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ELECTROMAGNETIC TRANSMISSION THROUGH AN APERTURE OF ARBITRARY SHAPE IN A CONDUCTING SCREEN

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this work the problem of electromagnetic transmission through an arbitrarily-shaped aperture (uncovered) or window (covered with a thin lossy dielectric sheet) in a perfectly conducting plane is treated. The method of moments is used to solve numerically the integral equation for the equivalent magnetic current. Triangular patching is used to conform to the arbitrary shape. Local position vectors are chosen as both the expansion functions and the testing functions. The centroid-pair matching is utilized to complete the approximation. A set of computer codes is presented and briefly described.

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20. ABSTRACT (Continued).

To illustrate the solutions, computations are given for various apertures (different shapes), windows (different dielectric materials), and half spaces (different media). Numerical results are also compared with other data, if available. For windows with proper thickness or half spaces with proper media, the phenomenon of aperture resonance is demonstrated.

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I. INTRODUCTION

Ever since the generalized network formulation for aperture problems was given in terms of the method of moments [1], solutions for particular problems have been obtained using particular subsections. For example, rectangular patches were used for rectangular apertures [2], and annular subsections were used for annular apertures [3]. Babinet's principle plus the wire-grid model of the complementary conducting plate have been used for arbitrarily-shaped apertures [4]. This approach is often satisfactory for far-field quantities and transmission coefficients, but is not appropriate for computing near-field quantities. This is because there are difficulties in relating computed wire currents to equivalent surface magnetic currents. Also, the accuracy of the wire-grid approximation can be questioned on theoretical grounds.

In this report, the problem of electromagnetic transmission through an arbitrarily-shaped aperture in an infinite conducting screen of zero thickness is investigated using triangular patches to model the aperture. The method of solution is, in general, a specialization of that for bodies of arbitrary shape by Rao [5]. In the formulation, the equivalence principle and image theory [6] are used to derive an integral equation for the equivalent magnetic currents. The moment method [7,8] is used to metricize this integral equation. The expansion functions are chosen to be local position vectors inside each triangular patch.

Extensions of the basic problem are also given. One extension is two half spaces with different media. Another is a lossy dielectric window covering the aperture. Computer programs are written and numerical results for the magnetic currents, transmission cross section patterns and transmission coefficients are given for several sample cases.

II. STATEMENT OF THE PROTOTYPE PROBLEM

The problem configuration to be considered is shown in Fig. 1. An infinite conducting screen with an arbitrarily-shaped aperture covers the entire xy-plane. The excitation of this aperture is an arbitrarily-polarized plane wave incident from the region $z > 0$ at an angle θ^i to the z-axis. The quantities to be computed are the equivalent magnetic current distribution and the transmission characteristics of the aperture.

As described in [1], we use the equivalence principle and image theory to obtain equivalent situations for both regions. The solution is expressed in terms of the equivalent magnetic current $\underline{M} = \underline{E} \times \hat{z}$ in the aperture. To compute $\underline{M}$, we use a linear expansion of basis functions $\underline{M}_n$ and moment methods to evaluate the coefficients. Hence, we have to determine a generalized admittance matrix and an excitation vector. To predict the transmission characteristics, we need a measurement vector. Since, the incident field is a plane were, the excitation vector is of the same form as the measurement vector.

III. FUNDAMENTAL FORMULATION

Refer to the generalized network formulation for aperture problems [1]. Define $\underline{M} = \sum_n V_n \underline{M}_n$ over the aperture region. Then

$$\vec{V} = [Y^a + Y^b]^{-1} \vec{I}^i \tag{1}$$

where

$$[Y^a] = [Y^b] = [Y^{hs}] = [<- W_m, H_t^{hs}(\underline{M}_n)>]_{N\times N}$$

$$= 2[Y^{fs}] = 2[< - W_m, H_t^{fs}(\underline{M}_n)>]_{N\times N} \tag{2}$$

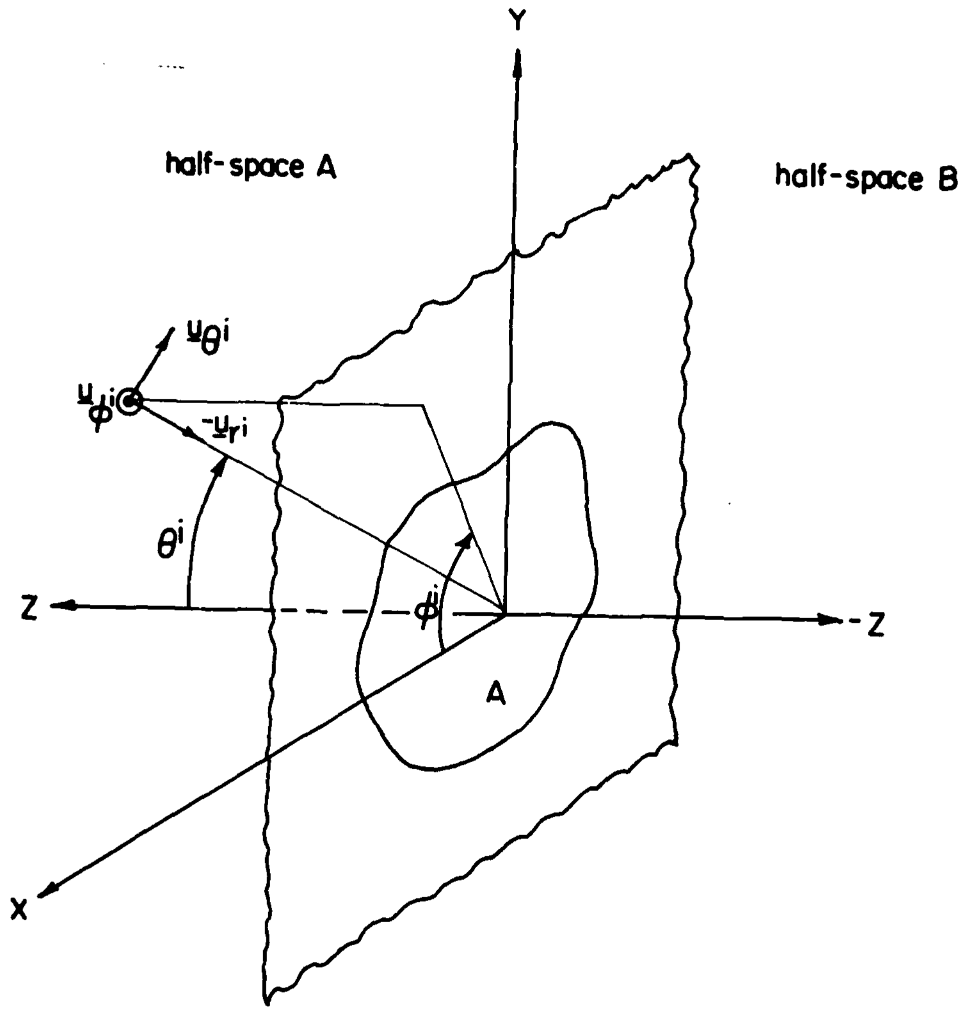


Fig. 1. Prototype problem configuration.

$$\vec{I}^{i} = [\langle W_m, H_t^{i}\rangle]_{N\times 1} = 2[\langle W_m, H_t^{io}\rangle]_{N\times 1} = 2\,\vec{I}^{io} \tag{3}$$

$$\vec{V} = [V_n]_{N\times 1}$$

Hence,

$$\vec{V} = \frac{1}{2}\,[\langle -W_m, H_t^{fs}(\underline{M}_n)\rangle]_{N\times N}^{-1}\,[\langle W_m, H_t^{io}\rangle]_{N\times 1} \tag{4}$$

In free space, the magnetic field produced by a source $\underline{M}_n$ is

$$\underline{H}(\underline{M}_n) = -\,j\omega\underline{F}_n - \nabla\Phi_n \tag{5}$$

where $\bar{F}_n$ and Φ_n are the electric vector potential and the magnetic scalar potential related to $\underline{M}_n$ as follows [6]

$$\underline{F}_n = \frac{\varepsilon}{4\pi}\iint_A \underline{M}_n \cdot G(k,\underline{r},\underline{r}')ds'$$

$$\Phi_n = \frac{1}{4\pi\mu}\iint_A m_n\, G(k,\underline{r},\underline{r}')ds'$$

$$m_n = \frac{-1}{j\omega}\nabla\cdot\underline{M}_n$$

where the free space Green's function is

$$G(k,\underline{r},\underline{r}') = \frac{e^{-jk|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|}$$

Hence, the element Y_{mn} in the admittance matrix is

$$\begin{aligned} Y_{mn}^{a} + Y_{mn}^{b} &= 4\,\langle -W_m, H_t^{fs}(\underline{M}_n)\rangle \\ &= 4\iint_{A_m} \underline{W}_m \cdot (j\omega\underline{F}_n + \nabla\Phi_n)ds' \\ &= 4\iint_{A_m} (j\omega\underline{W}_m \cdot \underline{F}_n - \Phi_n\nabla\cdot\underline{W}_m)ds' \\ &= 4j\omega\iint_{A} (\underline{W}_m \cdot \underline{F}_n + w_m\Phi_n)ds' \end{aligned} \tag{6}$$

IV. TRIANGULAR PATCHES AND BASIS FUNCTIONS

Different approaches to model apertures with some simple, or highly symmetrical shapes (e.g. slot, rectangle, circle) have been developed. Here we use triangular patches for the sake of being able to conform closely to arbitrarily-shaped apertures. There are other advantages: First, this triangular patch scheme is easily inputed to the computer, since the vertices can be independently specified. Second, it also provides the flexibility of having greater patch densities on those portions of the aperture where more resolution is desired, e.g. when we are concerned about the edge effect.

The presence of derivatives on the magnetic current and on the scalar magnetic potential suggests that we have to be careful in selecting the expansion functions and testing procedures in the method of moments. As Rao did for a scattering body [5],we choose a set of basis functions which yield a continuous magnetic current and a piecewise constant magnetic charge representation.

Assume that a suitable triangulation defined by an appropriate set of patches, edges, vertices, and boundary edges, such as shown in Fig. 2, has been found to approximate the aperture region A.

We associate $\underline{M}_n$ with the nth edge. As Fig. 3 shows, there are two triangles, T_n^+ and T_n^-, related to the nth edge (assumed not on the boundary) of a triangulated area modeling the aperture. The global position vector $\underline{r}$ and the local position vectors $\underline{\rho}_n^+$, $\underline{\rho}_n^-$ are defined as shown. The plus or minus designation of the triangles is determined by the choice of a positive current reference direction for the nth edge, which is assumed to be from T_n^+ to T_n^-. Define

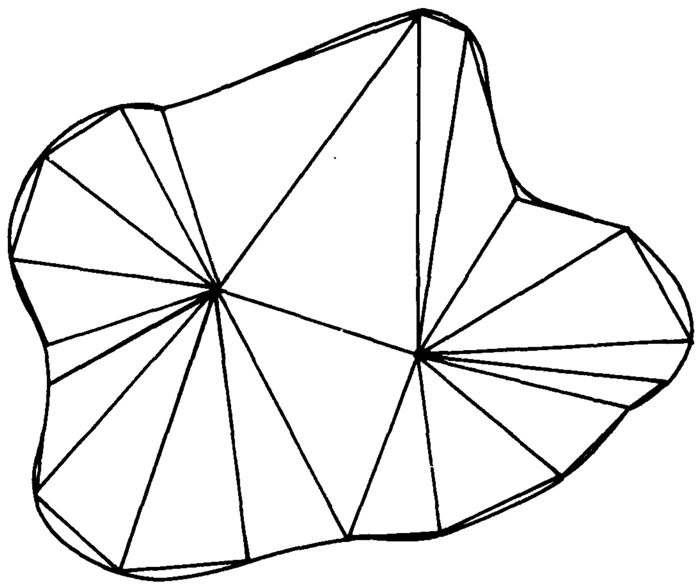


Fig. 2. Triangulation Example.

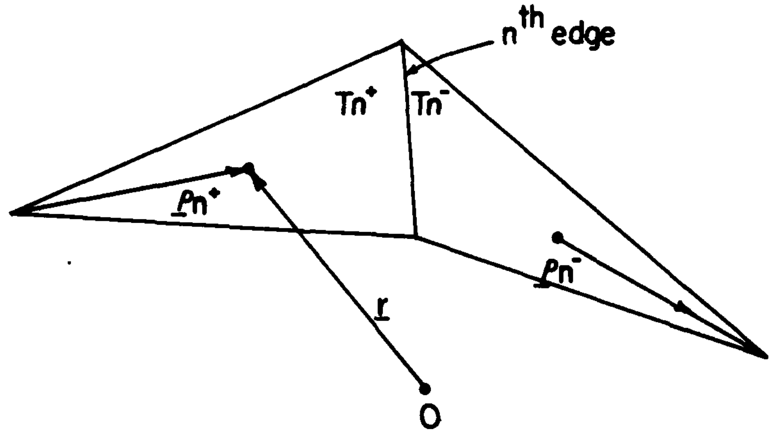


Fig. 3. Expansion function.

$$\underline{M}_n = \begin{cases} \dfrac{\ell_n}{2A_n^{\pm}} \underline{\rho}_n^{\pm}, & \underline{r} \text{ in } T_n^{\pm} \\ \\ \underline{0}, & \text{elsewhere} \end{cases} \tag{7}$$

where ℓ_n is the length of the edge n and $A_n^{\pm}$ is the area of triangle $T_n^{\pm}$.

As pointed out, $\underline{M}_n$ is related to the nth edge, which is not on the boundary of the aperture. Since the magnetic current must not have a normal component on the boundary, we need not define basis functions for any boundary edges. (See the reason in the following section.)

Using the basis functions above, we see that all edges of T_n^+ and T_n^- are free of magnetic line charges. For the common edge n, the normal component of the magnetic current is constant and continuous across the edge (see Fig. 4), shown as follows:

$$M^+_{n,\text{ normal}} = \frac{\ell_n}{2A_n^+} \frac{A_n^+}{\ell_n/2} = 1$$

$$M^-_{n,\text{ normal}} = \frac{\ell_n}{2A_n^-} \frac{A_n^-}{\ell_n/2} = 1$$

Hence, V_n may be interpreted as the normal component of the magnetic current density crossing the nth edge. For the other conjoined edges, $\underline{M}_n$ has no component normal to them, and hence no magnetic line charges exist along those edges.

With basis function $\underline{M}_n$ defined as above, the associated magnetic surface charge density is of the form of pulse doublets:

$$\begin{aligned} m_n &= -\frac{1}{j\omega} \nabla \cdot \underline{M}_n \\ &= -\frac{1}{j\omega} \frac{\pm 1}{\rho_n^{\pm}} \frac{\partial(\rho_n^{\pm} M_n)}{\partial \rho_n^{\pm}} \\ &= \frac{\mp \ell_n}{j\omega A_n^{\pm}}, \quad \underline{r} \text{ in } T_n^{\pm} \end{aligned} \tag{8}$$

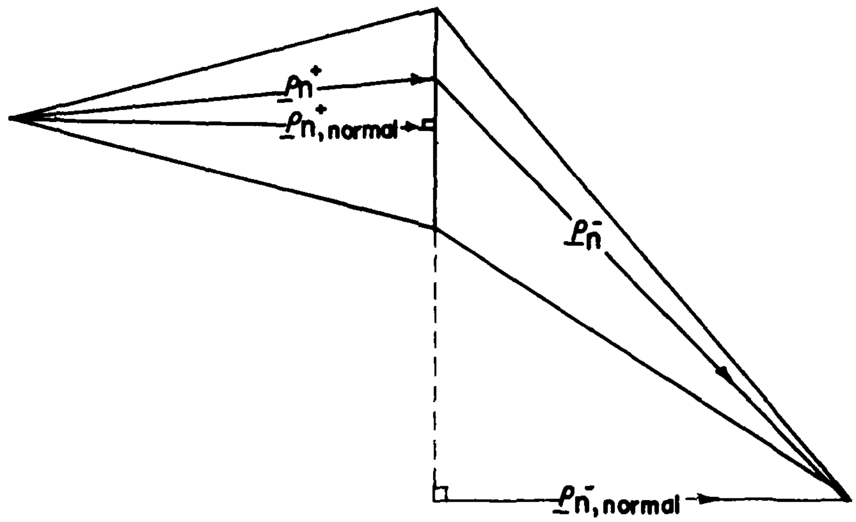


Fig. 4. Normal component crossing the edge.

Also, it can be proved that a superposition of the basis functions within a triangle is capable of representing a constant current flowing in an arbitrary direction within the triangle.

V. ADMITTANCE MATRIX

We start with Galerkin's method, then approximate the surface integral by averaging the integral with its value at the centroid of each triangle, i.e., with $\underline{W}_m = \underline{M}_m$.

$$\iint_{T_m} \Phi_n w_m ds = \iint_{T_m^+} \Phi_n \frac{-\ell_m}{j\omega A_m^+} dS + \iint_{T_m^-} \Phi_n \frac{\ell_m}{j\omega A_m^-} dS$$

$$= \frac{-\ell_m}{j\omega} \left[\frac{1}{A_m^+} \iint_{T_m^+} \Phi_n dS - \frac{1}{A_m^-} \iint_{T_m^-} \Phi_n dS\right]$$

$$\cong \frac{-\ell_m}{j\omega} \left[\Phi_n(\underline{r}_m^{c+}) - \Phi_n(\underline{r}_m^{c-})\right] \tag{9}$$

$$\iint_{T_m} \underline{W}_m \cdot \underline{F}_n \, dS = \iint_{T_m^+} \frac{\ell_m}{2A_m^+} \underline{\rho}_m^+ \cdot \underline{F}_n dS + \iint_{T_m^-} \frac{\ell_m}{2A_m^-} \underline{\rho}_m^- \cdot \underline{F}_n dS$$

$$= \frac{\ell_m}{2} \left[\frac{1}{A_m^+} \iint_{T_m^+} \underline{F}_n \cdot \underline{\rho}_m^+ dS + \frac{1}{A_m^-} \iint_{T_m^-} \underline{F}_n \cdot \underline{\rho}_m^- dS\right]$$

$$\cong \frac{\ell_m}{2} \left[\underline{F}_n(\underline{r}_m^{c+}) \cdot \underline{\rho}_m^{c+} + \underline{F}_n(\underline{r}_m^{c-}) \cdot \underline{\rho}_m^{c-}\right] \tag{10}$$

Here $\underline{r}_m = (\underline{r}_m^{1\pm} + \underline{r}_m^{2\pm} + \underline{r}_m^{3\pm})/3$ is the centroid of $T_m^{\pm}$. After these manipulations, element Y_{mn} of the admittance matrix becomes

$$Y_{mn} = 4j\omega \iint_{T_m} (\underline{W}_m \cdot \underline{F}_n + \Phi_n w_m) dS$$

$$\cong 4 \{ j\omega \ell_m [\underline{F}_n(\underline{r}_m^{c+}) \cdot \frac{\underline{\rho}_m^{c+}}{2} + \underline{F}_n(\underline{r}_m^{c-}) \frac{\underline{\rho}_m^{c-}}{2}] + \ell_m [\Phi_n(\underline{r}_m^{c-}) - \Phi_n(\underline{r}_m^{c+})] \}$$

$$= 4\ell_m \{ j\omega [\underline{F}_n(\underline{r}_m^{c+}) \cdot \frac{\underline{\rho}_m^{c+}}{2} + \underline{F}_n(\underline{r}_m^{c-}) \cdot \frac{\underline{\rho}_m^{c-}}{2}] + \Phi_n(\underline{r}_m^{c-}) - \Phi_n(\underline{r}_m^{c+}) \}$$

where

$$\underline{F}_n(\underline{r}_m^{c\pm}) = \frac{\varepsilon}{4\pi} \iint_{T_n^{\pm}} \underline{M}_n(\underline{r}') \cdot G(k, \underline{r}_m^{c\pm}, \underline{r}') dS'$$

$$\Phi_n(\underline{r}_m^{c\pm}) = \frac{-1}{4\pi j\omega\mu} \iint_{T_n^{\pm}} \nabla_s' \cdot \underline{M}_n(\underline{r}') \, G(k, \underline{r}_m^{c\pm}, \underline{r}') dS'$$

To evaluate $\underline{F}_n(\underline{r}_m^{c\pm})$ and $\Phi_n(\underline{r}_m^{c\pm})$, we proceed face by face for the sake of efficiency. Now,let us look into the case shown in Fig. 5, with observation triangle T_p and source triangle T_q. The number of basis functions for T_q is less than or equal to three.

$$\underline{\rho}_i = \pm (\underline{r}' - \underline{r}_i)$$

$$\underline{F}_i^{pq} \triangleq \underline{F}_{q_i}(\underline{r}_p^c)$$

$$\Phi_i^{pq} \triangleq \Phi_{q_i} (\underline{r}_p^c)$$

where i = 1,2,3. Hence,

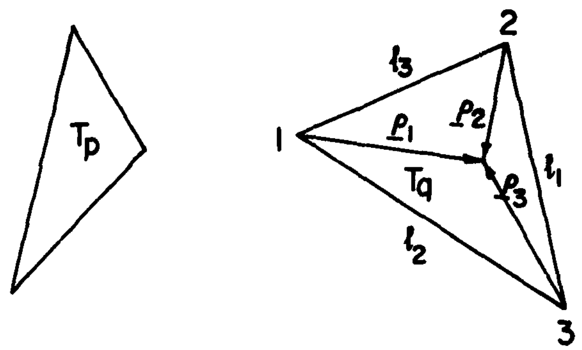


Fig. 5. Local index for source triangle.

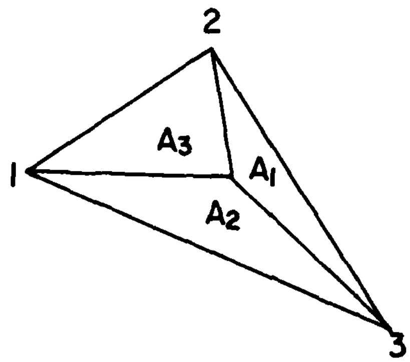


Fig. 6. Area coordinate.

$$\underline{F}_i^{pq} = \frac{\varepsilon}{4\pi} \iint_{T_q} \underline{M}_{q_i}(\underline{r}')\, G(k, \underline{r}_p^c, \underline{r}')dS'$$

$$= \pm \frac{\varepsilon}{4\pi} \iint_{T_q} \frac{\ell_i}{2A_q} \underline{\rho}_i\, G(k, \underline{r}_p^c, \underline{r}')dS \qquad i = 1,2,3$$

$$\Phi_i^{pq} = \frac{-1}{4\pi j\omega\mu} \iint_{T_q} \nabla_S' \cdot \underline{M}_{q_i}(\underline{r}')\, G(k, \underline{r}_p^c, \underline{r}')dS'$$

$$= \frac{\mp 1}{4\pi j\omega\mu} \iint_{T_q} \frac{\ell_i}{A_q} G(k, \underline{r}_p^c, \underline{r}')dS' \qquad i = 1,2,3$$

Now, make use of the area coordinate [8] for triangle T_q. (Check Fig. 6.)

$$\underline{r}' = \xi\underline{r}_1 + \eta\underline{r}_2 + \zeta\underline{r}_3$$

i.e.

$$(x,y) = \xi(x_1, y_1) + \eta(x_2, y_2) + \zeta(x_3, y_3)$$

where

$$\xi = A_1/A_q$$

$$\eta = A_2/A_q$$

$$\zeta = A_3/A_q = 1 - \xi - \eta$$

We transform the surface integrals for $\underline{F}_i^{pq}$ and Φ_i^{pq} into double integrals by the following formula

$$\iint_{T_q} f(\underline{r}')dS' = 2A_q \int_0^1 \int_0^{1-\eta} f(\xi\underline{r}_1 + \eta\underline{r}_2 + (1-\xi-\eta)\underline{r}_3)d\xi d\eta \qquad (12)$$

Here the limits come from the definition of area coordinate, and the constant factor $2A_q$ can be easily proved by using the constant integrand $f(\underline{r}') = 1$. We now have the following

$$\underline{F}_i^{pq} = \frac{\pm\,\varepsilon\ell_i}{4\pi}\,\frac{1}{2A_q}\iint_{T_q}(\xi\underline{r}_1 + \eta\underline{r}_2 + \zeta\underline{r}_3 - \underline{r}_i)\cdot G(k,\,R_p)dS'$$

$$= \frac{\pm\,\varepsilon\ell_i}{4\pi}\{\underline{r}_1\int_0^1\int_0^{1-\eta}\xi\,G(k,\,R_p)d\xi d\eta + \underline{r}_2\int_0^1\int_0^{1-\eta}\eta G(k,R_p)d\xi d\eta$$

$$+\,\underline{r}_3\int_0^1\int_0^{1-\eta}(1-\xi-\eta)\,G(k,\,R_p)d\xi d\eta - \underline{r}_i\int_0^1\int_0^{1-\eta}G(k,\,R_p)\,d\xi d\eta\}$$

$$\triangleq \frac{\pm\,\varepsilon\ell_i}{4\pi}(\underline{r}_1\,I_\xi^{pq} + \underline{r}_2\,I_\eta^{pq} + \underline{r}_3\,I_\zeta^{pq} - \underline{r}_i\,I^{pq}) \qquad i = 1,2,3 \tag{13}$$

$$\Phi_i^{pq} = \frac{\mp\,\ell_i}{4\pi j\omega\mu}\,\frac{1}{A_q}\iint_{T_q}G(k,R_p)\,dS'$$

$$= \frac{\mp\,\ell_i}{2\pi j\omega\mu}\int_0^1\int_0^{1-\eta}G(k,\,R_p)\,d\xi d\eta$$

$$\triangleq \mp\,\frac{\ell_i}{2\pi j\omega\mu}\,I^{pq} \qquad i = 1,2,3 \tag{14}$$

where

$$G(k,\,R_p) = \frac{e^{-jkR_p}}{R_p}$$

$$R_p = |\underline{r}_p^c - \xi\underline{r}_1 - \eta\underline{r}_2 - (1 - \xi - \eta)\,\underline{r}_3|$$

and

$$I^{pq} \triangleq \int_0^1\int_0^{1-\eta}G(k,\,R_p)\,d\xi d\eta$$

$$I_{\xi}^{pq} \triangleq \int_0^1 \int_0^{1-\eta} \xi G(k, R_p)\, d\xi d\eta$$

$$I_{\eta}^{pq} \triangleq \int_0^1 \int_0^{1-\eta} \eta G(k,R_p)\, d\xi d\eta$$

$$I_{\zeta}^{pq} \triangleq I^{pq} - I_{\xi}^{pq} - I_{\eta}^{pq}$$

For the numerical integration of these I's, one can refer to [9] and Rao's work [5].

Actually, many derivations above are parallel to those of [5]. The simplest way of showing this is by duality. Then the matrix equations $\vec{V} = Y^{-1}\, \vec{I}^i$ here and $\vec{I} = Z^{-1}\, \vec{V}^i$ in the body scattering problem are mathematically equivalent in the following way:

$$\vec{I}^i \rightarrow 2\vec{V}^i$$

$$Y \rightarrow 4Z$$

Hence

$$\vec{V} \leftarrow \frac{1}{2}\vec{I}$$

VI. EXCITATION AND MEASUREMENT VECTORS

For plane wave incidence, $\vec{I}^i$ and $\vec{I}^m$ are of the same form except for a minus sign. Therefore, we can evaluate both of them in a similar way.

With procedures similar to those in the previous section, i.e. Galerkin's method and centroid approximation, we have from (3)

$$\iint_A \underline{W}_m \cdot \underline{H}_t^{io} dS = \ell_m \left[\frac{1}{2A_m^+}\int_{T_m^+} \underline{H}_t^{io} \cdot \underline{\rho}_m^+ \, dS + \frac{1}{2A_m^-}\int_{T_m^-} \underline{H}_t^{io} \cdot \underline{\rho}_m^- \, dS\right]$$

$$\cong \frac{\ell_m}{2}\left[\underline{H}^{io}(\underline{r}_m^{c+}) \cdot \underline{\rho}_m^{c+} + \underline{H}_t^{io}(\underline{r}_m^{c-}) \cdot \underline{\rho}_m^{c-}\right] \qquad (15)$$

Hence

$$I^i_m = 2 \iint_A \underline{W}_m \cdot \underline{H}^{io}_t \, dS$$

$$\cong 2\ell_m [\underline{H}^{io}_t(\underline{r}^{c+}_m) \cdot \frac{\underline{\rho}^{c+}_m}{2} + \underline{H}^{io}_t(\underline{r}^{c-}_m) \cdot \frac{\underline{\rho}^{c-}_m}{2}] \tag{16}$$

where

$$\underline{H}^{io}_t(\underline{r}^{c\pm}_m) = (\underline{U}_{\theta^i} H^i_\theta + \underline{U}_{\phi^i} H^i_\phi)\, e^{j\underline{k}^i \cdot \underline{r}^{c\pm}_m}$$

$$\underline{k}^i = - \underline{U}_{r^i} k^i$$

$$= - k^i(\hat{\underline{x}} \sin \theta^i \cos \phi^i + \hat{\underline{y}} \sin \theta^i \sin \phi^i + \hat{\underline{z}} \cos \theta^i)$$

By a similar formulation, we have

$$I^m_n = - 2 \iint_A \underline{M}_n \cdot \underline{H}^{mo}_t \, dS$$

$$\cong - 2\ell_n [\underline{H}^{mo}_t(\underline{r}^{c+}_n) \cdot \frac{\underline{\rho}^{c+}_n}{2} + \underline{H}^{mo}_t(\underline{r}^{c-}_n) \cdot \frac{\underline{\rho}^{c-}_n}{2}] \tag{17}$$

where

$$\underline{H}^{mo}_t(\underline{r}^{c\pm}_n) = (\underline{U}_{\theta^m} H^m_\theta + \underline{U}_{\phi^m} H^m_\phi) e^{-j\underline{k}^m \cdot \underline{r}^{c\pm}_n}$$

$$\underline{k}^m = - \underline{U}_{r^m} k^m$$

$$= - k^m(\hat{\underline{x}} \sin \theta^m \cos \phi^m + \hat{\underline{y}} \sin \theta^m \sin \phi^m + \hat{\underline{z}} \cos \theta^m)$$

VII. REPRESENTATIVE QUANTITIES TO BE CALCULATED

We first calculate the equivalent current $\underline{M} = \sum_n V_n \underline{M}_n$, then the equivalent charge density $m = \frac{-1}{j\omega} \nabla \cdot \underline{M}$, the far-field H_m, the incident power P_{inc}, and the power transmitted P_{trans}. We can find the transmission cross section patterns τ, the transmission coefficient T, and the transmission area TA. They are listed as follows: (most of the derivations are in [1].)

$$\vec{V} = [Y^a + Y^b]^{-1}\,\vec{I}^i$$

$$= \frac{1}{4}\,[Y^{fs}]^{-1}\cdot 2\,\vec{I}^{io}$$

$$= \frac{1}{2}\,[Y^{fs}]^{-1}\,\vec{I}^{io} \tag{18}$$

$$m_n = \begin{cases} \dfrac{\mp\,\ell_n}{j\omega A_n^{\pm}}\,, & \text{in } T_n^{\pm} \\ 0\,, & \text{elsewhere} \end{cases}$$

$$H_m = \frac{-j\omega\varepsilon}{8\pi r_m}\,e^{-jkr_m}\,\tilde{P}^m\,[Y^{hs}]^{-1}\,\vec{P}^i$$

$$= \frac{-j\omega\varepsilon}{8\pi r_m}\,e^{-jkr_m}\cdot 2\,\tilde{I}^{mo}\,\frac{1}{2}\,[Y^{fs}]^{-1}\cdot 2\,\vec{I}^{io}$$

$$= \frac{-j\omega\varepsilon}{4\pi r_m}\,e^{-jkr_m}\,\tilde{I}^{mo}\,[Y^{fs}]^{-1}\,\vec{I}^{io}$$

$$\tau = 2\pi r_m^2\cdot\eta|H_m|^2/\eta|H^{io}|^2$$

$$= \frac{\omega^2\varepsilon^2}{32\pi}\,|\tilde{P}^m\quad[Y^{hs}]^{-1}\,\vec{P}^i|^2/|H^{io}|^2$$

$$= \frac{\omega^2\varepsilon^2}{8\pi}\,|\tilde{I}^m\,\vec{V}|^2/|H^{io}|^2 \tag{19}$$

$$P_{inc} = \eta|H^{io}|^2\,S\cos\theta^i$$

$$P_{trans} = \mathrm{Re}(\tilde{V}[Y^{hs}]^{*}\vec{V}^{*}$$

$$= \mathrm{Re}(\frac{1}{2}\,\tilde{V}\cdot\vec{I}^{i*})$$

$$T = \frac{\frac{1}{2}\,\mathrm{Re}(\tilde{V}\,\vec{I}^{i*})}{\eta|H^{io}|^2\,S\cos\theta^i} \tag{20}$$

$$TA = T \cdot S \cos \theta^{i}$$

$$= \mathrm{Re}(\tilde{V}\, \vec{I}^{i*})/2\eta |H^{io}|^2 \qquad (21)$$

VIII. NUMERICAL RESULTS AND DISCUSSION FOR THE PROTOTYPE CASES

Using the previous formulation and adopting quite a few subroutines from [5], we have developed a versatile computer program. This program can solve not only the prototype problems, but also both extensions mentioned in Section I. It is described and listed in Section XIV. Some representative computations for the prototype cases are given in this section. To ensure the validity of our formulation, we examined several special examples which are available in the literature. As we will see, the results agree very well.

The first example is a narrow slot, width $\lambda/20$ and variable length L, lying on the x-y plane with axis in the y direction. This slot aperture is illuminated by a normally incident plane wave with unit magnetic field polarized in the ϕ direction. Figure 7 shows the configuration. As shown in Fig. 8, it is triangulated into 40 patches. Figure 9 shows the transmission cross section patterns in two principal planes, i.e. τ_θ at ($\phi = 90°$, $\theta = 90 \rightarrow 180°$) and ($\phi = 270°$, $\theta = 180° \rightarrow 90°$), τ_ϕ at ($\phi = 0°$, $\theta = 90° \rightarrow 180°$) and ($\phi = 180°$, $\theta = 180° \rightarrow 90°$). Figure 10 plots the equivalent magnetic current in the aperture region. Table 1 shows the physical area A, transmission coefficient T, transmission area TA of the corresponding cases. From our data, we found good agreement with earlier results. The only case of significant discrepancy is the far-field for $L = \lambda/4$, and it is believed that our result is correct.

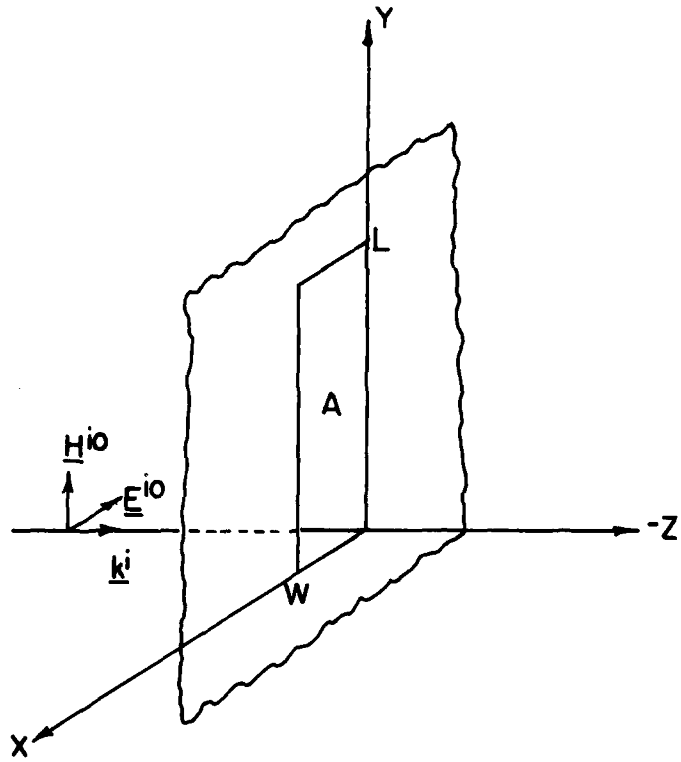


Fig. 7. Slot aperture under unit $\underline{H}^{i}$ normal incidence.

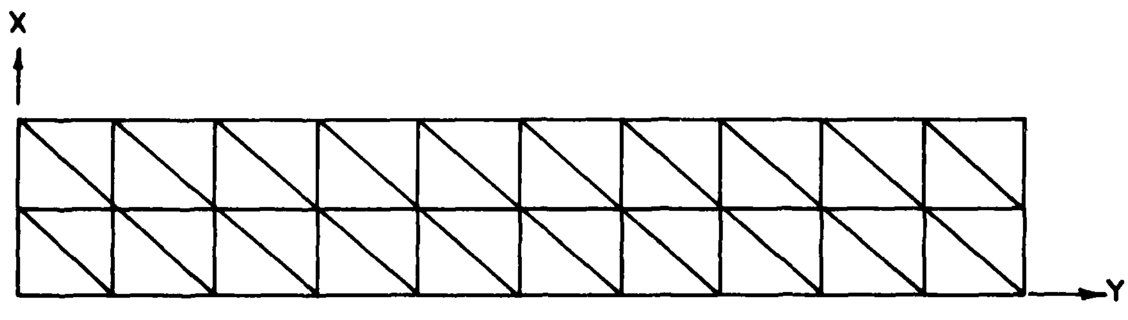


Fig. 8. Triangulation of the slot aperture.

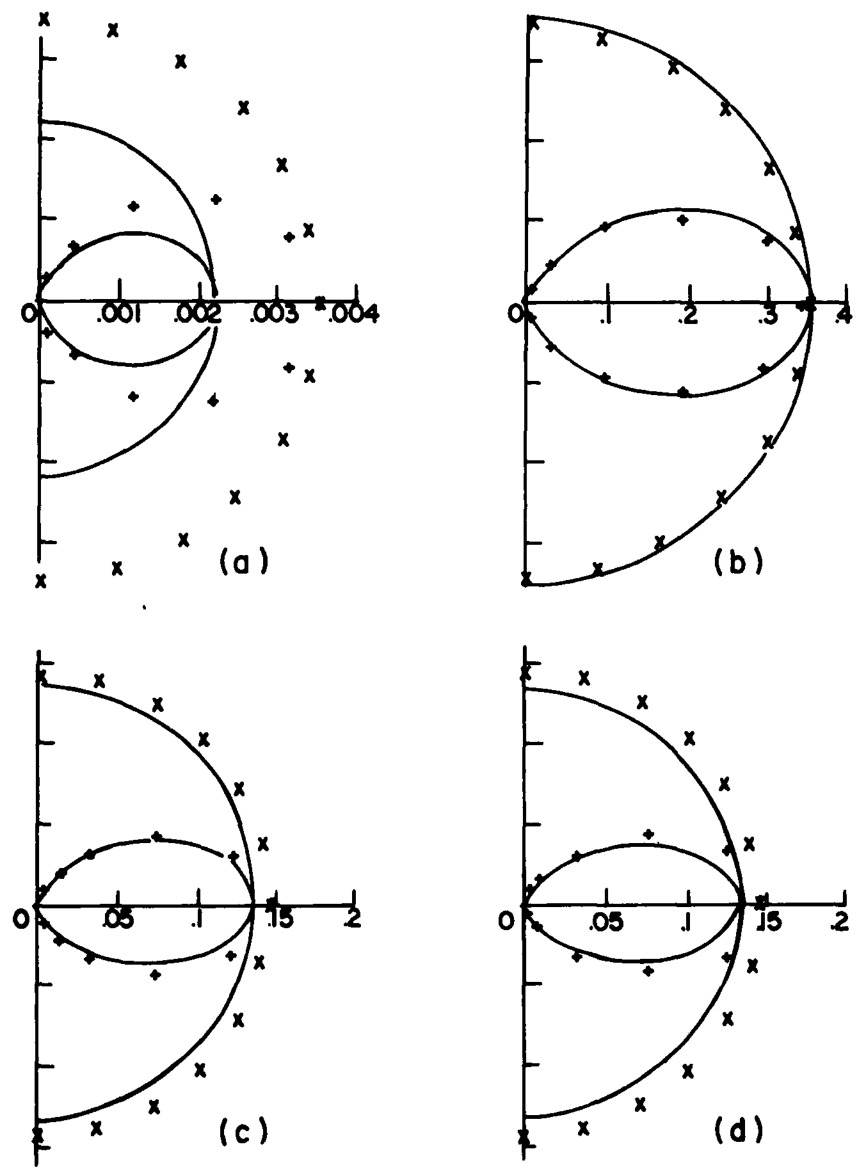


Fig. 9. Transmission cross section for slot aperture of Fig. 7.
$W = \lambda/20$; $L = \lambda/4$, $\lambda/2$, $3\lambda/4$, λ in (a), (b), (c), (d).

+: computed τ_θ/λ^2 in principal plane.

×: computed τ_ϕ/λ^2 in principal plane.

-: corresponding results from [2].

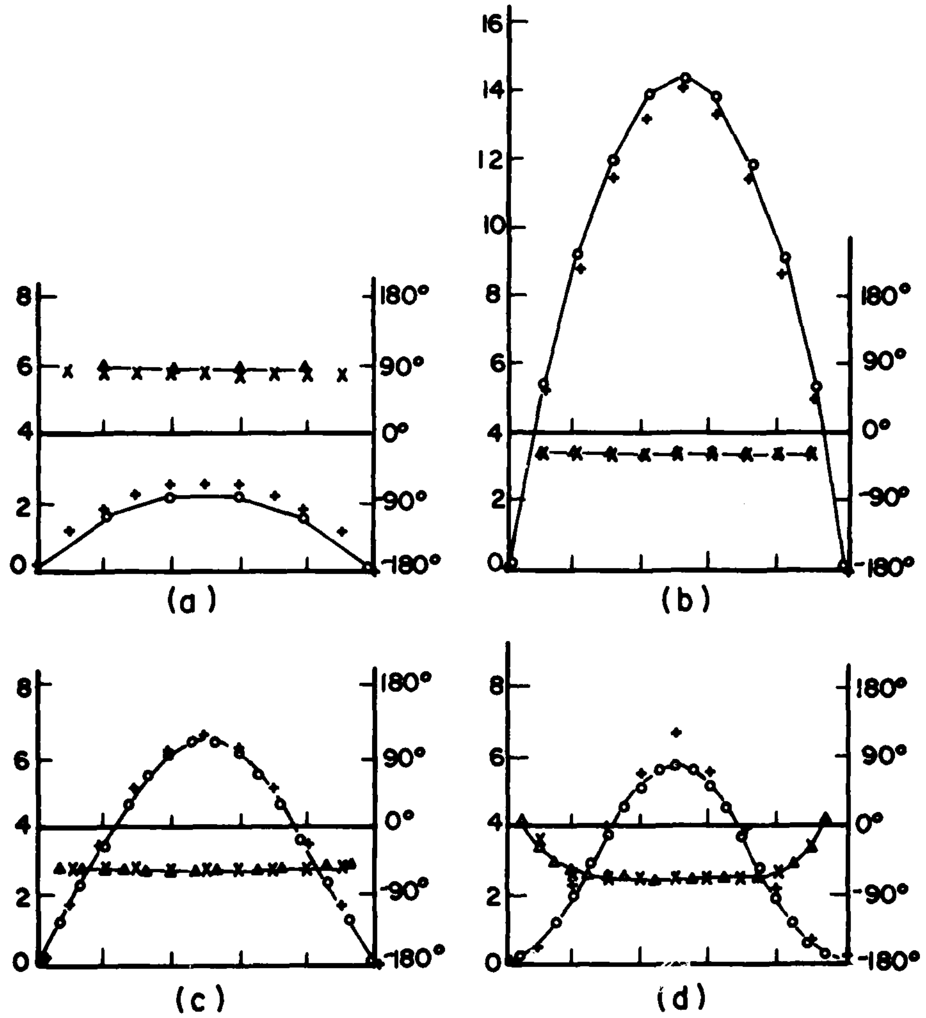


Fig. 10. Equivalent magnetic currents for slot of Fig. 7.

$\underline{M}$ is in y-direction.

$W = \lambda/20$; $L = \lambda/4, \lambda/2, 3\lambda/4, \lambda$ in (a), (b), (c), (d).

+: computed magnitude of $|\underline{M}/\underline{E}^{io}|$.

×: computed phase of $|\underline{M}/\underline{E}^{io}|$.

0,Δ: corresponding results from [2].

Table 1. Transmission coefficient for slot of Fig. 7, $W = \lambda/20$.

$L = \lambda/4$	A	$0.12500E - 01\ (\lambda^2)$
	T	$0.17950E + 00$
	TA	$0.22438E - 02\ (\lambda^2)$
$L = \lambda/2$	A	$0.25000E - 01\ (\lambda^2)$
	T	$0.81829E + 01$
	TA	$0.20457E + 00\ (\lambda^2)$
$L = 3\lambda/4$	A	$0.37500E - 01\ (\lambda^2)$
	T	$0.21401E + 01$
	TA	$0.80254E - 01\ (\lambda^2)$
$L = \lambda$	A	$0.50000E - 01\ (\lambda^2)$
	T	$0.15163E + 01$
	TA	$0.75815E - 01\ (\lambda^2)$

The second check is made for a square aperture lying in the x-y plane. It is illuminated by a normally incident plane wave with unit magnetic field polarized in the ϕ direction. This square aperture is triangulated into 32 patches as shown in Fig. 11. The computed far-field patterns match almost perfectly the results from [2], which are plotted in Fig. 12. For near field quantities, there is no available data to compare with. Nevertheless, our results shown in Fig. 13, seem to be reasonable.

For a third check, we consider a circular aperture which is triangulated into 24 patches as shown in Fig. 14. There are two things

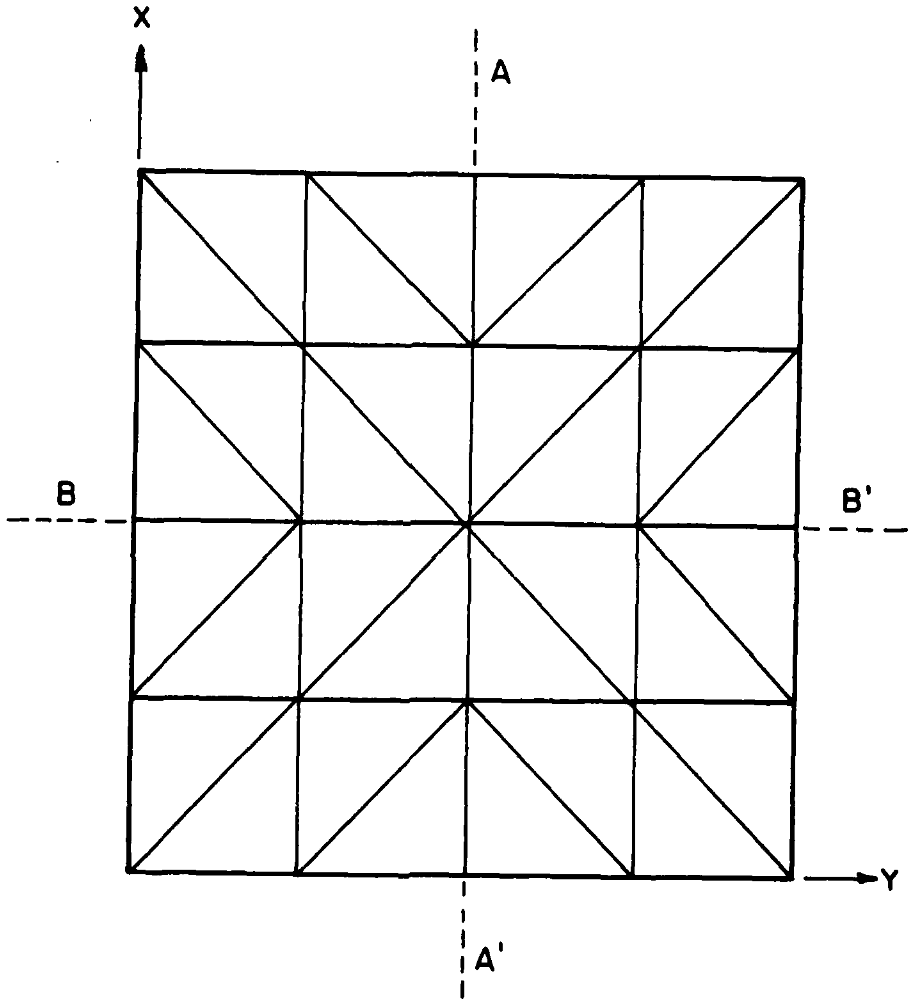
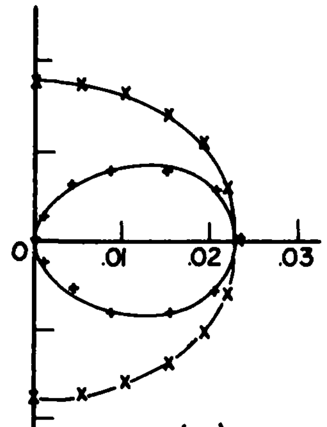


Fig. 11. Triangulation of the square aperture.

Illuminated by a normal incident plane wave with unit magnetic field polarized in ϕ-direction.

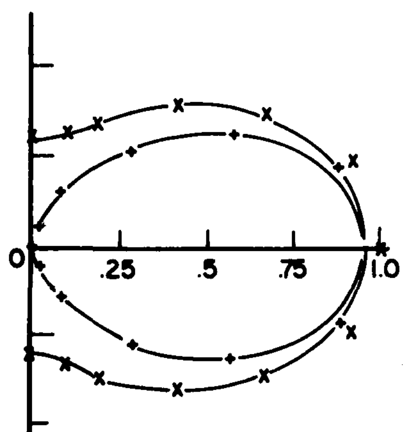


A = 0.62500E-01 (λ^2)

T = 0.21483 E+00

TA= 0.13427E-01 (λ^2)

(a)



A = 0.25000 E+00(λ^2)

T = 0.15673 E+01

TA= 0.39183 E+00 (λ^2)

(b)

Fig. 12. Transmission characteristics for square aperture in Fig. 11.

All the notation and marks are similar to Fig. 9 and Table 1. But L = $\lambda/4$, $\lambda/2$ in (a), (b).

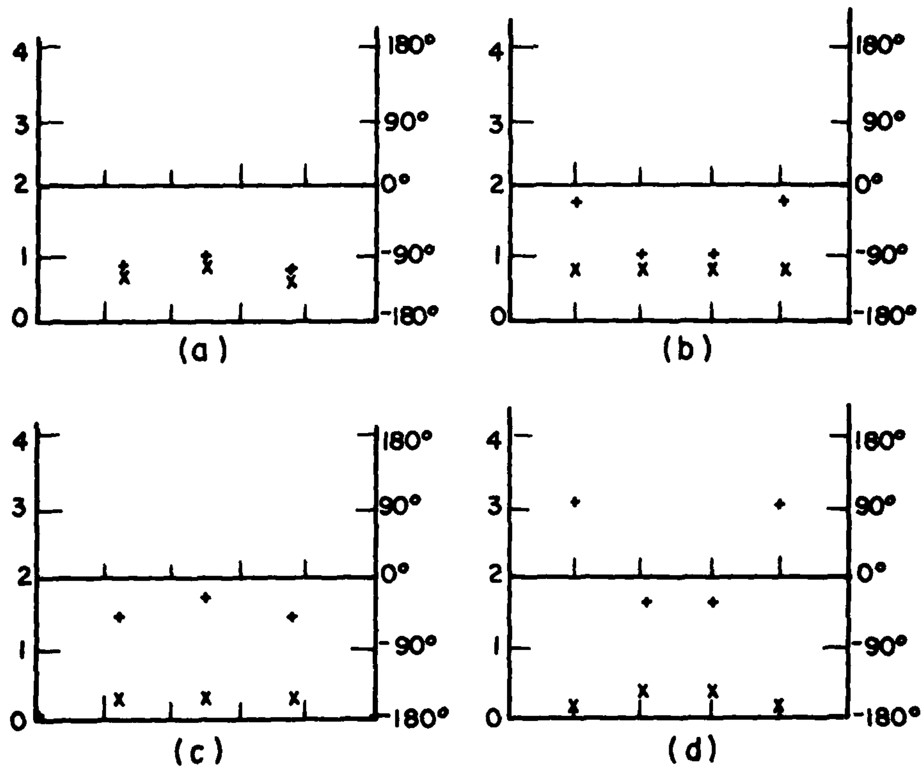


Fig. 13. Magnitude and phase of $|\underline{M}/\underline{E}^{io}|$ of a square aperture.

(a), $L = \lambda/4$; cut at BB'

(b), $L = \lambda/4$; cut at AA'

(c), $L = \lambda/2$; cut at BB'

(d), $L = \lambda/2$; cut at AA'

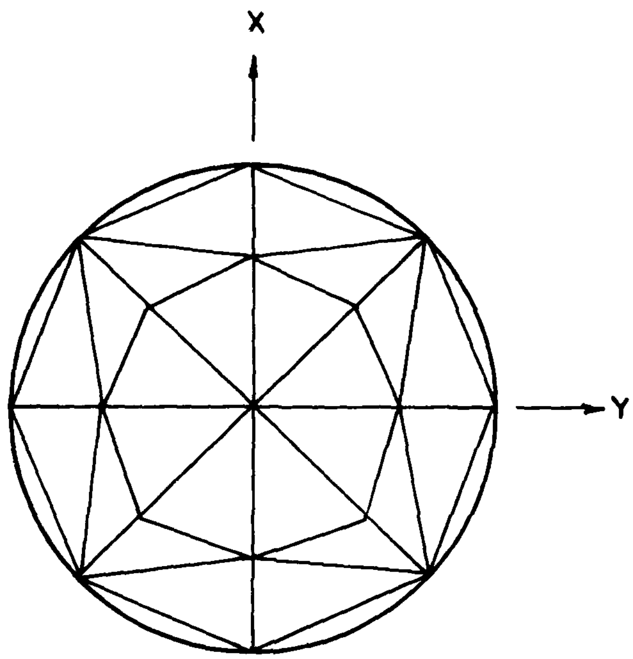


Fig. 14. Triangulation of a circular aperture.

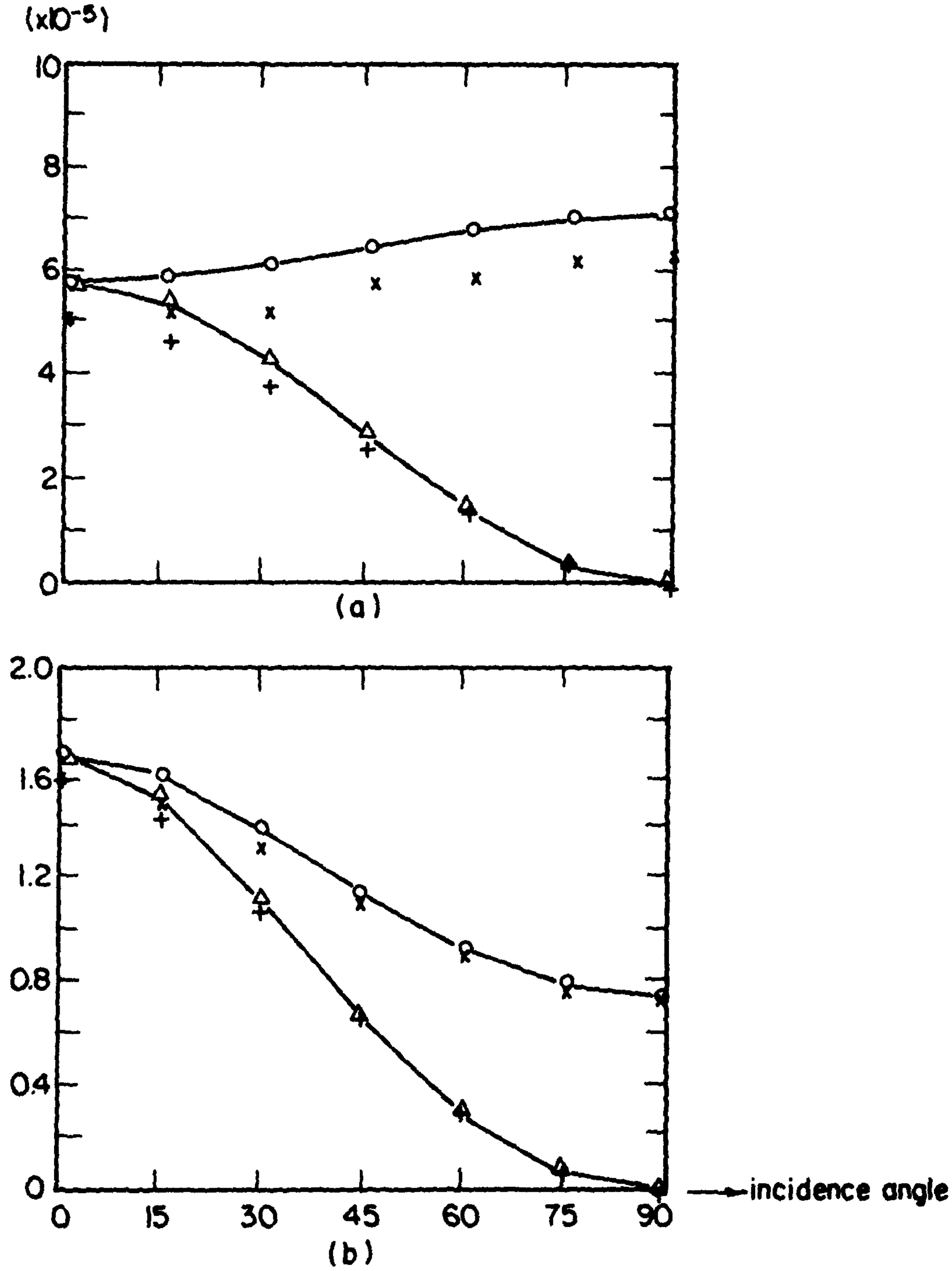


Fig. 15. Transmission coefficient for a circular aperture.
$R = 0.02\lambda$, 0.25λ in (a), (b).
+: computed result for $E_{\perp}$-polarization.
×: computed result for $E_{\parallel}$-polarization
Δ,O: corresponding data from [3].

to notice. First, to compensate for the loss in total area, we should put the boundary vertices outside the circle so that we get the correct total aperture area. Second, to take care of the edge effect, we need a higher patch density around the boundary than in the center. Now this circular aperture is excited by an obliquely incident plane wave with either parallel or perpendicular polarization [3]. To compare with the data available, we redefine the transmission coefficient, denoted as TCHA, instead of the previous T. It is normalized with respect to the incident power density at normal incidence rather than the actual incident power density at oblique incidence. As can be seen from Fig. 15, our computed data agree well with the previous literature [3]. The slight discrepancy probably is due to the edge effect plus the difficulty in matching the exactly circular boundary with straight line segments.

So far, all the cases we have tried are just validity checks. Obviously, for any rectangular aperture (including the narrow slot, the square aperture, etc.), triangular patching is not superior to the rectangular patching in [2]. Also, for any annular aperture (including circular aperture), triangular patching is not superior to the annular subsections used in [3].

To show the versatility of the triangulation method, we try two other shapes. These are a diamond-shaped aperture and a cross-shaped aperture. Our formulation treats them without difficulty . Figures 16 and 17 demonstrate some simple ways to triangulate the diamond aperture and cross aperture into 12 patches and 20 patches.

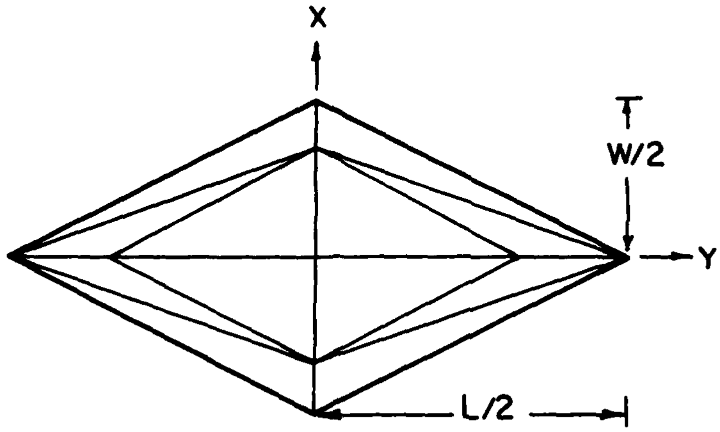


Fig. 16. Triangulation of a diamond aperture.

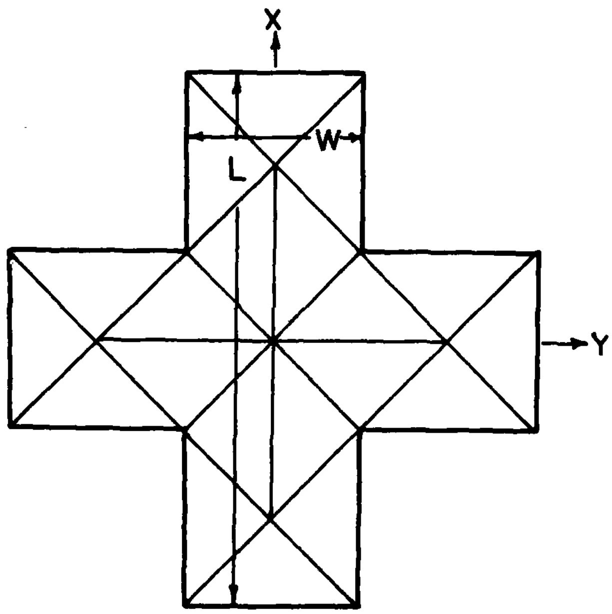


Fig. 17. Triangulation of a cross aperture.

The examples worked out are for small apertures. The transmission cross section patterns in the principal planes are the usual transmission patterns, only the magnitude is small (peak values are between 10^{-5} and 10^{-7}). We do not plot the pattern. Instead, a list of the transmission coefficient and transmission area are given in Tables 2 and 3.

Table 2. Transmission characteristics for a diamond-shaped aperture as in Fig. 16.

W = 0.5L	L = 0.1λ	A	0.25000E - 02	(λ^2)
		T	0.47906E - 03	
		TA	0.11976E - 05	(λ^2)
	L = 0.2λ/3	A	0.11111E + 00	(λ^2)
		T	0.92387E - 04	
		TA	0.10265E - 04	(λ^2)
	L = 0.05λ	A	0.62500E - 03	(λ^2)
		T	0.28988E - 04	
		TA	0.18118E - 07	(λ^2)
W = 0.25L	L = 0.1λ	A	0.12500E - 02	(λ^2)
		T	0.49212E - 03	
		TA	0.61515E - 06	(λ^2)
	L = 0.2λ/3	A	0.55556E - 01	(λ^2)
		T	0.94904E - 04	
		TA	0.52725E - 05	(λ^2)
	L = 0.05λ	A	0.31250E - 03	(λ^2)
		T	0.29776E - 04	
		TA	0.93050E - 08	(λ^2)

Table 3. Transmission Characteristics for a cross-shaped aperture as in Fig. 17.

W = L/3	L = 0.1λ	A	0.55556E - 02	(λ^2)
		T	0.13420E - 02	
		TA	0.74556E - 05	(λ^2)
	L = 0.2λ/3	A	0.24691E + 00	(λ^2)
		T	0.25184E - 03	
		TA	0.62182E - 04	(λ^2)
	L = 0.05λ	A	0.13889E - 02	(λ^2)
		T	0.78284E - 04	
		TA	0.10873E - 06	(λ^2)
W = 0.25L	L = 0.1λ	A	0.43750E - 02	(λ^2)
		T	0.11586E - 02	
		TA	0.50689E - 05	(λ^2)
	L = 0.2λ/3	A	0.19444E + 00	(λ^2)
		T	0.21751E - 03	
		TA	0.42932E - 04	(λ^2)
	L = 0.05λ	A	0.10938E - 02	(λ^2)
		T	0.67616E - 04	
		TA	0.73955E - 07	(λ^2)

IX. EXTENSION I: HALF SPACES WITH DIFFERENT MEDIA

Assume two half spaces with different media, ε^a and ε^b, separated by an infinite conducting plane with an arbitrarily shaped aperture. The method of solution will be almost the same, except for some minor differences.

First, instead of $Y_{mn} = 4 <-\underline{W}_m, \underline{H}_t^{fs}(\underline{M}_n)>$, the elements in the admittance matrix will be

$$Y_{mn}^a + Y_{mn}^b = 2<-\underline{W}_m, \underline{H}_t^{as}(\underline{M}_n) + \underline{H}_t^{bs}(\underline{M}_n)>$$

$$\cong 2\ell_m\{j\omega[\underline{F}_n^a(\underline{r}_m^{c+}) \cdot \frac{\underline{\rho}_m^{c+}}{2} + \underline{F}_n^a(\underline{r}_m^{c-}) \cdot \frac{\underline{\rho}_m^{c-}}{2}] + \Phi_n^a(\underline{r}_m^{c-}) - \Phi_n^a(\underline{r}_m^{c+})\}$$

$$+ 2\ell_m\{j\omega[\underline{F}_n^b(\underline{r}_m^{c+}) \cdot \frac{\underline{\rho}_m^{c+}}{2} + \underline{F}_n^b(\underline{r}_m^{c-}) \cdot \frac{\underline{\rho}_m^{c-}}{2}] + \Phi_n^b(\underline{r}_m^{c-}) - \Phi_n^b(\underline{r}_m^{c+})\} \quad (22)$$

where

$$\underline{F}_n^{\overset{a}{b}}(\underline{r}_m^{c\pm}) = \frac{\varepsilon^{\overset{a}{b}}}{4\pi} \iint_{T_n^\pm} \underline{M}_n(\underline{r}') \cdot G(k^{\overset{a}{b}}, |\underline{r}_m^{c\pm} - \underline{r}'|)ds'$$

$$\Phi_n^{\overset{a}{b}}(\underline{r}_m^{c\pm}) = \frac{-1}{4\pi j\omega\mu} \iint_{T_n^\pm} \nabla_s' \cdot \underline{M}_n(\underline{r}')\, G(k^{\overset{a}{b}}, |\underline{r}_m^{c\pm} - \underline{r}'|)ds'$$

Hence

$$\underline{F}_i^{pq\overset{a}{b}} = \pm \frac{\varepsilon^{\overset{a}{b}}\ \ell_i}{4\pi} (\underline{r}_1\, I_\xi^{pq} + \underline{r}_2\, I_\eta^{pq} + \underline{r}_3\, I_\zeta^{pq} - \underline{r}_i\, I^{pq})$$

where

$$k^{\overset{a}{b}} = \frac{\omega}{v^{\overset{a}{b}}} = \frac{\omega}{C}\sqrt{\varepsilon_{\overset{a}{b}}}$$

$$\varepsilon_{\overset{a}{b}} = \varepsilon^{\overset{a}{b}}/\varepsilon_o$$

Second, use $\underline{k}_a^i = \sqrt{\varepsilon_a}\,\underline{k}^i$ for the excitation vector $\vec{I}^i$, and $\underline{k}_b^m = \sqrt{\varepsilon_b}\,\underline{k}^m$ for the measurement vector $\vec{I}^m$. Then all the remaining calculations are exactly the same as in the prototype problem.

X. NUMERICAL RESULTS AND DISCUSSION FOR EXTENSION I

Some arbitrary but interesting combinations of different ε_a and ε_b are used as examples. Tables 4, 5, and 6 give the peak values of the dominant component of $\underline{M}$, the transmission coefficients, the transmission areas, and the maxima of the transmission cross section patterns in the principal planes.

The sources are normally incident plane waves, with the magnetic field polarized along the largest dimension of the aperture, for the diamond-shape, cross-shape, circular, etc. All the largest dimensions are chosen to be a quarter wavelength, and the relative dielectric constants are chosen to be combinations of 1 and 4. The results show an interesting phenomenon; i.e. after changing the dielectric constant on one side, the aperture appears to be resonant with respect to that half space. Hence both the equivalent current and the transmission characteristics have significant increases.

Table 4. Characteristic quantities of a diamond aperture with respect to some combinations of ε_a and ε_b
Aperture area = 0,0036 m^2, L = 0.25λ_o, W = L/2 where λ_o is the free space wavelength of the incident plane wave. (λ_o = 0.48m).

$(\varepsilon_a, \varepsilon_b)$	(1, 1)	(1, 4)	(4, 1)	(4, 4)
$\lvert M/E \rvert_{max}$	0.58000E 00 0.91000E 00	0.72000E 00 0.11450E 01	0.14300E 01 0.22890E 01	0.18000E 01 0.28680E 01
T	0.23621E -01	0.15036E 00	0.12028E 01	0.33212E 01
$TA(\lambda^2)$	0.36908E -03	0.23493E -02	0.18794E -01	0.51892E -01
$(\tau_\theta/\lambda^2)_{max}$	0.57259E -03	0.56648 -01	0.88513 -03	0.89017E -01
$(\tau_\phi/\lambda^2)_{max}$	0.57259E -03	0.14162E -01	0.22128E -03	0.89017E -01
$(\tau_\theta/\lambda^2)_{min}$	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
$(\tau_\phi/\lambda^2)_{min}$	0.55987E -03	0.12919E -01	0.21629E -03	0.81170E -01

Table 5. Characteristic quantities of a cross aperture with respect to some combinations of ε_a and ε_b.
Aperture area = 0.008 m^2, L = 0.25λ_o, W = L/3 where λ_o is the free space wavelength, (λ_o = 0.48 m).

$(\varepsilon_a, \varepsilon_b)$	(1, 1)	(1, 4)	(4; 1)	(4, 4)
$\lvert M/E \rvert_{max}$	0.78000E 00	0.11280E 01	0.22560E 01	0.19000E 01
T	0.89157E -01	0.79526E 00	0.63621E 01	0.84532E 01
$TA^{(\lambda^2)}$	0.30957E -02	0.27613E -01	0.22091E +00	0.29351E +00
$(\tau_\theta/\lambda^2)_{max}$	0.49090E -02	0.71640E 00	0.11194E -01	0.54156E 00
$(\tau_\phi/\lambda^2)_{max}$	0.49090E -02	0.17910E 00	0.27984E -02	0.54156E 00
$(\tau_\theta/\lambda^2)_{min}$	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
$(\tau_\phi/\lambda^2)_{min}$	0.46000E -02	0.14100E 00	0.26300E -02	0.43650E 00

Table 6. Characteristic quantities of a circular aperture with respect to some combinations of ε_a and ε_b.

Aperture area = 0.19635 m^2, R = 0.25λ_o, λ_o = 1m (free space)

(Triangulized aperture area $\cong$ 0.17678 m^2, $\lambda_o \cong$ 0.94886 m.)

$(\varepsilon_a, \varepsilon_b)$	(1, 1)	(1, 4)	(4, 1)	(4, 4)
$\lvert M/E\rvert_{max}$	0.18860E 01 0.44320E 01	0.11212E 01 0.16350E 01	0.22420E 01 0.32700E 01	0.15430E 01 0.19740E 01
T	0.15820E 01	0.89937E 01	0.71949E 01	0.46297E 01
$TA^{(\lambda^2)}$	0.27967E 00	0.15899E 01	0.12719E 01	0.81844E 00
$(\tau_\theta/\lambda^2)_{max}$	0.72745E 00	0.12550E 02	0.19610E 00	0.52419E 01
$(\tau_\phi/\lambda^2)_{max}$	0.72745E 00	0.31375E 01	0.49024E -01	0.52419E 01
$(\tau_\theta/\lambda^2)_{min}$	0.00000E 00	0.00000E 00	0.00000E 00	0.00000E 00
$(\tau_\phi/\lambda^2)_{min}$	0.32401E 00	0.68000E -01	0.24698E -01	0.14000E 00

XI. EXTENSION II: LOSSY DIELECTRIC WINDOW

Assume now that the arbitrarily-shaped aperture is covered with a non-magnetic material sheet ($\mu = \mu_o$) which is very thin. Then, we can treat this aperture as a continuously loaded case. The addition of one more term to the admittance matrix of the prototype problem will suffice to give the solution in a straightforward way. Both theoretical and numerical derivations will be given in moderate detail.

Since the aperture region is covered with a lossy dielectric (σ, ε) there will be current [10] consisting of conduction current and polarization current. This gives a total increase of volume current density

$$\Delta \underline{J} = \underline{J}_m = \sigma \underline{E} + \frac{\partial \underline{p}}{\partial t}$$

$$= \sigma \underline{E} + j\omega (\varepsilon - \varepsilon_o)\underline{E} \tag{23}$$

Since this window is assumed to be very thin, the electric fields on both sides are still continuous. Hence equivalent currents can still be $\underline{M}$ in region a and $-\underline{M}$ in region b. The current has no normal component and is tangential to the window. The increase in surface current density at the aperture region should be

$$\Delta \underline{J}_s = d\,\underline{J}_m \qquad d < \lambda/10$$

where d is the thickness of the window.

Now, instead of $H_t^a = H_t^b$ as in the prototype problem the boundary condition at the aperture region is

$$\underline{n} \times (\underline{H}^b - \underline{H}^a) = \Delta \underline{J}_s$$

$$= [\sigma + j\omega\,(\varepsilon - \varepsilon_o)]d\underline{E}$$

$$= [\sigma + j\omega\Delta\varepsilon)d \cdot (- \hat{\underline{n}} \times \underline{M})$$

$$\therefore \qquad \underline{H}_t^b - \underline{H}_t^a = - y_\ell \underline{M}$$

where

$$y_\ell = (\sigma + j\omega\Delta\varepsilon)d \tag{24}$$

In general, for a good dielectric, the conduction current is much smaller than the polarization current. Then $j_\ell = j\omega\Delta\varepsilon d$, i.e. purely susceptive.

Equation (24) can be rewritten as follows:

$$\underline{H}_t^b(-\underline{M}) - \underline{H}_t^a(\underline{M}) - \underline{H}_t^i = - y_\ell \underline{M}$$

$$- \underline{H}_t^b(\underline{M}) - \underline{H}_t^a(\underline{M}) + y_\ell \underline{M} = \underline{H}_t^i$$

i.e.,

$$[Y^a + Y^b + Y^\ell]\vec{V} = \vec{I}^i \tag{25}$$

where

$$[Y^\ell] = [\langle \underline{W}_m,\ y_\ell \underline{M}_n \rangle]_{N\times N}$$

If the window is isotropic and homogeneous, then

$$[Y^\ell] = y_\ell [\langle \underline{W}_m,\ \underline{M}_n \rangle]_{N\times N} \tag{26}$$

and $\vec{V} = [Y^a + Y^b + Y^\ell]^{-1}\vec{I}^i$ can be solved by analogy to the prototype problem as soon as we have Y^ℓ computed.

To evaluate $Y_{mn} = y_\ell \langle \underline{W}_m, \underline{M}_n \rangle$, referring to Fig. 18, we can see that corresponding to m, only five n's can make Y_{mn} non-zero, i.e., $n = m, m_1^+, m_2^+, m_1^-, m_2^-$.

$n = m$:

$$\iint_{T_m} \underline{W}_m \cdot \underline{M}_n ds = \iint_{T_m^+} \left(\frac{\ell_m}{2A_m^+}\right)^2 \underline{\rho}_m^+ \cdot \underline{\rho}_m^+ ds + \iint_{T_m^-} \left(\frac{\ell_m}{2A_m^-}\right)^2 \underline{\rho}_m^- \cdot \underline{\rho}_m^- ds$$

$$= \frac{\ell_m^2}{4} \left[\frac{1}{A_m^{+2}} \iint_{T_m^+} |\rho_m^+|^2 ds + \frac{1}{A_m^{-2}} \iint_{T_m^-} |\rho_m^-|^2 ds\right]$$

$$\cong \frac{\ell_m^2}{4} \left[\frac{1}{A_m^+} |\rho_m^{c+}|^2 + \frac{1}{A_m^-} |\rho_m^{c-}|^2\right]$$

$n = m_i^\pm$, $i = 1,2,$:

$$\iint_{T_m} \underline{W}_m \cdot \underline{M}_n ds = \iint_{T_m^\pm} \left(\frac{\ell_m}{2A_m^\pm} \underline{\rho}_m^\pm\right) \cdot \left(\frac{\ell_n}{2A_m^\pm} \underline{\rho}_n\right) ds$$

$$= \frac{\ell_m \ell_n}{4A_m^{\pm 2}} \iint_{T_m^\pm} \underline{\rho}_m^\pm \cdot \underline{\rho}_n ds$$

$$\cong \frac{\ell_m \ell_n}{4A_m^\pm} (\underline{\rho}_m^{c\pm} \cdot \underline{\rho}_n^c)$$

Therefore

$$Y_{mn}^\ell = (\sigma + j\omega\Delta\varepsilon) \cdot d \cdot L_{mn}$$

where

$$L_{mn} \cong \begin{cases} \dfrac{\ell_m^2}{4} \left[\dfrac{1}{A_m^+} |\rho_m^{c+}|^2 + \dfrac{1}{A_m^-} |\rho_m^{c-}|^2\right], & n = m \\[2ex] \dfrac{\ell_m \ell_n}{4A_m^\pm} \underline{\rho}_m^{c\pm} \cdot \underline{\rho}_n^c, & n = m_1^\pm, m_2^\pm \\[2ex] 0, & \text{otherwise} \end{cases}$$

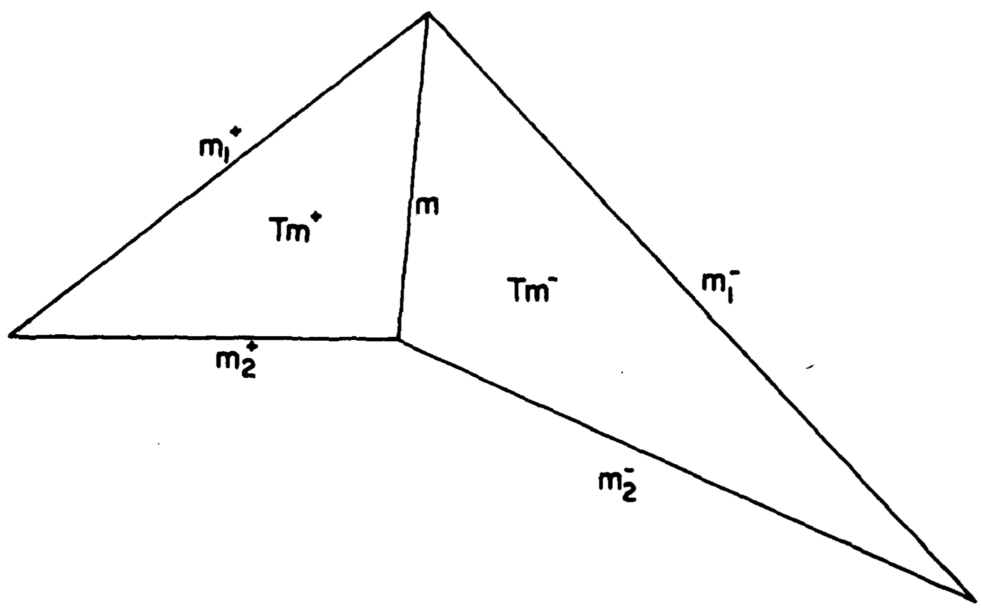


Fig. 18. Demonstration for evaluating Y_{mn}^{ℓ}

XII. NUMERICAL RESULTS AND DISCUSSION FOR EXTENSION II

As a first example, let us consider some good dielectric sheets (low loss: $\sigma = 10^{-4}$) with different dielectric constants ε_r being 900, 81, and 4, and variable thickness d ranging from zero (uncovered aperture) to some reasonable limits (e.g. around $0.1\lambda_d$, where $\lambda_d = \lambda_o/\sqrt{\varepsilon_r}$). We use these materials for diamond-shaped windows (major axis $0.45\lambda_o$, minor axis $0.1\lambda_o$). Each is illuminated by a normally incident plane wave.

On first thought, we might expect the transmission coefficient to become smaller after we cover the aperture with a lossy dielectric window. However, in the limiting case (magnetic dipole mode for a small aperture), the susceptance of a small aperture is inductive [17]. Now since the dielectric sheet we use is essentially a distributed capacitive loading, we might expect some kind of "resonance-like" behavior to occur. Our results support this expectation.

Figures 19, 20, and 21 show the transmission coefficients vs. thickness for the diamond-shaped windows. The incident wavelength λ_o is 0.2m. We see from these figures that when we start increasing d, the transmission coefficients drop from their original value (0.504) to almost zero ($\sim 10^{-2}$). But, then, instead of becoming exactly zero, there is a jump in each case. This resonance-like behavior can be explained as the result of the better match between the two half spaces provided by the dielectric sheet of proper thickness. Even though this resonance occurs at different d's for different materials, it always reaches the same maximum value (8.728), i.e., the T_{max} is independent of the material (ε_r). Also, this T_{max} is less than the optimum value for small apertures ($T_{opt} = \frac{3\lambda_o^2}{4\pi A} \cong 10.61$,

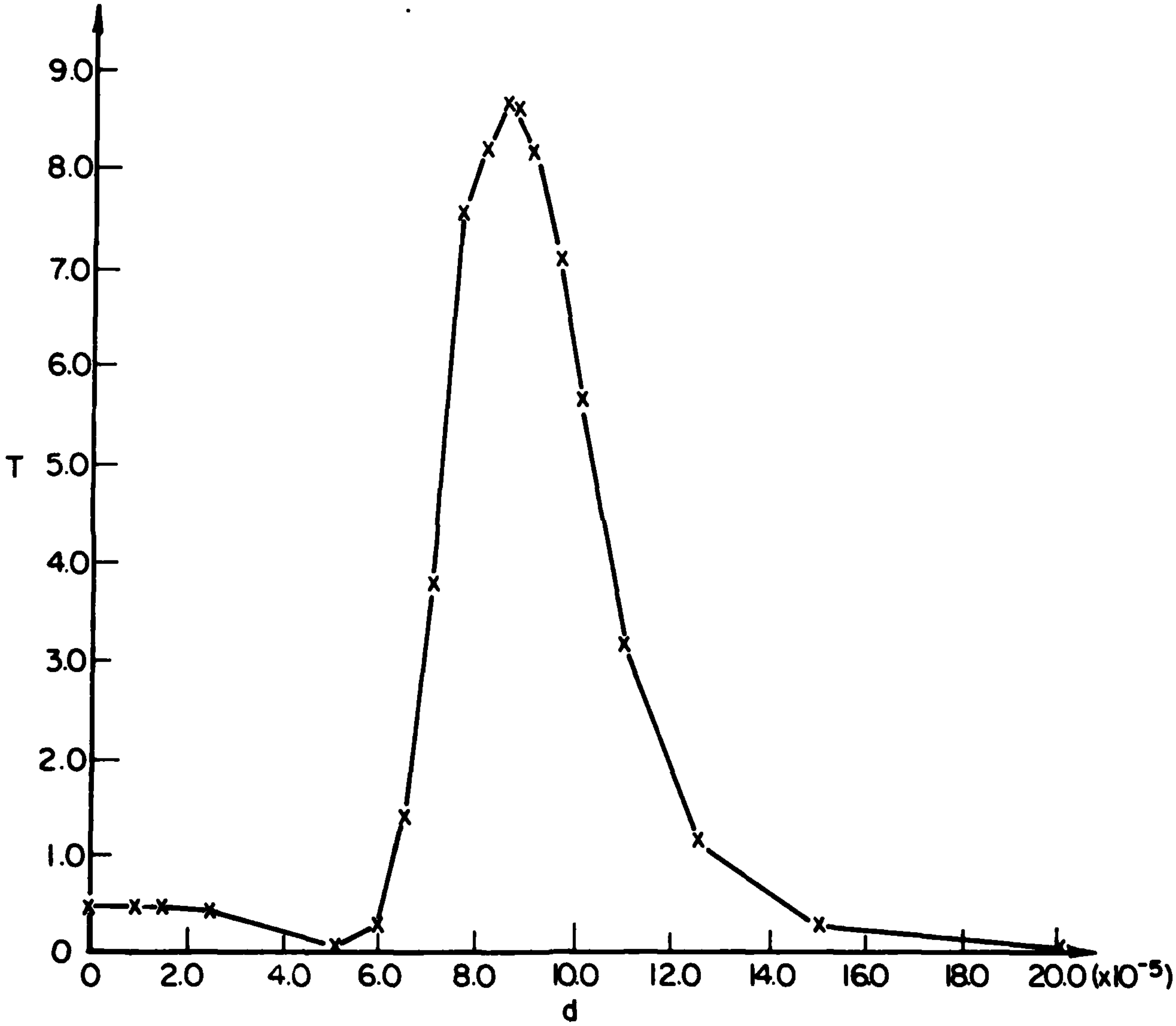


Fig. 19. Transmission coefficient of a diamond-shaped window, $\varepsilon_r = 900$. $\lambda_o = 0.2$; $\sigma = 10^{-4}$.

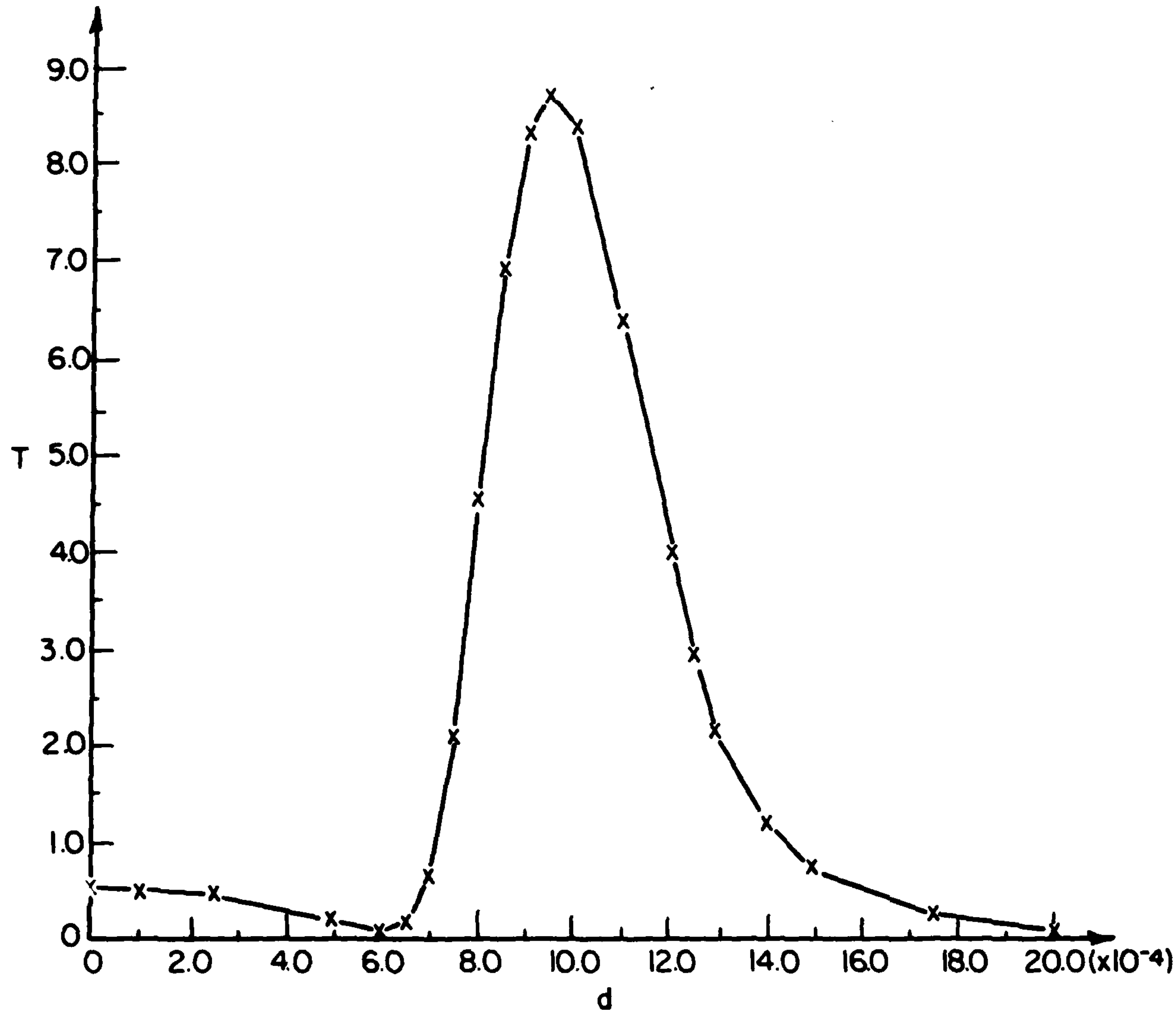


Fig. 20. Transmission coefficient of a diamond-shaped window, $\varepsilon_r = 81$. $\lambda_o = 0.2$; $\sigma = 10^{-4}$.

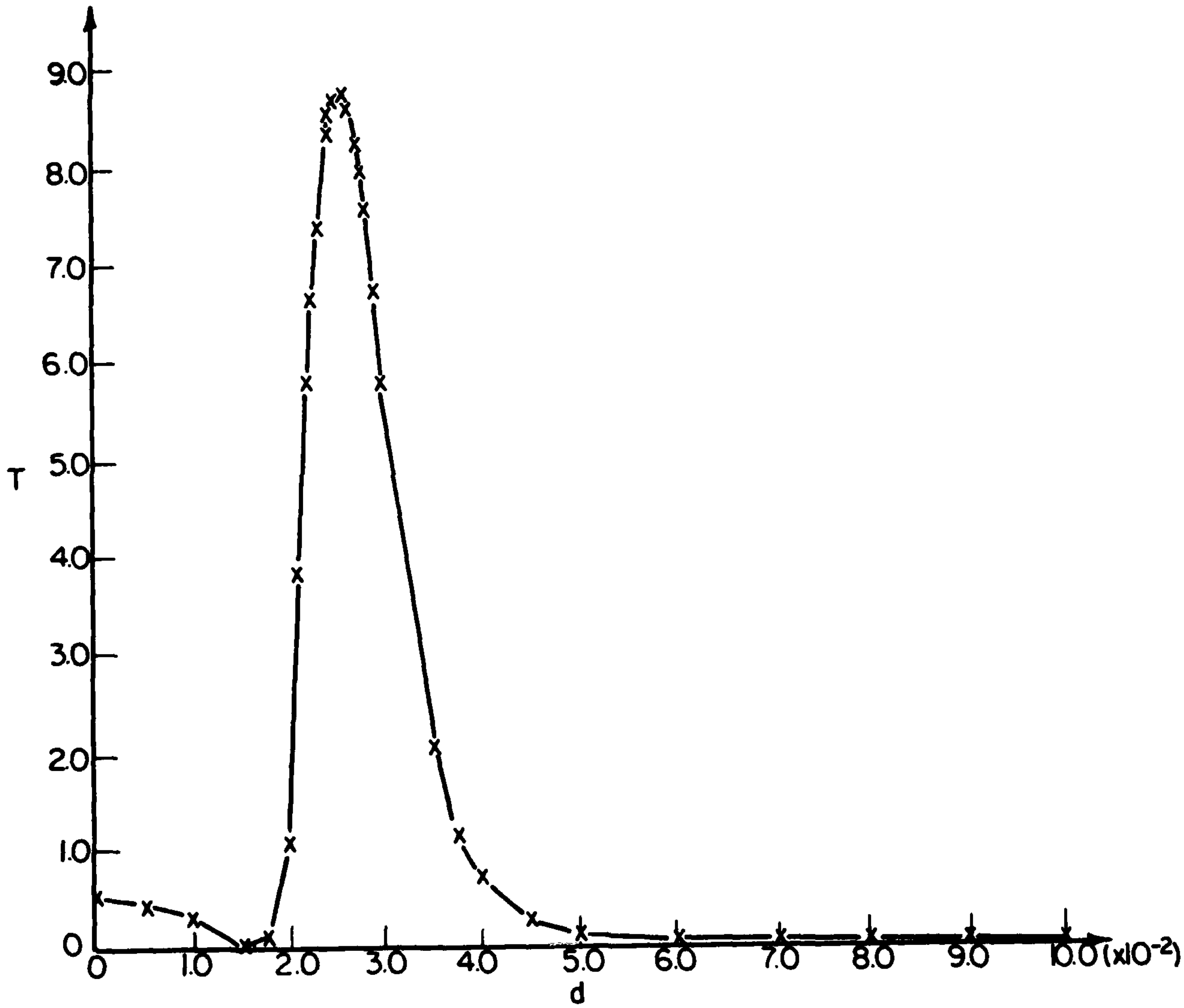


Fig. 21. Transmission coefficient of a diamond-shaped window, $\varepsilon_r = 4$. $\lambda_o = 0.2$; $\sigma = 10^{-4}$.

at resonance). This is due to the fact that the capacitive coupling through the dielectric sheet can never reach exactly perfect matching, since we are only modifying five elements of each row in the admittance matrix.

As a second example, we choose $\sigma = 10^{-4}$, $\varepsilon_r = 900$ for a rectangular window (actual slot $0.45\lambda_o$ by $0.05\lambda_o$) illuminated by a normally incident plane wave. Figure 22 shows the result. Since the slot is already in resonance, its transmission coefficient has the optimum value ($T_{max.} = 10.8 \cong T_{opt.}$) before we cover it. Then T monotonically decreases as we increase d. There is a small rise near $d = 0.5\lambda_d$, but that thickness is probably outside the range of our theory. We can say that the window provides a shielding effect in this case.

Finally, let us check the effect of high loss dielectric materials. With the same rectangular window and the same incidence as in the previous example, we choose four different cases:(σ, ε_r) = (5000, 900), (5000, 4), (10^4, 900), (10^4, 4). The results are shown in Figs. 23 and 24. These curves are interesting yet difficult to interpret, especially for the extremely high and narrow peak right before the dropping to zero. Regardless, they show very good shielding effect; i.e. the T already drops to zero at around $d \approx 2 \times 10^{-4}$, 10^{-4}. Actually, since the skin depths here (approximately 1.84×10^{-4} and 1.30×10^{-4}) are much smaller than one tenth of λ_d (6.67×10^{-4} and 10^{-2}), they play a more important role. As we can see, the unexplanable peaks occur near those thicknesses. It may be that our formulation doesn't work when the sheet thickness become comparable to its skin depth. Nevertheless, with high conductivities, changing dielectric constant doesn't change the curve much; while with the same dielectric constant, doubling the conductivity gives the same effect with only half of the thickness required. This is reasonable because there the factor $\sigma + j\omega\Delta\varepsilon$ is approximately $\sigma + j(\varepsilon_r - 1)/12$, which obviously has a dominant real part.

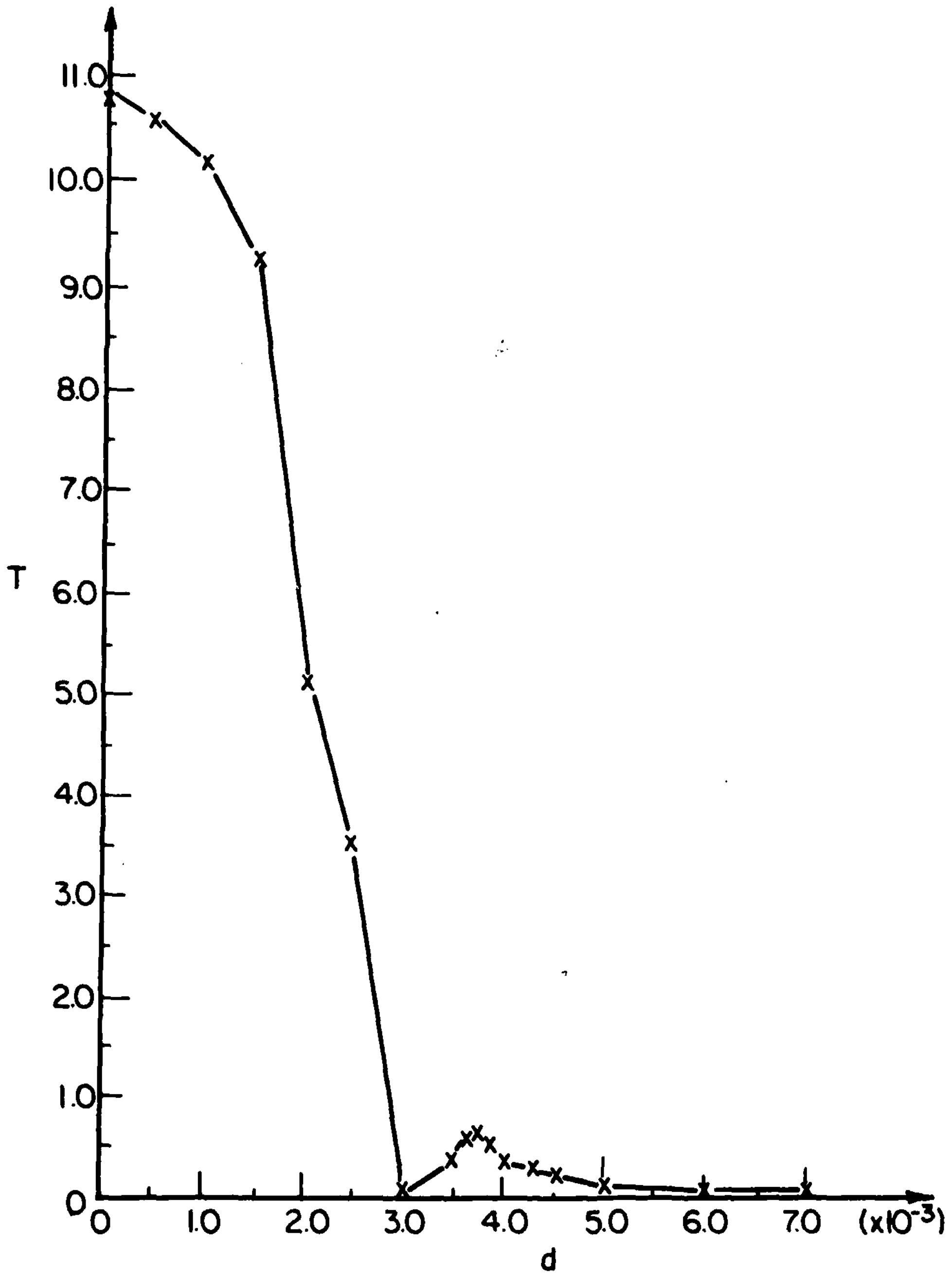


Fig. 22. Transmission coefficient of a rectangular window, $\varepsilon_r = 900$. $\lambda_o = 0.2$; $\sigma = 10^{-4}$.

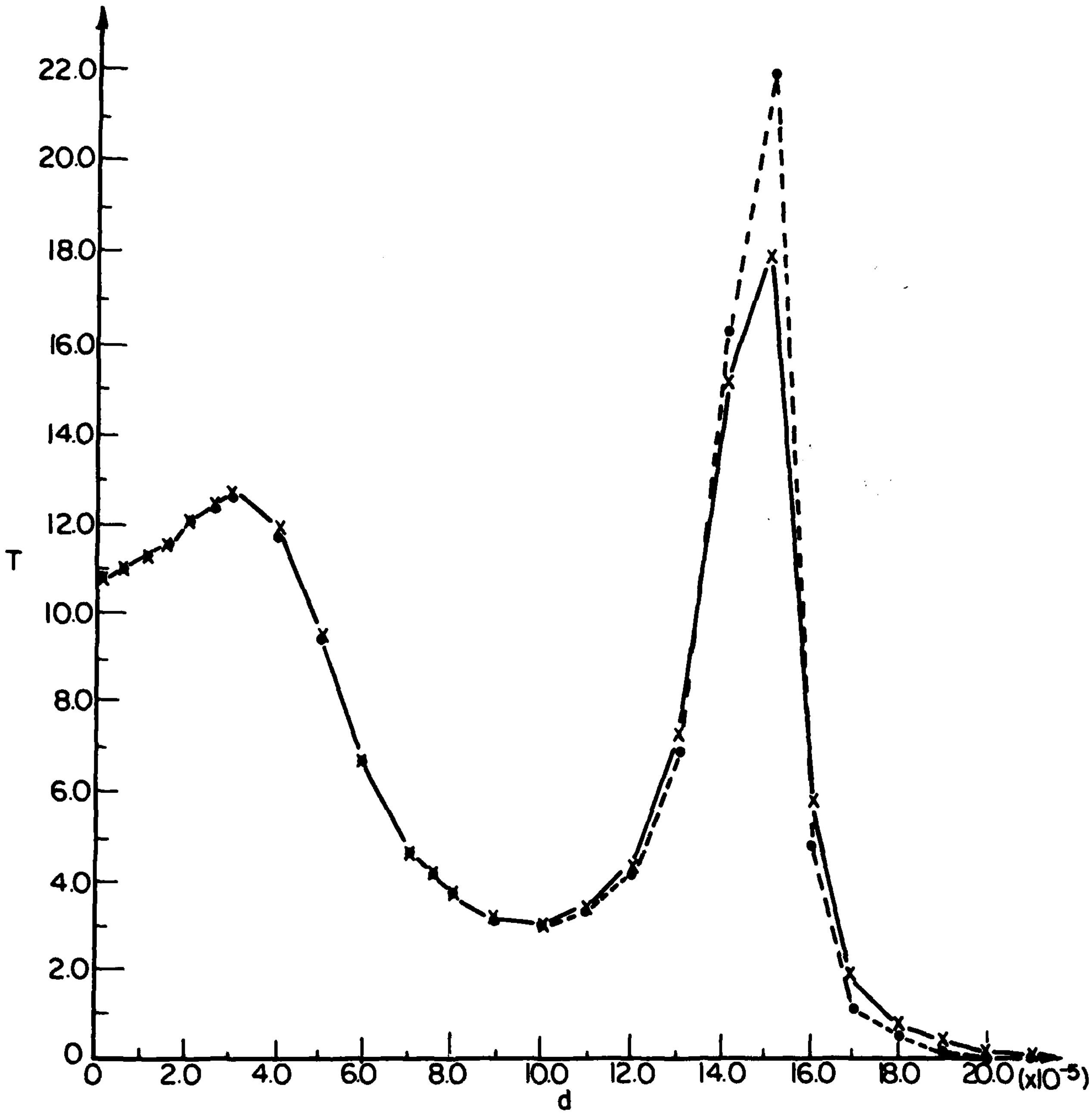


Fig. 23. Transmission coefficients of high loss dielectric windows.
$\lambda_o = 0.2$; $\sigma = 5000$.
$\times$: $\varepsilon_r = 900$
$\bullet$: $\varepsilon_r = 4$

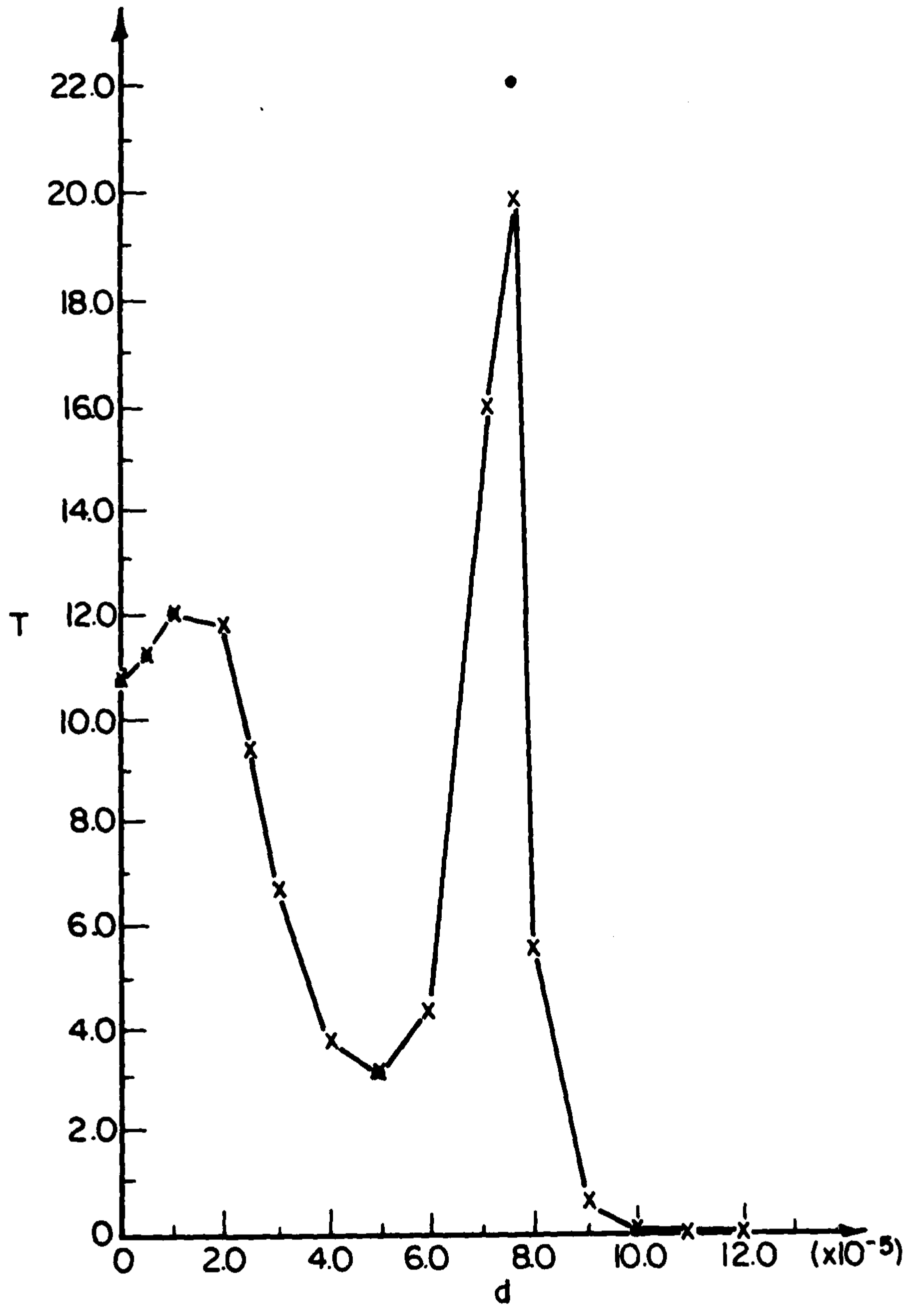


Fig. 24. Transmission coefficients of high loss rectangular windows.

$\lambda_o = 0.2$; $\sigma = 10^4$

× : $\varepsilon_r = 900$

• : $\varepsilon_r = 4$

XIII. CONCLUDING REMARKS

To obtain the transmission characteristics of arbitrarily-shaped apertures, the generalized network formulation for aperture problems was utilized. Triangular patching and local position vectors were used as bases for arbitrary 2-dimensional shapes. Extensions, such as different media in half spaces and lossy dielectric windows, were derived. Programs for calculating both far-field and near-field quantities were also developed.

The apertures considered in this report were for coupling between two half spaces. As further work, we could try some of the following: arbitrarily-shaped apertures backed by an infinitely-long wire, and arbitrarily-shaped aperture providing coupling between various combinations of half-spaces, waveguides, and cavities.

We should point out that there are systematic schemes for generating the triangular patching model [14, 15]. But for the sake of simplicity, it may still be preferable for the user to input all the nodes and meshes himself.

One more thing, magnetic and electric polarizabilities of small apertures have been solved with scalar bases and quadrilateral and triangular patches [16]. It would be interesting to find the polarizabilities for arbitrarily-shaped small apertures by using the vector bases.

XIV. COMPUTER PROGRAMS

This complete program treats the most general case; arbitrarily-shaped apertures on an infinite conducting plane, half spaces on both sides can have different media, the aperture can be covered with a lossy dielectric sheet. A thorough listing of the complete program will be given after a short description.

In addition to the main program, there are 18 subroutines and 1 function subprogram in this set. They are the following:

MAIN

SOLTN

INDATA

GEOM

AJUNC

CURDIR

BODPAR

YMATRX

YWINDO

MAGCHA

TRANS

MEASUR

SCAINT

VECINT

LININT

INTGRL

CA

EXPRN

CSMINV

DTRMNT

Brief descriptions of MAIN, SOLTN, AJUNC, YMATRX, YWINDO, MAGCHA, TRANS, MEASUR will be given; all the other subroutines were adopted from Rao's work and modified for 2-dimensional geometry.

MAIN program reads in the number of nodes NNODES, the number of edges NEDGES, and dielectric constants DIA and DIB. It calls subroutine INDATA to read in all the nodes DATNOD and meshes NCONN. By calling subroutines GEOM, AJUNC, CURDIR, BODPAR it obtains the triangular patching model of the aperture. Then subroutine SOLTN is called to find the transmission characteristics.

SOLTN subroutine reads in properties of the possible window; conductivity SIGMA, difference between the dielectric constant and 1 (free space) DELDIE, thickness THICK; window with zero thickness means aperture without covering. It also reads in the number of incident fields to be treated NFIELD, incident angles THETA and PHI, incident polarizations ETHETA and EPHI, and a flag IRCS for calculating the aperture transmission cross section. It calls subroutine YMATRX and YWINDO to get the admittance matrix CY and the excitation vector CI, inverts CY by calling CSMINV, then obtains the magnetic current coefficients CV and calculates the transmission area TS, transmission coefficients T and TCHA (the latter is normalized with respect to normal incident power density). As an option, subroutine MAGCHA can be called to give the corresponding magnetic charge distribution. Finally, IRCS serves as an indicator to show whether subroutine TRANS should be called and which polarizability of the transmitted field is to be considered; IRCS = 0 means the TCS pattern is not desired, 1 or 2 means theta- or phi-polarized TCS pattern is to be computed.

AJUNC subroutine finds the corresponding pair of triangles for each non-boundary edge, NJUNC. It serves as a preparation for YWINDO.

YMATRX subroutine calculates the summation of admittance matrices, for both half spaces,CY, the excitation current vector CI in half space A.

It evaluates the necessary integrals in the triangular area by calling SCAINT and VECINT.

YWINDO subroutine is called only when THICK is not zero. It computes the load admittance and adds it to CY.

MAGCHA calculates the equivalent magnetic charge densities. It can be dropped out without hurting the completeness of the general-purpose program set.

TRANS subroutine is called to find the theta- or phi-polarized TCS pattern when IRCS is 1 or 2. It reads in the initial angles THETA1 and PHI1, final angles THETA2 and PHI2, numbers of subangles NTHETA and NPHI to form the mesh nodes for desired TCS. It calls MEASUR to obtain the measurement vector CIM in half space B.

MEASUR subroutine computes the measurement current vector CIM in a similar way as YMATRX computes CI.

```
C THIS PROGRAM CALCULATES THE MAGNETIC CURRENT DISTRIBUTION
C FOR A ARBITRARY-SHAPE APERTURE ON A PERFECT CONDUCTING PLANE
C EXCITED BY A PLANE WAVE OF DESIRED AMPLITUDE E-FIELD.
C THE ARBITRARY APERTURE IS DESCRIBED BY
C A SET OF NODES AND THEIR COORDINATES,AND EDGES CONNECTING
C THESE NODES.THIS PROGRAM MANIPULATES THE GIVEN DATA TO
C OBTAIN TRIANGULAR PATCHES AND CURRENT DENSITY IS CALCULATED
C AT THE CENTER OF EACH EDGE.AS IT IS THIS PROGRAM CAN
C HANDLE 100 X 100 UNKNOWNS. BUT BY MODIFYING THE DIMENSION
C STATEMENTS ONE CAN USE STILL LARGER SIZE MATRICES.
C                   THE MAIN PROGRAM READS THE TOTAL NUMBER OF
C NODES AND TOTAL NUMBER OF EDGES.IT ALSO CALLS FIVE SUBROUTINES.
      IMPLICIT COMPLEX (C)
      COMPLEX CV(200),CI(200),CY(50,50),CIM(200)
      DIMENSION DATNOD(150,3)
      INTEGER NCONN(300,3),ITRAK(300),NBOUND(150,4),IMIN(300)
      INTEGER NJUNC(300,3)
      COMMON/IF/IFACE
      COMMON/DIELEC/DIA,DIB
C READ THE TOTAL NUMBER OF NODES(NNODES) AND EDGES(NEDGES).
      READ(1,45) NNODES,NEDGES
45    FORMAT(2I3)
      CALL INDATA(DATNOD,NCONN,NNODES,NEDGES)
      CALL GEOM(NCONN,NBOUND,ITRAK,IMIN,NEDGES,NNODES,NSE,NAPTRE,NHANDL)
C HERE WE CALULATE THE TOTAL NUMBER OF FACES(NFACES).
C WE ALSO CALCULATE THE NUMBER OF UNKNOWNS(NUNKNS).
C THE NUMBER OF UNKNOWNS ARE OBTAINED BY SUBSTRACTING THE
C NUMBER OF SURFACE EDGES(NSE) FROM TOTAL NUMBER OF EDGES.
      NFACES=IFACE
      NUNKNS=NEDGES-NSE
      CALL AJUNC(NJUNC,NBOUND,NEDGES,NFACES)
      CALL CURDIR(NCONN,NBOUND,NFACES,NEDGES,IMIN,NSE)
50    CALL BODPAR(DATNOD,NCONN,NBOUND,NNODES,NEDGES,NFACES,
     $NUNKNS,NSE,NAPTRE,NHANDL)
      READ(1,55) DIA,DIB
55    FORMAT(2F8.3)
      WRITE(3,60) DIA,DIB
60    FORMAT('1',///1X,'DIA=',F8.3,//,1X,'DIB=',F8.3)
      CALL SOLTN(CY,CV,CI,NUNKNS,DATNOD,NCONN,NBOUND,NEDGES,
     $NFACES,NNODES,IMIN,ITRAK,CIM,NJUNC)
      STOP
      END
```

```
      SUBROUTINE SOLTN(CY,CV,CI,NUNKNS,DATNOD,NCONN,NBOUND,NEDGES,
     $NFACES,NNODES,IMIN,ITRAK,CIM,NJUNC)
C IN THIS SUBROUTINE ,THE MATRIX EQUATION YV=I IS SOLVED.
C Y-MATRIX AND I-MATRIX ARE OBTAINED BY CALLING THE
C SUBROUTINE YMATRX. USING SUBROUTINE CSMINV, WE INVERT
C THE Y-MATRIX AND THEN MULTIPLY BY CURRENT VECTOR TO GET THE
C VOLTAGE COEFFICIENTS.THIS SUBROUTINE ALSO CALLS TWO OTHER
C SUBROUTINES, NAMELY MAGCHA AND TRANCS, TO COMPUTE MAGNETIC CHARGE
C DISTRIBUTION AND TRANS CROSS SECTION RESPECTIVELY.
      IMPLICIT COMPLEX (C)
      REAL CABS,COS
      COMPLEX CY(NUNKNS,NUNKNS),CV(NUNKNS),CI(NUNKNS),CIM(NUNKNS)
      COMPLEX HTHETA,HPHI,ETHETA,EPHI,HMT,HMP
      DIMENSION DATNOD(NNODES,3)
      INTEGER NCONN(NEDGES,3),NBOUND(150,4),ITRAK(NEDGES),IMIN(NEDGES)
      INTEGER NJUNC(NEDGES,3)
      COMMON/PARAM/THETA,PHI,IFIELD
      COMMON/FIELD/ETHETA,EPHI,ALAMDA
      COMMON/MOD1/AREAT,RAV,IKMAX,IKMIN,RMAX,RMIN
      COMMON/MOD2/AVAREA,JMAX,ARMAX,JMIN,ARMIN,KMIN,RATIO
      COMMON/FHINCI/FHINC
      COMMON/KKK/AK,PI
      COMMON/DIELEC/DIA,DIB
      COMMON/PCLARM/HMT,HMP
      COMMON/FRE/AOMEGA
C SPECIFY THE WAVE LENGTH (ALAMDA) IN METERS.
      PI=3.14159265
      ALAMDA=0.2
      DIE=SQRT(DIA)
      ALAMDA=ALAMDA/DIE
      AK=2.0*PI/ALAMDA
      VEL=3.0E+08/DIE
      FREQ=(VEL/ALAMDA)*1.0E-06
      AOMEGA=2.0*PI*1.0E+06*FREQ
      AMU=4.0*PI*1.0E-07
      AIMP=120.0*PI/DIE
      EPSLON=1.0/(VEL**2*AMU)
      AREAT=AREAT/(ALAMDA**2)
      RAV=RAV/ALAMDA
      RMAX=RMAX/ALAMDA
      RMIN=RMIN/ALAMDA
      AVAREA=AVAREA/(ALAMDA**2)
      ARMAX=ARMAX/(ALAMDA**2)
      ARMIN=ARMIN/(ALAMDA**2)
      FACESM=1.0/AVAREA
      WRITE(3,205)
 205  FORMAT('1'//////////25X,'MODELING PARAMETER LIST @ REGION A'///)
      WRITE(3,206) AREAT
 206  FORMAT(/10X,'SURFACE AREA OF THE APERTURE= ',1E13.5,1X,'SQ.
     $ WAVE LENGTHS')
      WRITE(3,209) RAV,IKMAX,RMAX,IKMIN,RMIN
 209  FORMAT(/10X,'AVERAGE EDGE LENGTH=',1E13.5,1X,'WAVE LENGTHS',
     $//10X,'MAXIMUM EDGE LENGTH(EDGE NO.',I3,1X,')=',1E13.5,1X,
     $'WAVE LENGTHS'//10X,'MINIMUM EDGE LENGTH(EDGE NO.',I3,
     $1X,')=',1E13.5,1X,'WAVE LENGTHS')
      WRITE(3,210) AVAREA,JMAX,ARMAX,JMIN,ARMIN
 210  FORMAT(/10X,'AVERAGE FACE AREA =',1E13.5,1X,'SQ.WAVE LENGTHS',
     $//10X,'MAXIMUM FACE AREA (FACE NO.',I3,1X,')=',1E13.5,1X,
     $'SQ.WAVE LENGTHS',//10X,'MINIMUM FACE AREA (FACE NO.',I3,
     $1X,')=',1E13.5,1X,'SQ.WAVE LENGTHS')
```

```
      WRITE(3,211) KMIN,RATIO,FACESM
 211  FORMAT(/10X,'MINIMUM FACE HEIGHT TO BASE RATIO (
     $FACE NO.',I3,1X,')=',1E13.5,//10X,'AVERAGE NUMBER OF FACES PER
     $ SQUARE WAVE LENGTH=',1E13.5)
      WRITE(3,212)
 212  FORMAT('1',//20X,'ELECTRICAL PARAMETERS')
      WRITE(3,213) FREQ,AOMEGA,AK,ALAMDA
 213  FORMAT(/10X,'FREQUENCY=',1E13.5,1X,'MHZ',//,10X,'ANGULAR
     $FREQUENCY=',1E13.5,1X,'RADIANS/SEC',//,10X,'WAVE NUMBER=',1E13.5,
     $1X,'(1/METERS)',//,10X,'WAVE LENGTH=',1E13.5,1X,'METERS')
      WRITE(3,214) EPSLON,AMU,VEL,AIMP
 214  FORMAT(/10X,'PERMITTIVITY=',1E13.5,1X,'FARADS/METER',
     $//,10X,'PERMEABILITY=',1E13.5,1X,'HENRYS/METER',
     $//,10X,'VELOCITY OF LIGHT=',1E13.5,1X,'METERS/SECOND',
     $//,10X,'INTRINSIC IMPEDANCE=',1E13.5,1X,'OHMS')
C INITIALIZE THE Y-MATRIX
      DO 10 I=1,NUNKNS
      DO 10 J=1,NUNKNS
      CY(I,J)=CMPLX(0.0,0.0)
 10   CONTINUE
C READ THE NUMBER OF INCIDENT FIELDS FOR WHICH THE MAGNETIC CURRENT
C DISTRIBUTION  NEEDS TO BE COMPUTED. IT IS NECESSARY TO
C SPECIFY THE INCIDENT FIELD PARAMETERS ON SEPARATE CARDS
C SO THAT THE PROGRAM EXECUTES ONE SET OF PARAMETERS AT A TIME.
      READ(1,699)SIGMA,DELDIE,THICK
699   FORMAT(E10.3,F10.5,E10.3)
      READ(1,15)NFIELD
 15   FORMAT(I3)
      DO 499 IJ=1,NFIELD
      IFIELD=IJ
C INITIALIZE THE CURRENT AND VOLTAGE VECTORS.
      DO 498 I=1,NUNKNS
      CV(I)=CMPLX(0.0,0.0)
      CI(I)=CMPLX(0.0,0.0)
 498  CONTINUE
C READ THE INCIDENT FIELD PARAMETERS.THETA AND PHI REPRESENT
C THE USUAL SPHERICAL COORDINATE ANGLES WHICH DETERMINE THE
C DIRECTION OF PROPAGATION OF THE PLANEWAVE. ETHETA AND EPHI
C REPRESENT THE AMPLITUDE OF THE INCIDENT PLANEWAVE.
C           THE VARIABLE IRCS IS REFERRED TO THE COMPUTATION
C OF TRANS CROSS SECTION. IF IRCS=0, THEN RADAR CROSS SECTION IS
C NOT COMPUTED.
      READ(1,16)THETA,PHI,ETHETA,EPHI,IRCS
 16   FORMAT(6E10.3,I3)
      WRITE(3,17) THETA,PHI
 17   FORMAT(//5X,'ANGLE OF INCIDENCE',//,10X,'THETA=',1E13.5,1X,'DEGREE
     $S',//10X,'PHI=',1E13.5,1X,'DEGREES')
      HPHI=ETHETA/CMPLX(AIMP,0.0)
      HTHETA=EPHI/CMPLX(-AIMP,0.0)
      FHINC=CABS(CMPLX(CABS(HPHI),CABS(HTHETA)))
      WRITE(3,18) ETHETA,EPHI,HTHETA,HPHI
 18   FORMAT(//5X,'POLARIZATION',//,10X,'E-THETA= (',2E13.5,
     $') VOLTS/METER',
     $//,10X,'E-PHI= (',2E13.5,') VOLTS/METER',
     $//,10X,'H-THETA=(',2E13.5,') AMPS/METER',
     $//,10X,'H-PHI=(',2E13.5,') AMPS/METER')
      THETA=THETA*PI/180.0
      PHI=PHI*PI/180.0
C CALL THE YMATRX SUROUTINE TO FILL THE Y-MATRIX
C AND THE CURRENT VECTOR.
```

```
      CALL YMATRX(CY,CATNOD,NCONN,NBOUNC,NNODES,NEDGES,
     $NFACES,NUNKNS,ITRAK,CI)
      ZERC=.000E+00
      IF(THICK.EC.ZERC) GC TC 999
      CALL YWINDC(CY,CATNOD,NCONN,NBOUND,NNODES,NEDGES,NFACES,NUNKNS,
     $ITRAK,NJUNC,SIGMA,DELDIE,THICK)
999   IF(IJ.NE.1) GC TC 19
C OBTAIN THE INVERSE OF THE Y-MATRIX.
      CALL CSMINV(CY,NUNKNS,NUNKNS,CDTRM,ACCNC,IER)
C MULTIPLY THE Y-INVERSE MATRIX WITH CURRENT CCLUMN VECTOR
C TO OBTAIN VOLTAGE CCEFFIECIENTS.
 19   DC 20 I=1,NUNKNS
      DC 20 J=1,NUNKNS
      CV(I)=CV(I)+CY(I,J)*CI(J)
 20   CONTINUE
C WRITE THE MAGNETIC CURRENT DENSITY TABLE
      EABS=SQRT(CABS(ETHETA)**2+CABS(EPHI)**2)
      WRITE(3,22)
 22   FORMAT('1',///25X,'SURFACE MAGNETIC CURRENTS',///)
      WRITE(3,23)
 23   FORMAT(1X,'EDGE NUMBER',25X,'MAGNETIC CURRENT DENSITY (WEBS/M-S)')
      WRITE(3,24)
24    FORMAT(/20X,'REAL',8X,'IMAGINARY',6X,'MAGNITUDE',8X,'PHASE'
     $,8X,'|M/E| RATIC',6X,'PHASED')
      NSE=NEDGES-NUNKNS
      K1=0
      DC 50 I=1,NEDGES
      IF(NSE.EC.0) GC TC 31
      DO 30 J=1,NSE
      IF(I.EC.ITRAK(J)) GO TO 45
 30   CONTINUE
 31   K1=K1+1
      RA1=REAL(CV(K1))
      RA2=AIMAG(CV(K1))
      RA3=CABS(CV(K1))
      RA4=ATAN2(RA2,RA1)
      RA5=RA3/EABS
      RA6=RA4*18C.0/PI
      WRITE(3,101) I,RA1,RA2,RA3,RA4,RA5,RA6
 101  FORMAT(2X,I4,8X,1E13.5,2X,1E13.5,2X,1E13.5,2X,1E13.5,
     $2X,1E13.5,2X,1E13.5,/)
      GO TO 50
 45   C0=CMPLX(0.0,0.0)
      WRITE(3,101)I,CC,C0,C0
 50   CONTINUE
C CALL MAGCHA SUBROUTINE TO CALCULATE THE MAGNETIC CHARGE DISTRIBUTION
      CALL MAGCHA(CV,CATNOD,NCCNN,NBCUND,NNCDES,NEDGES,
     $NFACES,NUNKNS,ITRAK)
C     COMPUTE THE TRANSMISSION AREA AND THE TRANSMISSION COEFFICIENT
      CTS=CMPLX(C.0,C.0)
      DO 100 I=1,NUNKNS
100   CTS=CTS+CV(I)*CCNJG(CI(I))
      TS=REAL(CTS)/(2.0*AIMP)
      AREAIN=AREAT*COS(THETA)*ALAMDA**2
      T=TS/AREAIN
      WRITE(3,11C) TS,T
110   FORMAT('1',/////,10X,'TRANSMISSION AREA =',1E13.5,2X,'SQ. METERS',
     $///,10X,'TRANSMISSION COEFFICIENT =',1E13.5)
      TCHA=TS/(AREAT*ALAMDA**2)
      WRITE(3,120) TCHA
```

```
120   FORMAT (//,10X,'TCHA =',1E13.5)
      IF(IRCS.EQ.0) GO TO 499
C SINCE IRCS IS NOT EQUAL TO ZERO,COMPUTE THE TRANSMISSION
C CROSS SECTION
      IF(IRCS.EQ.1) GO TO 299
      IF(IRCS.EQ.2) GO TO 399
      GO TO 499
299   HMT=CMPLX(1.0,0.0)
      HMP=CMPLX(0.0,0.0)
      GO TO 450
399   HMT=CMPLX(0.0,0.0)
      HMP=CMPLX(1.0,0.0)
450   CALL TRANCS(DATNOD,NCONN,NBOUND,NNODES,NEDGES,NFACES,
     $NUNKNS,CV,ITRAK,CIM)
 499  CONTINUE
      RETURN
      END
```

```
      SUBROUTINE INDATA(DATNOD,NCONN,NNODES,NEDGES)
C     THIS SUBROUTINE READS TWO SETS OF INPUT DATA AND ARRANGES THEM
C     IN NUMERICAL ORDER.THE FIRST SET OF DATA CONTAINS NODE NUMBERS
C     AND THEIR COORDINATES.EACH NODE ALONG WITH IT'S THREE COORDINATES
C     IS READ AND STORED IN THE MATRIX DATNOD.
C                   THE SECOND SET OF DATA CONTAINS EDGE NUMBERS AND
C     THE NODES TO WHICH THIS PARTICULAR EDGE IS CONNECTED. THIS
C     INFORMATION IS STORED IN THE MATRIX NCONN.
      DIMENSION DATNOD(NNODES,3)
      INTEGER NCONN(NEDGES,3)
      DO 10 I=1,NNODES
      READ(1,5) NODE,X,Y
5     FORMAT(I3,2F8.5)
      AN=FLOAT(NODE)
      DATNOD(NODE,1)=AN
      DATNOD(NODE,2)=X
      DATNOD(NODE,3)=Y
 10   CONTINUE
      DO 20 I=1,NEDGES
      READ(1,15) NE,NF,NT
15    FORMAT(3I3)
      NCONN(NE,1)=NE
      NCONN(NE,2)=NF
      NCONN(NE,3)=NT
 20   CONTINUE
      WRITE(3,18)
 18   FORMAT('1')
      WRITE(3,19)
 19   FORMAT(20X,'VERTEX  COORDINATE  LIST')
      WRITE(3,21)
 21   FORMAT(/20X,'ALL DIMENSIONS ARE IN METERS')
      WRITE(3,22)
 22   FORMAT(//1X,'VERTEX NUMBER',5X,'X-COORDINATE',5X,
     $'Y-COORDINATE')
      DO 30 I=1,NNODES
      IDUMMY=IFIX(DATNOD(I,1))
      WRITE(3,23)IDUMMY,DATNOD(I,2),DATNOD(I,3)
 23   FORMAT(/3X,I3,11X,1E13.5,2X,1E13.5)
 30   CONTINUE
      WRITE(3,28)
 28   FORMAT('1')
      WRITE(3,29)
 29   FORMAT(/10X,'EDGE-VERTEX CONNECTION LIST')
      DO 40 I=1,NEDGES
      WRITE(3,31)NCONN(I,1),NCONN(I,2),NCONN(I,3)
 31   FORMAT(/3X,'EDGE',I3,1X,'IS CONNECTED FROM VERTEX',1X,I3,1X,
     $'TO VERTEX',1X,I3)
 40   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE GEOM(NCONN,NBOUND,ITRAK,IMIN,NEDGES,NNODES,
     $NSE,NAPTRE,NHANDL)
C    THIS SUBROUTINE USES THE INPUT DATA TO FORM TRIANGULAR PATCHES.
C    THE INFORMATION CONCERNING THE TRIANGLE AND ITS ASSOCIATED
C    EDGES IS STORED IN THE MATRIX NBOUND. ITRAK AND IMIN ARE TWO
C    AUXILIARY VECTORS NEEDED IN THE PROGRAM.IN CASE OF AN OPEN BODY,
C    IT CALCULATES NUMBER OF APERTURES AND NUMBER OF HANDLES.
C    IT ALSO LISTS THE SURFACE EDGES ASSOCIATED WITH EACH APERTURE.
      INTEGER NCONN(NEDGES,3),NBOUND(150,4),ITRAK(NEDGES)
      INTEGER IMIN(NEDGES)
      COMMON/IF/IFACE
      IFACE=0
      NF1=0
      NF2=0
      DO 100 IJ=1,NEDGES
      ICOUNT=0
      N1=NCONN(IJ,2)
      N2=NCONN(IJ,3)
      DO 10 I=1,NEDGES
      DO 10 J=2,3
      IF(I.EQ.IJ) GO TO 10
      NA=NCONN(I,J)
      IF(NA.EQ.N1.OR.NA.EQ.N2) GO TO 6
      GO TO 10
 6    ICOUNT=ICOUNT+1
      ITRAK(ICOUNT)=I
 10   CONTINUE
      MARK1=0
      MARK2=0
 75   CONTINUE
      K1=1
      I1=ITRAK(K1)
      DO 15 I=2,ICOUNT
      IF(ITRAK(I).LT.I1) GO TO 12
      GO TO 15
 12   I1=ITRAK(I)
      K1=I
 15   CONTINUE
      IF(MARK1.EQ.ICOUNT) GO TO 100
      IF(I1.GT.IJ) GO TO 20
      GO TO 31
 20   CONTINUE
      N3=NCONN(I1,2)
      N4=NCONN(I1,3)
      IF(N3.EQ.N1.OR.N3.EQ.N2) GO TO 21
      IF(N4.EQ.N1.OR.N4.EQ.N2) GO TO 22
 21   NB=N4
      GO TO 23
 22   NB=N3
 23   CONTINUE
      ICO=0
      DO 25 I=1,NEDGES
      DO 25 J=2,3
      IF(I.EQ.I1) GO TO 25
      NC=NCONN(I,J)
      IF(NC.EQ.NB) GO TO 24
      GO TO 25
 24   ICO=ICO+1
      IMIN(ICO)=I
 25   CONTINUE
```

```
      DO 30 I=1,ICO
      IA=IMIN(I)
      IF(N1.EQ.NCONN(IA,2).OR.N1.EQ.NCONN(IA,3)) GO TO 29
      IF(N2.EQ.NCONN(IA,2).OR.N2.EQ.NCONN(IA,3)) GO TO 29
      GO TO 30
29    I2=IA
      GO TO 32
30    CONTINUE
31    CONTINUE
      ITRAK(K1)=NEDGES+1
      MARK1=MARK1+1
      GO TO 75
32    IF(I2.LT.IJ) GO TO 74
      IF(IFACE.EQ.0) GO TO 35
      NF1=NBOUND(IFACE,2)
      NF2=NBOUND(IFACE,3)
35    IF(IJ.EQ.NF1.AND.I2.EQ.NF2) GO TO 74
      IFACE=IFACE+1
      NBOUND(IFACE,1)=IFACE
      NBOUND(IFACE,2)=IJ
      NBOUND(IFACE,3)=I1
      NBOUND(IFACE,4)=I2
      MARK2=MARK2+1
      MARK1=MARK1+1
      IF(MARK2.EQ.2) GO TO 100
      ITRAK(K1)=NEDGES+1
      GO TO 75
74    CONTINUE
      ITRAK(K1)=NEDGES+1
      MARK1=MARK1+1
      GO TO 75
100   CONTINUE
      NSE=0
      DO 120 I=1,IFACE
      DO 120 J=2,4
      ISEDGE=NBOUND(I,J)
      NCOUNT=0
      DO 125 K=1,IFACE
      DO 125 M=2,4
      IF(I.EQ.K.AND.J.EQ.M) GO TO 125
      IF(ISEDGE.EQ.NBOUND(K,M)) NCOUNT=NCOUNT+1
125   CONTINUE
      IF(NCOUNT.EQ.0) GO TO 119
      GO TO 120
119   NSE=NSE+1
      ITRAK(NSE)=ISEDGE
120   CONTINUE
      NAPTRE=0
      IF(NSE.EQ.0) GO TO 991
      DO 147 K1=1,NSE
      I1=ITRAK(K1)
      DO 145 J=K1,NSE
      IF(J.EQ.K1) GO TO 145
      IF(ITRAK(J).GT.ITRAK(K1)) GO TO 145
      IDUMMY=ITRAK(K1)
      ITRAK(K1)=ITRAK(J)
      ITRAK(J)=IDUMMY
145   CONTINUE
147   CONTINUE
      DO 159 I=1,NSE
```

```
      IMIN(I)=ITRAK(I)
159   CONTINUE
      WRITE(3,1599)
1599  FORMAT('1',//,10X,'BOUNDARY CONTOUR LIST')
161   DO 900 J=1,NSE
      IF(IMIN(J).NE.0) GO TO 169
      GO TO 900
169   IJ=IMIN(J)
      IMIN(J)=0
      NAPTRE=NAPTRE+1
      WRITE(3,1601) NAPTRE
1601  FORMAT(//1X,'APERTURE',I3,2X,'CONSISTS OF THE
     $ FOLLOWING BOUNDARY EDGES')
      WRITE(3,1602) IJ
1602  FORMAT(15X,I3)
      N1=NCONN(IJ,2)
      N2=NCONN(IJ,3)
175   DO 600 K=1,NSE
      IF(IMIN(K).NE.0) GO TO 179
      GO TO 600
179   IK=IMIN(K)
      NK1=NCONN(IK,2)
      NK2=NCONN(IK,3)
      IF(N2.EQ.NK1) GO TO 190
      IF(N2.EQ.NK2) GO TO 195
      GO TO 600
190   WRITE(3,1603) IK
1603  FORMAT(15X,I3)
      IMIN(K)=0
      N2=NK2
      IF(N2.EQ.N1) GO TO 900
      GO TO 600
195   WRITE(3,1603) IK
      IMIN(K)=0
      N2=NK1
      IF(N2.EQ.N1) GO TO 900
600   CONTINUE
      IF(N2.NE.N1) GO TO 175
900   CONTINUE
      DO 901 I=1,NSE
      IF(IMIN(I).NE.0) GO TO 161
901   CONTINUE
991   NHANDL=1-(IFACE-NEDGES+NNODES+NAPTRE)/2
      RETURN
      END
```

```
      SUBROUTINE AJUNC(NJUNC,NBOUND,NEDGES,NFACES)
C THIS SUBROUTINE TABULIZES THE JUNCTIONS.
      INTEGER NJUNC(NEDGES,3),NBOUND(150,4)
      DO 10 I=1,NEDGES
      NJUNC(I,1)=I
      NJUNC(I,2)=0
      NJUNC(I,3)=0
10    CONTINUE
      DO 70 I=1,NFACES
      N2=NBOUND(I,2)
      N3=NBOUND(I,3)
      N4=NBOUND(I,4)
      IF(NJUNC(N2,2).EQ.0) GO TO 20
      NJUNC(N2,3)=I
      GO TO 30
20    NJUNC(N2,2)=I
30    IF(NJUNC(N3,2).EQ.0) GO TO 40
      NJUNC(N3,3)=I
      GO TO 50
40    NJUNC(N3,2)=I
50    IF(NJUNC(N4,2).EQ.0) GO TO 60
      NJUNC(N4,3)=I
      GO TO 70
60    NJUNC(N4,2)=I
70    CONTINUE
      WRITE(3,80)
80    FORMAT('1',20X,'JUNCTION LIST',//)
      DO 100 I=1,NEDGES
      WRITE(3,90) (NJUNC(I,J),J=1,3)
90    FORMAT(/3X,'EDGE',I3,1X,'IS THE JUNCTION OF FACE',1X,I3,1X,'AND FA
     $CE',1X,I3)
100   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CURDIR(NCONN,NBOUND,NFACES,NEDGES,IMIN,NSE)
C IN THIS SUROUTINE ,NORMAL VECTOR TO THE SURFACE IS CALCULATED.
C THE NORMAL VECTOR IS OBTAINED BY LISTING THE EDGES ASSOCIATED
C WITH EACH TRIANGLE IN A SEQUENTIAL MANNER.HERE THE USER HAS
C THE CHOICE OF SELECTING THE DIRECTION OF NORMAL.BUT IN THIS CASE,
C THE USER SHOULD SUPPLY THE INFORMATION IN A PRESCRIBED MANNER.
C FOR DETAILS,PLEASE REFER TO THE REFERENCE SITED IN THE NOTE.
      INTEGER NCONN(NEDGES,3),NBOUND(150,4),IMIN(NEDGES)
      INTEGER IMAX(6)
      IM=1
      IMIN(IM)=1
      M1=0
      DO 999 IJK=1,NFACES
      IK=IJK
      DO 2 I=1,6
      IMAX(I)=0
2     CONTINUE
      IFLAG=0
      DO 4 J=1,IM
      IF(IJK.EQ.IMIN(J)) GO TO 1
      IFLAG=1
4     CONTINUE
      IF(IFLAG.EQ.1) GO TO 999
1     J1=0
      I1=0
      L1=0
      I2=NBOUND(IK,2)
      I3=NBOUND(IK,3)
      I4=NBOUND(IK,4)
      N1=NCONN(I2,2)
      N2=NCONN(I2,3)
      N3=NCONN(I3,2)
      IF(N3.EQ.N1.OR.N3.EQ.N2) GO TO 5
      GO TO 10
5     N3=NCONN(I3,3)
10    CONTINUE
      I1=1
      ITEMP=I2
11    DO 20 IJ=1,NFACES
      DO 12 J=1,IM
      IF(IJ.EQ.IMIN(J)) GO TO 20
12    CONTINUE
      J2=NBOUND(IJ,2)
      J3=NBOUND(IJ,3)
      J4=NBOUND(IJ,4)
      IF(ITEMP.EQ.J2.OR.ITEMP.EQ.J3.OR.ITEMP.EQ.J4) GO TO 15
      GO TO 20
15    IL=IJ
      GO TO 25
20    CONTINUE
      IF(I1.EQ.1.AND.J1.EQ.1) GO TO 21
      J1=1
      ITEMP=I3
      GO TO 11
21    IF(I1.EQ.1.AND.J1.EQ.1.AND.L1.EQ.1) GO TO 23
      L1=1
      ITEMP=I4
      GO TO 11
23    IF(M1.EQ.1) GO TO 999
      M1=M1+1
```

```
      IK=IJK
      GO TO 1
25    KN1=NCONN(ITEMP,2)
      KN2=NCONN(ITEMP,3)
      IF(N1.EQ.KN1.OR.N1.EQ.KN2) GO TO 35
      KN3=N1
      GO TO 40
35    IF(N2.EQ.KN1.OR.N2.EQ.KN2) GO TO 36
      KN3=N2
      GO TO 40
36    KN3=N3
40    J2=NBOUND(IL,2)
      J3=NBOUND(IL,3)
      J4=NBOUND(IL,4)
      IF(J2.EQ.ITEMP) GO TO 59
      IF(NCONN(J2,2).EQ.KN1.OR.NCONN(J2,2).EQ.KN2) GO TO 57
      KN4=NCONN(J2,2)
      GO TO 68
57    KN4=NCONN(J2,3)
      GO TO 68
59    IF(NCONN(J3,2).EQ.KN1.OR.NCONN(J3,2).EQ.KN2) GO TO 61
      KN4=NCONN(J3,2)
      GO TO 68
61    KN4=NCONN(J3,3)
68    CONTINUE
      IF(IM.EQ.1) GO TO 115
      IF(ITEMP.EQ.IMAX(6)) GO TO 109
      IMAX(1)=IMAX(5)
      IMAX(2)=IMAX(4)
      IMAX(3)=IMAX(6)
      GO TO 115
109   IMAX(1)=IMAX(4)
      IMAX(2)=IMAX(6)
      IMAX(3)=IMAX(5)
115   IF(M1.NE.1) GO TO 175
      IF(ITEMP.EQ.NBOUND(IJK,4)) GO TO 165
      IMAX(1)=NBOUND(IJK,4)
      IMAX(2)=NBOUND(IJK,2)
      IMAX(3)=NBOUND(IJK,3)
      M1=0
      GO TO 175
165   IMAX(1)=NBOUND(IJK,3)
      IMAX(2)=NBOUND(IJK,4)
      IMAX(3)=NBOUND(IJK,2)
      M1=0
175   KDUMMY=KN3
      DO 100 I=1,2
      IF(I.EQ.1.AND.IM.NE.1) GO TO 99
      ID=I+(I-1)*2
      IF(ITEMP.EQ.I2) GO TO 79
      IF(ITEMP.EQ.I3) GO TO 89
      IF(N1.EQ.KN3.AND.N2.EQ.KN1) GO TO 69
      IF(N1.EQ.KN1.AND.N2.EQ.KN3) GO TO 69
      IMAX(ID)=I3
      IMAX(ID+2)=I2
      GO TO 99
69    IMAX(ID)=I2
      IMAX(ID+2)=I3
      GO TO 99
79    NN1=NCONN(I3,2)
```

```
      NN2=NCONN(I3,3)
      IF(NN1.EQ.KN1.AND.NN2.EQ.KN3) GO TO 81
      IF(NN1.EQ.KN3.AND.NN2.EQ.KN1) GO TO 81
      IMAX(ID)=I4
      IMAX(ID+2)=I3
      GO TO 99
 81   IMAX(ID)=I3
      IMAX(ID+2)=I4
      GO TO 99
 89   IF(N1.EQ.KN3.AND.N2.EQ.KN1) GO TO 91
      IF(N1.EQ.KN1.AND.N2.EQ.KN3) GO TO 91
      IMAX(ID)=I4
      IMAX(ID+2)=I2
      GO TO 99
 91   IMAX(ID)=I2
      IMAX(ID+2)=I4
 99   KN3=KN4
      I2=J2
      I3=J3
      I4=J4
 100  CONTINUE
      KN3=KDUMMY
      NA1=NCONN(IMAX(1),2)
      NA2=NCONN(IMAX(1),3)
      NB1=NCONN(IMAX(4),2)
      NB2=NCONN(IMAX(4),3)
      IF(NB1.EQ.NA1.OR.NB1.EQ.NA2) GO TO 125
      IF(NB2.EQ.NA1.OR.NB2.EQ.NA2) GO TO 125
      IDUMMY=IMAX(6)
      IMAX(6)=IMAX(4)
      IMAX(4)=IDUMMY
 125  IMAX(2)=ITEMP
      IMAX(5)=ITEMP
      IF(IM.NE.1) GO TO 149
      NBOUND(IK,2)=IMAX(1)
      NBOUND(IK,3)=IMAX(2)
      NBOUND(IK,4)=IMAX(3)
 149  NBOUND(IL,2)=IMAX(6)
      NBOUND(IL,3)=IMAX(5)
      NBOUND(IL,4)=IMAX(4)
      IM=IM+1
      IMIN(IM)=IL
      IK=IL
      IF(IM.EQ.NFACES) GO TO 1000
      GO TO 1
 999  CONTINUE
 1000 CONTINUE
      IF(NSE.EQ.0) GO TO 1001
      WRITE(3,98)
 98   FORMAT('1')
      WRITE(3,102)
 102  FORMAT(10X,'LIST OF EDGES AND VERTICES BOUNDING EACH FACE')
      DO 1999 IJK=1,NFACES
      I2=NBOUND(IJK,2)
      I3=NBOUND(IJK,3)
      I4=NBOUND(IJK,4)
      IF(NCONN(I2,2).EQ.NCONN(I3,2)) GO TO 1005
      IF(NCONN(I2,2).EQ.NCONN(I3,3)) GO TO 1005
      N1=NCONN(I2,3)
      GO TO 1006
```

```
1005 N1=NCCNN(I2,2)
1006 IF(NCONN(I3,2).EQ.NCONN(I4,2)) GO TO 1010
     IF(NCONN(I3,2).EQ.NCONN(I4,3)) GO TO 1010
     N2=NCONN(I3,3)
     GO TO 1011
1010 N2=NCONN(I3,2)
1011 IF(NCONN(I4,2).EQ.NCONN(I2,2)) GO TO 1015
     IF(NCONN(I4,2).EQ.NCONN(I2,3)) GC TO 1015
     N3=NCONN(I4,3)
     GO TO 1016
1015 N3=NCONN(I4,2)
1016 CCNTINUE
     WRITE(3,1050) IJK,I2,I3,I4,N1,N2,N3
1050 FORMAT(/3X,'FACE',I3,1X,'IS BCUNDED BY EDGES',1X,I3,1X,I3,
    $1X,I3,2X,'AND VERTICES',1X,I3,1X,I3,1X,I3)
1999 CCNTINUE
1001 RETURN
     ENC
```

```
      SUBROUTINE BCOPAR(DATNOD,NCONN,NBOUND,NNODES,NEDGES,NFACES,
     $NUNKNS,NSE,NAPTRE,NHANDL)
      DIMENSION DATNOD(NNODES,3),AL(3),H(3),RBH(3)
      INTEGER NCONN(NEDGES,3),NBOUND(150,4),IS(3)
      COMMON/VOL/VOLUME
      COMMON/MOD1/AREAT,RAV,IKMAX,IKMIN,RMAX,RMIN
      COMMON/MOD2/AVAREA,JMAX,ARMAX,JMIN,ARMIN,KMIN,RATIO
      WRITE(3,110)
  110 FORMAT('1',///25X,'BODY PARAMETER LIST',///)
      WRITE(3,111) NNODES,NEDGES,NFACES,NSE,NUNKNS,NAPTRE,NHANDL
  111 FORMAT(/10X,'NUMBER OF VERTICES=',1X,I3,
     $//10X,'NUMBER OF EDGES=',1X,I3,
     $//10X,'NUMBER OF FACES=',1X,I3,
     $//10X,'NUMBER OF BOUNDARY EDGES=',1X,I3,
     $//10X,'NUMBER OF INTERIOR EDGES(NO.OF UNKNOWNS) =',1X,I3,
     $//10X,'NUMBER OF APERTURES(BOUNDARY CONTOURS) =',1X,I3,
     $//10X,'NUMBER OF HANDLES =',1X,I3)
  107 AREAT=0.0
      ALT=0.0
      DO 199 IJK=1,NFACES
      I2=NBOUND(IJK,2)
      I3=NBOUND(IJK,3)
      I4=NBOUND(IJK,4)
      IS(1)=I4
      IS(2)=I2
      IS(3)=I3
      IF(NCONN(I2,2).EQ.NCONN(I3,2)) GO TO 5
      IF(NCONN(I2,2).EQ.NCONN(I3,3)) GO TO 5
      N1=NCONN(I2,3)
      GO TO 6
    5 N1=NCONN(I2,2)
    6 IF(NCONN(I3,2).EQ.NCONN(I4,2)) GO TO 10
      IF(NCONN(I3,2).EQ.NCONN(I4,3)) GO TO 10
      N2=NCONN(I3,3)
      GO TO 11
   10 N2=NCONN(I3,2)
   11 IF(NCONN(I4,2).EQ.NCONN(I2,2)) GO TO 15
      IF(NCONN(I4,2).EQ.NCONN(I2,3)) GO TO 15
      N3=NCONN(I4,3)
      GO TO 16
   15 N3=NCONN(I4,2)
   16 CONTINUE
      X1=DATNOD(N1,2)
      Y1=DATNOD(N1,3)
      X2=DATNOD(N2,2)
      Y2=DATNOD(N2,3)
      X3=DATNOD(N3,2)
      Y3=DATNOD(N3,3)
      AR3=(X2-X1)*(Y3-Y1)-(X3-X1)*(Y2-Y1)
      AREA=ABS(AR3)/2.0
      AREAT=AREAT+AREA
      AL(3)=SQRT((X2-X1)**2+(Y2-Y1)**2)
      AL(1)=SQRT((X3-X2)**2+(Y3-Y2)**2)
      AL(2)=SQRT((X1-X3)**2+(Y1-Y3)**2)
      ALT=ALT+AL(1)+AL(2)+AL(3)
      H(1)=2.0*AREA/AL(1)
      H(2)=2.0*AREA/AL(2)
      H(3)=2.0*AREA/AL(3)
      RBH(1)=H(1)/AL(1)
      RBH(2)=H(2)/AL(2)
```

```
      RBH(3)=H(3)/AL(3)
      IF(IJK.EQ.1) GC TC 19
      DO 17 I=1,3
      IF(AL(I).LE.RMIN) GO TO 19
17    CONTINUE
      GO TO 21
19    IMIN=IS(1)
      ALMIN=AL(1)
      DC 20 I=1,2
      AR=AL(I+1)
      IF(ALMIN.LE.AR) GC TO 20
      ALMIN=AL(I+1)
      IMIN=IS(I+1)
20    CCNTINUE
      RMIN=ALMIN
21    IF(IJK.EQ.1) GO TC 24
      DC 22 I=1,3
      IF(AL(I).GE.RMAX) GC TC 24
22    CONTINUE
      GC TC 26
24    IMAX=IS(1)
      ALMAX=AL(1)
      DC 25 I=1,2
      AR=AL(I+1)
      IF(ALMAX.GE.AR) GC TO 25
      ALMAX=AL(I+1)
      IMAX=IS(I+1)
25    CONTINUE
      RMAX=ALMAX
26    IF(IJK.EC.1) GC TC 35
      DO 28 I=1,3
      IF(RBH(I).LE.RATIO) GO TO 35
28    CCNTINUE
      GO TC 41
35    KMIN=IJK
      RATIC1=RBH(1)
      DO 40 I=1,2
      IF(RATIO1.LE.RBH(I+1)) GC TC 40
      RATIO1=RBH(I+1)
40    CONTINUE
      RATIC=RATIC1
41    IF(IJK.EQ.1) GC TC 198
      IF(AREA.LE.ARMIN) GO TO 191
      GC TC 192
191   ARMIN=AREA
      JMIN=IJK
192   IF(AREA.GE.ARMAX) GO TO 196
      GC TC 199
196   ARMAX=AREA
      JMAX=IJK
      GC TC 199
198   ARMAX=AREA
      JMAX=IJK
      JMIN=IJK
      ARMIN=AREA
199   CONTINUE
      RAV=ALT/FLOAT(NECGES+NEDGES-NSE)
      AVAREA=AREAT/FLCAT(NFACES)
      FACESM=1.0/AVAREA
      IKMAX=IMAX
```

```
      IKMIN=IMIN
      WRITE(3,205)
205   FORMAT(//////////25X,'MODELING PARAMETER LIST'///)
      WRITE(3,206) AREAT
206   FORMAT(/10X,'SURFACE AREA OF THE SCATTERER=',1E13.5,1X,
     $'SQ.METERS')
208   WRITE(3,209) RAV,IMAX,RMAX,IMIN,RMIN
209   FORMAT(/10X,'AVERAGE EDGE LENGTH=',1E13.5,1X,'METERS',
     $//10X,'MAXIMUM EDGE LENGTH(EDGE NO.',I3,1X,')=',1E13.5,1X,
     $'METERS',
     $//10X,'MINIMUM EDGE LENGTH(EDGE NO.',I3,1X,')=',1E13.5,1X,
     $'METERS')
      WRITE(3,210) AVAREA,JMAX,ARMAX,JMIN,ARMIN
210   FORMAT(/10X,'AVERAGE FACE AREA =',1E13.5,1X,'SQ.METERS',
     $//10X,'MAXIMUM FACE AREA (FACE NO.',I3,1X,')=',1E13.5,1X,
     $'SQ.METERS',
     $//10X,'MINIMUM FACE AREA (FACE NO.',I3,1X,')=',1E13.5,1X,
     $'SQ.METERS')
      WRITE(3,211) KMIN,RATIO,FACESM
211   FORMAT(/10X,'MINIMUM FACE HEIGHT TO BASE RATIO (
     $FACE NO.',I3,1X,')=',1E13.5,//10X,'AVERAGE NUMBER OF FACES PER
     $ SQUARE METER=',1E13.5)
      RETURN
      END
```

```
      SUBROUTINE YMATRX(CY,DATNOD,NCONN,NBOUND,NNODES,NEDGES,
     $NFACES,NUNKNS,ITRAK,CI)
C IN THIS SUBROUTINE,WE COMPUTE THE MATRIX ELEMENTS AS DESCRIBED
C IN THE REFERENCE SITED IN THE NOTE.FIRST WE CALCULATE THE
C POTENTIAL QUANTITIES OVER A SOURCE TRIANGULAR REGION AT
C THE CENTROID OF A FIELD TRIANGLE.THEN WE PUT THIS
C QUANTITIES WITH APPROPRIATE MULTYPLING FACTORS IN DIFFERENT
C ROWS AND COLUMNS OF Y-MATRIX.
      IMPLICIT COMPLEX (C)
      REAL COS,CABS
      COMPLEX CY(NUNKNS,NUNKNS),CI(NUNKNS),HTHETA,HPHI
      COMPLEX CYTEMP(50,50),CITEMP(50)
      COMPLEX CS(3),ETHETA,EPHI,HX,HY,HZ,HDOTT
      DIMENSION DATNOD(NNODES,3),TMAT(3,2)
      INTEGER NCONN(NEDGES,3),NBOUND(150,4),ITRAK(NEDGES)
      COMMON/KKK/AK,PI
      COMMON/PARAM/THETA,PHI,IFIELD
      COMMON/FIELD/ETHETA,EPHI,ALAMDA
      COMMON/DIELEC/DIA,DIB
      NFIELD=IFIELD
      C1=CMPLX(1.0,0.0)
      C2=CMPLX(2.0,0.0)
2     DO 1040 IDIE=1,2
      DIE=SQRT(DIA)
      IF(IDIE.EQ.2) DIE=SQRT(DIB)
C CALCULATE ELECTRICAL PARAMETERS
      PI=3.14159265
      AK=2.0*PI/ALAMDA/SQRT(DIA)*DIE
      VEL=3.0E+08/DIE
      AOMEGA=AK*VEL
      CONST1=CMPLX(1.0E-07,0.0)*CMPLX(0.0,AOMEGA)
      CONST2=CMPLX(9.0E+09,0.0)*CMPLX(0.0,1.0/AOMEGA)/DIE**2
      AIMP=120.0*PI/DIE
      CIMP=CMPLX(AIMP,0.0)
      CONST1=CONST1/(CIMP*CIMP)
      CONST2=CONST2/(CIMP*CIMP)
C CALCULATE HTHETA AND HPHI
      HPHI=ETHETA/CIMP
      HTHETA=-EPHI/CIMP
      CT1=CMPLX(COS(THETA),0.0)
      CT2=CMPLX(SIN(THETA),0.0)
      CPHI1=CMPLX(COS(PHI),0.0)
      CPHI2=CMPLX(SIN(PHI),0.0)
C COMPUTE THE AMPLITUDE OF INCIDENT MAGNETIC FIELD IN
C TERMS OF HX,HY AND HZ.
      HX=HTHETA*CT1*CPHI1-HPHI*CPHI2
      HY=HTHETA*CT1*CPHI2+HPHI*CPHI1
      HZ=-HTHETA*CT2
      NSE=NEDGES-NUNKNS
      DO 999 IJK=1,NFACES
      IF(IJK.NE.1.AND.NFIELD.GT.1) GO TO 999
C OBTAIN THE EDGE NUMBERS OF THE SOURCE TRIANGLE.
      I2=NBOUND(IJK,2)
      I3=NBOUND(IJK,3)
      I4=NBOUND(IJK,4)
C OBTAIN THE VERTICES OF THE SOURCE TRIANGLE.
      IF(NCONN(I2,2).EQ.NCONN(I3,2)) GO TO 5
      IF(NCONN(I2,2).EQ.NCONN(I3,3)) GO TO 5
      N1=NCONN(I2,3)
      GO TO 6
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 5     N1=NCONN(I2,2)
 6     IF(NCONN(I3,2).EQ.NCONN(I4,2)) GO TO 10
       IF(NCONN(I3,2).EQ.NCONN(I4,3)) GO TO 10
       N2=NCONN(I3,3)
       GO TO 11
 10    N2=NCONN(I3,2)
 11    IF(NCONN(I4,2).EQ.NCONN(I2,2)) GO TO 15
       IF(NCONN(I4,2).EQ.NCONN(I2,3)) GO TO 15
       N3=NCONN(I4,3)
       GO TO 16
 15    N3=NCONN(I4,2)
 16    CONTINUE
C OBTAIN THE COORDINATES OF THE VERTICES OF THE SOURCE
C TRIANGLE.
       X1=DATNOD(N1,2)
       Y1=DATNOD(N1,3)
       X2=DATNOD(N2,2)
       Y2=DATNOD(N2,3)
       X3=DATNOD(N3,2)
       Y3=DATNOD(N3,3)
C CALCULATE THE AREA OF THE TRIANGLE BY TAKING THE
C MAGNITUDE OF THE VECTOR CROSS PRODUCT OF THE TWO SIDES.
       AR3=(X2-X1)*(Y3-Y1)-(X3-X1)*(Y2-Y1)
       AREA=ABS(AR3)/2.0
C OBTAIN THE LENGTHS OF EACH SIDE.
       R2MR1M=SQRT((X2-X1)**2+(Y2-Y1)**2)
       R3MR2M=SQRT((X3-X2)**2+(Y3-Y2)**2)
       R1MR3M=SQRT((X1-X3)**2+(Y1-Y3)**2)
C OBTAIN THE HEIGHTS OF EACH VERTEX WITH RESPECT TO
C THE CORRESPONDING OPPOSITE EDGE.
       CH1=CMPLX(2.0*AREA/R3MR2M,0.0)
       CH2=CMPLX(2.0*AREA/R1MR3M,0.0)
       CH3=CMPLX(2.0*AREA/R2MR1M,0.0)
C NOW CALCULATE THE PARAMETERS OF THE FIELD TRIANGLE.
       DO 499 IJ=1,NFACES
C OBTAIN THE EDGES OF THE FIELD TRIANGLE.
       J2=NBOUND(IJ,2)
       J3=NBOUND(IJ,3)
       J4=NBOUND(IJ,4)
C OBTAIN THE VERTICES OF THE FIELD TRIANGLE.
       IF(NCONN(J2,2).EQ.NCONN(J3,2)) GO TO 250
       IF(NCONN(J2,2).EQ.NCONN(J3,3)) GO TO 250
       NJ1=NCONN(J2,3)
       GO TO 255
 250   NJ1=NCONN(J2,2)
 255   IF(NCONN(J3,2).EQ.NCONN(J4,2)) GO TO 256
       IF(NCONN(J3,2).EQ.NCONN(J4,3)) GO TO 256
       NJ2=NCONN(J3,3)
       GO TO 258
 256   NJ2=NCONN(J3,2)
 258   IF(NCONN(J4,2).EQ.NCONN(J2,2)) GO TO 259
       IF(NCONN(J4,2).EQ.NCONN(J2,3)) GO TO 259
       NJ3=NCONN(J4,3)
       GO TO 260
 259   NJ3=NCONN(J4,2)
 260   CONTINUE
C OBTAIN THE CENTROID OF THE FIELD TRIANGLE.
       X=(DATNOD(NJ1,2)+DATNOD(NJ2,2)+DATNOD(NJ3,2))/3.0
       Y=(DATNOD(NJ1,3)+DATNOD(NJ2,3)+DATNOD(NJ3,3))/3.0
C CALCULATE COMPONENTS OF THE TESTING VECTOR
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C CORRESPONDING TO EACH SIDE.
      TMAT(1,1)=(DATNOD(NJ2,2)+DATNOD(NJ3,2))/2.0-X
      TMAT(1,2)=(DATNOD(NJ2,3)+DATNOD(NJ3,3))/2.0-Y
      TMAT(2,1)=(DATNOD(NJ3,2)+DATNOD(NJ1,2))/2.0-X
      TMAT(2,2)=(DATNOD(NJ3,3)+DATNOD(NJ1,3))/2.0-Y
      TMAT(3,1)=(DATNOD(NJ1,2)+DATNOD(NJ2,2))/2.0-X
      TMAT(3,2)=(DATNOD(NJ1,3)+DATNOD(NJ2,3))/2.0-Y
C OBTAIN THE COORDINATES OF THE FIELD TRIANGLE.
      XJ1=DATNOD(NJ1,2)
      YJ1=DATNOD(NJ1,3)
      XJ2=DATNOD(NJ2,2)
      YJ2=DATNOD(NJ2,3)
      XJ3=DATNOD(NJ3,2)
      YJ3=DATNOD(NJ3,3)
C OBTAIN THE LENGTH OF THE EACH SIDE.
      CS(1)=CMPLX(SQRT((XJ2-XJ3)**2+(YJ2-YJ3)**2),0.0)
      CS(2)=CMPLX(SQRT((XJ3-XJ1)**2+(YJ3-YJ1)**2),0.0)
      CS(3)=CMPLX(SQRT((XJ1-XJ2)**2+(YJ1-YJ2)**2),0.0)
      IF(NFIELD.NE.1) GO TO 281
C CALL THE SUBROUTINES TO COMPUTE THE INTEGRALS.
      CALL SCAINT(X1,Y1,X2,Y2,X3,Y3,X,Y,CPHI,AREA)
      CALL VECINT(X1,Y1,X2,Y2,X3,Y3,X,Y,
     $CAXSI,CAETA,AREA)
 281  IV=0
C COMPUTE THE VECTOR AND SCALAR POTENTIALS ASSOCIATED WITH
C EACH EDGE OF THE SOURCE TRIANGLE.IF ANY OF THE THREE EDGES
C IS A BOUNDARY EDGE,THEN THE CURRENT COEFFICIENT OF THIS EDGE
C IS ZERO AND HENCE THE POINTER JUMPS OUT OF THE LOOP.
C       HERE CAX,CAY AND CAZ ARE THE VECTOR POTENTIALS
C IN THE X,Y AND Z-DIRECTIONS RESPECTIVELY.
      DO 460 IK=1,3
      IF(IK.EQ.1) I1=I4
      IF(IK.EQ.2) I1=I2
      IF(IK.EQ.3) I1=I3
      K1=0
      IF(NSE.EQ.0) GO TO 288
      DO 285 J=1,NSE
      IF(I1.EQ.ITRAK(J)) GO TO 460
 285  CONTINUE
      DO 287 K=1,NSE
      IF(I1.GT.ITRAK(K)) GO TO 286
      GO TO 288
 286  K1=K1+1
 287  CONTINUE
 288  IF(IK.EQ.2) GO TO 300
      IF(IK.EQ.1) GO TO 310
      CFLAG=C1
      IF(NCONN(I1,2).EQ.N2.AND.NCONN(I1,3).EQ.N1) CFLAG=-C1
      CAX=CFLAG*(CMPLX(X1-X3,0.0)*CPHI+CMPLX(X2-X1,0.0)*CAXSI
     $+CMPLX(X3-X1,0.0)*CAETA)/CH3
      CAY=CFLAG*(CMPLX(Y1-Y3,0.0)*CPHI+CMPLX(Y2-Y1,0.0)*CAXSI
     $+CMPLX(Y3-Y1,0.0)*CAETA)/CH3
      CSPOT=CFLAG*CONST2*CPHI*C2/CH3
      GO TO 375
 300  CFLAG=C1
      IF(NCONN(I1,2).EQ.N1.AND.NCONN(I1,3).EQ.N3) CFLAG=-C1
      CAX=CFLAG*(CMPLX(X1-X2,0.0)*CPHI+CMPLX(X2-X1,0.0)*CAXSI
     $+CMPLX(X3-X1,0.0)*CAETA)/CH2
      CAY=CFLAG*(CMPLX(Y1-Y2,0.0)*CPHI+CMPLX(Y2-Y1,0.0)*CAXSI
     $+CMPLX(Y3-Y1,0.0)*CAETA)/CH2
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      CSPOT=CFLAG*CONST2*CPHI*C2/CH2
      GO TO 375
 310  CFLAG=C1
      IF(NCONN(I1,2).EQ.N3.AND.NCONN(I1,3).EQ.N2) CFLAG=-C1
      CAX=CFLAG*(CMPLX(X2-X1,0.0)*CAXSI+CMPLX(X3-X1,0.0)*CAETA)/CH1
      CAY=CFLAG*(CMPLX(Y2-Y1,0.0)*CAXSI+CMPLX(Y3-Y1,0.0)*CAETA)/CH1
      CSPOT=CFLAG*CONST2*CPHI*C2/CH1
 375  CONTINUE
      IV=IV+1
C COMPUTE THE TESTING VECTOR ASSOCIATED WITH EACH EDGE.
C AGAIN IF ANY OF THE EDGES OF FIELD TRIANGLE IS A BOUNDARY
C EDGE THEN THE POINTER JUMPS OUT OF THE LOOP.
      DO 450 IR=1,3
      IF(IR.EQ.1) J1=J4
      IF(IR.EQ.2) J1=J2
      IF(IR.EQ.3) J1=J3
      L1=0
      IF(NSE.EQ.0) GO TO 405
      DO 390 J=1,NSE
      IF(J1.EQ.ITRAK(J)) GO TO 450
 390  CONTINUE
      DO 399 K=1,NSE
      IF(J1.GT.ITRAK(K)) GO TO 397
      GO TO 405
 397  L1=L1+1
 399  CONTINUE
 405  CT1=CMPLX(TMAT(IR,1),0.0)
      CT2=CMPLX(TMAT(IR,2),0.0)
C COMPUTE THE DOT PRODUCT BETWEEN THE VECTOR POTENTIAL
C AND THE TESTING VECTOR.
      CTEMP=CONST1*(CAX*CT1+CAY*CT2)
      ARGMNT=X*SIN(THETA)*COS(PHI)+Y*SIN(THETA)*SIN(PHI)
      CARG=CMPLX(0.0,-AK*ARGMNT)
      HDOTT=HX*CT1+HY*CT2
      CHTEMP=HDOTT*CEXP(CARG)
      IF(IR.EQ.1) GO TO 420
      IF(IR.EQ.2) GO TO 430
      CFLAG=C1
      IF(NCONN(J1,2).EQ.NJ2.AND.NCONN(J1,3).EQ.NJ1) CFLAG=-C1
      GO TO 440
 420  CFLAG=C1
      IF(NCONN(J1,2).EQ.NJ3.AND.NCONN(J1,3).EQ.NJ2) CFLAG=-C1
      GO TO 440
 430  CFLAG=C1
      IF(NCONN(J1,2).EQ.NJ1.AND.NCONN(J1,3).EQ.NJ3) CFLAG=-C1
 440  IF(NFIELD.NE.1) GO TO 442
      CY(J1-L1,I1-K1)=CY(J1-L1,I1-K1)+CFLAG*(CTEMP-CSPOT)*CS(IR)
 442  IF(IJK.NE.1.OR.IV.GT.1) GO TO 450
      CI(J1-L1)=CI(J1-L1)+CFLAG*CS(IR)*CHTEMP
 450  CONTINUE
 460  CONTINUE
 499  CONTINUE
 999  CONTINUE
      IF(NFIELD.NE.1) GO TO 1005
      DO 1000 I=1,NUNKNS
      DO 1000 J=1,NUNKNS
1000  CY(I,J)=CY(I,J)*CMPLX(4.0,0.0)
1005  DO 1010 I=1,NUNKNS
1010  CI(I)=CI(I)*CMPLX(2.0,0.0)
      IF(DIA.EQ.DIB) GO TO 2000
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      IF(IDIE.EQ.2) GO TO 1025
      DO 1020 I=1,NUNKNS
      DO 1015 J=1,NUNKNS
      CYTEMP(I,J)=CY(I,J)/C2
 1015 CY(I,J)=CMPLX(0.0,0.0)
      CITEMP(I)=CI(I)
 1020 CI(I)=CMPLX(0.0,0.0)
      GO TO 1040
 1025 DO 1035 I=1,NUNKNS
      DO 1030 J=1,NUNKNS
 1030 CY(I,J)=CY(I,J)/C2+CYTEMP(I,J)
 1035 CI(I)=CITEMP(I)
 1040 CONTINUE
 2000 RETURN
      END
```

```
      SUBROUTINE YWINDO(CY,DATNOD,NCONN,NBOUND,NNODES,NEDGES,NFACES,NUNK
     $NS,ITRAK,NJUNC,SIGMA,DELDIE,THICK)
C THIS SUBROUTINE CALCULATES THE ADDITIONAL TERM IN THE ADMITTANCE
C MATRIX WHEN WE COVER THE APERTURE WITH A (LOSSY) DIELECTRIC SHEET.
      IMPLICIT COMPLEX (C)
      REAL COS,CABS
      COMPLEX CY(NUNKNS,NUNKNS),CS(3)
      DIMENSION DATNOD(NNODES,3),TMAT(3,2)
      INTEGER NCONN(NEDGES,3),NBOUND(150,4),ITRAK(NEDGES)
      INTEGER NJUNC(NEDGES,3)
      COMMON/PARAM/THETA,PHI,IFIELD
      COMMON/FRE/AOMEGA
      PI=3.14159265
      CONST=CMPLX(SIGMA,AOMEGA*DELDIE/(36*PI*1.0E+09))
      CONST=CONST*CMPLX(THICK,0.0)
      C1=CMPLX(1.0,0.0)
      NFIELD=IFIELD
      IV=0
      NSE=NEDGES-NUNKNS
      DO 999 I=1,NEDGES
      NF1=NJUNC(I,2)
      NF2=NJUNC(I,3)
      IF(NF2.NE.0) GO TO 888
      IV=IV+1
      GO TO 999
888   DO 499 M=1,2
      J2=NBOUND(NF1,2)
      IF(M.EQ.2) J2=NBOUND(NF2,2)
      J3=NBOUND(NF1,3)
      IF(M.EQ.2) J3=NBOUND(NF2,3)
      J4=NBOUND(NF1,4)
      IF(M.EQ.2) J4=NBOUND(NF2,4)
C OBTAIN THE VERTICES OF THE FIELD TRIANGLE.
      IF(NCONN(J2,2).EQ.NCONN(J3,2)) GO TO 250
      IF(NCONN(J2,2).EQ.NCONN(J3,3)) GO TO 250
      NJ1=NCONN(J2,3)
      GO TO 255
 250  NJ1=NCONN(J2,2)
 255  IF(NCONN(J3,2).EQ.NCONN(J4,2)) GO TO 256
      IF(NCONN(J3,2).EQ.NCONN(J4,3)) GO TO 256
      NJ2=NCONN(J3,3)
      GO TO 258
 256  NJ2=NCONN(J3,2)
 258  IF(NCONN(J4,2).EQ.NCONN(J2,2)) GO TO 259
      IF(NCONN(J4,2).EQ.NCONN(J2,3)) GO TO 259
      NJ3=NCONN(J4,3)
      GO TO 260
 259  NJ3=NCONN(J4,2)
 260  CONTINUE
C OBTAIN THE CENTROID OF THE FIELD TRIANGLE.
      X=(DATNOD(NJ1,2)+DATNOD(NJ2,2)+DATNOD(NJ3,2))/3.0
      Y=(DATNOD(NJ1,3)+DATNOD(NJ2,3)+DATNOD(NJ3,3))/3.0
C CALCULATE COMPONENTS OF THE TESTING VECTOR
C CORRESPONDING TO EACH SIDE.
      TMAT(1,1)=(DATNOD(NJ2,2)+DATNOD(NJ3,2))/2.0-X
      TMAT(1,2)=(DATNOD(NJ2,3)+DATNOD(NJ3,3))/2.0-Y
      TMAT(2,1)=(DATNOD(NJ3,2)+DATNOD(NJ1,2))/2.0-X
      TMAT(2,2)=(DATNOD(NJ3,3)+DATNOD(NJ1,3))/2.0-Y
      TMAT(3,1)=(DATNOD(NJ1,2)+DATNOD(NJ2,2))/2.0-X
      TMAT(3,2)=(DATNOD(NJ1,3)+DATNOD(NJ2,3))/2.0-Y
```

```
      IF(I.EQ.J2) IIV=1
      IF(I.EQ.J3) IIV=2
      IF(I.EQ.J4) IIV=3
C OBTAIN THE COORDINATES OF THE FIELD TRIANGLE.
      XJ1=DATNOD(NJ1,2)
      YJ1=DATNOD(NJ1,3)
      XJ2=DATNOD(NJ2,2)
      YJ2=DATNOD(NJ2,3)
      XJ3=DATNOD(NJ3,2)
      YJ3=DATNOD(NJ3,3)
C OBTAIN THE LENGTH CF THE EACH SIDE.
      CS(1)=CMPLX(SQRT((XJ2-XJ3)**2+(YJ2-YJ3)**2),0.0)
      CS(2)=CMPLX(SQRT((XJ3-XJ1)**2+(YJ3-YJ1)**2),0.0)
      CS(3)=CMPLX(SQRT((XJ1-XJ2)**2+(YJ1-YJ2)**2),0.0)
      AR3=(XJ2-XJ1)*(YJ3-YJ1)-(XJ3-XJ1)*(YJ2-YJ1)
      AREA=ABS(AR3)/2.0
C COMPUTE THE TESTING VECTOR ASSOCIATED WITH EACH EDGE.
C AGAIN IF ANY OF THE EDGES OF FIELD TRIANGLE IS A BOUNDARY
C EDGE THEN THE POINTER JUMPS OUT OF THE LOOP.
      DO 450 IR=1,3
      IF(IR.EQ.1) J1=J4
      IF(IR.EQ.2) J1=J2
      IF(IR.EQ.3) J1=J3
      L1=0
      IF(NSE.EQ.0) GO TO 405
      DO 390 J=1,NSE
      IF(J1.EQ.ITRAK(J)) GO TC 450
 390  CONTINUE
      DC 399 K=1,NSE
      IF(J1.GT.ITRAK(K)) GO TC 397
      GO TO 405
 397  L1=L1+1
 399  CONTINUE
405   CT1=CMPLX(TMAT(IR,1)*TMAT(IIV,1),0.0)
      CT2=CMPLX(TMAT(IR,2)*TMAT(IIV,2),0.0)
      CLOA=CS(IR)*CS(IIV)/CMPLX(4*AREA,0.0)
      IF(IR.EQ.1) GO TC 420
      IF(IR.EQ.2) GC TO 430
      CFLAG=C1
      IF(NCONN(J1,2).EQ.NJ2.AND.NCONN(J1,3).EQ.NJ1) CFLAG=-C1
      GO TO 440
 420  CFLAG=C1
      IF(NCONN(J1,2).EQ.NJ3.AND.NCONN(J1,3).EQ.NJ2) CFLAG=-C1
      GO TO 440
 430  CFLAG=C1
      IF(NCONN(J1,2).EQ.NJ1.AND.NCONN(J1,3).EQ.NJ3) CFLAG=-C1
 440  IF(NFIELD.NE.1) GO TO 450
      CY(I-IV,J1-L1)=CY(I-IV,J1-L1)+CFLAG*CONST*CLOA*(CT1+CT2)
 450  CONTINUE
 460  CONTINUE
 499  CCNTINUE
 999  CONTINUE
      RETURN
      END
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      SUBROUTINE MAGCHA(CV,DATNOD,NCONN,NBOUND,NNODES,NEDGES,
     $NFACES,NUNKNS,ITRAK)
C THIS SUBROUTINE COMPUTES THE MAGNETIC CHARGE DISTRIBUTION ON THE APERTURE
C AREA.THE CHARGE DENSITY IS COMPUTED AT THE CENTROID OF
C EACH TRIANGLE.
      IMPLICIT COMPLEX (C)
      REAL COS,CABS
      COMPLEX CV(NUNKNS),CS(3)
      DIMENSION DATNOD(NNODES,3)
      INTEGER NCONN(NEDGES,3),NBOUND(150,4),ITRAK(NEDGES)
      COMMON/KKK/AK,PI
      COMMON/DIELEC/DIA,DIB
      C1=CMPLX(1.0,0.0)
      VEL=3.0E+08/SQRT(DIB)
      AOMEGA=AK*VEL
      CONST1=CMPLX(0.0,1.0/AOMEGA)
      CHARGE=CMPLX(0.0,0.0)
      WRITE(3,101)
 101  FORMAT('1',///25X,'SURFACE MAGNETIC CHARGE',///)
      WRITE(3,102)
 102  FORMAT(1X,'FACE NUMBER',25X,'MAGNETIC CHARGE DENSITY (WEBS/M-M)')
      WRITE(3,103)
 103  FORMAT(/20X,'REAL',11X,'IMAGINARY',8X,'MAGNITUDE',10X,'PHASE')
      NSE=NEDGES-NUNKNS
      DO 999 IJK=1,NFACES
C OBTAIN THE EDGES OF THE TRIANGLE.
      I2=NBOUND(IJK,2)
      I3=NBOUND(IJK,3)
      I4=NBOUND(IJK,4)
C OBTAIN THE VERTICES CONNECTED TO THESE EDGES.
      IF(NCONN(I2,2).EQ.NCONN(I3,2)) GO TO 5
      IF(NCONN(I2,2).EQ.NCONN(I3,3)) GO TO 5
      N1=NCONN(I2,3)
      GO TO 6
 5    N1=NCONN(I2,2)
 6    IF(NCONN(I3,2).EQ.NCONN(I4,2)) GO TO 10
      IF(NCONN(I3,2).EQ.NCONN(I4,3)) GO TO 10
      N2=NCONN(I3,3)
      GO TO 11
 10   N2=NCONN(I3,2)
 11   IF(NCONN(I4,2).EQ.NCONN(I2,2)) GO TO 15
      IF(NCONN(I4,2).EQ.NCONN(I2,3)) GO TO 15
      N3=NCONN(I4,3)
      GO TO 16
 15   N3=NCONN(I4,2)
 16   CONTINUE
C COMPUTE THE COORDINATES OF EACH VERTEX.
      X1=DATNOD(N1,2)
      Y1=DATNOD(N1,3)
      X2=DATNOD(N2,2)
      Y2=DATNOD(N2,3)
      X3=DATNOD(N3,2)
      Y3=DATNOD(N3,3)
C CALCULATE THE AREA OF THE TRIANGLE.
      AR3=(X2-X1)*(Y3-Y1)-(X3-X1)*(Y2-Y1)
      AREA=ABS(AR3)/2.0
C CALCULATE THE LENGTHS OF EACH SIDE.
      R2WR1W=SQRT((X2-X1)**2+(Y2-Y1)**2)
      R3WR2W=SQRT((X3-X2)**2+(Y3-Y2)**2)
      R1WR3W=SQRT((X1-X3)**2+(Y1-Y3)**2)
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      CS(1)=CMPLX(R3MR2M,0.0)
      CS(2)=CMPLX(R1MR3M,0.0)
      CS(3)=CMPLX(R2MR1M,0.0)
C COMPUTE THE MAGNETIC CHARGE DENSITY ON THE TRIANGLE.
      CSUM=CMPLX(0.0,0.0)
      DO 460 IK=1,3
      IF(IK.EQ.1) I1=I4
      IF(IK.EQ.2) I1=I2
      IF(IK.EQ.3) I1=I3
      K1=0
      IF(NSE.EQ.0) GO TO 288
      DO 285 J=1,NSE
      IF(I1.EQ.ITRAK(J)) GO TO 460
  285 CONTINUE
      DO 287 K=1,NSE
      IF(I1.GT.ITRAK(K)) GO TO 286
      GO TO 288
  286 K1=K1+1
  287 CONTINUE
  288 IF(IK.EQ.2) GO TO 300
      IF(IK.EQ.1) GO TO 310
      CFLAG=C1
      IF(NCONN(I1,2).EQ.N2.AND.NCONN(I1,3).EQ.N1) CFLAG=-C1
      GO TO 375
  300 CFLAG=C1
      IF(NCONN(I1,2).EQ.N1.AND.NCONN(I1,3).EQ.N3) CFLAG=-C1
      GO TO 375
  310 CFLAG=C1
      IF(NCONN(I1,2).EQ.N3.AND.NCONN(I1,3).EQ.N2) CFLAG=-C1
  375 CONTINUE
      CSUM=CSUM+CFLAG*CONST1*CS(IK)*CV(I1-K1)
  460 CONTINUE
      CHDEN=CSUM/CMPLX(AREA,0.0)
      RA1=REAL(CHDEN)
      RA2=AIMAG(CHDEN)
      RA3=CABS(CHDEN)
      RA4=ATAN2(RA2,RA1)
      WRITE(3,501) IJK,RA1,RA2,RA3,RA4
  501 FORMAT(2X,I4,8X,1E13.5,2X,1E13.5,2X,1E13.5,2X,1E13.5,/)
      CHARGE=CHARGE+CSUM
  999 CONTINUE
      WRITE(3,502) CHARGE
  502 FORMAT(////10X,'TOTAL MAGNETIC CHARGE ON THE AREA= (',2E13.5,1X,
     $' WEBERS')
      RETURN
      END
```

```
      SUBROUTINE TRANCS(DATNOD,NCONN,NBOUND,NNODES,NEDGES,NFACES,
     $NUNKNS,CV,ITRAK,CIM)
C IN THIS SUBROUTINE, THE TRANS CROSS SECTION IS COMPUTED AS A
C FUNCTION OF SPHERICAL COORDINATE ANGLES THETA AND PHI.
C PHI1 AND PHI2 REPRESENT THE INITIAL AND FINAL VALUES OF PHI
C AND NPHI REPRESENTS THE NUMBER OF DIVISIONS IN PHI DIRECTION.
C SIMILARLY THETA1 AND THETA2 REPRESENT THE INITIAL AND FINAL
C VALUES OF THETA AND NTHETA REPRESENTS THE NUMBER OF DIVISIONS IN
C THETA DIRECTION.THUS, THIS SUBROUTINE COMPUTES TRANS CROSS SECTION
C FOR NPHI X NTHETA VALUES OF THETA AND PHI.
      IMPLICIT COMPLEX (C)
      REAL COS,CABS,CCNST
      COMPLEX CIM(NUNKNS),CV(NUNKNS),HMT,HMP
      DIMENSION DATNOD(NNODES,3)
      INTEGER NCONN(NEDGES,3),NBOUND(150,4),ITRAK(NEDGES)
      COMMON/FHINCI/FHINC
      COMMON/KKK/AK,PI
      COMMON/POLARM/HMT,HMP
      COMMON/DIELEC/CIA,DIB
      READ(1,195)PHI1,PHI2,NPHI,THETA1,THETA2,NTHETA
 195  FORMAT(2F7.2,I3,2F7.2,I3)
      WRITE(3,196)
 196  FORMAT('1',////20X,'APERTURE TRANS CROSS SECTION/SQ.W.L.',///)
      WRITE(3,197)
 197  FORMAT(5X,'THETA(DEGREES)',5X,'PHI(DEGREES)',5X,
     $3X,'TRANS CROSS SECTION (SQ.METERS/SQ.METERS)')
      DPHI=(PHI2-PHI1)/FLOAT(NPHI)
      DTHETA=(THETA2-THETA1)/FLOAT(NTHETA)
      NTHET=NTHETA+1
      NPH=NPHI+1
      DO 999 II=1,NTHET
      THETA=THETA1+FLOAT(II-1)*DTHETA
      THETA=THETA*PI/180.0
      DO 998 JJ=1,NPH
      PHI=PHI1+FLOAT(JJ-1)*DPHI
      PHI=PHI*PI/180.0
      ALAMDA=2*PI/AK
      AOMEGA=AK*3.0E+08/SQRT(DIB)
      CONST=(AOMEGA/(36*PI))*1.0E-09*DIB
      CONST=CONST**2/(8*PI)
      CALL MEASUR(CIM,THETA,PHI,DATNOD,NCONN,NBOUND,NNODES,
     $NEDGES,NFACES,NUNKNS,ITRAK)
      CTCS=CMPLX(0.0,0.0)
      DO 499 IJK=1,NUNKNS
      CTCS=CTCS+CIM(IJK)*CV(IJK)
499   CONTINUE
      TCS=(CABS(CTCS))**2*CCNST
      TCS=TCS/ALAMDA**2
      TCS=TCS/FHINC**2
      THETAD=THETA*180.0/PI
      PHID=PHI*180.0/PI
      WRITE(3,991) THETAD,PHID,TCS
 991  FORMAT(/3X,1E13.5,4X,1E13.5,12X,1E13.5)
 998  CONTINUE
 999  CONTINUE
      IF(CABS(HMT).EQ.FLOAT(1)) GO TO 1000
      IF(CABS(HMP).EQ.FLOAT(1)) GO TO 1010
      GO TO 1020
1000  WRITE(3,1005)
1005  FORMAT(//3X,'TCS/SQ.W.L. OF THETA POLARIZATION MEASUREMENT.')
```

```
      GO TO 1020
1010  WRITE(3,1015)
1015  FORMAT(//3X,'TCS/SQ.W.L. OF PHI POLARIZATION MEASUREMENT.')
1020  RETURN
      END
```

```
      SUBROUTINE MEASUR(CIM,THETA,PHI,DATNOD,NCONN,NBOUND,NNODES,
     $NEDGES,NFACES,NUNKNS,ITRAK)
C THIS SUBROUTINE COMPUTE THE MEASUREMENT CURRENT VECTOR.
      IMPLICIT COMPLEX (C)
      REAL COS,CABS
      COMPLEX CIM(NUNKNS),HMT,HMP
      COMPLEX HX,HY,HZ,HDOTT,CS(3)
      DIMENSION DATNOD(NNODES,3),TMAT(3,2)
      INTEGER NCONN(NEDGES,3),NBOUND(150,4),ITRAK(NEDGES)
      COMMON/KKK/AK,PI
      COMMON/POLARM/HMT,HMP
      COMMON/DIELEC/DIA,DIB
      CI=CMPLX(1.0,0.0)
C
      DO 2 I=1,NUNKNS
      CIM(I)=CMPLX(0.0,0.0)
2     CONTINUE
      CT1=CMPLX(COS(THETA),0.0)
      CT2=CMPLX(SIN(THETA),0.0)
      CPHI1=CMPLX(COS(PHI),0.0)
      CPHI2=CMPLX(SIN(PHI),0.0)
C
      HX=HMT*CT1*CPHI1-HMP*CPHI2
      HY=HMT*CT1*CPHI2+HMP*CPHI1
      HZ=-HMT*CT2
      NSE=NEDGES-NUNKNS
C
C NOW CALCULATE THE PARAMETERS OF THE FIELD TRIANGLE.
      DO 499 IJ=1,NFACES
C OBTAIN THE EDGES OF THE FIELD TRIANGLE.
      J2=NBOUND(IJ,2)
      J3=NBOUND(IJ,3)
      J4=NBOUND(IJ,4)
C OBTAIN THE VERTICES OF THE FIELD TRIANGLE.
      IF(NCONN(J2,2).EQ.NCONN(J3,2)) GO TO 250
      IF(NCONN(J2,2).EQ.NCONN(J3,3)) GO TO 250
      NJ1=NCONN(J2,3)
      GO TO 255
 250  NJ1=NCONN(J2,2)
 255  IF(NCONN(J3,2).EQ.NCONN(J4,2)) GO TO 256
      IF(NCONN(J3,2).EQ.NCONN(J4,3)) GO TO 256
      NJ2=NCONN(J3,3)
      GO TO 258
 256  NJ2=NCONN(J3,2)
 258  IF(NCONN(J4,2).EQ.NCONN(J2,2)) GO TO 259
      IF(NCONN(J4,2).EQ.NCONN(J2,3)) GO TO 259
      NJ3=NCONN(J4,3)
      GO TO 260
 259  NJ3=NCONN(J4,2)
 260  CONTINUE
C OBTAIN THE CENTROID OF THE FIELD TRIANGLE.
      X=(DATNOD(NJ1,2)+DATNOD(NJ2,2)+DATNOD(NJ3,2))/3.0
      Y=(DATNOD(NJ1,3)+DATNOD(NJ2,3)+DATNOD(NJ3,3))/3.0
C CALCULATE COMPONENTS OF THE TESTING VECTOR
C CORRESPONDING TO EACH SIDE.
      TMAT(1,1)=(DATNOD(NJ2,2)+DATNOD(NJ3,2))/2.0-X
      TMAT(1,2)=(DATNOD(NJ2,3)+DATNOD(NJ3,3))/2.0-Y
      TMAT(2,1)=(DATNOD(NJ3,2)+DATNOD(NJ1,2))/2.0-X
      TMAT(2,2)=(DATNOD(NJ3,3)+DATNOD(NJ1,3))/2.0-Y
      TMAT(3,1)=(DATNOD(NJ1,2)+DATNOD(NJ2,2))/2.0-X
```

```
      TMAT(3,2)=(DATNOD(NJ1,3)+DATNOD(NJ2,3))/2.0-Y
C OBTAIN THE COORDINATES OF THE FIELD TRIANGLE.
      XJ1=DATNOD(NJ1,2)
      YJ1=DATNOD(NJ1,3)
      XJ2=DATNOD(NJ2,2)
      YJ2=DATNOD(NJ2,3)
      XJ3=DATNOD(NJ3,2)
      YJ3=DATNOD(NJ3,3)
C OBTAIN THE LENGTH OF THE EACH SIDE.
      CS(1)=CMPLX(SQRT((XJ2-XJ3)**2+(YJ2-YJ3)**2),0.0)
      CS(2)=CMPLX(SQRT((XJ3-XJ1)**2+(YJ3-YJ1)**2),0.0)
      CS(3)=CMPLX(SQRT((XJ1-XJ2)**2+(YJ1-YJ2)**2),0.0)
C COMPUTE THE TESTING VECTOR ASSOCIATED WITH EACH EDGE.
C AGAIN IF ANY OF THE EDGES OF FIELD TRIANGLE IS A BOUNDARY
C EDGE THEN THE POINTER JUMPS OUT OF THE LOOP.
      DO 450 IR=1,3
      IF(IR.EQ.1) J1=J4
      IF(IR.EQ.2) J1=J2
      IF(IR.EQ.3) J1=J3
      L1=0
      IF(NSE.EQ.0) GO TO 405
      DO 390 J=1,NSE
      IF(J1.EQ.ITRAK(J)) GO TO 450
 390  CONTINUE
      DO 399 K=1,NSE
      IF(J1.GT.ITRAK(K)) GO TO 397
      GO TO 405
 397  L1=L1+1
 399  CONTINUE
 405  CT1=CMPLX(TMAT(IR,1),0.0)
      CT2=CMPLX(TMAT(IR,2),0.0)
C
      ARGMNT=X*SIN(THETA)*COS(PHI)+Y*SIN(THETA)*SIN(PHI)
      CARG=CMPLX(0.0,-AK*ARGMNT)
      HDOTT=HX*CT1+HY*CT2
      CHTEMP=HDOTT*CEXP(CARG)
      IF(IR.EQ.1) GO TO 420
      IF(IR.EQ.2) GO TO 430
      CFLAG=C1
      IF(NCONN(J1,2).EQ.NJ2.AND.NCONN(J1,3).EQ.NJ1) CFLAG=-C1
      GO TO 440
 420  CFLAG=C1
      IF(NCONN(J1,2).EQ.NJ3.AND.NCONN(J1,3).EQ.NJ2) CFLAG=-C1
      GO TO 440
 430  CFLAG=C1
      IF(NCONN(J1,2).EQ.NJ1.AND.NCONN(J1,3).EQ.NJ3) CFLAG=-C1
440   CONTINUE
      CIM(J1-L1)=CIM(J1-L1)+CFLAG*CS(IR)*CHTEMP
450   CONTINUE
460   CONTINUE
499   CONTINUE
999   CONTINUE
      DO 1000 I=1,NUNKNS
1000  CIM(I)=CIM(I)*CMPLX(2.0,0.0)
      RETURN
      END
```

```
      SUBROUTINE SCAINT(X1,Y1,X2,Y2,X3,Y3,X,Y,CPHI,AREA)
C THIS SUBROUTINE,WITH THE HELP OF SUBROUTINE INTGRL,
C EVALUATES THE SCALAR POTENTIAL INTEGRAL OVER A
C TRIANGULAR REGION. FOR DETAILS,PLEASE REFER TO THE NOTE.
      IMPLICIT COMPLEX (C)
      REAL CABS,COS
      COMMON/KKK/AK,PI
      COMMON/VEC/XSI(7),ETA(7)
      XSI(1)=1.0/3.0
      XSI(2)=0.05971587
      XSI(3)=0.47014206
      XSI(4)=XSI(3)
      XSI(5)=0.79742699
      XSI(6)=0.10128651
      XSI(7)=XSI(6)
      ETA(1)=XSI(1)
      ETA(2)=XSI(3)
      ETA(3)=XSI(2)
      ETA(4)=XSI(4)
      ETA(5)=XSI(6)
      ETA(6)=XSI(5)
      ETA(7)=XSI(7)
      CF=CMPLX(0.0,0.0)
      DO 120 I=1,7
      R1=((X-X1)-(X2-X1)*XSI(I)-(X3-X1)*ETA(I))**2
      R2=((Y-Y1)-(Y2-Y1)*XSI(I)-(Y3-Y1)*ETA(I))**2
      R=SQRT(R1+R2)
      CR=CMPLX(0.0,-1.0*AK*R)
      IF(CABS(CR).LE.1.0E-06) GO TO 102
      CF1=(CEXP(CR)-CMPLX(1.0,0.0))/CMPLX(R,0.0)
      GO TO 103
  102 CF1=CMPLX(0.0,-AK)
  103 IF(I.EQ.1) GO TO 105
      IF(I.EQ.2.OR.I.EQ.3.OR.I.EQ.4) GO TO 110
      CF=CF+CF1*CMPLX(.1259392,0.0)
      GO TO 120
  105 CF=CF+CF1*CMPLX(0.225,0.0)
      GO TO 120
  110 CF=CF+CF1*CMPLX(.1323942,0.0)
  120 CONTINUE
      CALL INTGRL(X1,Y1,X2,Y2,X3,Y3,X,Y,POT,AREA)
      CPHI=CF*CMPLX(AREA,0.0)+CMPLX(POT,0.0)
  150 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE VECINT(X1,Y1,X2,Y2,X3,Y3,
     $X,Y,CAXSI,CAETA,AREA)
C THIS SUBROUTINE,WITH THE HELP OF SUBROUTINE LININT,
C EVALUATES THE VECTOR POTENTIAL INTEGRALS OVER A
C TRIANGULAR REGION.FOR DETAILS,PLEASE REFER TO THE NOTE.
      IMPLICIT COMPLEX (C)
      REAL CABS,CCS
      COMMON/KKK/AK,PI
      COMMON/VEC/XSI(7),ETA(7)
      CF=CMPLX(0.0,0.0)
      CG=CMPLX(0.0,0.0)
      DO 120 I=1,7
      R1=((X-X1)-(X2-X1)*XSI(I)-(X3-X1)*ETA(I))**2
      R2=((Y-Y1)-(Y2-Y1)*XSI(I)-(Y3-Y1)*ETA(I))**2
      R=SQRT(R1+R2)
      CR=CMPLX(0.0,-1.0*AK*R)
      IF(CABS(CR).LE.1.0E-06) GO TO 102
      CA=(CEXP(CR)-CMPLX(1.0,0.0))/CMPLX(R,0.0)
      CF1=CMPLX(XSI(I),0.0)*CA
      CG1=CMPLX(ETA(I),0.0)*CA
      GO TO 103
 102  CF1=CMPLX(0.0,-AK*XSI(I))
      CG1=CMPLX(0.0,-AK*ETA(I))
 103  IF(I.EQ.1) GO TO 105
      IF(I.EQ.2.OR.I.EQ.3.OR.I.EQ.4) GO TO 110
      CF=CF+CF1*CMPLX(.1259392,0.0)
      CG=CG+CG1*CMPLX(.1259392,0.0)
      GO TO 120
 105  CF=CF+CF1*CMPLX(0.225,0.0)
      CG=CG+CG1*CMPLX(0.225,0.0)
      GO TO 120
 110  CF=CF+CF1*CMPLX(.1323942,0.0)
      CG=CG+CG1*CMPLX(.1323942,0.0)
 120  CONTINUE
      CALL LININT(X1,Y1,X2,Y2,X3,Y3,X,Y,POTXSI,POTETA,AREA)
      CAXSI=CF*CMPLX(AREA,0.0)+CMPLX(POTXSI,0.0)
      CAETA=CG*CMPLX(AREA,0.0)+CMPLX(POTETA,0.0)
 150  CONTINUE
      RETURN
      END
```

```
      SUBROUTINE LININT(X1,Y1,X2,Y2,X3,Y3,X,Y,
     $POTXSI,POTETA,AREA)
C THIS SUBROUTINE,WITH THE HELP OF SUBROUTINE INTGRL,EVALUATES
C XSI/R AND ETA/R INTEGRALS OVER A TRIANGULAR REGION. THE
C QUANTITIES DEFINED HERE ARE SAME AS THOSE USED IN THE
C REFERENCE SITED IN THE NOTE.
      COMMON/POTEN/POT1
      A=(X2-X1)**2+(Y2-Y1)**2
      B=(X3-X1)**2+(Y3-Y1)**2
      C=-2.0*((X-X1)*(X2-X1)+(Y-Y1)*(Y2-Y1))
      D=-2.0*((X-X1)*(X3-X1)+(Y-Y1)*(Y3-Y1))
      E=2.0*((X2-X1)*(X3-X1)+(Y2-Y1)*(Y3-Y1))
      F=(X-X1)**2+(Y-Y1)**2
      A1=(2.0*B-C+D-E)*SQRT(B+D+F)+(2.0*A+C-D-E)*SQRT(A+C+F)
      A2=4.0*(A+B-E)
      A3=A1/A2
      A4=4.0*(A+C)*(B+D+F)+4.0*F*(E-C-E)-(C+D+E)**2
      A5=8.0*SQRT((A+B-E)**3)
      A6=A4/A5
      IF(ABS(A6).LE.1.0E-04) GO TO 5
      AL1=2.0*SQRT(A+B-E)*SQRT(B+D+F)
      AL2=2.0*SQRT(A+B-E)*SQRT(A+C+F)
      AL3=2.0*B-C+D-E
      AL4=2.0*A+C-D-E
      AJ1=A3+A6*ALOG(ABS((AL1+AL3)/(AL2-AL4)))
      AJ3=A3+A6*ALOG(ABS((AL2+AL4)/(AL1-AL3)))
      GO TO 6
5     AJ1=A3
      AJ3=A3
6     B1=SQRT(A+C+F)
      B2=((2.0*A+C)*B1-C*SQRT(F))/(4.0*A)
      ANUM=ABS(2.0*SQRT(A)*B1+2.0*A+C)
      DEN=ABS(2.0*SQRT(A*F)+C)
      IF(ANUM.LE.1.0E-04) GO TO 10
      IF(DEN.LE.1.0E-04) GO TO 10
      B3=ABS((2.0*SQRT(A)*B1+2.0*A+C)/(2.0*SQRT(A*F)+C))
      AB3=ALOG(B3)
      AJ4=B2+(4.0*A*F-C**2)*AB3/(8.0*SQRT(A**3))
      GO TO 11
10    AJ4=B2
11    B4=SQRT(B+D+F)
      B5=((2.0*B+D)*B4-D*SQRT(F))/(4.0*B)
      ANUM=ABS(2.0*SQRT(B)*B4+2.0*B+D)
      DEN=ABS(2.0*SQRT(B*F)+C)
      IF(ANUM.LE.1.0E-04) GO TO 15
      IF(DEN.LE.1.0E-04) GO TO 15
      B6=ABS((2.0*SQRT(B)*B4+2.0*B+D)/(2.0*SQRT(B*F)+D))
      AB6=ALOG(B6)
      AJ2=B5+(4.0*B*F-D**2)*AB6/(8.0*SQRT(B**3))
      GO TO 16
15    AJ2=B5
16    CONTINUE
      POT=POT1/(2.0*AREA)
      AR1=2.0*B*(AJ1-AJ2)-E*(AJ3-AJ4)
      AR2=(2.0*AR1-(2.0*B*C-E*D)*POT)/(4.0*A*B-E**2)
      POTXSI=2.0*AREA*AR2
      AR3=4.0*A*(AJ3-AJ4)-2.0*E*(AJ1-AJ2)-(2.0*A*D-E*C)*POT
      POTETA=(2.0*AREA*AR3)/(4.0*A*B-E**2)
      RETURN
      END
```

```
      SUBROUTINE INTGRL(XA1,YA1,XA2,YA2,XA3,YA3,
     $X,Y,POT,AREA)
C THIS SUBROUTINE,WITH THE HELP OF SUBROUTINE CA,EVALUATES
C THE 1/R INTEGRAL OVER A TRIANGULAR REGION.
        COMMON/POTEN/POT1
        PI=2.0*ARSIN(1.0)
        X1=XA1
        Y1=YA1
        X2=XA2
        Y2=YA2
        X3=XA3
        Y3=YA3
5       AR3=(X2-X1)*(Y3-Y1)-(Y2-Y1)*(X3-X1)
      AREA=ABS(AR3)/2.0
        UNZ=AR3/(2.0*AREA)
      X0=X
      Y0=Y
        RM=SQRT((X1-X0)**2+(Y1-Y0)**2)
        IF(RM.GT.1.0E-06) GO TO 12
        XDUMMY=X2
        YDUMMY=Y2
        X2=X1
        Y2=Y1
        X1=X3
        Y1=Y3
        X3=XDUMMY
        Y3=YDUMMY
        GO TO 5
 12     URX=(X1-X0)/RM
        URY=(Y1-Y0)/RM
      UTX=-UNZ*URY
      UTY=UNZ*URX
        XT1=(X1-X0)*URX+(Y1-Y0)*URY
        YT1=(X1-X0)*UTX+(Y1-Y0)*UTY
        XT2=(X2-X0)*URX+(Y2-Y0)*URY
        YT2=(X2-X0)*UTX+(Y2-Y0)*UTY
        XT3=(X3-X0)*URX+(Y3-Y0)*URY
        YT3=(X3-X0)*UTX+(Y3-Y0)*UTY
        DETRM=2.0*AREA
        XSI=(XT3*YT1-XT1*YT3)/DETRM
        ETA=(XT1*YT2-XT2*YT1)/DETRM
        ZETA=1.0-XSI-ETA
        SIDE1=SQRT((XT2-XT1)**2+(YT2-YT1)**2)
        SIDE2=SQRT((XT3-XT2)**2+(YT3-YT2)**2)
        SIDE3=SQRT((XT1-XT3)**2+(YT1-YT3)**2)
        TEMP=(XT2-XT1)*(XT3-XT1)+(YT2-YT1)*(YT3-YT1)
        ANGLE1=ARCOS(TEMP/(SIDE1*SIDE3))
        TEMP=(XT3-XT2)*(XT1-XT2)+(YT3-YT2)*(YT1-YT2)
        ANGLE2=ARCOS(TEMP/(SIDE2*SIDE1))
        ANGLE3=PI-ANGLE1-ANGLE2
        ER1=1.0E-06
        FLAG=0.0
        ADD=ABS(XSI)+ABS(ETA)+ABS(ZETA)
        IF(ADD.GT.(1.0+ER1)) GO TO 50
        IF(XSI.GE.(1.0-ER1).AND.XSI.LE.(1.0+ER1)) GO TO 15
        IF(ETA.GE.(1.0-ER1).AND.ETA.LE.(1.0+ER1)) GO TO 20
        IF(ZETA.GE.(1.0-ER1).AND.ZETA.LE.(1.0+ER1)) GO TO 25
        IF(XSI.GE.-ER1.AND.XSI.LE.ER1) GO TO 30
        IF(ETA.GE.-ER1.AND.ETA.LE.ER1) GO TO 35
        IF(ZETA.GE.-ER1.AND.ZETA.LE.ER1) GO TO 40
```

```
      FLAG=1.0
      GO TO 50
15    CALL CA(XT3,YT3,XT1,YT1,VAL1)
      VAL=VAL1
      GO TO 100
20    CALL CA(XT1,YT1,XT2,YT2,VAL1)
      VAL=VAL1
      GO TO 100
25    CALL CA(XT2,YT2,XT3,YT3,VAL1)
      VAL=VAL1
      GO TO 100
30    CALL CA(XT1,YT1,XT2,YT2,VAL1)
      CALL CA(XT2,YT2,XT3,YT3,VAL2)
      VAL=VAL1+VAL2
      GO TO 100
35    CALL CA(XT2,YT2,XT3,YT3,VAL1)
      CALL CA(XT3,YT3,XT1,YT1,VAL2)
      VAL=VAL1+VAL2
      GO TO 100
40    CALL CA(XT3,YT3,XT1,YT1,VAL1)
      CALL CA(XT1,YT1,XT2,YT2,VAL2)
      VAL=VAL1+VAL2
      GO TO 100
50    CALL CA(XT1,YT1,XT2,YT2,VAL1)
      CALL CA(XT2,YT2,XT3,YT3,VAL2)
      CALL CA(XT3,YT3,XT1,YT1,VAL3)
      VAL=VAL1+VAL2+VAL3
100   CONTINUE
      POT=VAL
      POT1=POT
      RETURN
      END
```

```
      SUBROUTINE CA(X1,Y1,X2,Y2,VAL)
      COMMON/ERROR/ERR1
      RA=SQRT(X1**2+Y1**2)
      RB=SQRT(X2**2+Y2**2)
      AL=SQRT((X2-X1)**2+(Y2-Y1)**2)
      DOT=((-X1)*(X2-X1)+(-Y1)*(Y2-Y1))/AL
      XD=X1+DOT*(X2-X1)/AL
      YD=Y1+DOT*(Y2-Y1)/AL
      RNOT=SQRT(XD**2+YD**2)
      ERR1=RNOT/AL
      ZERO=1.0E-06
      IF(ERR1.LE.ZERO) GO TO 10
      PHI1=ATAN2(Y1,X1)
      PHI2=ATAN2(Y2,X2)
      PHINOT=ATAN2(YD,XD)
      F1=EXPRN(RNOT,PHINOT,PHI1)
      F2=EXPRN(RNOT,PHINOT,PHI2)
      VAL=F2-F1
      GO TO 11
10    VAL=0.0
11    RETURN
      END
```

```
      FUNCTION EXPRN(RNCT,PHINOT,PHI)
      COMMON/ERROR/ERR1
    TEMP1=SCRT(RNOT**2)
      TEMP2=RNOT*SIN(PHINOT-PHI)
      IF(ERR1.LE.1.0E-06) GO TO 10
      ALPHA1=PHINOT-PHI
      ALPHA2=ARSIN(1.0)
      ERR2=ABS(ALPHA1**2-ALPHA2**2)
      IF(ERR2.LE.1.0E-06) GO TC 10
      TEMP3=ALCG((TEMP1+TEMP2)/(TEMP1-TEMP2))
      GC TC 11
 10   TEMP3=0.0
 11   EXPRN=-(RNCT*TEMP3)/2.0
      RETURN
      END
```

```
      SUBROUTINE CSMINV(A,NDIM,N,DETERM,COND,IERR)
      COMPLEX A(NDIM,NDIM),PIVOT(250),AMAX,T,SWAP,DETERM,U,CMPLX,CONJG
      INTEGER*4 IPIVCT(250),INDEX(250,2)
      REAL    TEMP,ALPHA(250),CABS
      COMPLEX CTEMP,CALPHA(250)
      IERR=0
      IF(NDIM.LE.250) GC TO 5
      IERR=1
      WRITE(3,4) NDIM
4     FORMAT('OCSMINV ERROR.  ATTEMPT TO INVERT A MATRIX 'I4,
     1' CN A SIDE.'/' WHEN 250 X 250 IS THE MAXIMUM ALLOWED.')
      RETURN
5     CONTINUE
      DETERM = CMPLX(1.0,0.0)
      SUMAXA=0.
      DO 20 J=1,N
      ALPHA(J)=0.0
      CALPHA(J)=(0.0,C.0)
      SUMROW=0.
      DC 10 I=1,N
      CALPHA(J)=CALPHA(J)+A(J,I)* CCNJG(A(J,I))
      ALPHA(J)=REAL(CALPHA(J))
10    SUMRCW=SUMROW + CABS(A(J,I))
      ALPHA(J)= SQRT(ALPHA(J))
      IF(SUMROW.GT.SUMAXA) SUMAXA=SUMRCW
20    IPIVCT(J)=0
      DO 600 I=1,N
      AMAX=CMPLX(0.0,0.0)
      DO 105 J=1,N
      IF (IPIVOT(J)-1) 60, 105, 60
60    DO 100 K=1,N
      IF (IPIVOT(K)-1) 80, 100, 740
80    CTEMP=AMAX* CONJG(AMAX)-A(J,K)* CONJG(A(J,K))
      TEMP=REAL(CTEMP)
      IF(TEMP)85,85,100
85    IRCW=J
      ICOLUM=K
      AMAX=A(J,K)
100   CONTINUE
105   CONTINUE
       IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
      IF (IROW-ICOLUM) 140, 260, 140
140   DETERM=-DETERM
      DO 200 L=1,N
      SWAP=A(IRCW,L)
      A(IROW,L)=A(ICCLUM,L)
200   A(ICOLUM,L)=SWAP
      SWAP=ALPHA(IROW)
      ALPHA(IROW)=ALPHA(ICOLUM)
      CALPHA(ICOLUM)=SWAP
      ALPHA(ICOLUM)=REAL(CALPHA(ICOLUM))
260   INDEX(I,1)=IROW
      INDEX(I,2)=ICOLUM
      PIVOT(I)=A(ICOLUM,ICOLUM)
      U = PIVOT(I)
      ALPHAI=ALPHA(ICCLUM)
      CALL DTRMNT(DETERM,U,ALPHAI)
      CTEMP=PIVOT(I)* CONJG(PIVOT(I))
      TEMP=REAL(CTEMP)
      IF(TEMP)330,720,330
```

```
330   A(ICOLUM,ICOLUM) = CMPLX(1.0,0.0)
      DO 350 L=1,N
      U = PIVCT(I)
350   A(ICOLUM,L) = A(ICOLUM,L)/U
380   DO 550 L1=1,N
      IF(L1-ICOLUM) 400, 550, 400
400   T=A(L1,ICOLUM)
      A(L1,ICOLUM)= CMPLX(0.0,0.0)
      DO 450 L=1,N
      U = A(ICOLUM,L)
450   A(L1,L) = A(L1,L)-U*T
550   CONTINUE
600   CONTINUE
620   DO 710 I=1,N
      L=N+1-I
      IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
630   JROW=INDEX(L,1)
      JCOLUM=INDEX(L,2)
      DO 705 K=1,N
      SWAP=A(K,JROW)
      A(K,JROW)=A(K,JCOLUM)
      A(K,JCOLUM)=SWAP
705   CONTINUE
710   CONTINUE
      SUMAXI=0.
      DO 910 I=1,N
      SUMROW=0.
      DO 900 J=1,N
900   SUMROW=SUMROW + CABS(A(I,J))
      IF(SUMROW.GT.SUMAXI) SUMAXI=SUMROW
910   CONTINUE
      COND = 1./(SUMAXA*SUMAXI)
      RETURN
720   WRITE(3,730)
730   FORMAT('0',10('********')/'0MATRIX IS SINGULAR'/'0',10('********'))
740   RETURN
      END
```

```
      SUBROUTINE DTRMNT(DETERM,U,A)
        REAL CABS
      COMPLEX DETERM,U,CMPLX
      COMMON/SCAFAC/ISCALE
      DATA ISCALE/0/
      IF(CABS(DETERM) .GT. 1.E-10) GO TO 100
      DETERM=DETERM*1.E10
      ISCALE=ISCALE+1
100   DETERM=DETERM*U/CMPLX(A,0.0)
      RETURN
      END
```

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