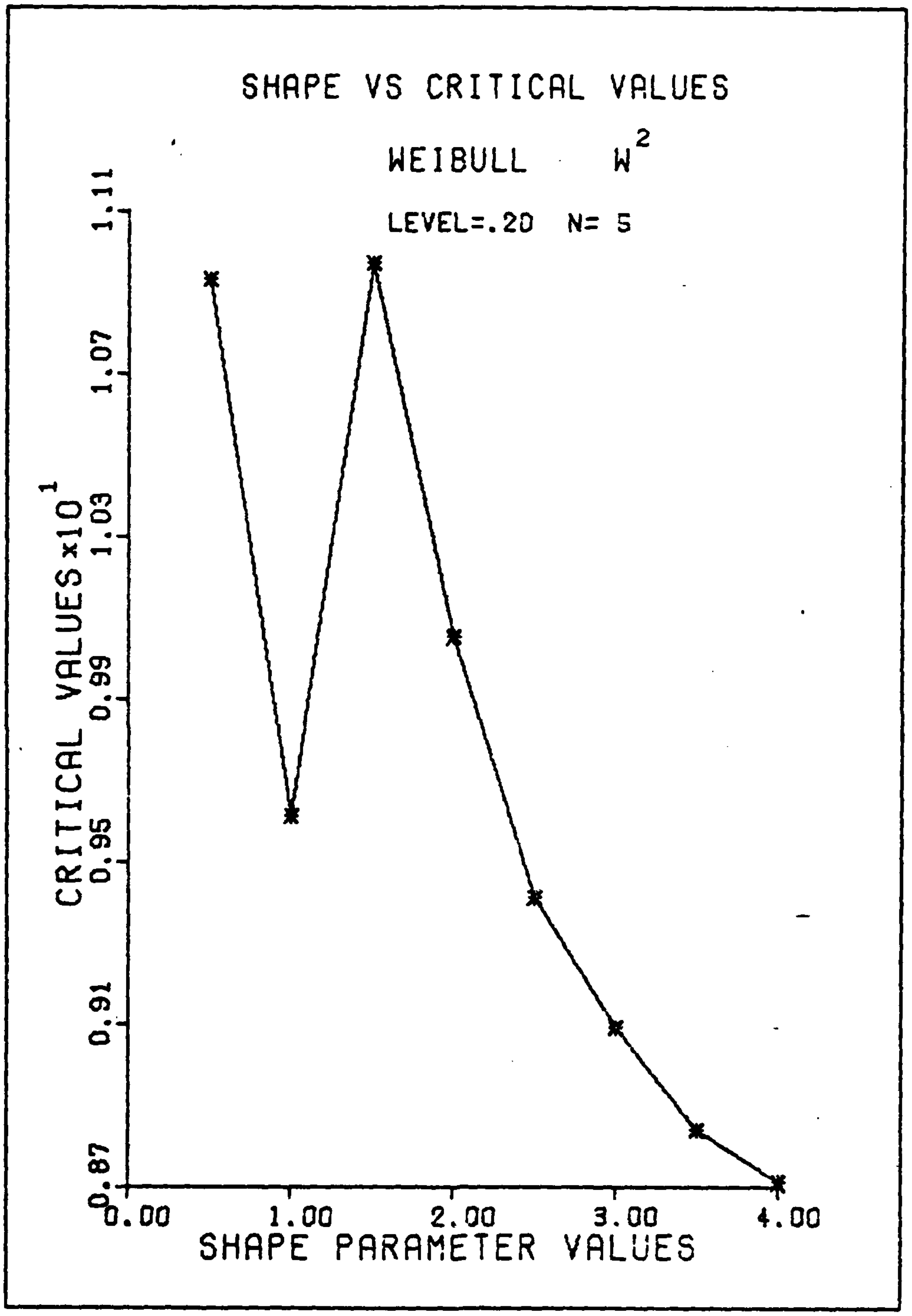

Fig. 9. Shape vs W^2 Critical Values, Level=.20, n=5

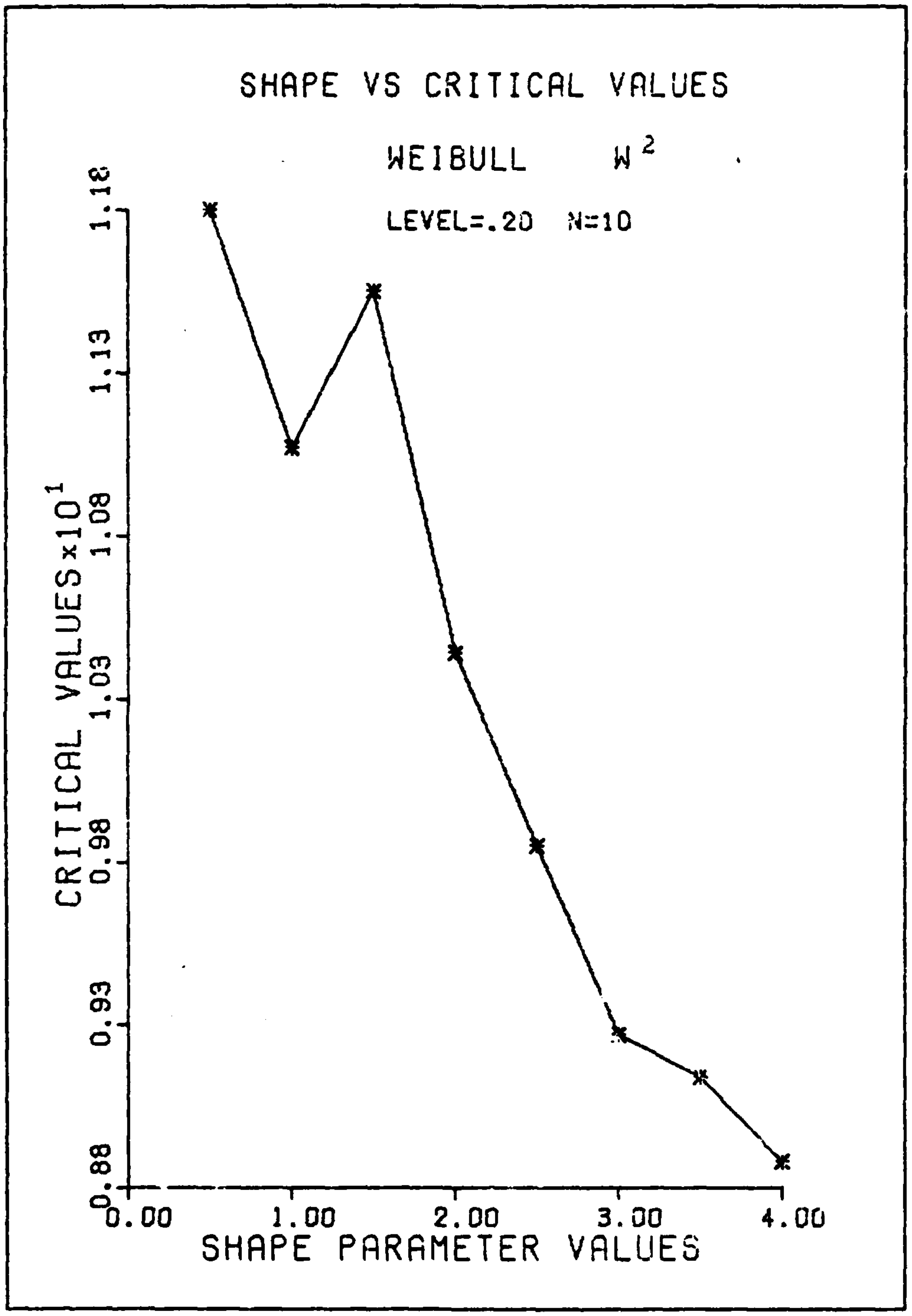

Fig. 10. Shape vs W^2 Critical Values, Level=.20, n=10

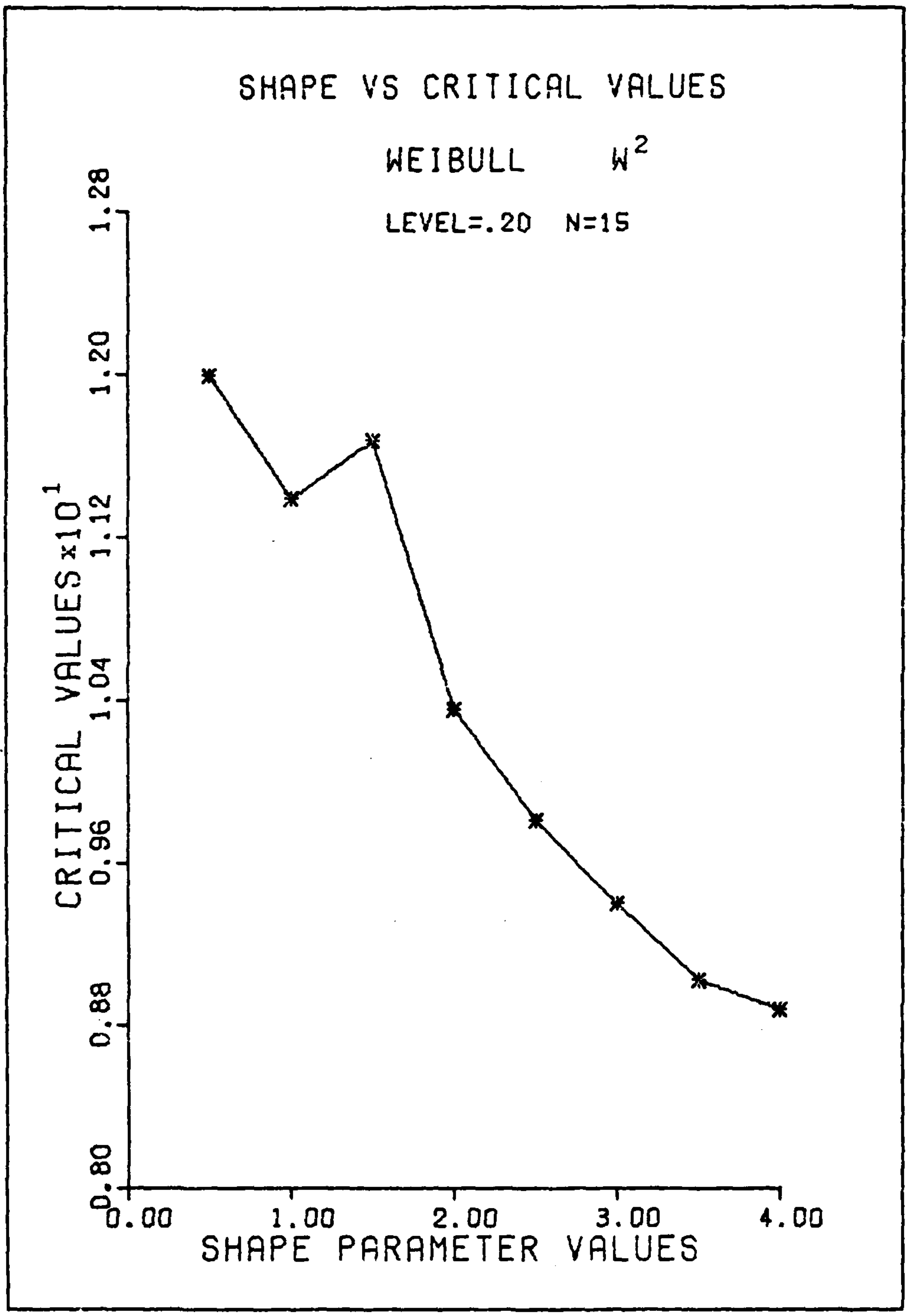

Fig. 11. Shape vs W^2 Critical Values, Level=.20, n=15

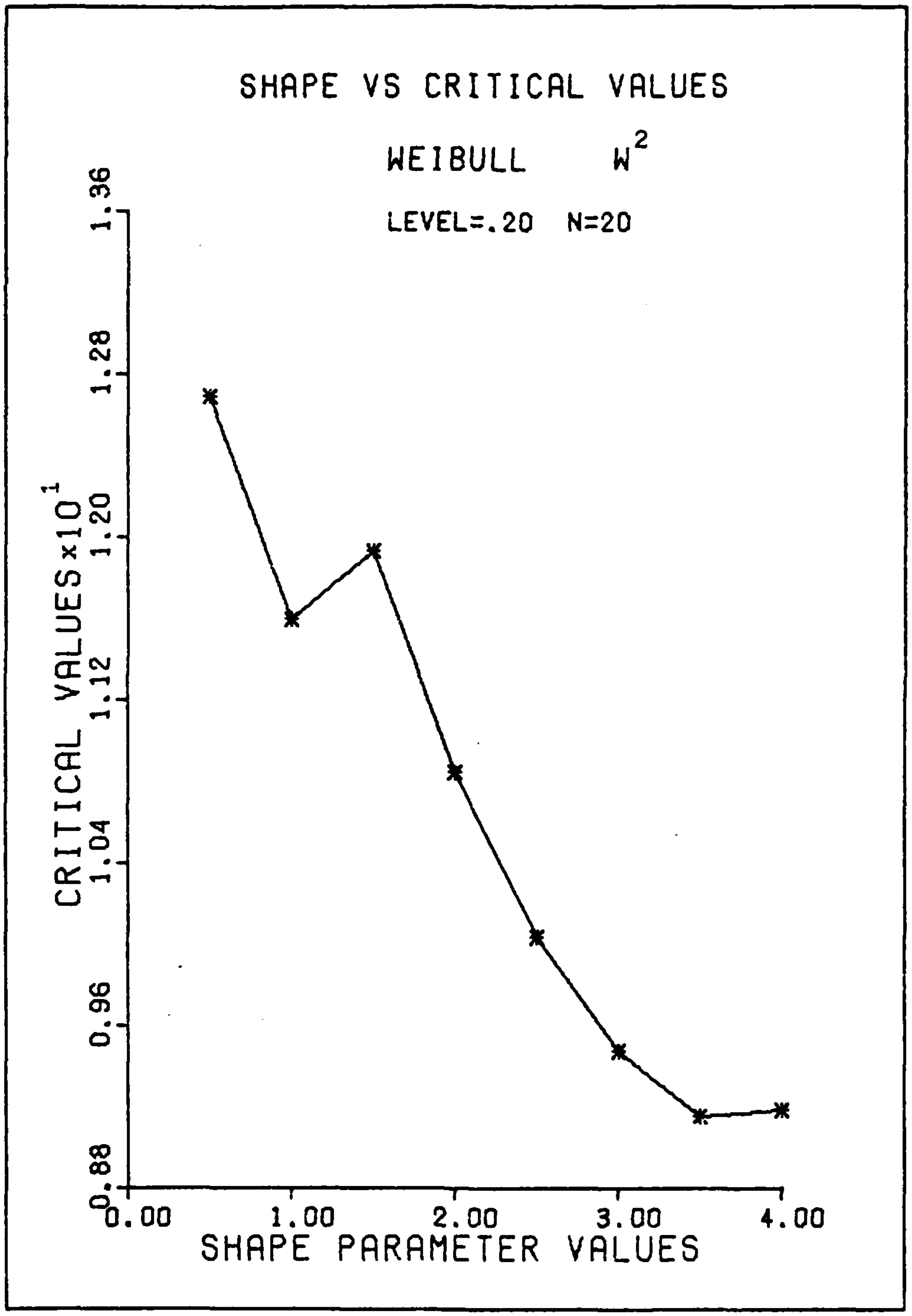

Fig. 12. Shape vs W^2 Critical Values, Level=.20, n=20

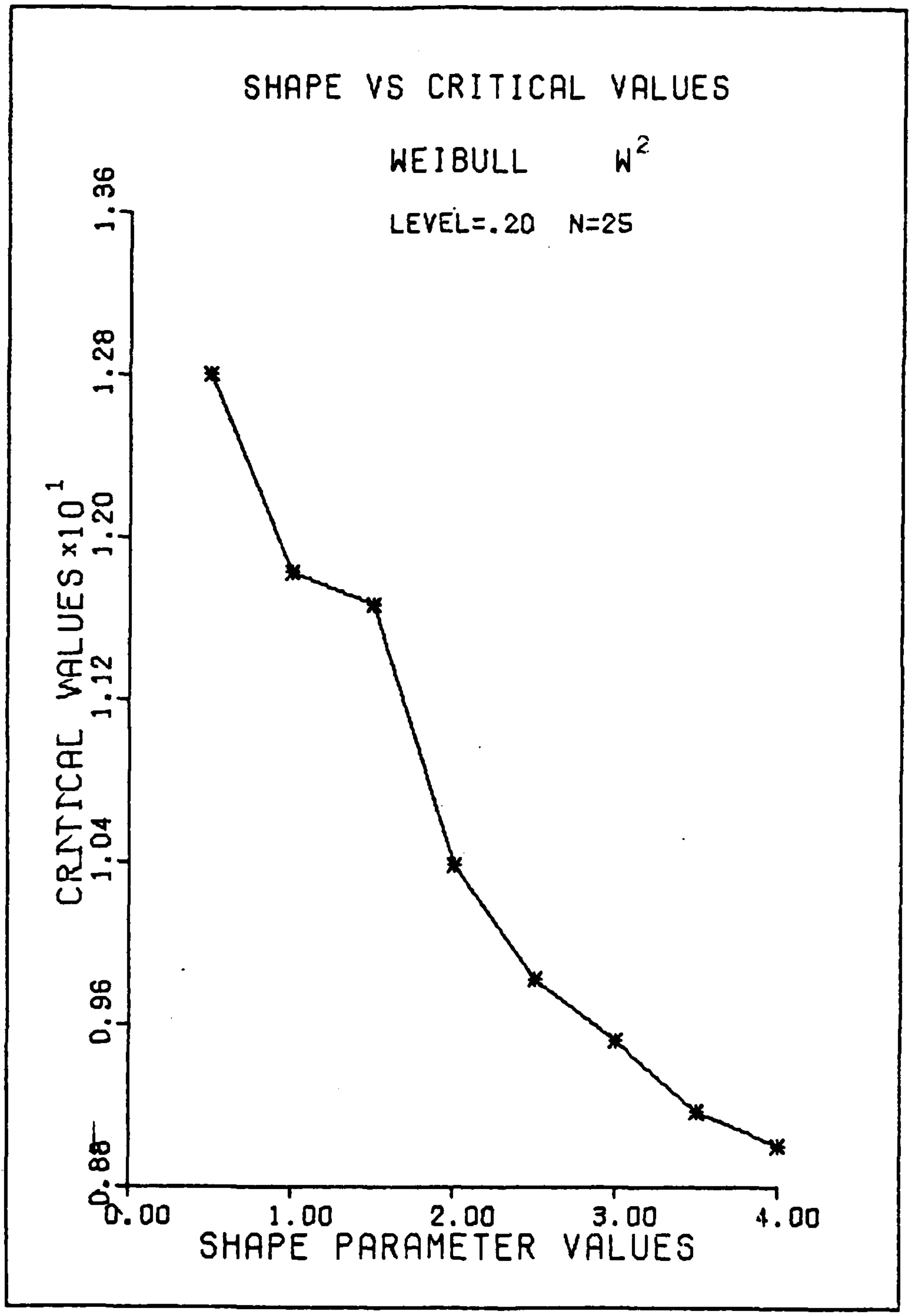

Fig. 13. Shape vs W^2 Critical Values, Level=.20, n=25

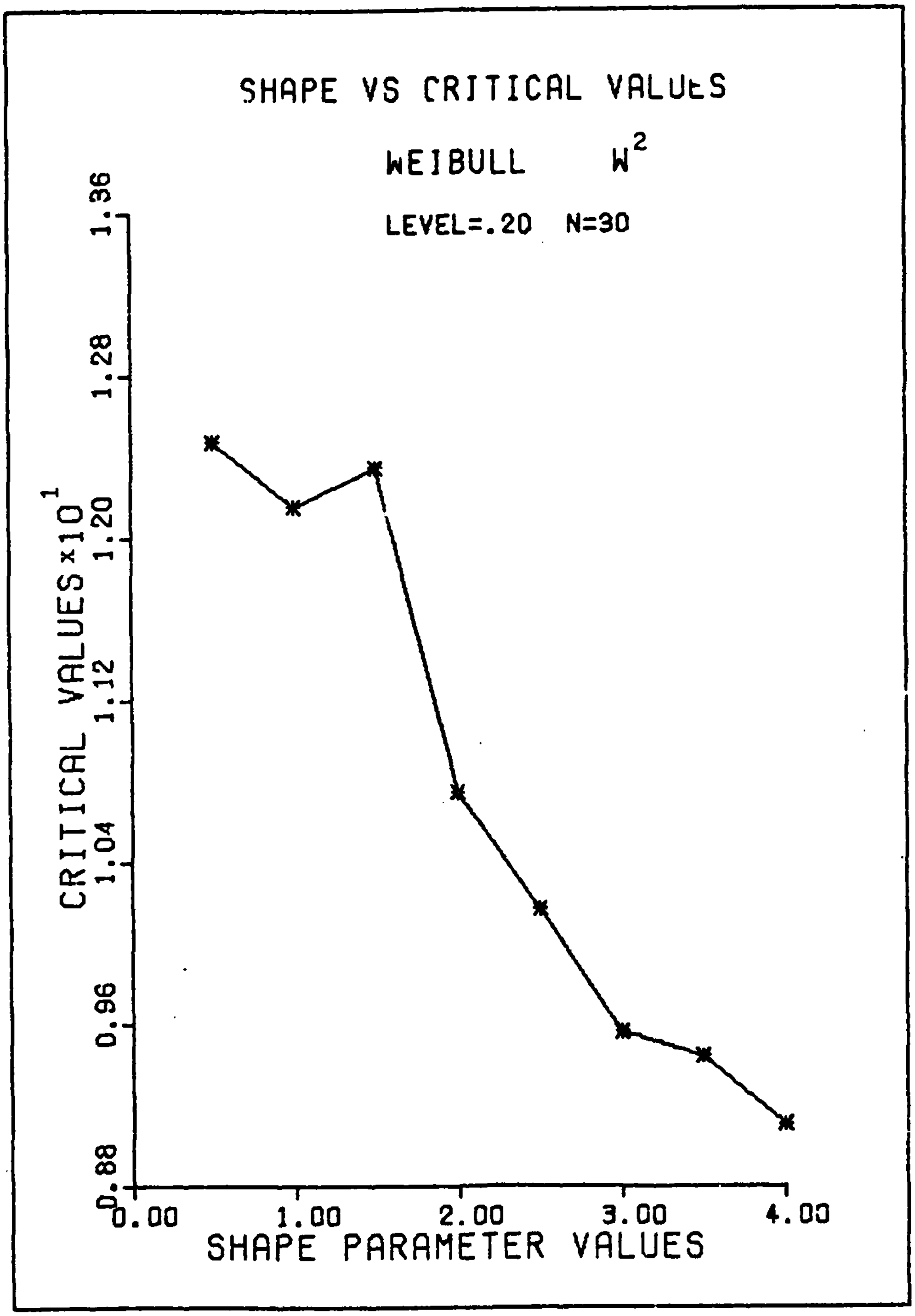

Fig. 14. Shape vs W^2 Critical Values, Level=.20, n=30

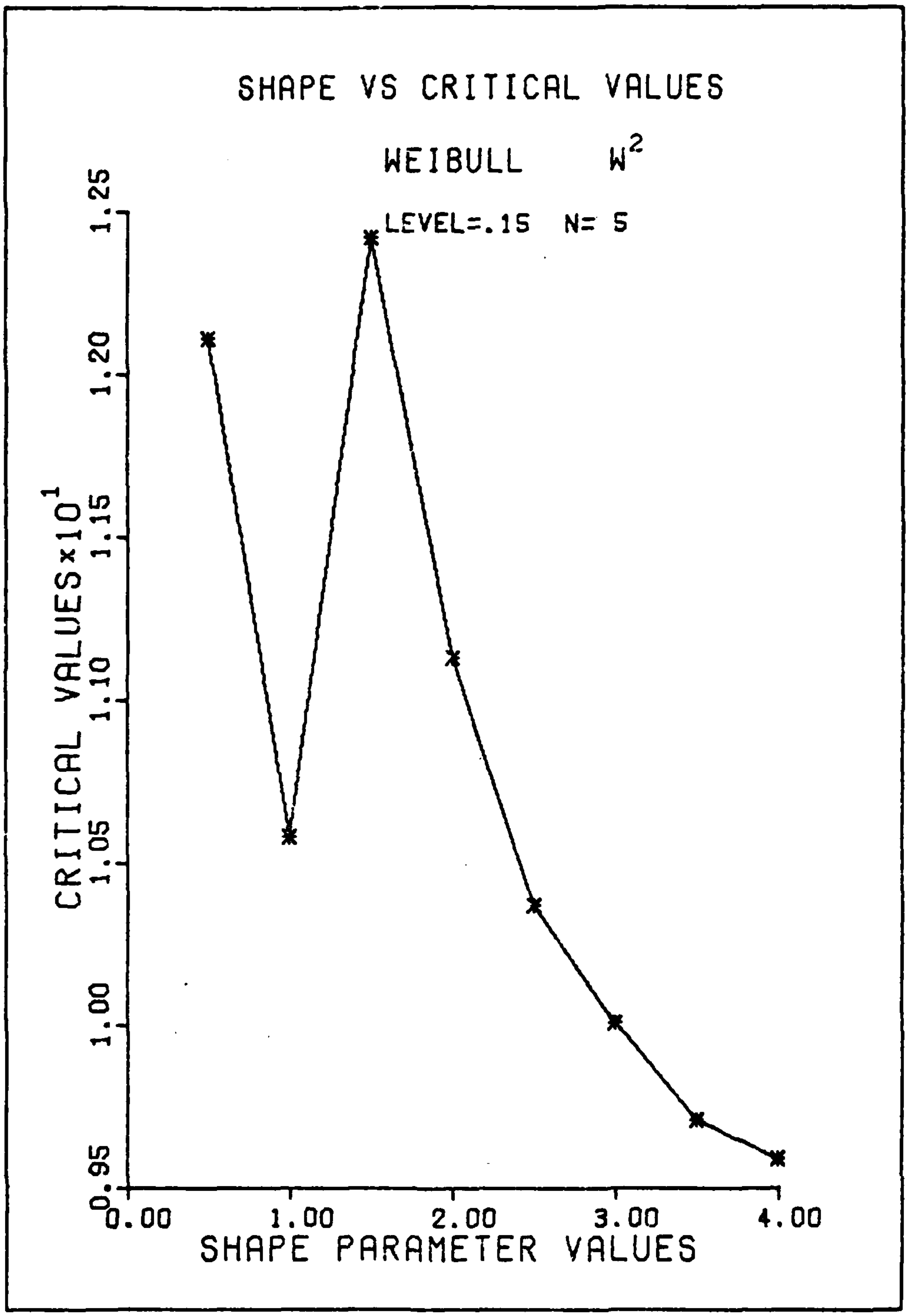

Fig. 15. Shape vs W^2 Critical Values, Level=.15, n=5

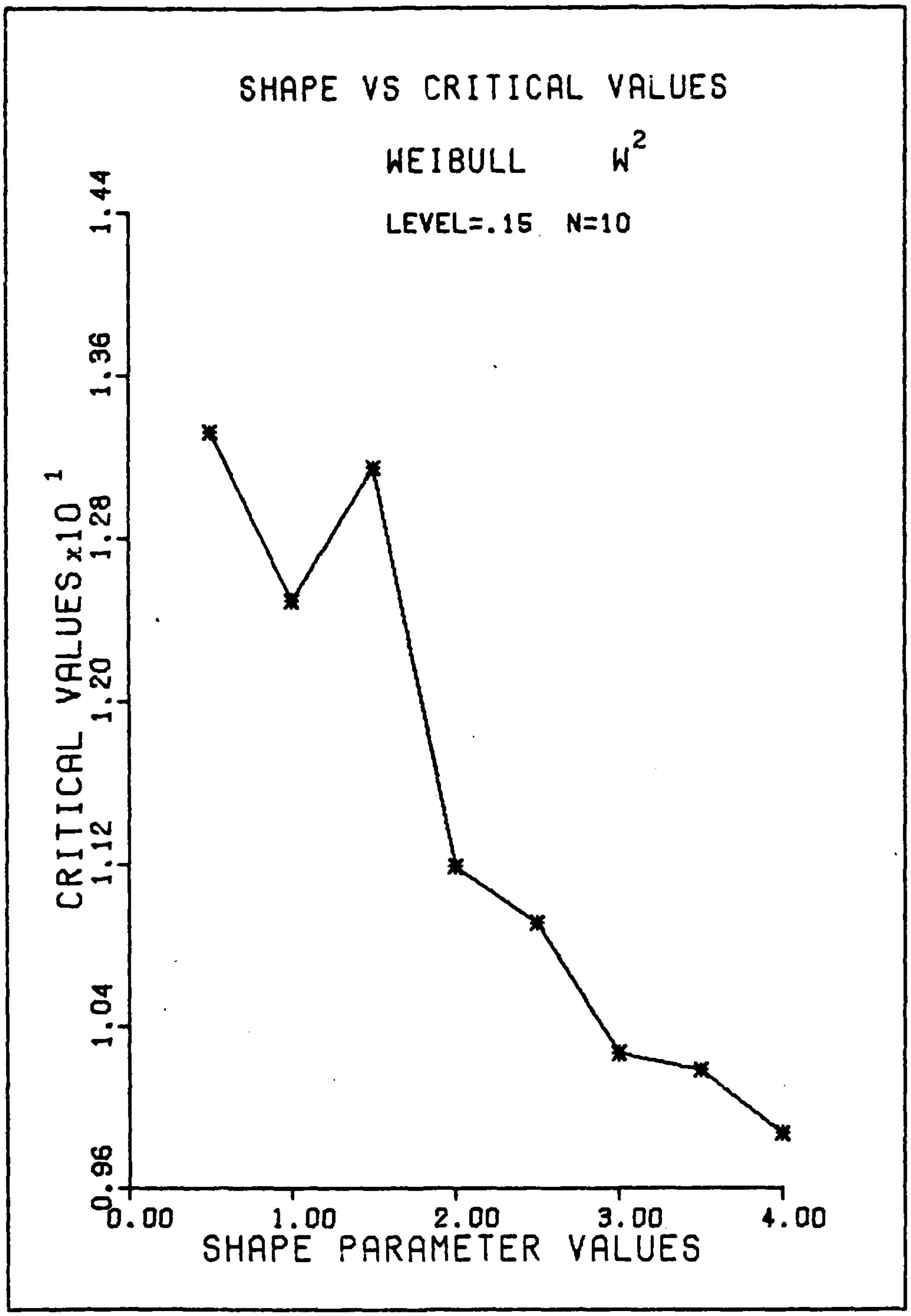

Fig. 16. Shape vs W^2 Critical Values, Level=.15, n=10

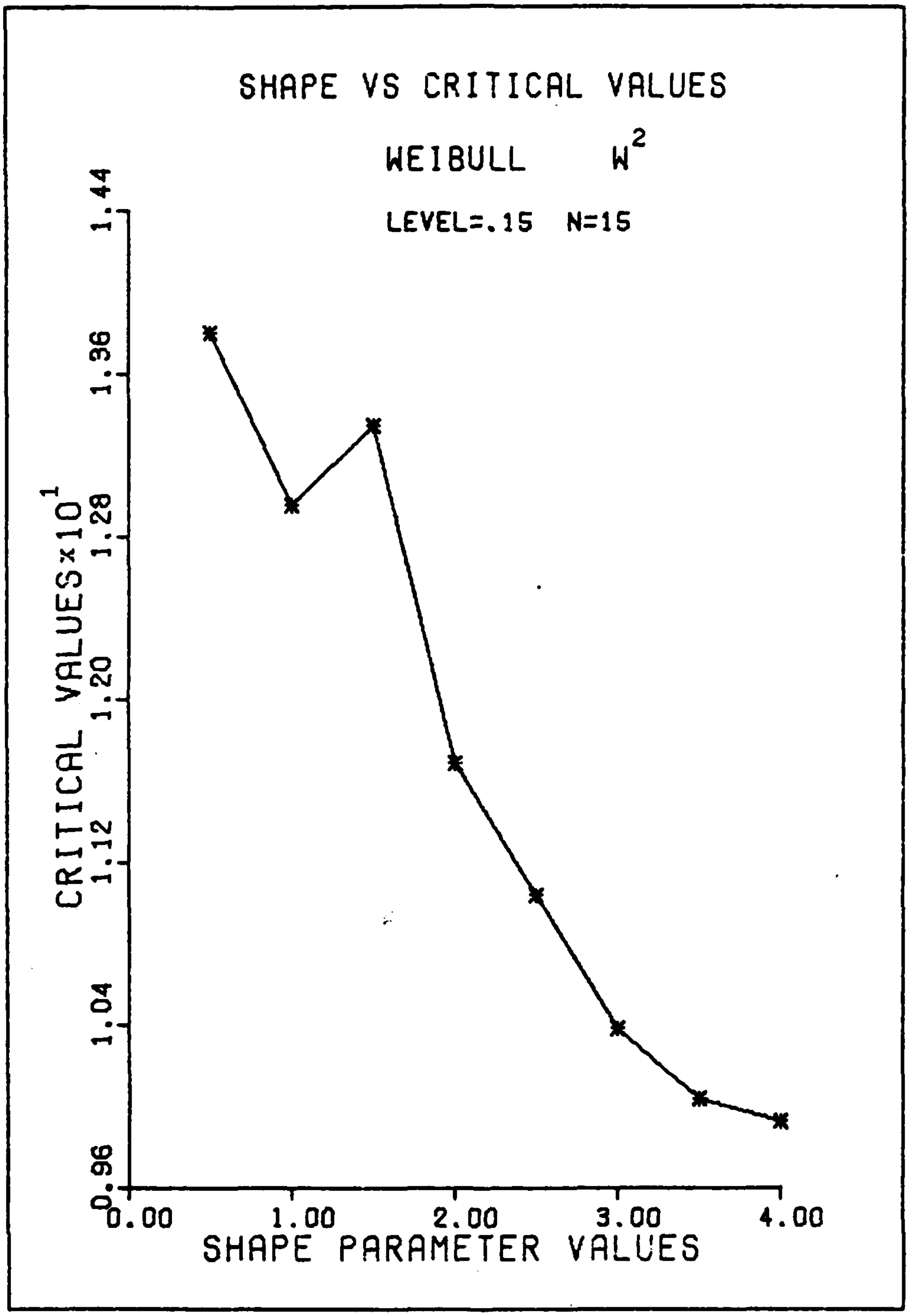

Fig. 17. Shape vs W^2 Critical Values, Level=.15, n=15

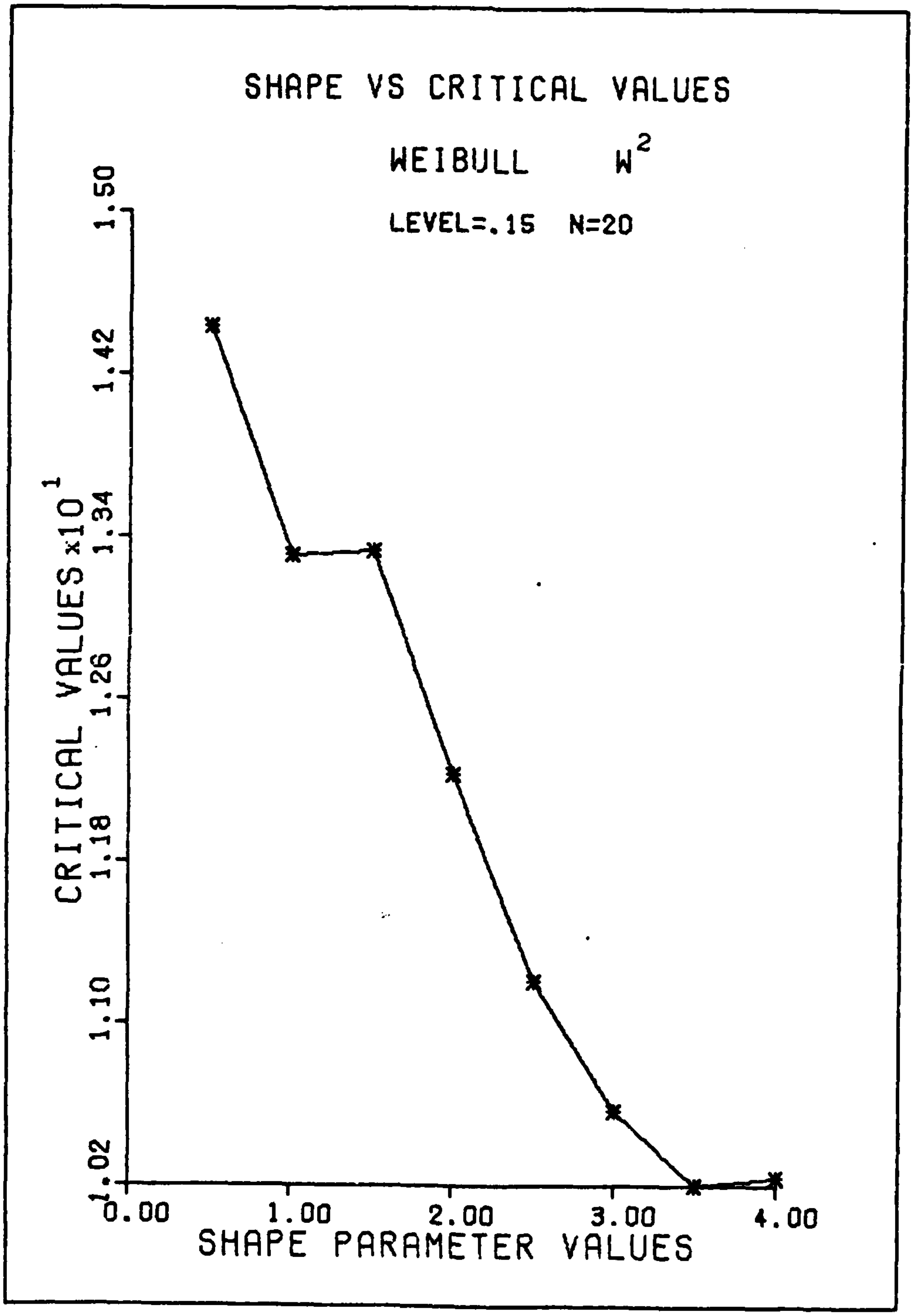

Fig. 18. Shape vs W^2 Critical Values, Level=.15, n=20

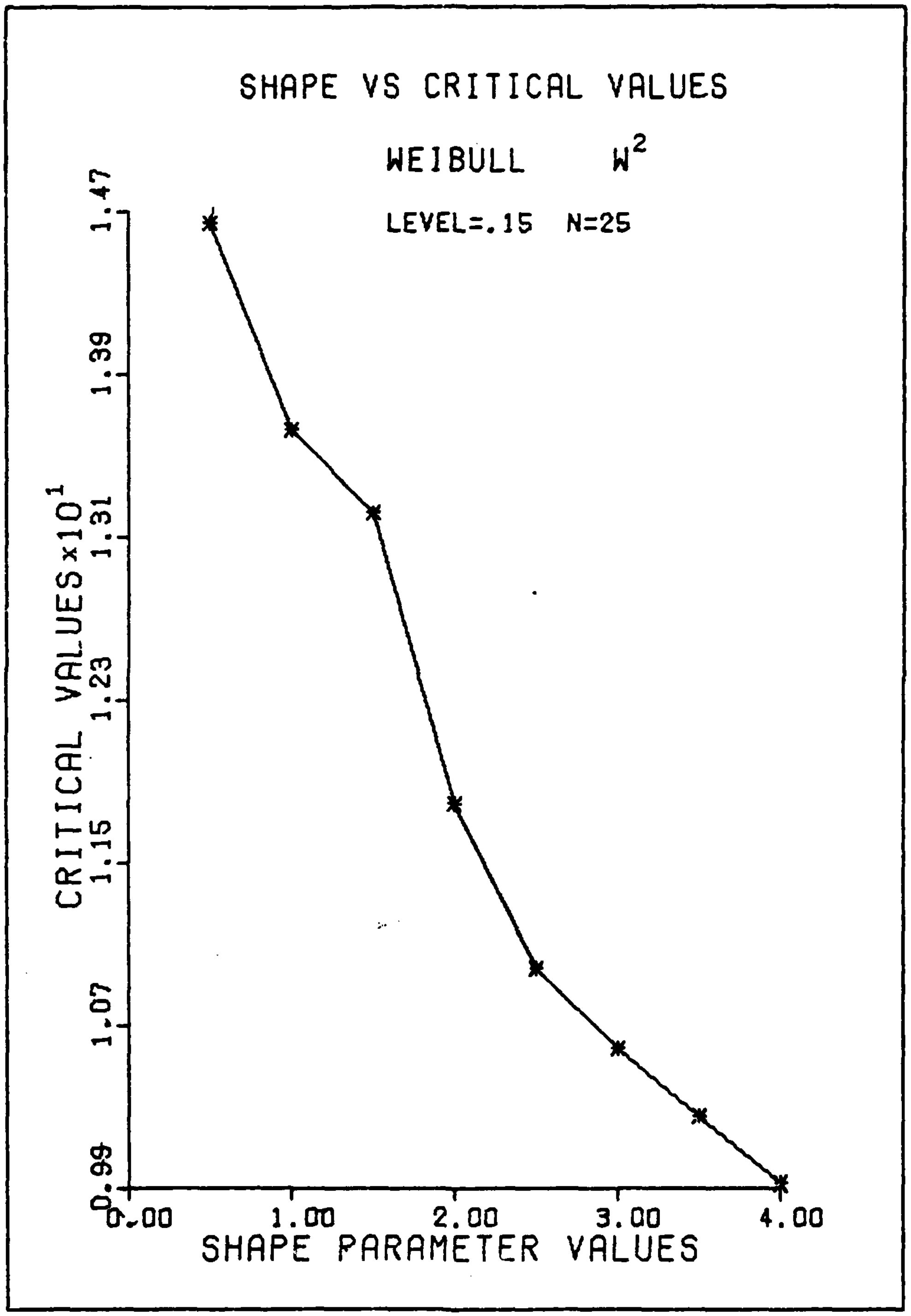

Fig. 19. Shape vs W^2 Critical Values, Level=.15, n=25

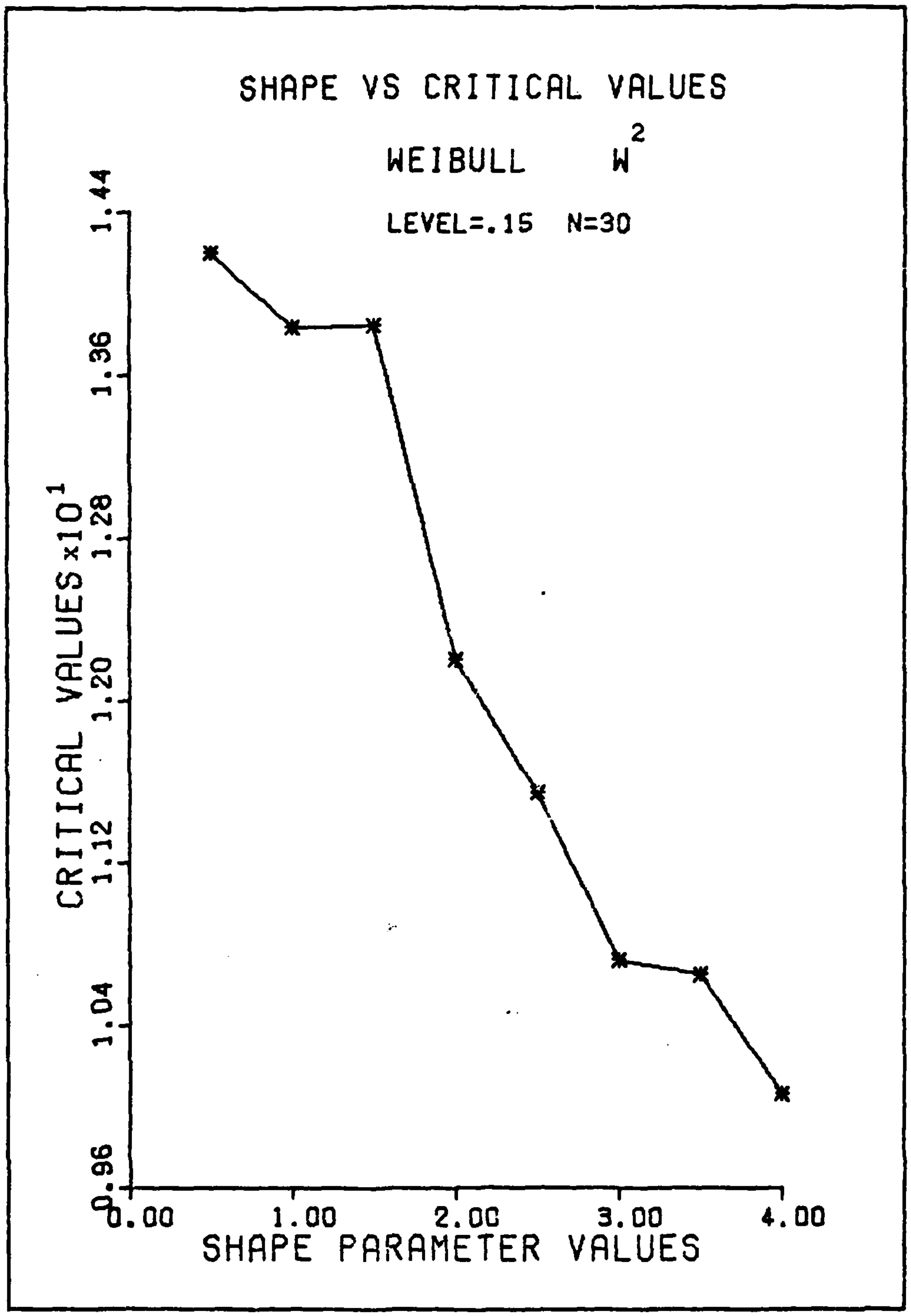

Fig. 20. Shape vs W^2 Critical Values, Level=.15, n=30

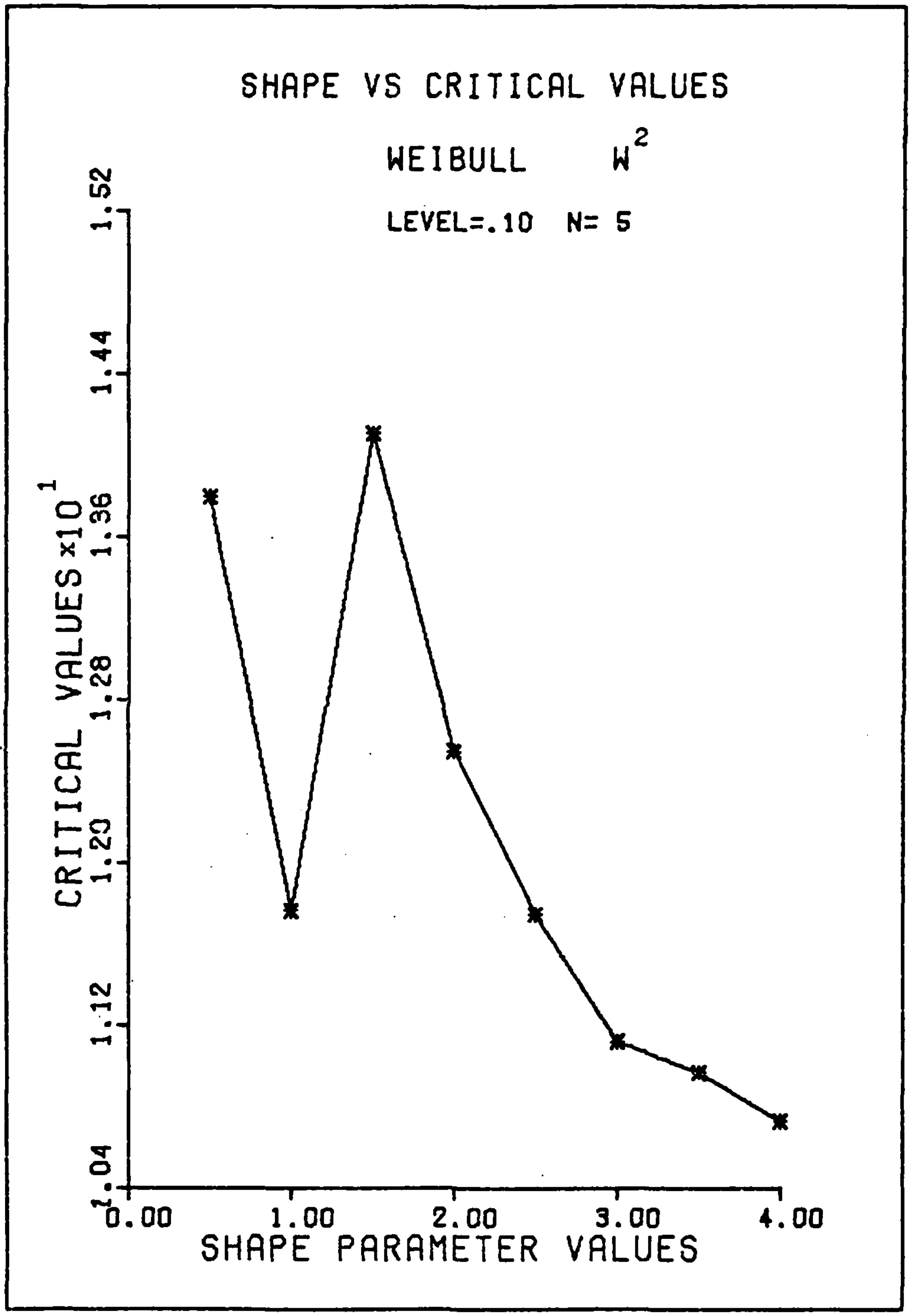

Fig. 21. Shape vs W^2 Critical Values, Level=.10, n=5

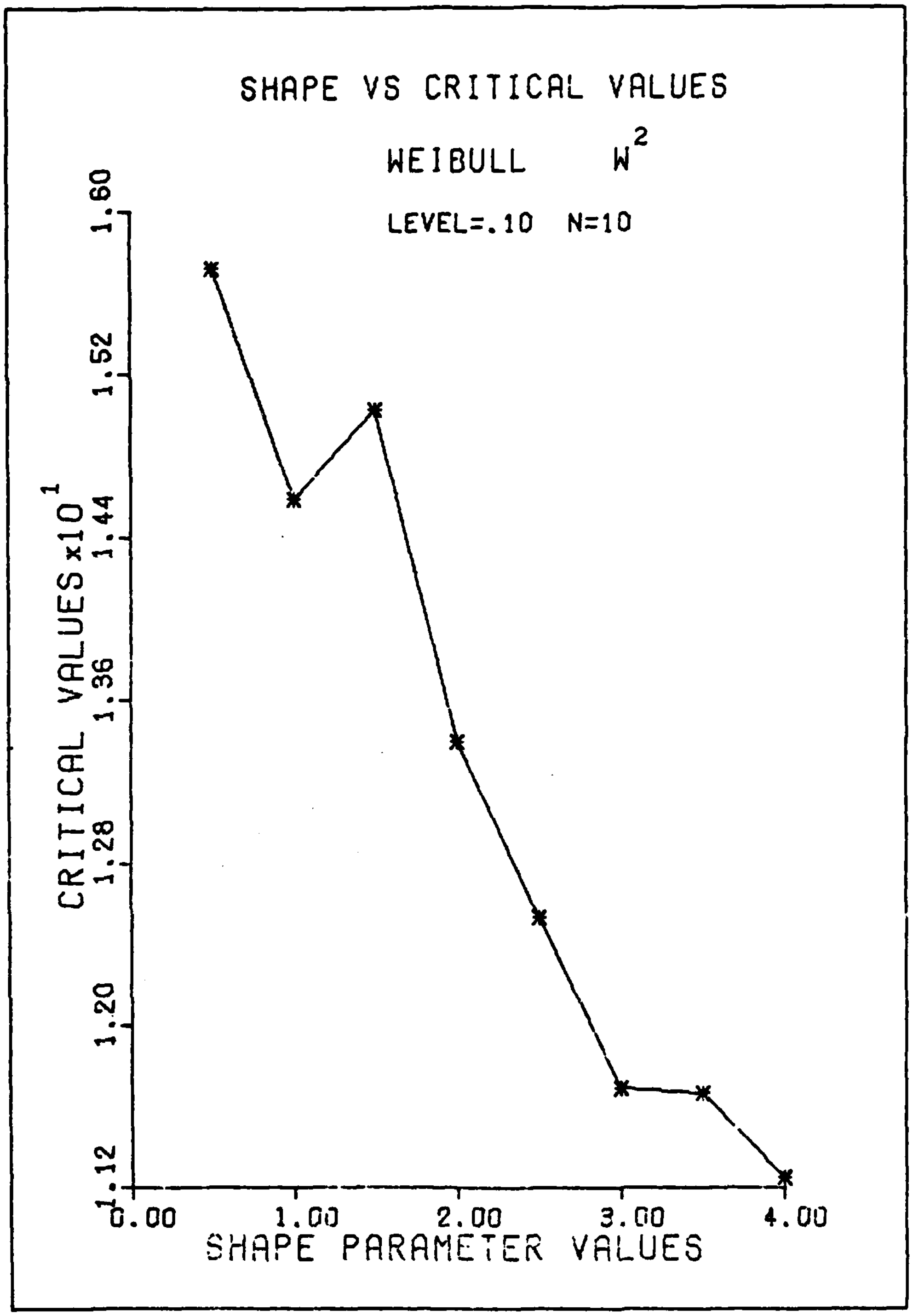

Fig. 22. Shape vs W^2 Critical Values, Level=.10, n=10

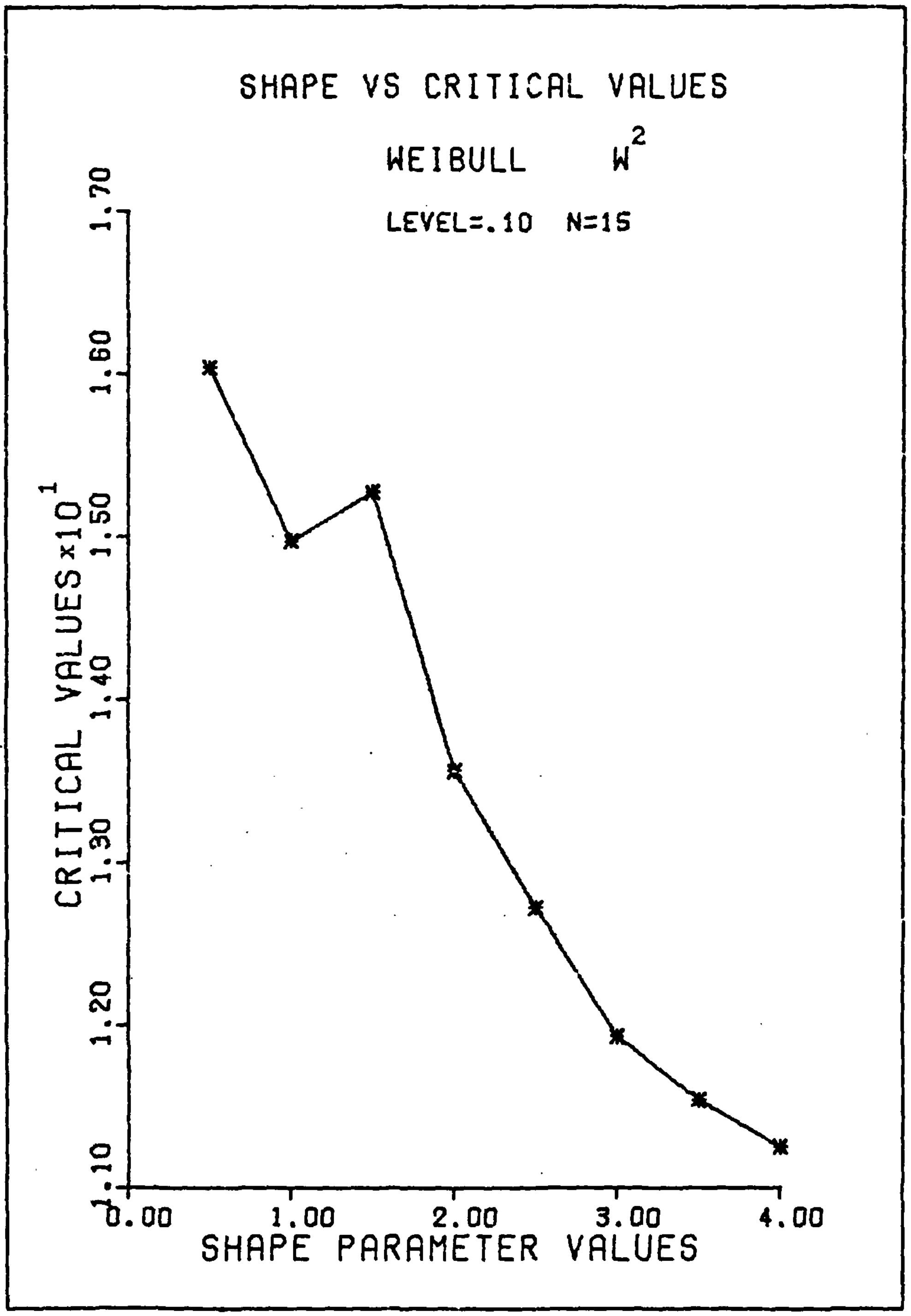

Fig. 23. Shape vs W^2 Critical Values, Level=.10, n=15

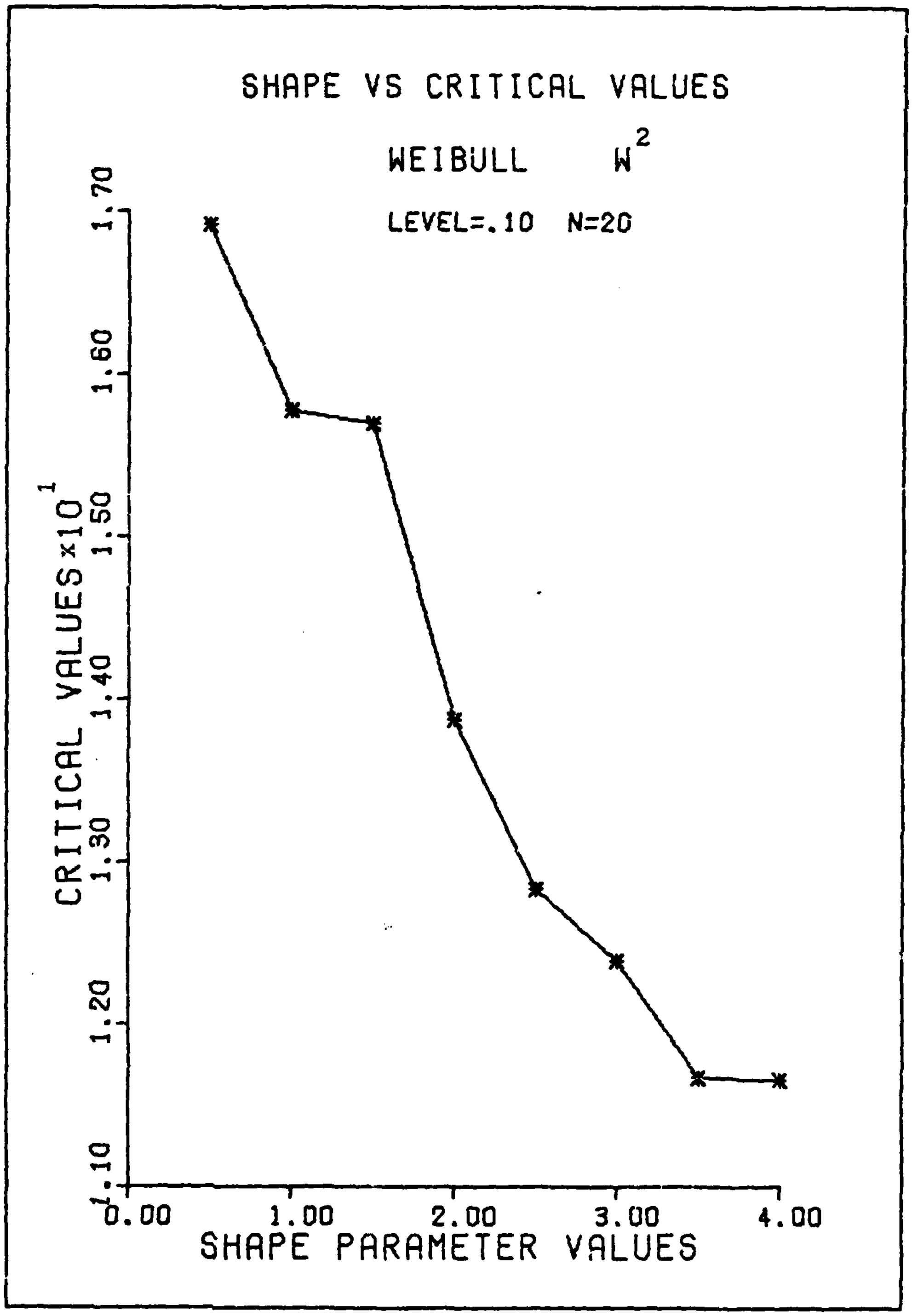

Fig. 24. Shape vs W^2 Critical Values, Level=.10, n=20

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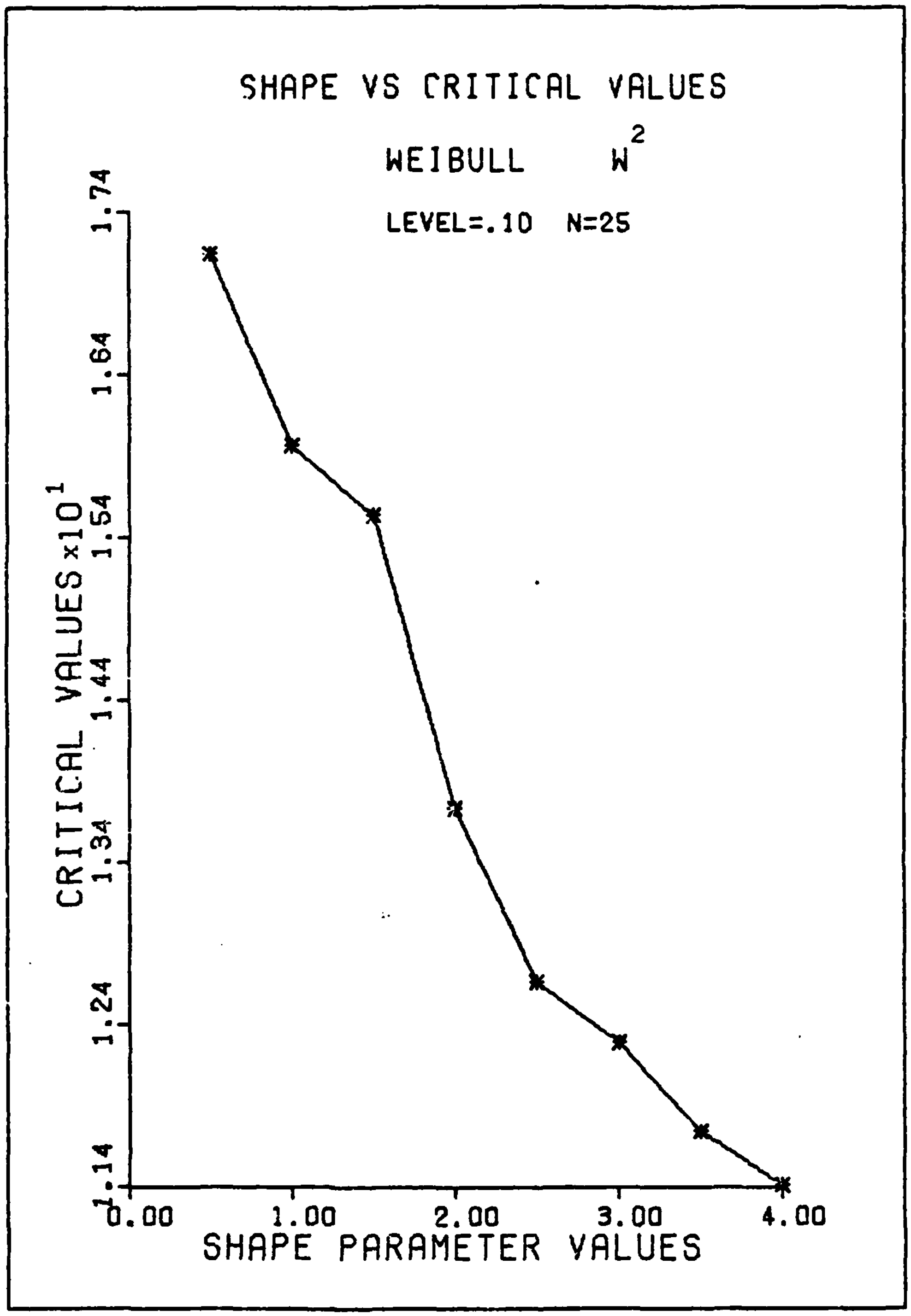

Fig. 25. Shape vs W^2 Critical Values, Level=.10, n=25

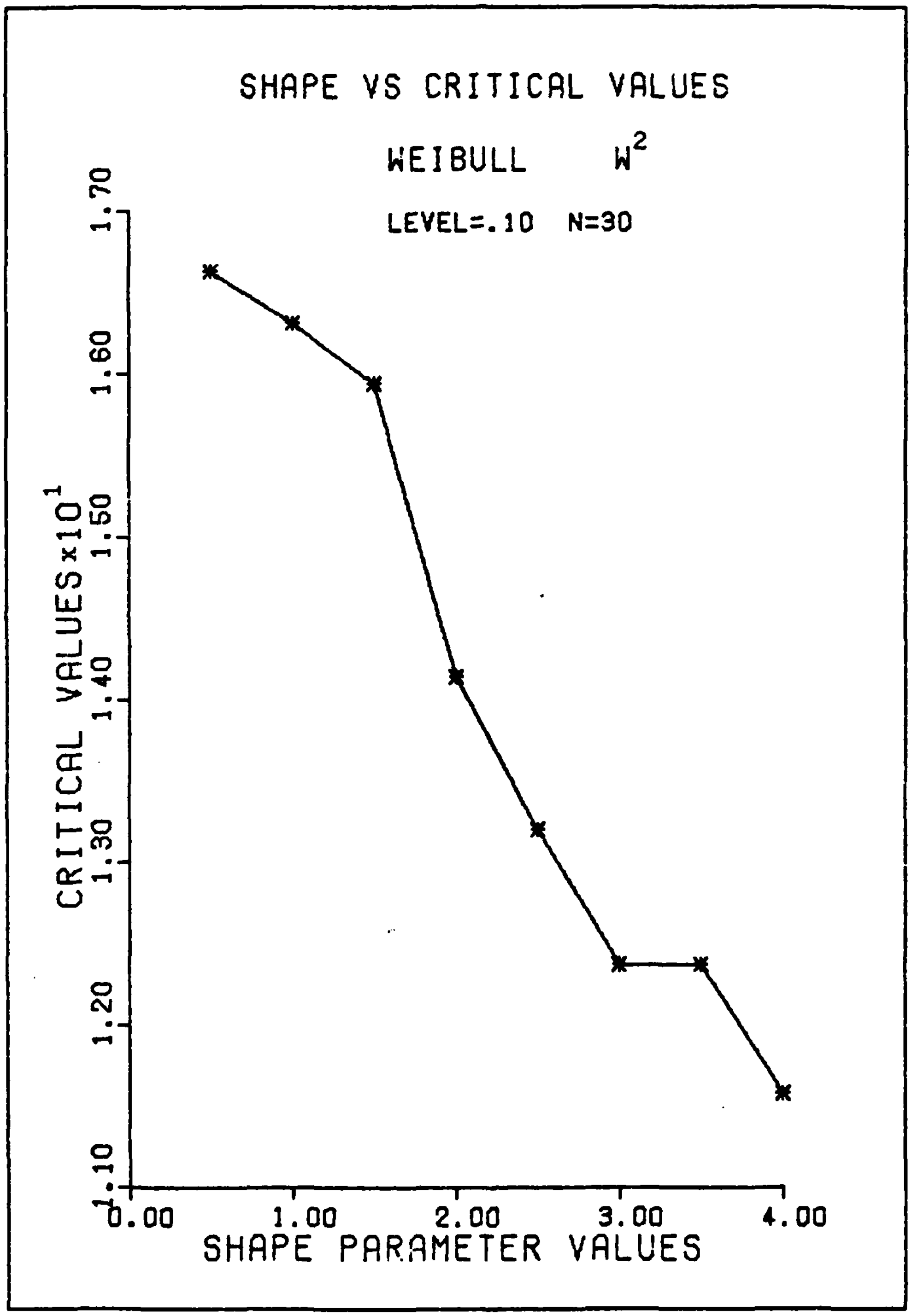

Fig. 26. Shape vs W^2 Critical Values, Level=.10, n=30

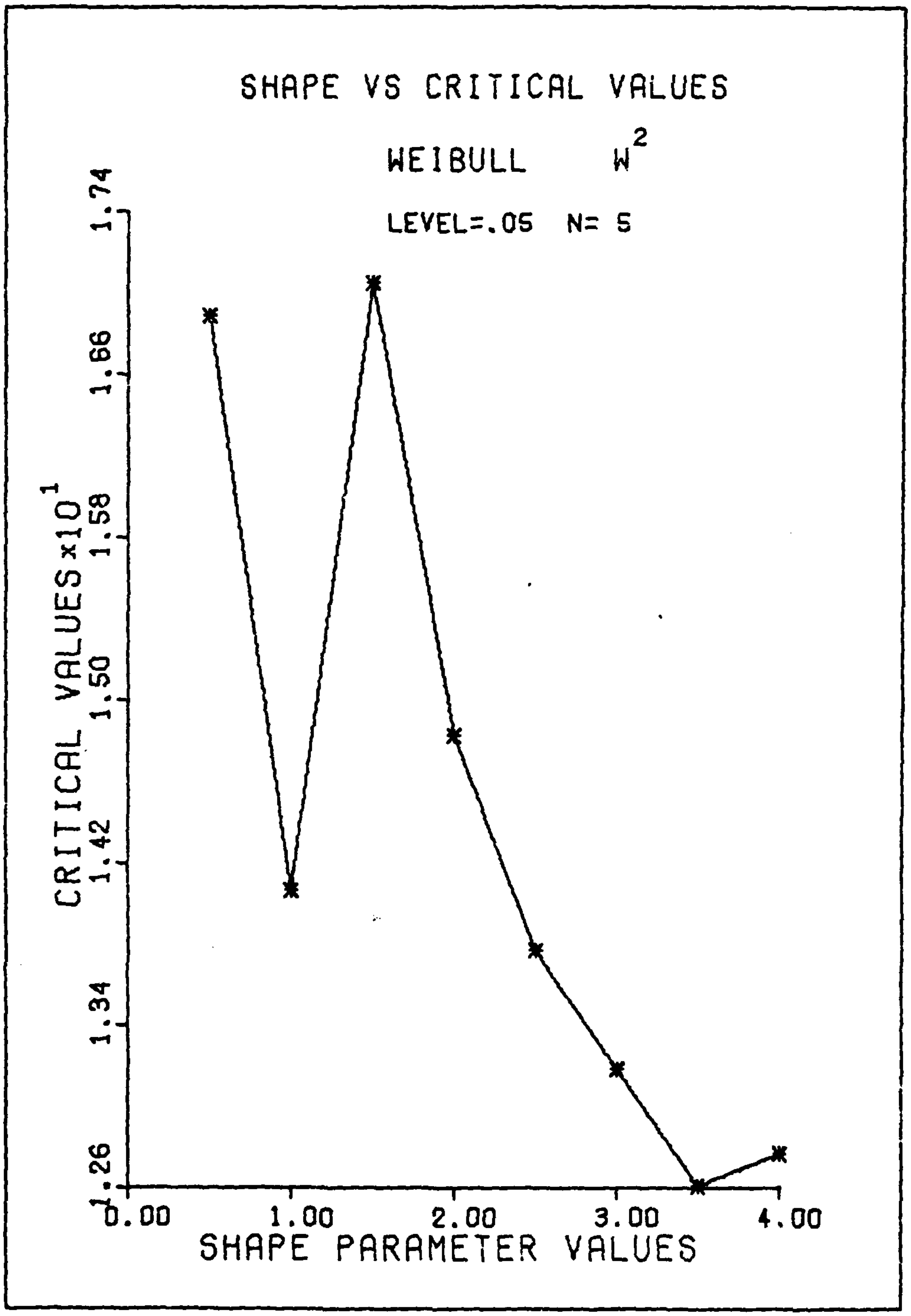

Fig. 27. Shape vs W^2 Critical Values, Level=.05, n=5

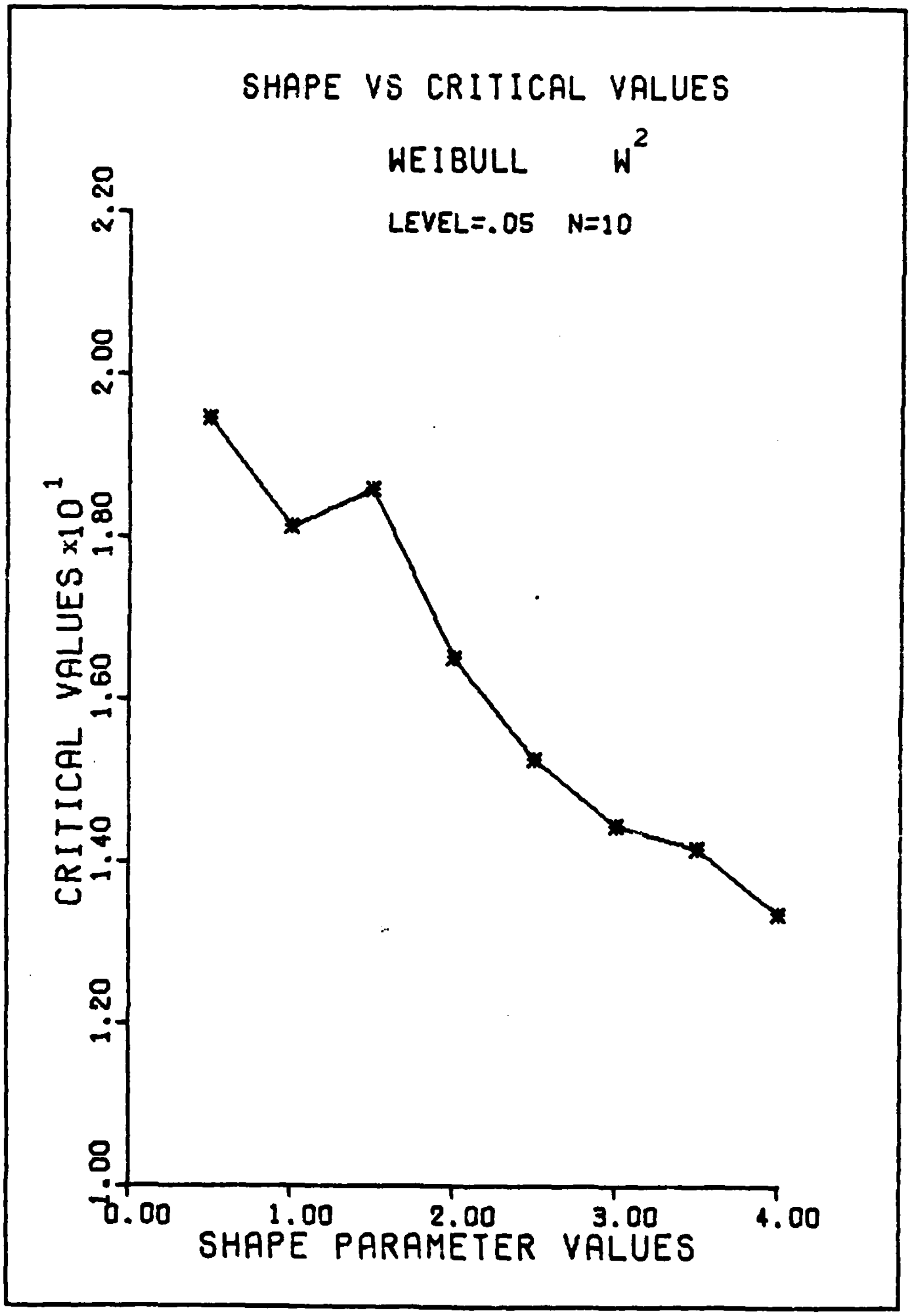

Fig. 28. Shape vs W^2 Critical Values, Level=.05, n=10

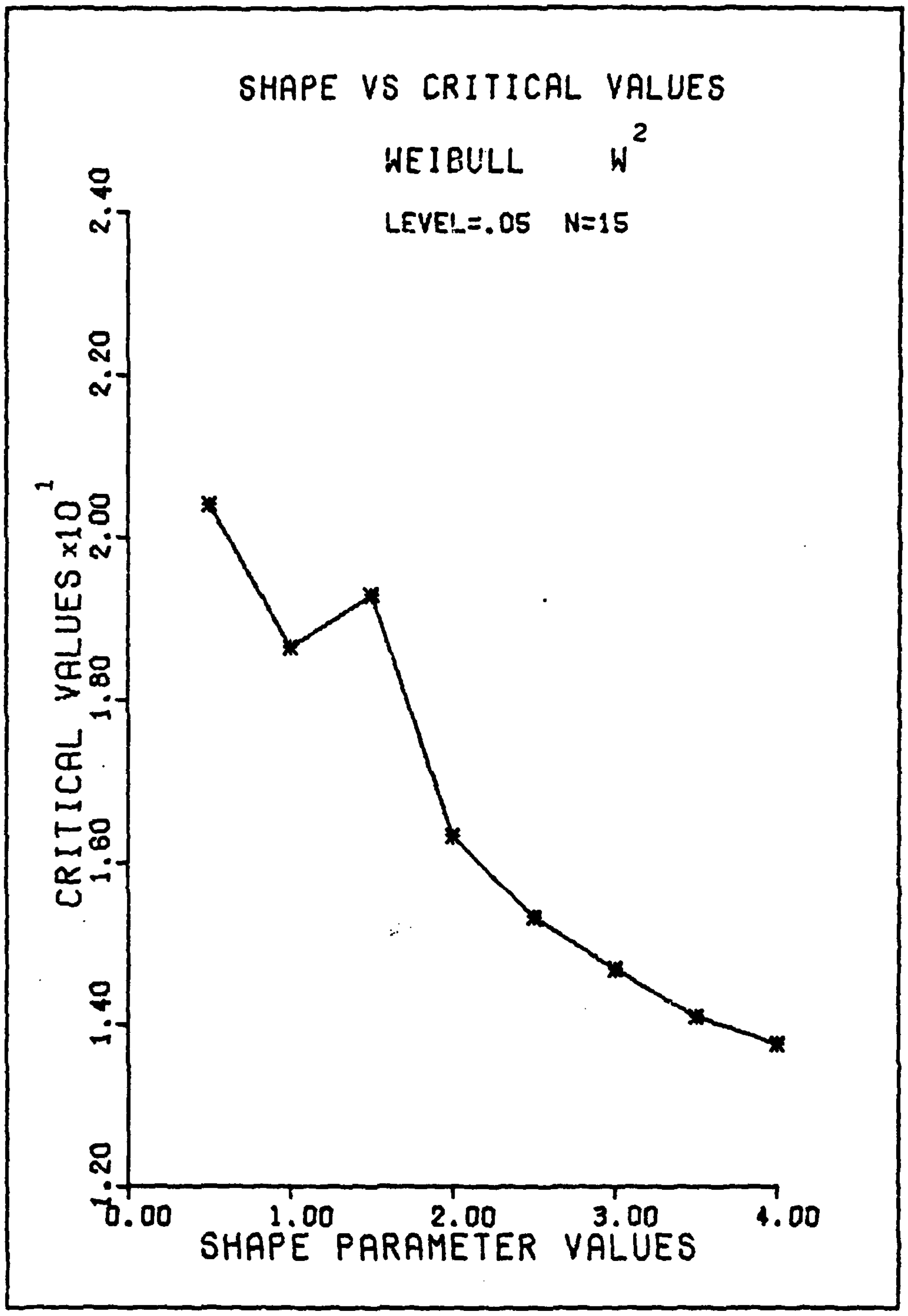

Fig. 29. Shape vs W^2 Critical Values, Level=.05, n=15

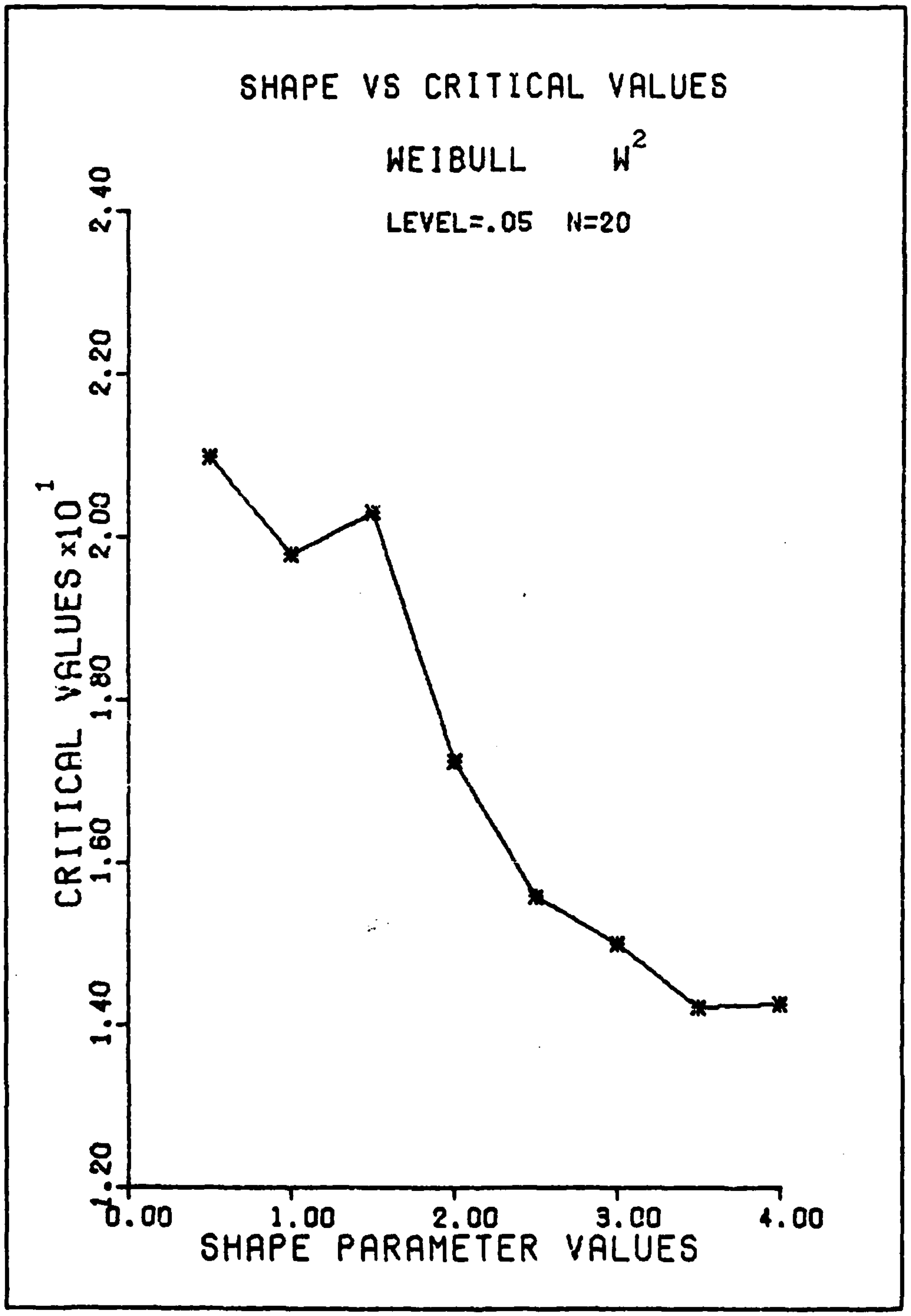

Fig. 30. Shape vs W^2 Critical Values, Level=.05, n=20

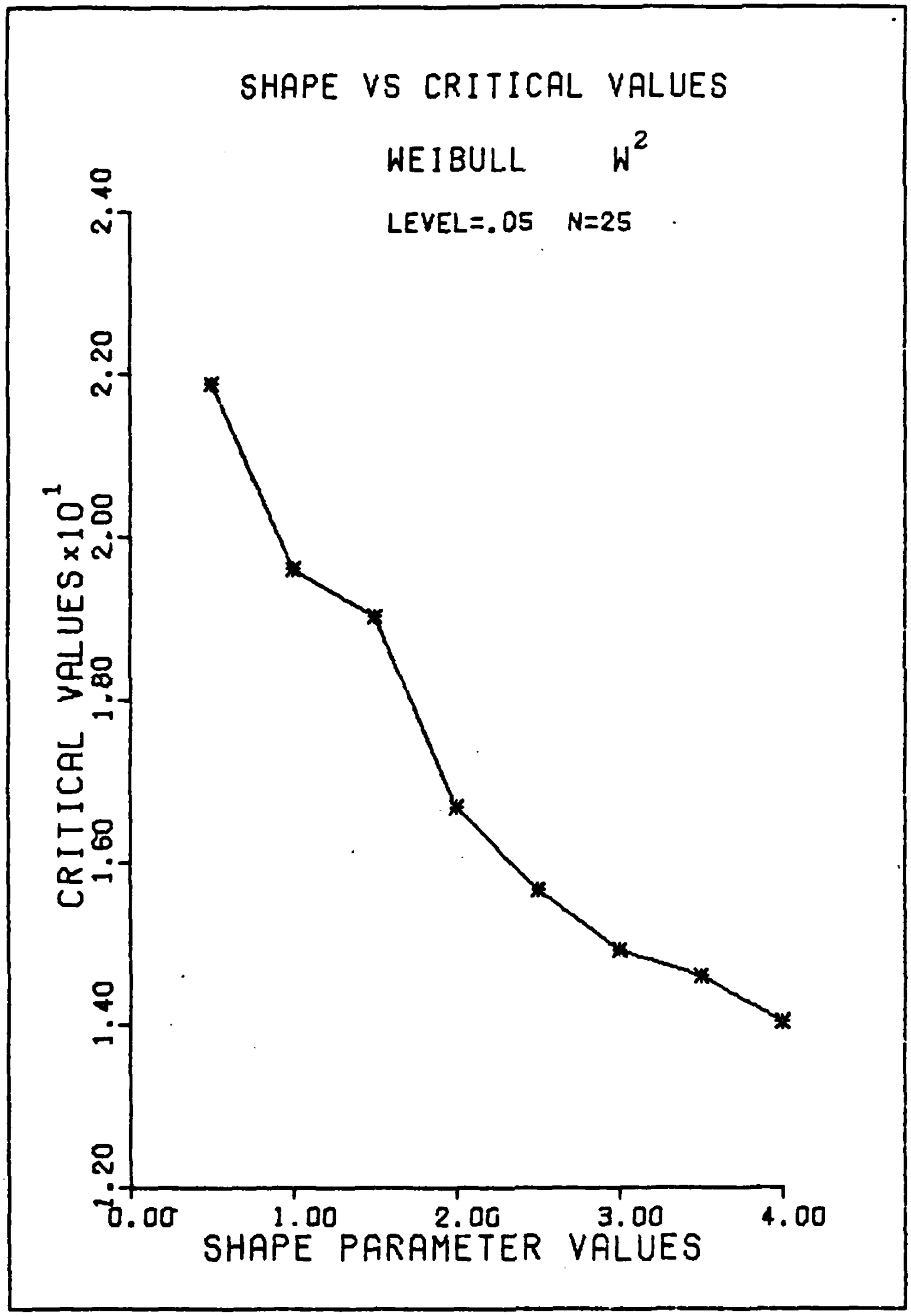

Fig. 31. Shape vs W^2 Critical Values, Level=.05, n=25

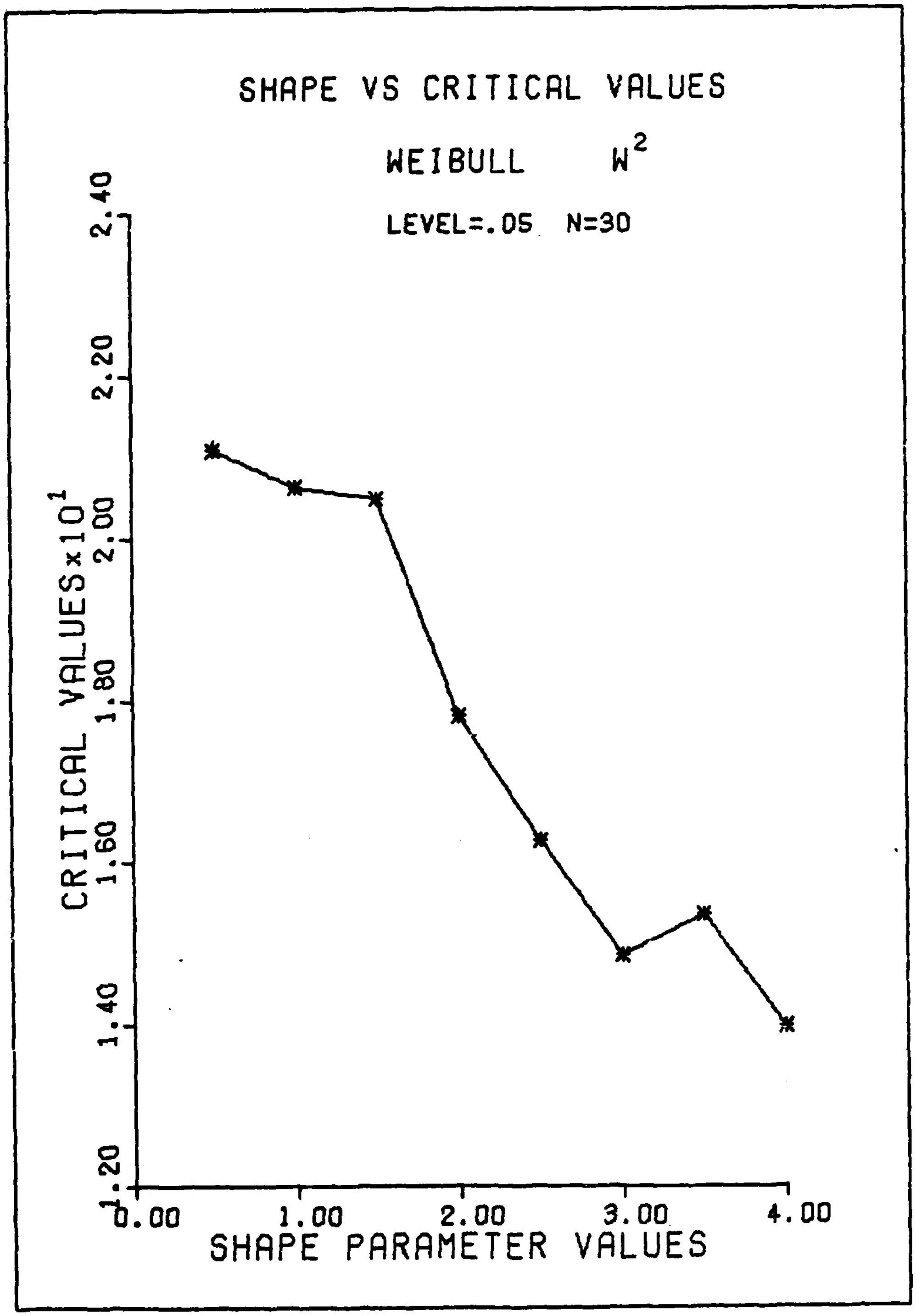

Fig. 32. Shape vs W^2 Critical Values, Level=.05, n=30

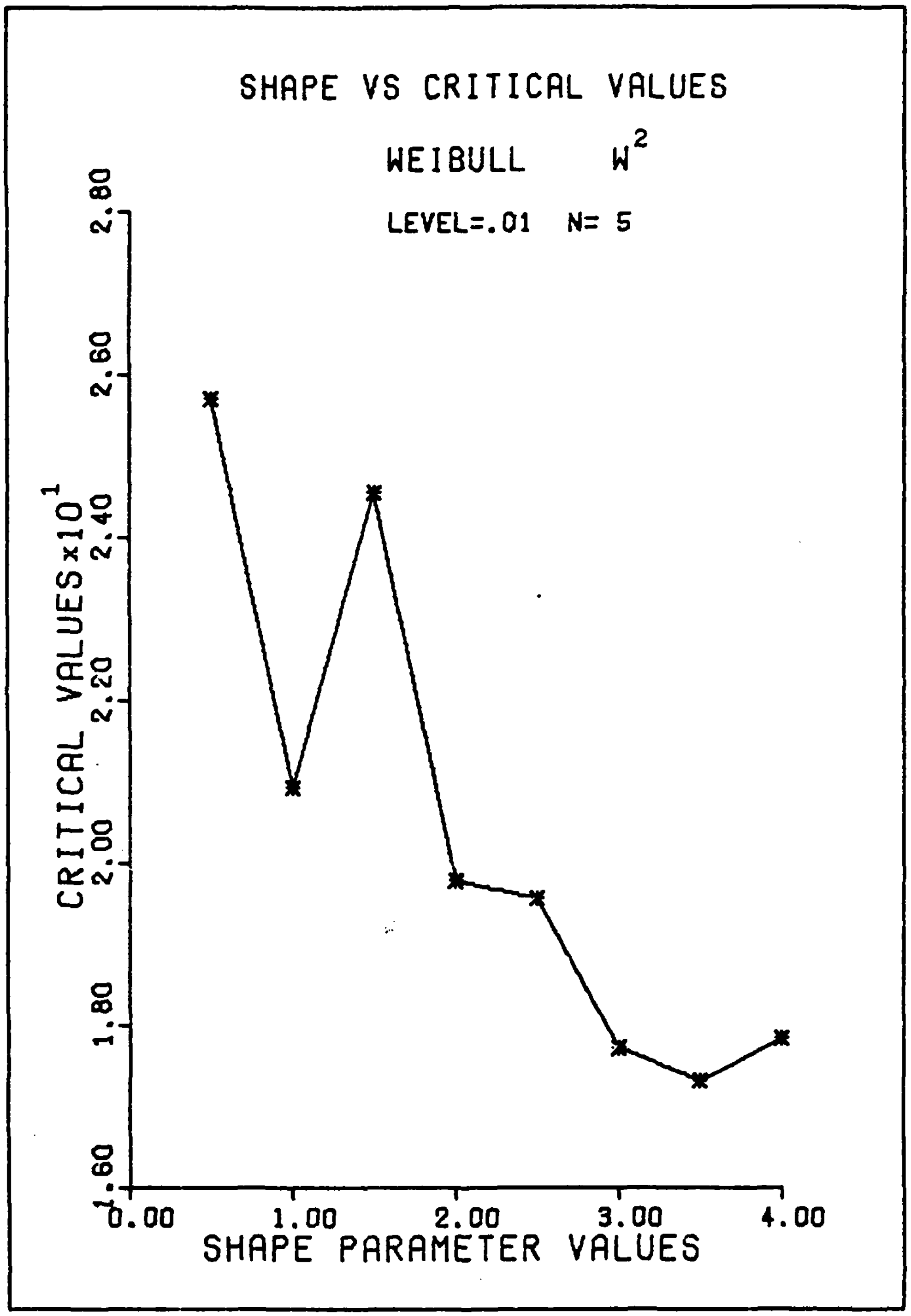

Fig. 33. Shape vs W^2 Critical Values, Level=.01, n=5

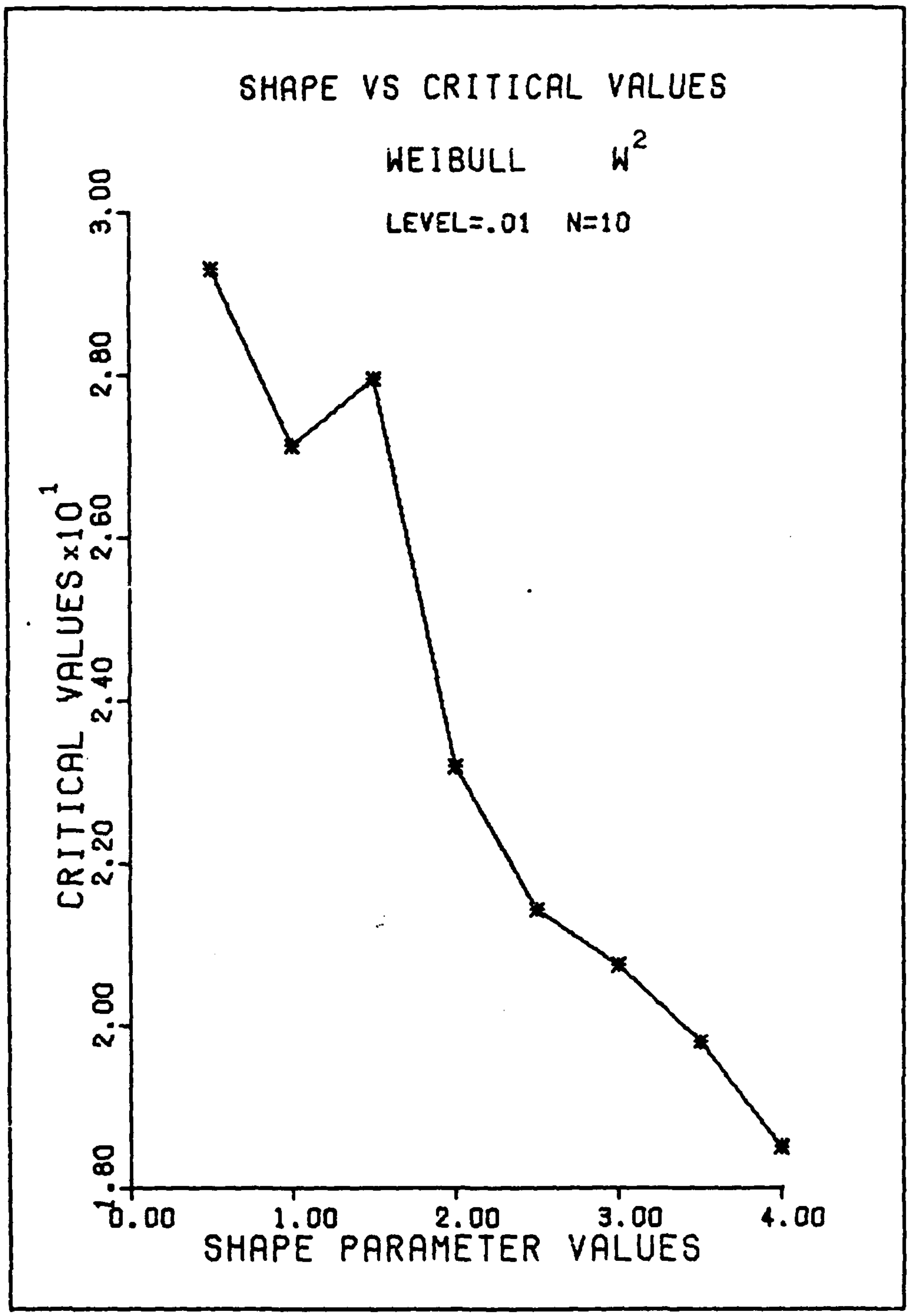

Fig. 34. Shape vs W^2 Critical Values, Level=.01, n=10

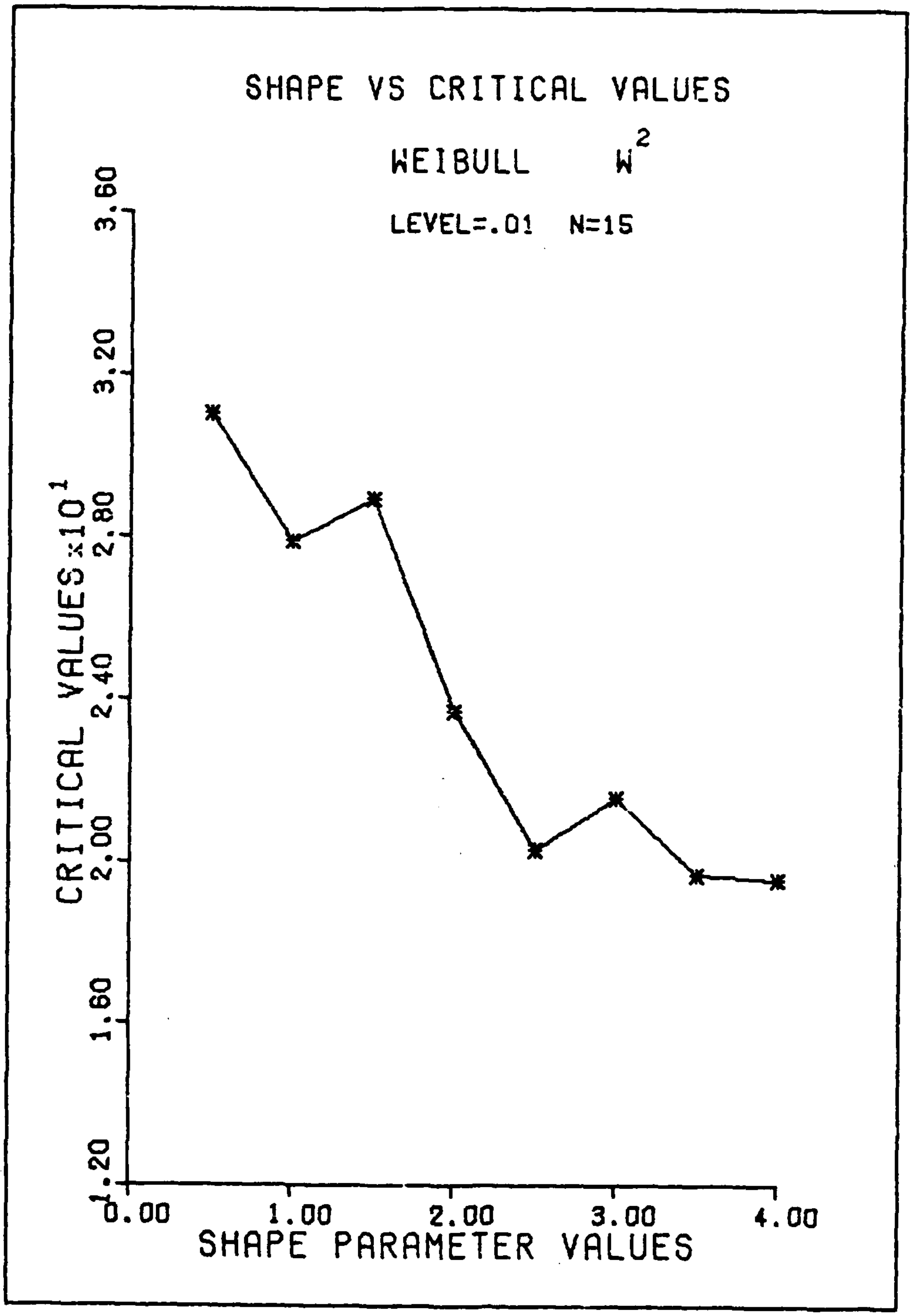

Fig. 35. Shape vs W^2 Critical Values, Level=.01, n=15

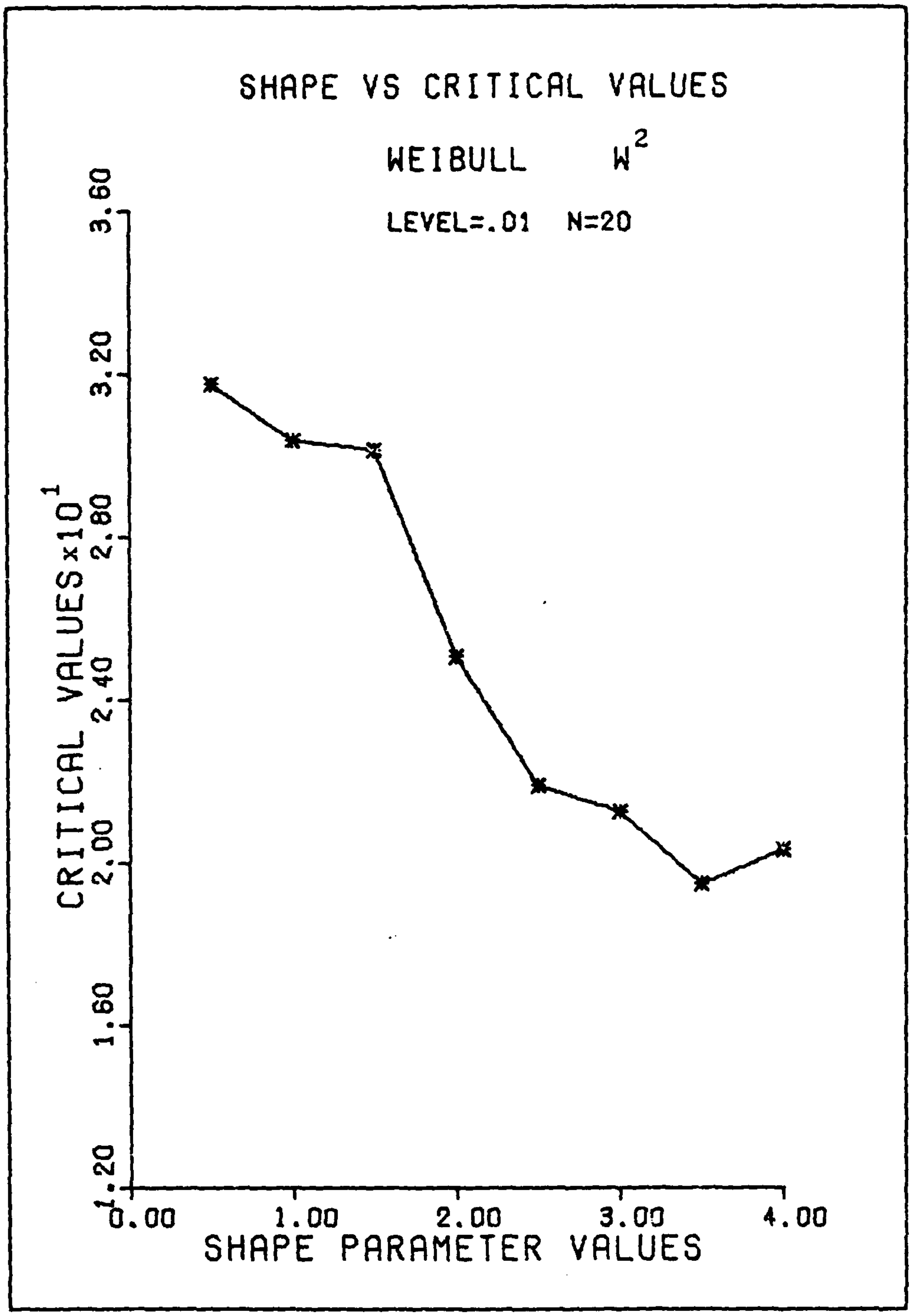

Fig. 36. Shape vs W^2 Critical Values, Level=.01, n=20

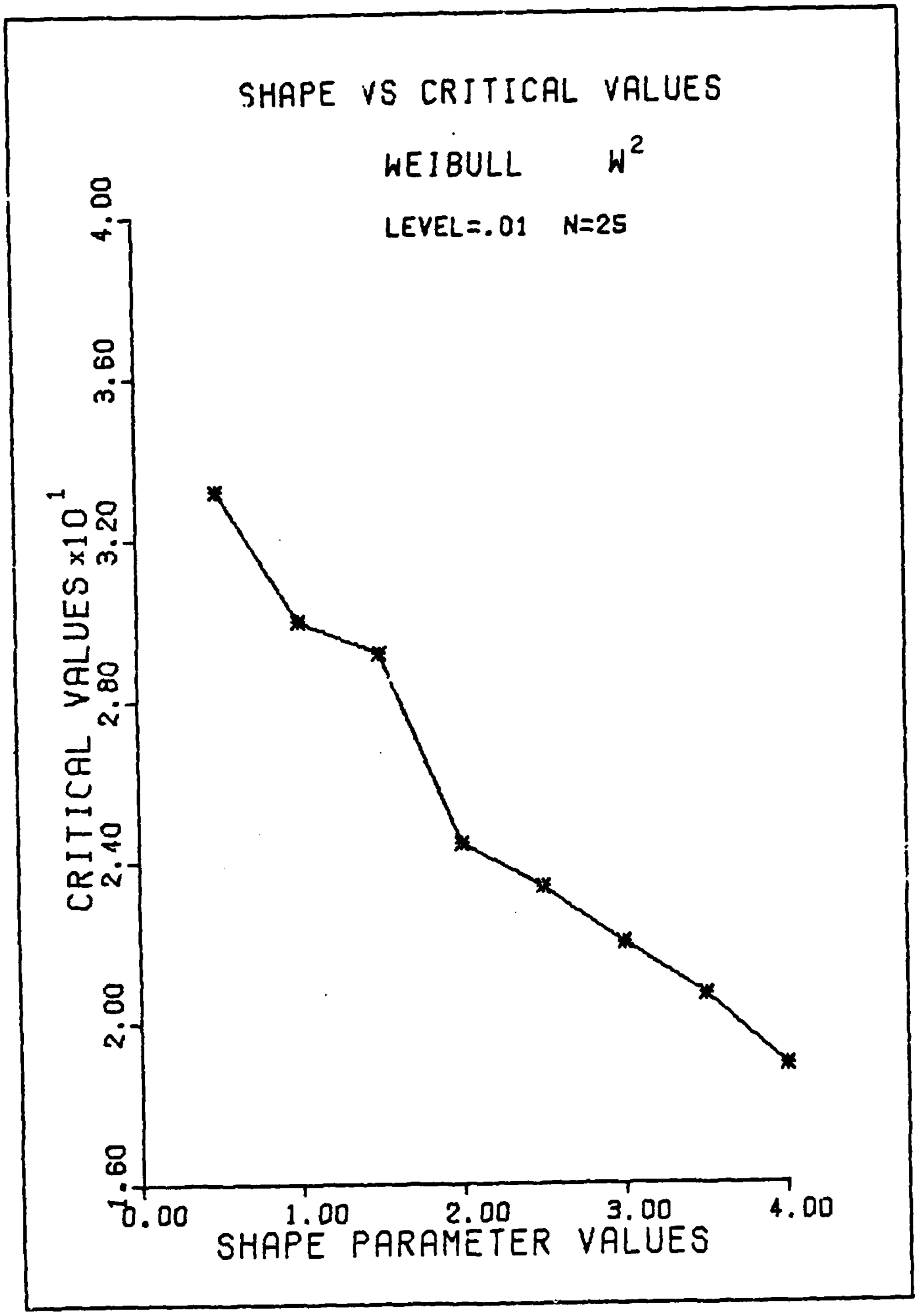

Fig. 37. Shape vs W^2 Critical Values, Level=.01, n=25

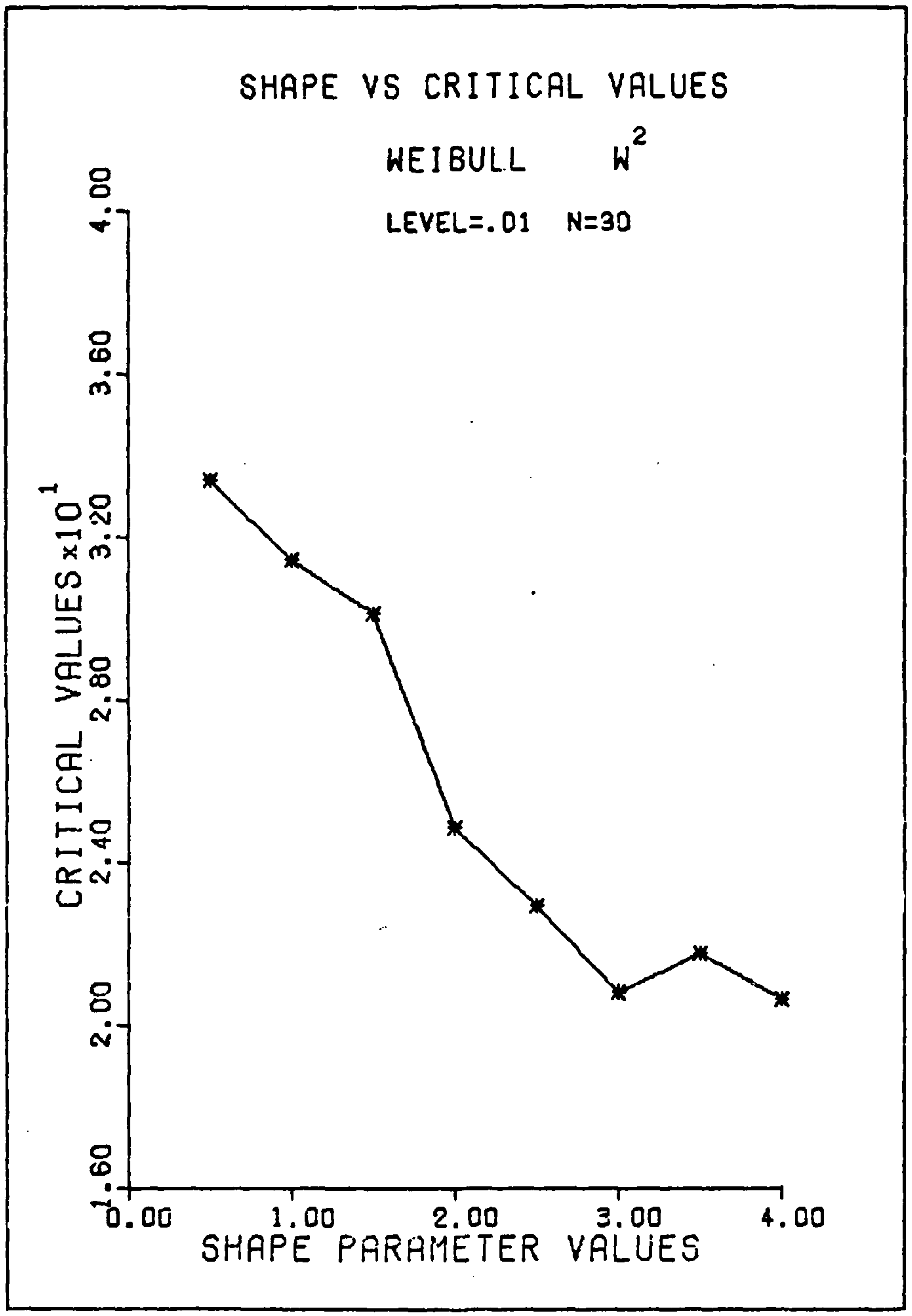

Fig. 38. Shape vs W^2 Critical Values, Level=.01, n=30

APPENDIX D

Graphs of the Anderson-Darling Critical

Values Versus the Weib ll

Shape Parameters

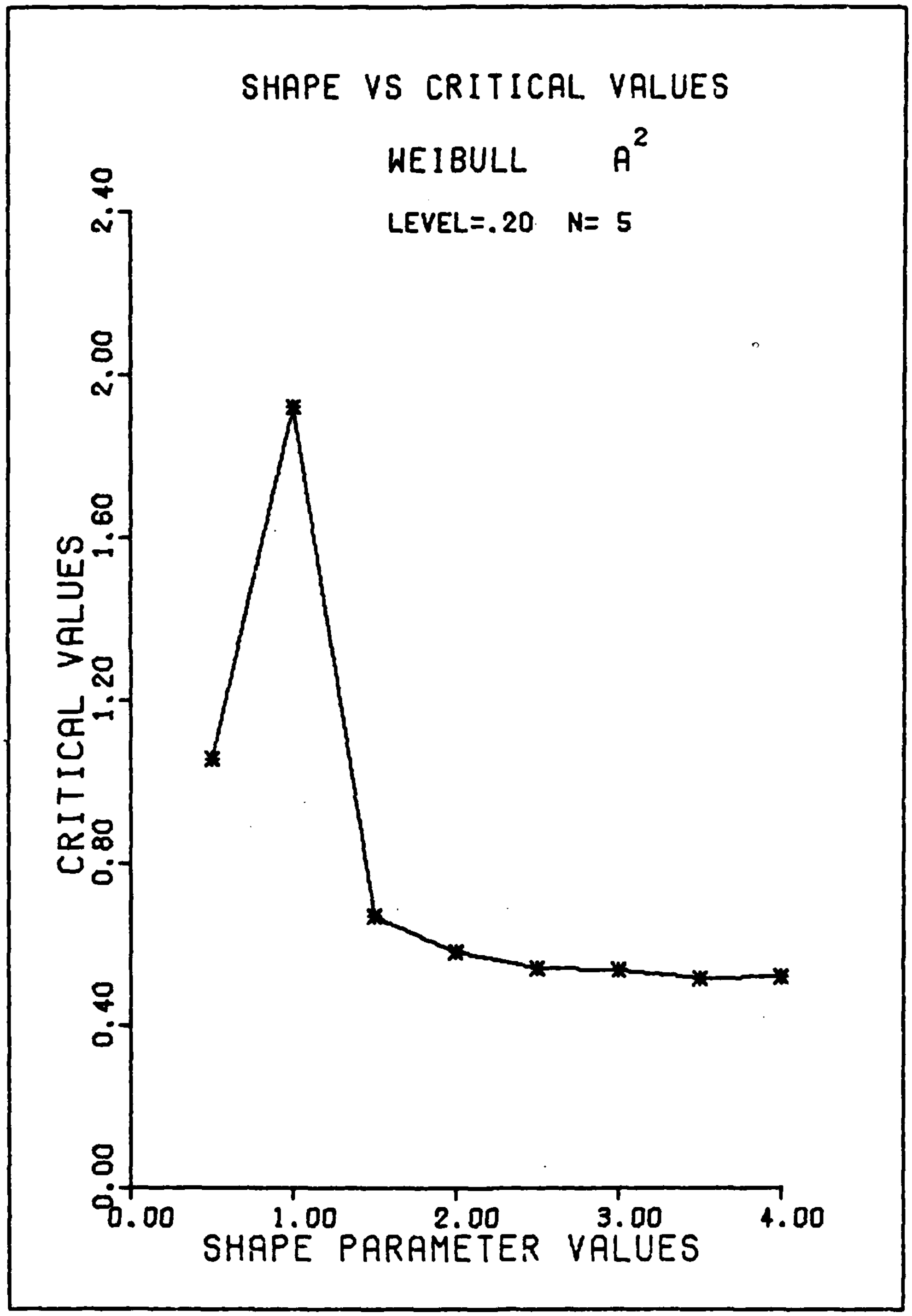

Fig. 39. Shape vs A^2 Critical Values, Level=.20, n=5

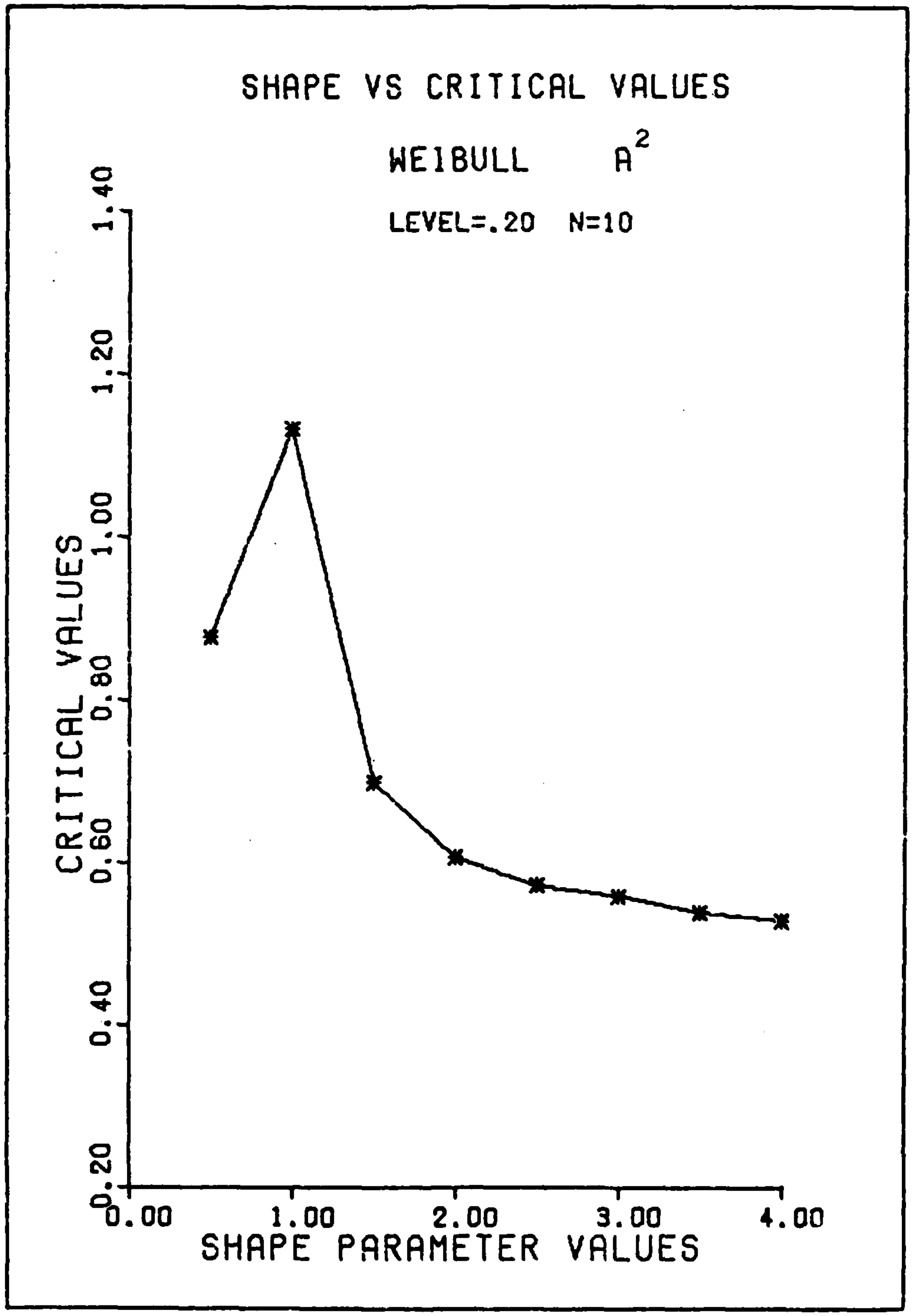

Fig. 40. Shape vs A^2 Critical Values, Level=.20, n=10

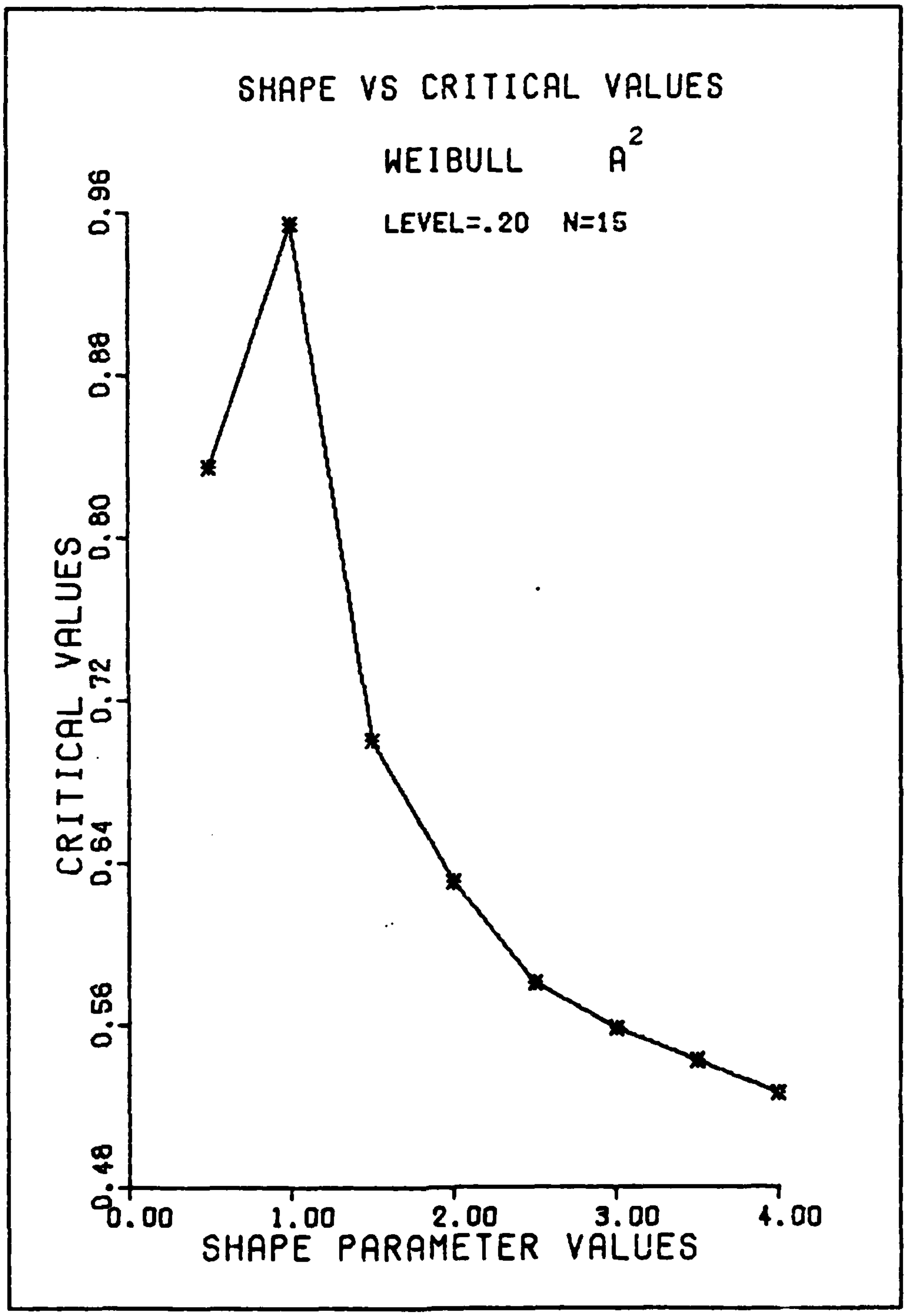

Fig. 41. Shape vs A^2 Critical Values, Level=.20, n=15

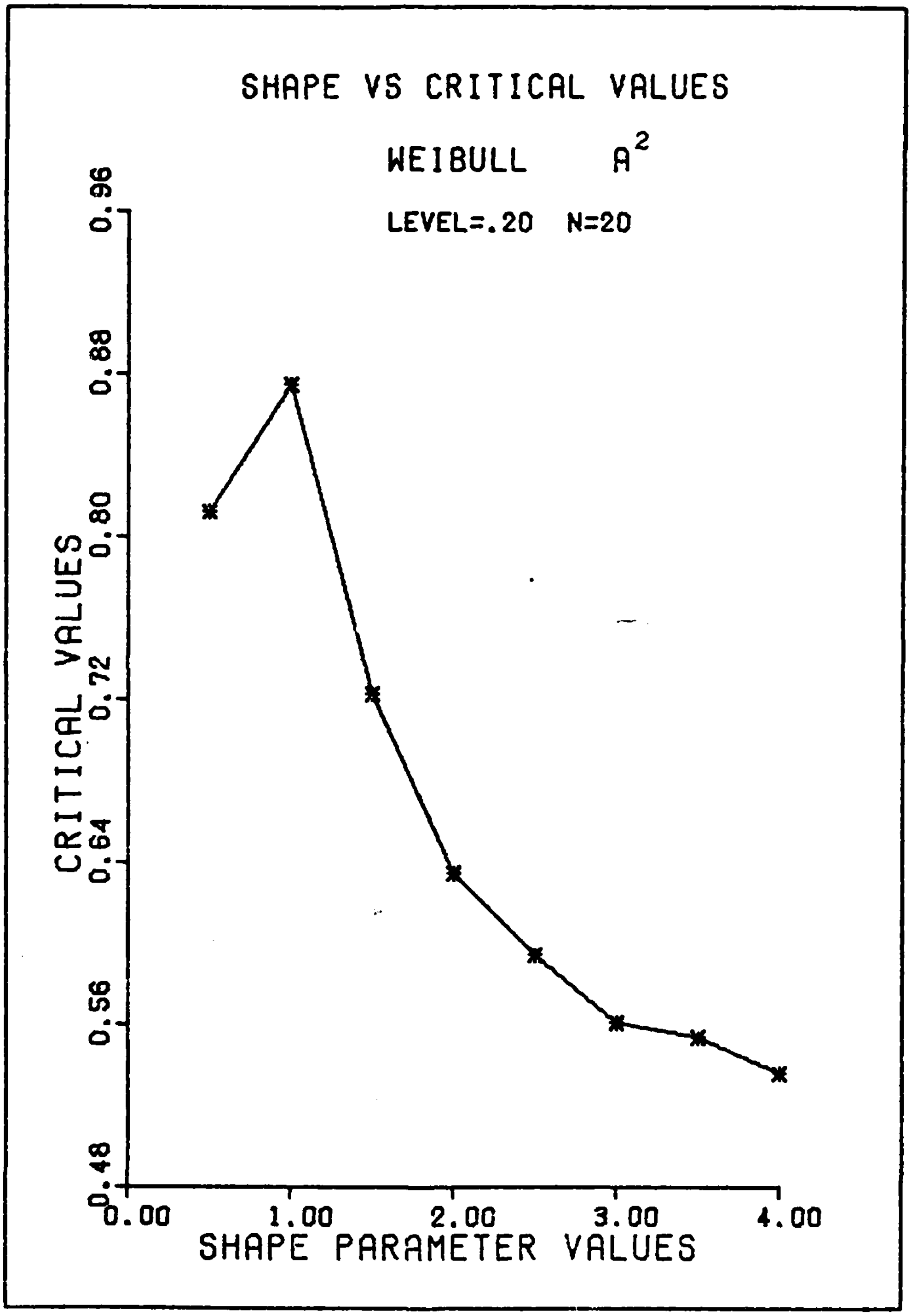

Fig. 42. Shape vs A^2 Critical Values, Level=.20, n=20

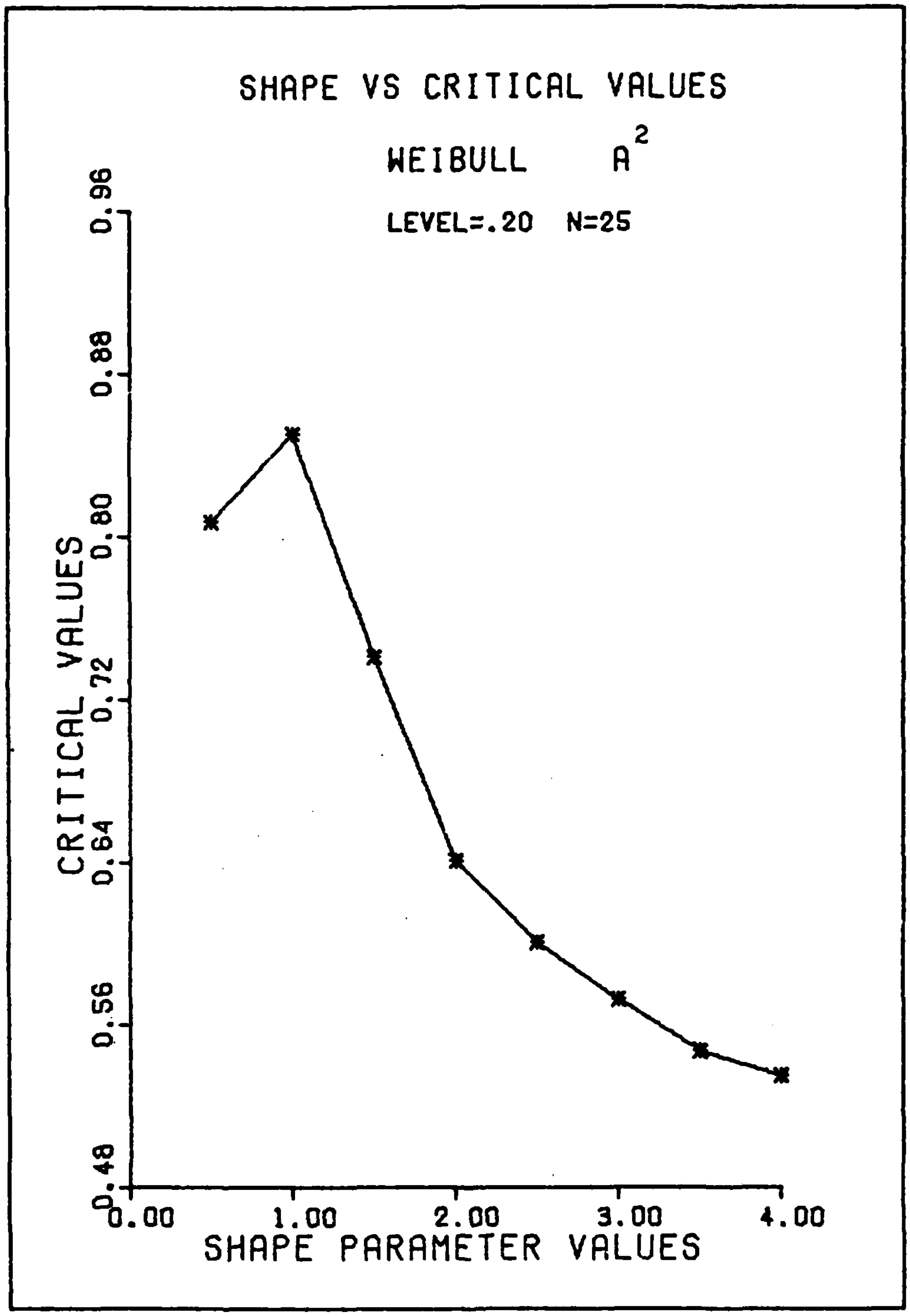

Fig. 43. Shape vs A^2 Critical Values, Level=.20, n=25

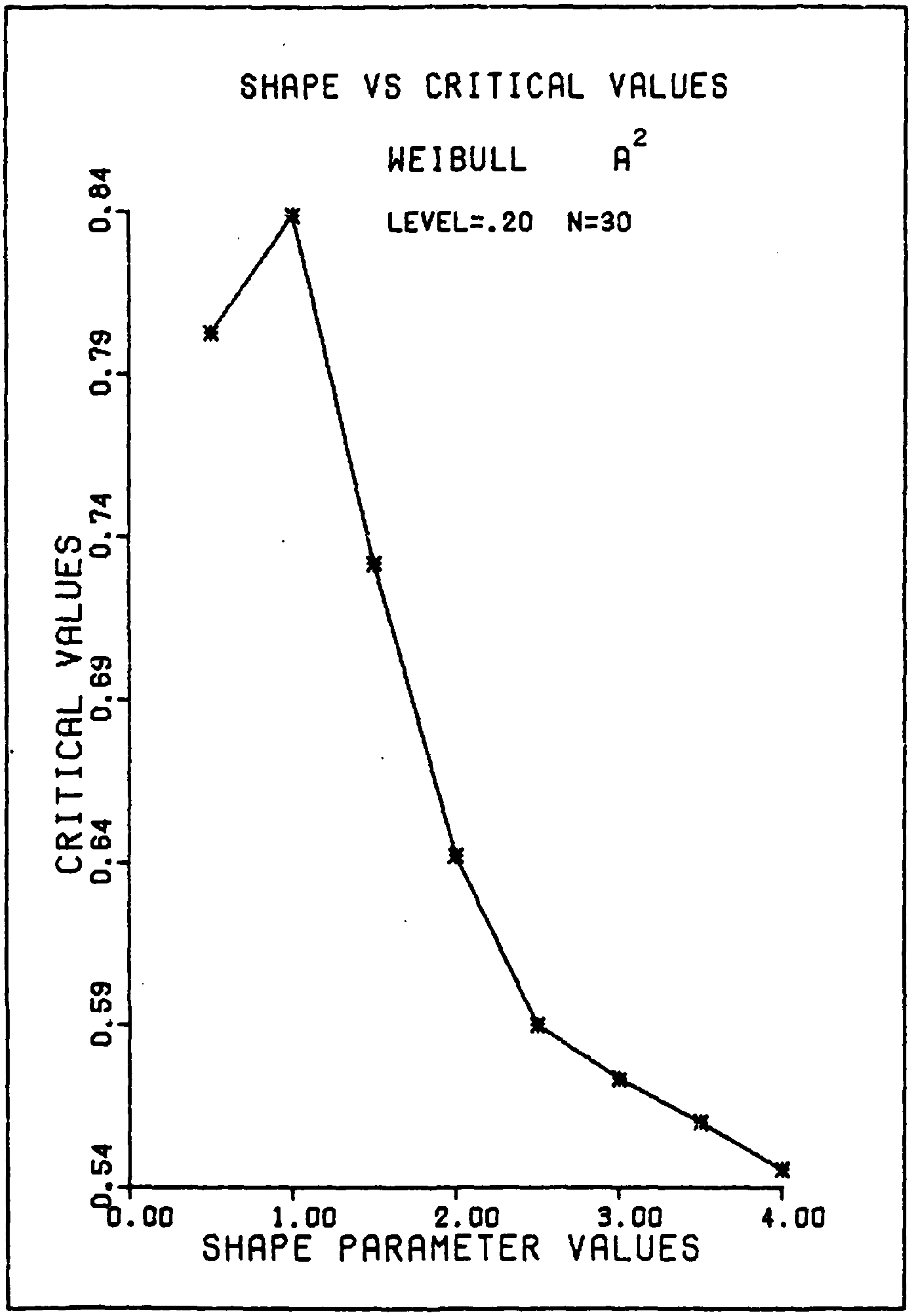

Fig. 44. Shape vs A^2 Critical Values, Level=.20, n=30

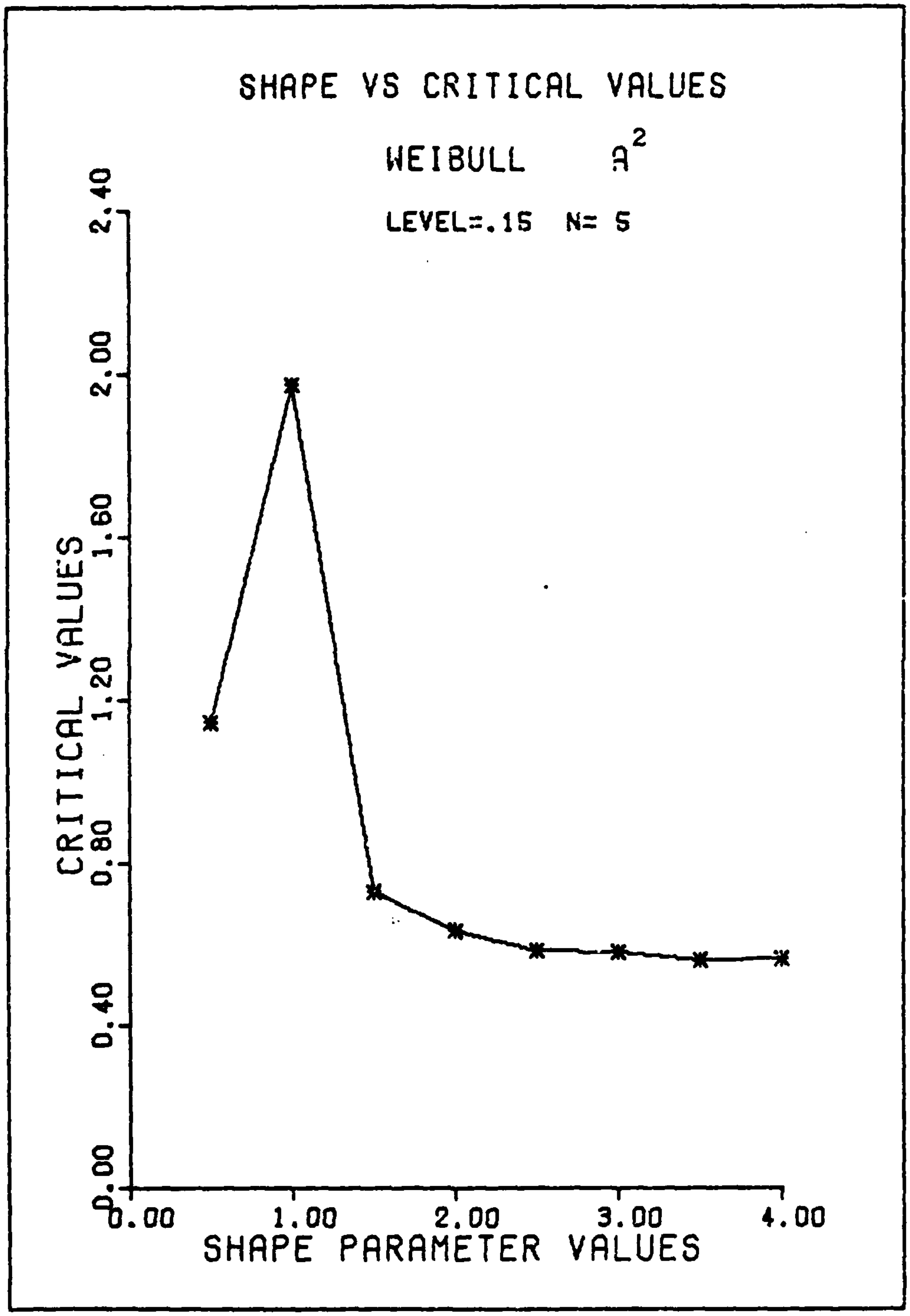

Fig. 45. Shape vs A^2 Critical Values, Level=.15, n=5

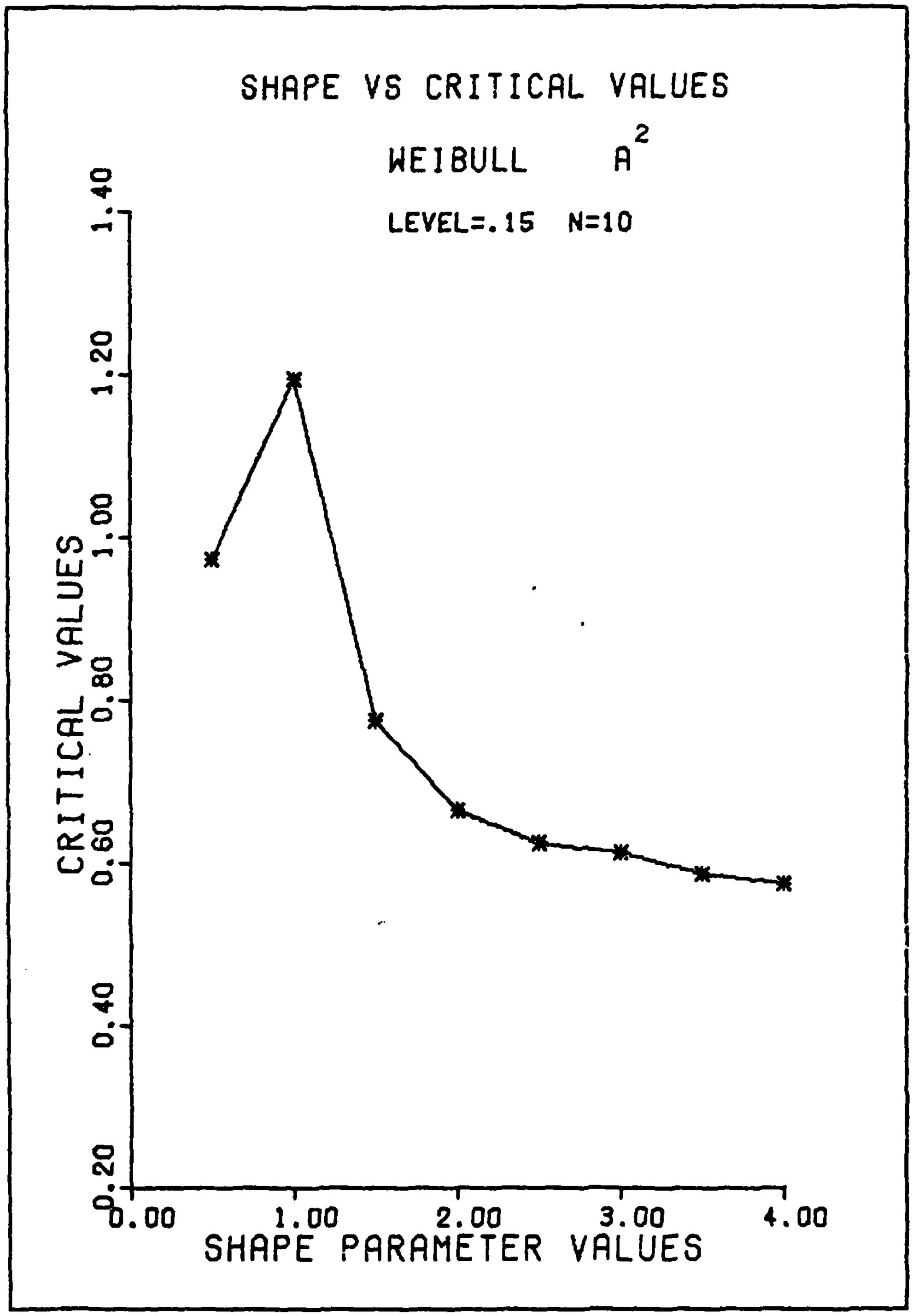

Fig. 46. Shape vs A^2 Critical Values, Level=.15, n=10

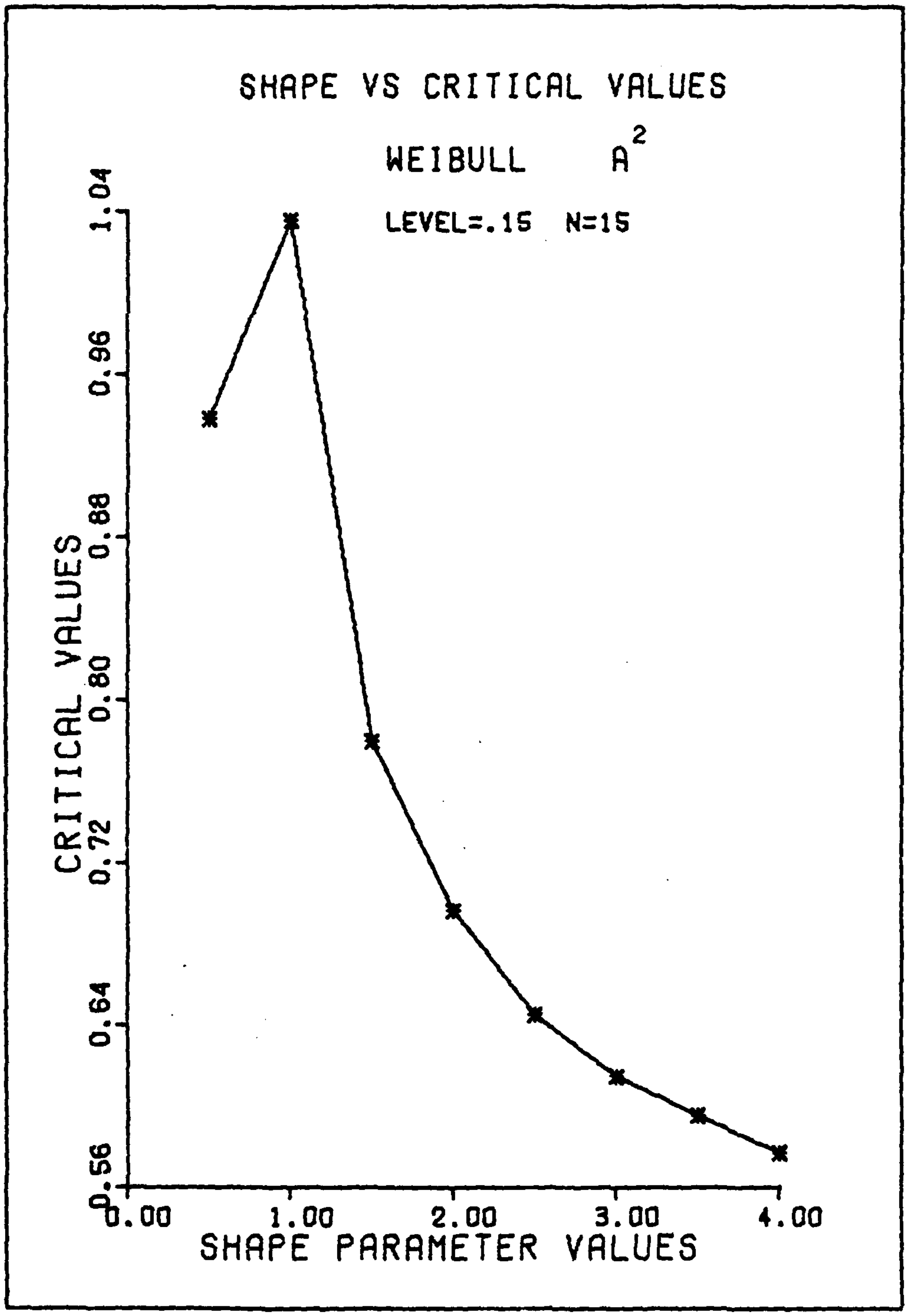

Fig. 47. Shape vs A^2 Critical Values, Level=.15, n=15

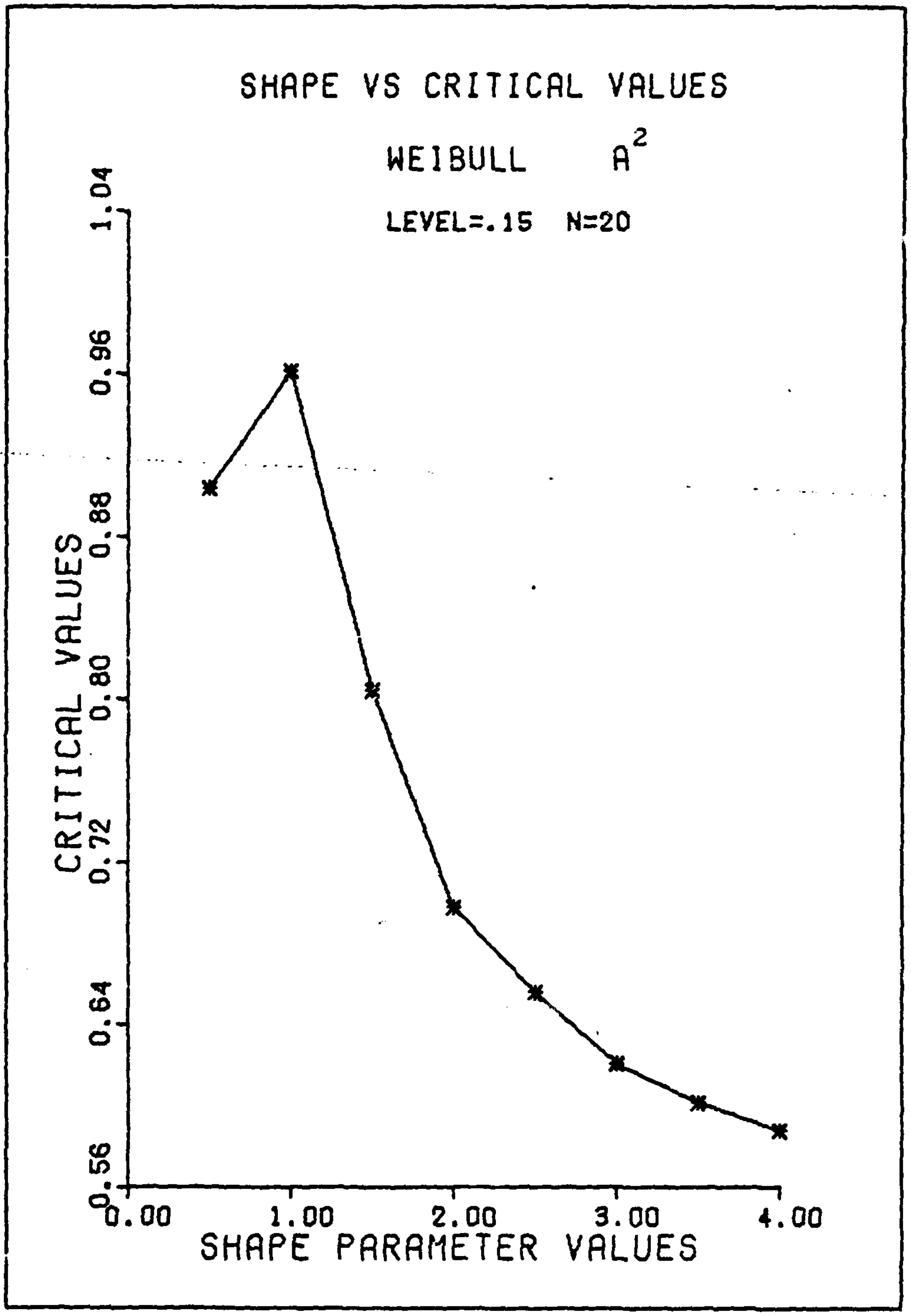

Fig. 48. Shape vs A^2 Critical Values, Level=.15, n=20

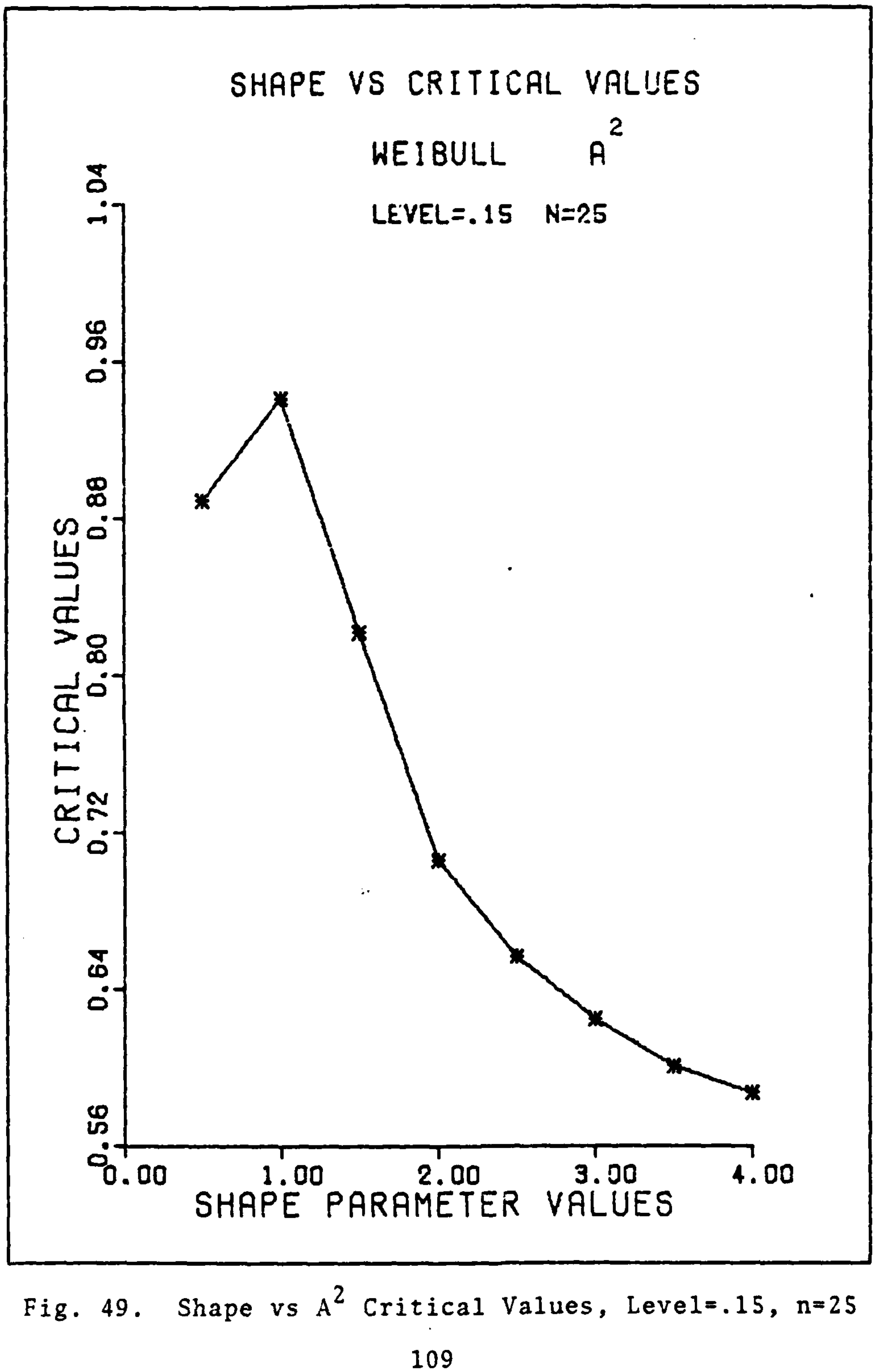

Fig. 49. Shape vs A^2 Critical Values, Level=.15, n=25

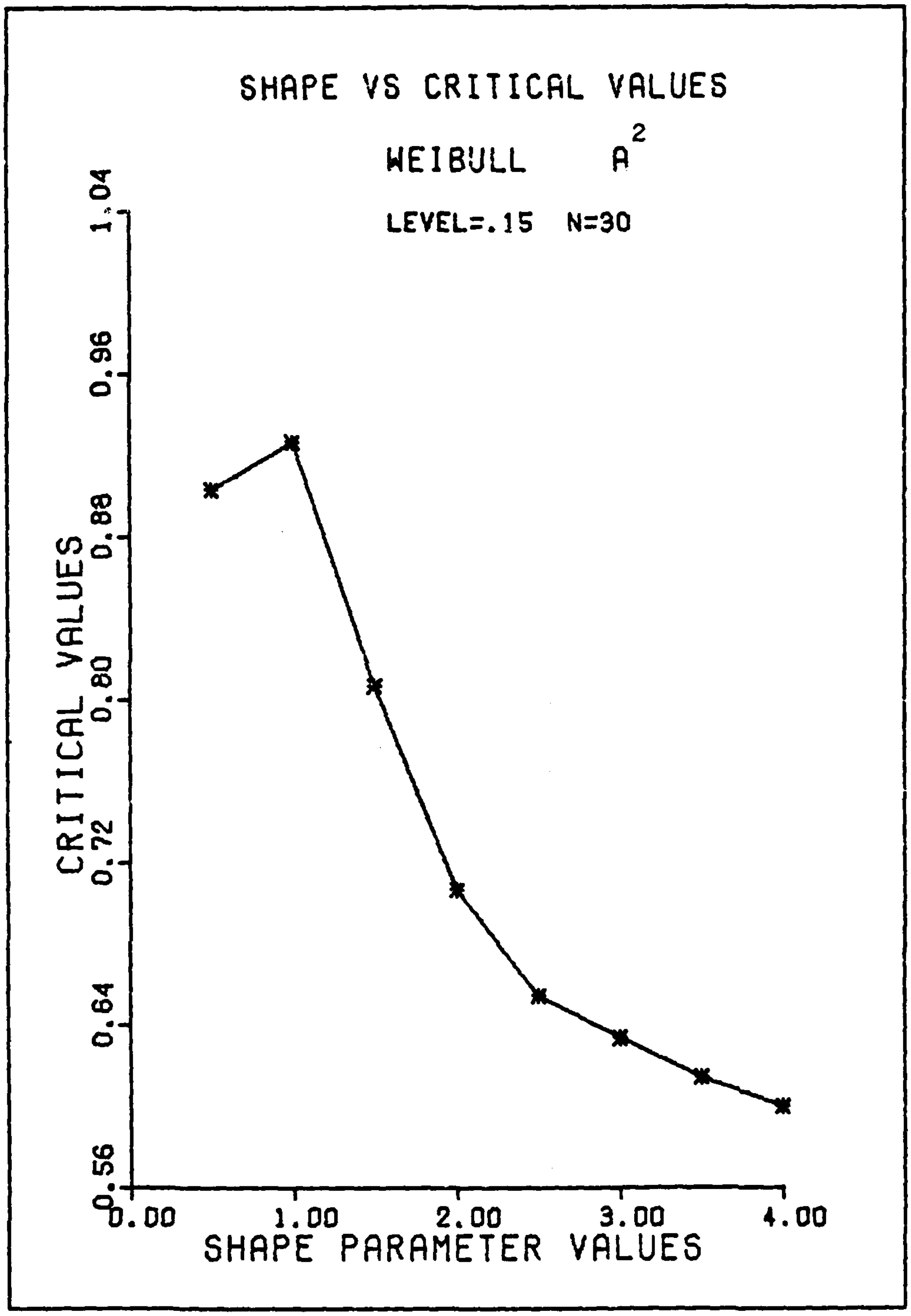

Fig. 50. Shape vs A^2 Critical Values, Level=.15, n=30

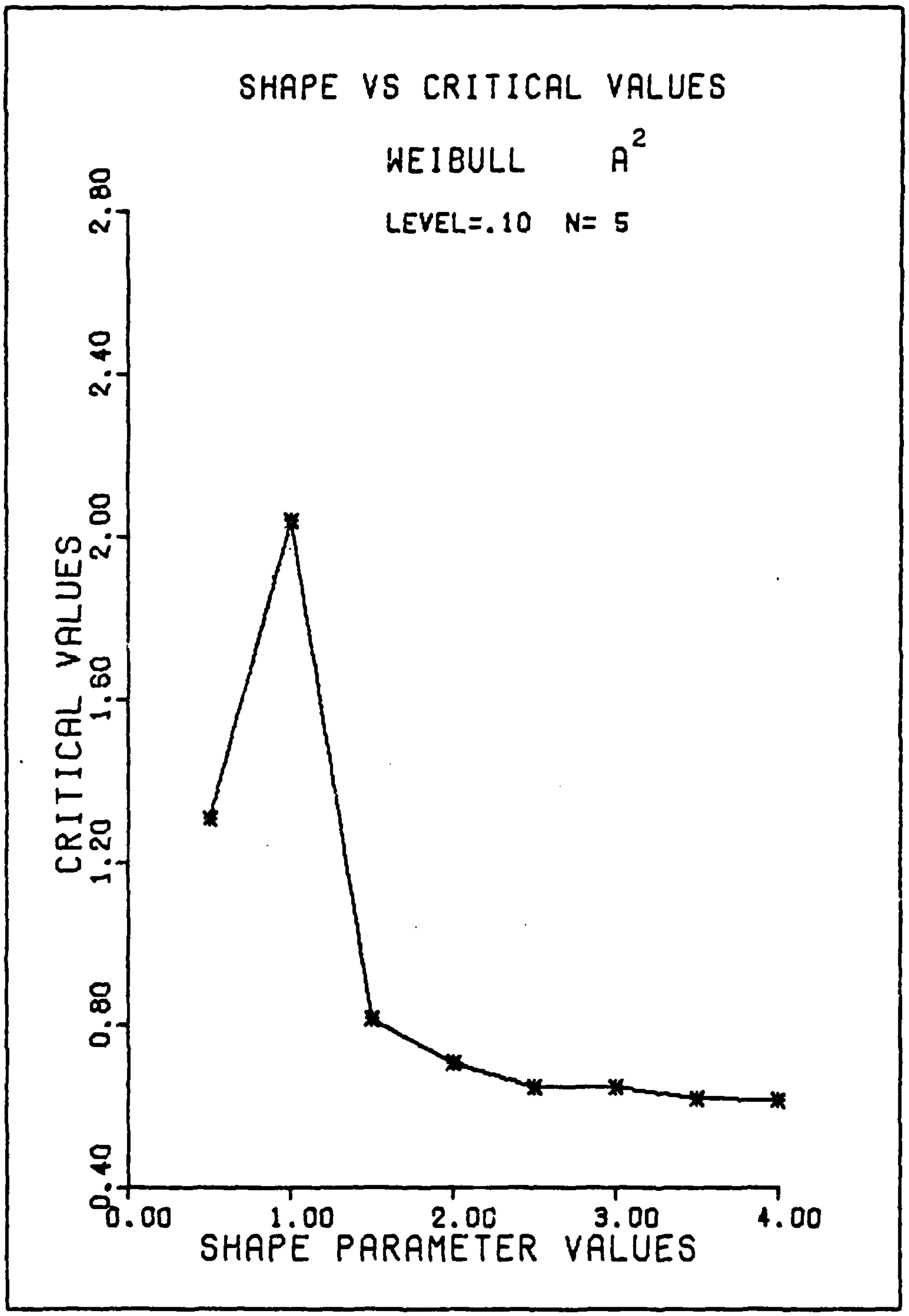

Fig. 51. Shape vs A^2 Critical Values, Level=.10, n=5

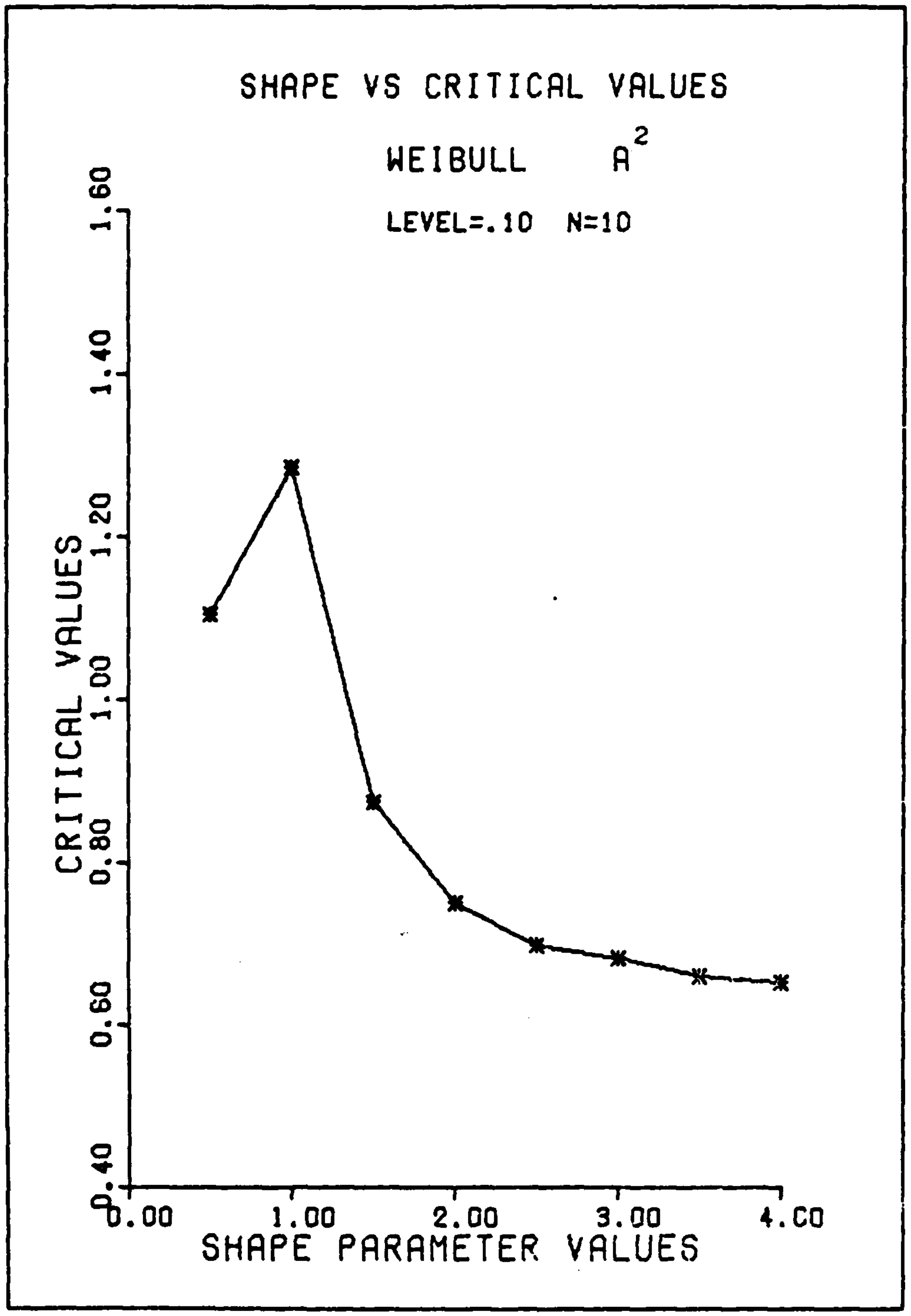

Fig. 52. Shape vs A^2 Critical Values, Level=.10, n=10

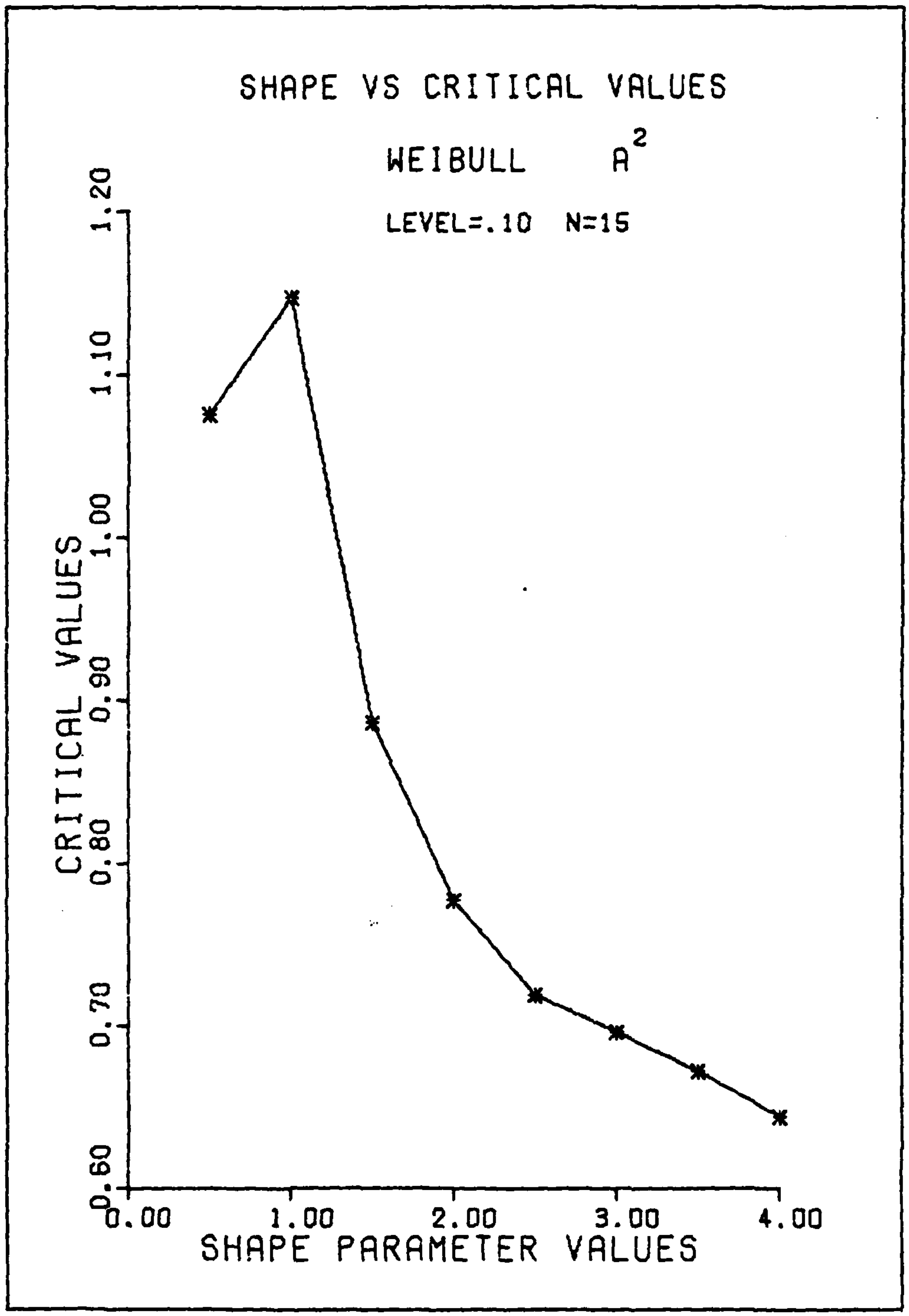

Fig. 53. Shape vs A^2 Critical Values, Level=.10, n=15

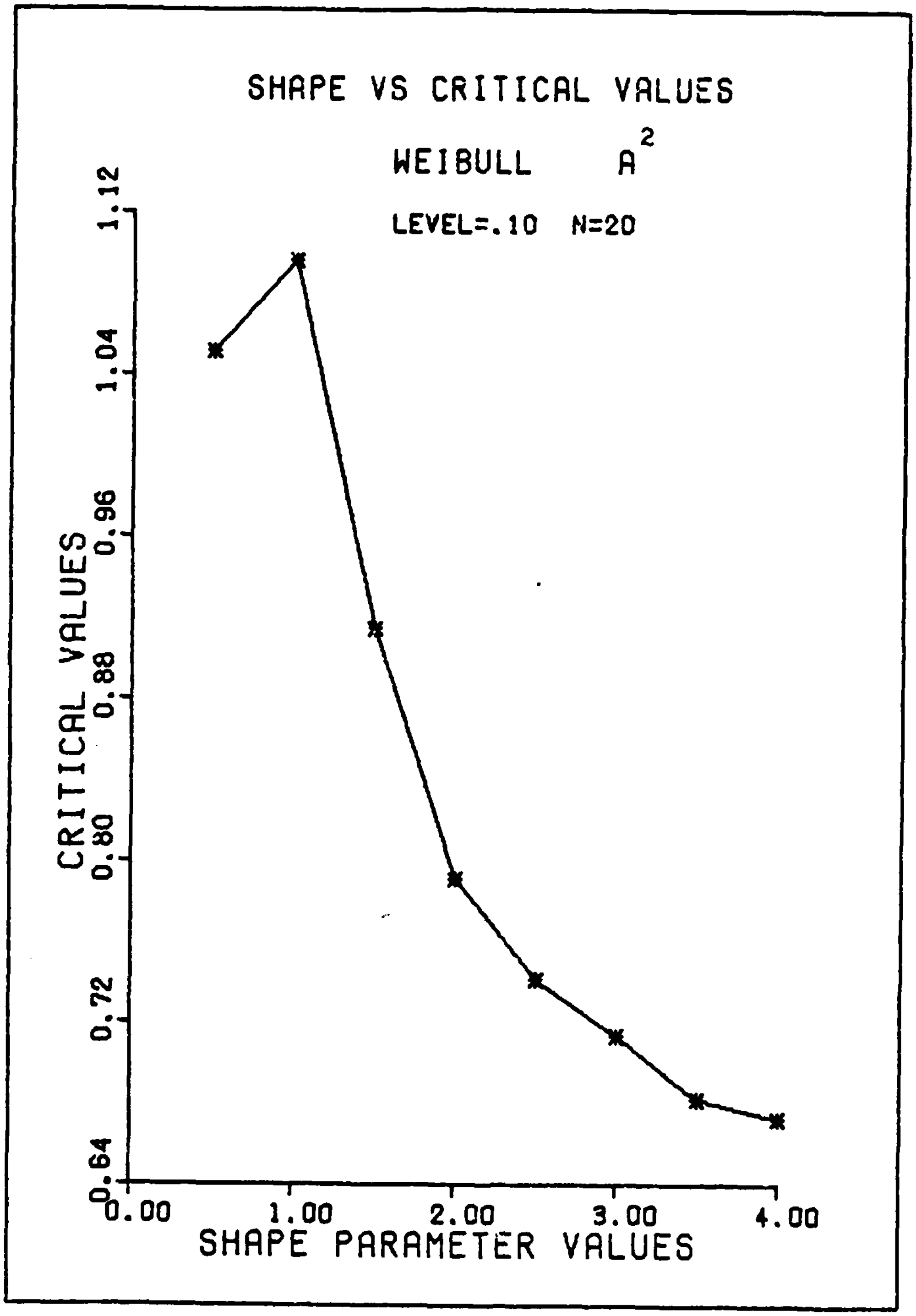

Fig. 54. Shape vs A^2 Critical Values, Level=.10, n=20

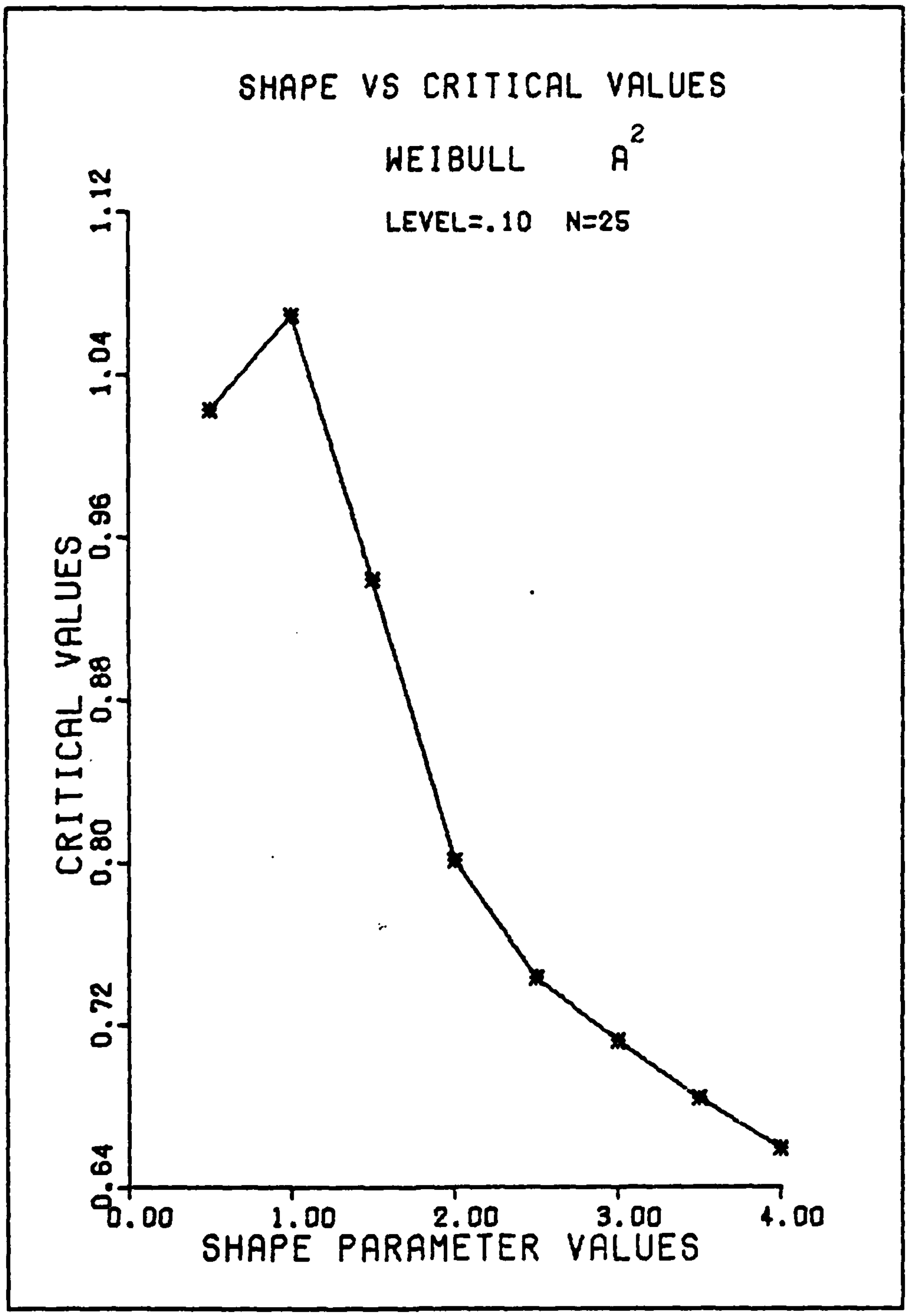

Fig. 55. Shape vs A^2 Critical Values, Level=.10, n=25

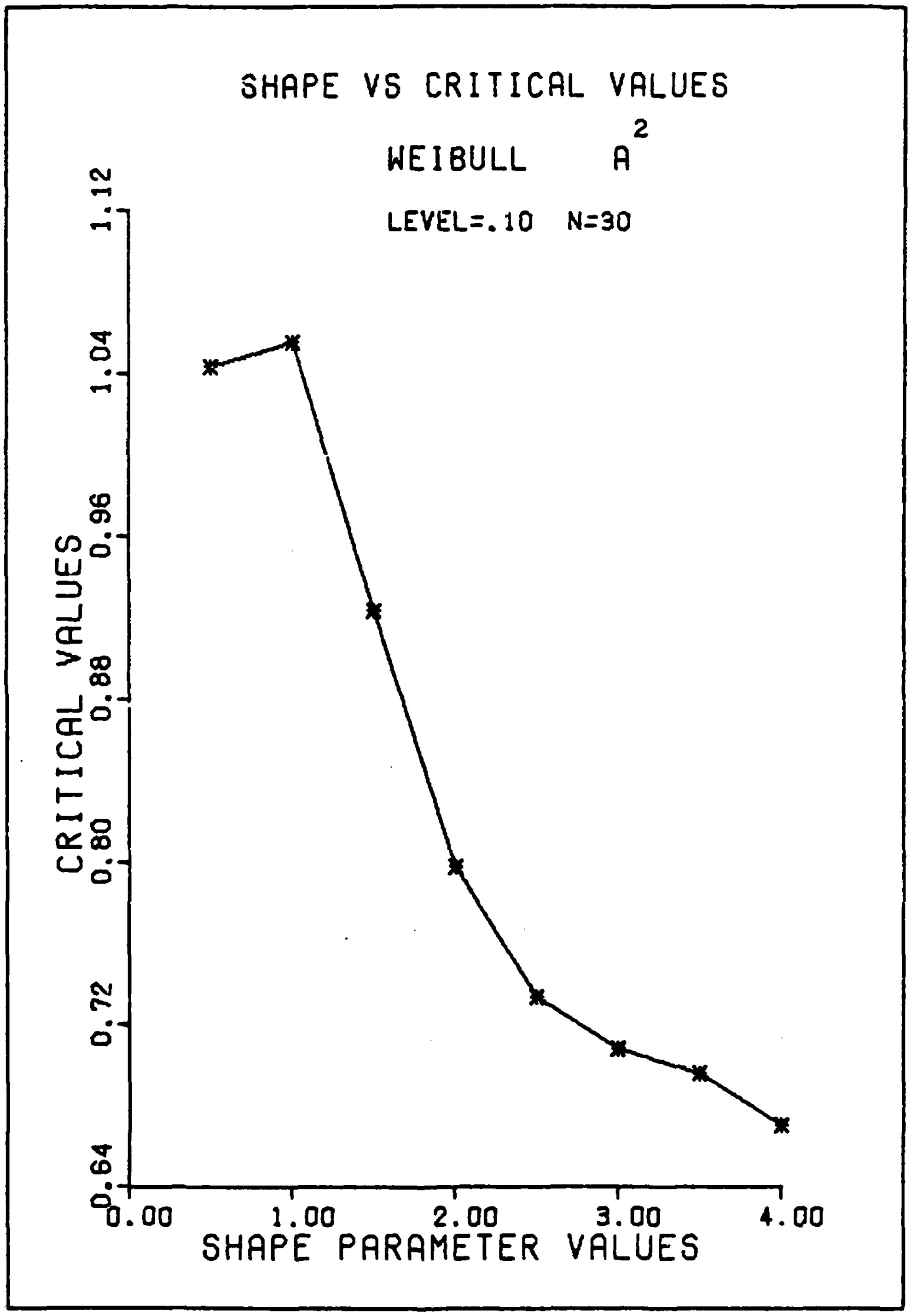

Fig. 56. Shape vs A^2 Critical Values, Level=.10, n=30

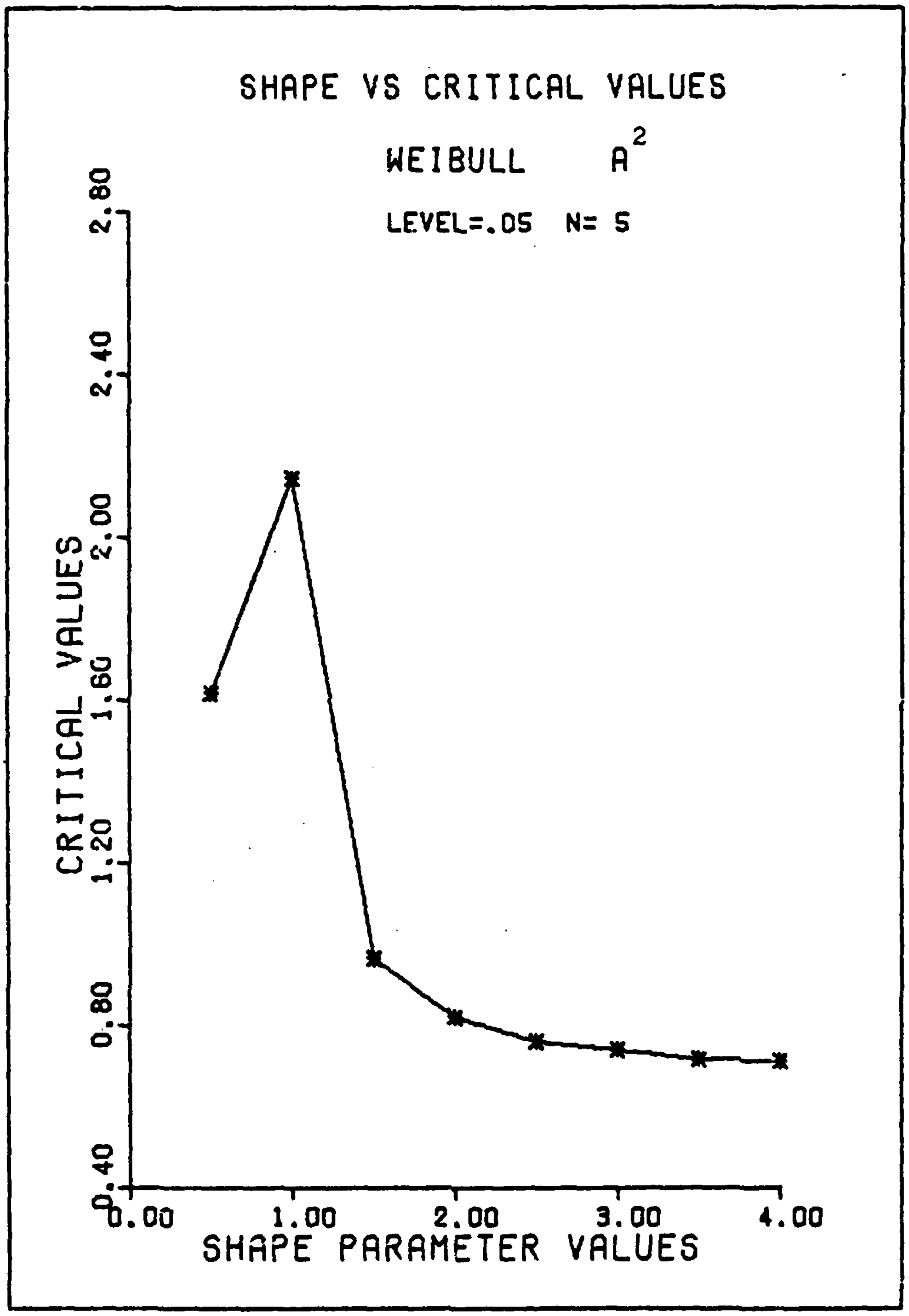

Fig. 57. Shape vs A^2 Critical Values, Level=.05, n=5

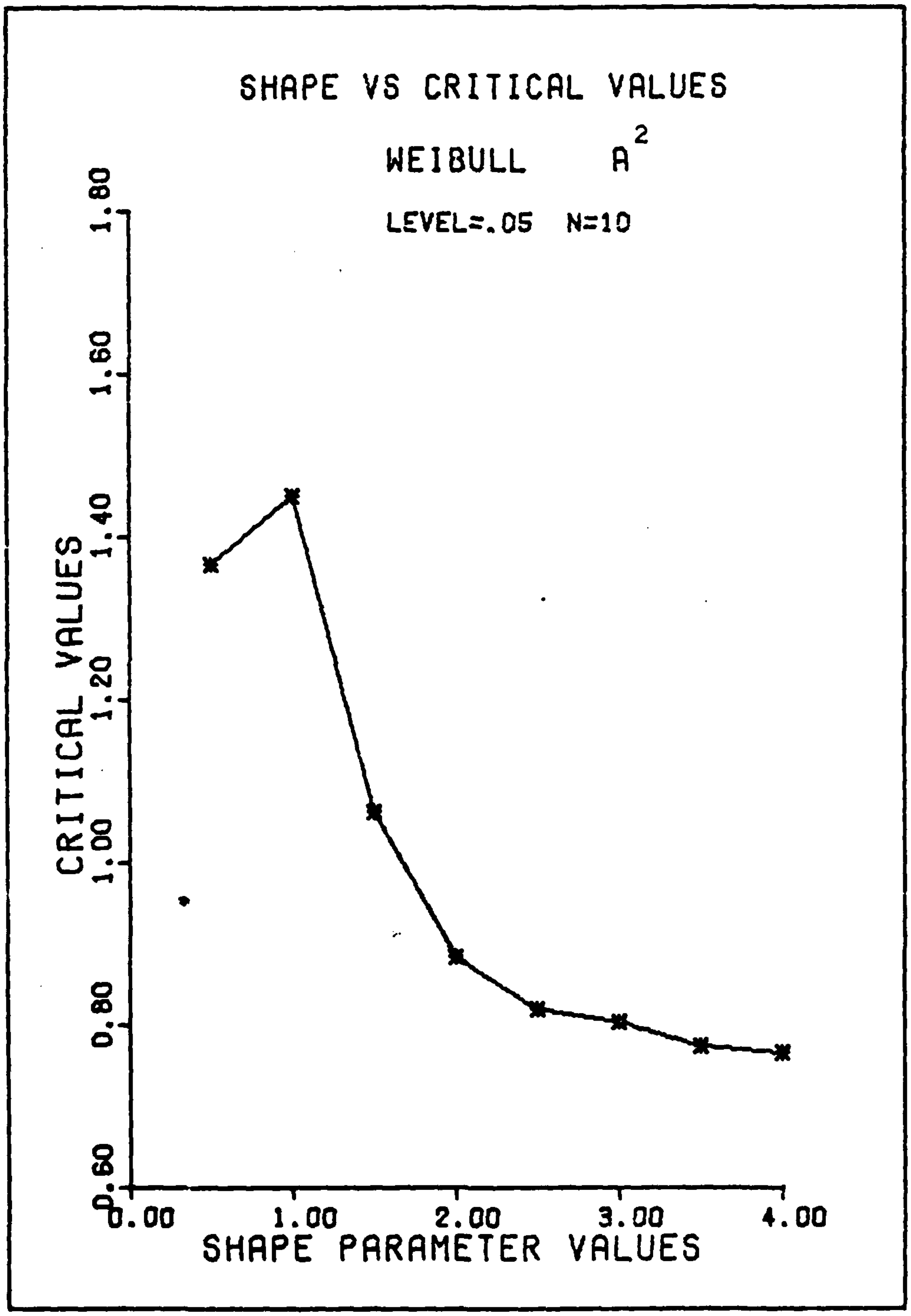

Fig. 58. Shape vs A^2 Critical Values, Level=.05, n=10

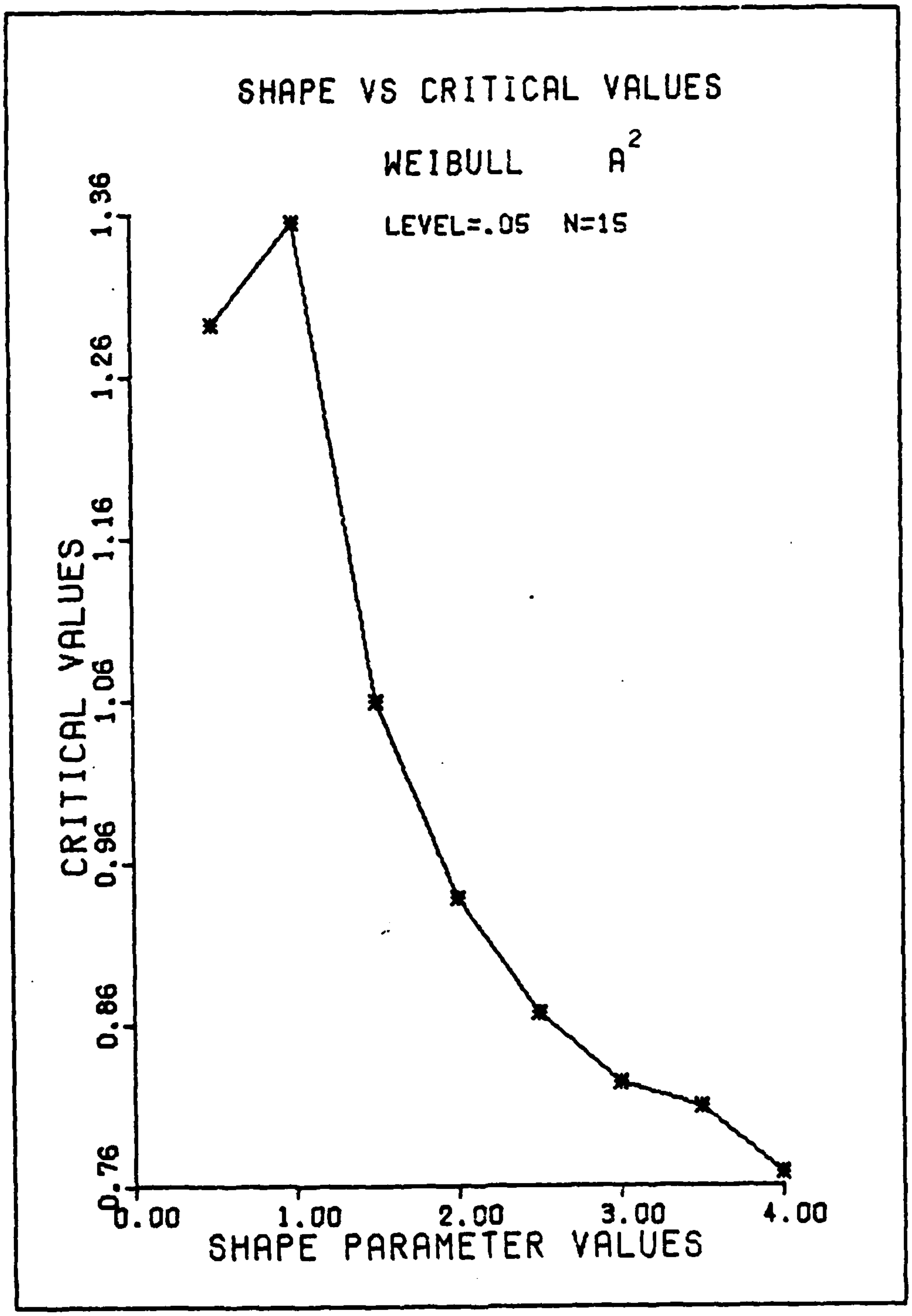

Fig. 59. Shape vs A^2 Critical Values, Level=.05, n=15

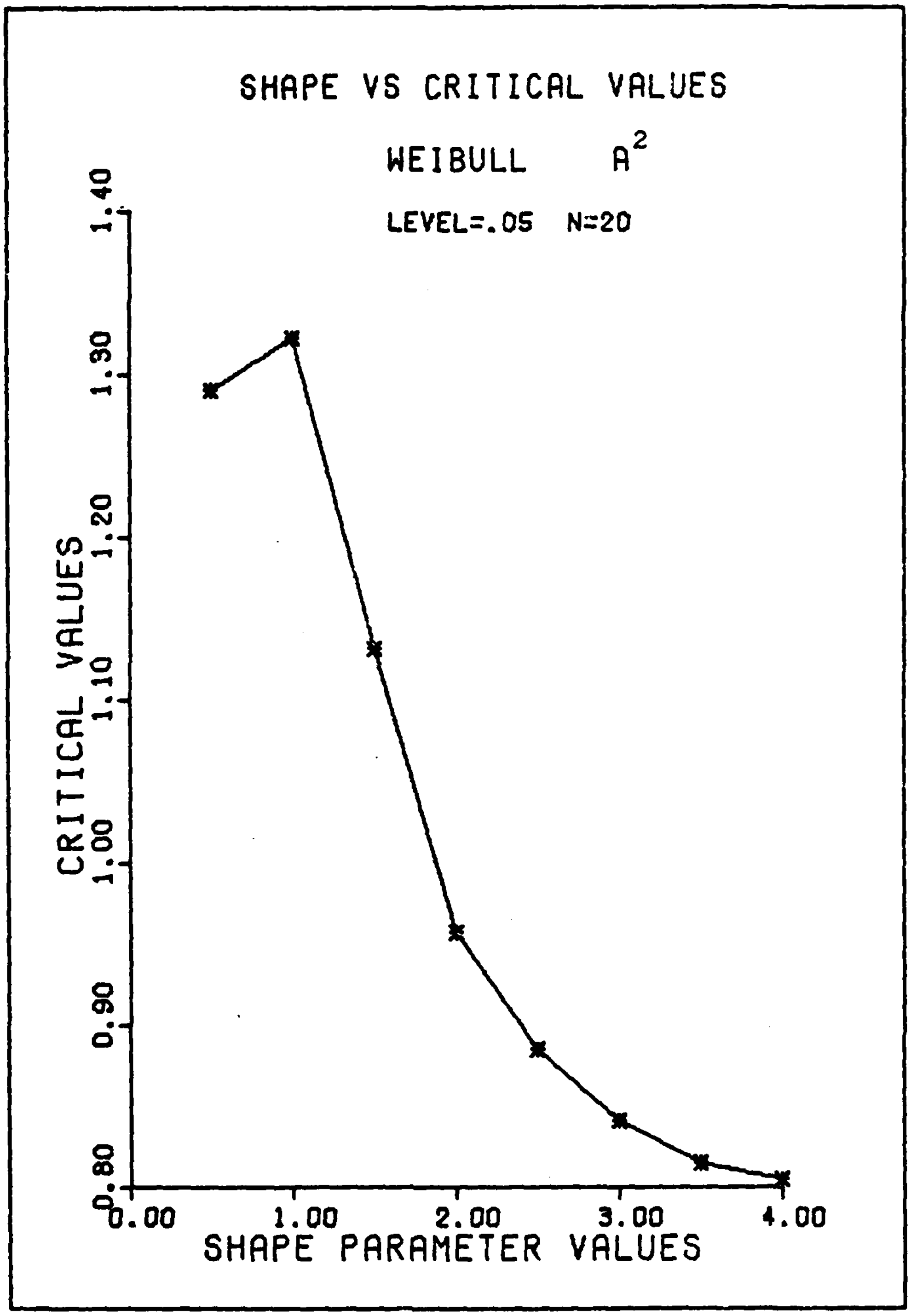

Fig. 60. Shape vs A^2 Critical Values, Level=.05, n=20

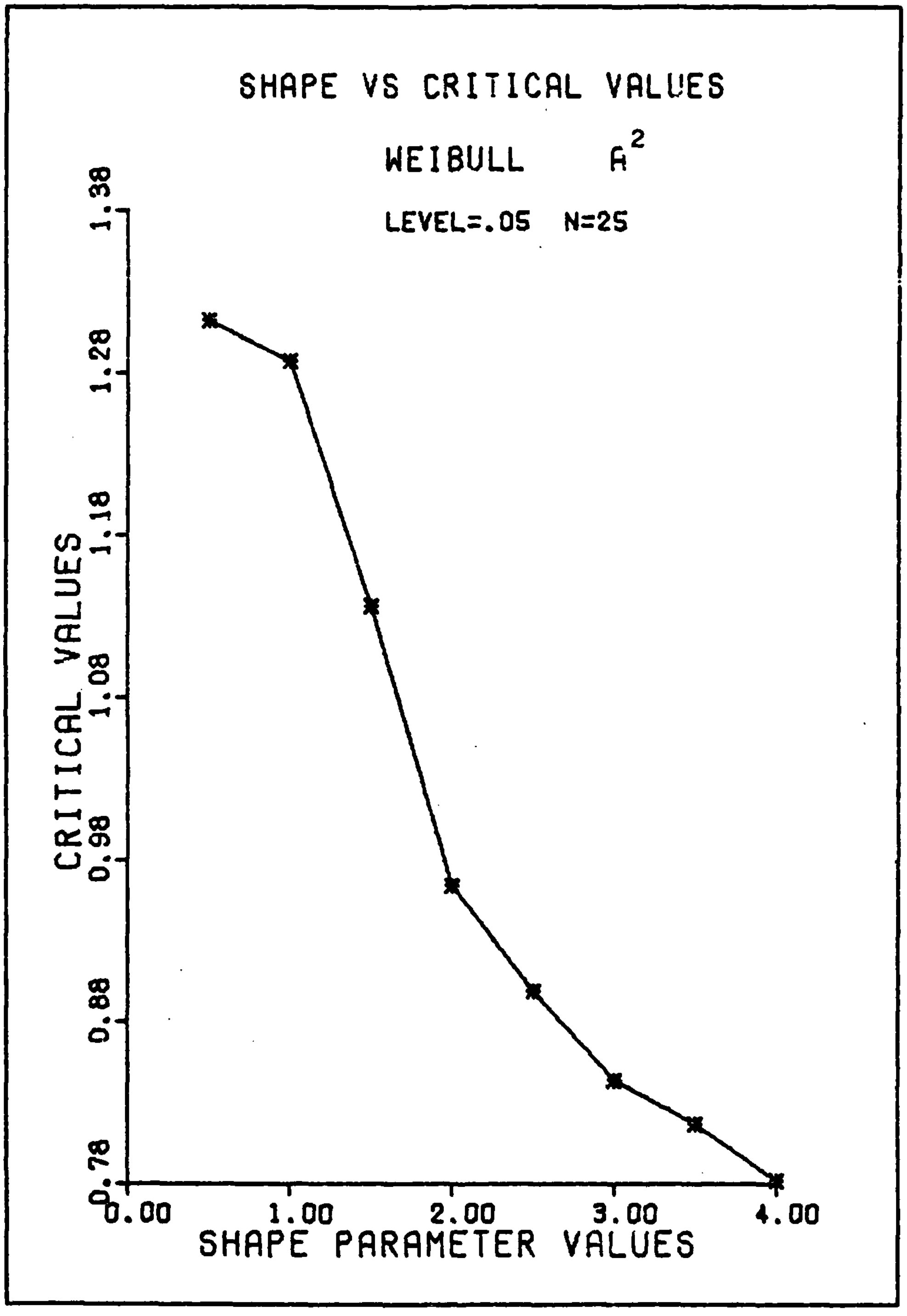

Fig. 61. Shape vs A^2 Critical Values, Level=.05, n=25

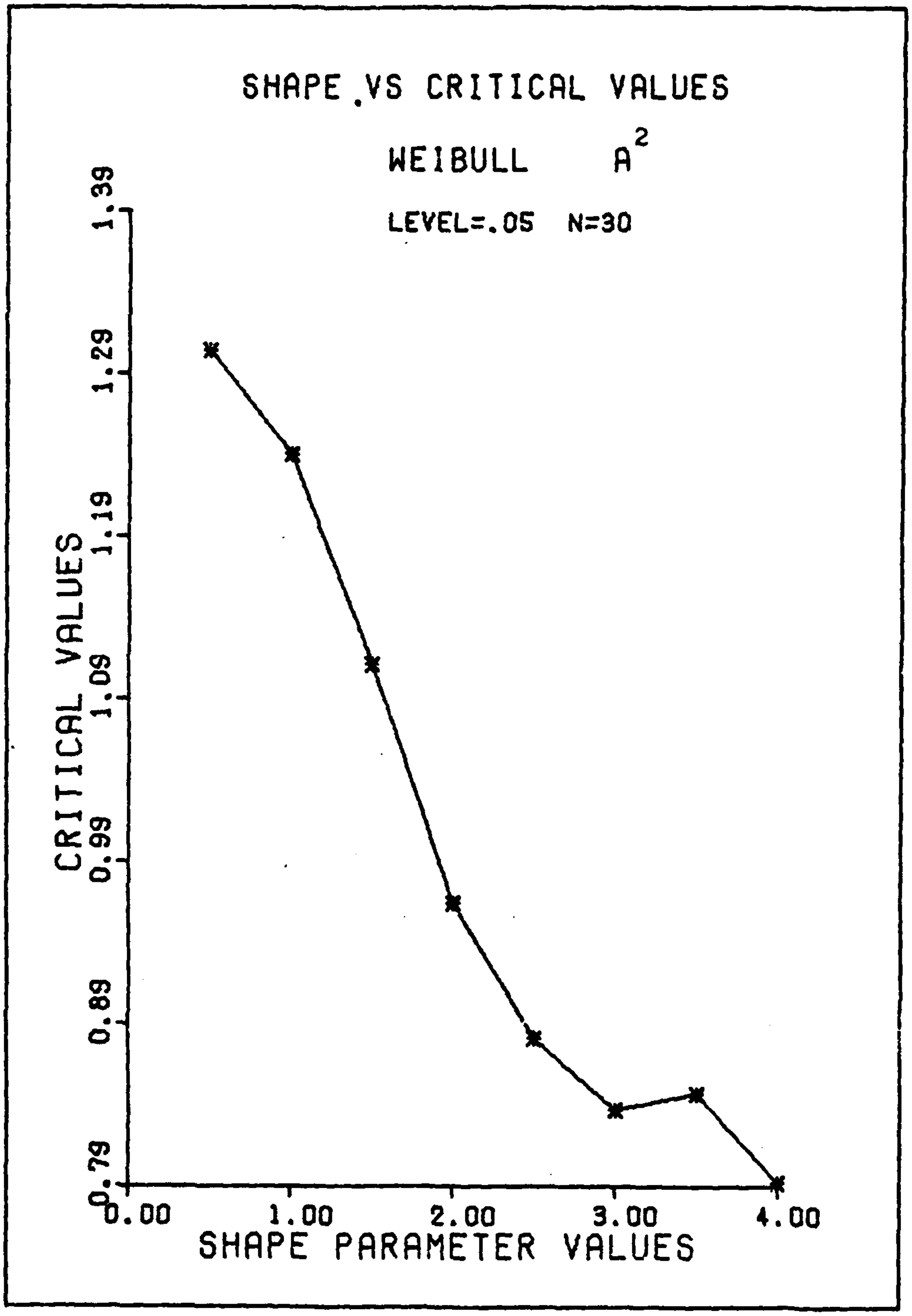

Fig. 62. Shape vs A^2 Critical Values, Level=.05, n=30

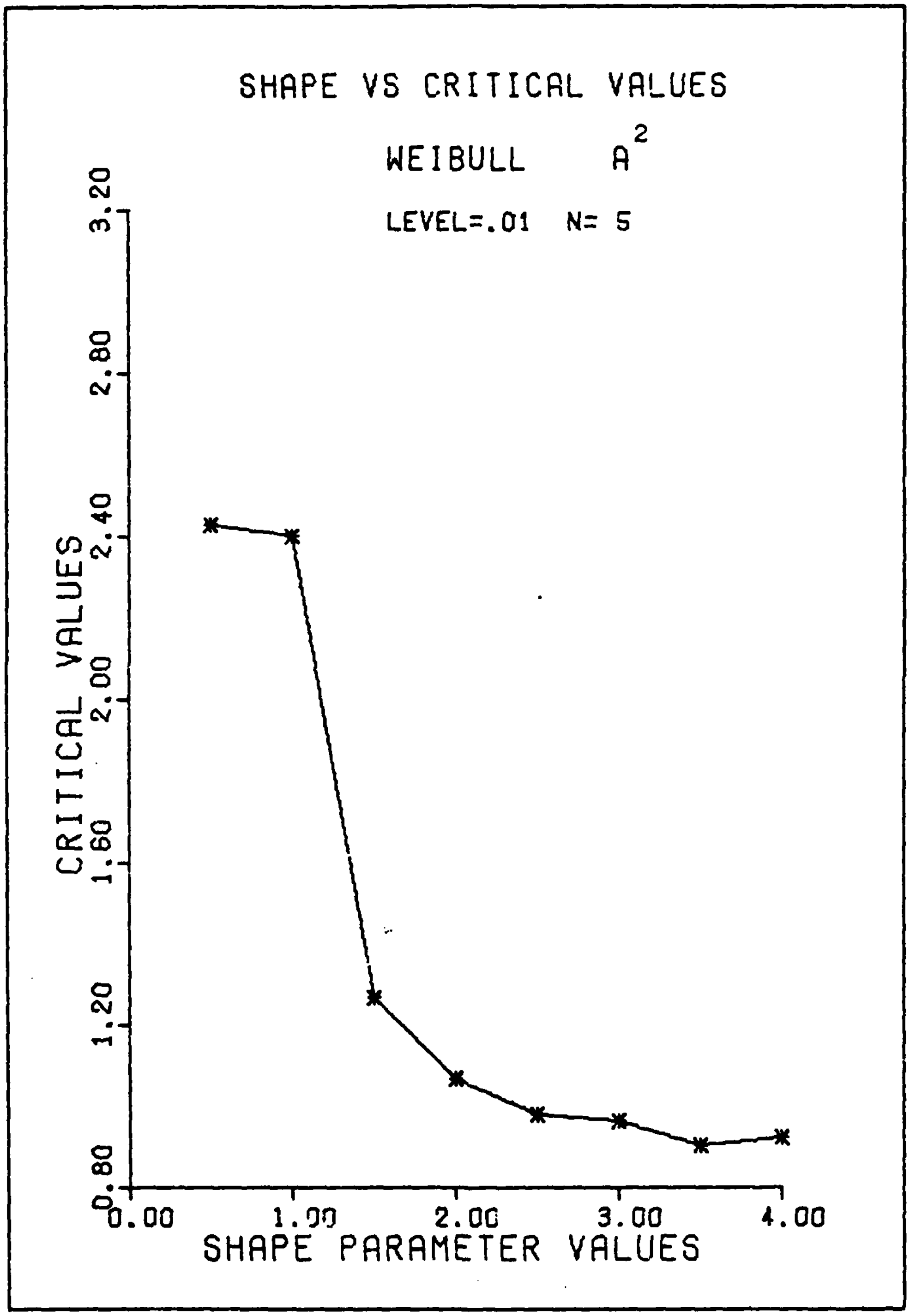

Fig. 63. Shape vs A^2 Critical Values, Level=.01, n=5

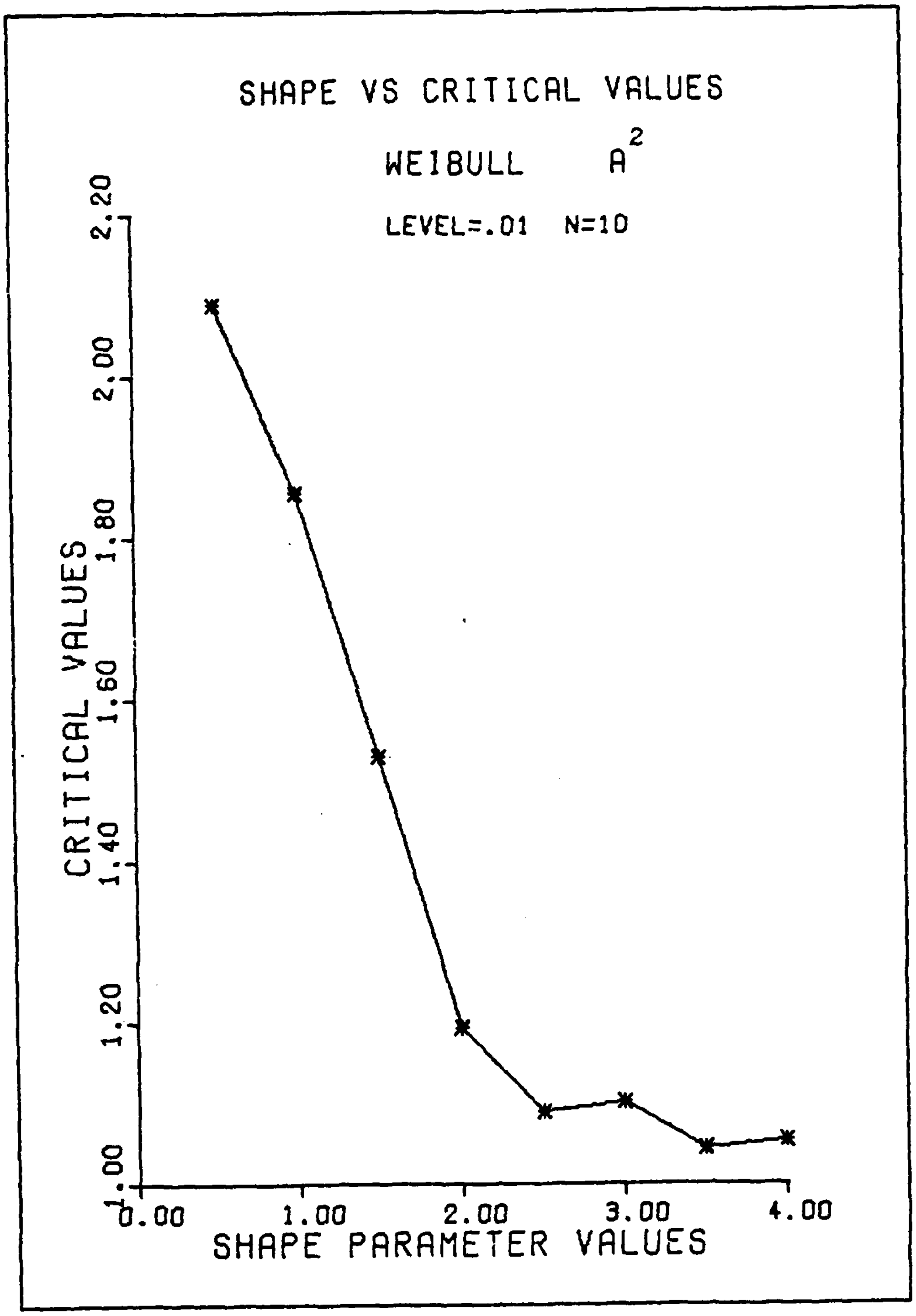

Fig. 64. Shape vs A^2 Critical Values, Level=.01, n=10

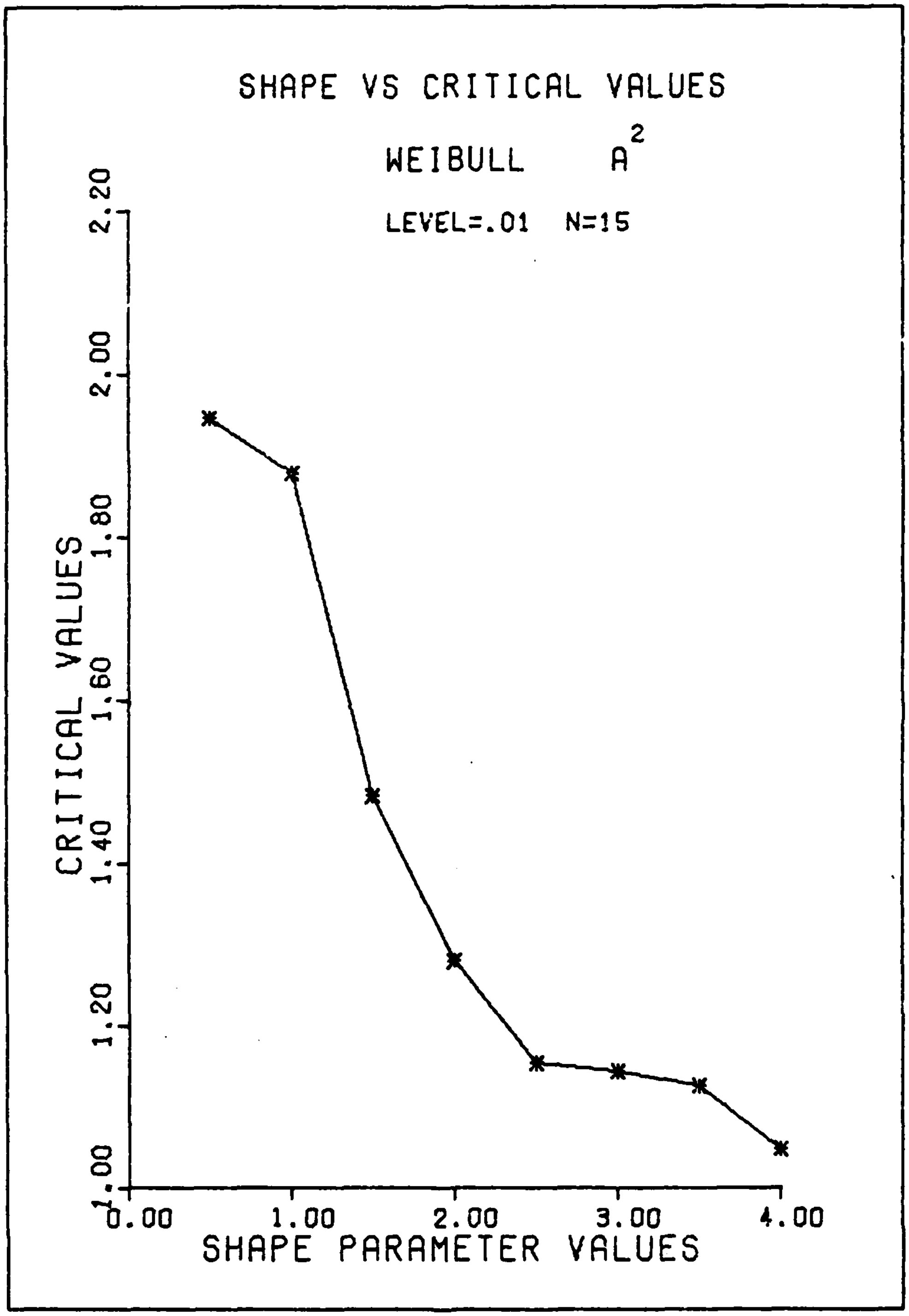

Fig. 65. Shape vs A^2 Critical Values, Level=.01, n=15

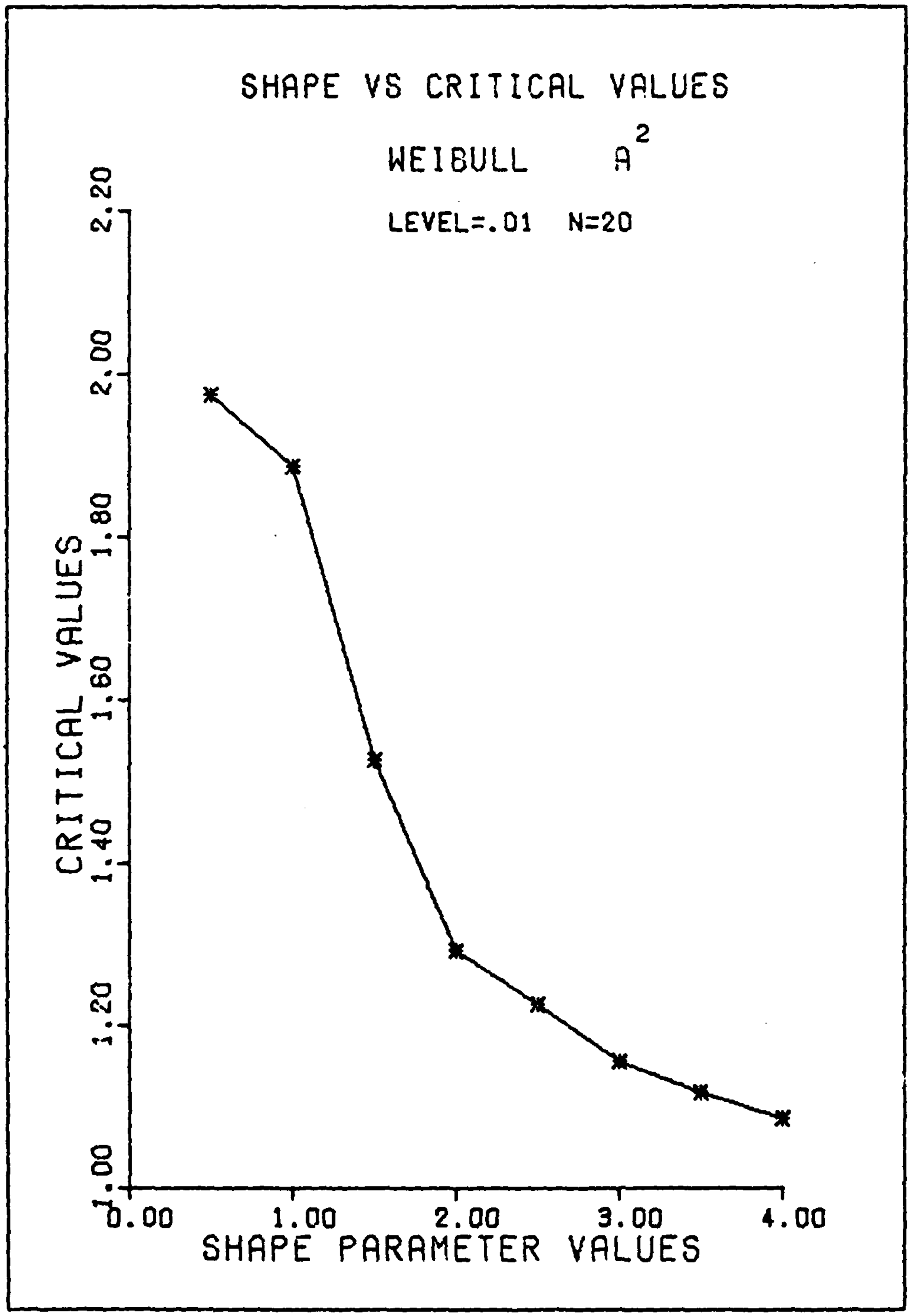

Fig. 66. Shape vs A^2 Critical Values, Level=.01, n=20

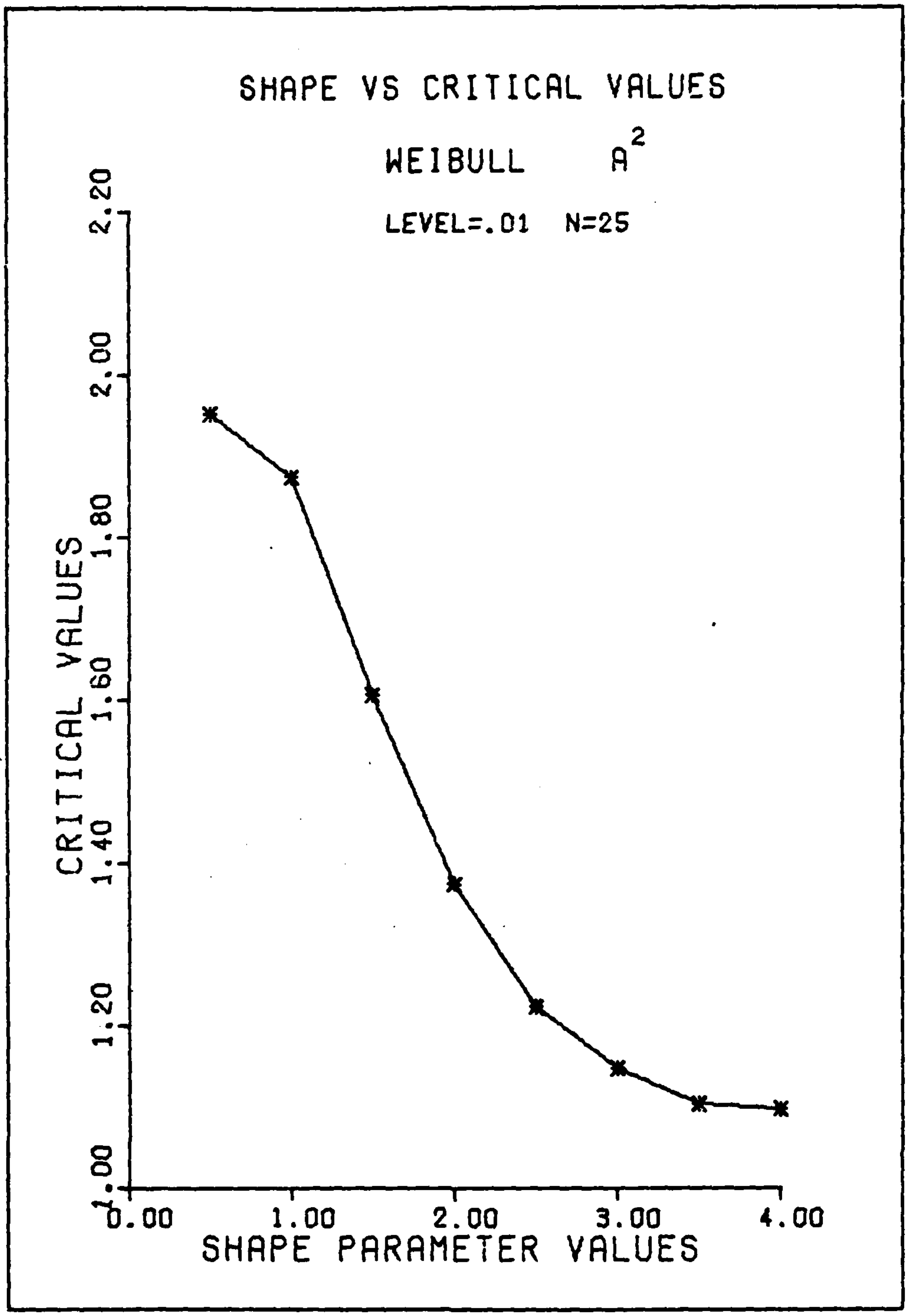

Fig. 67. Shape vs A^2 Critical Values, Level=.01, n=25

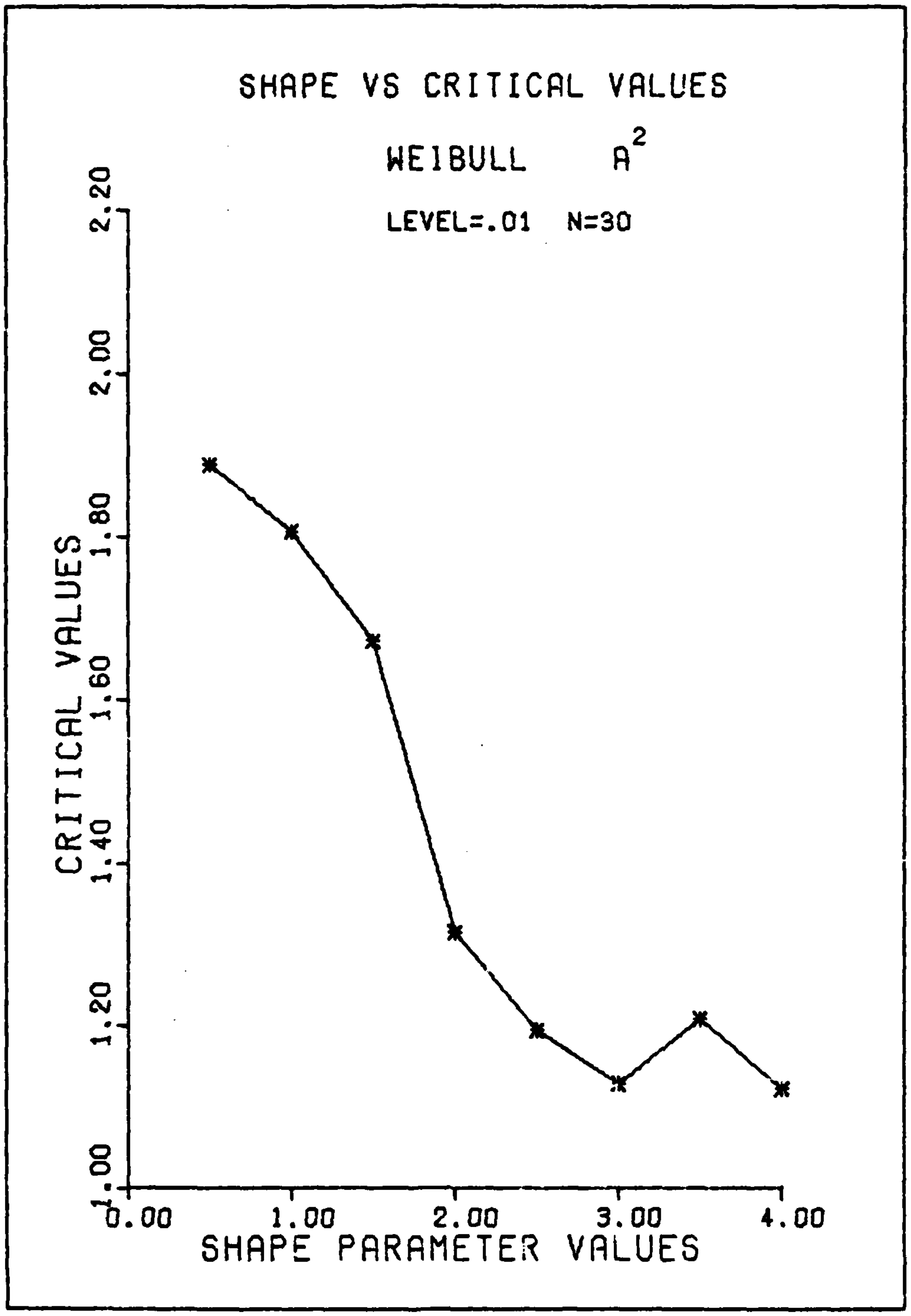

Fig. 68. Shape vs A^2 Critical Values, Level=.01, n=30

APPENDIX E

Computer Programs

Program to Calculate the Cramer-von Mises Critical Values

```
J10,T2000,I070,CM100000.T800613,BUSH,4022
ATTACH,IMSL,ID=LIBRARY,SN=ASD.
LIBRARY,IMSL.
FTN5,ANSI=0.
LGO.
*EOR
       PROGRAM CVM
C      ***************************************************************
C      ***************************************************************
C      *THIS PROGRAM GENERATES THE CVM STATISTICS                    *
C      *5000 REPS                                                    *
C      *THE TABLES GENERATED ARE VALID FOR THE WEIBULL DISTRIBUTION  *
C      *N=SAMPLE SIZE = 5,(5),30                                     *
C      *SS1=0 IF SCALE PARAMETER (THETA) IS KNOWN                    *
C      *SS1=1 IF THETA IS TO BE ESTIMATED                            *
C      *SS2=0 IF SHAPE(  K  ) IS KNOWN                               *
C      *SS2=1 IF ( K ) IS TO BE ESTIMATED                            *
C      *SS3=0 IF LOCATION(C) IS KNOWN                                *
C      *SS3=1 IF C IS TO BE ESTIMATED                                *
C      *C1=INITIAL ESTIMATION OF C (OR KNOWN VALUE)                  *
C      *T1=INITIAL ESTIMATION OF THETA (OR KNOWN VALUE)              *
C      *EK1=INITIAL ESTIMATION OF ALPHA (OR KNOWN VALUE)             *
C      ***************************************************************
C      ***************************************************************
       COMMON/RAY/Z(100),N
       COMMON/SAND/SS1,SS2,SS3,M,C1,T1,EK1,MR
       DOUBLE PRECISION DSEED
       DIMENSION FX(60),AA(5000),XX(5002),YY(5002)
       INTEGER REP,PP
       DSEED=10300.0D0
       MR=0
       REP=5002
       NOS=REP-2
       NUM=REP-2
C
C      * CALCULATES (I-.5)/N RANKS
C
       YY(1)=0
       YY(REP)=1
       DO 405 L=2,REP-1
       YY(L)=((L-1)-.5)/NOS
405    CONTINUE
       READ*,SS1,SS2,SS3,C1,T1,EK1
```

```
      PRINT*,SS1,SS2,SS3,C1,T1,EK1
      PRINT*
      PRINT*
      PRINT*
      PRINT '(2X,A,F5.1)','SHAPE = ',EK1
      PRINT '(2X,A)'    ,'---------------'
      PRINT*
      DO 100 PP=5,30,5
      N=PP
      M=N
      DO 99 KK=1,5000
      CALL GGWIB(DSEED,EK1,N,Z)
      DO 719 IK=1,N
      Z(IK) = 1.*Z(IK)+2.
719   CONTINUE
      CALL VSRTA(Z,N)
      CALL WEIBULL(CSJ,TSJ,EKSJ)
      DO 88 L=1,N
      FX(L)=1.-EXP(-((Z(L)-CSJ)/TSJ)**EKSJ)
88    CONTINUE
      WCVM = 0.
      XN=N
      DO 500 I=1,N
      XI = I
      WCVM = WCVM + (FX(I)-(2.*XI-1.) / (2.*XN))**2
500   CONTINUE
      WCVM = WCVM+1./(12.*XN)
      AA(KK) = WCVM
99    CONTINUE
      CALL VSRTA(AA,5000)
      DO 400 L = 1,REP-2
      XX(L+1) = AA(L)
400   CONTINUE
      CALL ENDPT(XX,YY,REP,NUM)
C
C     * PRINTS PERCENTILES
C
      PRINT '(2X,A,I2)','FOR N = ',PP
      PRINT '(2X,A)','---------------'
      PRINT*
      DO 410 J= 80,95,5
      DO 420 II= 1,REP
      I= REP + 1-II
      IF (YY(I).LT.(J/100.0))THEN
      SLOPE = (YY(I+1) - YY(I)) / (XX(I+1) - XX(I))
      ZZ= -SLOPE * XX(I) + YY(I)
```

```
      PRINT '(2X,A,I2,A,F9.4)',
     C'THE',J,'TH PERCENTILE IS',
     C((J/100.)-ZZ)/SLOPE
      PRINT*
      GOTO 410
      ENDIF
420   CONTINUE
410   CONTINUE
      DO 430 AK=1,REP
      K = REP+1-AK
      IF (YY(K).LT..99) THEN
      GOTO 999
      ENDIF
430   CONTINUE
999   SLOPE = (YY(K+1)-YY(K)) / (XX(K+1)-XX(K))
      ZZ= -SLOPE*XX(K) + YY(K)
      PRINT '(2X,A,F9.4)',
     C'THE 99TH PERCENTILE IS',
     C(.99-ZZ) / SLOPE
      PRINT*
      PRINT*
      PRINT*
      PRINT*
100   CONTINUE
      PRINT '(2X,F9.4,4X,F9.4,4X,F9.4)', CSJ,TSJ,EKSJ
      END
```

Program to Calculate the Anderson-Darling Critical Values

```
J20,T6000,IO70,CM100000,T800613,BUSH,4022
ATTACH,IMSL,ID=LIBRARY,SN=ASD.
LIBRARY,IMSL.
FTN5,ANSI=0.
LGO.
*EOR
      PROGRAM CVM
C     ****************************************************************
C     ****************************************************************
C     *THIS PROGRAM GENERATES THE A-D STATISTICS                     *
C     *5000 REPS                                                     *
C     *THE TABLES GENERATED ARE VALID FOR THE WEIBULL DISTRIBUTION   *
C     *N=SAMPLE SIZE= 5,(5),30                                       *
C     *SS1=0 IF SCALE PARAMETER (THETA) IS KNOWN                     *
C     *SS1=1 IF THETA IS TO BE ESTIMATED                             *
C     *SS2=0 IF SHAPE(  K  ) IS KNOWN                                *
C     *SS2=1 IF ( K ) IS TO BE ESTIMATED                             *
C     *SS3=0 IF LOCATION(C) IS KNOWN                                 *
C     *SS3=1 IF C IS TO BE ESTIMATED                                 *
C     *C1=INITIAL ESTIMATION OF C (OR KNOWN VALUE)                   *
C     *T1=INITIAL ESTIMATION OF THETA (OR KNOWN VALUE)               *
C     *EK1=INITIAL ESTIMATION OF ALPHA (OR KNOWN VALUE)              *
C     ****************************************************************
C     ****************************************************************
      COMMON/RAY/Z(100),N
      COMMON/SAND/SS1,SS2,SS3,M,C1,T1,EK1,MR
      DOUBLE PRECISION DSEED
      DIMENSION FX(60),AA(5000),XX(5002),YY(5002)
      INTEGER REP,PP
      DSEED=10000.0D0
      MR=0
      NZERO=0
      NONE=0
      REP=5002
      NUM=REP-2
      NOS=REP-2
C
C     *CALCULATES I-.5/N RANKS
C
      YY(1)=0
      YY(REP)=1
      DO 405 L=2,REP-1
      YY(L)=((L-1)-.5)/NOS
```

```
405   CONTINUE
      READ*,SS1,SS2,SS3,C1,T1,EK1
      PRINT*,SS1,SS2,SS3,C1,T1,EK1
      PRINT*
      PRINT*
      PRINT*
      PRINT '(2X,A,F5.1)','SHAPE = ',EK1
      PRINT '(2X,A)'     ,'---------------'
      PRINT*
      DO 100 PP=5,30,5
      N=PP
      M=N
      DO 99 KK=1,5000
      CALL GGWIB(DSEEF,EK1,N,Z)
      DO 719 IK=1,N
      Z(IK)=1.*Z(IK)+2.
719   CONTINUE
      CALL VSRTA(Z,N)
      CALL WEIBULL(CSJ,TSJ,EKSJ)
      DO 88 L=1,N
      FX(L)=1.-EXP(-((Z(L)-CSJ)/TSJ)**EKSJ)
      IF (FX(L).EQ.0.) THEN
      FX(L)=FX(L)+.0001
      NZERO=NZERO+1
      ENDIF
      IF (FX(L).EQ.1.) THEN
      FX(L) = FX(L) - .0001
      NONE=NONE+1
      ENDIF
88    CONTINUE
      WAD = 0.
      XN=N
      DO 500 I=1,N
      XI = I
      WAD=WAD+ (2.*XI-1)*(LOG(FX(I)) + LOG(1-FX(N+1-I)))
500   CONTINUE
      WAD = (-WAD/XN) - XN
      AA(KK) = WAD
99    CONTINUE
      CALL VSRTA(AA,5000)
      PRINT*
      PRINT*
      PRINT*,'NZERO= ',NZERO
      PRINT*,'NONE= ',NONE
      DO 400 L = 1,REP-2
      XX(L+1) = AA(L)
```

```
400   CONTINUE
      CALL ENDPT(XX,YY,REP,NUM)
C
C      * PRINTS PERCENTILES
C
      PRINT '(2X,A,I2)','FOR N = ',PP
      PRINT '(2X,A)','---------------'
      PRINT*
      DO 410 J= 80,95,5
      DO 420 II= 1,REP
      I= REP + 1-II
      IF (YY(I).LT.(J/100.0))THEN
      SLOPE = (YY(I+1) - YY(I)) / (XX(I+1) - XX(I))
      ZZ= -SLOPE * XX(I) + YY(I)
       PRINT '(2X,A,I2,A,F9.4)',
     C'THE',J,'TH PERCENTILE IS',
     C((J/100.)-ZZ)/SLOPE
      PRINT*
      GOTO 410
      ENDIF
420   CONTINUE
410   CONTINUE
      DO 430 AK=1,REP
      K = REP+1-AK
      IF (YY(K).LT..99) THEN
      GOTO 999
      ENDIF
430   CONTINUE
999   SLOPE = (YY(K+1)-YY(K)) / (XX(K+1)-XX(K))
      ZZ= -SLOPE*XX(K) + YY(K)
      PRINT '(2X,A,F9.4)',
     C'THE 99TH PERCENTILE IS',
     C(.99-ZZ) / SLOPE
      PRINT*
      RRINT*
      PRINT*
      PRINT*
100   CONTINUE
      PRINT '(2X,F9.4,4X,F9.4,4X,F9.4)', ESJ,TSJ,EKSJ
      END
```

Power Study

```
J50,T2000,I070,CM200000.T820041,BUSH,4022
ATTACH,IMSL,ID=LIBRARY,SN=ASD.
LIBRARY.IMSL.
FTN5,ANSI=0.
LGO,PL=10000.
*EOR
      PROGRAM POWER
C    ******************************************************************
C    ******************************************************************
C    *THIS PROGRAM GENERATES A POWER STUDY BETWEEN THE FOLLOWING:     *
C     *KS, CRAMER VON MISES, ANDERSON DARLING, AND CHI-SQUARE STAT.   *
C    *5000 REPS                                                       *
C    *THIS POWER STUDY IS VALID FOR THE WEIBULL DISTRIBUTION          *
C    *N = SAMPLE SIZE = 25                                            *
C    *SS1=0 IF SCALE PARAMETER THETA IS KNOWN                         *
C    *SS1=1 IF THETA IS TO BE ESTIMATED                               *
C    *SS2=0 IF SHAPE(  K  ) IS KNOWN                                  *
C    *SS2=1 IF ( K ) IS TO BE ESTIMATED                               *
C    *SS3=0 IF LOCATION(C) IS KNOWN                                   *
C    *SS3=1 IF C IS TO BE ESTIMATED                                   *
C    *C1=INITIAL ESTIMATION OF C (OR KNOWN VALUE)                     *
C    *T1=INITAL ESTIMATION OF THETA (OR KNOWN VALUE)                  *
C    *EK1=INITIAL ESTIMATION OF K (OR KNOWN VALUE)                    *
C    ******************************************************************
C    ******************************************************************
      COMMON/RAY/Z(100),N
      COMMON/SAND/SS1,SS2,SS3,M,C1,T1,EK1,MR
      DOUBLE PRECISION DSEED
      DIMENSION FX(60), FIX(60)
     C,FFX(10),ZZ(10),CELL(5) ,
     CAAWCVM(5000),AAWAD(5000),AAKS(5000),AACHI(5000)
      INTEGER PP
      DSEED=20000.0D0
      MR=0
      RWCVM=0.
      RWKS=0.
      RWAD=0.
      RCHISC=0.
      NZERO=0
      NONE=0
      DO 710 IN=1,6
      FFX(IN) = 0. + .2*(IN-1)
710   CONTINUE
      READ*,SS1,SS2,SS3,C1,T1,EK1
```

```
      PRINT*,SS1,SS2,SS3,C1,T1,EK1
      PRINT*
      PRINT*
      PRINT*
      PRINT '(2X,A,F5.1)','SHAPE = ',EK1
      PRINT '(2X,A)'    ,'---------------'
      PRINT*
      PP=25
      N=PP
      M=N
      DO 99 KK=1,5000
      CALL GGWIB(DSEED,2.0,N,Z)
      DO 719 IK = 1,N
      Z(IK) = 1.*Z(IK) + 2.
719   CONTINUE
      CALL VSRTA(Z,N)
      CALL WEIBULL(CSJ,TSJ,EKSJ)
      DO 88 L=1,N
      FX(L)=1.-EXP(-((Z(L)-CSJ)/TSJ)**EKSJ)
      FIX(L) = FX(L)
      IF (FIX(L).EQ. 0.) THEN
      FIX(L)=FIX(L)+.0001
      NZERO=NZERO+1
      ENDIF
      IF (FIX(L).EQ.1.) THEN
      FIX(L) = FIX(L) - .0001
      NONE=NONE+1
      ENDIF
88    CONTINUE
      DO 711 IN = 1,5
      CELL(IN) = 0.
711   CONTINUE
      DO 712 KI = 2,5
      ZZ(KI)=(((LOG(1./(1.-FFX(KI))))**(1./EKSJ))*TSJ)+CSJ
712   CONTINUE
      WCVM = 0.
      XN=N
      WAD = 0.
      TOP=0.0
      BOT=0.0
      DO 500 I=1,N
      XI = I
      RL=I
      IF(RL/XN-FIX(I) .GT. TOP)TOP=RL/XN-FIX(I)
      IF(FIX(I)-(RL-1)/XN .GT. BOT) BOT=FIX(I)-(RL-1)/XN
      WCVM = WCVM + (FX(I)-(2.*XI-1.) / (2.*XN))**2
      WAD=WAD+ (2.*XI-1)*(LOG(FIX(I)) + LOG(1-FIX(N+1-I)))
```

```
      IF (Z(I).LE.ZZ(2)) THEN
      CELL(1) = CELL(1) + 1
      ELSEIF  (Z(I).LE.ZZ(3)) THEN
      CELL(2) = CELL(2) + 1
      ELSEIF  (Z(I).LE.ZZ(4)) THEN
      CELL(3) = CELL(3) + 1
      ELSEIF  (Z(I).LE.ZZ(5)) THEN
      CELL(4) = CELL(4) + 1
      ELSE
      CELL(5) = CELL(5) + 1
      ENDIF
500   CONTINUE
      DIF=TOP
      IF(BOT .GT. DIF)DIF=BOT
      WKS = DIF
      AAKS(KK) = DIF
      IF (WKS.GT..2071) RWKS = RWKS + 1
      WCVM = WCVM+1./(12.*XN)
      AAWCVM(KK) = WCVM
      IF(WCVM.GT..1959) RWCVM = RWCVM + 1
      WAD = (-WAD/XN) - XN
      AAWAD(KK) = WAD
      IF (WAD.GT.1.2873) RWAD = RWAD + 1
      CHISQ=((CELL(1)-5.)**2)/5. + ((CELL(2)-5.)**2)/5. +
     C((CELL(3)-5.)**2)/5. + ((CELL(4)-5.)**2)/5. +
     C((CELL(5)-5.)**2)/5.
      AACHI(KK) = CHISQ
      IF(CHISQ.GT. 7.6 ) RCHISQ = RCHISQ +1
99    CONTINUE
      CALL VSRTA(AAKS,5000)
      CALL VSRTA(AAWCVM,5000)
      CALL VSRTA(AAWAD,5000)
      CALL VSRTA(AACHI,5000)
      PRINT*
      PRINT*
      PRINT*,'NZERO= ',NZERO
      PRINT*,'NONE= ',NONE
      PRINT*
      PRINT*, 'SAMPLE SIZE = ', PP
      PRINT*
      PRINT*
      PRINT*, 'TOTAL REJECTION % FOR K-S= ', RWKS/5000
      PRINT*
      PRINT*, 'TOTAL REJECTION % FOR WCVM= ', RWCVM/5000
      PRINT*
      PRINT*,'TOTAL REJECTION % FOR WAD= ', RWAD/5000
      PRINT*,'TOTAL REJECTION % FOR CHISQUARE= ', RCHISQ/5000
```

```
      PRINT*
      PRINT*, 'AAKS. AAWCVM,AAWAD,AACHI'
      DO 765 IJ = 1,5000
      PRINT '(2X,I4,4(F9.4))', IJ,AAKS(IJ), AAWCVM(IJ), AAWAD(IJ),
     CAACHI(IJ)
765   CONTINUE
       PRINT '(2X,F9.4,4X,F9.4,4X,F9.4)', CSJ,TSJ,EKSJ
       END
```

Harter and Moore Weibull Parameter Estimation Routine

```
C
C
C
C     COMPUTES MLE FOR WEIBULL
C
      SUBROUTINE WEIBULL(CSJ,TSJ,EKSJ)
C     *****************************
C     *****************************
      DIMENSION T(100),C(550),THETA(550),EK(550),X(56), Y(55)
      COMMON/RAY/Z(100),N
      COMMON/SAND/SS1,SS2,SS3,M,C1,T1,EK1,MR
      REAL EKSJ
      EN=N
      DO 2 I = 1,N
2     T(I)=Z(I)
      C(1)=C1
      THETA(1) = T1
      EK(1) = EK1
      IF (M) 66,66,32
32    CONTINUE
      EM=M
31    ELNM=0.
      EMR=MR
      MRP= MR+1
33    NM=N-M+1
      DO 34 I=NM,N
      EI=I
34    ELNM=ELNM+ALOG(EI)
      IF (MR) 66,35,74
74    DO 75 I=1,MR
      EI=I
75    ELNM=ELNM-ALOG(EI)
35    DO 30 J=1,550
      IF (J-1) 66,25,37
37    JJ=J-1
      SK=0.
      SL=0.
      DO 6 I=MRP,M
6     SK=SK+(T(I)-C(JJ))**EK(JJ)
      IF (SS1) 7,7,8
7     THETA(J)=THETA(JJ)
      GOTO 9
8     IF (MR) 66,19,20
```

```
19    THETA(J)=((SK+(EN-EM)*(T(M)-C(JJ))**EK(JJ))/EM)**(1./EK(JJ))
      GOTO 9
20    X(1)=THETA(JJ)
      LS=0
      DO 21 L=1,55
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-C(JJ))/X(L))**EK(JJ)
      Y(L)=-EK(JJ)*(EM-EMR)/X(L)+EK(JJ)*SK/X(L)**(EK(JJ)+1.)+EK(JJ)*(EN-
     1EM)*(T(M)-C(JJ))**EK(JJ)/X(L)**(EK(JJ)+1.)-EMR*EK(JJ)*ZRK*EXP(-ZRK
     2)/(X(L)*(1.-EXP(-ZRK)))
      IF (Y(L)) 53,73,54
53    LS=LS-1
      IF (LS+L) 58,55,58
54    LS=LS+1
      IF (LS-L) 58,56,58
55    X(LP)=.5*X(L)
      GOTO 61
56    X(LP)=1.5*X(L)
      GOTO 61
58    IF (Y(L)*Y(LL)) 60,73,59
59    LL=LL-1
      GOTO 58
60    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
61    IF (ABS(X(LP)-X(L))-1.E-4) 73,73,21
21    CONTINUE
73    THETA(J)=X(LP)
9     EK(J)=EK(JJ)
10    IF (SS2) 12,12,11
11    DO 17 I=MRP,M
17    SL=SL+ALOG(T(I)-C(JJ))
      X(1)=EK(J)
      LS=0
      DO 51 L=1,55
      SLK=0.
      DO 18 I=MRP,M
18    SLK=SLK+(ALOG(T(I)-C(JJ))-ALOG(THETA(J)))*(T(I)-C(JJ))**X(L)
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-C(JJ))/THETA(J))**X(L)
      Y(L)=(EM-EMR)*(1./X(L)-ALOG(THETA(J)))+SL-SLK/THETA(J)**X(L)+(EN-
     1EM)*(ALOG(THETA(J))-ALOG(T(M)-C(JJ)))*(T(M)-C(JJ))**X(L)/THETA(J)
     2**X(L)+EMR*ZRK*(ALOG(ZRK)/X(L))*EXP(-ZRK)/(1.-EXP(-ZRK))
      IF (Y(L)) 43,52,44
```

```
43    LS=LS-1
      IF (LS+L) 47,45,47
44    LS=LS+1
      IF (LS-L) 47,46,47
45    X(LP)=.5*X(L)
      GOTO 50
46    X(LP)=1.5*X(L)
      GOTO 50
47    IF (Y(L)*Y(LL)) 49,52,48
48    LL=LL-1
      GOTO 47
49    X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
50    IF (ABS(X(LP)-X(L))-1.E-4) 52,52,51
51    CONTINUE
52    EK(J)=X(LP)
12    C(J)=C(JJ)
62    IF (SS3) 25,25,14
14    IF (1.-EK(J)) 16,78,78
78    IF (SS1+SS2) 57,57,16
16    X(1)=C(J)
      LS=0
      DO 23 L=1,55
      SK1=0.
      SR=0.
      DO 15 I=MRP,M
      SK1=SK1+(T(I)-X(L))**(EK(J)-1.)
15    SR=SR+1./(T(I)-X(L))
      LL=L-1
      LP=L+1
      X(LP)=X(L)
      ZRK=((T(MRP)-X(L))/THETA(J))**EK(J)
      Y(L)=(1.-EK(J))*SR+EK(J)*(SK1+(EN-EM)*(T(M)-X(L))**(EK(J)-1.))
     1/THETA(J)**EK(J)-EMR*EK(J)*ZRK*EXP(-ZRK)/((T(MRP)-X(L))*(1.-EXP
     2(-ZRK)))
      IF (Y(L)) 39,24,40
39    LS=LS-1
      IF (LS+L) 70,41,70
40    LS=LS+1
      IF (LS-L) 70,42,70
41    X(LP)=.5*X(L)
      GOTO 22
42    X(LP)=.5*X(L)+.5*T(1)
      GOTO 22
70    IF (Y(L)*Y(LL)) 72,24,71
71    LL=LL-1
      GOTO 70
```

```
72      X(LP)=X(L)+Y(L)*(X(L)-X(LL))/(Y(LL)-Y(L))
22      IF(ABS(X(LP)-X(L))-1.E-4) 24,24,23
23      CONTINUE
24      C(J)=X(LP)
        GOTO 25
57      C(J)=T(1)
25      IF(MR) 66,38,69
38      DO 63 I=1,M
        IF(C(J)+1.E-4-T(I)) 68,67,67
67      MR=MR+1
63      C(1)=T(1)
68      IF(MR) 66,69,31
69      SK=0.
        SL=0.
        DO 36 I=MRP,M
        SK=SK+(T(I)-C(J))**EK(J)
36      SL=SL+ALOG(T(I)-C(J))
        ZRK=((T(MRP)-C(J))/THETA(J))**EK(J)
        EL=ELNM+(EM-EMR)*(ALOG(EK(J))-EK(J)*ALOG(THETA(J)))+(EK(J)-1.)*SL-
       1(SK+(EN-EM)*(T(M)-C(J))**EK(J))/(THETA(J)**EK(J))+EMR*ALOG(1.-EXP
       2(-ZRK))
        IF(J-3) 30,27,27
27      IF(ABS(C(J)-C(JJ))-1.E-4) 28,28,30
28      IF(ABS(THETA(J)-THETA(JJ))-1.E-4) 29,29,30
29      IF(ABS(EK(J)-EK(JJ))-1.E-4) 4,4,30
30      CONTINUE
4       CONTINUE
        CSJ=C(J)
        TSJ=THETA(J)
        EKSJ=EK(J)
66      RETURN
        END
*EOR
1.,0.,0.,0.,1.,1.
*EOR
*EOF
```

Program to Evaluate the Endpoints

```
C
C       *SUBROUTINE TO EVALUATE ENDPOINTS
C
        SUBROUTINE ENDPT(XX,YY,REP,NUM)
        INTEGER REP
        DIMENSION XX(REP), YY(REP)
        SLOPE=(YY(2)-YY(3)) / (XX(2)-XX(3))
        B = YY(2) - SLOPE*XX(2)
        V1 = -B/SLOPE
        IF (V1.LT.0.) THEN
        V1=0.
        ENDIF
        XX(1)=V1
        SLOPE=(YY(NUM)-YY(NUM+1)) / (XX(NUM)-XX(NUM+1))
        B1=YY(NUM) - SLOPE*XX(NUM)
        V2=(1.-B1)/SLOPE
        XX(REP)=V2
        RETURN
        END
```

Vita

John Gregory Bush was born on 29 December 1947 in Schenectady, New York. He graduated from Conestoga High School in Berwyn, Pennsylvania in 1965. He attended Old Dominion University in Norfolk, Virginia, and received a bachelor's degree in mathematics in 1969. He then entered the U.S. Air Force and attended Undergraduate Navigator Training School at Mather AFB, California. After receiving his wings in 1970, he flew in both the C-123 and OV-10 aircraft in Southeast Asia. He then flew the C-141 aircraft for two years at Charleston AFB, South Carolina. After spending a tour of duty as a Rescue Control Coordination Officer in the Panama Canal Zone, he returned to the U.S. to fly C-141 aircraft at McGuire AFB, New Jersey. He then entered the School of Engineering, Air Force Institute of Technology in June 1980.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The Anderson-Darling and Cramer-von Mises critical values are generated for the three-parameter Weibull distribution. The critical values are used for testing whether a set of observations follows a Weibull distribution when the scale and location parameters are unspecified and are estimated from the sample. A Monte Carlo simulation, with 5000 repetitions, is used to generate

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Block 20:

critical values for sample sizes 5(5)30 and Weibull shape parameters .5(.5)4.0.

A Monte Carlo power investigation of the Anderson-Darling and Cramer-von Mises tests is made using 5, 15, and 25 observations from ten alternate distributions. The power of the two tests are compared to the Kolmogorov-Smirnov and the Chi-Square tests. The power of all the tests are low with a sample size of five. When the hypothesized distribution is the Weibull with shape equal 1.0, the power of the tests in decreasing order are: Cramer-von Mises, Anderson-Darling, Kolmogorov-Smirnov, and Chi-Square. When the hypothesized distribution is the Weibull with shape equal 3.5, the power of the tests are the Anderson-Darling, followed by the Cramer-von Mises, Kolmogorov-Smirnov, and the Chi-Square test.

A relationship between the Anderson-Darling and Cramer-von Mises critical values with the Weibull shape parameters is investigated. The critical values of both of the tests are found to be a function of the inverse of the shape parameters.

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