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ON THE TEMPERATURE DISTRIBUTION IN AN AIR-VENTILATED SNOW LAYER--ETC(U)
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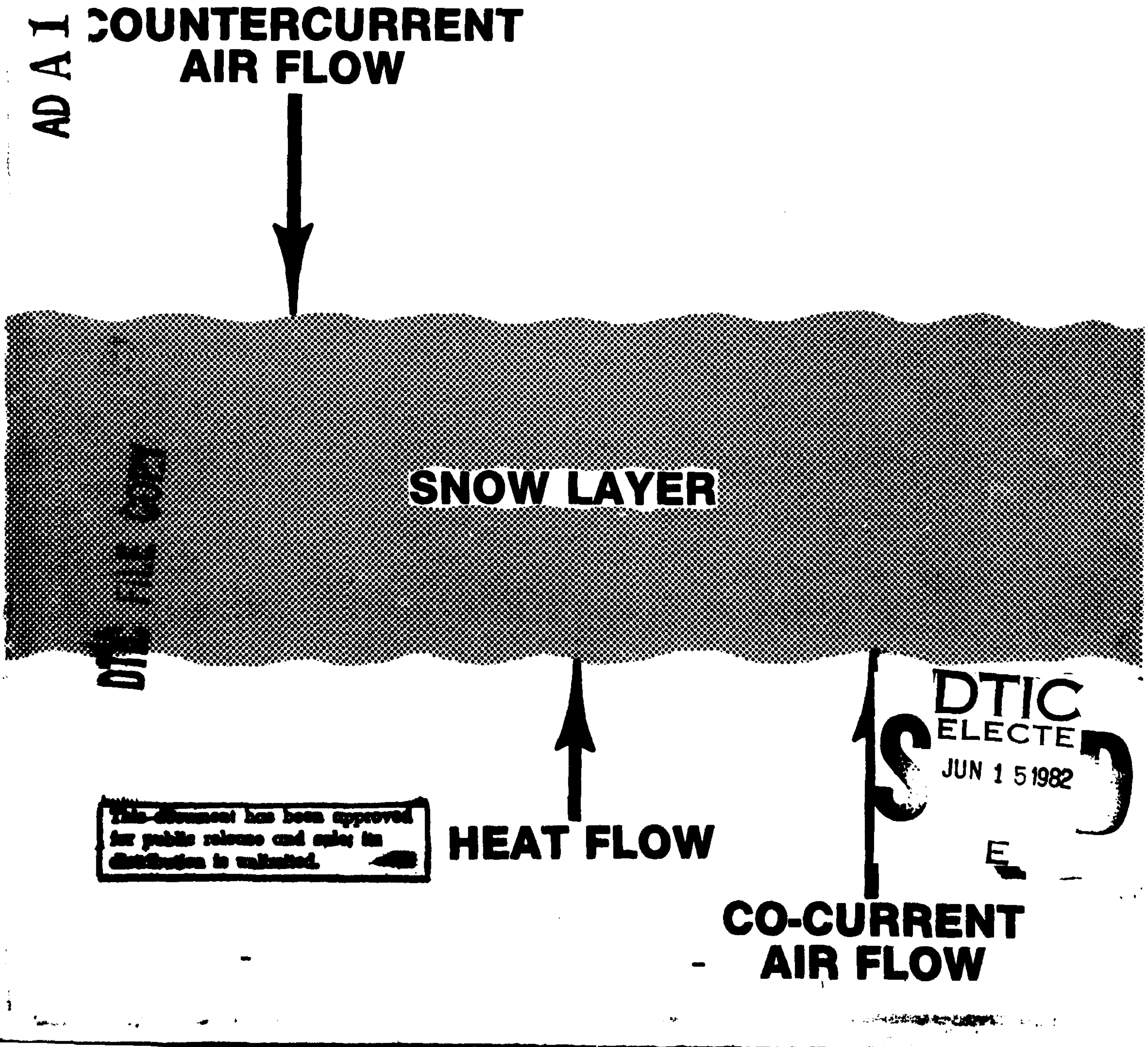

For conversion of SI metric units to U.S./British customary units of measurement consult ASTM Standard E380, Metric Practice Guide, *published by the American Society for Testing and Materials, 1916 Race St., Philadelphia, Pa. 19103.*

CRREL Report 82-5

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On the temperature distribution in an air-ventilated snow layer

Yin-Chao Yen

Prepared for
OFFICE OF THE CHIEF OF ENGINEERS

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The problem of simultaneous heat and mass transfer in a homogeneous snow layer, with one side kept at its initial temperature and the other side with a step temperature increase, was solved for the case of constant through-flow conditions. An experimentally determined effective thermal conductivity function, i.e. $K_e = 0.0014 + 0.58\, G$ (where G is dry mass flow rate of air in g/cm^{2}s), was employed in the solution. The computed nondimensional temperature distribution agreed quite well with experimental data taken under pseudo-steady state conditions with the exception of the temperature for the lowest flow rate used in the experiment. The pronounced nonlinearity of the temperature distribution was found to be a strong function of the flow rate. For sinusoidal variation of atmospheric pressure, the responding flow in the snow medium was also found to be sinusoidal. In conjunction with the diurnal temperature change, this variation

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20. Abstract (Cont'd)

facilitated the process of repeated sublimation and condensation in alternate directions and thereby produced a surface layer of approximately constant snow density.

PREFACE

This report was prepared by Dr. Yin-Chao Yen, Chief, Physical Sciences Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory. Funding for this research was provided by DA Project 4A161102AT24, *Research in Snow, Ice and Frozen Ground,* Task A, *Properties of Cold Regions Materials,* Work Unit 005, *Thermophysics and Mechanics of Cold Regions Materials.*

The manuscript of this report was technically reviewed by Dr. Samuel Colbeck and Dr. Yoshisuke Nakano of CRREL.

NOMENCLATURE

C_p	heat capacity
D	vapor diffusivity
k	permeability
K	thermal conductivity
L	latent heat
ℓ	layer depth
M	molecular weight
p	partial pressure of water vapor, vapor pressure, air pressure
P	defined as $p^2 - p_0^2$
T	temperature
t	time
v	defined by $(T - T_0)$
w	defined by eq 12
α_e	defined by eq 8
β	defined by eq 9
γ	defined by eq 20
η	dimensionless distance (y/ℓ); dimensionless number defined as $(\omega/2\sigma)^{1/2}$
π	system pressure; a constant = 3.1416
ρ	density
ϕ	dimensionless temperature $(T-T_0)/(T_\ell-T_0)$; also as defined in eq 26
σ	defined as kp_0/ϵ
ω	period

Subscripts

a	air
e	effective; porosity
ℓ	at $y = \ell$
s	snow; sublimation
w	water
0	at $y = 0$; initial

ON THE TEMPERATURE DISTRIBUTION IN AN AIR-VENTILATED SNOW LAYER

Yin-Chao Yen

INTRODUCTION

In nature, the surface layers of a snow cover are affected by the constantly changing temperature of the ambient air. During the winter months, the bottom layers have a significantly higher and more constant temperature than the atmosphere. The temperature difference between the upper and lower layers gives rise to various processes within the snow cover that are caused, in part, by the different vapor pressures between the snow grains. Kondrat́eva [1945] reported on the thermal conductivity of artificially compressed snow. She indicated qualitatively the extent of the vaporization-sublimation processes occurring in the snow sample by comparing the snow density of each layer determined at the end of an experiment with those at the beginning. Yosida (1950) experimentally determined the macroscopic diffusion coefficient D (or the effective diffusion coefficient D_e) of water vapor through snow by measuring the weight change of a snow sample under a constant temperature differential. (This coefficient includes all the possible mass transfer mechanisms occurring in a snow medium, i.e. migration of water molecules along the snow grains, sublimation and condensation, and diffusion through the void space between the snow grains.) He concluded 1) that the diffusion coefficient of water vapor through snow is independent of snow density, 2) that it is four to five times larger than the diffusion coefficient D_0 of water vapor through air ($0.22\ cm^2/s$ at 0°C and atmospheric pressure) and 3) that gravity has no effect on the value of the diffusion coefficient.

In these two investigations, the snow samples were subjected to a constant temperature gradient with stagnant fluid (air in this case) in the void space between the snow grains. In nature, however, a certain degree of convective transfer of heat and mass occurs near the surface layer of snow cover owing to wind currents. Yen (1962, 1963) studied the effect of air flow on the effective thermal conductivity K_e and vapor diffusivity D_e and found the following relations:

$$K_e = 0.0014 + 0.589\,G \qquad (1)$$

and

$$D_e = 95.38\,(G + 0.456\times10^{-4})^{1/2} \qquad (2)$$

for snow densities from 0.376 to 0.472 g/cm^3 and a mass flow rate of dry air G from 2×10^{-4} to $40\times10^{-4}\ g/cm^2\ s$. Adivarahan (1961) extensively investigated the effective thermal conductivity of porous rocks under the influence of fluid flow but without the accompanying effect of mass transfer.

The purpose of this study is to demonstrate analytically the resultant effect of an imposed temperature differential and a constant through-flow of air on the temperature distribution of an initially homogeneous snow layer. The results will be compared with reported data on flow through consolidated and unconsolidated porous media with or without mass transport. To consider the more practical case of variable flow situations, a sinusoidal atmospheric pressure variation on the snow surface is considered and the characteristic nature of the penetrating wave into the medium is derived. The combined effects of diurnal temperature change and propagating wave characteristics on the processes of heat and mass transfer on snow mass redistribution are assessed.

FORMULATION OF THE PROBLEM

Constant flow

If we consider a homogeneous snow layer with constant countercurrent or cocurrent air flow rates, the energy balance equation for a differential layer thickness dy can be written as

$$K_e \frac{d^2T}{dy^2} \pm G\,C_{pa} \frac{dT}{dy} \pm \frac{GM_w}{M_a} \frac{d}{dy}\left(\frac{p}{\pi - p}\right) = (\rho_s C_{ps}) \frac{\partial T}{\partial t} \tag{3}$$

where

y = distance measured from the direction of heat flow
K_e = effective thermal conductivity
p = partial pressure of the water vapor
π = system pressure
M_w and M_a = the molecular weight of water vapor and dry air
C_{pa} and C_{ps} = the heat capacities of air and snow
ρ_s = snow density.

This equation is derived from the assumptions that 1) the flow is unidirectional at a constant rate, 2) the density of the air is constant, and 3) the temperatures of the air and of the snow are the same at the same point in the medium and time. Based on the work of Bader et al. (1939), the partial pressure of water vapor in air can reasonably be replaced by the saturation vapor pressure of snow p_s which, as reported by Yosida (1950), can be represented by

$$p_s = 6.1 \exp(0.0857\,T) \tag{4}$$

where p_s is in millibars and T is in degrees Celsius. For a temperature range of $-17° < T < -7°$C, eq 4 can well be approximated (with an average error of ±4.0%) by

$$p_s = 4.636 + 0.195\,T\,. \tag{5}$$

Substituting eq 1 and 5 into 3 and considering the case that $\pi >> p_s$, eq 3 becomes

$$(0.0014 + 0.58G) \frac{d^2T}{dy^2} \pm G\left(C_{pa} + 0.195\,\frac{M_w L_s}{M_a \pi}\right) \frac{dT}{dy} = (\rho_s C_{ps}) \frac{\partial T}{\partial t} \tag{6}$$

where the + sign indicates that air flow is in the opposite direction of the heat flow, and L_s is the latent heat of sublimation. The term $0.195\,M_w L_s / M_a \pi$ represents the contribution associated with net water vapor sublimation. The initial and boundary conditions to be considered are

$$T(y,0) = T_0$$

$$T(0,t) = T_0$$

$$T(\ell, t) = T_\ell \tag{7}$$

By using the transformation of $v(y, t) = T - T_0$ and defining

$$\alpha_e = \frac{0.0014 + 0.58\,G}{(C_{ps}\rho_s)} \tag{8}$$

and

$$\beta = \frac{\pm G\left(C_{pa} + 0.195\,\dfrac{M_w L_s}{M_a \pi}\right)}{C_{ps}\rho_s} \tag{9}$$

eq 6 becomes

$$\alpha_e \frac{\partial^2 v}{\partial y^2} + \beta \frac{\partial v}{\partial y} = \frac{\partial v}{\partial t} \tag{10}$$

with corresponding initial and boundary conditions

$$v(y,0) = 0 \tag{11}$$

$$v(0,t) = 0$$

$$v(\ell,t) = T_\ell - T_0\,. \tag{12}$$

By using the further transformation of

$$v = w \exp\left(\frac{\beta y}{2\alpha_e} + \frac{\beta^2 t}{4\alpha_e}\right) \tag{13}$$

eq 10 and 11 become

$$\alpha_e \frac{\partial^2 w}{\partial y^2} = \frac{\partial w}{\partial t}$$

and

$$w(y,0) = 0 \tag{14}$$

$$w(0,t) = 0$$

$$w(\ell,t) = (T_\ell - T_0) \exp\left(\frac{\beta y}{2\alpha_e} + \frac{\beta^2 t}{4\alpha_e}\right).$$

The solution of eq 13 with the initial and boundary conditions of eq 14 can be obtained through the use of Duhamal's formula (Hildebrand 1956) and is given as

$$\frac{T-T_0}{T_\ell-T_0} \exp\left[-\frac{\beta}{2\alpha_e}\,(\ell-y)\right] = \frac{y}{\ell} + \frac{2}{\pi}\sum \frac{(-1)^n}{n}\,\frac{\beta^2\ell^2}{\beta^2\ell^2+4\alpha_e{}^2\pi^2 n^2}\,\sin\frac{n\pi y}{\ell}$$

$$+ \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\,\frac{4\alpha_e^2 n^2\pi^2}{\beta^2\ell^2 + 4\alpha_e^2\pi^2 n^2}\,\exp\left[-\left(\frac{\beta^2}{4\alpha_e} + \frac{\alpha_e\pi^2 n^2}{\ell^2}\right)t\right]\sin\frac{n\pi y}{\ell}\,. \tag{15}$$

The last term accounts for the transient state behavior. But as time goes by it becomes less significant, and the temperature distribution for the steady state is

$$\frac{T-T_0}{T_\ell-T_0} \exp\left[-\frac{\beta}{2\alpha_e}(\ell-y)\right] = \frac{y}{\ell} + \frac{2}{\pi}\sum_{n=0}^{\infty} \frac{(-1)^n\beta^2\ell^2}{n(\beta^2\ell^2+4\alpha_e^2\pi^2n^2)} \sin\frac{n\pi y}{\ell} . \tag{16}$$

Equation 16 is hard to evaluate because of the slow convergence of the series. However, in the case where $\beta^2\ell^2$ is much smaller than $4\alpha_e^2\pi^2$, the following approximation can then be made:

$$\frac{T-T_0}{T_\ell-T_0} \exp\left[-\frac{\beta}{2\alpha_e}(\ell-y)\right] = \frac{y}{\ell}\left\{1-\frac{\beta^2\ell^2}{24\alpha_e^2}\left[1-\left(\frac{y}{\ell}\right)^2\right]\right\} \tag{17}$$

or

$$\frac{T-T_0}{T_\ell-T_0} = \exp\left[\frac{\beta}{2\alpha_e}(\ell-y)\right]\frac{y}{\ell}\left\{1-\frac{\beta^2\ell^2}{24\alpha_e^2}\left[1-\left(\frac{y}{\ell}\right)^2\right]\right\} . \tag{18}$$

A positive value of β indicates that the direction of air flow is opposite that of heat flow (or countercurrent flow), and a negative value of β shows the same direction of heat and air flow (cocurrent flow).

For the steady-state case, eq 6 is a linear second-order differential equation with constant coefficients, and its solution with its initial and boundary conditions shown in eq 7 can be written as

$$\frac{T-T_0}{T_\ell-T_0} = \frac{1-\exp(-\gamma y)}{1-\exp(-\gamma\ell)} \tag{19}$$

where γ is defined as

$$\gamma = \pm G\,\frac{C_{pa}+0.195\,(M_w L_s)/M_a\pi}{0.0014+0.58\,G} \tag{20}$$

with positive and negative values of γ indicating countercurrent and cocurrent flows respectively. For the case where $\beta^2\ell^2$ is much smaller than $4\alpha_e^2\pi^2$ (as derived in eq 18), the temperature distributions computed from eq 18 and 19 should be comparable.

Variable flow

The one-dimensional pressure variation within the snow layer due to atmospheric changes of a sinusoidal nature can be written as

$$\frac{1}{\sigma}\frac{\partial P'}{\partial t} = \left(\frac{\partial^2 P'}{\partial y^2}\right) \tag{21}$$

where $\sigma = kp_0/\epsilon$.

p_0 = zero pressure within the snow layer at $t = 0$

ϵ = porosity

k = permeability defined by Darcy's law; i.e.

$$v_y = -k\,\frac{\partial P'}{\partial y} \tag{22}$$

P' = the difference between $p^2 - p_0^2$

$p = p_0\,[1 + A'/2p_0)\sin\omega t]$

$A'/2p_0$ = amplitude of the pressure waves

ω = wave period.

Equation 21 is solved with the following initial and boundary conditions:

$$P'(y,0) = 0 \tag{23a}$$

$$P'(0,t) = 0 \tag{23b}$$

$$P'(\ell,t) = p^2 - p_o^2 \tag{23c}$$

with $p = p_0\,[1 + (A'/2p_0)\sin\omega t)]$. Equation 23c becomes

$$P'(\ell,t) \cong A\sin\omega t \tag{24}$$

by neglecting the term $A'2/4\ sin^2\ \omega t$ and with $A = A'p_o$. The solution of eq 21 with the conditions of eq 23 (Carslaw and Jaeger 1959) can be expressed as

$$P' = A\left[\frac{\cosh 2\eta y - \cos 2\eta y}{\cosh 2\eta\ell - \cos 2\eta\ell}\right]\sin(\omega t + \phi) + 2\pi\sigma \sum_{n=1}^{\infty} \frac{n(-1)^{n+1}\omega\ell^2}{\sigma^2 n^2\pi^2 + \omega^2\ell^2}\sin\frac{n\pi y}{\ell}\ .$$
$$\exp(-\sigma n^2\pi^2/\ell^2) \tag{25}$$

where $\eta = (\omega/2\sigma)^{1/2}$ and ϕ is defined as

$$\phi = \mathrm{Arg}\,\frac{\sinh\eta y\,(1+i)}{\sinh\eta\ell\,(1+i)}\ . \tag{26}$$

Rewriting eq 25 and considering the steady-state case, it becomes

$$\frac{p^2 - p_0^2}{A} = \left[\frac{\cosh 2\eta y - \cos 2\eta y}{\cosh 2\eta\ell - \cos 2\eta\ell}\right]\sin(\omega t + \phi)\ . \tag{27}$$

To evaluate the velocity through the surface, the pressure gradient is found from eq 27 as

$$\frac{2p}{A}\left(\frac{\partial p}{\partial y}\right) = \frac{2\eta\,(\sinh 2\eta y + \sin 2\eta y)}{(\cosh 2\eta\ell - \cos 2\eta\ell)^{1/2}\,(\cosh 2\eta y - \cos 2\eta y)^{1/2}}\sin(\omega t + \phi)$$
$$+ \left[\frac{\cosh 2\eta y - \cos 2\eta y}{\cosh 2\eta\ell - \cos 2\eta\ell}\right]^{1/2}\cos(\omega t + \phi)\,\frac{d\phi}{dy}\ . \tag{28}$$

The value of $d\phi/dy$ at $y = \ell$ is found from eq 26 and

$$\frac{d\phi}{dy} = \frac{\eta(\sin\eta\ell\cos\eta\ell - \sinh\eta\ell\cosh\eta\ell)}{(\sinh\eta\ell\cos\eta\ell)^2 + (\cosh\eta\ell + \sin\eta\ell)^2}\ . \tag{29}$$

Substituting eq 29 into 28 and simplifying, the value of dp/dy at $y = \ell$ is

$$\left.\frac{dp}{dy}\right|_{y=\ell} = \frac{A\eta}{2p}\left[\frac{2(\sinh 2\eta\ell + \sin 2\eta\ell)}{(\cosh 2\eta\ell - \cos 2\eta\ell)}\sin\omega t\right.$$
$$\left. + \frac{(\sin\eta\ell\cos\eta\ell - \sinh\eta\ell\cosh\eta\ell)}{(\sinh\eta\ell\cos\eta\ell)^2 + (\cosh\eta\ell\sin\eta\ell)^2}\cos\omega t\right]\ . \tag{30}$$

For small values of $\eta \ell$, eq 30 can be much simplified and the velocity at the upper surface (i.e. $y = \ell$) can be expressed as

$$v_y\Big|_{y=\ell} = \frac{A}{2p}\left(2\frac{k}{\ell}\sin\omega t - \frac{k y^2 \ell}{2}\cos\omega t\right). \tag{31}$$

Since the value of η is generally rather small the second term in the bracket can be considered insignificant in comparison with the first term. Therefore the velocity at the surface is

$$v_y\Big|_{y=\ell} = \frac{Ak}{p\ell}\sin\omega t. \tag{32}$$

By substituting $A = A'\rho_0$, and $p = \rho_0\,[1 + (A'/2\rho_0)\sin\omega t]$, eq 32 becomes

$$v_y\Big|_{y=\ell} = \frac{kA'}{\ell}\left[\frac{\sin\omega t}{1 + (A'/2\rho_0)\sin\omega t}\right]. \tag{33}$$

Equation 33 indicates that the air flow rate is sinusoidal and varies in the same manner as the imposed atmospheric pressure. The results from eq 17 and 18 have shown (see next section) that the temperature within the snow layer, when there is a constant flow of air, would either be greater or smaller than that with heat transfer by conduction only, the difference depending on whether the fluid flow is a countercurrent or cocurrent in relation to the direction of heat flow. If the imposed flow is of a periodical nature, the snow temperature would be expected to oscillate about the neutral point. The repeated alternate changes in the direction of heat and mass transfer account for the almost constant snow densities observed near the surface layers.

COMPARISON AND DISCUSSION OF RESULTS

Experimental

Temperature profiles under pseudo-steady state conditions were taken in connection with the study of the effective thermal conductivity of snow (Yen 1962). In that study, snow fractions from sieve analysis were used with densities ranging from 0.376 to 0.472 g/cm^3 and a snow particle nominal diameter of 0.22 to 0.07 cm. To eliminate or reduce the radial heat transfer, the snow bed was contained in a Dewar flask, 6.5 cm i.d. and 15.24 cm long, which was made of a double-walled glass tube, the annular space being evacuated and the internal surfaces plated with silver. Saturated air at constant temperature was introduced into the top of the vertical bed and passed downward, counter to the upward flow of heat energy. The heat source used to obtain a uniform temperature across the bottom of the snow bed was a copper cylinder immersed in a constant temperature bath below the bed. Two sets of 30-guage copper-constantan thermocouples (7 in each set and equally spaced) were placed along the center axis and along the wall of the bed, respectively. Any difference between corresponding center and wall temperatures would indicate a radial flow of heat through the walls of the Dewar flask.

The constant temperature bath had a maximum temperature variation of ± 0.05°C, and minor fluctuations were smoothed out by the mass of the large copper cylinder. The temperature readings through the snow bed were taken at regular intervals until the pseudo-steady state was arrived at*. When this condition was reached, the air flow rate was measured. To ensure that a steady state

*In actuality a steady state will never be achieved because of the simultaneous heat and mass transfers. Due to the mass transfer of sublimation and subsequent condensation, the snow mass was constantly redistributed as the experiments progressed, thus affecting the conductive heat transfer or the snow bed temperature distribution. However, for the small temperature differences imposed between the top and bottom of the snow bed, the redistribution of snow mass within the snow bed was a rather slow process, so that a pseudo-steady state can be considered to have occurred.

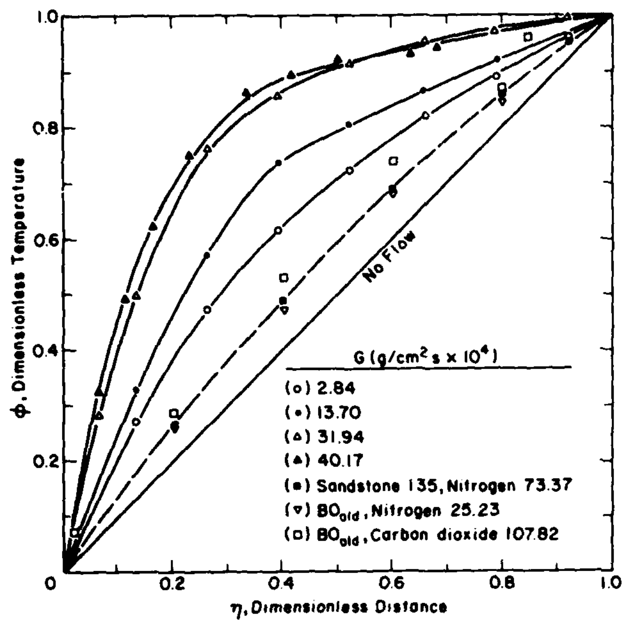

Figure 1. Experimental data on dimensionless temperature ϕ as a function of dimensionless distance η.

had been attained, all the readings were taken again after a period of 30 minutes. The time required to reach the steady state varied with the rate of air flow and was about 3 to 6 hours for a mass air flow rate of 2.84×10^{-4} to 40.17×10^{-4} g/cm^2 s. Figure 1 shows some typical dimensionless temperature distributions ($\phi = T - T_0/T_g - T_0$) within the snow bed of density 0.472 g/cm^2 s. It shows a significant deviation from the normally linear temperature distribution. (A linear temperature distribution will occur only for the case in which the sole heat transfer mechanism taking place within the medium is pure conduction without any involvement of mass transport; therefore, it is expected that this non-linear distribution will hold even at no-through-flow conditions.) The deviation apparently has been enhanced by simultaneous mass transfer via the repeated process of sublimation and condensation.

In this study, the heat is conducted upward and the air flow is downward, and it clearly demonstrates that the temperature of the snow is closer to that of the colder surface and is considerably affected by the rate of flow. The steep temperature gradient in the warm region in Figure 1 is a result of the continued depletion of water vapor from a warm region and deposition in a cold region (prior to the onset of the experiment, the snow bed had uniform density and temperature). This metamorphic phenomenon and its effect on the redistribution of the snow mass have been reported by Kondratéva (1945), Yosida (1950), de Quervain (1963) and Yen (1963). If the steep temperature gradient is sustained for a long duration, the continued transport of water vapor will eventually lead to the formation of faceted, cohesionless cyrstals in the warmer area. A surface layer of nearly constant snow density observed in the Arctic and Antarctic was duplicated in the laboratory experiments and is believed to be caused by the alternate upward and downward water vapor movement associated with diurnal temperature change. Three sets of data on counterflow of heat and fluid (heat flow downwards and fluid flow upwards, just the reverse of the direction in the snow studies reported by Yen [1962]) in consolidated porous media (sandstones) are included in Figure 1 [Adivarahan 1961]. Adivarahan's experiments were also conducted until steady-state conditions were reached. However, in his study, no complications arose from the simultaneous mass transfer via the process of sublimation-condensation under a constantly changing temperature gradient. One set of the data was taken with sandstone 135 with a porosity of 0.195, a mean pore size of 9.967×10^{-3} mm, and a nitrogen flow rate of $G = 73.37 \times 10^{-4}$ g/cm² s. The other two sets were taken with sandstone BO_{old} of porosity 0.267, mean pore size of 1.88×10^{-2} mm, and with nitrogen and carbon dioxide flow rates of 25.23×10^{-4} and 107×10^{-4} g/cm² s respectively.

Contrary to the case of air flow through snow, the temperature distribution in the sandstone did not deviate much from the linear distribution. In the case of BO_{old}, it had a greater porosity, and though the nitrogen flow rate was lower, it produced a temperature distribution almost identical with sandstone 135 with about a flow rate three times greater. Since BO_{old} is of greater porosity, the intricate fluid velocity is lower in comparison with sandstone 135, but the increase in the heat transfer area per unit volume of the medium counters the effect of the more intensive turbulent characteristic in sandstone 135. In the case of BO_{old}, with carbon dioxide flowing at a rate about four times greater than the nitrogen, there exists a much smaller difference (in comparison with the case of air flow through snow) in the dimensionless temperature distribution, indicating the more turbulent nature of flow. From these, as well as the studies on snow, it is clear that, irregardless of flow direction, the temperature is always closer to that of the colder surface of the medium as long as there is counterflow of heat and fluid.

Theoretical

For the case when $\beta^2\ell^2/4\alpha_e^2\pi^2$ (a term in the denominator of eq 16) is much less than one and for the case of steady state, eq 18 can be used to compute the dimensionless temperature distribution ϕ. In the case of the snow experiments, the snow sample has a length of 15.24 cm, and the value of β is evaluated from eq 9 to be $\beta = 1.3758G$. α_e is computed from eq 8 by using K_e values given by eq 1. For the flow rate of $G = 2.84 \times 10^{-4}$ g/cm² s, the ratio of $\beta^2\ell^2$ to $4\alpha_e^2\pi^2$ was found to be 0.021, a value much less than one. Therefore for $G = 2.84 \times 10^{-4}$ g/cm² s, eq 18 becomes

$$\phi = \frac{T-T_0}{T_\ell-T_0} = \exp\left(\pm\, 0.02961\, (\ell-y)\right) \frac{y}{\ell}\left[1 - 0.0339\left(1-\left(\frac{y}{\ell}\right)^2\right)\right]. \tag{34}$$

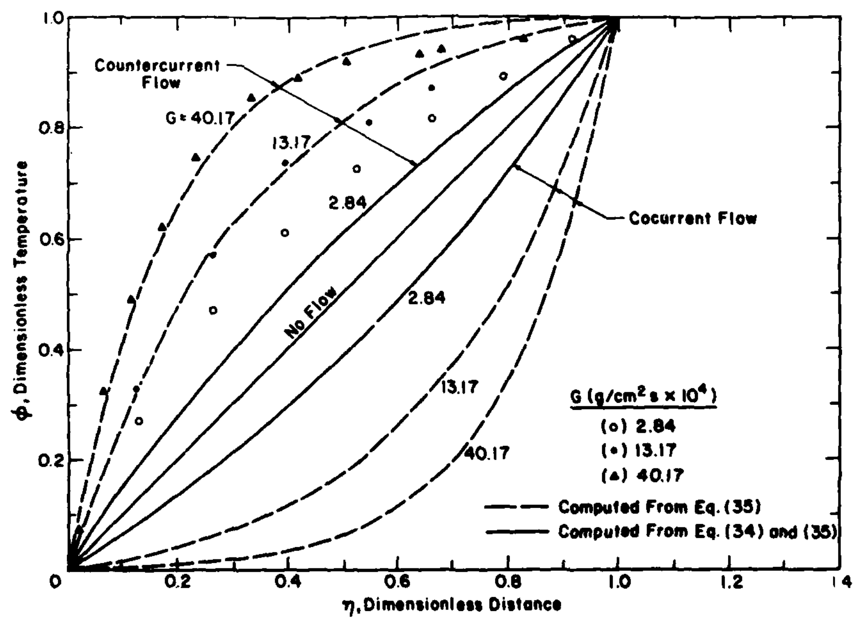

Figure 2. Comparison of experimental data with theoretical predictions.

Table 1. Dimensionless temperature distributions in a snow bed with countercurrent ϕ and cocurrent ϕ' flow of heat and fluid ($G = 2.84 \times 10^{-4}$ g/cm^2 s).

	ϕ		ϕ'	
y/ℓ	*eq 34*	*eq 35*	*eq 34*	*eq 35*
0	0	0	0	0
0.1	0.1450	0.1450	0.0644	0.0645
0.2	0.2776	0.2775	0.1349	0.1352
0.3	0.3988	0.3986	0.2120	0.2124
0.4	0.5094	0.5093	0.2964	0.2970
0.5	0.6106	0.6105	0.3889	0.3895
0.6	0.7031	0.7029	0.4901	0.4906
0.7	0.7876	0.7876	0.6008	0.6014
0.8	0.8648	0.8648	0.7220	0.7225
0.9	0.9338	0.9381	0.8591	0.8601
1.0	1.000	1.000	1.000	1.000

For the same flow rate, eq 19 reduces to

$$\phi = \frac{T-T_0}{T_\ell-T_0} = \frac{1-\exp\,(\pm\,0.059y)}{1-\exp\,(\pm\,0.059\ell)} \quad . \tag{35}$$

As shown in Figure 2 and Table 1, for a pore velocity of approximately $v_e = 0.48$ cm/s. the calculated values of ϕ and ϕ' from eq 34 and 35 are almost identical, indicating the validity of eq 34 at least up to this pore velocity ($v_e = G/\epsilon\rho_a$, where ϵ is the porosity $\epsilon = 1 - \rho_s/\rho_i$, and ρ_s and ρ_i are densities of snow and ice respectively). As indicated in Figure 2, for cocurrent flow of heat and fluid, the temperature of the medium is, on the contrary, closer to that of the warmer end of the sample. The ϕ values evaluated from the experimental results are shown in the figure; a consistent variation from the values computed from eq 34 and 35 seems to occur for $G = 2.84 \times 10^{-4}$ cm/s. However, the comparison between the experimental results and computed values of ϕ from eq 35 for $G = 13.17$ and 40.17×10^{-4} g/cm^{2}s, respectively, appears in good agreement for a greater portion of the sample lengths. (Equation 34 cannot be used for comparison because in these cases the values of $\beta^2\ell^2$ and $4\alpha_e^2\pi^2$ are of the same order of magnitude.)

CONCLUSIONS

From this investigation, it can be concluded that for counterflow systems and for those involving vapor diffusion through the process of sublimation and condensation, the dimensionless temperature ϕ is always greater than the value from the linear temperature distribution for no-through-flow and no-moisture-transport situations. The temperature gradient is found to be much steeper at the warm region than at the colder regions and the accompanying moisture transfer processes under a temperature differential significantly influence the redistribution of snow mass, i.e. a net loss in the warm zone and a net gain in the cold zone. The extent of this mass redistribution depends on the overall temperature difference imposed, the temperature level, and the mass flow rate.

For counterflow systems without accompanying simultaneous vapor transport, the nonlinearity in the dimensionless temperature ϕ is remarkably reduced. For a cocurrent flow system, the reverse of the observed temperature distribution exhibited in a counterflow system is true; i.e. temperature gradients are steeper in the colder zones, and temperature gradients increase very slightly in the warm zones, for a great portion of the sample depth.

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