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UNITED STATES AIR FORCE

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Wright-Patterson Air Force Base, Ohio

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AFIT/GE/EE/81D-14

MODELING OF A TRACKING RADAR IN TERMS OF A NON-LINEAR SECOND ORDER PHASE LOCK LOOP

THESIS

AFIT/GE/EE/81D-14 PETER M. CRONK
FLTLT RAAF

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AFIT/GE/EE/81D-14

MODELING OF A TRACKING RADAR IN TERMS OF A NON-LINEAR SECOND ORDER PHASE LOCK LOOP

THESIS

Presented to the Faculty of the
School of Engineering of the
Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

By

Peter M. Cronk, Dip Com Eng
FLTLT RAAF

Graduate Electrical Engineering

December 1981

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Preface

Accurate predictions of the signal to noise ratios that will cause an Amplitude Comparison Monopulse Radar to break angle tracking lock, are at present, not available. This thesis models the Amplitude Comparison Monopulse Radar in terms of a tracking loop using the Maximum Likelihood Criteria. From this model, probability density functions are desired for the estimated angle off boresight. A criteria for predicting break lock is derived from the probability density functions.

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Peter M. Cronk

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Glossary of Terms

Roman Letter Symbols

A	Unknown Amplitude of Incoming Signal
$\hat{A}$	Estimate of Amplitude of Incoming Signal
a	Width of Antenna Aperture
c	Coefficient in Gram-Charlier Series
d	Distance Between Antenna Centers
E	Energy of Incoming Signal
$g(\theta)$	Antenna Function
Ko	Antenna Constant
m	Central Moment of $p(\theta)$
No	Noise Spectral Density
$n(t)$	Noise Signal
$p(\theta)$	Probability Density Function
$S(t)$	Transmitted Signal
$\frac{S}{N}$	Signal to Noise Ratio
$X(t)$	Incoming Signals

Greek Letter Symbols

Σ	Sum Channel
Δ	Difference Channel
θ	Angle Off Boresight
$\hat{\theta}$	Estimate Angle Off Boresight
η	Mean
σ^2	Variance
ϵ	Error Voltage

Abstract

An amplitude comparison monopulse radar is modeled using additive channel and system noise to the received signals. The amplitude of the incoming signal and the angle off boresight are estimated under the Maximum Likelihood Criteria. An ensemble of estimates of the angle off boresight are used to derive probability density functions for the estimate angle off boresight. From these probability density functions, a criteria for predicting breaklock is derived.

I INTRODUCTION

Background

Accurate predictions of the signal to noise ratios that will cause an Amplitude Comparison Monopulse Radar to break angle tracking lock, are at present not available. To enable accurate predictions to be made, probability density functions for the estimated angle off boresight need to be derived for various signal to noise ratios.

Techniques for analysis and prediction of break lock for Phase Lock Loops have been developed and well documented (References 11 and 12). Also, the analysis of a Phase Lock Loop system modeled under Maximum Likelihood Estimate Criteria for simple estimation problems, has been documented (Reference 11).

The Amplitude Comparison Monopulse Radar has been modeled under an approximation to the Maximum Likelihood Criteria by Bakut (Reference 1). Bakut shows how all system noise and channel noise can be lumped to one normally distributed noise source which is additive to the received signal.

Cohen and Steinmetz (Reference 3) have modeled the antenna functions of the Amplitude Comparison Monopulse Radar as the sum of sines and cosines but this is an approximation of the antenna pattern within the 3db beam width.

Objective

To gain accurate probability density functions of the angle off boresight estimate ($\hat{\theta}$), an accurate model of the radar needs to be derived. The antenna functions are to be modeled without any approximations. The radar will be modeled under the Maximum Likelihood Estimate Criteria. A discrimination model to convert error voltage to angle estimate off boresight is to be modeled. The resulting tracking loop probability density functions of the angle estimate with their relevant mean and variances are to be plotted.

Assumptions

The scope of this thesis was restricted by the following assumptions in relation to, the noise, the radar platform, the radar model and the target movement.

The noise is assumed to be white gaussian distributed noise. As the noise is additive to the sum and difference channels of the radar it is also assumed to be independent from pulse to pulse and from channel to channel.

The model is restricted to one dimension only and total decoupling from a second dimension is assumed. In deriving the model, noise is assumed to be zero mean as well as white independent gaussian distributed. The radar platform is assumed stationary and consequently eliminates modulation of the radar input due to platform movement.

The target is assumed to have two separate movements; stationary and a constant velocity. When the target is assumed to be stationary, then this also includes the case where the radar is tracking a target which is moving slowly in relation to the total number of pulses integrated.

II MODEL OF AMPLITUDE COMPARISON MONOPULSE RADAR

General

Through the use of two (minimum number) offset antennas, Figure 1, an Amplitude Comparison Monopulse Radar provides range and angle information. The range information is derived from the signal return delay. Angle information is derived by comparing the addition and subtraction of the return signal's output from the two offset antenna, and then estimating the angular error of the target to the antenna system boresight axis. The general block diagram of an Amplitude Comparison Monopulse Radar is shown in Figure 2.

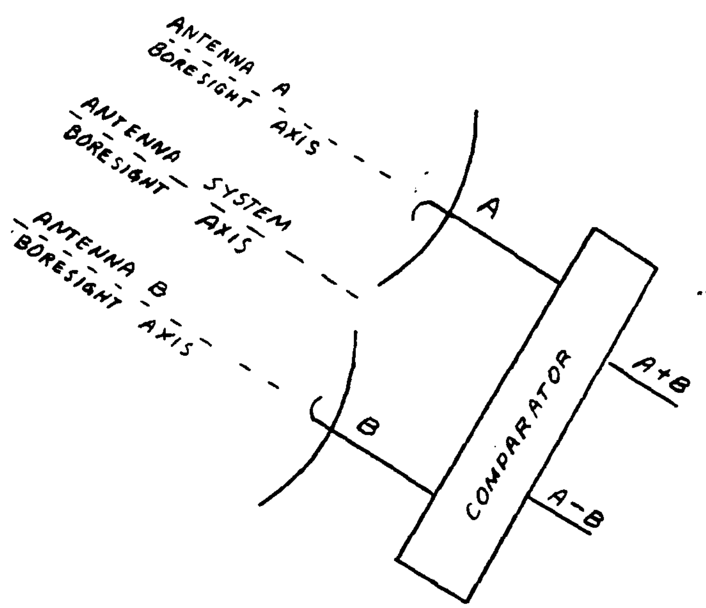


Figure 1. Offset Antenna Configuration

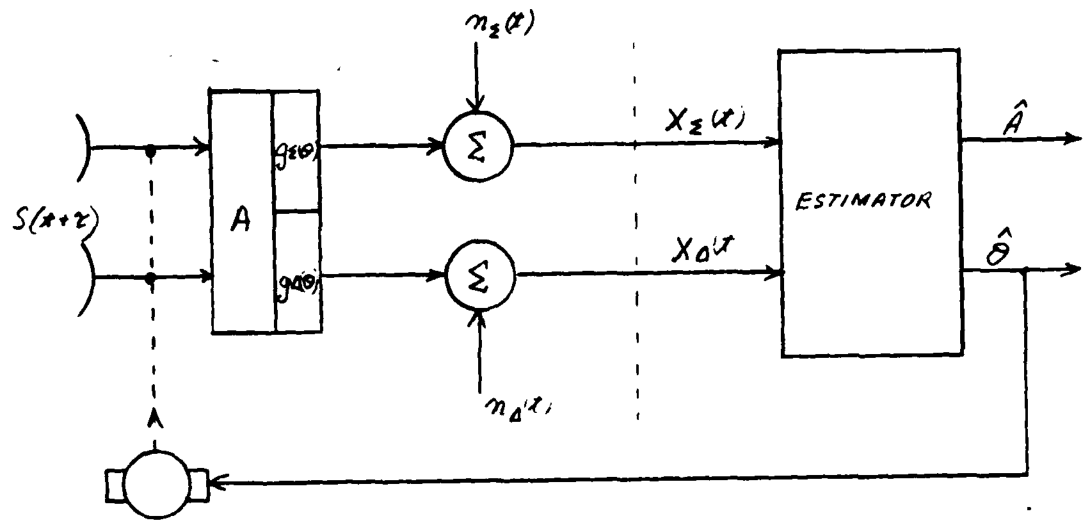


Figure 2. General Block Diagram of Amplitude Comparison Monopulse Radar

<u>Noise</u>

To account for system noise and channel noise, noise is lumped and modeled as independent gaussian, zero mean and added independently to the sum channel and difference channel to form the X signals. The input signals to the estimator, the $X\Sigma(t)$ and $X\Delta(t)$ signals contain amplitude, range and angle off system boresight information and are modeled as follows:

$$X\Sigma(t) = Ag\Sigma(\theta)S(t+\tau_1) + n\Sigma(t)$$

$$X\Delta(t) = Ag\Delta(\theta)S(t+\tau_2) + n\Delta(t)$$

<u>Received Signal $S(t+\tau)$</u>

As this thesis is concerned primarily with angle tracking the τ (range estimate) problem will be ignored. This consequently enables the $S(t+\tau)$ portion of the X signals to be evaluated as $S(t)$. Therefore, the X signals become:

$$X\Sigma(t) = Ag\Sigma(\theta)S(t) + n\Sigma(t)$$

$$X\Delta(t) = Ag\Delta(\theta)S(t) + n\Delta(t)$$

The A component of the received signal is a random process of unknown density. The S(t) signal will be modeled in two methods.

(1) As a transmitted pulse of unit energy with the received function constant

$$S(t) = \begin{cases} \frac{1}{T}\,u(t) & 0 \le t \le T \\ 0 & \text{else} \end{cases}$$

This represents a return from a stationary target.

(2) As a transmitted pulse of unit energy with angular movement and, consequently, the received antenna functions as a function of $\Theta(t)$

$$S(t) = \begin{cases} \frac{1}{T}\,u(t) & 0 \le t \le T \\ 0 & \text{else} \end{cases}$$

$$\Theta(t) = Pt + Q$$

Where P is the rate of change and

Q is the starting angle.

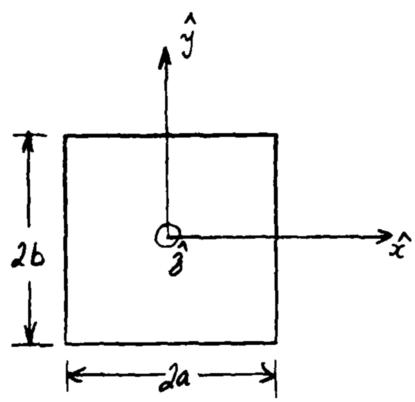
In both cases the amplitude of the incoming signal A is the random variable.

The typical pulse duration of a monopulse tracking radar is 0.2μs. To aid in the computer implementation of the radar model, the pulse has been scaled to 20ms with all other timing scaled accordingly.

$$S(t);\ 0 \leq t \leq 20\text{ms}.$$

Antenna functions $g\Sigma(\theta)$, $g\Delta(\theta)$

A pencil beam of 7.5° for each antenna was assumed. To create the pencil beam pattern, a square aperture with uniform aperture field distribution was also assumed. A representation of the field is Figure 3.



$$\tilde{E}a = Eo\hat{x}/x/\leq a;(y)\leq b$$
$$0 \quad \text{else}$$

Figure 3. Square aperture with uniform aperture field distribution

The fourier transform (Reference 4) of this aperture field gives the function ft. as:

$$ft = \frac{x\ 4abEo\ \sin\ (koa\ \sin\ \theta\ \cos\ \phi)\ \sin\ (kob\ \sin\ \theta\ \sin\ \phi)}{(koa\ \sin\ \theta\ \cos\ \phi)\ (kob\ \sin\ \theta\ \sin\phi)}$$

In the far field,

$$E(\bar{r}) = j\ \frac{ko}{2\pi r}\ e^{-jkor}\ \phi\ \cos\ \theta \sin\ \phi(-fx) + \theta\ fx\ \cos\ \phi$$

$$= \frac{\pm jko}{2\pi r}\ e^{-jkor}\quad fx\ \cos\ \phi\ \theta + fx\ \cos\ \theta\ \sin\ \phi\ \phi$$

$$= \frac{jkoe^{-jkor}}{2\pi r} \quad fx \quad \cos\phi\ \theta - \cos\theta\ \sin\phi\ \phi$$

$$fx = \frac{4abEo\ \sin[koa\ \sin\theta]\cos\phi\ \ \sin[kob\ \sin\theta]\sin\phi}{koa\ \sin\theta\ \cos\phi\quad kob\ \sin\theta\ \sin\phi}$$

in $\phi = \frac{\pi}{2}, \frac{3\pi}{2}$ plane;

$$fx = \frac{4abEo\ \sin\ (kob\ \sin\ \theta)}{kob\ \sin\ \theta}$$

and;

$$E(\bar{r}) = \frac{jkoe^{-jkor}}{2\pi r} \quad fx \quad \mp \cos\theta\ \phi$$

$$\text{where } \phi = \frac{\pi}{2};\ \frac{3\pi}{2}$$

for the high gain antenna case;

$$E(\bar{r}) = \frac{jkoe^{-jkor}}{2\pi r}\ \frac{4abEo\ \sin\ (kob\ \sin\ \theta)}{kob\ \sin\ \theta}$$

Normalizing for E(o);

$$\frac{E(r)}{E(o)} = \frac{\sin[kob\ \sin\ \theta]}{kob\ \sin\ \theta} \tag{1}$$

The normalized radiation pattern for each antenna is

Figure 4.

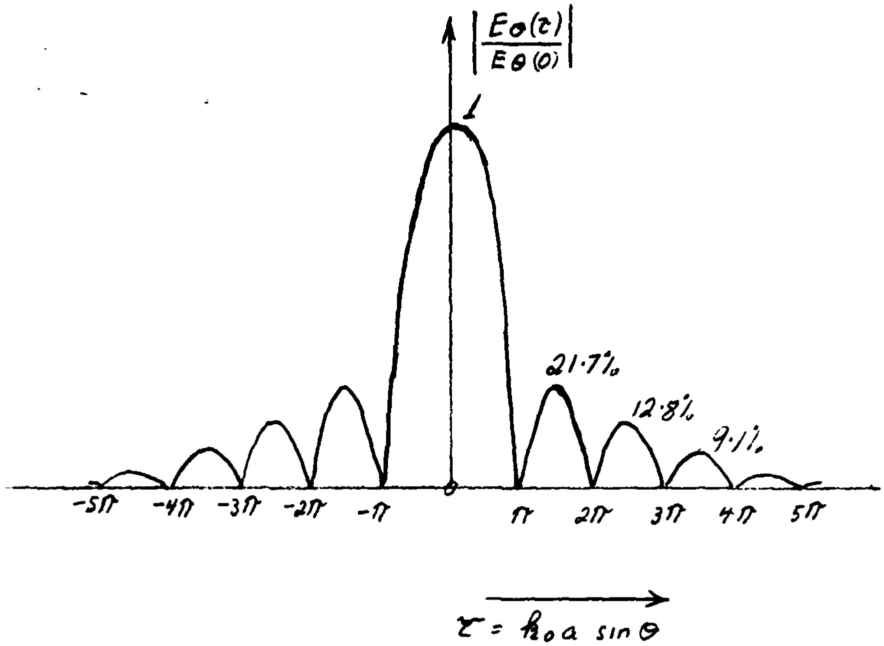


Figure 4. Normalized radiation pattern for each antenna 3db beam width is assumed as 7.5°.

Therefore,

$$.707 = \frac{\sin\,(\text{koa} \sin 3.75^\circ)}{\text{koa} \sin 3.75^\circ}$$

$$.707 = \text{sine}\ (\text{koa}\ .0654)$$

This gives koa = 13.6874π

$$\text{koa} = 43.0003097 \tag{2}$$

The two antennas form relative main radiation beams as in Figure 5. The two antenna boresight axes have a separation of d. For this analysis both beams will be referenced to the antenna system boresight axis.

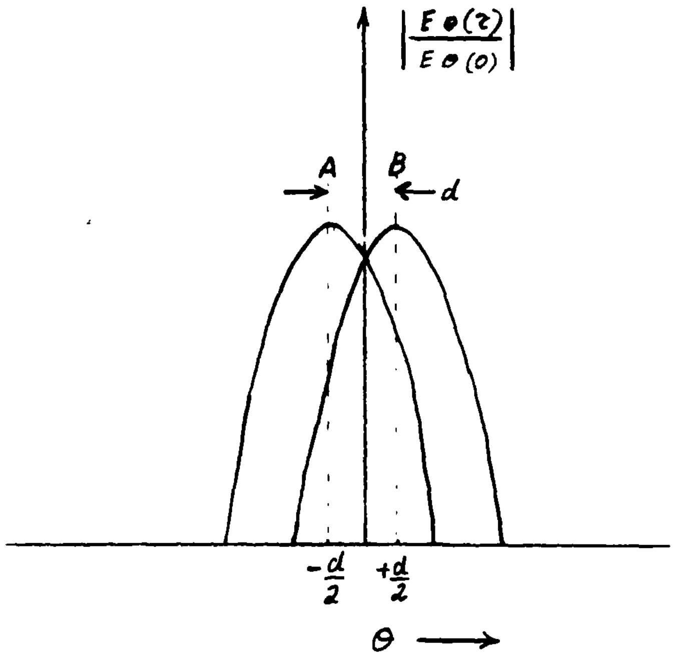


Figure 5. Relative Antenna A and B main radiation beams

From Equation 1

$$\text{Antenna A} \quad \frac{E\theta\ (\theta)}{E\theta\ (0)} = \frac{\sin\left[koa \sin\left(\theta + \frac{d}{2}\right)\right]}{koa \sin\left(\theta + \frac{d}{2}\right)} \quad (3)$$

$$\text{Antenna B} \quad \frac{E\theta\ (\theta)}{E\theta\ (0)} = \frac{\sin\left[koa \sin\left(\theta - \frac{d}{2}\right)\right]}{koa \sin\left(\theta - \frac{d}{2}\right)} \quad (4)$$

If the separation between the two lobes A and B is small compared to the focal length, then squint angle $\frac{d}{2}$ can be computed approximately from the antenna parameters (References 3 and 6) by the following:

$$d = Kp \tan^{-1} \frac{n}{f}$$

$$d = .6 \tan^{-1} \frac{.4}{5.546}$$

$= 2.475$

$$\frac{d}{2} = 1.2375^{o} \qquad (5)$$

$= 0.0216$ rad

Kp is a constant which is a function of the size and shape of the reflector and aperture 0.6 (References 3 and 6); n = distance between centers (40 cms); f = focal length (5.5m)

Antenna function $g\Sigma(\theta)$

$$g\Sigma(\theta) = A + B$$

from equation 4 and 5

$$g\Sigma(\theta) = \frac{\sin\left[koa \sin\left(\theta + \frac{d}{2}\right)\right]}{koa \sin\left(\theta + \frac{d}{2}\right)} + \frac{\sin\left[koa \sin\left(\theta - \frac{d}{2}\right)\right]}{koa \sin\left(\theta - \frac{d}{2}\right)} \qquad (6)$$

This produces a radiation pattern as Figure 6.

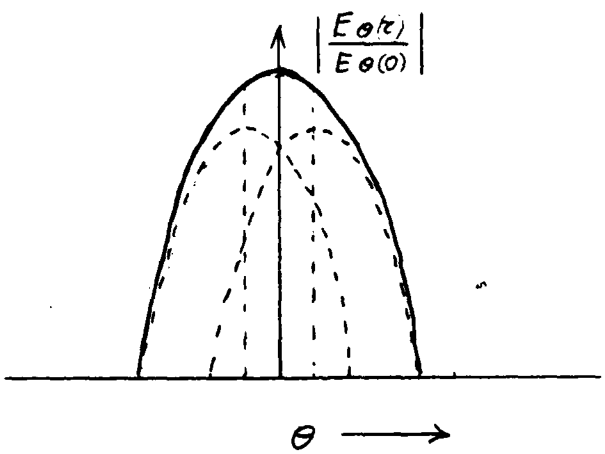


Figure 6. Radiation pattern for $g\Sigma(\theta)$

Antenna function $g\Delta(\theta)$

$$g\Delta(\theta) = A - B$$

from equation 4 and 5

$$g\Delta(\theta) = \frac{\sin\left[koa \sin\left(\theta + \frac{d}{2}\right)\right]}{koa \sin\left(\theta + \frac{d}{2}\right)} - \frac{\sin\left[koa \sin\left(\theta - \frac{d}{2}\right)\right]}{koa \sin\left(\theta - \frac{d}{2}\right)} \qquad (7)$$

This produces a radiation pattern as Figure 7.

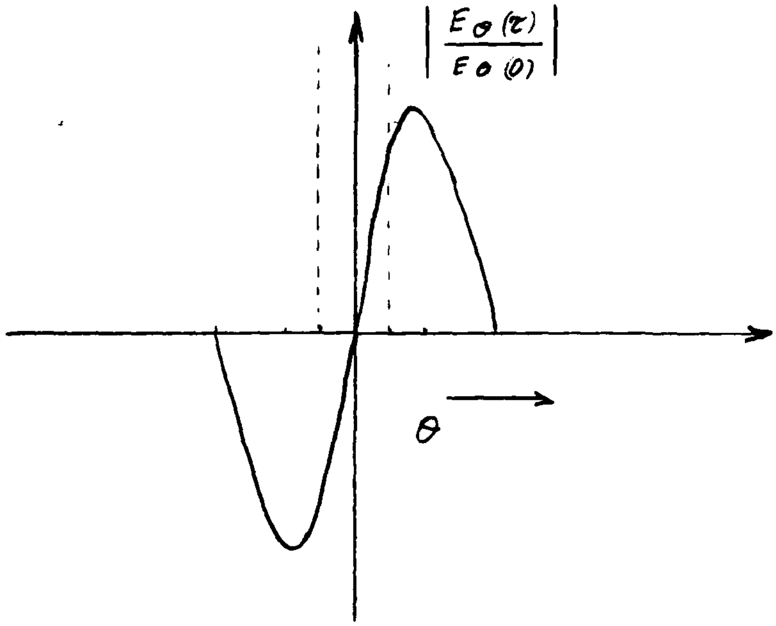


Figure 7. Radiation pattern for $g\Delta(\theta)$

<u>Estimator θ</u>

At this point in the Monopulse Radar, (output of the I.F. section, input to the estimator), the information signals are the $X\Sigma(t)$ and $X\Delta(t)$ signals. From these two signals θ, (angle the target is off boresight), must be estimated.

A way of estimating θ is to maximize the a posteriori density of $p\theta/f(X\Sigma, X\Delta, a)\ (\Theta/f(X\Sigma, X\Delta, A))$, (References 10 and 11), as in Figure 8. If the maximum is interior to the allowable range of θ and ln $p\theta/f(X\Sigma, X\Delta, a)\ (\Theta/f(X\Sigma, X\Delta, A))$ has a continuous first derivative then a condition for a maximum can be obtained by differentiating ln $p\ \theta/f(X\Sigma, X\Delta, a)\ (\Theta/f(X\Sigma, X\Delta, A))$ with respect to θ and setting the result to zero.

$$\left.\frac{\partial \ln p\ \theta/f(X\Sigma, X\Delta, a)\ (\Theta/f(X\Sigma, X\Delta, A))}{\partial\Theta}\right|_{\Theta = \theta\ (f(X\Sigma, X\Delta, A)} = 0$$

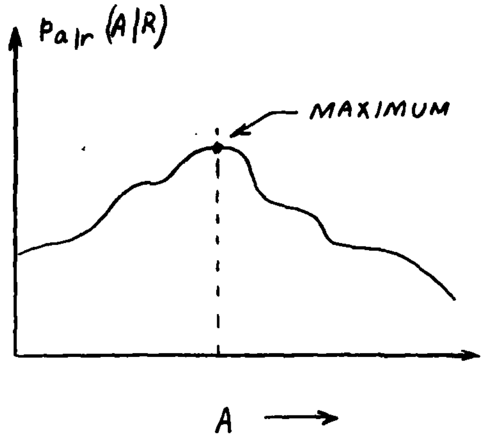


Figure 8. An a posteriori density

Let $f(X\Sigma, X\Delta, A) = Y$

From Bayes Rule (Reference Van Trees)

$$p\theta/y\ (\Theta/Y) = \frac{y/\theta\ (Y/\Theta)\ \ \theta(\Theta)}{y(Y)}$$

Therefore,

$$\ln p\theta/y\ (\Theta/Y) = \ln y/\theta\ (Y/\Theta) + \ln\ \theta(\Theta) - \ln y\ (Y)$$

As the main point of interest here is to find the value of Θ where the left hand side is maximum, the last term on the right hand side is not a function of Θ and can be ignored.

Therefore, to maximize the a posteriori density $\ln\ \theta/y\ (\theta/Y)$, need only maximize;

$$\ln p\ y/\theta\ (Y/\Theta) + \ln\ \theta\ (\theta)$$

Nothing is known about the a priori knowledge of $\theta(\theta)$ and therefore if this is assumed to be a uniform distribution within the range of θ then the a priori knowledge is a constant K.

But $\frac{\partial \ln}{\partial \theta}$ K = 0

Therefore, to maximize ln θ/y (Θ/Y) is equivalent to maximizing ln $y/\theta\,(Y/\Theta)$.

Therefore,

$$\left.\frac{\partial \ln p\ \theta/y\ (\Theta/Y)}{\partial\Theta}\right|_{\Theta = \theta ML(Y)} = \left.\frac{\partial \ln p\ y/\theta\ (Y/\Theta)}{\partial\Theta}\right|_{\Theta = \theta\ ML(Y)}$$

This is refered to as the Maximum Log Likelihood Estimation of θ

$$\left.\frac{\partial \ln p\, y/\theta\ (Y/\Theta)}{\partial\Theta}\right|_{\Theta = \theta_{ML}(Y)} = 0$$

This estimation of θ will now lend itself to a form of tracking loop as demonstrated by Verterbi (Reference 11).

The probability density of the noise, $(Ni(t),\ i = \Sigma,\ \Delta)$ is known, (independent, white, zero mean, gaussian with identical independent variances V), and therefore, the joint probability density $pf(X\Sigma(t),\ X\Delta(t)/a,\theta)$ $(f(X\Sigma(t),\ X\Delta(t)/A,\Theta)$ can be derived.

The Gram Schmidt or ıogonalization procedure (Reference 8) was used for this derivation.

<u>Gram Schmidt Orthogonalization Procedure</u>

As $Ni(t)$, $i = \Sigma,\Delta$), are independent white gaussian noise with independent identical variances then the noise can be represented (Reference 8) as;

$$n(t) = \sum_{i=1}^{\infty} Ni\phi i(t)$$

Where $\phi i(t)$ is any complete orthonormal set over the interval 0,T and the Ni are independent random variables with variance V as $Ni(t)$, $(i = \Sigma,\Delta)$ are white and independent with independent variances

$$V = \frac{No}{2}$$

Also expanding x(t) on the same basis set

$$X(t) = \sum_{i=1}^{\infty} X_i \phi_i(t) \quad \text{for } 0 \leq t \leq T$$

and as $X\Sigma(t) = Ag\Sigma(\theta)S(t) + N\Sigma(t)$

$$X\Delta(t) = Ag\Delta(\theta)S(t) + N\Delta(t) \quad \text{then;}$$

$$X\Sigma i = \int_0^T X\Sigma(t)\phi i(t)dt \text{ and } N\Sigma i = \int_0^T N\Sigma(t)\phi i(t)dt$$

$$X\Delta i = \int_0^T X\Delta(t)\phi i(t)dt \text{ and } N\Delta i = \int_0^T N\Delta(t)\phi i(t)dt$$

$$\text{Let } \phi_1(t) = \frac{S(t)}{\sqrt{E}} \qquad E = B^2T$$

$$B = 1$$

$$= \frac{S(t)}{T}$$

$$X\Sigma_1 = \int_0^T AS(t)g\Sigma(\theta)\frac{S(t)}{\sqrt{E}}dt + \int_0^T N\Sigma(t)\frac{S(t)}{\sqrt{E}}dt$$

$$= \frac{Ag\Sigma(\theta)}{E} \int_0^T S^2(t)dt + N\Sigma_1$$

$$= Ag\Sigma(\theta) \quad E \quad + N\Sigma_1 \tag{8}$$

Similarly for $X\Delta_1$

$$X\Delta_1 = \int_0^T AS(t)g\Delta(\theta)\frac{S(t)}{\sqrt{E}}dt + \int_0^T N\Delta(t)\frac{S(t)}{\sqrt{E}}dt$$

$$X\Delta_1 = Ag\Delta(\theta)\sqrt{E} + N\Delta_1 \tag{9}$$

<u>Statistics of $X\Sigma_i$ and $X\Delta_i$</u>

$X\Sigma_i$ and $X\Delta_i$ are both the sums of deterministic signals and gaussian random variables. Consequently, both $X\Sigma_i$ and $X\Delta_i$ are gaussian.

$$E\left[N\Sigma_i\right] = \int_0^T E\left[N\Sigma(t)\right] \frac{S(t)}{\sqrt{E}} \, dt = 0$$

$$E\left[N\Delta_i\right] = \int_0^T E\left[N\Delta(t)\right] \frac{S(t)}{\sqrt{E}} \, dt = 0$$

$$\mathrm{Var}\left[N\Sigma_i\right] = E\left[\int_0^T \int_0^T N\Sigma(t)N\Sigma(\gamma) \frac{S(t)}{\sqrt{E}} \frac{S(\gamma)}{\sqrt{E}} \, dt \, d\gamma\right]$$

$$= \frac{No}{2} \int_0^T S^2(t)dt$$

$$= \frac{No}{2}$$

Similarly

$$\mathrm{Var}\left[N\Delta_i\right] = E\left[\int_0^T \int_0^T N\Delta(t)N\Delta(\gamma) \frac{S(t)}{\sqrt{E}} \frac{S(\gamma)}{\sqrt{E}} \, dt \, d\gamma\right]$$

$$= \frac{No}{2} \int_0^T S^2(t)dt$$

$$= \frac{No}{2}$$

$$E\left[X\Sigma_1/\Theta,A\right] = E\left[Ag\Sigma(\theta)\sqrt{E}\right] + E\left[N\Sigma_1\right] \tag{10}$$
$$= Ag\Sigma(\theta)\sqrt{E}$$

$$E\left[X\Delta_1/\Theta,A\right] = E\left[Ag\Delta(\theta)\sqrt{E}\right] + E\left[N\Delta_1\right] \tag{11}$$
$$= Ag\Delta(\theta)\sqrt{E}$$

$$\mathrm{Var}\left[X\Sigma_1/\Theta,A\right] = E\left[(Ag\Sigma(\theta)\sqrt{E} + N\Sigma_1)^2\right] - (Ag\Sigma(\theta)\sqrt{E})^2$$
$$= E\left[(Ag\Sigma(\theta)\sqrt{E})^2 + 2Ag\Sigma(\theta)\sqrt{E}N\Sigma_1 + (N\Sigma_1)^2\right] - (Ag\Sigma(\theta)\sqrt{E})^2$$
$$= (Ag\Sigma(\theta)\sqrt{E})^2 + \frac{No}{2} - (Ag\Sigma(\theta)\sqrt{E})^2$$
$$= \frac{No}{2}$$

Similarly Var $X\Delta i/\Theta,A = \frac{No}{2}$

$$X\Sigma = Ag\Sigma(\theta)\sqrt{E} + N\Sigma$$

$$X\Delta = Ag\Delta(\theta)\sqrt{E} + N\Delta$$

(At this stage, analysis will refer to first pulse only.)

Where the $N\Sigma$ and $N\Delta$ are independent and identically distributed gaussian random variables with mean zero and variance $\frac{No}{2}$. The joint density function $(f(X\Sigma, X\Delta)/\Theta,A)$ is given by

$$p\,(f(X\Sigma, X\Delta)/\Theta,A) = \prod_i (Xi/\Theta,A) \qquad i = \Sigma,\Delta$$

$$= \frac{1}{2\,\frac{No}{2\pi}^{1}} \exp\left[- \frac{(X\Sigma - Ag\Sigma(\theta)\sqrt{E})^2}{2\frac{No}{2}} + \frac{(X\Delta - Ag\Delta(\theta)\sqrt{E})^2}{2\frac{No}{2}} \right]$$

$$= K \exp\left[- \frac{(X\Sigma - Ag\Sigma(\theta)\sqrt{E})^2 + (X\Delta - Ag\Delta(\theta)\sqrt{E})^2}{No} \right]$$

Maximum Likelihood Estimator

$p\,(f(X\Sigma, X\Delta)/\Theta,A)$ can be maximized by setting the partial derivative of $p\,(f(X\Sigma, X\Delta)/\Theta,A)$ with respect to θ to zero.

The partial derivative of $(f(X\Sigma, X\Delta)/\Theta,A)$ will be aided if first the $\ln p\ (f(X\Sigma, X\Delta)/\Theta,A)$ is found.

$$\ln p(f(X\Sigma, X\Delta)/\Theta,A) = \ln K - \frac{(X\Sigma - Ag\Sigma(\theta)\sqrt{E})^2 + (X\Delta - Ag\Delta(\theta)\sqrt{E})^2}{No}$$

$$= \left[\ln K - \frac{X\Sigma^2 - 2X\Sigma Ag\Sigma(\theta)\sqrt{E} + (Ag\Sigma(\theta)\sqrt{E})^2}{No} - \frac{X\Delta^2 - 2X\Delta Ag\Sigma(\theta)\sqrt{E} + (Ag\Delta(\theta)\sqrt{E})^2}{No} \right]$$

$$= \ln K - \frac{1}{No}\ X\Sigma^2 + X\Delta^2 - 2A\sqrt{E}(g\Sigma(\theta)X\Sigma + g\Delta(\theta)X\Delta) + EA^2(g\Sigma^2(\theta) + g\Delta^2(\theta)) \quad (12)$$

$$\frac{\partial}{\partial\Theta}\ln p(f(X\Sigma, X\Delta)/A,\Theta) = \left[- \frac{1}{No} \left[-2A\ \ E(X\Sigma g\Sigma(\theta) + X\Delta g\Delta(\theta)) + 2EA^2(g\Sigma(\theta)g\dot{\Sigma}(\theta) + g\Delta(\theta)g\dot{\Delta}(\theta)) \right] \right]$$

$$= -\frac{2A\sqrt{E}}{No}\left[-\left(X\Sigma g\Sigma(\theta)+X\Delta g\Delta(\theta)\right) + EA\left(g\Sigma(\theta)g\Sigma(\theta) + g\Delta(\theta)\, g\Delta(\theta)\right)\right]$$

Set

$$\frac{\partial}{\partial\theta} \ln p\ \left(f(X\Sigma,\ X\Delta)/A,\Theta\right) = 0$$

Therefore,

$$\frac{\partial}{\partial\theta} \ln p\ \left(f(X\Sigma,\ X\Delta)/A,\Theta\right) = -\left(X\Sigma g\Sigma(\theta) + X\Delta g\Delta(\theta)\right) + EA\left(g\Sigma(\theta)g\Sigma(\theta)+g\Delta(\theta)g\Delta(\theta)\right) = 0$$

Therefore;

$$X\Sigma g\Sigma(\theta) + X\Delta g\Delta(\theta) = EA\left(g\Sigma(\theta)g\Sigma(\theta)+g\Delta(\theta)g\Delta(\theta)\right) \qquad (13)$$

and

$$\frac{\partial}{\partial A} \ln p\ \left(f(X\Sigma,\ X\Delta)/A,\Theta\right) = -\frac{1}{No}\left[2\sqrt{E}\left(g\Sigma(\theta)X\Sigma + g\Delta(\theta)X\Delta\right) + 2EA\left(g\Sigma^2(\theta) + g\Delta^2(\theta)\right)\right]$$

setting $\frac{\partial}{\partial A} \ln p\ \left(f(X\Sigma,X\Delta)/A,\theta\right) = 0$

Therefore;

$$2\ E\left(g\Sigma(\theta)X\Sigma + g\Delta(\theta)X\Delta\right) = 2\ EA\ \left(g\Sigma^2(\theta) + g\Delta^2\ (\theta)\right)$$

$$\hat{A} = \frac{\left(g\Sigma(\theta)X\Sigma+g\Delta(\theta)X\Delta\right)}{E\left(g\Sigma^2(\theta) + g\Delta^2(\theta)\right)} \qquad (14)$$

Substituting equation 14 into equation 13;

$$X\Sigma\dot{g}\Sigma(\theta) + X\Delta\dot{g}\Delta(\theta) = \frac{\left(g\Sigma(\theta)X\Sigma+g\Delta(\theta)X\Delta\right)\left(g\Sigma(\theta)\dot{g}\Sigma(\theta)+g\Delta(\theta)\dot{g}\Delta(\theta)\right)}{\left(g\Sigma^2(\theta)\ +\ g\Delta^2(\theta)\right)}$$

$$\left[X\Sigma\dot{g}\Sigma(\theta)+X\Delta\dot{g}\Delta(\theta)\right]\cdot\left[g\Sigma^2(\theta)+g\Delta^2(\theta)\right] = \left[g\Sigma(\theta)X\Sigma+g\Delta(\theta)X\Delta\right]\cdot\left[\dot{g}\Sigma(\theta)g\Sigma(\theta) + \dot{g}\Delta(\theta)g\Delta(\theta)\right]$$

Expanding;

$$\left[g\Sigma^2(\theta)X\Sigma\dot{g}\Sigma(\theta) + g\Delta(\theta)X\Delta g\Sigma(\theta)\dot{g}\Sigma(\theta) + g\Sigma(\theta)X\Sigma g\Delta(\theta)\dot{g}\Delta(\theta) + g^2\Delta(\theta)\dot{g}\Delta(\theta)X\Delta\right] =$$

$$\left[g\Sigma^2(\theta)\dot{g}\Sigma(\theta)X\Sigma + \dot{g}\Delta(\theta)X\Delta g\Sigma^2(\theta) + g^2\Delta(\theta)\dot{g}\Sigma(\theta)X\Sigma+\dot{g}\Delta(\theta)g^2\Delta(\theta)X\Delta\right]$$

Therefore, after cancellation;

$$g\Delta(\theta)g\Sigma(\theta)\dot{g}\Sigma(\theta)X\Delta + \dot{g}\Delta(\theta)g\Sigma(\theta)g\Delta(\theta)X\Sigma = \dot{g}\Delta(\theta)g^2\Sigma(\theta)X\Delta + g^2\Delta(\theta)\dot{g}\Sigma(\theta)X\Sigma$$

Collecting like terms;

$$(\dot{g}\Delta(\theta)g\Sigma(\theta)g\Delta(\theta) - g\Delta^2(\theta)\dot{g}\Sigma(\theta))X\Sigma = -(\dot{g}\Sigma(\theta)g\Sigma(\theta)\dot{g}\Delta(\theta) - g\Sigma^2(\theta)g\Delta(\theta))X\Delta$$

$$g\Delta(\theta)(\dot{g}\Delta(\theta)g\Sigma(\theta)-g\Delta(\theta)\dot{g}\Sigma(\theta))X\Sigma = -g\Sigma(\theta)(\dot{g}\Sigma(\theta)g\Delta(\theta) - g\Sigma(\theta)\dot{g}\Delta(\theta))X\Delta$$

$$g\Delta(\theta)X\Sigma = -g\Sigma(\theta)X\Delta\,\frac{(\dot{g}\Sigma(\theta)g\Delta(\theta) - g\Sigma(\theta)\dot{g}\Delta(\theta))}{-(\dot{g}\Sigma(\theta)g\Delta(\theta)-g\Sigma(\theta)\dot{g}\Delta(\theta))}$$

Therefore;

$$g\Delta(\theta)X\Sigma = g\Sigma(\theta)X\Delta$$

and for the maximum likelihood estimate of θ

$$g\Delta(\hat{\theta})X\Sigma_1 - g\Sigma(\hat{\theta})X\Delta_1 = 0 \qquad (15)$$

From equations 8 and 9

$$X\Sigma_1 = Ag\Sigma(\theta)\sqrt{E} + N\Sigma_1$$

$$X\Delta_1 = Ag\Delta(\theta)\sqrt{E} + N\Delta_1$$

and

$$X\Sigma_1 = \int_0^T \frac{S(t)}{\sqrt{E}}\, X\Sigma(t)\, dt$$

$$X\Delta_1 = \int_0^T \frac{S(t)}{\sqrt{E}}\, X\Delta(t)\, dt$$

This is equivalent to performing the estimation process after the incoming signals have been coherently demodulated.

Error Voltage ε produced by the maximum likelihood equation

Equation 15 states that when $\hat{\theta}_{ML}$ is equal to θ (angle off boresight of the target) then;

$g\Delta(\hat{\theta})X\Sigma_1 - g\Sigma(\hat{\theta})X\Delta_1$ is equal to zero. When $\hat{\theta}_{ML}$ does not equal θ then equation 15 will equal some number other than zero. This number is the error voltage and through the discriminator (ε to $\hat{\theta}$ mapping) a $\hat{\theta}$ estimate of the angle off boresight can be made. Setting equation 15 to ε

$$g\Delta(\hat{\theta})X\Sigma_1 - g\Sigma(\hat{\theta})X\Delta = \varepsilon \tag{15a}$$

$$\text{where } \varepsilon = 0 \text{ when } \hat{\theta} = \theta$$

$$\varepsilon \neq 0 \text{ when } \hat{\theta} \neq \theta$$

Statistics of ε

$$E\left[\varepsilon\right] = g\Delta(\hat{\theta})\ E\left[X\Sigma\right] - g\Sigma(\hat{\theta})\ E\ \left[X\Delta\right]$$

from equations 10 and 11

$$E\left[\varepsilon\right] = g\Delta(\hat{\theta})Ag\Sigma(\theta)\sqrt{E} - g\Sigma(\hat{\theta})Ag\Delta(\theta)\sqrt{E}$$

$$E\left[\varepsilon\right] = A\sqrt{E}(g\Delta(\hat{\theta})g\Sigma(\theta) - g\Sigma(\hat{\theta})g\Delta(\theta))$$

If $\theta = \theta$ then

$$E\left[\varepsilon\right] = 0 \tag{16}$$

$$\begin{aligned} \mathrm{Var}\left[\varepsilon\right] &= E\left[(g\Delta(\theta)X\Sigma - g\Sigma(\theta)X\Delta)^2\right] - A^2E(g\Delta(\theta)g\Sigma(\theta) - g\Sigma(\theta)g\Delta(\theta))^2 \\ &= E\left[g\Delta^2(\theta)X\Sigma^2 - 2g\Delta(\theta)g\Sigma(\theta)X\Sigma X\Delta + g^2\Sigma(\theta)X\Delta^2\right] - \varepsilon^{-2} \\ &= \Big[EA^2g\Delta^2(\theta)g\Sigma^2(\theta) + \frac{No}{2}g\Delta^2(\theta) + \frac{No}{2}g\Sigma^2(\theta) - \\ &\quad 2g\Delta(\theta)g\Sigma(\theta)E\left[X\Sigma X\Delta\right] - \varepsilon^{-2} + EA^2g^2\Sigma(\theta)g\Delta^2(\theta)\Big] \end{aligned}$$

$$\begin{aligned} E\left[X\Sigma X\Delta\right] &= E\left[(Ag\Sigma(\theta)\sqrt{E} + N\Sigma)(Ag\Delta(\theta)\sqrt{E} + N\Delta)\right] \\ &= E\Big[A^2Eg\Sigma(\theta)g\Delta(\theta) + Ag\Sigma(\theta)\sqrt{E}\,N\Delta + Ag\Delta(\theta)\sqrt{E}\,N\Sigma + \\ &\qquad N\Sigma N\Delta\Big] \end{aligned}$$

$$= A^2 E g\Sigma(\theta) g\Delta(\theta) \quad + \quad E\left[N\Sigma N\Delta\right]$$

after cancellation of zero terms; $Ag\Sigma(\theta)$ E $N\Delta$, $Ag\Delta(\theta)$ E $N\Sigma$ and $E\left[N\Sigma N\Delta\right]$

$$\mathrm{Var}\left[\varepsilon\right] = EA^2 g\Sigma^2(\theta) g\Delta^2(\hat{\theta}) + \frac{No}{2}\left[g\Delta^2(\hat{\theta}) + g\Sigma^2(\hat{\theta})\right] -$$
$$2EA^2 g\Sigma(\theta) g\Delta(\theta) g\Delta(\hat{\theta}) g\Sigma(\hat{\theta}) \quad -$$
$$\varepsilon{-2} \quad + EA^2 g\Delta^2(\theta) g\Sigma^2(\hat{\theta})$$

$$= EA^2\left[g\Sigma^2(\theta) g\Delta^2(\hat{\theta}) + g\Sigma^2(\hat{\theta}) g\Delta^2(\theta)\right] - 2\, EA^2 g\Sigma(\theta) g\Delta(\theta) g\Delta(\hat{\theta}) g\Sigma(\hat{\theta})$$
$$+ \frac{No}{2}\left[g\Delta^2(\hat{\theta}) + g\Sigma^2(\hat{\theta})\right] - \varepsilon^{-2}$$

$$\varepsilon^{-2} = A^2 E(g\Delta(\hat{\theta}) g\Sigma(\theta) - g\Sigma(\hat{\theta}) g\Delta(\theta))^2$$
$$= A^2 E(g\Delta^2(\hat{\theta}) g\Sigma^2(\theta) - 2g\Delta(\hat{\theta}) g\Sigma(\theta) g\Sigma(\hat{\theta}) g\Delta(\theta) + g\Sigma^2(\hat{\theta}) g\Delta^2(\theta))$$

$$\varepsilon^{-2} = -A^2 E(g\Delta^2(\hat{\theta}) g\Sigma^2(\theta) = g\Sigma^2(\hat{\theta}) g\Delta^2(\theta)) + 2A^2 E g\Delta(\hat{\theta}) g\Sigma(\theta) g\Sigma(\hat{\theta}) g\Delta(\theta)$$

Therefore other cancellation;

$$\mathrm{Var}\ \varepsilon = \frac{No}{2}\left[g\Delta^2(\hat{\theta}) + g\Sigma^2(\hat{\theta})\right] \tag{17}$$

The density function of ε is the sum of two gaussian functions that are multiplied by constants $g\Delta(\theta)$ and $g\Sigma(\theta)$, and is consequently gaussian with the following statistics with $\theta = \theta$;

$$\overline{\varepsilon} = 0 \tag{16}$$

$$\overline{\varepsilon}^2 = \frac{No}{2}\left[g\Delta^2(\hat{\theta}) + g\Sigma^2(\hat{\theta})\right] \tag{17}$$

<u>Statistics of A estimate</u>

from Equation 14

$$\hat{A} = \frac{(g\Sigma(\theta)X\Sigma + g\Delta(\theta)X\Delta)}{\sqrt{E}\,(g\Sigma^2(\theta) + g\Delta^2(\theta))}$$

$$E\left[\hat{A}\right] + \frac{1}{E(g\Sigma^2(\hat{\theta}) + g\Delta^2(\hat{\theta}))}\left[g\Sigma(\hat{\theta})E \quad X\Sigma \quad + \quad g\Delta(\hat{\theta})E \quad [X\Delta]\right]$$

let

$$K = \frac{1}{E(g\Sigma^2(\hat{\theta}) + g\Delta^2(\hat{\theta}))}$$

from

$$E\left[\hat{A}/A\right] = K\left[g\Sigma(\hat{\theta})Ag\Sigma(\theta)\sqrt{E} + g\Delta(\hat{\theta})Ag\Delta(\theta)\sqrt{E}\right]$$

$$= K \quad A\sqrt{E}\left[g\Sigma(\hat{\theta}) \; g\Sigma(\theta) \; + \; g\Sigma(\hat{\theta})g\Delta(\theta)\right]$$

$$E\left[\hat{A}/A\right] = \frac{A\left[g\Sigma(\theta)g\Sigma(\theta) \; + \; g\Sigma(\theta)g\Delta(\theta)\right]}{g\Sigma^2(\hat{\theta}) \; + \; g\Delta^2(\hat{\theta})} \tag{18}$$

Under the condition of $\hat{\theta} = \theta$; then

$$E\left[\hat{A}/A\right] = A \tag{19}$$

Therefore, if $\hat{\theta} = \theta$, then the estimate of A is conditionally

unbiased.

$$\mathrm{Var}\left[\hat{A}/A\right] = E\left[\left[\frac{g\Sigma(\hat{\theta})X\Sigma \; + \; g\Delta(\hat{\theta})X\Delta)}{E\;(g\Sigma^2(\hat{\theta}) \; + g\Delta^2(\hat{\theta})}\right]^2 /A\right] -A^2$$

$$= \frac{1}{E(g\Sigma^2(\hat{\theta}) + g\Delta^2(\hat{\theta}))^2} \; E\left[(g\Sigma(\hat{\theta})X\Sigma + g\Delta(\hat{\theta})X\Delta)^2/A\right] -A^2$$

$$= \frac{1}{E(g\Sigma^2(\hat{\theta}) \; + g\Delta^2(\hat{\theta}))^2} \; E\left[(g^2\Sigma(\hat{\theta})X^2\Sigma + 2g\Sigma(\hat{\theta})g\Delta(\hat{\theta})X\Sigma X\Delta + g\Delta^2(\hat{\theta})X\Delta^2)/A\right] -A^2$$

let;

$$K = \frac{1}{E(g\Sigma^2(\hat{\theta}) + g\Delta^2(\hat{\theta}))^2}$$

$$\begin{aligned}\mathrm{Var}\left[\hat{A}/A\right] = KE \Big[& g^2\Sigma(\hat{\theta})(A^2g^2\Sigma(\theta)E + 2Ag\Sigma(\theta)\sqrt{E}\,N\Sigma + (N\Sigma)^2) \; + \\ & 2g\Sigma(\hat{\theta})g\Delta(\hat{\theta})(Ag\Sigma(\theta)\sqrt{E} \; + \; N\Sigma)(Ag\Delta(\theta)\sqrt{E} + N\Delta) \; + \\ & g^2\Delta(\hat{\theta})(A^2g^2\Delta(\theta)E + 2Ag\Delta(\theta)\sqrt{E}\,N\Delta + (N\Delta)^2\Big] \; -A^2\end{aligned}$$

$$= K\left[A^2Eg^2\Sigma(\hat{\theta})g^2\Sigma(\theta) + g^2\Sigma(\hat{\theta})\frac{No}{2} + 2g\Sigma(\hat{\theta})g\Delta(\hat{\theta})A^2Eg\Sigma(\theta)g\Delta(\theta) + A^2Eg^2\Delta(\hat{\theta})g^2\Delta(\theta) + g^2\Delta\hat{\theta}\frac{No}{2}\right] - A^2$$

$$= K\left[A^2E\ \ g^2\Sigma(\hat{\theta})g^2\Sigma(\theta) + g^2\Delta(\theta)g^2\Delta(\theta) + 2g\Sigma(\hat{\theta})g\Delta(\hat{\theta})g\Sigma(\theta)g\Delta(\theta)) + \frac{No}{2}(g^2\Sigma(\hat{\theta}) + g\Delta^2(\hat{\theta}))\right] - A^2$$

again let $\hat{\theta} = \theta$;

$$\mathrm{Var}\left[\hat{A}/A\right] = \frac{1}{E(g\Sigma^2(\theta) + g\Delta^2(\theta))^2}\ A^2E(g\Sigma^2(\theta) + g\Delta^2(\theta))^2 + \frac{No}{2}\ \frac{(g^2\Sigma(\theta) + g\Delta^2(\theta)}{(g^2\Sigma(\theta) + g\Delta^2(\theta)^2} - A^2 \tag{20}$$

after cancellation;

$$\mathrm{Var}\left[\hat{A}/A\right] = \frac{No}{2}\ \frac{1}{(g\Sigma^2(\theta) + g\Delta^2(\theta))} \tag{21}$$

To determine if the conditionally unbiased estimate of A is an efficient estimate then the conditional variance equation 20 needs to be equal to an established lower bound, (Cramer-Rao Bound, Reference 8).

Cramer-Rao Bound (Reference 8)

$$\mathrm{Var}\left[A/A\right] \geq \left(E\left[\frac{\partial}{\partial A}\left[\ln p\,(f(X\Sigma,\ X\Delta)/A\theta)\right]^2\right]\right)^{-1}$$

or

$$\geq \left(-E\left[\frac{\partial^2}{\partial A^2}\ \ln p\,(f(X\Sigma,\ X\Delta)/A\theta)\right]\right)^{-1}$$

from equation 12;

$$\ln p\,(f(X\Sigma,\ X\Delta)/\Theta,A) = \ln K - \frac{1}{No}\ \ X\Sigma^2 + X\Delta^2 - 2A\ \ E\ \ (g\Sigma(\theta)X\Sigma + g\Delta(\theta)X\Delta) + EA^2(g\Sigma^2(\theta) + g\Delta^2(\theta)) \tag{12}$$

$$\frac{\partial}{\partial A}\ln p(f(X\Sigma,X\Delta)/\Theta,A) = \frac{-1}{No}\ \ 2\ \ E(g\Sigma(\theta)X\Sigma + g\Delta(\theta)X\Delta) + 2EA(g\Sigma^2(\theta) + g\Delta^2(\theta))$$

$$\frac{\partial^2}{\partial A^2} \ln p\ (f(X\Sigma,\ X\Delta)/\Theta,A) = -\frac{1}{No}\ 2\ E(g^2\Sigma(\theta) + g\Delta^2(\theta))$$

$$-E\left[\frac{\partial^2}{\partial A^2}\ \ln p\ (f(X\Sigma,\ X\Delta)/\Theta,A)\right] = \frac{2E}{No}\ (g^2\Sigma(\theta) + g\Delta^2(\theta))$$

$$-E\left[\frac{\partial}{\partial A^2}\ \ln p\ (f(X\Sigma,\ X\Delta)/\Theta,A)\right] = \frac{No}{2E}\ (g^2\Sigma\ (\theta) + g\Delta^2(\theta)) \qquad (22)$$

Equation 22 is equal to equation 21 consequently, the estimate of $\hat{A}$ is an efficient conditionally unbiased estimate and can be used in the implementation model to determine $\hat{\theta}$.

Maximum likelihood estimate using two pulses

The analysis of the system has been done assuming only one pulse. The analysis will now be extended to two pulses as in Figure 9.

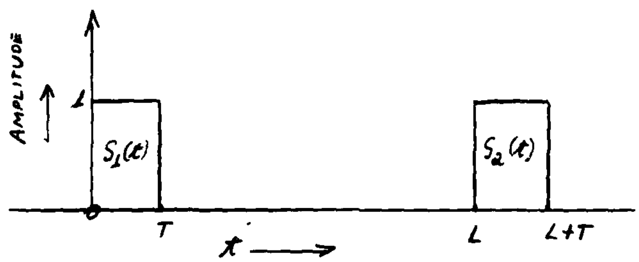


Figure 9. Two Receive Pulses

$$X\Sigma(t) = Ag\Sigma(\theta)S_1(t) + Ag\Sigma(\theta)S_2(t) + N\Sigma(t)$$

$$X\Delta(t) = Ag\Delta(\theta)S_1(t) + Ag\Delta(\theta)S_2(t) + N\Sigma(t)$$

Using Gram Schmidt Orthogonalization procedure, decompose the input signals into two dimensional random variables.

$$X\Sigma(t) = X\Sigma_1 S_1\phi_1(t) + X\Sigma_2 S_2\phi_2(t)$$

$$X\Delta(t) = X\Delta_1 S_1\phi_1(t) + X\Delta_2 S_2\phi_2(t)$$

Where $\underline{X\Sigma} = (X\Sigma_1,\ X\Sigma_2)$

$\underline{X\Delta} = (X\Delta_1,\ X\Delta_2)$

$\underline{S} = (S_1,\ S_2)$

Given that $\theta(t)$ is constant during pulse and during the pulse repetition interval.

Therefore,

$$\frac{d\theta(t)}{dt} = 0$$

Then (θ) will not change during the pulse or from pulse to pulse so $g_i(\theta)$ $(i=\Sigma,\Delta)$ will remain constant from pulse to pulse for Σ channel.

$$S_1 = Ag\Sigma(\theta) \int_0^T S(t)\phi_1(t)dt$$

$$= Ag\Sigma(\theta) \int_0^T S(t) \frac{1}{E} dt$$

$$= Ag\Sigma(\theta)\sqrt{E}$$

$$S_2 = Ag\Sigma(\theta) \int_0^T S(t)\phi_2(t)dt$$

$$= Ag\Sigma(\theta) \int_0^T S(t) \frac{1}{E} dt$$

$$= Ag\Sigma(\theta)\sqrt{E}$$

Similarly for Δ channel

$$S_1 = Ag\Delta(\theta)\sqrt{E}$$

$$S_2 = Ag\Delta(\theta)\sqrt{E}$$

Therefore,

$$\underline{X\Sigma} = (Ag\Sigma(\theta)\sqrt{E} + N\Sigma_1,\ Ag\Sigma(\theta)\sqrt{E} + N\Sigma_2)$$

$$\underline{X\Delta} = (Ag\Delta(\theta)\sqrt{E} + N\Delta_1,\ Ag\Delta(\theta)\sqrt{E} + N\Delta_2)$$

Now to write $p(\underline{X\Sigma}, \underline{X\Delta}/\Theta,A)$

Since $\underline{N\Sigma}$ and $\underline{N\Delta}$ are independent

$$p(\underline{X\Sigma}, \underline{X\Delta}/\Theta,A) = p(\underline{X\Sigma}/\Theta,A)\ p(\underline{X\Delta}/\Theta A)$$

and since $N\Sigma_1$ and $N\Sigma_2$ are independent and $N\Delta_1$ and $N\Delta_2$ are independent;

$$p(\underline{X}\Sigma, \underline{X}\Delta/\Theta,A) = p(X\Sigma_1/\Theta,A)\, p(X\Sigma_2/\Theta,A)\, p(X\Delta_1/\Theta,A)\, p(X\Delta_2/\Theta,A)$$

therefore,

$$p(\underline{X}\Sigma, \underline{X}\Delta/\Theta,A) = K \exp\left[-\frac{1}{2}\frac{2}{No}\ (X\Sigma_1 - Ag\Sigma(\theta)\sqrt{E}) + (X\Sigma_2 - Ag\Sigma(\theta)\sqrt{E})^2 \right.$$
$$\left. (X\Delta_1 - Ag\Delta(\theta)\quad E)^2 + (X\Delta_2 - Ag\Delta(\theta)\sqrt{E})^2\right]$$

$$\ln p\ (\underline{X}\Sigma, \underline{X}\Delta/\Theta,A) = \ln K - \frac{1}{No}\left[(X\Sigma_1 - Ag\Sigma(\theta)\sqrt{E})^2 + (X\Sigma_2 - Ag\Sigma(\theta)\sqrt{E})^2 \right.$$
$$\left. + (X\Delta_1 - Ag\Delta(\theta)\sqrt{E})^2 + (X\Delta_2 - Ag\Delta(\theta)\sqrt{E})^2\right] \qquad (23)$$

<u>For Estimate of $\hat{A}$</u>

$$\frac{\partial \ln}{\partial A} p\quad (\underline{X}\Sigma, \underline{X}\Delta/\Theta,A) = -\frac{1}{No}\left[2(X\Sigma_1 - Ag\Sigma(\theta)\quad E)(-)g\Sigma(\theta)\sqrt{E} + \right.$$
$$2(X\Sigma_2 - Ag\Sigma(\theta)\quad E)(-)g\Sigma(\theta)\sqrt{E} +$$
$$2(X\Delta_1 - Ag\Delta(\theta)\quad E)(-)g\Delta(\theta)\sqrt{E} +$$
$$\left. 2(X\Delta_2 - Ag\Delta(\theta)\quad E)(-)g\Delta(\theta)\sqrt{E}\right]$$

setting $\frac{\partial}{\partial A}\ \ln p\ (\underline{X}\Sigma, \underline{X}\Delta/\Theta,A) = 0$

$$0 = -X\Sigma_1 g\Sigma(\theta)\sqrt{E} + Ag\Sigma^2(\theta)E - X\Sigma_2 g\Sigma(\theta)\sqrt{E} + Ag\Sigma^2(\theta)E -$$
$$- X\Delta_1 g\Delta(\theta)\sqrt{E} + Ag^2\Delta(\theta)E - X\Delta_2 g\Delta(\theta)\sqrt{E} + Ag\Delta^2(\theta)E$$

$$2AE(g\Sigma^2(\theta) + g\Delta^2(\theta)) = \sqrt{E}\left[g\Sigma(\theta)(X\Sigma_1 + X\Sigma_2) + g\Delta(\theta)(X\Delta_1 + X\Delta_2)\right]$$

Therefore;

$$\hat{A} = \frac{g\Sigma(\theta)(X\Sigma_1 + X\Sigma_2) + g\Delta(\theta)(\Delta_1 + X\Delta_2)}{2\quad E(g\Sigma^2(\theta) + g\Delta^2(\theta))} \qquad (24)$$

__For Estimate of $\hat{\theta}$__

From equation 23;

$$\frac{\partial}{\partial\theta} \ln p\,(\underline{X}\Sigma,\ \underline{X}\Delta/\Theta,A) = -\frac{1}{No}\Big[\ 2(X\Sigma_1 - Ag\Sigma(\theta)\quad E)(-)A\dot{g}\Sigma(\theta)\sqrt{E} +$$

$$2(X\Sigma_2 - Ag\Sigma(\theta)\quad E)(-)A\dot{g}\Sigma(\theta)\sqrt{E} +$$

$$2(X\Delta_1 - Ag\Delta(\theta)\quad E)(-)A\dot{g}\Delta(\theta)\sqrt{E} +$$

$$2(X\Delta_2 - Ag\Delta(\theta)\quad E)(-)A\dot{g}\Delta(\theta)\sqrt{E} + \Big]$$

setting $\frac{\partial}{\partial\theta}\ln p(\underline{X}\Sigma,\ \underline{X}\Delta/\Theta,A) = 0$;

$$0 = -A\sqrt{E}\ X\Sigma_1\dot{g}\Sigma(\theta) + A^2E\ g\Sigma(\theta)\dot{g}\Sigma\theta -$$

$$-A\sqrt{E}\ X\Sigma_2\dot{g}\Sigma(\theta) + A^2E\ g\Sigma(\theta)\dot{g}\Sigma\theta -$$

$$-A\sqrt{E}\ X\Delta_1\dot{g}\Delta(\theta) + A^2E\ g\Delta(\theta)\dot{g}\Delta(\theta) -$$

$$-A\sqrt{E}\ X\Delta_2\dot{g}\Delta(\theta) + A^2E\ g\Delta(\theta)\dot{g}\Delta(\theta)$$

$$0 = -\dot{g}\Sigma(\theta)(X\Sigma_1+X\Sigma_2) - \dot{g}\Delta(\theta)(X\Delta_1+X\Delta_2) +$$

$$2A\quad E(g\Sigma(\theta)\dot{g}\Sigma(\theta)+g\Delta(\theta)\dot{g}\Delta(\theta))$$

using equation 24;

$$\frac{2\sqrt{E}\left[g\Sigma(\theta)X\Sigma_1+X\Sigma_2)\ +g\Delta(\theta)(X\Delta_1+X\Delta_2)\right]}{2\sqrt{E}\ (g\Sigma^2(\theta) + g\Delta^2(\theta)}\cdot\left[g\Sigma(\theta)\dot{g}\Sigma(\theta)+g\Delta(\theta)\dot{g}\Delta(\theta)\right] =$$

$$\dot{g}\Sigma(\theta)(X\Sigma_1+X\Sigma_2) + \dot{g}\Delta(\theta)(X\Delta_1+X\Delta_2) \tag{25}$$

Cross multiplying;

$$\Big[\Big[g\Sigma^2(\theta)\dot{g}\Sigma(\theta)(X\Sigma_1+X\Sigma_2) + g\Delta(\theta)g\Sigma(\theta)\dot{g}\Sigma(\theta)(X\Delta_1+X\Delta_2) +$$

$$g\Sigma(\theta)\dot{g}\Delta(\theta)g\Delta(\theta)(X\Sigma_1+X\Sigma_2) + g\Delta^2(\theta)\dot{g}\Delta(\theta)(X\Delta_1+X\Delta_2)\Big] =$$

$$\left[g\Sigma^2(\theta)\dot{g}\Sigma(\theta)(X\Sigma_1+X\Sigma_2) + g\Sigma^2(\theta)\dot{g}\Delta(\theta)(X\Delta_1+X\Delta_2) + g\Delta^2(\theta)\dot{g}\Sigma(\theta)(X\Sigma_1+X\Sigma_2) + g\Delta^2(\theta)\dot{g}\Delta(\theta)(X\Delta_1+X\Delta_2)\right]$$

$$\left[\left[g\Delta(\theta)g\Sigma(\theta)\dot{g}\Sigma(\theta)(X\Delta_1+X\Delta_2) + g\Sigma(\theta)\dot{g}\Delta(\theta)g\Delta(\theta)(X\Sigma_1+X\Sigma_2)\right]\right] =$$

$$\left[g\Sigma^2(\theta)\dot{g}\Delta(\theta)(X\Delta_1+X\Delta_2) + g\Delta^2(\theta)\dot{g}\Sigma(\theta)(X\Sigma_1+X\Sigma_2)\right]$$

$$\left[(X\Delta_1+X\Delta_2)g\Sigma(\theta)(g\Delta(\theta)\dot{g}\Sigma(\theta)-g\Sigma(\theta)\dot{g}\Delta(\theta)) + +(X\Sigma_1+X\Sigma_2)g\Delta(\theta)(g\Sigma(\theta)\dot{g}\Delta(\theta)-g\Delta(\theta)\dot{g}\Sigma(\theta))\right] = 0$$

$$(g\Delta(\theta)\dot{g}\Sigma(\theta)-g\Sigma(\theta)\dot{g}\Delta(\theta)) \quad (X\Delta_1+X\Delta_2)g\Sigma(\theta)-(X\Sigma_1+X\Sigma_2)g\Delta(\theta) = 0$$

therefore,

$$(X\Delta_1+X\Delta_2)g\Sigma(\theta)-(X\Sigma_1+X\Sigma_2)g\Delta(\theta) = 0$$

or;

$$(X\Sigma_1+X\Sigma_2)g\Delta(\theta)-(X\Delta_1+X\Delta_2)g\Sigma(\theta) = 0 \tag{26}$$

$$(X\Sigma_1+X\Sigma_2)g\Delta(\theta)-(X\Delta_1+X\Delta_2)g\Sigma(\theta) = \varepsilon \tag{26a}$$

Each received pulse is processed individually and no cross terms exist. The analysis of the single pulse consequently holds for more than one pulse.

<u>The effect on error voltage ε from using more than one pulse</u>

Using equation 26a;

let;

$$\frac{X\Sigma_1+X\Sigma_2}{2}\, g\Delta(\hat{\theta})-\frac{(X\Delta_1+X\Delta_2)}{2}\, g\Sigma(\hat{\theta}) = \varepsilon$$

$$\tfrac{1}{2}X\Sigma_1 g\Delta(\hat{\theta}) + \tfrac{1}{2}X\Sigma_2 g\Delta\hat{\theta} - \frac{X\Delta_1}{2} g\Sigma\hat{\theta} - \frac{X\Delta_2}{2} g\Sigma(\hat{\theta}) = \varepsilon$$

$$\tfrac{1}{2}X\Sigma_1 g\Delta(\hat{\theta}) - \tfrac{1}{2}X\Delta_1 g\Sigma(\hat{\theta}) + \tfrac{1}{2}X\Sigma_2 g\Delta(\hat{\theta}) - \tfrac{1}{2}X\Delta_2 g\Sigma(\hat{\theta}) = \varepsilon$$

$$\tfrac{1}{2}\ X\Sigma_1 g\Delta(\hat{\theta}) - X\Delta_1 g\Sigma(\hat{\theta}) + \tfrac{1}{2}\ X\Sigma_2 g\Delta(\hat{\theta}) - X\Delta_2 g\Sigma(\hat{\theta}) = \varepsilon$$

from equation 15a;

$$\tfrac{1}{2}\varepsilon_1 + \tfrac{1}{2}\varepsilon_2 = \varepsilon \tag{27}$$

This states that the error from one pulse can be added to the error from the second pulse and the final error averaged to obtain a new error. As the mapping of error ε to estimate angle $\hat{\theta}$ is a one to one mapping, each $\hat{\theta}$ estimate obtained for each pulse can be added to the next $\hat{\theta}$ estimate for the next pulse and then averaged.

<u>Implementation of the error ε to θ estimate discrimination</u>

Using equation 15;

$$g\Delta(\hat{\theta})\ X\Sigma + g\Sigma(\hat{\theta})\ X\Delta = 0 \tag{15}$$

Assuming no noise, $N\Sigma = N\Delta = 0$;

$$g\Delta(\hat{\theta})Ag\Sigma(\theta)\ E + g\Sigma(\hat{\theta})Ag\Delta(\theta)\ E = \varepsilon$$

Using equations 6 and 7;

$$\frac{\sin koa \sin(\hat{\theta} - \frac{d}{2})}{koa \sin(\hat{\theta} - \frac{d}{2})} \left[\frac{\sin koa \sin(\theta + \frac{d}{2})}{koa \sin(\theta + \frac{d}{2})} - \frac{\sin koa \sin(\theta - \frac{d}{2})}{koa \sin(\theta - \frac{d}{2})}\right] \tag{28}$$

As this equation is a combination of sine functions and cannot be reduced effectively, the digital computer will be used to produce a series of $\hat{\theta}$ estimates for various errors ε . The error voltage resulting from the estimator will then be compared to the various errors and a corresponding $\hat{\theta}$ estimate chosen. This estimate will then be averaged with preceding estimates to account for multiple pulse processing.

From equation 28 EA scales the set of $\varepsilon/\hat{\theta}$ curves. As A is unknown, (A is contained in the incoming signal), then the estimate of $\hat{A}$, equation 14, is used to estimate A and is used to scale the $\varepsilon/\hat{\theta}$ curves.

From equation 14;

$$\hat{A} = \frac{(g\Sigma(\hat{\theta})X\Sigma + g\Delta(\hat{\theta})X\Delta)}{E(g^2\Sigma(\hat{\theta}) + g\Delta^2(\hat{\theta}))}$$

$\hat{A}$ is a function of both $\hat{\theta}$ and the amplitude of the incoming signal. Only when $\hat{\theta} = \theta$ does $\hat{A} = A$, therefore, the scaling of the $\varepsilon/\hat{\theta}$ curves will change for every pulse.

<u>The tracking Loop</u>

The system at this stage has estimated an angle $\hat{\theta}$ from one or more pulses. From equation 27 and the discussion following equation 27, the $\hat{\theta}$ estimate from successive pulses could be averaged.

The tracking loop can be completed in two methods as follows:

a. 50 pulses can be used to estimate 50 $\hat{\theta}_i$ estimates. These estimates can then be averaged to produce a $\hat{\theta}$ estimate angle off boresight, (Figure 10) as follows:

$$\hat{\theta} = \frac{1}{50} \; {}^{50}_{i=1} \, \theta_i$$

The $\hat{\theta}$ *estimate* can then be used to steer the antenna.

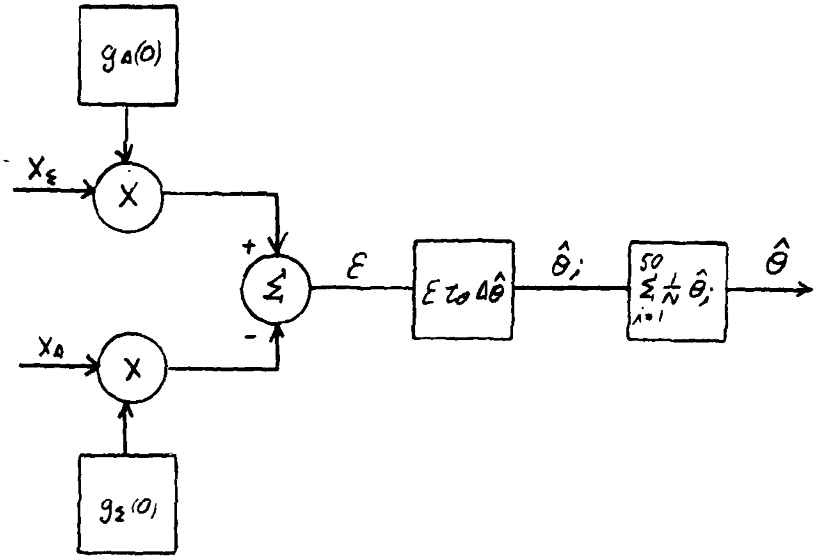


Figure 10. Implementation of System (a)

b. Over 50 pulses, each pulse can be used to derive a $\Delta\hat{\theta}$ estimate and this $\Delta\hat{\theta}$ estimate averaged with the preceding $\Delta\theta$ estimates. This average $\Delta\hat{\theta}$ estimate is then added to the current estimate to form a new estimate which is fed back to the estimate antenna functions $g\Sigma(\hat{\theta})$ and $g\Delta(\hat{\theta})$. The next estimate is then derived using these updated antenna functions, (Figure 11).

$$\Delta\hat{\theta}\quad i = \frac{1}{i}\ (i-1)\Delta\hat{\theta}_{i-1}+\Delta\hat{\theta}_i \quad ;\ i = 1,2,\ldots 50$$

$$\hat{\theta}_i = \frac{1}{i}\ (i-1)\Delta\hat{\theta}_{i-1}+\Delta\hat{\theta}_i\ +\hat{\theta}_{i-1} \quad ;\ i=1,2,\ldots 50$$

where - = 0

$$\hat{\theta} = \underset{i=1}{\overset{50}{}}\ \frac{1}{i}\ (i-1)\Delta\hat{\theta}_{i-1}+\Delta\hat{\theta}_i \qquad (30)$$

This system is equivalent to a tracking loop (Reference 11) which updates the estimate of $\hat{\theta}$ estimate based on the error ε produced from the preceding estimate.

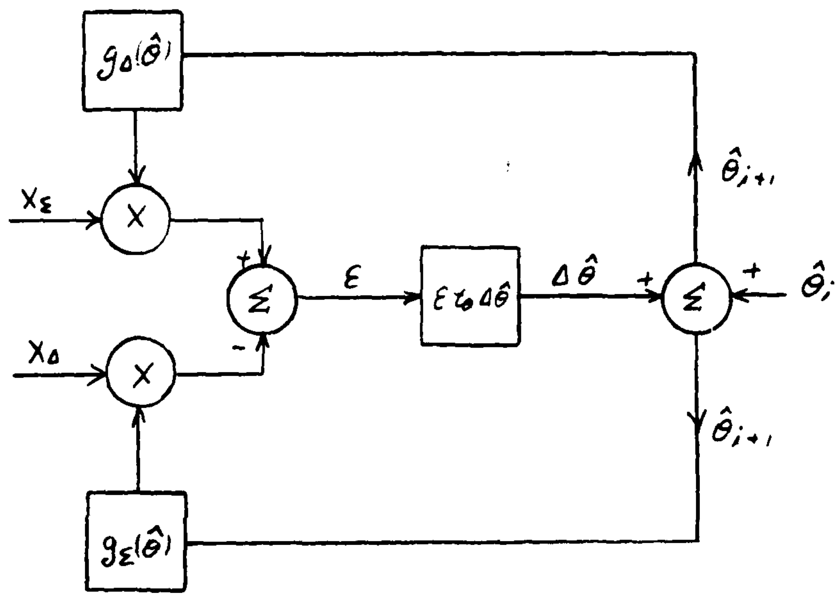


Figure 11. Implementation of System (b)

Implementation of Tracking Loop

Using equation 14, a digital computer program was written to implement each block of Figure 12.

The error ε voltage was determined by implementing equation 11a directly using the digital computer. The pulse width T was taken to be 20ms and the number of noise samples from a gaussian noise generator was 100. The first estimate value of $\hat{\theta}$ used in the feedback or estimate antenna functions $g\Sigma(\hat{\theta})$ and $g\Delta(\hat{\theta})$, was arbitrarily taken to be zero. The error voltage ε was then compared to a set of numbers generated by the discriminator. Each of the discriminator

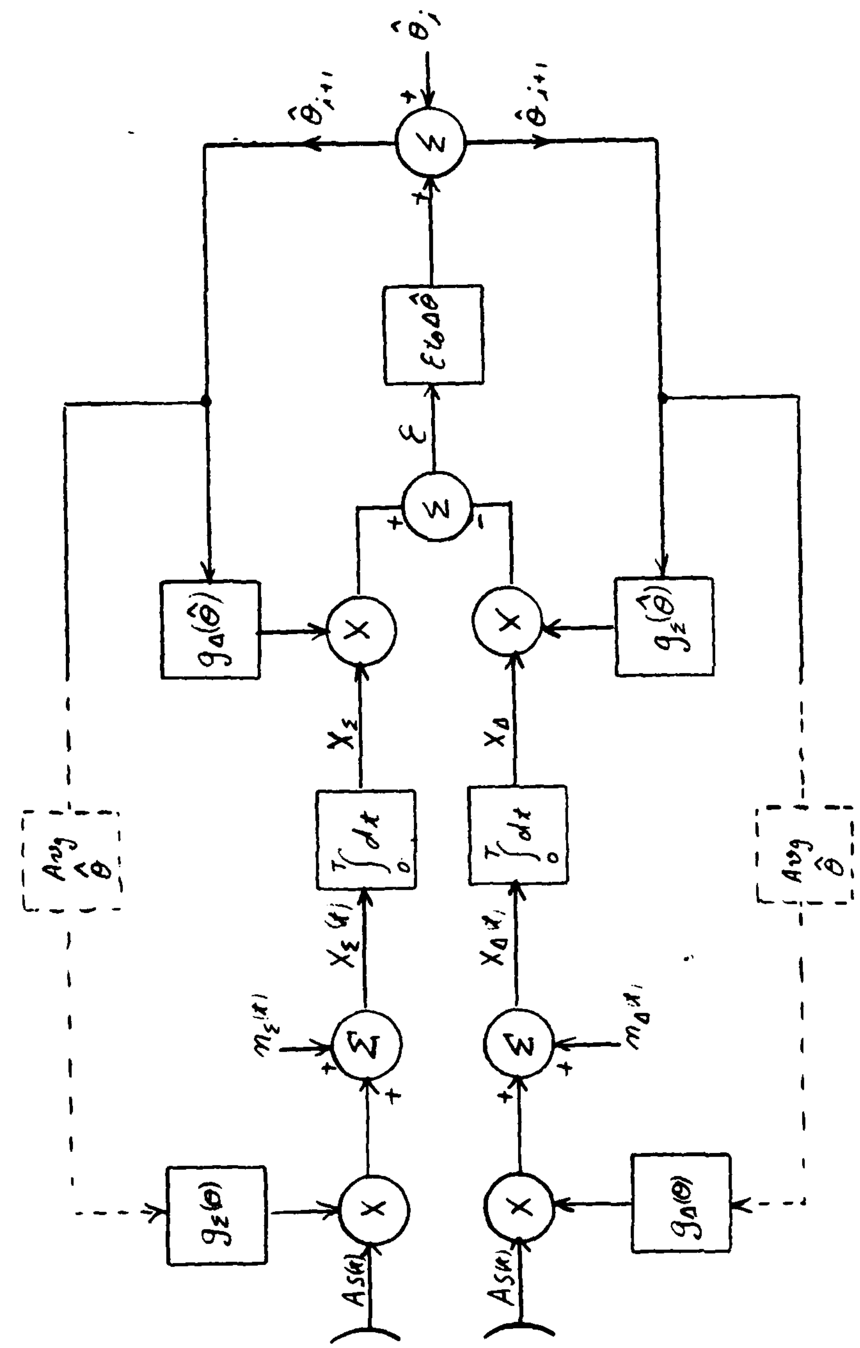


Figure 12. Overall Block Diagram of the Monopulse Radar System

boresight.

The probability density function of $\hat{\theta}$ is unknown. Also the expression for $\hat{\theta}$ is unknown as $\hat{\theta}$ is contained in the expression for ε which is the sum of sine functions. The lower bound for the variance of $\hat{\theta}/\theta$ can be determined using the Cramer-Rao Bound (Reference 8).

$$\mathrm{Var}\left[\hat{\theta}/\theta\right] \geq \left(E\left[\frac{\partial}{\partial\theta} \ln p\,(f(X\Sigma,\ X\Delta)/A\theta)^{2}\right]\right)^{-1}$$

$$\geq \left(-E\left[\frac{\partial}{\partial A^{2}} \ln p\,(f(X\Sigma,\ X\Delta)/A\theta)\right]\right)^{-1}$$

$$\mathrm{Var}\left[\hat{\theta}/\theta\right] \geq \left(-E\left[\frac{\partial}{\partial\theta} - \frac{1}{No} -2A\sqrt{E}\,(X\Sigma\dot{g}(\hat{\theta}) + X\Delta\dot{g}\Delta(\hat{\theta})) + 2EA^{2}\, g\Sigma(\hat{\theta})\dot{g}\Sigma(\hat{\theta}) + g\Delta(\hat{\theta})\dot{g}\Delta(\hat{\theta})\right]\right)^{-1}$$

$$\geq \left(-E\left[-\frac{1}{No} - 2A\,E\,(X\Sigma\ddot{g}\Sigma(\theta) + X\Delta\ddot{g}\Delta(\theta)) + 2EA^{2}\,(g\Sigma(\theta)\ddot{g}\Sigma(\hat{\theta}) + (\dot{g}\Sigma(\hat{\theta}))^{2} + g\Delta(\theta)\ddot{g}\Delta(\hat{\theta}) + (\dot{g}\Delta(\hat{\theta}))^{2})\right]\right)^{-1}$$

Using $E\left[X\Sigma\right] = Ag\Sigma(\theta)\sqrt{E}$ and $E\left[X\Delta\right] = Ag\Sigma(\theta)\sqrt{E}$

$$\mathrm{Var}\left[\hat{\theta}/\theta\right] \geq +\frac{1}{No}\,2\,A^{2}\,E\,(-g\Sigma(\theta)\ddot{g}\Sigma(\hat{\theta}) - g\Delta(\theta)\ddot{g}\Delta(\hat{\theta}) + g\Sigma(\hat{\theta})\ddot{g}\Sigma(\hat{\theta}) + (\dot{g}\Sigma(\hat{\theta}))^{2} + g\Delta(\hat{\theta})\ddot{g}\Delta(\hat{\theta}) + (\dot{g}\Delta(\hat{\theta}))^{2} \tag{34}$$

Equation 34 was derived for the lower bound of the variance of $\hat{\theta}$ or the best possible case of the variance of $\hat{\theta}$. In this case, the best possible estimate is when $\hat{\theta} = \theta$. Using $\hat{\theta} = \theta$.

$$\mathrm{Var}\left[\hat{\theta}/\theta\right] \geq \left[\frac{2EA^{2}}{No}(-g\Sigma(\theta)\ddot{g}\Sigma(\theta) - g\Delta(\theta)\ddot{g}\Delta(\theta) + g\Sigma(\theta)\ddot{g}\Sigma(\theta) + \dot{g}\Sigma^{2}(\theta) + g\Delta(\theta)\ddot{g}\Delta(\theta) + \dot{g}\Delta^{2}(\theta))\right]^{-1}$$

$$\geq \left[\frac{2EA^2}{No} \left(\dot{g}\Sigma^2(\theta) + \dot{g}\Delta^2(\theta)\right)\right]^{-1}$$

$$\geq \frac{No}{2EA^2 \left(\dot{g}\Sigma^2(\theta) + \dot{g}\Delta^2(\theta)\right)} \qquad (35)$$

Using equations 6 and 6a; need to find $g\Sigma^2(\theta)$ and $g\Delta^2(\theta)$

$$g\underset{\Delta}{\Sigma}(\theta) = \frac{\sin\left[koa \sin\left(\theta + \frac{d}{2}\right)\right]}{koa \sin\left(\theta + \frac{d}{2}\right)} \pm \frac{\sin\left[koa \sin\left(\theta - \frac{d}{2}\right)\right]}{koa \sin\left(\theta - \frac{d}{2}\right)}$$

$$= \frac{\sin\left[koa \sin\theta \cos\frac{d}{2} + koa \cos\theta \sin\frac{d}{2}\right]}{koa \sin\theta \cos\frac{d}{2} + koa \cos\theta \sin\frac{d}{2}} \pm$$

$$\frac{\sin\left[koa \sin\theta \cos\frac{d}{2} - koa \cos\theta \sin\frac{d}{2}\right]}{koa \sin\theta \cos\frac{d}{2} - koa \cos\theta \sin\frac{d}{2}}$$

Using equations 5 and 6

$koa = 43.0003097$ and $\frac{d}{2} = 1.2375^o$

The value of $(g^2\Sigma(\theta) + g^2\Delta(\theta))$ is evaluated in Annex A.

From Annex A; $\text{Var}\left[\theta/1.5\right] \geq \frac{No}{2E\,A^2\; 4.7276 \times 10^2}$

where $\frac{No}{2} = \sigma_\eta^2$ (variance of noise)

$A = 1$

$E = T = 2 \times 10^{-2}$

TABLE I. Computer runs conducted with a constant target at 1.5^{o} angle off boresight

	1st Pulse	50th Pulse
$\frac{No}{2}$	Var $\left[\theta/1.5^{o}\right]$	Var $\theta/1.5^{o}$
.01	1.0576×10^{-3}	2.1152×10^{-5}
.09	9.5186×10^{-3}	1.904×10^{-4}
.25	2.644×10^{-2}	5.288×10^{-4}
1	1.05×10^{-1}	2.115×10^{-3}
4	4.23×10^{-1}	8.46×10^{-2}
9	9.5186×10^{-1}	1.904×10^{-2}
25	2.644	5.288×10^{-2}

for 50th pulse;

$$V\left[\theta/1.5^{o}\right]_{\text{50th pulse}} = \frac{\frac{No}{2}}{50EA^{2} \times 4.7276 \times 10^{2}}$$

$$V\left[\theta/1.5^{o}\right]_{\text{Nth pulse}} \quad \frac{\frac{No}{2}}{N\ EA^{2} \times 4.7276 \times 10^{2}}$$

From equation 35 the lower bound of the variance of θ estimate is a function of the noise power, the amplitude of the incoming signal and the change in the two antenna functions.

In the computer model runs, the noise power and the amplitude of the incoming signal are held constant for each individual run but in the case of a moving target, the antenna functions will change with change in angle. This will cause the variance estimate to change also with each new angle.

Moving Target

All theory investigated so far has assumed a constant target. However, a tracking radar needs also to track a moving target. For the computer runs with a moving target, the target will assume to move from -1.9^{o} to 3^{o} in 50 pulses.

In the constant target model progressive averaging of the $\Delta\theta$ was done over the 50 pulses. As the assumed moving target is almost covering the whole radar's angular limits within 50 pulses, progressive averaging over the entire 50 pulses cannot be done. To determine how many pulses to average and what type of averaging is to be done is a control problem and is not within the scope of this thesis. Consequently, no averaging will be done and each estimate will be fed back to the estimate antenna functions to be used to correlate with the new pulse. The investigation of the moving target situation will be limited to investigating the radar's ability to track a moving target with no averaging of $\Delta\theta$.

Output of the tracking loop $\hat{\theta}$

The estimate $\hat{\theta}$ of the tracking loop was investigated in three methods for each of the target models.

For the constant target, the three methods were as follows:

a. For a given noise seed starting value the estimate θ mean, variance and (mean/variance) square was plotted for a run of 50 pulses. This was repeated for signal to noise values of 40db, 20db, 14 and 6db.

b. A criteria based on variance and (mean/variance) square was chosen by experiemnt to determine whether lock or no lock was achieved during the

50 pulse run.

c. The probability density function of $\hat{\theta}$ was plotted using a histogram approach (Reference 7).

For the moving target, the three methods were as follows:

a. For a given noise seed starting value, the estimate $\hat{\theta}$ was plotted for a run of 50 pulses.

b. The probability density function of $\hat{\theta}$ was plotted using a histogram approach.

c. A criteria based on mean square error of the estimate to the non-noise tracking value was chosen to determine lock or no lock was achieved during the 50 pulse run.

The density functions of the constant target and the moving target can be derived in many ways. Two such methods are the Histogram Approach (Reference 7) and the Gram Charlier Series Approximation (Reference 5). However, only the Histogram Approach approximation will be used.

<u>Histogram Approach (Reference 7)</u>

The limits of θ were chosen to be -3^{o} and 3^{o}. This line was then divided into 30 points or 0.2^{o} increments. The occurrences of θ estimate in each increment was then counted with the resultant number divided by the number of runs (50).

Therefore; $$p(\theta)_{\theta_i} = \frac{\sum_{i=1}^{N} OCC_i}{N}$$

θ_i = cell center point = -2.9, -2.7;;;2.9

N = 50

OCC_i = 0; $-0.1 \geq \theta - \theta_i \geq 0.1$

1; $-0.1 \leq \theta - \theta_i \leq 0.1$

mean

$$E\left[\hat{\theta}\right] = \sum_{i=1}^{30} p(\theta)_{\theta_i} \theta_i$$

Variance

$$Var\left[\hat{\theta}\right] = \sum_{i=1}^{30} p(\theta)_{\theta_i} {}^{2} \theta_i - (E\,\theta\,)^2$$

<u>Gram-Charlier Series (Reference 5)</u>

When many random variables are summed, the control limit theorem of statistics states that the probability density of the sum random variable approaches the gaussian distribution. The Gram-Charlier series is an orthogonal polynominal expansion whose first term is the gaussian density function. In this case, the second order of approximation was used. Accuracy of the density function is increased if higher orders are used.

$$p(\theta) = \frac{1}{\sigma} \sum_{i=0}^{\infty} C_i \; \phi^{(i)} \frac{\hat{\theta} - \bar{\hat{\theta}}}{\sigma} \qquad (34)$$

$$C_i = \frac{(-1)^i}{i} \int_{\infty}^{\infty} p(\theta) \; H_i \frac{\hat{\theta} - \bar{\hat{\theta}}}{\sigma} d\hat{\theta}$$

Where when evaluated

$C = 1$ first order

$C_1 = C_2 = 0$

$C_3 = -\frac{1}{3}\ {}_3$ second order

$_3$ is the 3rd central moment of $p(\hat{\theta})$ normalized by σ^3

$$_3 = \frac{1}{\sigma^3} \int_{\infty}^{\infty} (\hat{\theta} - \bar{\hat{\theta}})^3 \; p(\hat{\theta}) \; d\hat{\theta}$$

if Mn denotes the n the central moment of $p(\hat{\theta})$

$$Mn = \int_{\infty}^{\infty} \theta^n \, p(\theta) \; d\theta$$

then $$_3 = \frac{m_3 - 3\, m_2 m_1 + 2\, m_1^3}{\sigma^3}$$

and $$H_3\left(\frac{\hat{\theta} - \bar{\hat{\theta}}}{\sigma}\right) = \left(\frac{\hat{\theta} - \bar{\hat{\theta}}}{\sigma}\right)^3 - 3\left(\frac{\hat{\theta} - \hat{\theta}}{\sigma}\right)$$

and $$\overset{(i)}{\phi}\left(\frac{\hat{\theta} - \bar{\hat{\theta}}}{\sigma}\right) = \frac{(-1)^i}{2\pi}\, e^{-\frac{1}{2}\left(\frac{\hat{\theta} - \bar{\hat{\theta}}}{\sigma}\right)^2} H_i\left(\frac{\hat{\theta} - \bar{\hat{\theta}}}{\sigma}\right)$$

Therefore after substitution in equation 34

$$p(\theta) = \frac{1}{2\pi\,\sigma}\, e^{-\frac{1}{2}\left(\frac{\hat{\theta} - \bar{\hat{\theta}}}{\sigma}\right)^2} +$$

$$+ \frac{1}{6}\, \frac{(m_3 - 3m_2 m_1 + 2m_1^3)}{\sigma^3}\; \frac{1}{2\pi}\; e^{-\frac{1}{2}\left(\frac{\hat{\theta} - \bar{\hat{\theta}}}{\sigma}\right)^2}\left(\left(\frac{\hat{\theta} - \bar{\hat{\theta}}}{\sigma}\right)^3 - 3\left(\frac{\hat{\theta} - \bar{\hat{\theta}}}{\sigma}\right)\right) \qquad (35)$$

III DIGITAL COMPUTER PROGRAM

Digital Computer Programing was utilized using FORTRAN 5. The main flow Flow Chart is in Annex B. The basic tracking loop was implemented using subroutines called from Program Main. Variations in the output were then conducted within Program Main. Listings for the basic program are in Annex C.

Basic Tracking Loop

Ten subroutines were used to implement the basic tracking loop. The subroutine ERID was used as a mainline calling subroutine to the other subroutines.

Subroutine FNE

This subroutine implemented the basic tracking loop with calls to all the relevant subroutines except noise. The purpose is to create a ε to $\hat{\theta}$ curve. This is only one curve which was created and the scaling of A estimate was not done within this program. With each change in $\hat{\theta}$ estimate within the tracking loop A estimate changes. Therefore, the basic ε to $\hat{\theta}$ curve need only be created once with scaling by the current A each time a ε to $\hat{\theta}$ curve is required. The ε to θ curve was created by holding the input angle at 0° and varying the θ estimate from -2.8° to 2.8° in 0.01° steps. This created a set of ε voltages which will later be compared with the ε from the actual tracking loop to determine a θ estimate which would cause the system to have 0° off boresight.

Subroutine ERID

Subroutine ERID implemented the basic tracking loop with calls to subroutines XPAT and ERROR. The basic loop included noise as well as some defined input signal. The resultant error voltage ε was scaled by A estimate and then compared to the ε to θ curves. The resultant θ was then returned to Program Main as the new θ estimate to be used on the next pulse processing.

Subroutine XPAT

XPAT created the $X\Sigma$ and $X\Delta$ signals and then derived the voltage which was handed back to Subroutine ERID. XPAT called subroutines NOISE, SDPAT and ESTAN. The input to this subroutine is EST which is the current value of θ estimate.

Subroutine ERROR

The ε produced by XPAT was scaled by A estimate in ERID and then compared to the set of ε to $\hat{\theta}$ (curves created in FNE) in ERROR. The resultant θ estimate going back to ERID.

Subroutine NOISE

An IMSL routine GGNML is called from this subroutine. GGNML creates a gaussian vector with a given seed starting point. The output vector (R(I)) is gaussian where

R(I) = (Y(I)S + M

where Y(I) is gaussian zero mean

unit variance

S is chosen variance

M is chosen mean

Subroutines SDPAT, SINC, ESTAN and ESTDEG

These subroutines implement the antenna functions $g\Sigma(\theta)$ and $g\Delta(\theta)$.

Subroutine MEVAR

This subroutine was used to derive the mean and variance of all past θ estimates for any given run of 50 pulses.

IV RESULTS

The results of the computer program runs are divided into two sections; a constant target with 50 pulses are sampled and progressive averaging of the incremental error estimate is done for each pulse, and, a moving target with constant velocity where no averaging of the incremental error is done over a run of 50 pulses.

Constant Target

A plot of mean, variance and (mean/variance) square was plotted, (Figure D1, D2, D3, D4), for signal to noise ratios of 20db, 14db, 10db and 6db. These plots are 'once only' runs in that the same noise seed starting point was used and consequently are meant to be typical examples of the radar model tracking in various noise conditions. Each of these plots demonstrates the radar model's ability (variance of the estimate) to estimate the angle off boresight (1.5° off boresight), of the target for the various signal to noise conditions. The mean shows the progressive mean of each of the estimates over the 50 pulses, and the progressive variance of the estimates over the 50 pulses, and the progressive variance shows the relative deviation of each of the estimates for pulses 1 to 50. The (mean/variance) square plot shows the relative goodness of the mean and variance combination. Due to the averaging of the incremental estimate $\Delta\theta$ in the discriminator, then the variance should be expected to decrease for each pulse and asymptote to a particular value. On the other hand (mean/variance) square should increase for each pulse. Both of these conditions are for a locked system. If variance increases and (mean/variance) square decreases, then the system is considered to be unlocked. These criteria are good measures for lock or non-lock conditions when the system is working well above threshold or well into threshold. Around threshold the (mean/variance) square approaches 1 for the locked condition, however (mean/variance) square must still increase for a locked system.

From Table I a lower bound variance is possible of 2.644 for a signal to noise ratio of 6db. However, an accuracy specification for the system needs to be assumed. Assuming an accuracy of $\pm$ 0.5°, then the lower bound on the variance needs to 1. This equates to a signal to noise ratio of 10db.

From the plots of mean, variance and (mean/variance) square, the 20 and 14db runs (Figure D1, D2) were both lock situations but the 10db (Figure D3) and the 6db (Figure D4) resulted in non-lock. The 6db run broke lock on the fourth pulse and did not demonstrate any indications of regaining lock. The 10db (Figure D3) run did not break lock until the 15th pulse. Although the mean did track the target, the increasing variance indicated that if the system was allowed to continue for more than the 50 pulses, the mean would also break away from the tracking value. The variance tended to oscillate with the mean of the variance increasing. Also, the value of (mean/variance) square had started to decrease suddenly by the 35th pulse. Under the criteria of increasing variance and decreasing (mean/variance) square indicating a break lock condition, the 10db run is considered to have broken lock. However, within the 50 pulses the mean tracked the target and the variance increased slowly (1.5 over 50 pulses). This run is an indication of threshold.

The plots of probability of θ versus θ (Figure E1 through E24) display runs of 50 different noise seed starting points for signal to noise ratios of 40db, 20db, 14db and 6db. Each plot demonstrates the probability of θ versus θ at pulses of 1, 10, 20, 30, 40 and 50 for a particular signal to noise ratio. All of these plots were derived by the Histogram Approach (Reference 7) using a cell width of (.4°).

The 40db plots (Figure E1 to E6) show the relative frequency (out of 50 runs) of a particular θ estimate. Figure E1 shows that all 50 runs chose an θ estimate within 1.0° and 1.4° with a center or mean of 1.3° as all values are within one cell width the variance is equal to zero. On the tenth pulse (Figure E2), the mean has now moved to 1.56° and the variance is 8.4×10^{-3}. This indicates an increase in variance but the 20, 30, 40 and 50th pulses (Figure E3 to E6) show a decrease in variance until the 50th pulse (Figure E6) shows a mean of 1.5° and a variance of zero (all θ estimates of the individual runs within one cell). This agrees with the computed variance estimate for the 50th pulse of 2.1152×10^{-5}. This is most certainly within the .4° cell width.

For the 20db plots (Figure E7 to E12) tracking variance close to the computed variance estimate was obtained.

TABLE II. 20db Comparison of Computed and Actual Var $[\theta/\theta]$

Pulse	Mean	Var $[\theta/\theta]$ actual	Var $[\theta/\theta]$ Computed
1	1.308	.0911	0.1058
10	1.56	.0515	0.0158
20	1.568	.03937	0.00529
30	1.668	.0742	0.00353
40	1.604	.0516	3.77×10^{-3}
50	1.52	.0612	2.115×10^{-3}

The pulse 1 plot (Figure E7) indicated a variance very close to the computed value (difference of .0147). As the number of pulses increases (Figure E12) the actual variance does not decrease as quickly as the computed lower bound of the variance estimate. However, this difference can be partially accounted for in that the computed variance estimate is a discrete lower bound estimate where as the actual estimate is based on a cell size of 0.4°. The final mean 1.52 (Figure E12) and variance of 0.0612 gives a final θ estimate of 1.27° or 1.77° which is well within the tolerance of $1.5^{\circ} \pm 0.5^{\circ}$.

Again, the initial variance of the 14db plot was very close to the computed variance estimate of Table I.

TABLE III. 14db Comparison of Computed and Actual Var $[\theta/\theta]$

Pulse	Mean	Var $[\theta/\theta]$ Actual	Var $[\theta/\theta]$ Computed
1	1.432	.4554	.423
10	1.524	.238	.0423
20	1.668	.181	.0212
30	1.584	.344	.0141
40	1.56	.216	.0106
50	1.556	.26	.0085

generated numbers corresponded to an angle off boresight. Once ε was found to be within the limits (equating to $\theta \pm 0.01^{\circ}$) of a discriminator generated number, then a corresponding angle off boresight was chosen. This angle was then added to the original estimate of zero and this new estimate was then used as (θ) in the estimate antenna functions $g\Sigma(\hat{\theta})$ and $g\Delta(\hat{\theta})$. The process was now repeated, but with the resulting angle off boresight from the discriminator being averaged with the previous angles from the discriminator before being added to the previous or existing value of θ. This new θ was then fed back and used to update the estimate $\hat{\theta}$ used in the antenna functions $g\Sigma(\hat{\theta})$ and $g\Delta(\hat{\theta})$.

The digital computer implementation will now process a number of returns from a target and produce an estimate value of θ, (averaged over the number of pulses processed), for the angle the target is off boresight. This angle of θ can now be used to update the angle θ in the antenna functions $g\Sigma(\theta)$ and $g\Delta(\theta)$. In a real system, a servo motor (as opposed to phased array) is usually used to steer the antenna. The effect of the servo is to average all incoming estimates with a turning to the resultant mean.

<u>Statistics of θ</u>

The problem is now to determine how many pulses to process, (mean of $\hat{\theta} = \theta$) and to what degree of certainty (variance of θ), that $\hat{\theta} = \theta$.

$\hat{\theta}$ is derived via the discriminator from ε. Under noise conditions, the mean of ε is equal to zero only when $\hat{\theta} = \theta$. As ε and $\hat{\theta}$ is a one to one mapping, then the mean of $\hat{\theta}$ in the limit as the number of pulses processed becomes large (infinity), is the angle off

As the number of pulses increase, the variance decreases from pulse 1 to pulse 10 but then oscillates around an average value of .28. This is a very good indication of threshold where the variance is not decreasing significantly but also is not increasing. Plots for pulses 1, 10, 20, 30, 40, and 50 (Figure E13 to E18) show the mean unchanged. The final mean 1.556 and variance of .26 produce an estimate θ , 1.556 $\pm$ .509 which is just outside the assumed limit of 1.5 $\pm$ .5^{o} (2.06^{o} and 1.04^{o}).

As can be expected, the 6db plot (Figure E19 to E24) demonstrated a system operating beyond threshold.

<u>TABLE IV</u>. 6db Comparison of Computed and Actual Var $[\theta/\theta]$

Pulse	Mean	Var $[\theta/\theta]$ Actual	Var $[\theta/\theta]$ Computed
1	1.46	1.44	2.644
10	1.396	1.25	.2644
20	1.656	1.71	.1322
30	1.564	1.26	.088
40	1.504	1.64	.066
50	1.632	.97	.05288

The system still demonstrated a mean tracking of the target. From Figures E19 to E24, the system displayed a correlation at 3^{o}. This was caused by a routine within the model which limits the range of the system to $\pm 3^{o}$. As the variance of this system was large (average of 1.38) each individual pulse numbers standard deviation took the estimate to the outer cell. Consequently, all estimates which exceeded this cell were lumped into its histogram.

Probability of Break Lock

To determine a probability of break lock, an assumption needs to be made as to exactly what constitutes a break lock. As has been shown, the system can still track the target but have out of limit estimates. This will only occur when the out of limit estimates do not exceed the discrimination limits of $\pm 2.8^{\circ}$. If the system tracks a target but the estimate is out of limit, then the system is not operating as specified, and, consequently will be considered to have broken lock.

From Tchebycheff inequality (Reference 9);

$$p\left(\theta - \eta\right) > k\sigma \leq \frac{1}{k^2}$$

assuming as before $k = 0.5^{\circ}$ $\quad \eta = E$

$$\sigma^2 = \text{Var} \quad \theta/\theta$$

Table V gives a comparison of the probability that the system will be within tolerance ($\pm 0.5^{\circ}$) at the 50th pulse. Table VI gives a comparison of the probability of break lock of the system at the 50th pulse.

$$p\left(\eta - \varepsilon < \theta < \eta + \varepsilon\right) \geq \frac{\sigma^2}{\varepsilon^2}$$

TABLE V. Comparison of Probability of Lock

$\frac{S}{N}$	η	σ^2	$\eta - .5 < \theta < \eta + .5 \geq$
40db	1.5	2.1152×10^{-5}	1
20db	1.52	0.0612	.98
14db	1.556	.26	.73
6db	1.632	.97	0

TAVLE VI. Comparison of Probability of Break Lock

$\frac{S}{N}$	η	Break Lock Probability
40db	1.5	0
20db	1.52	.02
14db	1.556	.27
6db	1.632	1

Moving Target

Plots of the estimate (Figure F1 to F4) over a run of 50 pulses for signal to noise conditions of no noise, 40db, 14db and 6db. These plots were made to give an indication of the tracking ability of the system for a moving target (-1.9° to 3.0° over 50 pulses) and no averaging from pulse to pulse. As was seen in the constant target case, the system should track the no noise value of the estimate on the average but in the moving target the variance of the estimate cannot be expected to decrease with the number of pulses (no averaging). In the above cases, the radar tracked the target but with varying degrees of accuracy. This accuracy will be determined from the probability of $\hat{\theta}$ estimate versus $\hat{\theta}$ plots.

The 40db plot (Figure G1 to G6) shows the mean changing from -1.1° to 2.9°. The maximum variance for any of the 50 pulses was 0.009984 on pulse 27. This equates to an accuracy of 0.1° at a mean value of 0.796°. The angle for the non-noise case at pulse 27 is 0.7. This gives an error at most of 0.196° which is well within the assumed accuracy criteria of $\pm 0.5^{\circ}$.

The 14db plots (Figure G7 to G12) show the mean changing from -1.196 to 2.752. The maximum variance for any of the 50 pulses was 0.374 on pulse 30. This equates to an accuracy of $\pm$.61 at a mean value of 1.028°. The angle for the non-noise case at pulse 30 is 1°. This gives an error at most of $.89^{\circ}$ which is outside the assumed accuracy criteria of $\pm 0.5^{\circ}$.

The 6db plots (Figure G13 to G18) show the density function as a relatively uniform function (variance greater than one). The mean of the estimate did track the target with indications of -1.092° to 2.416°, but with a maximum variance of 2.243 on pulse 17. This equates to an accuracy of $\pm 1.498^{\circ}$ at a mean value of $-.368^{\circ}$. The angle for the non-noise case at pulse 17 is 0.3°. This gives an error at most of -2.178° which is well outside the assumed accuracy criteria of $\pm 0.5^{\circ}$. Again, correlation occurred at the 3° cell. This causes an artificially smaller variance at these angles.

General

In all cases of computing accuracy, the control problem of the system has been overlooked. This problem is when to turn the antenna (how many pulses) and on what value to turn the antenna (estimate on some form of mean or average). As long as the system is in a lock condition, the threshold could be extended by using an average or progressive mean in the constant case. In the tracking case, this problem is outside the scope of this thesis.

The lock no lock conditions in general can be determined from the relative scope or change of the variance or (mean/variance) square. If the variance decreases and the (mean/variance) square increases, then the system is locked. For the moving target tracking problem, no relationship can be determined for the variance as the number of pulses increases. However, the lock, non-lock conditions can be determined by examining the mean square error of the estimate in relation to the non-noise tracking value. If this exceeds the specified limits, then the system is either operating under too low a signal to noise ratio (in our case, threshold is approximately 20db) or the system has broken lock and is no longer tracking. In this example, the specified mean square error limit was 0.25 ($\pm 0.5^2$).

V CONCLUSIONS AND RECOMMENDATIONS

Theory Conclusions

The estimation system is to be implemented to model the Amplitude Comparison Monopulse Radar is as follows;

$$g\Delta(\hat{\theta}) \int_0^T X\Sigma(t)S(t)dt - g\Sigma(\hat{\theta}) \int_0^T X\Delta(t)S(t)dt = \varepsilon$$

Where $\varepsilon = 0$ when $\hat{\theta} = \theta$

The amplitude estimate A to be implemented so as to estimate the amplitude of the incoming signal is as follows;

$$\hat{A} = \frac{(g\Sigma(\theta)X\Sigma + g\Delta(\theta)X\Delta)}{E\ (g\Sigma^2(\theta) + g\Delta^2(\theta))} \qquad \text{where; } \overline{\hat{A}} = A$$

$$\mathrm{Var}\left[\hat{A}/A\right] = \frac{No}{2}\left[\frac{1}{g\Sigma^2(\theta) + g\Delta^2(\theta)}\right]$$

This A estimate was found to inversely vary the ε to θ curves of the discriminator.

The antenna functions to be implemented were found to be as follows;

$$g\Sigma(\theta) = \frac{\sin\left[koa \sin(\theta + \frac{\varepsilon}{2})\right]}{koa \sin(\theta + \frac{\varepsilon}{2})} + \frac{\sin\left[koa \sin(\theta - \frac{\varepsilon}{2})\right]}{koa \sin(\theta - \frac{\varepsilon}{2})}$$

$$g\Delta(\theta) = \frac{\sin\left[koa \sin(\theta + \frac{\varepsilon}{2})\right]}{koa \sin(\theta + \frac{\varepsilon}{2})} - \frac{\sin\left[koa \sin(\theta - \frac{\varepsilon}{2})\right]}{koa \sin(\theta - \frac{\varepsilon}{2})}$$

Where koa = 43.0003097

$$\frac{\varepsilon}{2} = 1.2375^\circ$$

The $\Delta\hat{\theta}$ estimate output of the discriminator can be progressively averaged for a constant target. Averaging other than progressive averaging over the entire 50 pulses needs to be done for a moving target.

The lower bound of the variance estimate for the $\hat{\theta}$ estimate of a constant target is as follows;

$$\text{Var}\left[\hat{\theta}/\theta\right] \geq \frac{No}{2EA^2(\dot{g}\Sigma^2(\theta) + \dot{g}\Delta^2(\theta)}$$

Results Conclusions

For the constant target with progressive averaging of $\Delta\hat{\theta}$ over the 50 pulses, the threshold was found to be determined by examining the lock condition as well as if the $\hat{\theta}$ estimate has exceeded the specified accuracy limits. In this example, threshold was found to be 14db. The lock, non-lock criteria for this system is, a 'lock' is determined if the variance decreases and the (mean/variance) square increases over the 50 pulses.

For the moving target with no averaging of $\Delta\hat{\theta}$ over the 50 pulses, the threshold was determined by examining each individual mean square error of the estimate in relation to the non-noise tracking value. If this exceeds the specified limits, then the system is either operating under too low a signal to noise ratio (threshold) or the system has broken lock.

Recommendations

As this study was limited in scope on several areas, the following areas are recommended for future research;

a. Investigate the effects of varying the incoming signal amplitude A on the threshold and break lock frequency. The use of A in the discrimination curves suggests that a saturation level of A will cause poor tracking.

b. Reproduce the plots of probability density of $\hat{\theta}$ versus $\hat{\theta}$ using the Gram-Charlier Series approximation. This should produce a more accurate determination of threshold than the Histogram Approach approximation.

c. Extend the system to two dimensions and investigate the effect of coupling from one dimension to the other.

d. Investigate the control problem of when to steer the antenna. The type and amount of averaging needs to be determined.

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APPENDIX A

Evaluation of $(g\Sigma^2(\theta) + g\Delta^2(\theta))$

$$g\underset{\Delta}{\Sigma}(\theta) = \frac{\sin\left(42.993 \sin\theta + .92873 \cos\theta\right)}{42.993 \sin\theta + .92873 \cos\theta} \pm$$

$$\frac{\sin\left(42.993 \sin\theta - .92873 \cos\theta\right)}{42.993 \sin\theta - .92873 \cos\theta}$$

$$= \frac{\sin\left((42.993 \sin\theta) \cos(.92873 \cos\theta)\right)}{42.993 \sin\theta + .92873 \cos\theta} +$$

$$\frac{\cos\left((42.993 \sin\theta) \sin(.92873 \cos\theta)\right)}{42.993 \sin\theta + .92873 \cos\theta} \pm$$

$$\frac{\sin\left((42.993 \sin\theta) \cos(.92873 \cos\theta)\right)}{42.993 \sin\theta - .92873 \cos\theta} \mp$$

$$\frac{\cos\left((42.993 \sin\theta) \sin(.92873 \cos\theta)\right)}{42.993 \sin\theta - .92873 \cos\theta}$$

The values of $g\Sigma(\theta)$ and $g\Delta(\theta)$ depend on the value of θ. In the computer runs using the model with a constant target, an angle off boresight of 1.5° was used. To enable the lower bound of Var $\theta/\theta = 1.5^\circ$ to be determined, the four terms of $g\Sigma(\theta)$ and $g\Delta(\theta)$ need to be evaluated at $= 1.5^\circ$.

1st term	=	8.553×10^{-3}	+ 9.012	- 5.506	=	3.515
2nd term	=	-3.055×10^{-3}	- 15.12	- 3.5	=	-18.636
3rd term	=	8.916×10^{-2}	+ 56.302	- 599.00	=	-542.6
4th term	=	3.185×10^{-2}	+ 157.626	+ 382.14	=	+539.79
				$g\Sigma^2(1.5^\circ)$	=	321.2
				1st term	=	3.515
				2nd term	=	-18.636

$$\text{3rd term} = +542.6$$

$$\text{4th term} = -539.79$$

$$g\Delta^2(1.5^\circ) = 151.56$$

Using $g\Sigma(1.5^\circ)$ and $g\Delta(1.5^\circ)$

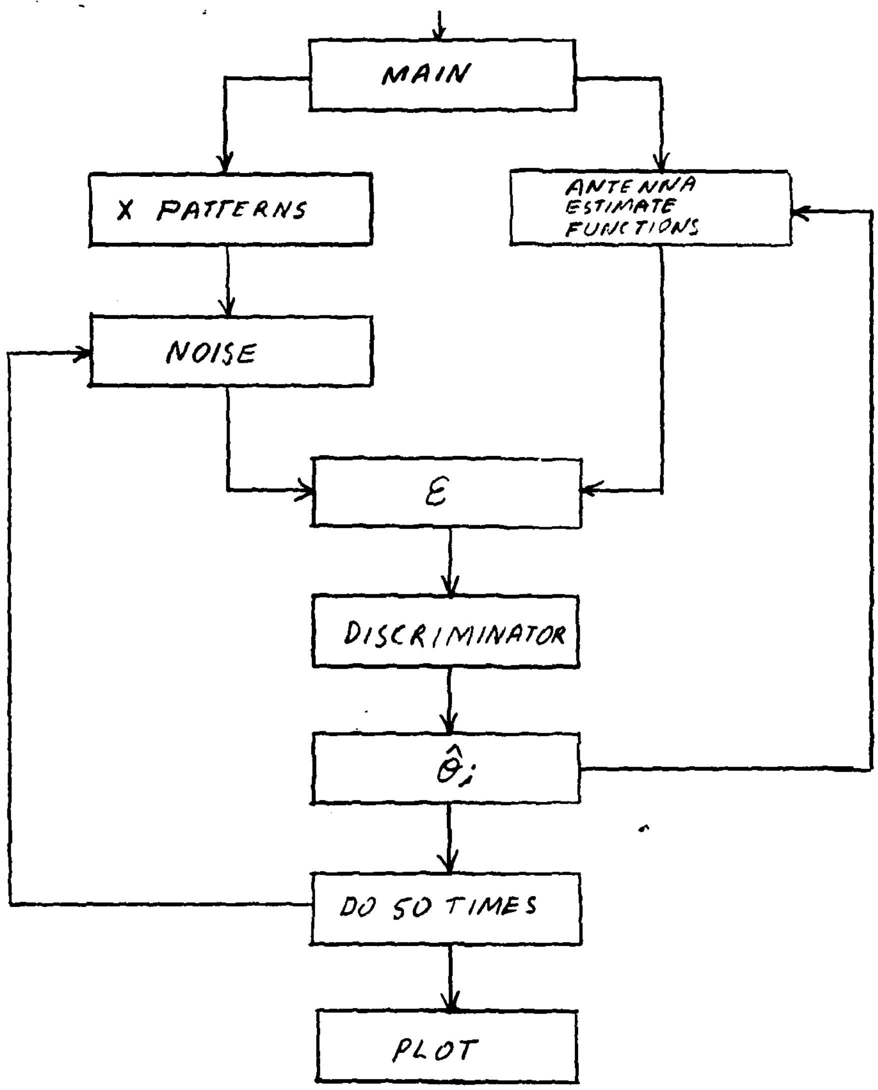
$$(g\Sigma^2(1.5^\circ) + g\Delta^2(1.5^\circ)) = 472.76$$

From equation 35

$$\text{Var}\left[\hat{\theta}/1.5^\circ\right] \geq \frac{No}{2EA^2 \;\; 4.7276 \times 10^2}$$

APPENDIX B

Program Flow Diagram



APPENDIX C

Program Listing

```
C          ********************************************
C          *                                          *
C          *    PROGRAM CONDEN                        *
C          *                                          *
C          *    THIS PROGRAM PLOTS THE PROBALITY      *
C          *    DENSITY FUNCTION OF THE ANGLE         *
C          *    OFF BORESIGHT  FOR A CONSTANT TARGET  *
C          *                                          *
C          ********************************************
      PROGRAM MAIN
C           THIS PROGRAM SIMULATES THE SUFFICIENT STASTIC
      INTEGER NR,NA,NB,NC
      REAL ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2,PI,PHI
     1  ,DEG,BETA,AMEAN,BMEAN,VAR1,VAR2,SNC,STF,OCC(50,30),SORT(33)
     2  ,EST,VNULL,SUM1,SUM2,EST1,EST2,A,MBOX(561),MHU(53)
     3       ,ETA(561),POS(561),BOX(561),DUMM(53),ALPHA(53)
     4  ,R(100),X1,X2,DMEAN(53),BVAR(53),X(33),Y(33),ORG
     5  ,EMEAN,CVAR
     6  ,SNR
      DOUBLEPRECISION DSEED
      DATA ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2,EST,PI
     1  ,PHI,VNULL,SUM1,SUM2,EST1,EST2,DEG,BETA,AMEAN,BMEAN,VAR1,VAR2
     2  ,SNC,STF,X1,X2,A,ORG
     3  /0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0/
      CALLPLOTS(0.,0.,9)
      CALL PLOT(0.,-11.,-3)
      CALL PLOT(2.,5.5,-3)
      NA=561
      NB=53
      NC=100
      DO 100 I=1,100
      R(I)=0.0
100   CONTINUE
      DO 108 I=1,50
      DO 105 J=1,30
      OCC(I,J)=0.0
105   CONTINUE
108   CONTINUE
      DO 110 I=1,53
      ALPHA(I)=0.0
      DUMM(I)=0.0
      DMEAN(I)=0.0
```

```
      BVAR(I)=0.0
      MHU(I)=0.0
110   CONTINUE
      DO 115 I=1,33
      SORT(I)=0.0
      Y(I)=0.0
      Y(I)=0.0
115   CONTINUE
      DO 120 I=1,561
      ETA(I)=0.0
      POS(I)=0.0
      BOX(I)=0.0
      MBOX(I)=0.0
120   CONTINUE
      DSEED=124457.D0
      PI=3.141592654
      CALL FNE(PI,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1 ,SUM1,SUM2,EST1,EST2,ETA,POS,MBOX,NA)
      DO 130 K=1,50
      EST=0.0
      BETA=0.0
      STF=1.0
      SNR=(LOG10(100.0/STF**2.0))*10.0
      DSEED=DSEED+FLDAT(K)
      DO 125 I=-30,30,2
      SORT((I+32)/2)=FLOAT(I)/10.0
125   CONTINUE
      DO 128 I=1,30
      X(I)=SORT(I)
      SORT(I)=(SORT(I+1)-SORT(I))/2.0+SORT(I)
128   CONTINUE
      CALL ERID(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1 ,R,X1,X2,DSEED,EST,VNULL,PI,POS,BOX,PHI,ALPHA,AMEAN,BMEAN
     2 ,VAR1,VAR2,DUMM,SNC,STF,NA,NB,NC,BETA,DMEAN,BVAR,MBOX,MHU)
      DO 440 I=1,50
      IF (DUMM(I) .GE. SORT(30)) THEN
      OCC(I,30)=OCC(I,30)+1.0
      GOTO 430
      ENDIF
      DO 420 J=1,30
      IF (DUMM(I) .LE. SORT(J)) THEN
      OCC(I,J)=OCC(I,J)+1.0
      GOTO 430
      ENDIF
420   CONTINUE
430   CONTINUE
440   CONTINUE
130   CONTINUE
      DO 480 L=1,50
      EMEAN=0.0
      CVAR=0.0
      DO 470 M=1,30
      EMEAN=OCC(L,M)/50.0*SORT(M)+EMEAN
      CVAR=OCC(L,M)/50.0*(SORT(M)**2.0)+CVAR
470   CONTINUE
      CVAR=CVAR-EMEAN**2.0
```

```
      PRINT'(//T5,A,T28,F5.2,T33,A///T13,A,T29,I2//
     1 T15,A,F10.2//T11,A,F10.2///T2,A/
     2 T9,A/T9,A,T35,F5.2,T41,A/T9,A,T21,I2///)',
     3 'SIGNAL TO NOISE RATIO =',SNR,'DB','AT PULSE',
     4 (L),'MEAN =',EMEAN,'VARIANCE =',CVAR,
     5 'FIG    PLOT OF THE PROBALITY DENSITY FUNCTION',
     6 'OF THETA ESTIMATE VERS THETA',
     7 'FOR SIGNAL TO NOISE RATIO OF',SNR,'DB',
     8 'AND AT PULSE',(L)
      X(31)=-3.
      Y(31)=0.
      X(32)=6./6.
      Y(32)=1./4.
      DO 455 J=1,30
      Y(J)=OCC(1,J)/50.0
455   CONTINUE
      CALL AXIS(0.,0.,'THETA',-5,6.,0.,X(31),X(32))
      CALL AXIS(3.,0.,'P(THETA)',+8,4.,90.,Y(31),Y(32))
      CALL PLOT(0.,0.,3)
      CALL LINE(X,Y,30,1,0,0)
      CALL PLOT(0.,0.,3)
      CALL PLOT(0.,-11.,-3)
      CALL PLOT(8.0,5.5,-3)
      DO 460 I=10,50,10
      DO 450 J=1,30
      Y(J)=OCC(I,J)/50.0
450   CONTINUE
      CALL AXIS(0.,0.,'THETA',-5,6.,0.,X(31),X(32))
      CALL AXIS(3.,0.,'P(THETA)',+8,4.,90.,Y(31),Y(32))
      CALL PLOT(0.,0.,3)
      CALL LINE(X,Y,30,1,0,0)
      CALL PLOT(0.,0.,3)
      CALL PLOT(0.,-11.,-3)
      CALL PLOT(8.0,5.5,-3)
460   CONTINUE
      CALL PLOTE(N)
480   CONTINUE
      END
      SUBROUTINE ERID(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1 ,R,X1,X2,DSEED,EST,VNULL,PI,POS,BOX,PHI,ALPHA,AMEAN,BMEAN
     2 ,VAR1,VAR2,DUMM,SNC,STF,NA,NB,NC,BETA,DMEAN,BVAR,MBOX,MHU)
      REAL POS(NA),BOX(NA),DUMM(NB),ALPHA(NB),MHU(NB)
     1 ,R(NC),DMEAN(NB),BVAR(NB),MBOX(NA)
      CALL XPAT(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1 ,R,X1,X2,DSEED,EST,VNULL,PI,NC,STF,A)
      DO 135 I=1,560
      BOX(I)=MBOX(I)*((GPAT1**2+GPAT2**2)*2.0E-2)/(GPAT1*X1+GPAT2*X2)
135   CONTINUE
      CALL ERROR(POS,BOX,PHI,VNULL,NA)
      ANGLE=PHI
      CALL ESTAN(ANGLE,PI)
      ALPHA(2)=ANGLE
      EST=ANGLE
      DEG=EST
      CALL ANDEG(DEG,PI)
```

```
      DUMM(1)=DEG
      DO 140 I=1,49
      CALL XPAT(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1 ,R,X1,X2,DSEED,EST,VNULL,PI,NC,STF,A)
      DO 130 N=1,560
      BOX(N)=MBOX(N)*((GPAT1**2+GPAT2**2)*2.0E-2)/(GPAT1*X1+GPAT2*X2)
130   CONTINUE
      CALL ERROR(POS,BOX,PHI,VNULL,NA)
      ANGLE=PHI
      CALL ESTAN(ANGLE,PI)
      ALPHA(I+2)=ANGLE
      BETA=BETA+ALPHA(I+2)
      EST=BETA/I+EST
      DEG=EST
      CALL ANDEG(DEG,PI)
      DUMM(I+1)=DEG
      SNC=I+1
      CALL MEVAR(ALPHA,AMEAN,BMEAN,VAR1,VAR2,SNC,DUMM,NB)
      DMEAN(I+1)=BMEAN
      BVAR(I+1)=VAR2
      MHU(I+1)=BMEAN**2/VAR2**2
140   CONTINUE
      RETURN
      END
      SUBROUTINE FNE(PI,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1 ,SUM1,SUM2,EST1,EST2,ETA,POS,MBOX,NA)
C        THIS PROGRAM ESTIMATES THE PHASE ERROR
      REAL ETA(NA),POS(NA),MBOX(NA)
      ANGLE=0.0
      CALL ESTAN(ANGLE,PI)
      CALL SDPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
      SUM1=GPAT1
      SUM2=GPAT2
      DO 220 I=-280,280
      ANGLE=FLOAT(I)/100.0
      POS(I+281)=I
      CALL ESTAN(ANGLE,PI)
      CALL SDPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
      EST1=GPAT1
      EST2=GPAT2
      ETA(I+281)=(SUM2*EST1-SUM1*EST2)*2.0E-2
220   CONTINUE
      DO 230 I=1,560
      MBOX(I)=(ETA(I+1)-ETA(I))/2.0+ETA(I)
230   CONTINUE
      DO 240 I=1,561
      POS(I)=POS(I)/100.0
240   CONTINUE
      RETURN
      END
      SUBROUTINE ESTAN(ANGLE,PI)
C        THIS PROGRAM CONVERTS DEG TO RAD
      ANGLE=ANGLE*PI/180
      RETURN
      END
      SUBROUTINE SDPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
C           THIS PROGRAM CALCULATES THE SUM AND DIFFERENCE AE PATTERNS
```

```
      THETA=ANGLE+.0215984495
      CALL  PATT(THETA,GPAT)
      GPATA=GPAT
      THETA=ANGLE-.0215984495
      CALL PATT(THETA,GPAT)
      GPATB=GPAT
      GPAT1=GPATA+GPATB
      GPAT2=GPATA-GPATB
      RETURN
      END
      SUBROUTINE PATT(THETA,GPAT)
      IF (THETA .EQ. 0) THEN
      GPAT=1.0
      ELSE
      CALL SINC(THETA,GPAT)
      ENDIF
      RETURN
      END
      SUBROUTINE SINC(THETA,GPAT)
C           THIS PROGRAM FINDS GPAT
      GPAT=SIN(43.003097*SIN(THETA))/(43.003097*SIN(THETA))
      RETURN
      END
      SUBROUTINE XPAT(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1  ,R,X1,X2,DSEED,EST,VNULL,PI,NC,STF,A)
C           THIS PROGRAM CALCULATES THE X1 , X2 SIGNALS
      REAL R(NC),STF
      A=1.0
      ANGLE=1.5
      CALL ESTAN(ANGLE,PI)
      CALL SDPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
      CALL NOISE(NR,R,DSEED)
      X1=0.0
      DO 150 I=1,100
      X1=((R(I)*STF+GPAT1)*A*2.0E-4)+X1
150   CONTINUE
      CALL NOISE(NR,R,DSEED)
      X2=0.0
      DO 160 I=1,100
      X2=((R(I)*STF+GPAT2)*A*2.0E-4)+X2
160   CONTINUE
      ANGLE=EST
      CALL SDPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
      VNULL=X1*GPAT2-X2*GPAT1
      RETURN
      END
      SUBROUTINE NOISE(NR,R,DSEED)
C           THIS PROGRAM GENERATES A NOISE VECTOR
      REAL R(100)
      DOUBLE PRECISION DSEED
      NR=100
      CALL GGNML (DSEED,NR,R)
      RETURN
      END
      SUBROUTINE ERROR(POS,BOX,PHI,VNULL,NA)
      REAL POS(NA),BOX(NA)
```

```
      IF (VNULL .GE. BOX(560)) THEN
      PHI=POS(561)
      GOTO 260
      ENDIF
      DO 250 I=1,560
      IF (VNULL .LE. BOX(I)) THEN
      PHI=POS(I)
      GOTO 260
      ENDIF
250   CONTINUE
260   CONTINUE
      RETURN
      END
      SUBROUTINE ANDEG(DEG,PI)
C          THIS PROGRAM CONVERTS RADIANS TO DEGREES
      DEG=DEG*180/PI
      RETURN
      END
      SUBROUTINE MEVAR(ALPHA,AMEAN,BMEAN,VAR1,VAR2,SNC,DUMM,NB)
      REAL ALPHA(NB),DUMM(NB)
      AMEAN=0.0
      BMEAN=0.0
      VAR1=0.0
      VAR2=0.0
      DO 135 I=1,SNC
      AMEAN=ALPHA(I+2)+AMEAN
      VAR1= ALPHA(I+2)**2+VAR1
      BMEAN=DUMM(I)+BMEAN
      VAR2=DUMM(I)**2+VAR2
135   CONTINUE
      AMEAN=AMEAN/SNC
      VAR1=VAR1/SNC-AMEAN**2
      BMEAN=BMEAN/SNC
      VAR2=VAR2/SNC-BMEAN**2
      RETURN
      END
```

```
C          *****************************************
C          *                                       *
C          *     PROGRAM VELMV                     *
C          *                                       *
C          *     THIS PROGRAM PLOTS THE ESTIMATE   *
C          *     ANGLE OFF BORESIGHT FOT A         *
C          *     MOVING TARGET                     *
C          *                                       *
C          *****************************************
      PROGRAM MAIN
C            THIS PROGRAM SIMULATES THE SUFFICIENT STASTIC
      INTEGER NR,NA,NB,NC
      REAL ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2,PI,PHI
     1  ,DEG,BETA,AMEAN,BMEAN,VAR1,VAR2,SNC,STF,VEL
     2  ,EST,VNULL,SUM1,SUM2,EST1,EST2,A,MBOX(561),MHU(53)
     3       ,ETA(561),POS(561),BOX(561),DUMM(53),ALPHA(53)
     4  ,R(100),X1,X2,DMEAN(53),BVAR(53),X(53),Y(53)
     5  ,SNR
      DOUBLEPRECISION DSEED
      DATA ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2,EST,PI
     1  ,PHI,VNULL,SUM1,SUM2,EST1,EST2,DEG,BETA,AMEAN,BMEAN,VAR1,VAR2
     2  ,SNC,STF,X1,X2,A,VEL
     3  /0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0/
      CALLPLOTS(0.,0.,9)
      CALL PLOT(0.,-11.,-3)
      CALL PLOT(2.,7.5,-3)
      NA=561
      NB=53
      NC=100
      DO 100 I=1,100
      R(I)=0.0
100   CONTINUE
      DO 110 I=1,53
      ALPHA(I)=0.0
      DUMM(I)=0.0
      DMEAN(I)=0.0
      BVAR(I)=0.0
      MHU(I)=0.0
      X(I)=0.0
      X(I)=0.0
110   CONTINUE
      DO 120 I=1,561
      ETA(I)=0.0
      POS(I)=0.0
      BOX(I)=0.0
      MBOX(I)=0.0
120   CONTINUE
      DSEED=124457.D0
      PI=3.141592654
      CALL FNE(PI,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1  ,SUM1,SUM2,EST1,EST2,ETA,POS,MBOX,NA)
      DO 130 I=1,5
      EST=0.0
      BETA=0.0
      STF=I
      SNR=(LOG10(100.0/STF**2.0))*10.0
```

```
      CALL ERID(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2,VEL
     1  ,R,X1,X2,DSEED,EST,VNULL,PI,POS,BOX,PHI,ALPHA,AMEAN,BMEAN
     2  ,VAR1,VAR2,DUMM,SNC,STF,NA,NB,NC,BETA,DMEAN,BVAR,X,MBOX,MHU)
      PRINT'(//T5,A,T28,F5.2,T33,A///T13,A,T35,I1,T44,I2,T54,I2//
     1  T14,A,F10.2,F10.2,F10.2///T2,A/T9,A,T17,F5.2,T22,A///)'
     3  ,'SIGNAL TO NOISE RATIO =',SNR,'DB','AT (I) PULSES',1,25,50,
     4  'ESTIMATE - =',DUMM(1),DUMM(25),DUMM(50),
     5  'FIG    PLOT OF ESTIMATE FOR SIGNAL TO NOISE'
     6  ,'RATIO OF',SNR,'DB AND A MOVING TARGET.'
 130  CONTINUE
      END
      SUBROUTINE ERID(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2,VEL
     1  ,R,X1,X2,DSEED,EST,VNULL,PI,POS,BOX,PHI,ALPHA,AMEAN,BMEAN
     2  ,VAR1,VAR2,DUMM,SNC,STF,NA,NB,NC,BETA,DMEAN,BVAR,X,MBOX,MHU)
      REAL POS(NA),BOX(NA),DUMM(NB),ALPHA(NB),MHU(NB)
     1  ,R(NC),DMEAN(NB),BVAR(NB),X(NB),MBOX(NA)
      VEL=1.0
      CALL XPAT(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1  ,R,X1,X2,DSEED,EST,VNULL,PI,NC,STF,A,VEL)
      DO 135 I=1,560
      BOX(I)=MBOX(I)*((GPAT1**2+GPAT2**2)*2.0E-2)/(GPAT1*X1+GPAT2*X2)
 135  CONTINUE
      CALL ERROR(POS,BOX,PHI,VNULL,NA)
      ANGLE=PHI
      CALL ESTAN(ANGLE,PI)
      ALPHA(2)=ANGLE
      EST=ANGLE
      DEG=EST
      CALL ANDEG(DEG,PI)
      DUMM(1)=DEG
      DO 140 I=1,49
      VEL=FLOAT(I)+1.0
      CALL XPAT(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1  ,R,X1,X2,DSEED,EST,VNULL,PI,NC,STF,A,VEL)
      DO 138 N=1,560
      BOX(N)=MBOX(N)*((GPAT1**2+GPAT2**2)*2.0E-2)/(GPAT1*X1+GPAT2*X2)
 138  CONTINUE
      CALL ERROR(POS,BOX,PHI,VNULL,NA)
      ANGLE=PHI
      CALL ESTAN(ANGLE,PI)
      ALPHA(I+2)=ANGLE
      EST=EST+ANGLE
      DEG=EST
      CALL ANDEG(DEG,PI)
      DUMM(I+1)=DEG
      SNC=I+1
      CALL MEVAR(ALPHA,AMEAN,BMEAN,VAR1,VAR2,SNC,DUMM,NB)
      DMEAN(I+1)=BMEAN
      BVAR(I+1)=VAR2
      MHU(I+1)=BMEAN**2/VAR2**2
      X(I+1)=I+1
      CALL LDO(Y,DUMM,NB)
      X(51)=0.
      Y(51)=-3.
      X(52)=50./5.
      Y(52)=6./4.
      CALL AXIS(0.,0.,'NUMBER',-6,5.,0.,X(51),X(52))
```

```
      CALL AXIS(0.,-2.,'MEAN VAR X',+10,4.,90.,Y(51),Y(52))
      CALL PLOT(0.,0.,3)
      CALL AXIS(5.,-2.,'MEANSQ/VAR',-10,4.,90.,Y(51),Y(52))
      CALL PLOT(0.,0.,3)
      CALL PLOT(0.,-2.,-3)
      CALL LINE(X,Y,50,1,0,0)
      CALL PLOT(0.,0.,3)
      CALL PLOT(10.0,-11.,-3)
      CALL PLOT(0.,7.5,-3)
140   CONTINUE
      CALL PLOTE(N)
      RETURN
      END
      SUBROUTINE FNE(PI,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1  ,SUM1,SUM2,EST1,EST2,ETA,POS,MBOX,NA)
C        THIS PROGRAM ESTIMATES THE PHASE ERROR
      REAL ETA(NA),POS(NA),MBOX(NA)
      ANGLE=0.0
      CALL ESTAN(ANGLE,PI)
      CALL SDPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
      SUM1=GPAT1
      SUM2=GPAT2
      DO 220 I=-280,280
      ANGLE=FLOAT(I)/100.0
      POS(I+281)=I
      CALL ESTAN(ANGLE,PI)
      CALL SDPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
      EST1=GPAT1
      EST2=GPAT2
      ETA(I+281)=(SUM2*EST1-SUM1*EST2)*2.0E-2
220   CONTINUE
      DO 230 I=1,560
      MBOX(I)=(ETA(I+1)-ETA(I))/2.0+ETA(I)
230   CONTINUE
      DO 240 I=1,561
      POS(I)=POS(I)/100.0
240   CONTINUE
      RETURN
      END
      SUBROUTINE ESTAN(ANGLE,PI)
C        THIS PROGRAM CONVERTS DEG TO RAD
      ANGLE=ANGLE*PI/180
      RETURN
      END
      SUBROUTINE SDPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
C           THIS PROGRAM CALCULATES THE SUM AND DIFFERENCE AE PATTERNS
      THETA=ANGLE+.0215984495
      CALL  PATT(THETA,GPAT)
      GPATA=GPAT
      THETA=ANGLE-.0215984495
      CALL PATT(THETA,GPAT)
      GPATB=GPAT
      GPAT1=GPATA+GPATB
      GPAT2=GPATA-GPATB
      RETURN
```

```
      END
      SUBROUTINE PATT(THETA,GPAT)
      IF (THETA .EQ. 0) THEN
      GPAT=1.0
      ELSE
      CALL SINC(THETA,GPAT)
      ENDIF
      RETURN
      END
      SUBROUTINE SINC(THETA,GPAT)
C             THIS PROGRAM FINDS GPAT
      GPAT=SIN(43.083897*SIN(THETA))/(43.083897*SIN(THETA))
      RETURN
      END
      SUBROUTINE XPAT(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1  ,R,X1,X2,DSEED,EST,VNULL,PI,NC,STF,A,VEL)
C             THIS PROGRAM CALCULATES THE X1 , X2 SIGNALS
      REAL R(NC),STF
      A=1.0
      ANGLE=VEL/10.0-2.0
      CALL ESTAN(ANGLE,PI)
      CALL SDPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
      CALL NOISE(NR,R,DSEED)
      X1=0.0
      DO 150 I=1,100
      X1=((R(I)*STF+GPAT1)*A*2.0E-4)+X1
150   CONTINUE
      CALL NOISE(NR,R,DSEED)
      X2=0.0
      DO 160 I=1,100
      X2=((R(I)*STF+GPAT2)*A*2.0E-4)+X2
160   CONTINUE
      ANGLE=EST
      CALL SDPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
      VNULL=X1*GPAT2-X2*GPAT1
      RETURN
      END
      SUBROUTINE NOISE(NR,R,DSEED)
C             THIS PROGRAM GENERATES A NOISE VECTOR
      REAL R(100)
      DOUBLE PRECISION DSEED
      NR=100
      CALL GGNML  (DSEED,NR,R)
      RETURN
      END
```

```
      SUBROUTINE ERROR(POS,BOX,PHI,VNULL,NA)
      REAL POS(NA),BOX(NA)
      IF (VNULL .GE. BOX(560)) THEN
      PHI=POS(561)
      GOTO 260
      ENDIF
      DO 250 I=1,560
      IF (VNULL .LE. BOX(I)) THEN
      PHI=POS(I)
      GOTO 260
      ENDIF
250   CONTINUE
260   CONTINUE
      RETURN
      END
      SUBROUTINE ANDEG(DEG,FI)
C          THIS PROGRAM CONVERTS RADIANS TO DEGREES
      DEG=DEG*180/PI
      RETURN
      END
      SUBROUTINE MEVAR(ALPHA,AMEAN,BMEAN,VAR1,VAR2,SNC,DUMM,NB)
      REAL ALPHA(NB),DUMM(NB)
      AMEAN=0.0
      BMEAN=0.0
      VAR1=0.0
      VAR2=0.0
      DO 135 I=1,SNC
      AMEAN=ALPHA(I+2)+AMEAN
      VAR1= ALPHA(I+2)**2+VAR1
      BMEAN=DUMM(I)+BMEAN
      VAR2=DUMM(I)**2+VAR2
135   CONTINUE
      AMEAN=AMEAN/SNC
      VAR1=VAR1/SNC-AMEAN**2
      BMEAN=BMEAN/SNC
      VAR2=VAR2/SNC-BMEAN**2
      RETURN
      END
      SUBROUTINE LDO(Y,DUMM,NB)
      REAL Y(NB),DUMM(NB)
      DO 270 I=1,50
      IF (DUMM(I) .LE. -3.0) THEN
      DUMM(I)=-3.0
      ELSE
      DUMM(I)=DUMM(I)
      ENDIF
      IF (DUMM(I) .GE. 3.0) THEN
      DUMM(I)=3.0
      ELSE
      DUMM(I)=DUMM(I)
      ENDIF
      Y(I)=DUMM(I)
270   CONTINUE
      RETURN
      END
```

```
C
C        *****************************************
C        *                                       *
C        *     PROGRAM CONMV                     *
C        *                                       *
C        *     THIS PROGRAM PLOTS THE MEAN , VARIANCE, *
C        *     AND THE (MEAN/VARIANCE)SQUARE FOR A *
C        *     CONSTANT TARGET                   *
C        *                                       *
C        *****************************************
      PROGRAM MAIN
C          THIS PROGRAM SIMULATES THE SUFFICIENT STASTIC
      INTEGER NR,NA,NB,NC
      REAL ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2,PI,PHI
     1 ,DEG,BETA,AMEAN,BMEAN,VAR1,VAR2,SNC,STF
     2 ,EST,VNULL,SUM1,SUM2,EST1,EST2,A,MBOX(561),MHU(53)
     3     ,ETA(561),POS(561),BOX(561),DUMM(53),ALPHA(53)
     4 ,R(100),X1,X2,DMEAN(53),BVAR(53),X(53),Y(53)
     5 ,SNR
      DOUBLEPRECISION DSEED
      DATA ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2,EST,PI
     1 ,PHI,VNULL,SUM1,SUM2,EST1,EST2,DEG,BETA,AMEAN,BMEAN,VAR1,VAR2
     2 ,SNC,STF,X1,X2,A
     3 /0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0/
      CALLPLOTS(0.,0.,9)
      CALL PLOT(0.,-11.,-3)
      CALL PLOT(2.,7.5,-3)
      NA=561
      NB=53
      NC=100
      DO 100 I=1,100
      R(I)=0.0
100   CONTINUE
      DO 110 I=1,53
      ALPHA(I)=0.0
      DUMM(I)=0.0
      DMEAN(I)=0.0
      BVAR(I)=0.0
      MHU(I)=0.0
      X(I)=0.0
      X(I)=0.0
110   CONTINUE
      DO 120 I=1,561
      ETA(I)=0.0
      POS(I)=0.0
      BOX(I)=0.0
      MBOX(I)=0.0
120   CONTINUE
      DSEED=124457.D0
      PI=3.141592654
      CALL FNE(PI,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1  ,SUM1,SUM2,EST1,EST2,ETA,POS,MBOX,NA)
      X6JZY6D6$6$5H72 E3RK6
      EST=0.0
      BETA=0.0
      STF=I
```

```
      SNR=(LOG10(100.0/STF**2.0))*10.0
      CALL ERID(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1 ,R,X1,X2,DSEED,EST,VNULL,PI,POS,BOX,PHI,ALPHA,AMEAN,BMEAN
     2 ,VAR1,VAR2,DUMM,SNC,STF,NA,NB,NC,BETA,DMEAN,BVAR,X,MBOX,MHU)
      CALL LDO(Y,DMEAN,NB)
      X(51)=0.
      Y(51)=-3.
      X(52)=50./5.
      Y(52)=6./4.
      CALL AXIS(0.,0.,'NUMBER',-6,5.,0.,X(51),X(52))
      CALL AXIS(0.,-2.,'MEAN VAR X',+10,4.,90.,Y(51),Y(52))
      CALL PLOT(0.,0.,3)
      CALL AXIS(5.,-2.,'MEANSQ/VAR',-10,4.,90.,Y(51),Y(52))
      CALL PLOT(2.,0.,3)
      CALL PLOT(0.,-2.,-3)
      CALL LINE(X,Y,50,1,0,0)
      CALL PLOT(0.,0.,3)
      CALL LDP(Y,BVAR,NB)
      CALL LINE(X,Y,50,1,5,4)
      CALL PLOT(0.,0.,3)
      CALL LDO(Y,MHU,NB)
      CALL LINE(X,Y,50,1,5,1)
      CALL PLOT(0.,0.,3)
      CALL PLOT(10.0,-11.,-3)
      CALL PLOT(0.,7.5,-3)
      PRINT'(//T6,A,T29,F5.2,T34,A///T14,A,T36,I1,T45,I2,T55,I2//
     1 T19,A,F10.2,F10.2,F10.2//T15,A,F10.2,F10.2,F10.2//
     2 T2,A,F10.2,F10.2,F10.2///T3,A,T38,A/T10,A,T36,F5.2,T41,A///)'
     3 ,'SIGNAL TO NOISE RATIO =',SNR,'DB','AT (I) PULSES',2,25,50,
     4 'MEAN - =',DMEAN(2),DMEAN(25),DMEAN(50),'VARIANCE X ='
     5 ,BVAR(2),BVAR(25),BVAR(50),'(MEAN/VARIANCE)SQUARE D ='
     6 ,MHU(2),MHU(25),MHU(50),'FIG    PLOT OF MEAN, VARIANCE AND (',
     7 'MEAN/VARIANCE)SQUARE','FOR SIGNAL TO NOISE RATIO OF',SNR,'DB'
130   CONTINUE
      CALL PLOTE(N)
      END
      SUBROUTINE ERID(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1 ,R,X1,X2,DSEED,EST,VNULL,PI,POS,BOX,PHI,ALPHA,AMEAN,BMEAN
     2 ,VAR1,VAR2,DUMM,SNC,STF,NA,NB,NC,BETA,DMEAN,BVAR,X,MBOX,MHU)
      REAL POS(NA),BOX(NA),CUMM(NB),ALPHA(NB),MHU(NB)
     1 ,R(NC),DMEAN(NB),BVAR(NB),X(NB),MBOX(NA)
      CALL XPAT(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1 ,R,X1,X2,DSEED,EST,VNULL,PI,NC,STF,A)
      DO 135 I=1,560
      BOX(I)=MBOX(I)*((GPAT1**2+GPAT2**2)*2.0E-2)/(GPAT1*X1+GPAT2*X2)
135   CONTINUE
      CALL ERROR(POS,BOX,PHI,VNULL,NA)
      ANGLE=PHI
      CALL ESTAN(ANGLE,PI)
      ALPHA(2)=ANGLE
      EST=ANGLE
      DEG=EST
      CALL ANDEG(DEG,PI)
      DUMM(1)=DEG
      PRINT *, 'EST = ',DEG
      DO 140 I=1,49
```

```
      CALL XPAT(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1  ,R,X1,X2,DSEED,EST,VNULL,PI,NC,STF,A)
      DO 130 N=1,560
      BOX(N)=MBOX(N)*((GPAT1**2+GPAT2**2)*2.0E-2)/(GPAT1*X1+GPAT2*X2)
130   CONTINUE
      CALL ERROR(POS,BOX,PHI,VNULL,NA)
      ANGLE=PHI
      CALL ESTAN(ANGLE,PI)
      ALPHA(I+2)=ANGLE
      BETA=BETA+ALPHA(I+2)
      EST=BETA/I+EST
      DEG=EST
      CALL ANDEG(DEG,PI)
      [illegible](I+1)=DEG
      [illegible]=I+1
      CALL [illegible](ALPHA,AMEAN,BMEAN,VAR1,VAR2,SNC,DUMM,NB)
      [illegible](I+1)=BMEAN
      [illegible](I+1)=VAR2
      [illegible](I+1)=[illegible]**2/VAR2**2
      [illegible]
140   CONTINUE
      RETURN
      END
      SUBROUTINE [illegible],THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1  ,SUM1,SUM2,[illegible],ETA,POS,MBOX,NA)
C         THIS PROGRAM [illegible] THE PHASE ERROR
      REAL ETA(NA),POS[illegible])
      ANGLE=0.0
      CALL ESTAN(ANGLE,PI)
      CALL SDPAT(ANGLE,THETA,[illegible],GPATB,GPAT1,GPAT2)
      SUM1=GPAT1
      SUM2=GPAT2
      DO 220 I=-280,280
      ANGLE=FLOAT(I)/100.0
      POS(I+281)=I
      CALL ESTAN(ANGLE,PI)
      CALL SDPAT(ANGLE,THETA,GPAT,GPATA,[illegible],GPAT2)
      EST1=GPAT1
      EST2=GPAT2
      ETA(I+281)=(SUM2*EST1-SUM1*EST2)*2.0E-[illegible]
220   CONTINUE
      DO 230 I=1,560
      MBOX(I)=(ETA(I+1)-ETA(I))/2.0+ETA(I)
230   CONTINUE
      DO 240 I=1,561
      POS(I)=POS(I)/100.0
240   CONTINUE
      RETURN
      END
      SUBROUTINE ESTAN(ANGLE,PI)
C         THIS PROGRAM CONVERTS DEG TO RAD
      ANGLE=ANGLE*PI/180
      RETURN
      END
      SUBROUTINE SDPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
C            THIS PROGRAM CALCULATES THE SUM AND DIFFERENCE AE PATTERNS
```

```
      THETA=ANGLE+.0215984495
      CALL  PATT(THETA,GPAT)
      GPATA=GPAT
      THETA=ANGLE-.0215984495
      CALL PATT(THETA,GPAT)
      GPATB=GPAT
      GPAT1=GPATA+GPATB
      GPAT2=GPATA-GPATB
      RETURN
      END
      SUBROUTINE PATT(THETA,GPAT)
      IF (THETA .EQ. 0) THEN
      GPAT=1.0
      ELSE
      CALL SINC(THETA,GPAT)
      ENDIF
      RETURN
      END
      SUBROUTINE SINC(THETA,GPAT)
C             THIS PROGRAM FINDS GPAT
      GPAT=SIN(43.003097*SIN(THETA))/(43.003097*SIN(THETA))
      RETURN
      END
      SUBROUTINE XPAT(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1  ,R,X1,X2,DSEED,EST,VNULL,PI,NC,STF,A)
C             THIS PROGRAM CALCULATES THE X1 , X2 SIGNALS
      REAL R(NC),STF
      A=1.0
      ANGLE=1.5
      CALL ESTAN(ANGLE,PI)
      CALL SDPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
      CALL NOISE(NR,R,DSEED)
      X1=0.0
      DO 150 I=1,100
      X1=((R(I)*STF+GPAT1)*A*2.0E-4)+X1
150   CONTINUE
      CALL NOISE(NR,R,DSEED)
      X2=0.0
      DO 160 I=1,100
      X2=((R(I)*STF+GPAT2)*A*2.0E-4)+X2
160   CONTINUE
      ANGLE=EST
      CALL SDPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
      VNULL=X1*GPAT2-X2*GPAT1
      RETURN
      END
      SUBROUTINE NOISE(NR,R,DSEED)
C             THIS PROGRAM GENERATES A NOISE VECTOR
      REAL R(100)
      DOUBLE PRECISION DSEED
      NR=100
      CALL GGNML (DSEED,NR,R)
      RETURN
      END
      SUBROUTINE ERROR(POS,BOX,PHI,VNULL,NA)
      REAL POS(NA),BOX(NA)
```

```
      IF (VNULL .GE. BOX(560)) THEN
      PHI=POS(561)
      GOTO 260
      ENDIF
      DO 250 I=1,560
      IF (VNULL .LE. BOX(I)) THEN
      PHI=POS(I)
      GOTO 260
      ENDIF
250   CONTINUE
260   CONTINUE
      RETURN
      END
      SUBROUTINE ANDEG(DEG,PI)
C           THIS PROGRAM CONVERTS RADIANS TO DEGREES
      DEG=DEG*180/PI
      RETURN
      END
      SUBROUTINE MEVAR(ALPHA,AMEAN,BMEAN,VAR1,VAR2,SNC,DUMM,NB)
      REAL ALPHA(NB),DUMM(NB)
      AMEAN=0.0
      BMEAN=0.0
      VAR1=0.0
      VAR2=0.0
      DO 135 I=1,SNC
      AMEAN=ALPHA(I+2)+AMEAN
      VAR1= ALPHA(I+2)**2+VAR1
      BMEAN=DUMM(I)+BMEAN
      VAR2=DUMM(I)**2+VAR2
135   CONTINUE
      AMEAN=AMEAN/SNC
      VAR1=VAR1/SNC-AMEAN**2
      BMEAN=BMEAN/SNC
      VAR2=VAR2/SNC-BMEAN**2
      RETURN
      END
      SUBROUTINE LOD(Y,DMEAN,NB)
      REAL Y(NB),DMEAN(NB)
      DO 270 I=1,50
      IF (DMEAN(I) .LE. -3.0) THEN
      DMEAN(I)=-3.0
      ELSE
      DMEAN(I)=DMEAN(I)
      ENDIF
      IF (DMEAN(I) .GE. 3.0) THEN
      DMEAN(I)=3.0
      ELSE
      DMEAN(I)=DMEAN(I)
      ENDIF
      Y(I)=DMEAN(I)
270   CONTINUE
      RETURN
      END
      SUBROUTINE LOP(Y,BVAR,NB)
      REAL Y(NB),BVAR(NB)
      DO 280 I=1,50
```

```
      IF (BVAR(I) .GE. 3.0) THEN
      BVAR(I)=3.0
      ELSE
      BVAR(I)=BVAR(I)
      ENDIF
      Y(I)=BVAR(I)
200   CONTINUE
      RETURN
      END
      SUBROUTINE LOO(Y,MHU,NB)
      REAL Y(NB),MHU(NB)
      DO 410 I=1,50
      IF (MHU(I) .GE. 3.0) THEN
      MHU(I)=3.0
      ELSE
      MHU(I)=MHU(I)
      ENDIF
      Y(I)=MHU(I)
410   CONTINUE
      RETURN
      END
```

```
C           ***************************************************
C           *                                                 *
C           *    PROGRAM VELDEN                               *
C           *                                                 *
C           *    THIS PROGRAM PLOTS THE PROBALITY             *
C           *    DENSITY FUNCTION OF THE ANGLE OFF            *
C           *    BORESIGHT FOR A MOVING TARGET                *
C           *                                                 *
C           ***************************************************
      PROGRAM MAIN
C           THIS PROGRAM SIMULATES THE SUFFICIENT STASTIC
      INTEGER NR,NA,NB,NC
      REAL ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2,PI,PHI
     1 ,DEG,BETA,AMEAN,BMEAN,VAR1,VAR2,SNC,STF,OCC(50,30),SORT(33)
     2 ,EST,VNULL,SUM1,SUM2,EST1,EST2,A,MBOX(561),MHU(53),VEL
     3      ,ETA(561),POS(561),BOX(561),DUMM(53),ALPHA(53)
     4 ,R(100),X1,X2,DMEAN(53),BVAR(53),X(33),Y(33),ORG
     5 ,EMEAN,CVAR
     6 ,SNR
      DOUBLEPRECISION DSEED
      DATA ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2,EST,PI
     1 ,PHI,VNULL,SUM1,SUM2,EST1,EST2,DEG,BETA,AMEAN,BMEAN,VAR1,VAR2
     2 ,SNC,STF,X1,X2,A,ORG,VEL
     3 /0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0/
      CALLPLOTS(0.,0.,9)
      CALL PLOT(0.,-11.,-3)
      CALL PLOT(2.,5.5,-3)
      NA=561
      NB=53
      NC=100
      DO 100 I=1,100
      R(I)=0.0
100   CONTINUE
      DO 108 I=1,50
      DO 105 J=1,30
      OCC(I,J)=0.0
105   CONTINUE
108   CONTINUE
      DO 110 I=1,53
      ALPHA(I)=0.0
      DUMM(I)=0.0
      DMEAN(I)=0.0
      BVAR(I)=0.0
      MHU(I)=0.0
110   CONTINUE
      DO 115 I=1,33
      SORT(I)=0.0
      Y(I)=0.0
      Y(I)=0.0
115   CONTINUE
      DO 120 I=1,561
      ETA(I)=0.0
      POS(I)=0.0
      BOX(I)=0.0
      MBOX(I)=0.0
120   CONTINUE
```

```
      DSEED=124457.D0
      PI=3.141592654
      CALL FNE(PI,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1 ,SUM1,SUM2,EST1,EST2,ETA,POS,MBOX,NA)
      DO 130 K=1,50
      EST=0.0
      BETA=0.0
      STF=5.
      SNR=(LOG10(100.0/STF**2.0))*10.0
      DSEED=DSEED+FLOAT(K)
      DO 125 I=-30,30,2
      SORT((I+32)/2)=FLOAT(I)/10.0
125   CONTINUE
      DO 128 I=1,30
      X(I)=SORT(I)
      SORT(I)=(SORT(I+1)-SORT(I))/2.0+SORT(I)
128   CONTINUE
      CALL ERID(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2,VEL
     1 ,R,X1,X2,DSEED,EST,NULL,PI,POS,BOX,PHI,ALPHA,AMEAN,BMEAN
     2 ,VAR1,VAR2,DUMM,SNC,STF,NA,NB,NC,BETA,DMEAN,BVAR,MBOX,MHU)
      DO 440 I=1,50
      IF (DUMM(I) .GE. SORT(30)) THEN
      OCC(I,30)=OCC(I,30)+1.0
      GOTO 430
      ENDIF
      DO 420 J=1,30
      IF (DUMM(I) .LE. SORT(J)) THEN
      OCC(I,J)=OCC(I,J)+1.0
      GOTO 430
      ENDIF
420   CONTINUE
430   CONTINUE
440   CONTINUE
130   CONTINUE
      DO 480 L=1,50
      EMEAN=0.0
      CVAR=0.0
      DO 470 M=1,30
      EMEAN=OCC(L,M)/50.0*SORT(M)+EMEAN
      CVAR=OCC(L,M)/50.0*(SORT(M)**2.0)+CVAR
470   CONTINUE
      CVAR=CVAR-EMEAN**2.0
      PRINT'(//T5,A,T28,F5.2,T33,A///T13,A,T29,I2//
     1 T15,A,F10.2//T11,A,F10.2///T2,A/
     2 T9,A/T9,A,T36,F5.2,T41,A/T9,A,T21,I2///)',
     3 'SIGNAL TO NOISE RATIO =',SNR,'DB','AT PULSE',
     4 (L),'MEAN =',EMEAN,'VARIANCE =',CVAR,
     5 'FIG     PLOT OF THE PROBALITY DENSITY FUNCTION',
     6 'OF THETA ESTIMATE VERS THETA',
     7 'FOR SIGNAL TO NOISE RATIO OF',SNR,'DB',
     8 'AND AT PULSE',(L)
      X(31)=-3.
      Y(31)=0.
      X(32)=6./6.
      Y(32)=1./4.
      DO 455 J=1,30
```

```
      Y(J)=OCC(1,J)/50.0
455   CONTINUE
      CALL AXIS(0.,0.,'THETA',-5,6.,0.,X(31),X(32))
      CALL AXIS(3.,0.,'P(THETA)',+8,4.,90.,Y(31),Y(32))
      CALL PLOT(0.,0.,3)
      CALL LINE(X,Y,30,1,0,0)
      CALL PLOT(0.,0.,3)
      CALL PLOT(0.,-11.,-3)
      CALL PLOT(8.0,5.5,-3)
      DO 460 I=10,50,10
      DO 450 J=1,30
      Y(J)=OCC(I,J)/50.0
450   CONTINUE
      CALL AXIS(0.,0.,'THETA',-5,6.,0.,X(31),X(32))
      CALL AXIS(3.,0.,'P(THETA)',+8,4.,90.,Y(31),Y(32))
      CALL PLOT(0.,0.,3)
      CALL LINE(X,Y,30,1,0,0)
      CALL PLOT(0.,0.,3)
      CALL PLOT(0.,-11.,-3)
      CALL PLOT(8.0,5.5,-3)
460   CONTINUE
480   CONTINUE
      CALL PLOTE(N)
      END
      SUBROUTINE ERID(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2,VEL
     1  ,R,X1,X2,DSEED,EST,VNULL,PI,POS,BOX,PHI,ALPHA,AMEAN,BMEAN
     2  ,VAR1,VAR2,DUMM,SNC,STF,NA,NB,NC,BETA,DMEAN,BVAR,MBOX,MMU)
      REAL POS(NA),BOX(NA),DUMM(NB),ALPHA(NB),MMU(NB)
     1  ,R(NC),DMEAN(NB),BVAR(NB),MBOX(NA)
      VEL=1.0
      CALL XPAT(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1  ,R,X1,X2,DSEED,EST,VNULL,PI,NC,STF,A,VEL)
      DO 135 I=1,560
      BOX(I)=MBOX(I)*((GPAT1**2+GPAT2**2)*2.0E-2)/(GPAT1*X1+GPAT2*X2)
135   CONTINUE
      CALL ERROR(POS,BOX,PHI,VNULL,NA)
      ANGLE=PHI
      CALL ESTAN(ANGLE,PI)
      ALPHA(2)=ANGLE
      EST=ANGLE
      DEG=EST
      CALL ANDEG(DEG,PI)
      DUMM(1)=DEG
      DO 140 I=1,49
      VEL=FLOAT(I)+1.0
      CALL XPAT(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1  ,R,X1,X2,DSEED,EST,VNULL,PI,NC,STF,A,VEL)
      DO 138 N=1,560
      BOX(N)=MBOX(N)*((GPAT1**2+GPAT2**2)*2.0E-2)/(GPAT1*X1+GPAT2*X2)
138   CONTINUE
      CALL ERROR(POS,BOX,PHI,VNULL,NA)
      ANGLE=PHI
      CALL ESTAN(ANGLE,PI)
      ALPHA(I+2)=ANGLE
      EST=EST+ANGLE
      DEG=EST
```

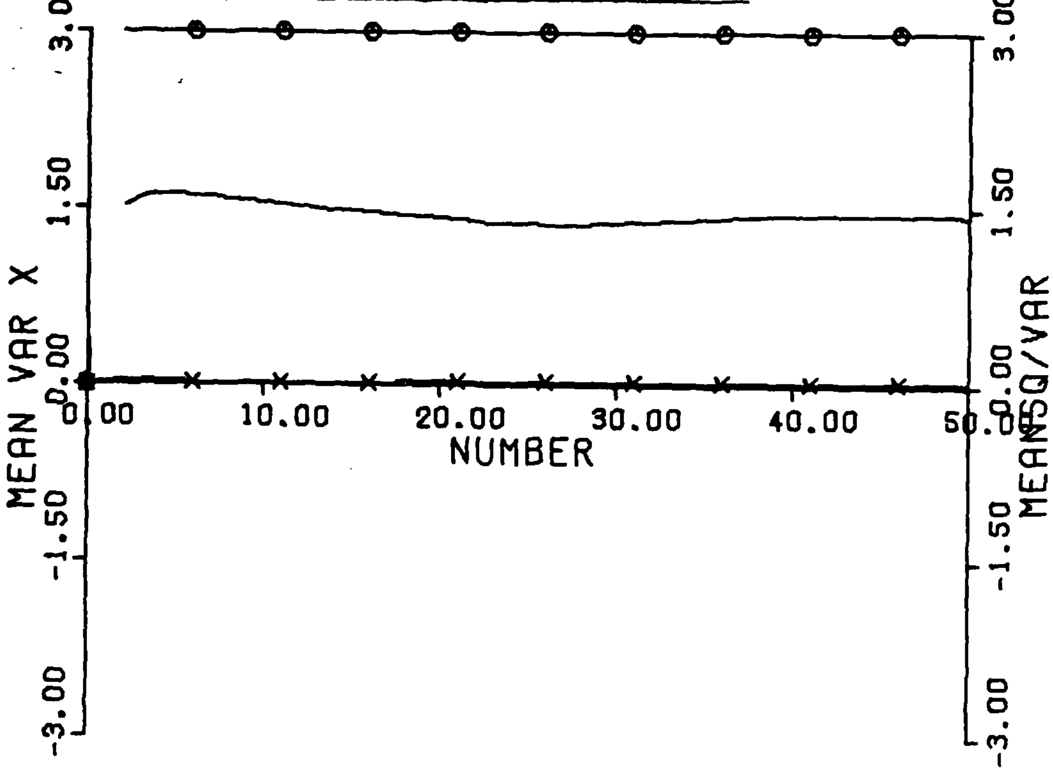
```
      CALL ANDEG(DEG,PI)
      DUMM(I+1)=DEG
      SNC=I+1
      CALL MEVAR(ALPHA,AMEAN,BMEAN,VAR1,VAR2,SNC,DUMM,NB)
      DMEAN(I+1)=BMEAN
      BVAR(I+1)=VAR2
      MHU(I+1)=BMEAN**2/VAR2**2
140   CONTINUE
      RETURN
      END
      SUBROUTINE FNE(PI,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1  ,SUM1,SUM2,EST1,EST2,ETA,POS,MBOX,NA)
C         THIS PROGRAM ESTIMATES THE PHASE ERROR
      REAL ETA(NA),POS(NA),MBOX(NA)
      ANGLE=0.0
      CALL ESTAN(ANGLE,PI)
      CALL SOPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
      SUM1=GPAT1
      SUM2=GPAT2
      DO 220 I=-280,280
      ANGLE=FLOAT(I)/100.0
      POS(I+281)=I
      CALL ESTAN(ANGLE,PI)
      CALL SOPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
      EST1=GPAT1
      EST2=GPAT2
      ETA(I+281)=(SUM2*EST1-SUM1*EST2)*2.0E-2
220   CONTINUE
      DO 230 I=1,560
      MBOX(I)=(ETA(I+1)-ETA(I))/2.0+ETA(I)
230   CONTINUE
      DO 240 I=1,561
      POS(I)=POS(I)/100.0
240   CONTINUE
      RETURN
      END
      SUBROUTINE ESTAN(ANGLE,PI)
C         THIS PROGRAM CONVERTS DEG TO RAD
      ANGLE=ANGLE*PI/180
      RETURN
      END
      SUBROUTINE SOPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
C            THIS PROGRAM CALCULATES THE SUM AND DIFFERENCE AE PATTERNS
      THETA=ANGLE+.0215984495
      CALL  PATT(THETA,GPAT)
      GPATA=GPAT
      THETA=ANGLE-.0215984495
      CALL PATT(THETA,GPAT)
      GPATB=GPAT
      GPAT1=GPATA+GPATB
      GPAT2=GPATA-GPATB
      RETURN
      END
      SUBROUTINE PATT(THETA,GPAT)
      IF (THETA .EQ. 0) THEN
      GPAT=1.0
```

```
      ELSE
      CALL SINC(THETA,GPAT)
      ENDIF
      RETURN
      END
      SUBROUTINE SINC(THETA,GPAT)
C             THIS PROGRAM FINDS GPAT
      GPAT=SIN(43.003097*SIN(THETA))/(43.003097*SIN(THETA))
      RETURN
      END
      SUBROUTINE XPAT(NR,ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2
     1  ,R,X1,X2,DSEED,EST,VNULL,PI,NC,STF,A,VEL)
C             THIS PROGRAM CALCULATES THE X1 , X2 SIGNALS
      REAL R(NC),STF
      A=1.0
      ANGLE=VEL/10.0-2.0
      CALL ESTAN(ANGLE,PI)
      CALL SDPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
      CALL NOISE(NR,R,DSEED)
      X1=0.0
      DO 150 I=1,100
      X1=((R(I)*STF+GPAT1)*A*2.0E-4)+X1
150   CONTINUE
      CALL NOISE(NR,R,DSEED)
      X2=0.0
      DO 160 I=1,100
      X2=((R(I)*STF+GPAT2)*A*2.0E-4)+X2
160   CONTINUE
      ANGLE=EST
      CALL SDPAT(ANGLE,THETA,GPAT,GPATA,GPATB,GPAT1,GPAT2)
      VNULL=X1*GPAT2-X2*GPAT1
      RETURN
      END
      SUBROUTINE NOISE(NR,R,DSEED)
C             THIS PROGRAM GENERATES A NOISE VECTOR
      REAL R(100)
      DOUBLE PRECISION DSEED
      NR=100
      CALL GGNML (DSEED,NR,R)
      RETURN
      END
      SUBROUTINE ERROR(POS,BOX,PHI,VNULL,NA)
      REAL POS(NA),BOX(NA)
      IF (VNULL .GE. BOX(561)) THEN
      PHI=POS(561)
      GOTO 260
      ENDIF
      DO 250 I=1,560
      IF (VNULL .LE. BOX(I)) THEN
      PHI=POS(I)
      GOTO 260
      ENDIF
250   CONTINUE
260   CONTINUE
```

```
      RETURN
      END
      SUBROUTINE ANDEG(DEG,PI)
C          THIS PROGRAM CONVERTS RADIANS TO DEGREES
      DEG=DEG*180/PI
      RETURN
      END
      SUBROUTINE MEVAR(ALPHA,AMEAN,BMEAN,VAR1,VAR2,SNC,DUMM,NB)
      REAL ALPHA(NB),DUMM(NB)
      AMEAN=0.0
      BMEAN=0.0
      VAR1=0.0
      VAR2=0.0
      DO 135 I=1,SNC
      AMEAN=ALPHA(I+2)+AMEAN
      VAR1= ALPHA(I+2)**2+VAR1
      BMEAN=DUMM(I)+BMEAN
      VAR2=DUMM(I)**2+VAR2
135   CONTINUE
      AMEAN=AMEAN/SNC
      VAR1=VAR1/SNC-AMEAN**2
      BMEAN=BMEAN/SNC
      VAR2=VAR2/SNC-BMEAN**2
      RETURN
      END
```

APPENDIX D

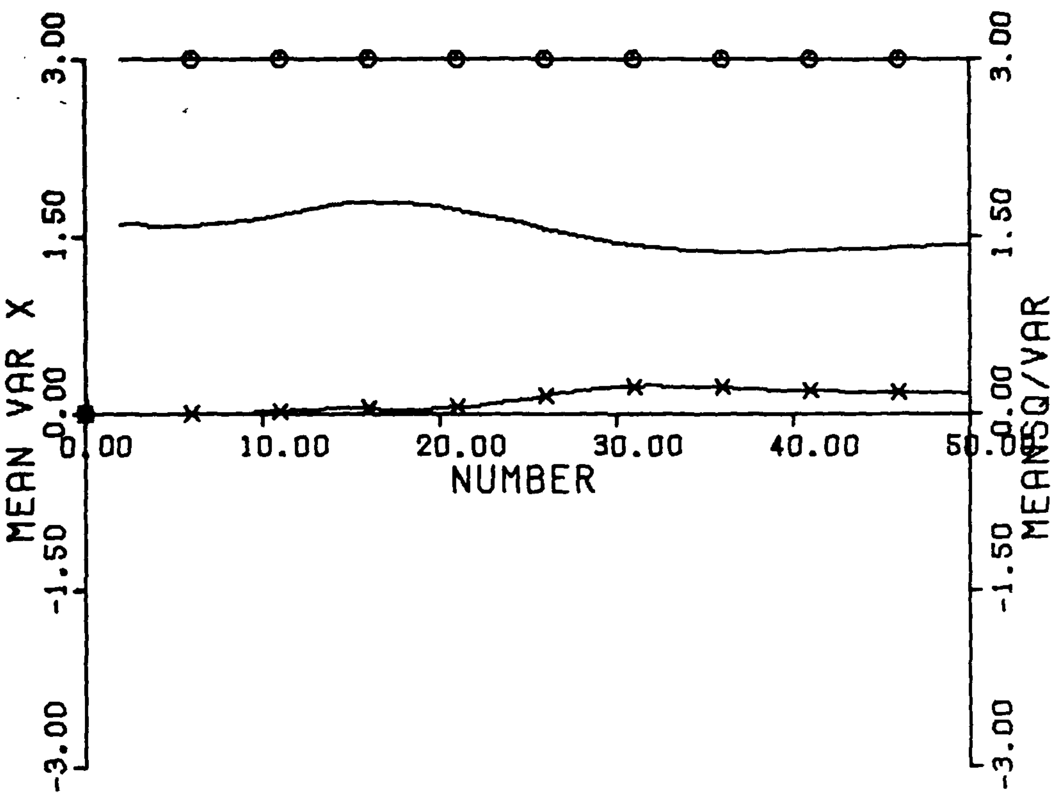
Constant Target Mean and Variance Results



SIGNAL TO NOISE RATIO = 20.00DB

AT (I) PULSES	2	25	50
MEAN - =	1.51	1.37	1.45
VARIANCE X =	.04	.03	.03
(MEAN/VARIANCE)SQUARE O =	1749.69	199[illegible].73	2216.90

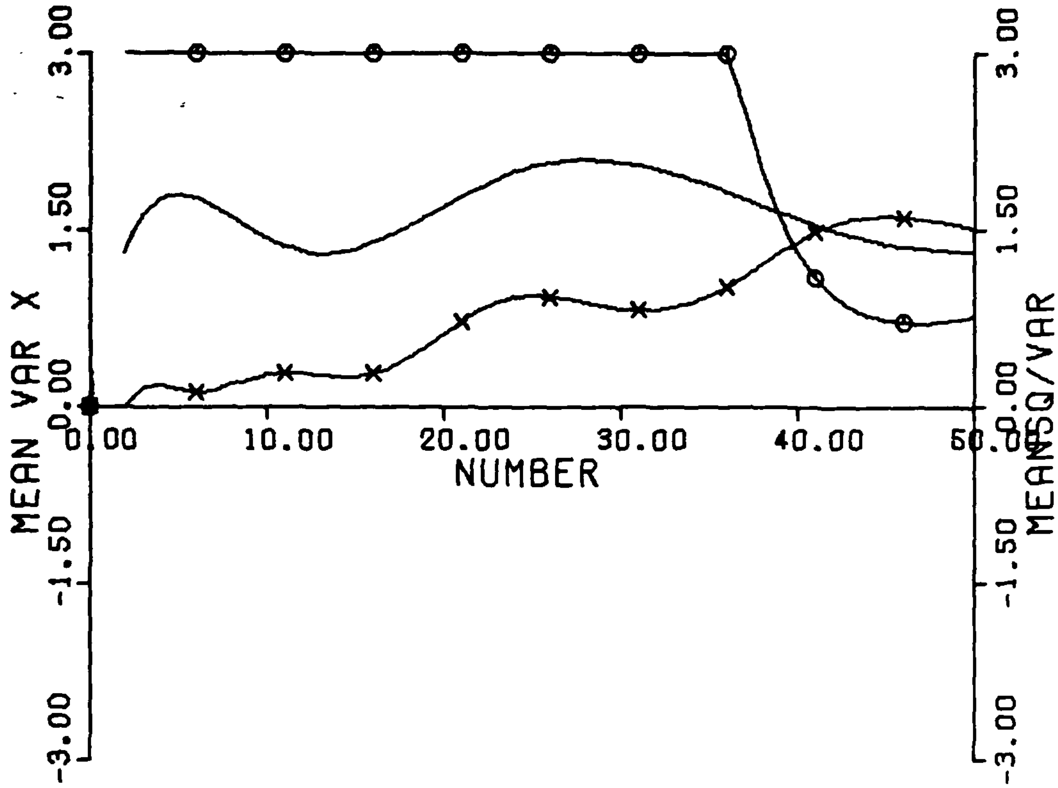
FIG D1 PLOT OF MEAN, VARIANCE AND (MEAN/VARIANCE) SQUARE FOR SIGNAL TO NOISE RATIO 20.00DB



SIGNAL TO NOISE RATIO =13.98DB

AT (I) PULSES	2	25	50
MEAN - =	1.61	1.60	1.43
VARIANCE K =	.11	.13	.17
(MEAN/VARIANCE)SQUARE D =	63283.69	158.30	7[illegible].42

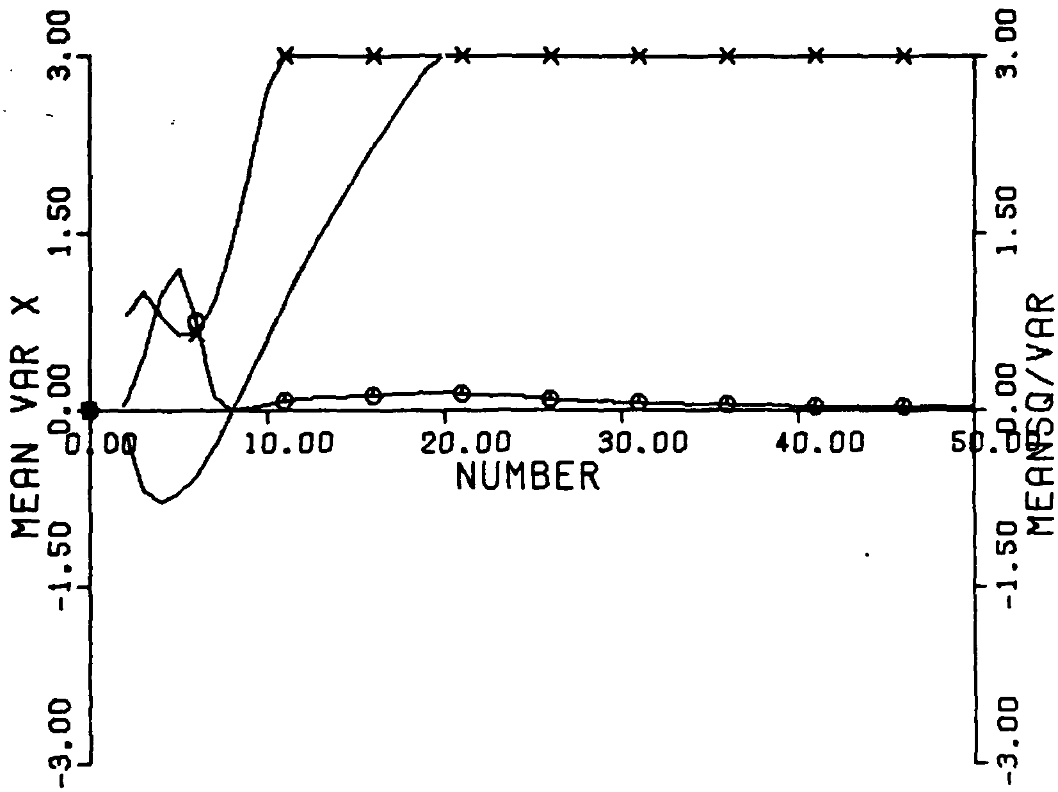
FIG D2 PLOT OF MEAN, VARIANCE AND (MEAN/VARIANCE) SQUARE FOR SIGNAL TO NOISE RATIO 13.98DB



SIGNAL TO NOISE RATIO =10.46DB

AT (I) PULSES	2	25	50
MEAN - =	1.35	2.64	1.32
VARIANCE X =	.00	.95	1.51
(MEAN/VARIANCE)SQUARE O =	**********	4.62	.76

FIG D3 PLOT OF MEAN, VARIANCE AND (MEAN/VARIANCE)SQUARE FOR SIGNAL TO NOISE RATIO 10.46DB



SIGNAL TO NOISE RATIO = 6.02DB

AT (I) PULSES	2	25	50
MEAN - =	-.16	4.45	12.25
VARIANCE X =	.79	13.87	76.83
(MEAN/VARIANCE) SQUARE O =	.15	.10	.02

FIG D4 PLOT OF MEAN, VARIANCE AND (MEAN/VARIANCE) SQUARE FOR SIGNAL TO NOISE RATIO 6.02DB

AD-A115 628 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOO--ETC F/G 17/9
MODELING OF A TRACKING RADAR IN TERMS OF A NON-LINEAR SECOND OR--ETC(U)
DEC 81 P M CRONK
UNCLASSIFIED AFIT/GE/EE/81D-14 NL

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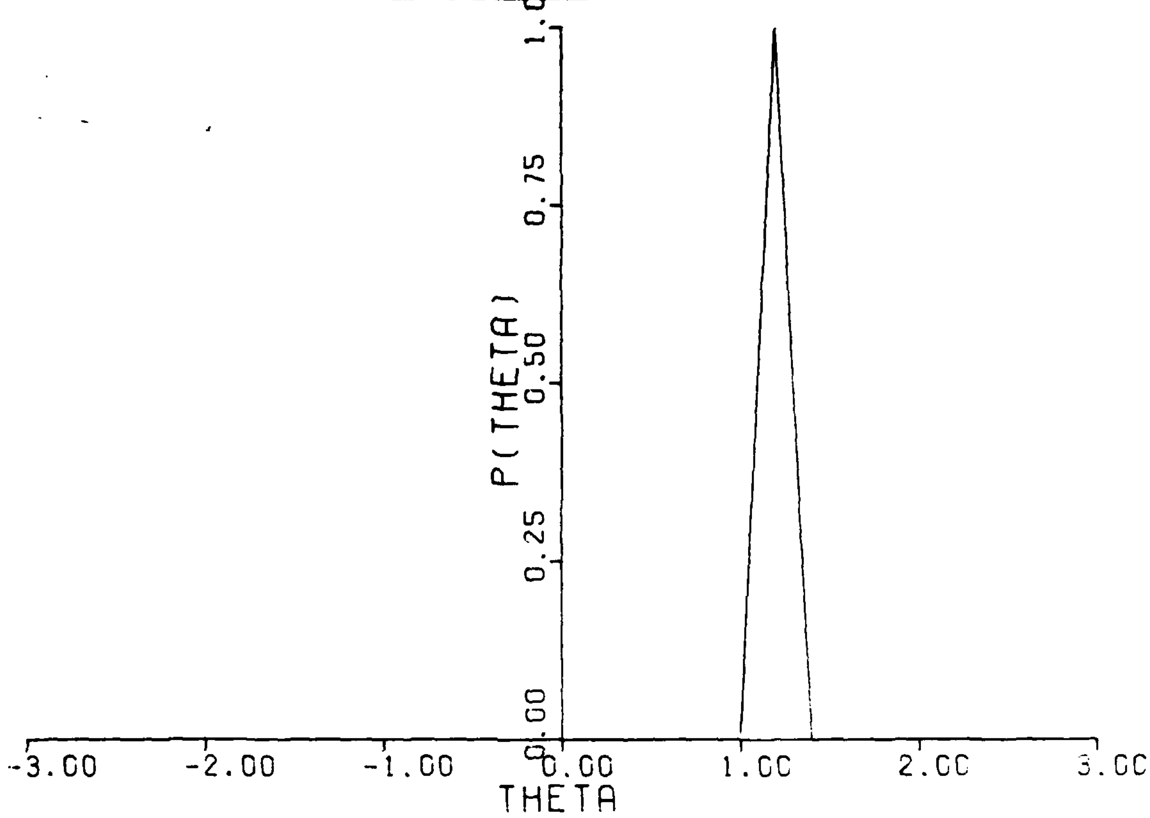
APPENDIX E

Constant Target Probability Density Function of Angle Off Boresight



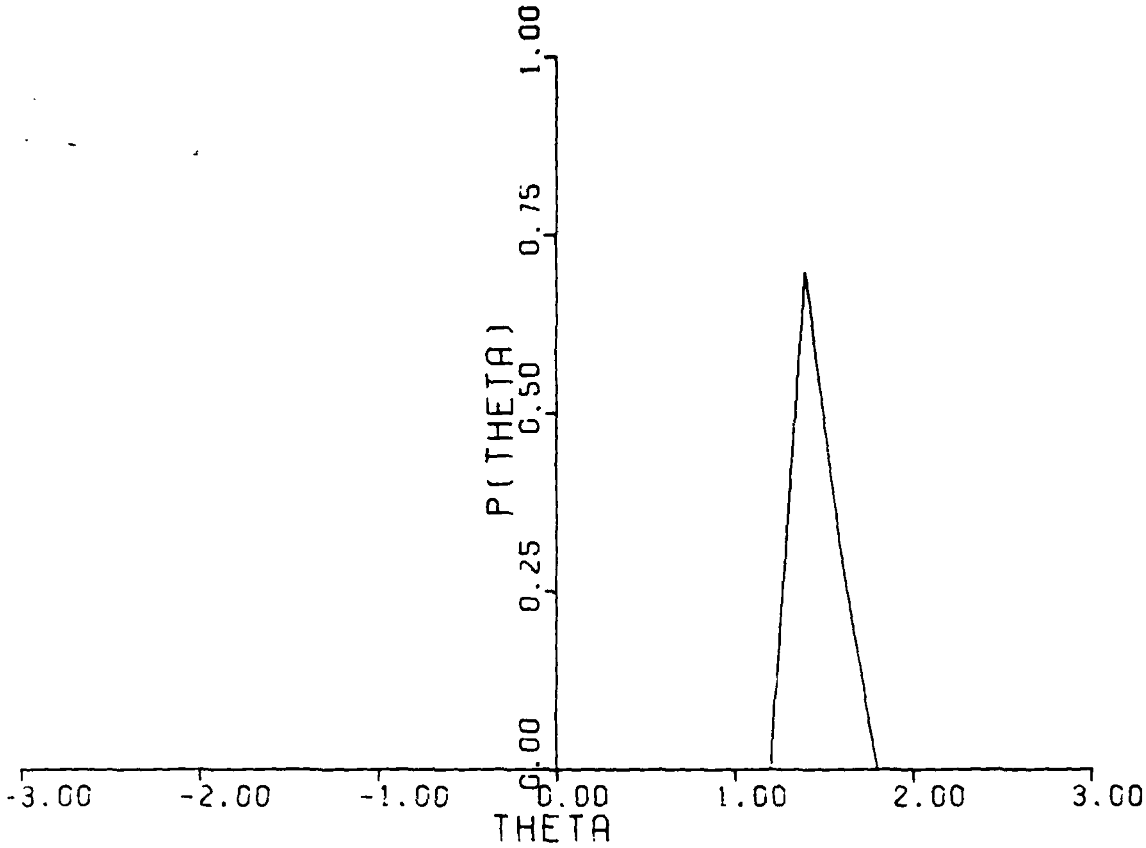
SIGNAL TO NOISE RATIO =40.00DB

AT PULSE 1

MEAN = 1.30

VARIANCE = 0.00

FIG E1 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 040.00DB
AND AT PULSE 1



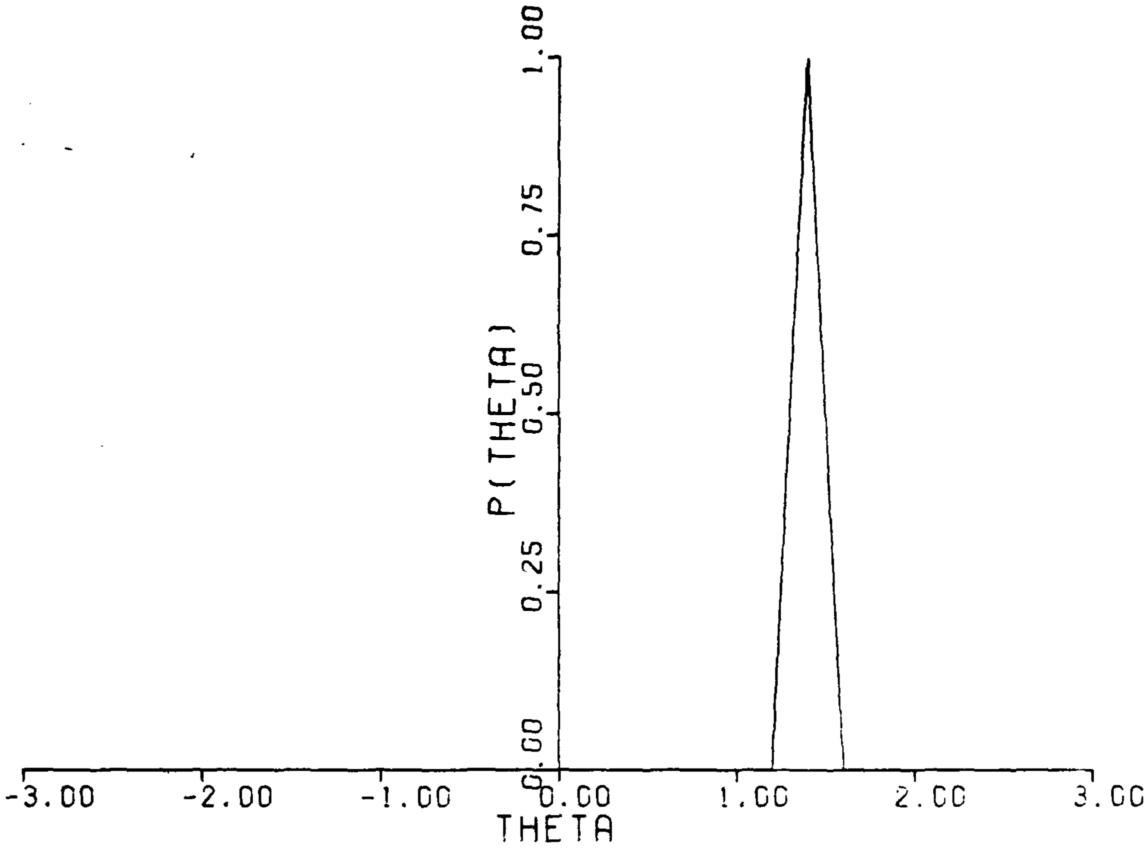
SIGNAL TO NOISE RATIO =4?.?ODB

AT PULSE 10

MEAN = 1.56

VARIANCE = .01

FIG E2 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 040.00DB
AND AT PULSE10



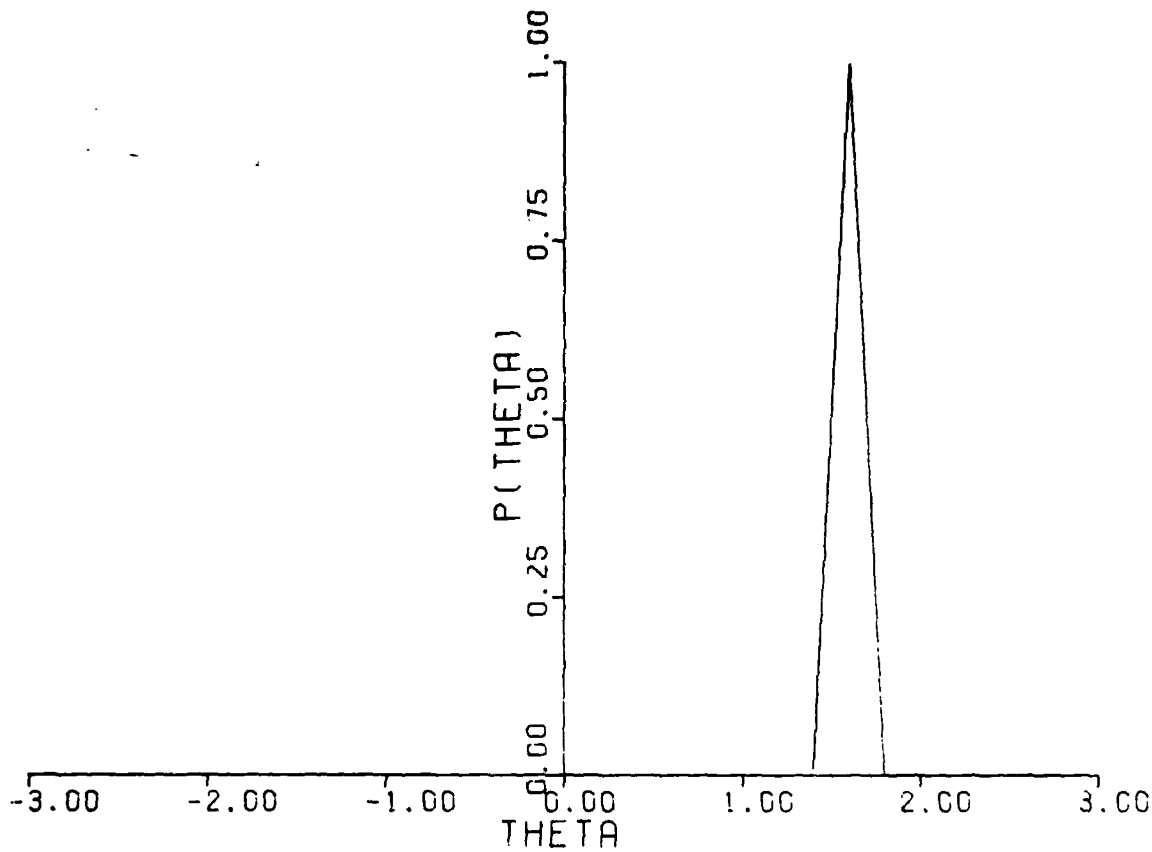
SIGNAL TO NOISE RATIO =40.00DB

AT PULSE 20

MEAN = 1.50

VARIANCE = 0.00

FIG E3 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 04..00DB
AND AT PULSE20



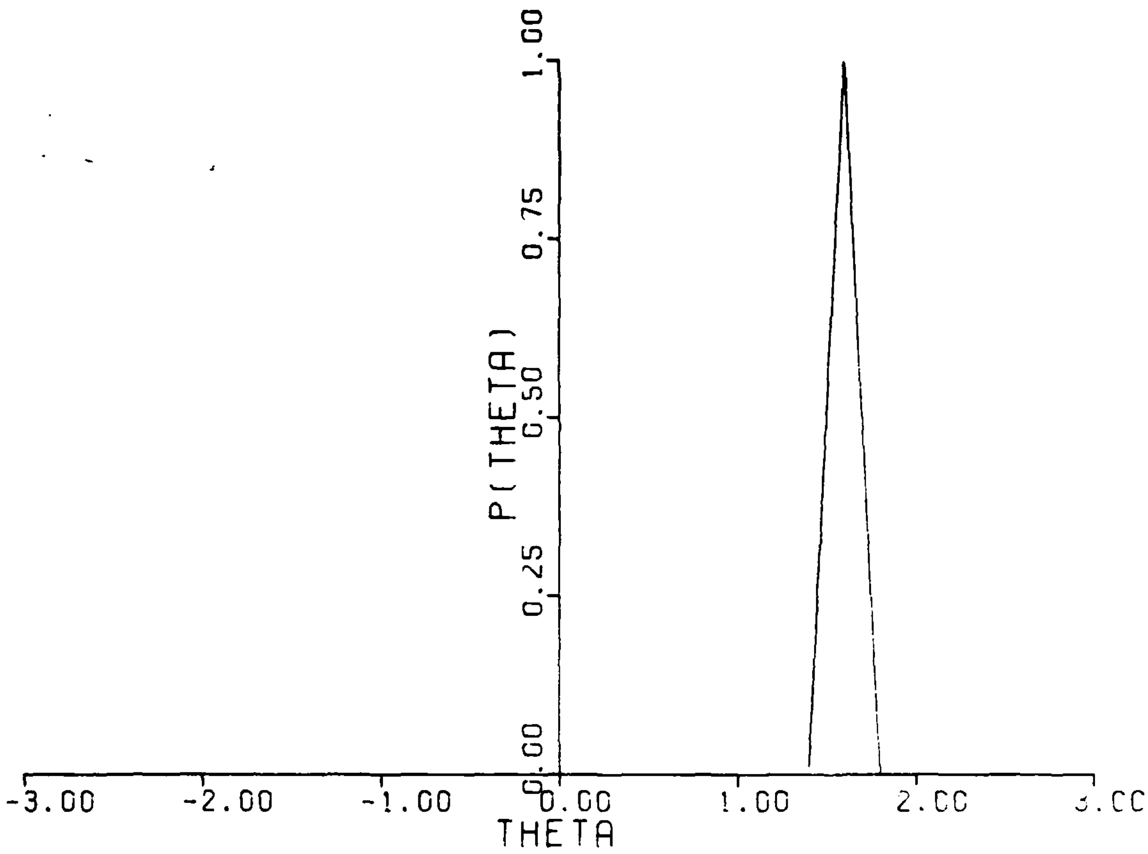
SIGNAL TO NOISE RATIO =40.00DB

AT PULSE 30

MEAN = 1.70

VARIANCE = 0.00

FIG E4 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 040.00DB
AND AT PULSE30



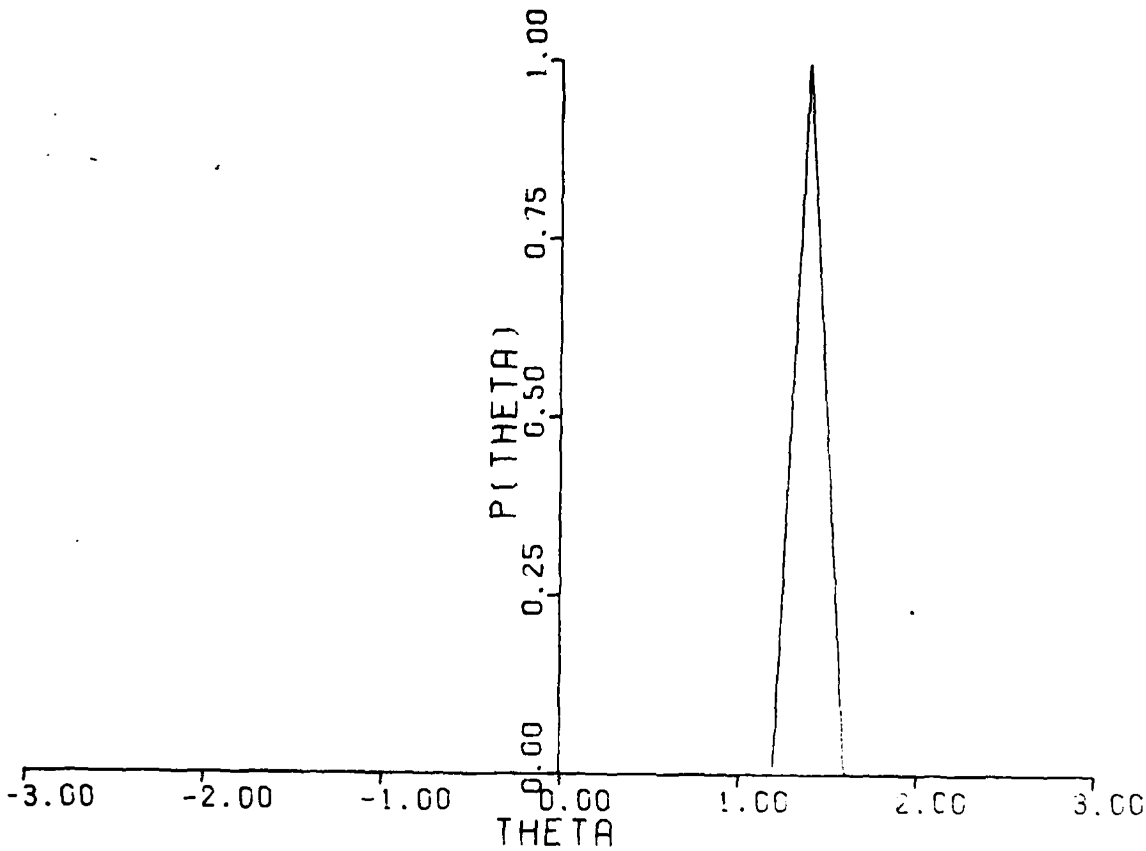
SIGNAL TO NOISE RATIO =40.30DB

AT PULSE 40

MEAN = 1.70

VARIANCE = 0.00

FIG E5 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 040.00DB
AND AT PULSE40



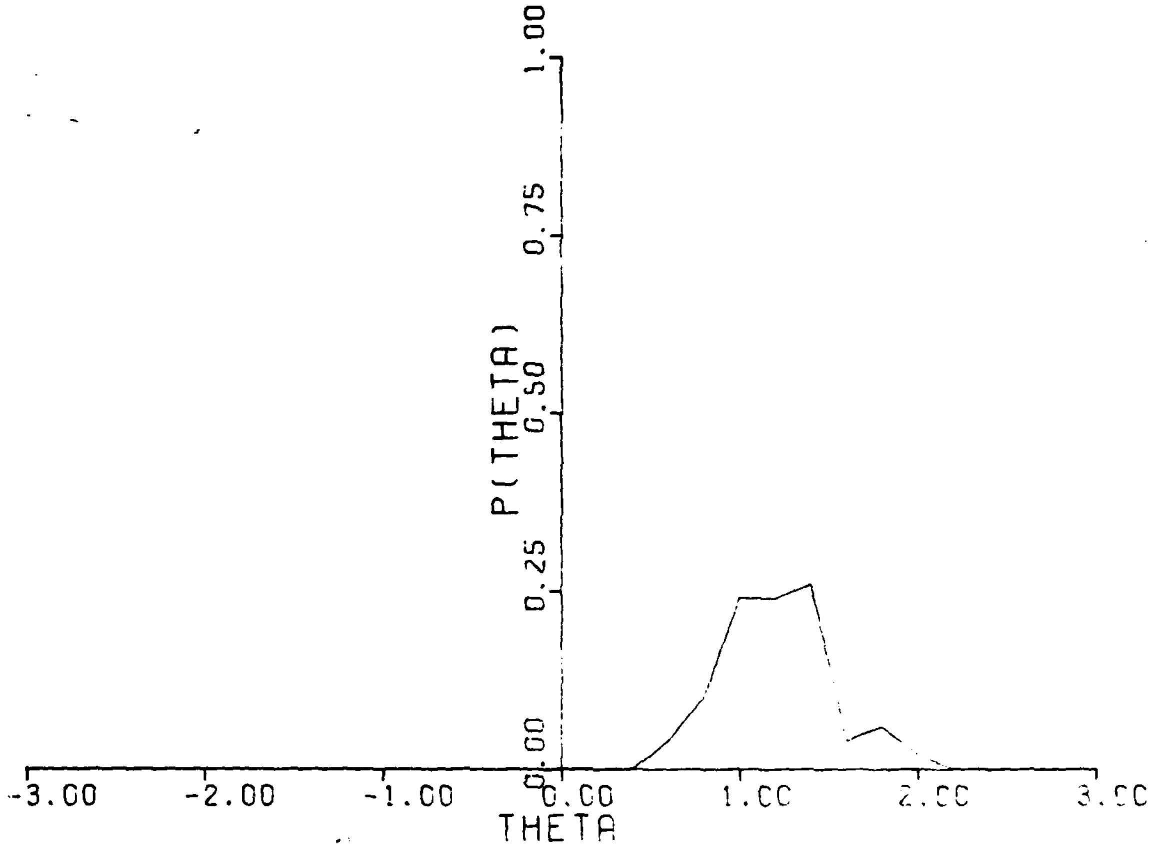
SIGNAL TO NOISE RATIO =40.0DB

AT PULSE 50

MEAN = 1.50

VARIANCE = 0.00

FIG E6 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 040.00DB
AND AT PULSE50



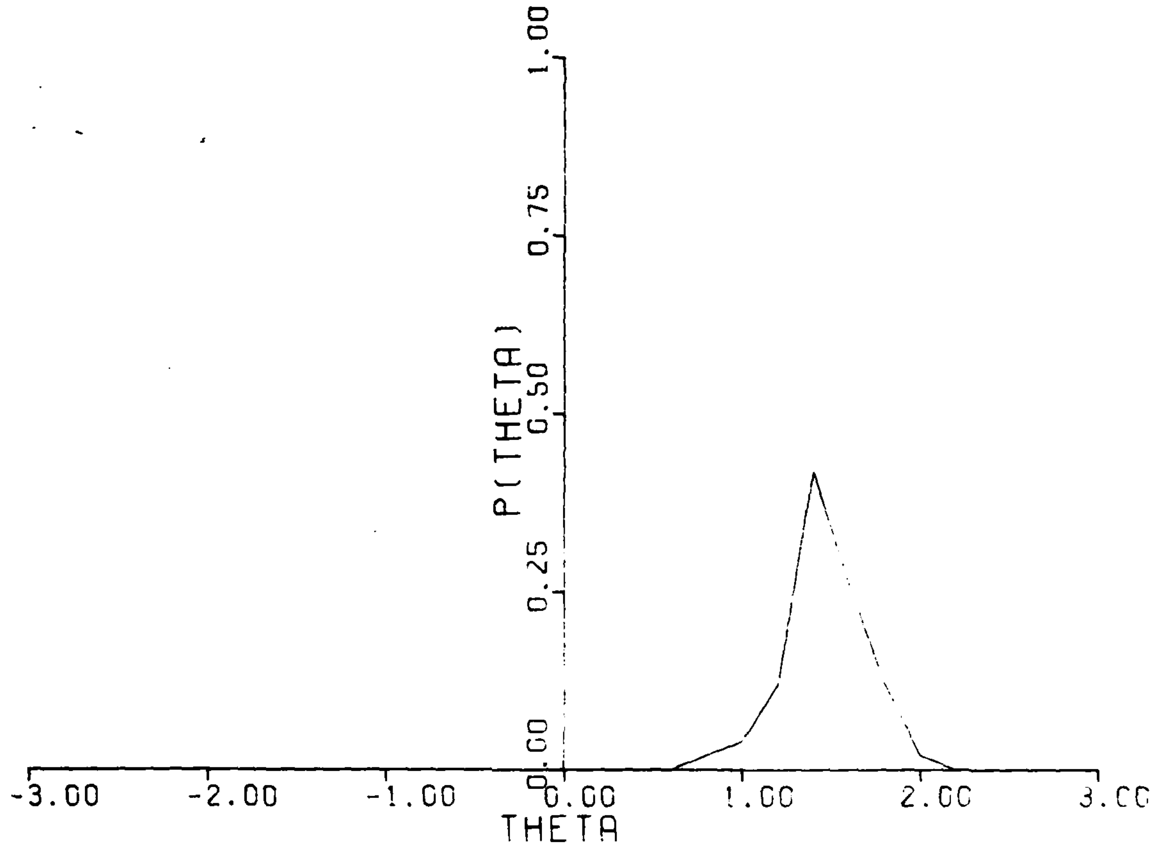
SIGNAL TO NOISE RATIO =20.[illegible]DB

AT PULSE 1

MEAN = 1.31

VARIANCE = .09

FIG E7 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 020.00DB
AND AT PULSE 1



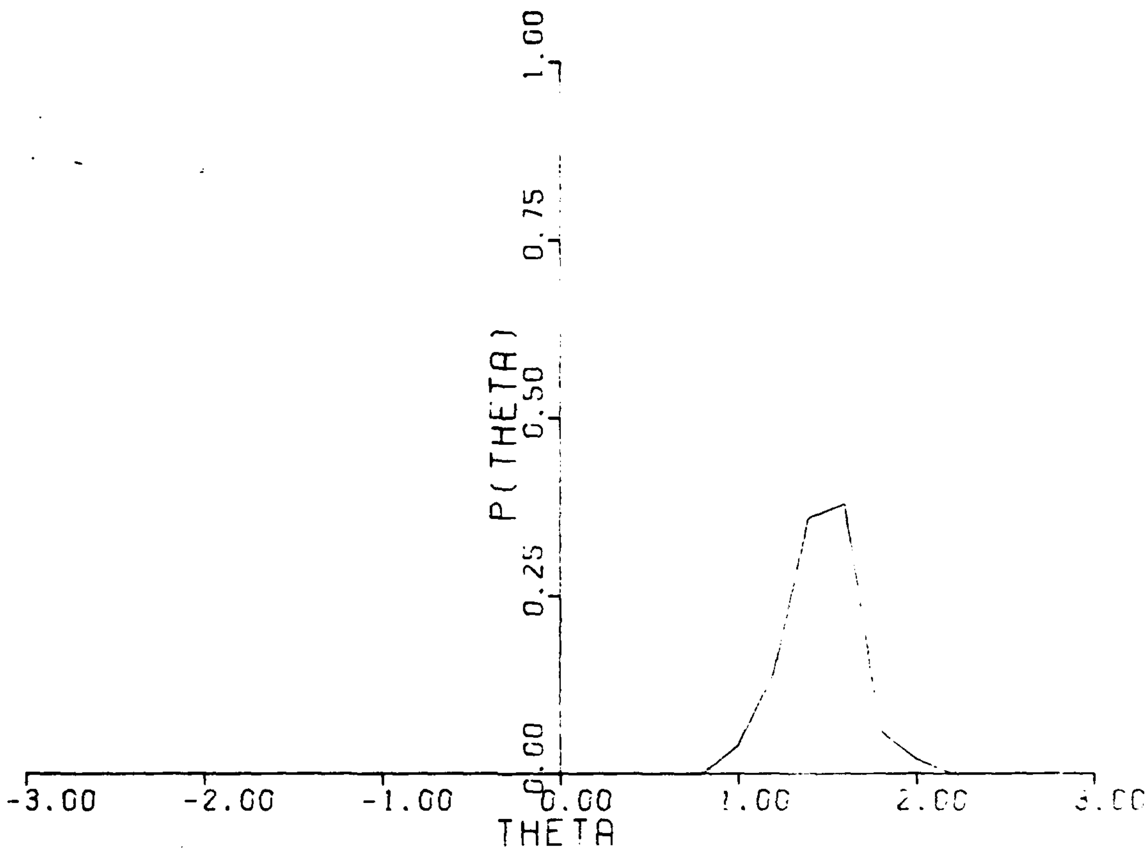
SIGNAL TO NOISE RATIO =20.00DB

AT PULSE 10

MEAN = 1.56

VARIANCE = .05

FIG E8 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 020.00DB
AND AT PULSE10



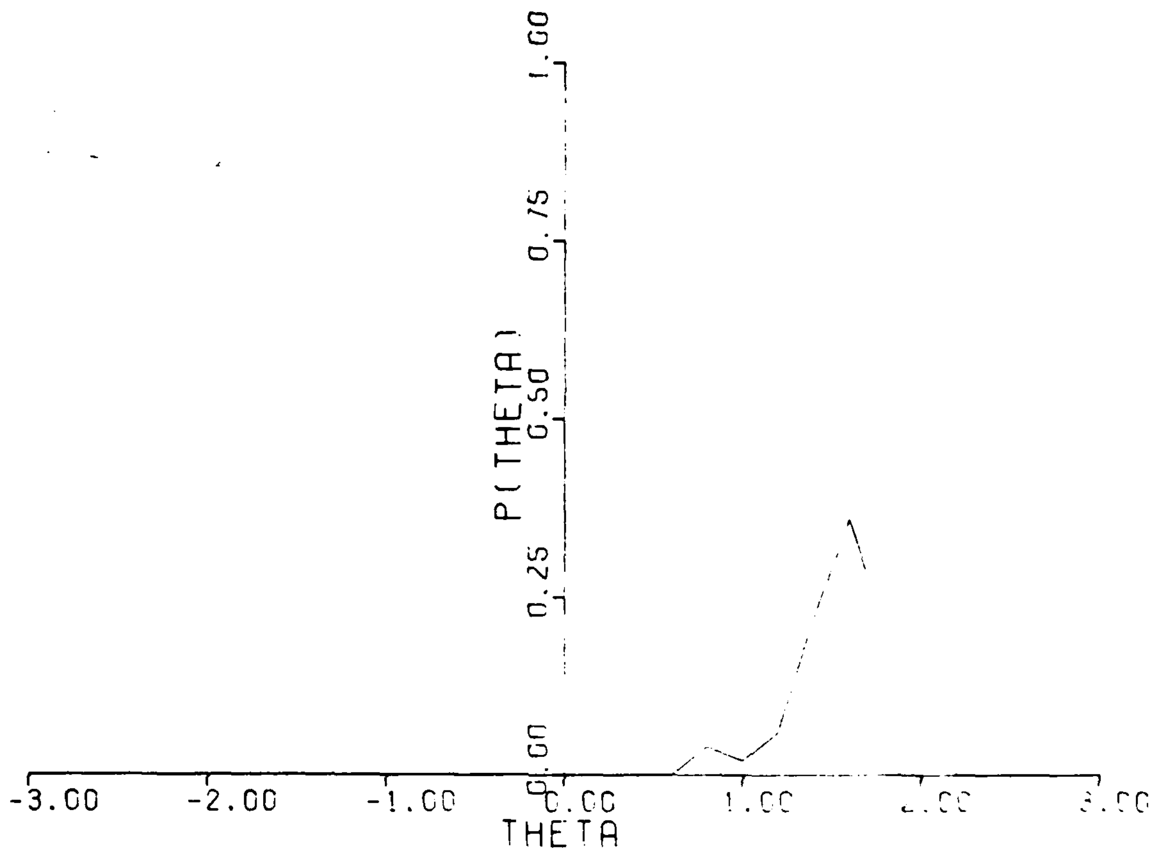
SIGNAL TO NOISE RATIO =2?. ?DB

AT PULSE 20

MEAN = 1.57

VARIANCE = .04

FIG E9 PLOT OF THE PROBALITY CENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 020.00DB
AND AT PULSE20



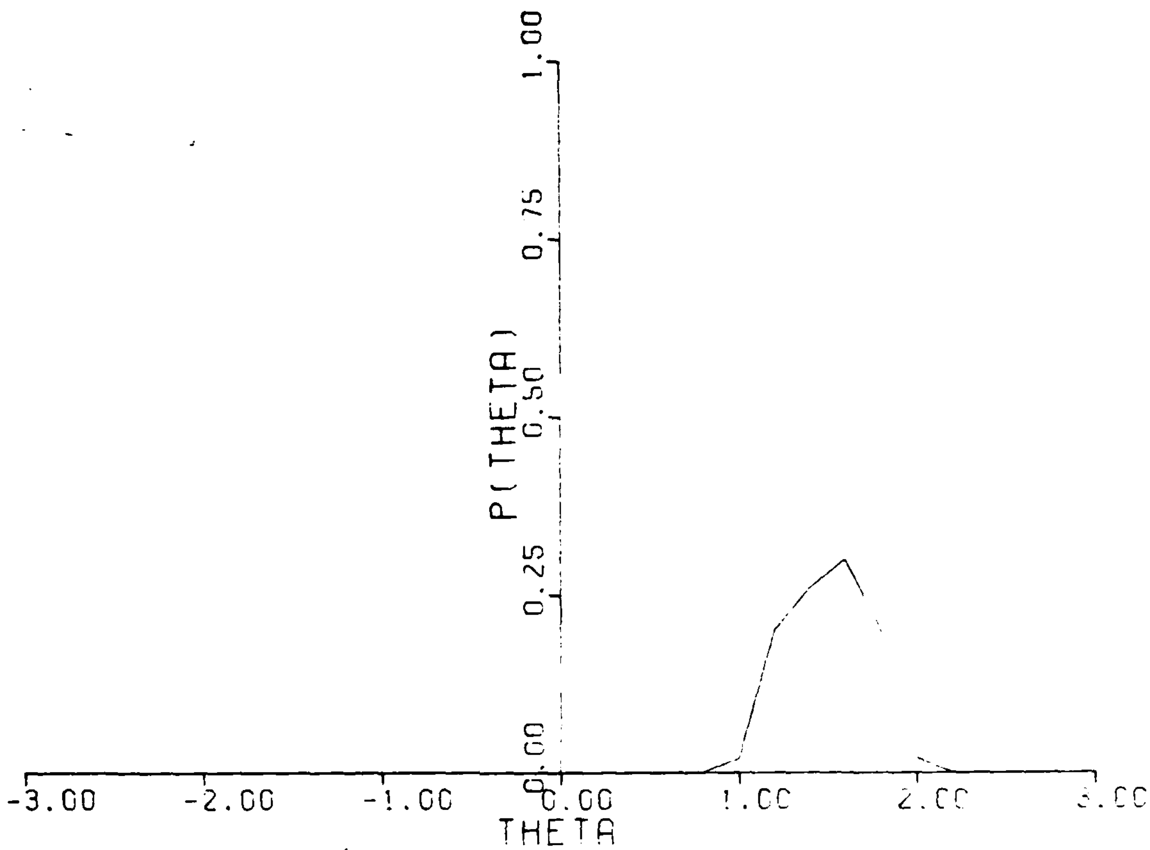
SIGNAL TO NOISE RATIO =20.00DB

AT PULSE 30

MEAN = 1.67

VARIANCE = .07

FIG E10 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 02..00DB
AND AT PULSE 30



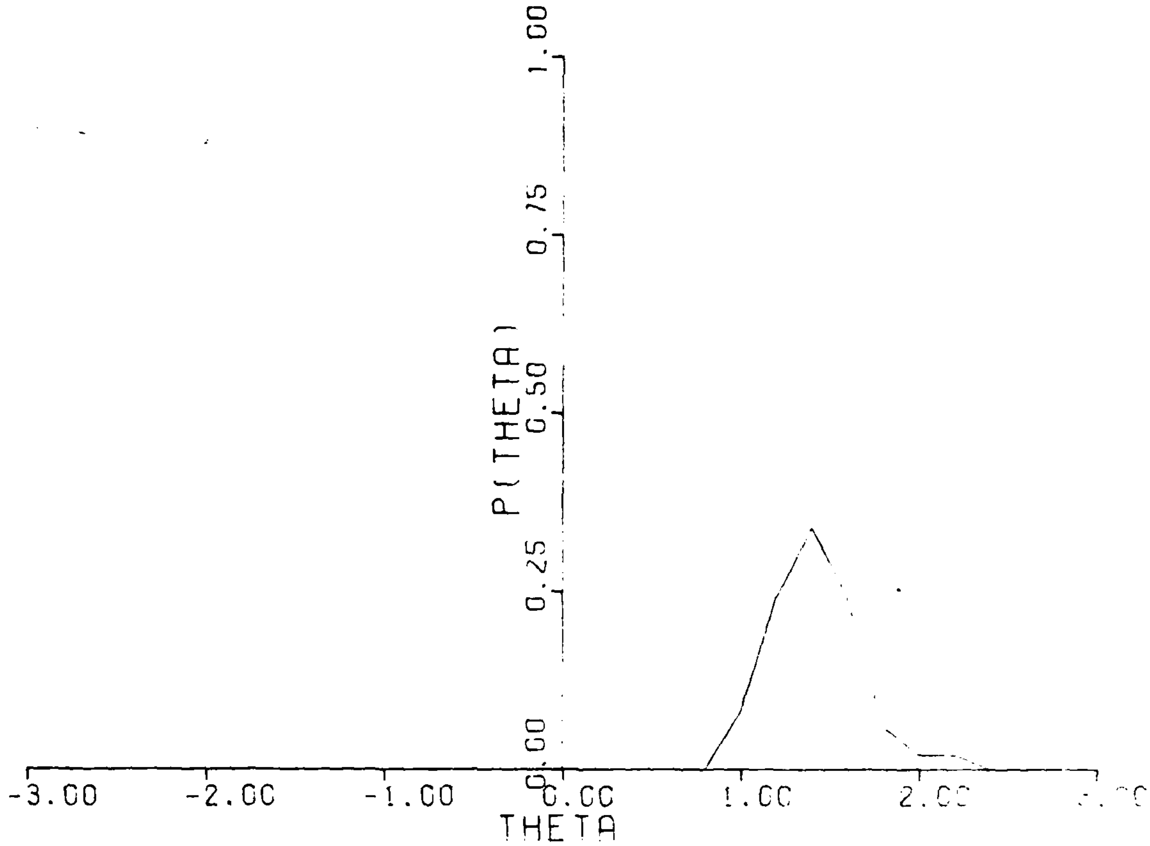
SIGNAL TO NOISE RATIO =20. 0DB

AT PULSE 40

MEAN = 1.60

VARIANCE = .05

FIG E11 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 020.00DB
AND AT PULSE40



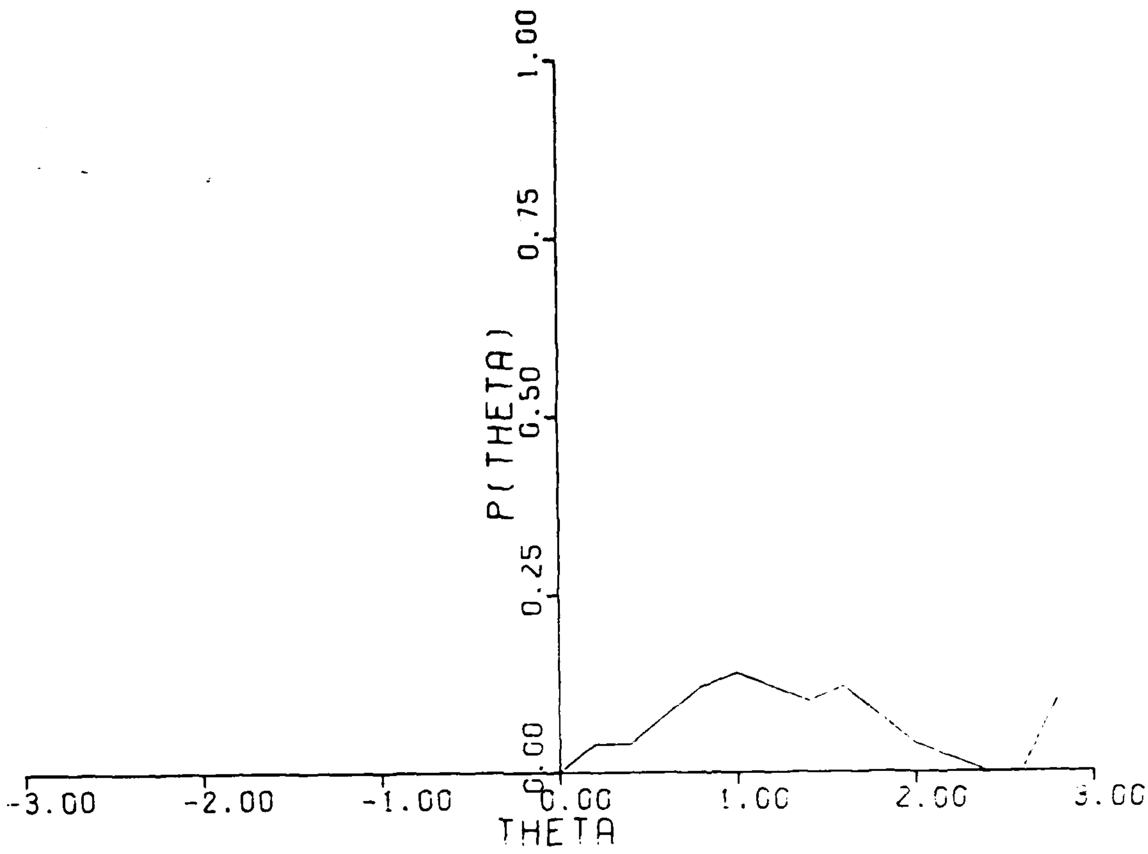
SIGNAL TO NOISE RATIO =2?. ?DB

AT PULSE 50

MEAN = 1.52

VARIANCE = .05

FIG E12 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 02...0DB
AND AT PULSE50



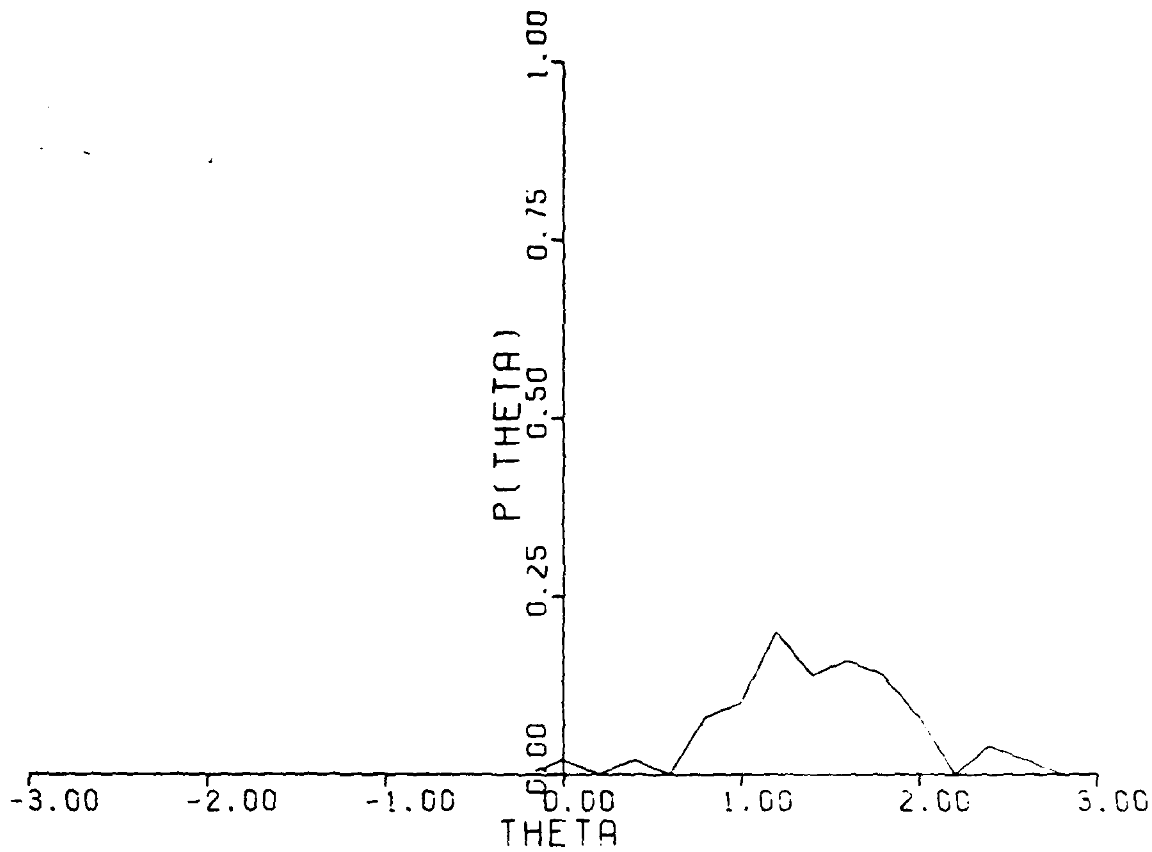
SIGNAL TO NOISE RATIO =13.98DB

AT PULSE 1

MEAN = 1.43

VARIANCE = .46

FIG E13 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 013.98DB
AND AT PULSE 1



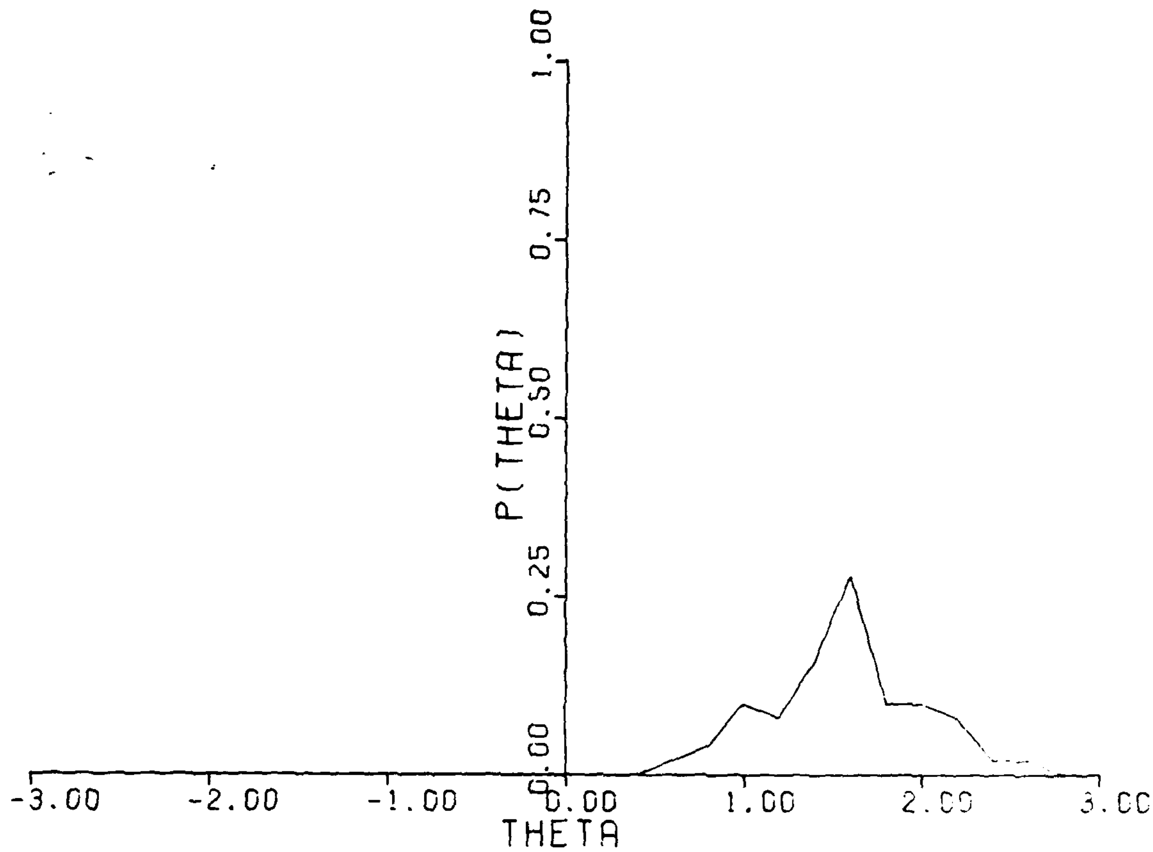
SIGNAL TO NOISE RATIO =13.98DB

AT PULSE 10

MEAN = 1.52

VARIANCE = .24

FIG E14 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO D13.98DB
AND AT PULSE10



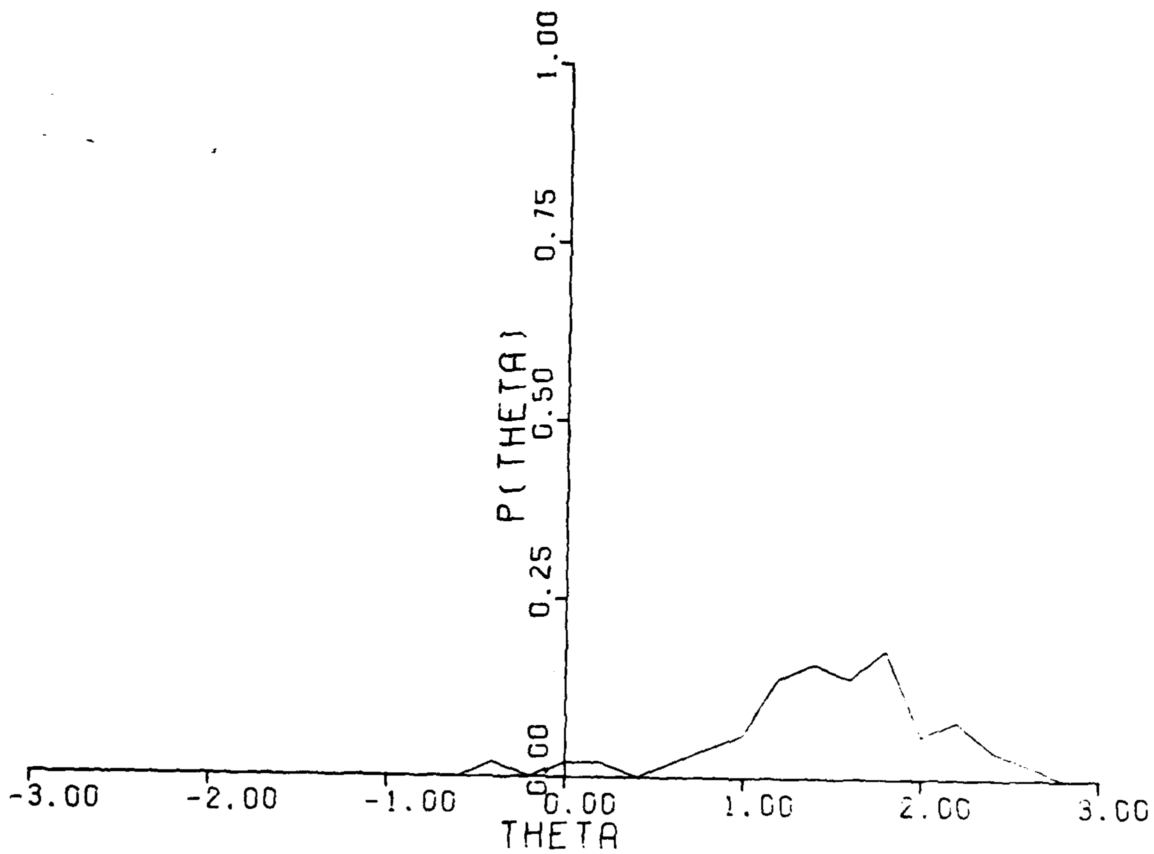
SIGNAL TO NOISE RATIO =13.98DB

AT PULSE 20

MEAN = 1.67

VARIANCE = .18

FIG E15 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO D13.96DB
AND AT PULSE20



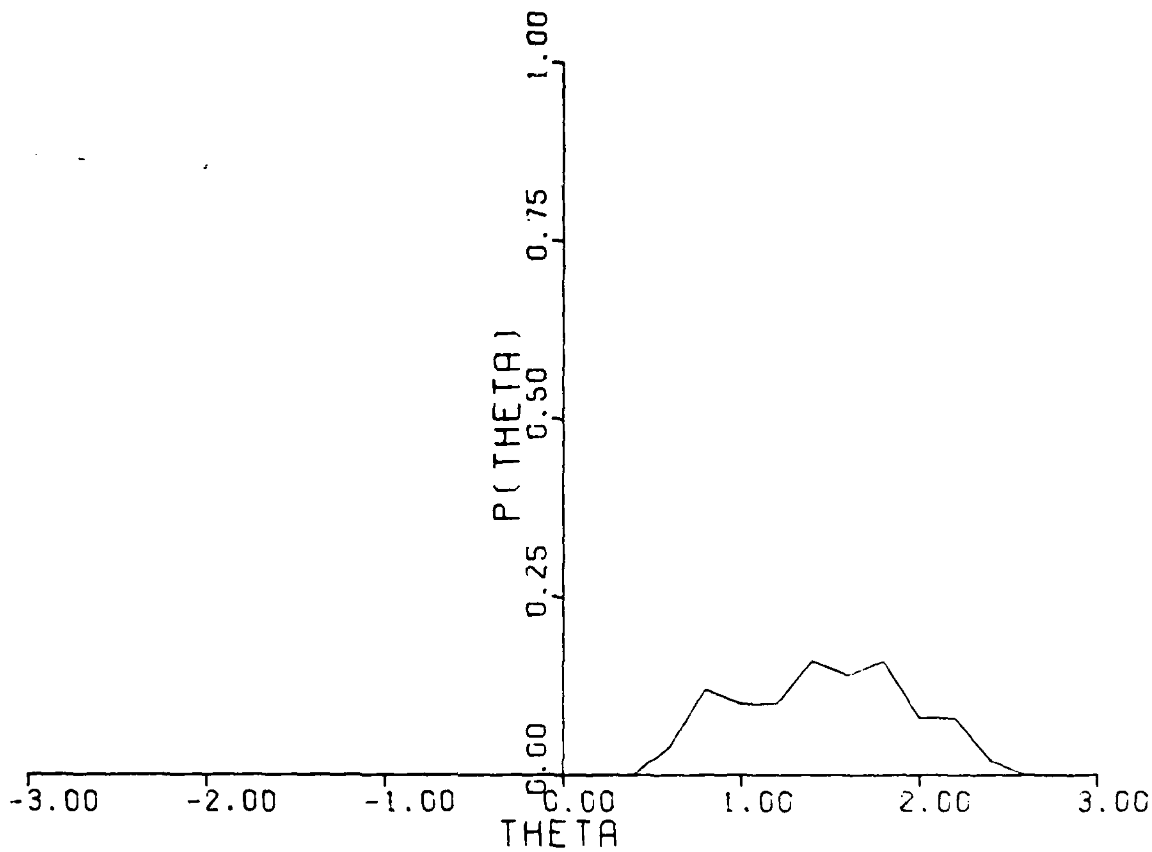
SIGNAL TO NOISE RATIO =13.98DB

AT PULSE 30

MEAN = 1.58

VARIANCE = .34

FIG E16 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO O13.98DB
AND AT PULSE30



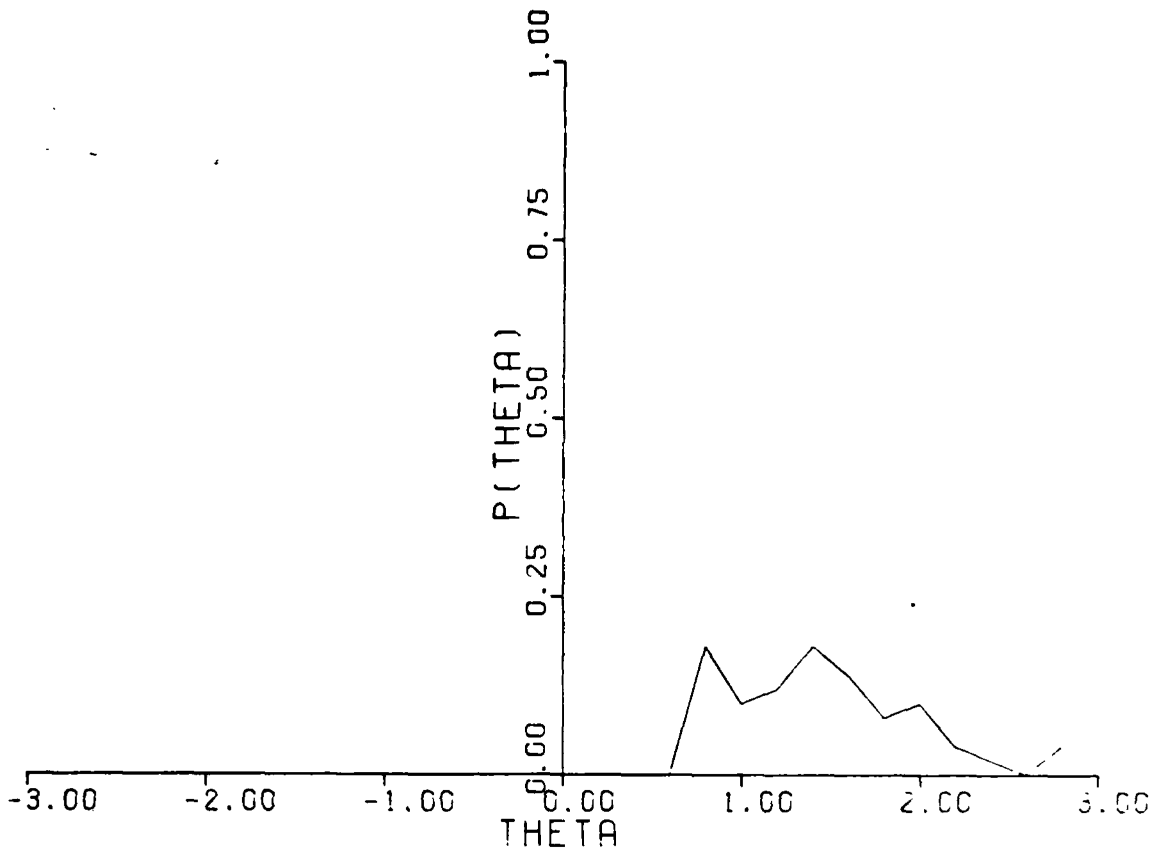
SIGNAL TO NOISE RATIO =13.98DB

AT PULSE 40

MEAN = 1.56

VARIANCE = .22

FIG E17 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO O13.98DB
AND AT PULSE40



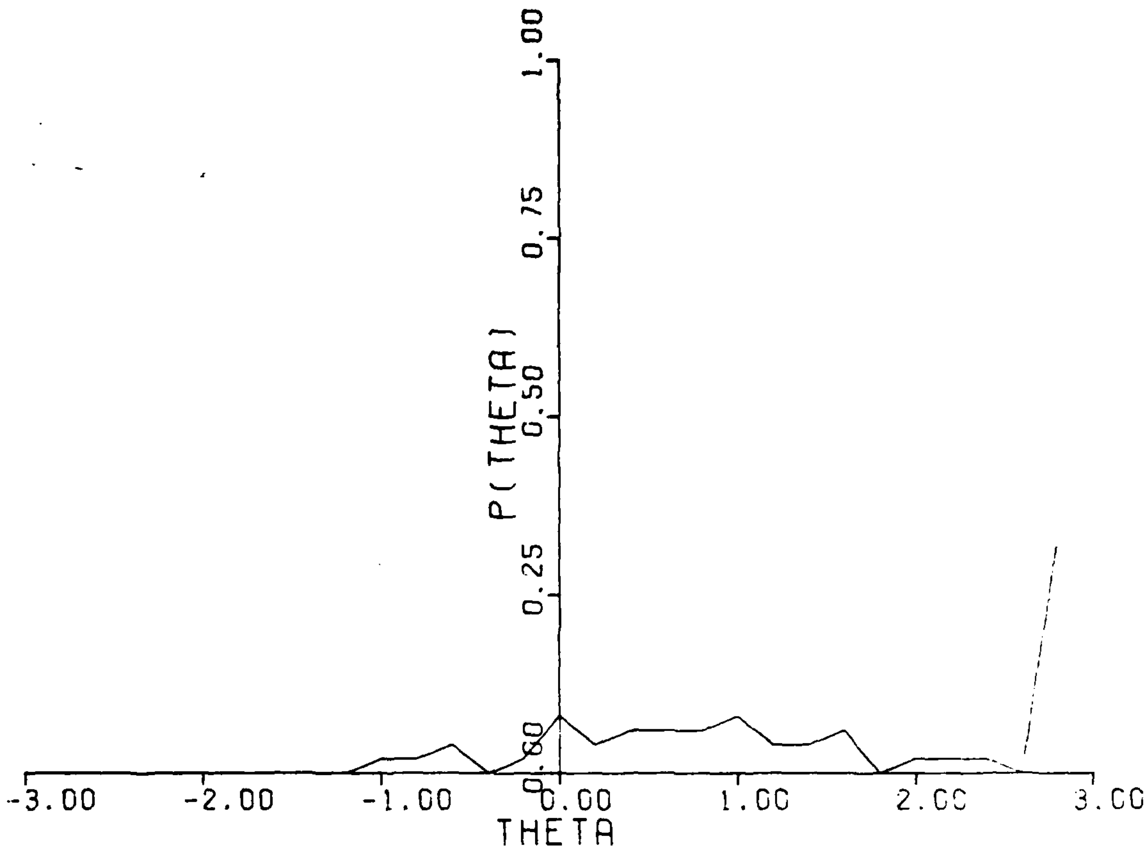
SIGNAL TO NOISE RATIO =13.98DB

AT PULSE 50

MEAN = 1.56

VARIANCE = .26

FIG E18 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO O13.98DB
AND AT PULSE50



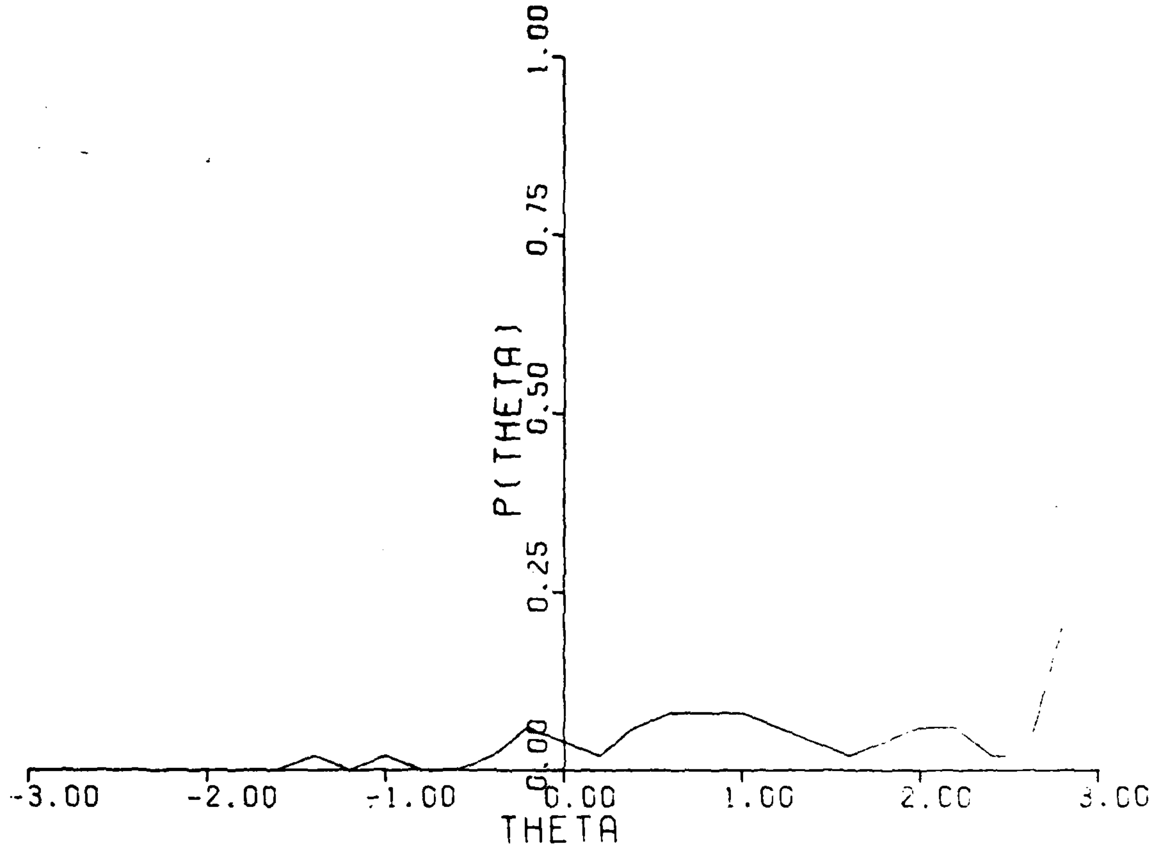
SIGNAL TO NOISE RATIO = 6.02DB

AT PULSE 1

MEAN = 1.46

VARIANCE = 1.44

FIG E19 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO D 6.02DB
AND AT PULSE 1



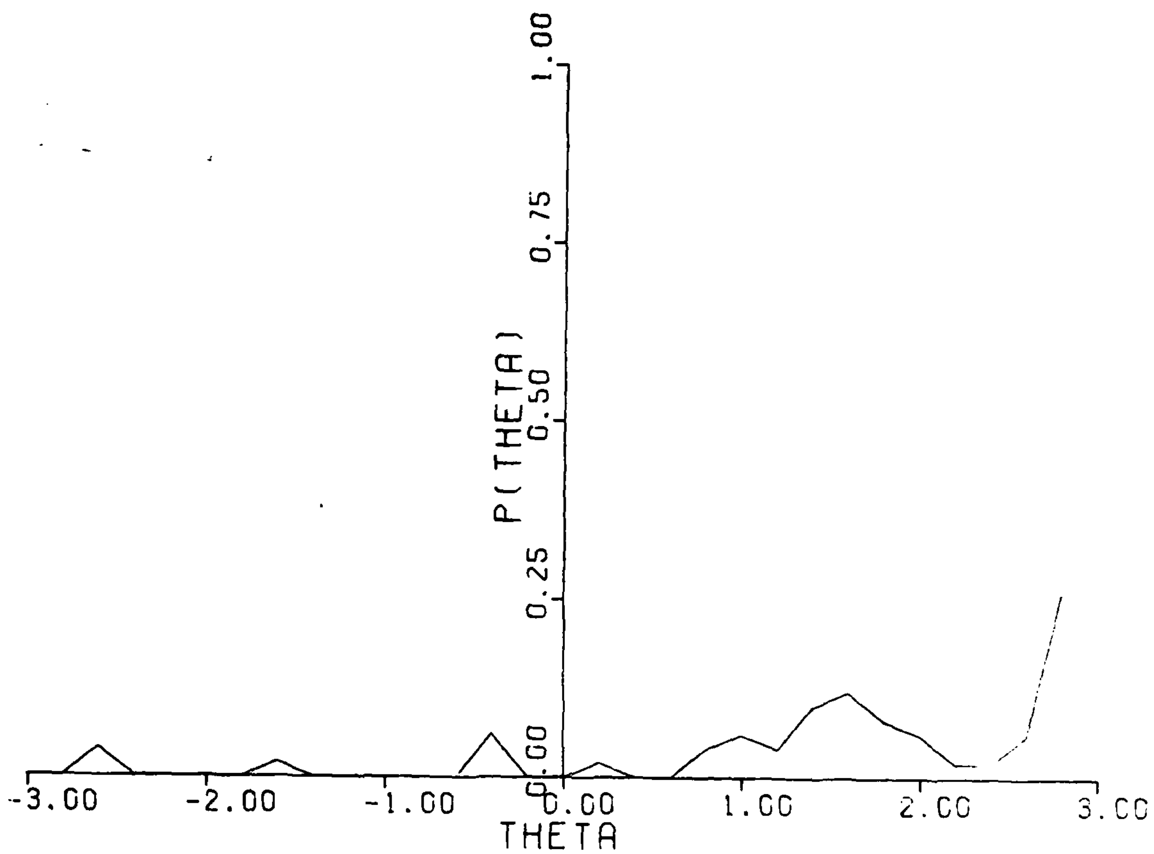
SIGNAL TO NOISE RATIO = 6.'2DB

AT PU_SE 10

MEAN = 1.40

VARIANCE = 1.25

FIG E20 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO O 6.02DB
AND AT PULSE10



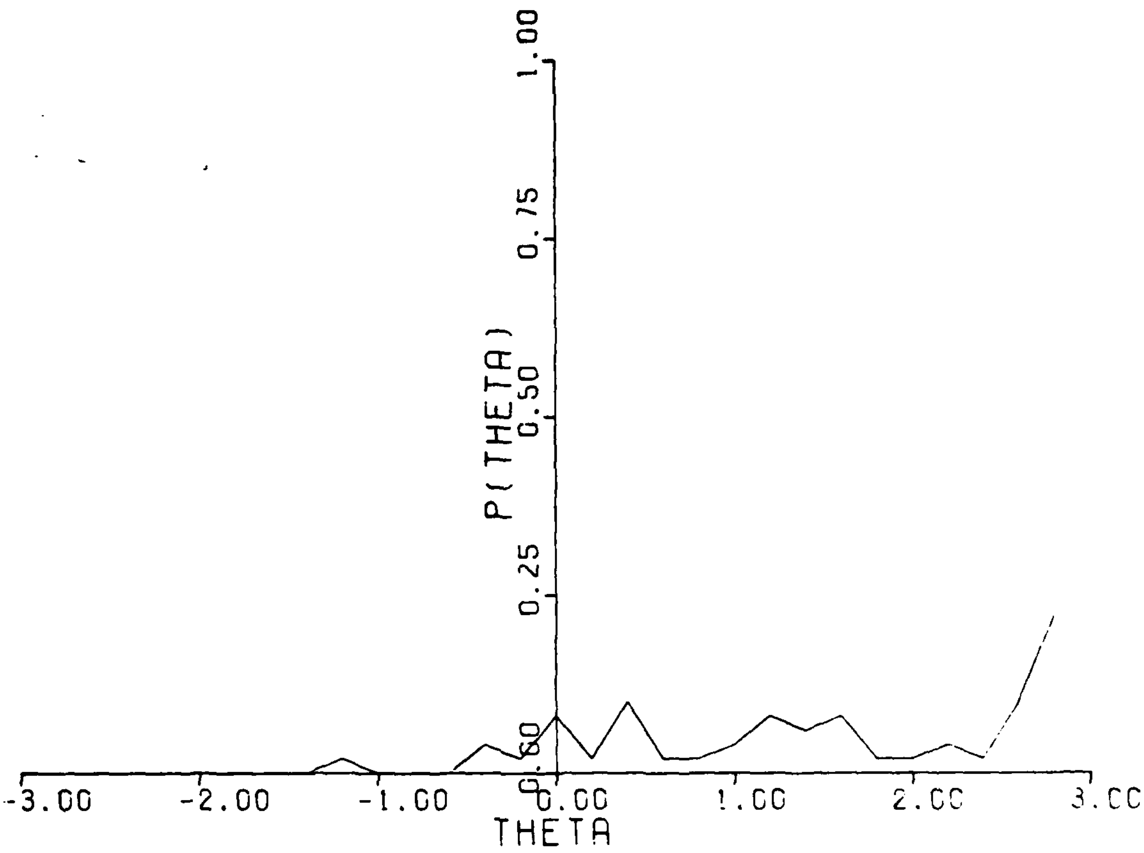
SIGNAL TO NOISE RATIO = 6.02DB

AT PULSE 20

MEAN = 1.66

VARIANCE = 1.71

FIG E21 PLOT OF THE PROBALITY DENSITY FUNCTION OF THETA ESTIMATE VERS THETA FOR SIGNAL TO NOISE RATIO D 6.02DB AND AT PULSE20



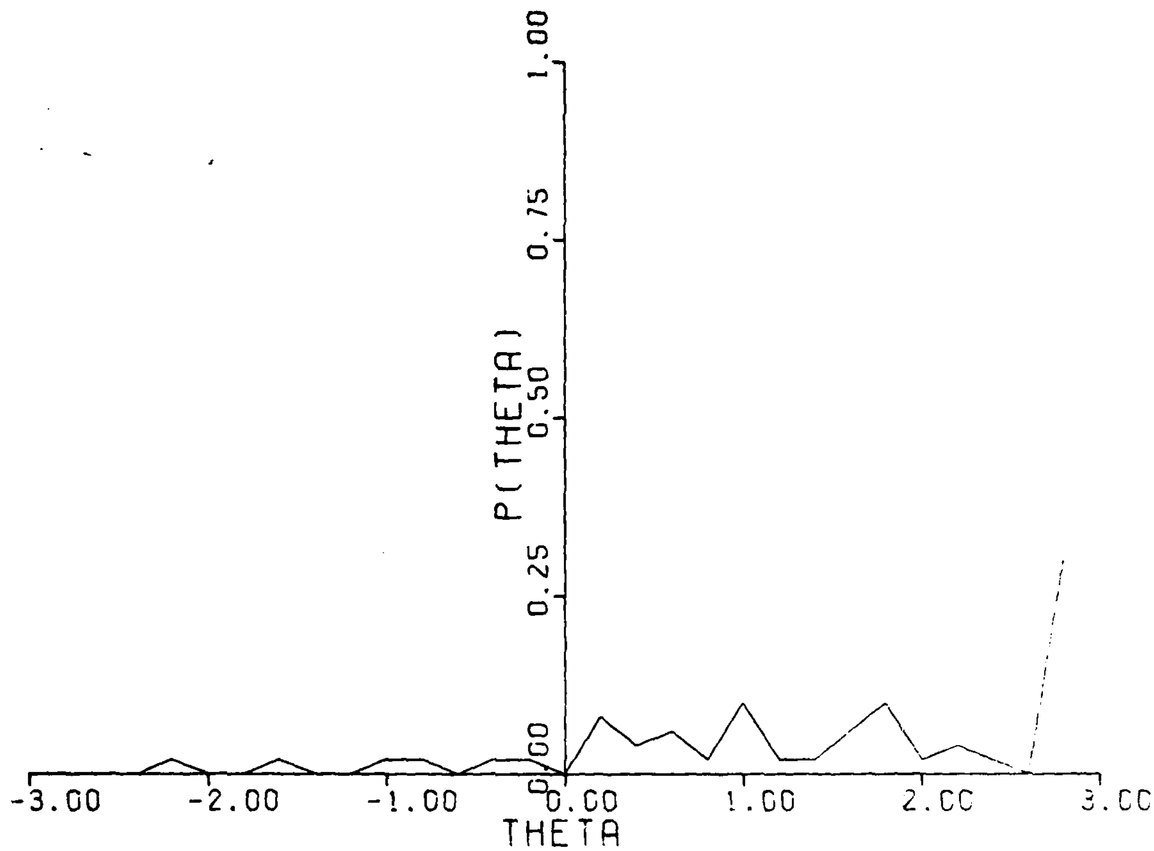
SIGNAL TO NOISE RATIO = 6.02DB

AT PULSE 30

MEAN = 1.56

VARIANCE = 1.26

FIG E22 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO O 6.02DB
AND AT PULSE 30



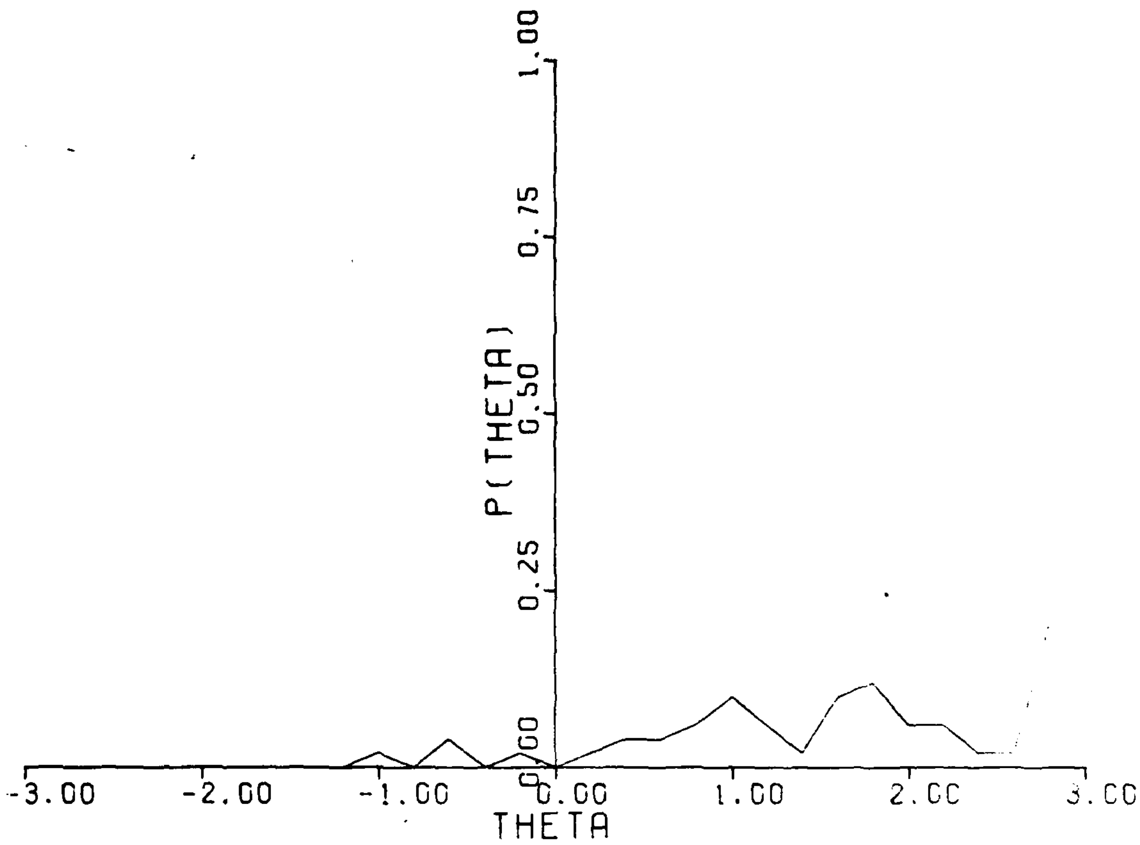
SIGNAL TO NOISE RATIO = 6.˜2DB

AT PULSE 4C

MEAN = 1.5C

VARIANCE = 1.64

FIG E23 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO D 5.620B
AND AT PULSE40



SIGNAL TO NOISE RATIO = 6.?2DB

AT PULSE 50

MEAN = 1.63

VARIANCE = .97

FIG E24 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO D 6.620B
AND AT PULSE50

APPENDIX F

Moving Target Estimate Results

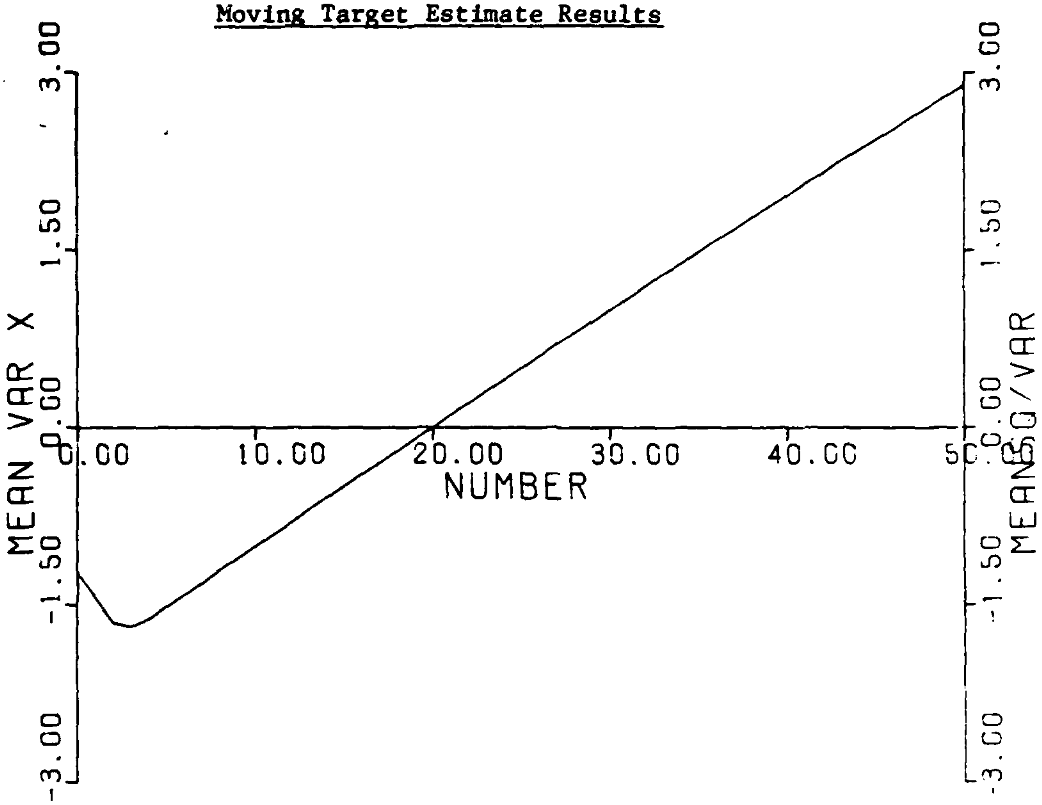
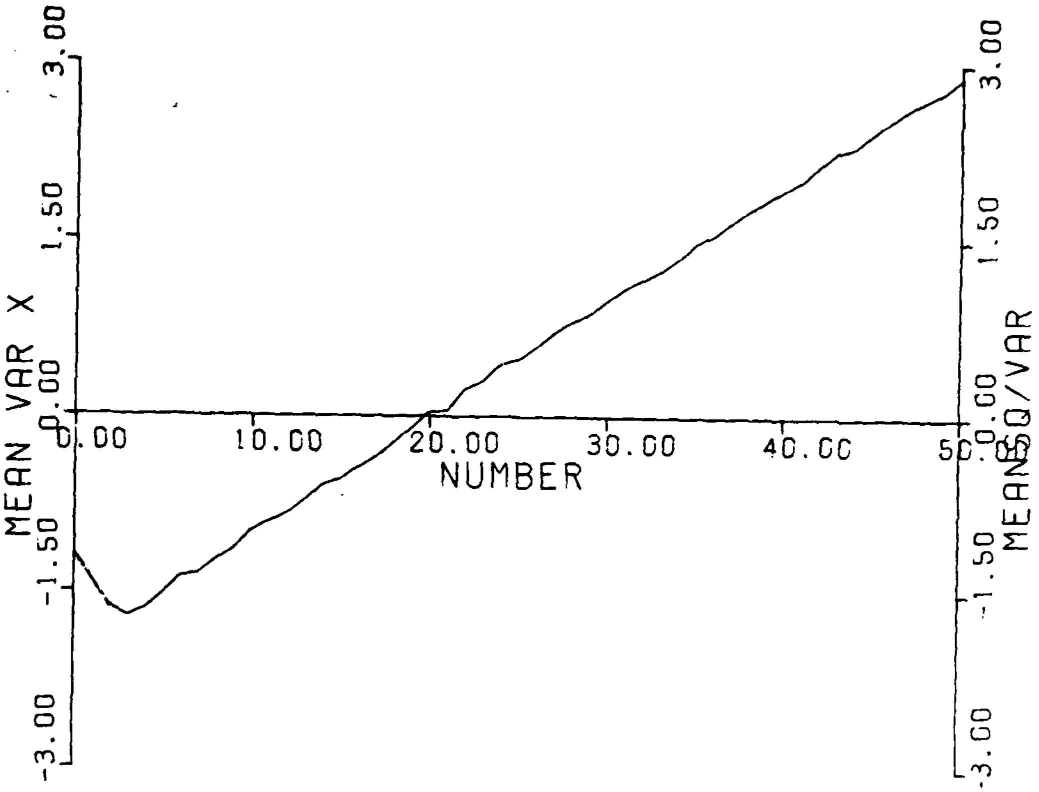


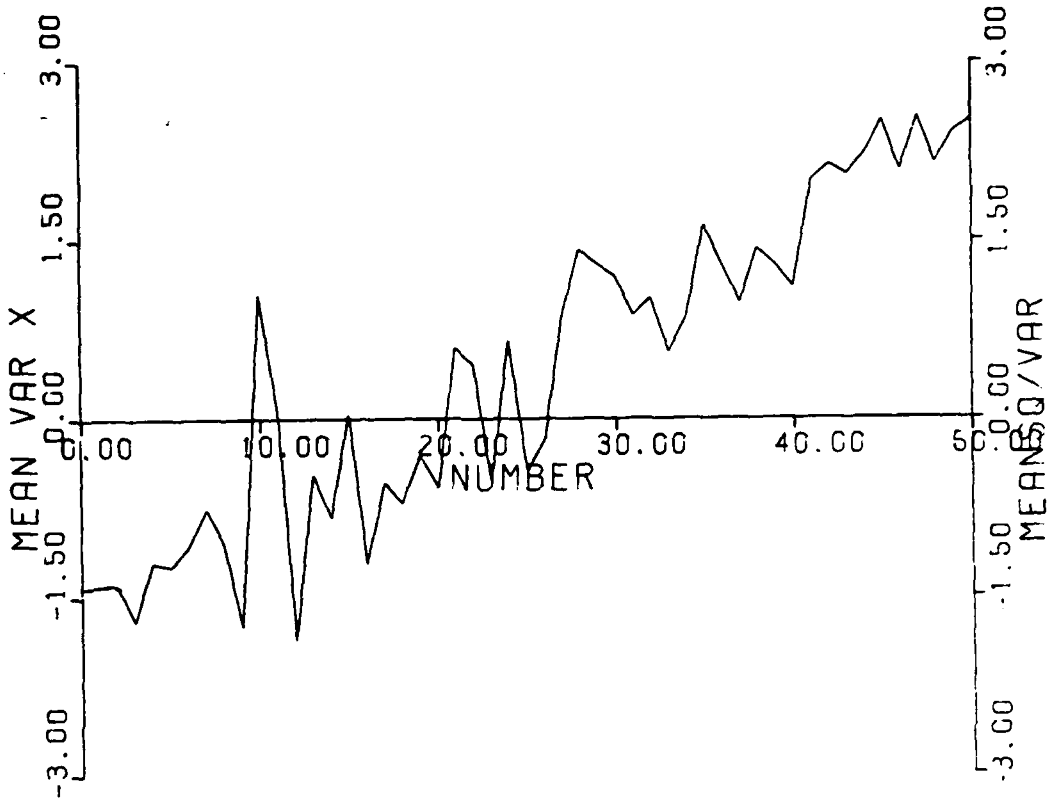
Fig F1



SIGNAL TO NOISE RATIO =40.[illegible]DB

AT (I) PULSES	1	25	5[illegible]
ESTIMATE - =	-1.2[illegible]	.49	2.89

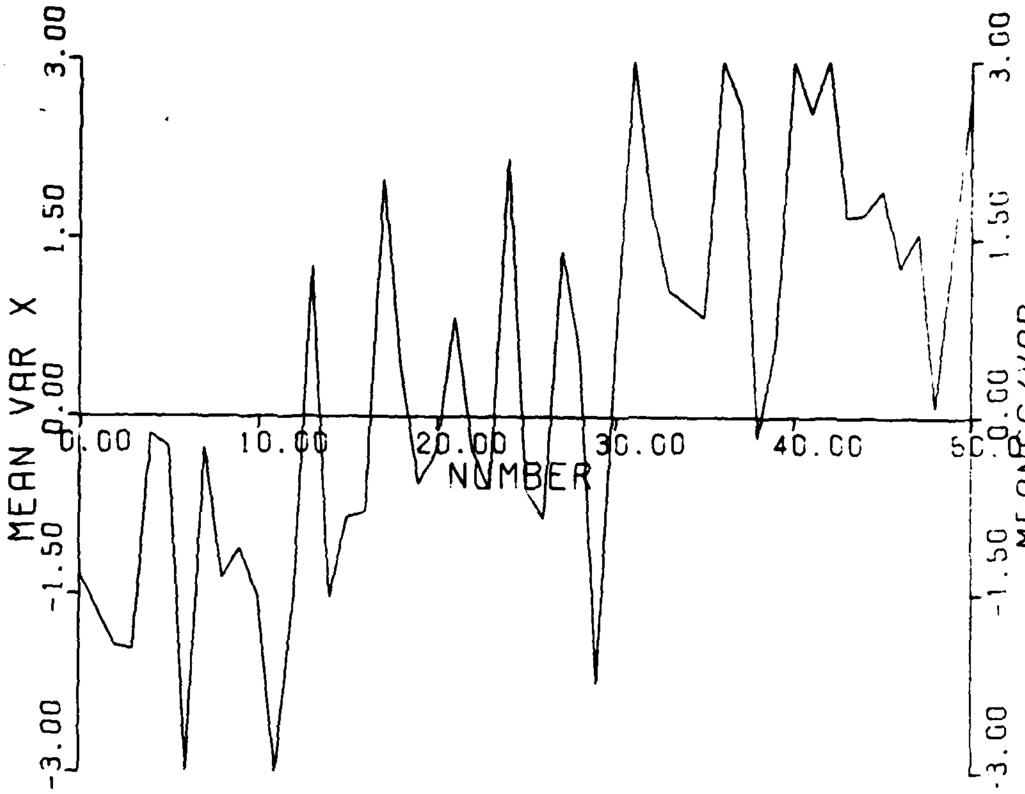
FIG F2 PLOT OF ESTIMATE FOR SIGNAL TO NOISE RATIO OF40.00DB AND A MOVING TARGET.



SIGNAL TO NOISE RATIO =13.98DB

AT (I) PULSES	1	25	50
ESTIMATE - =	-1.42	-.44	2.51

FIGF3 PLOT OF ESTIMATE FOR SIGNAL TO NOISE RATIO OF13.98DB AND A MOVING TARGET.



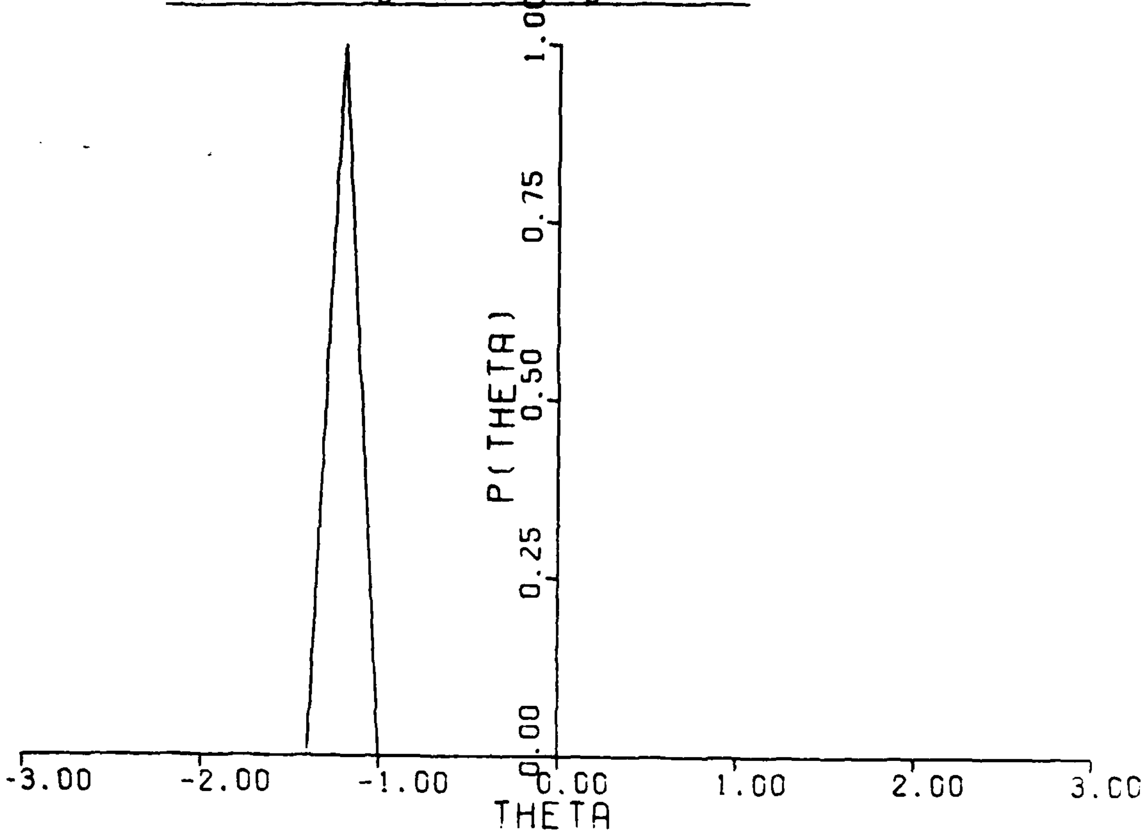
SIGNAL TO NOISE RATIO = 6.02DB

AT (I) PULSES	1	25	50
ESTIMATE - =	-1.33	-.62	2.68

FIGF4 PLOT OF ESTIMATE FOR SIGNAL TO NOISE RATIO OF 6.02DB AND A MOVING TARGET.

APPENDIX G

Moving Target Probability Density
Function of Angle Off Boresight Results



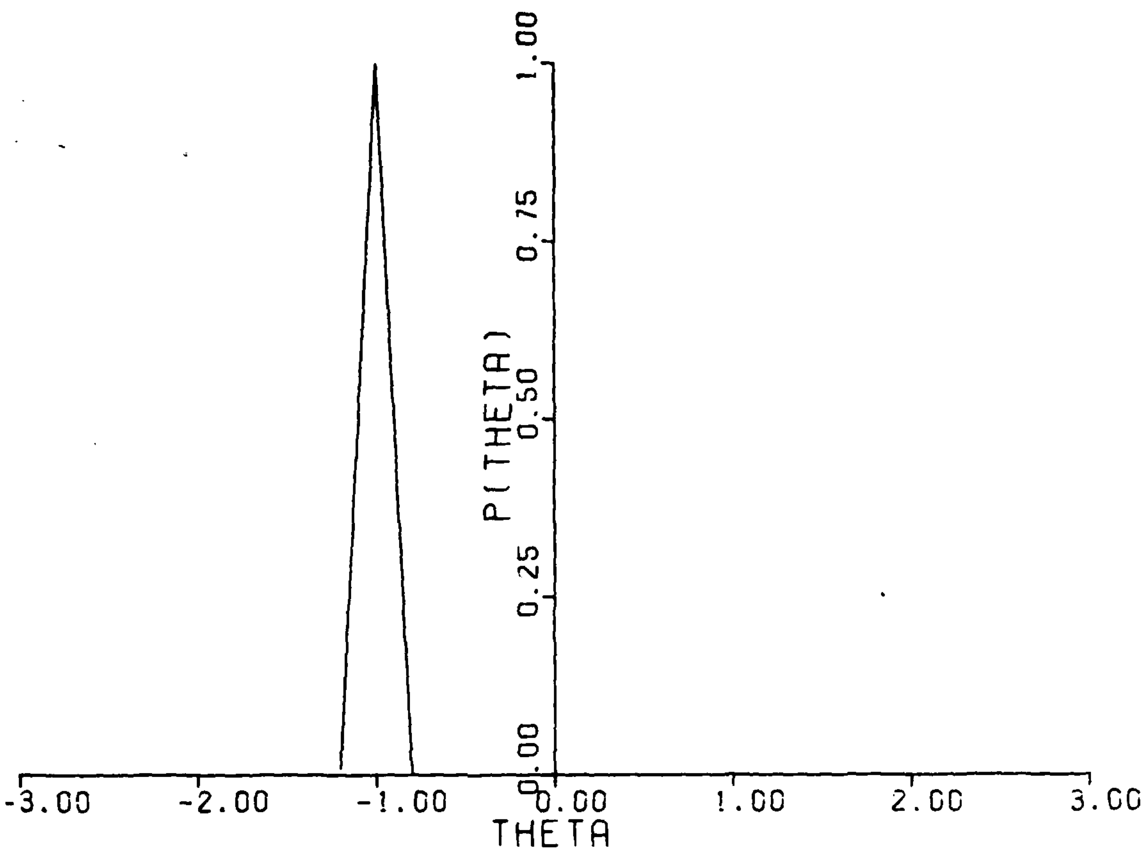
SIGNAL TO NOISE RATIO =49.00DB

AT PULSE 1

MEAN = -1.10

VARIANCE = 0.00

FIG G1 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 041.00DB
AND AT PULSE 1



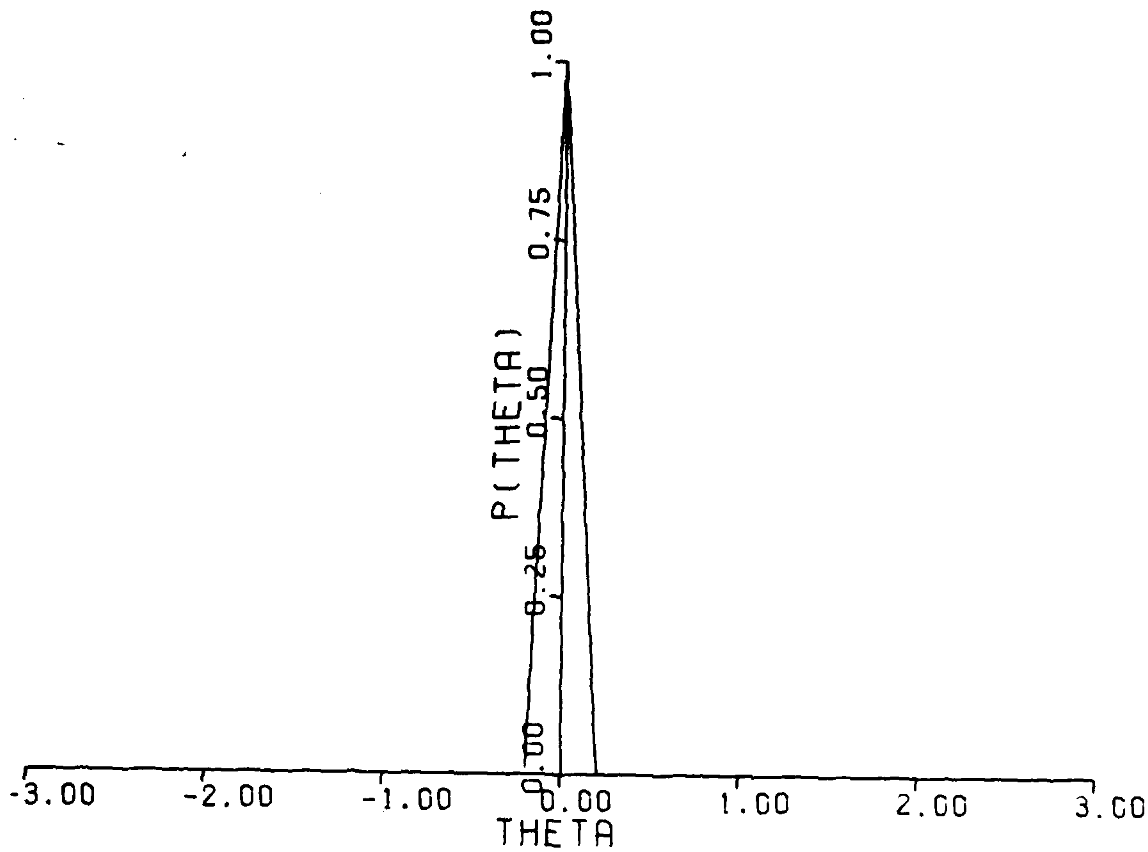
SIGNAL TO NOISE RATIO =4[illegible]DB

AT PULSE 10

MEAN = -.90

VARIANCE = 0.00

FIG G2 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 04[illegible]DB
AND AT PULSE10



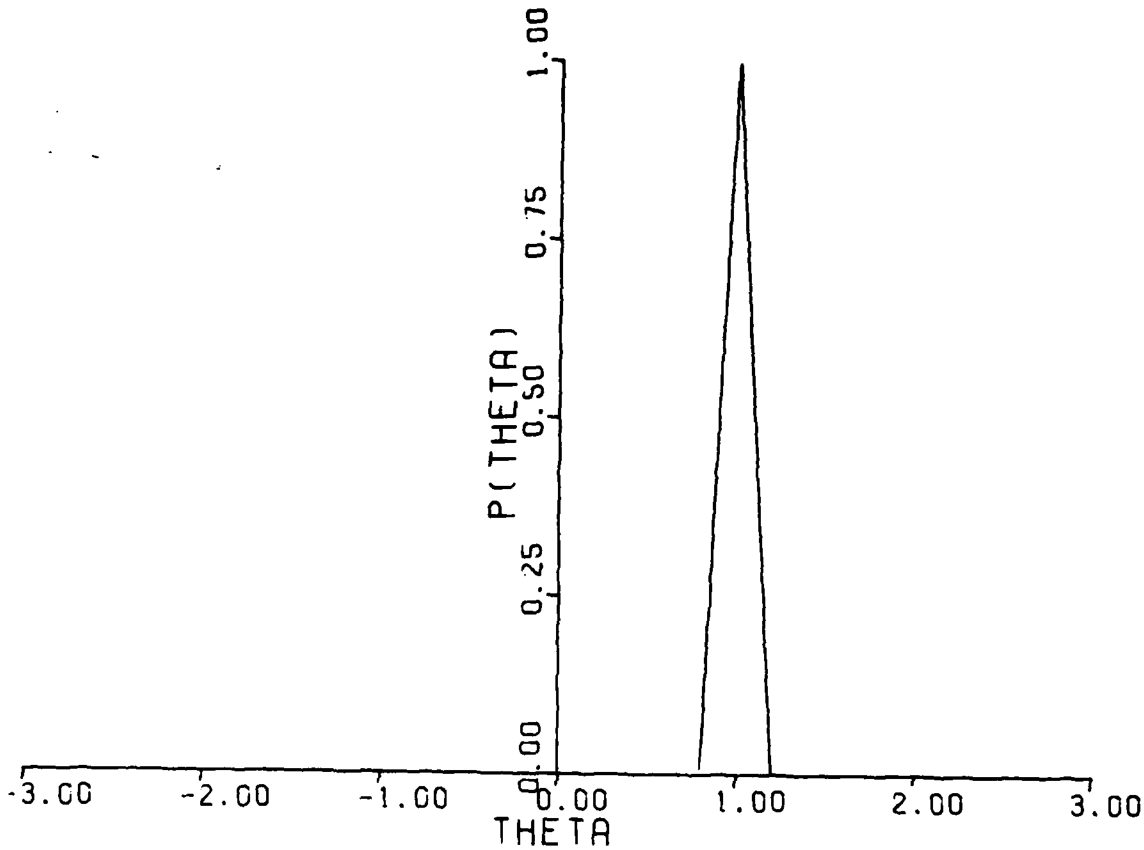
SIGNAL TO NOISE RATIO =4 9. '0DB

AT PULSE 20

MEAN = .10

VARIANCE = 0.00

FIG G3 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 04..00B
AND AT PULSE20



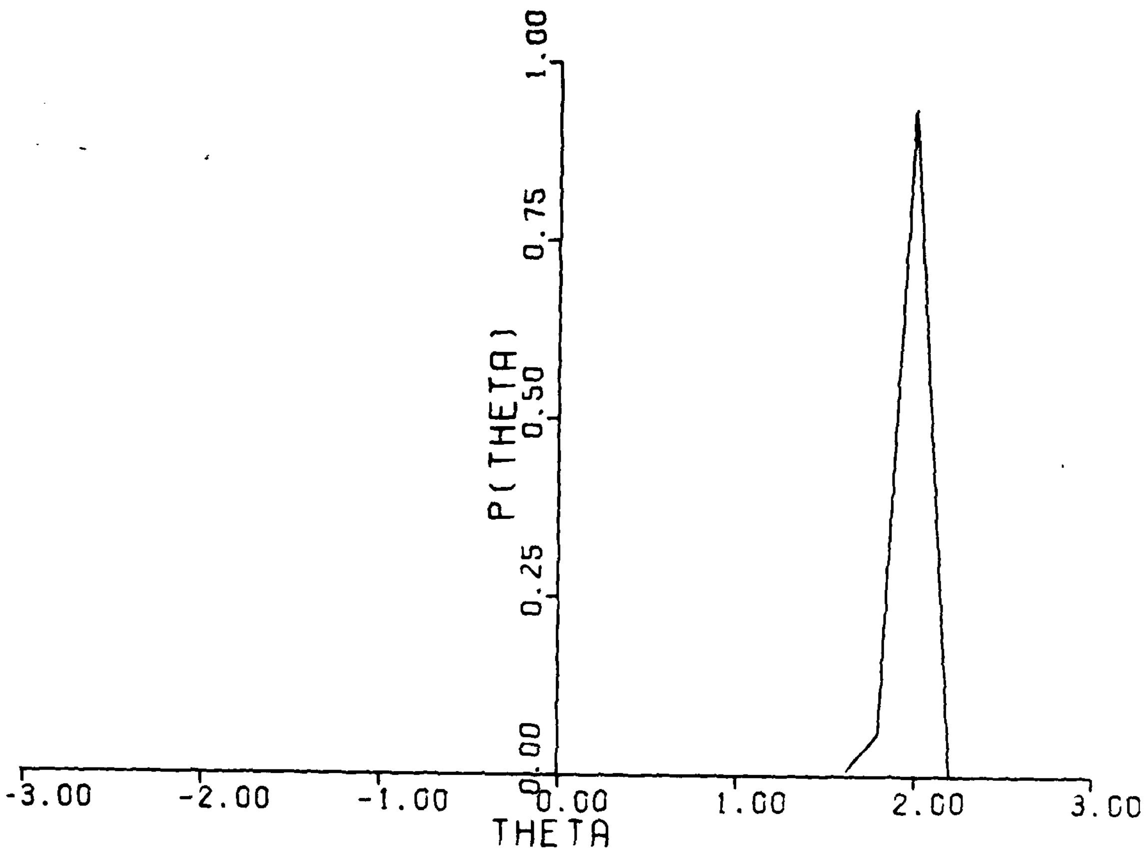
SIGNAL TO NOISE RATIO =40.'PDB

AT PULSE 30

MEAN = 1.10

VARIANCE = 0.00

FIG G4 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 040.CPDB
AND AT PULSE30



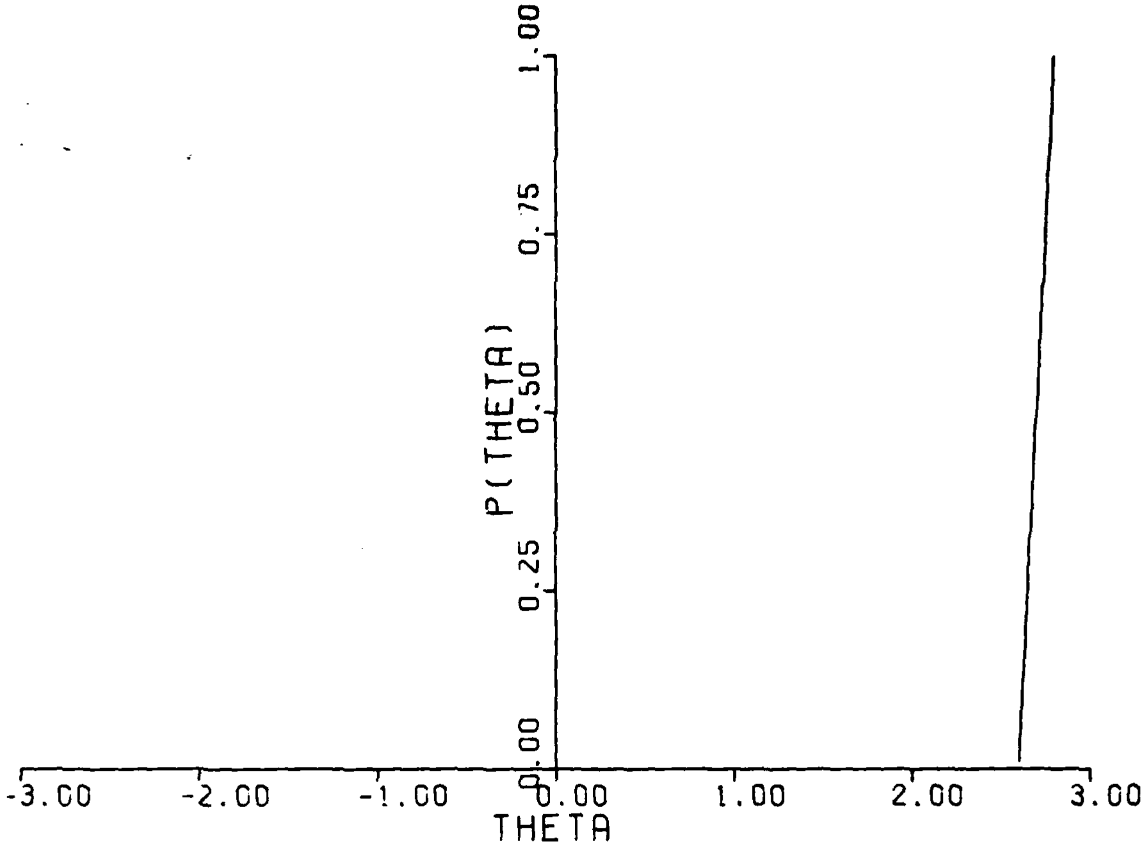
SIGNAL TO NOISE RATIO =40.00DB

AT PULSE 40

MEAN = 2.09

VARIANCE = .00

FIGG5 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 040.00DB
AND AT PULSE40



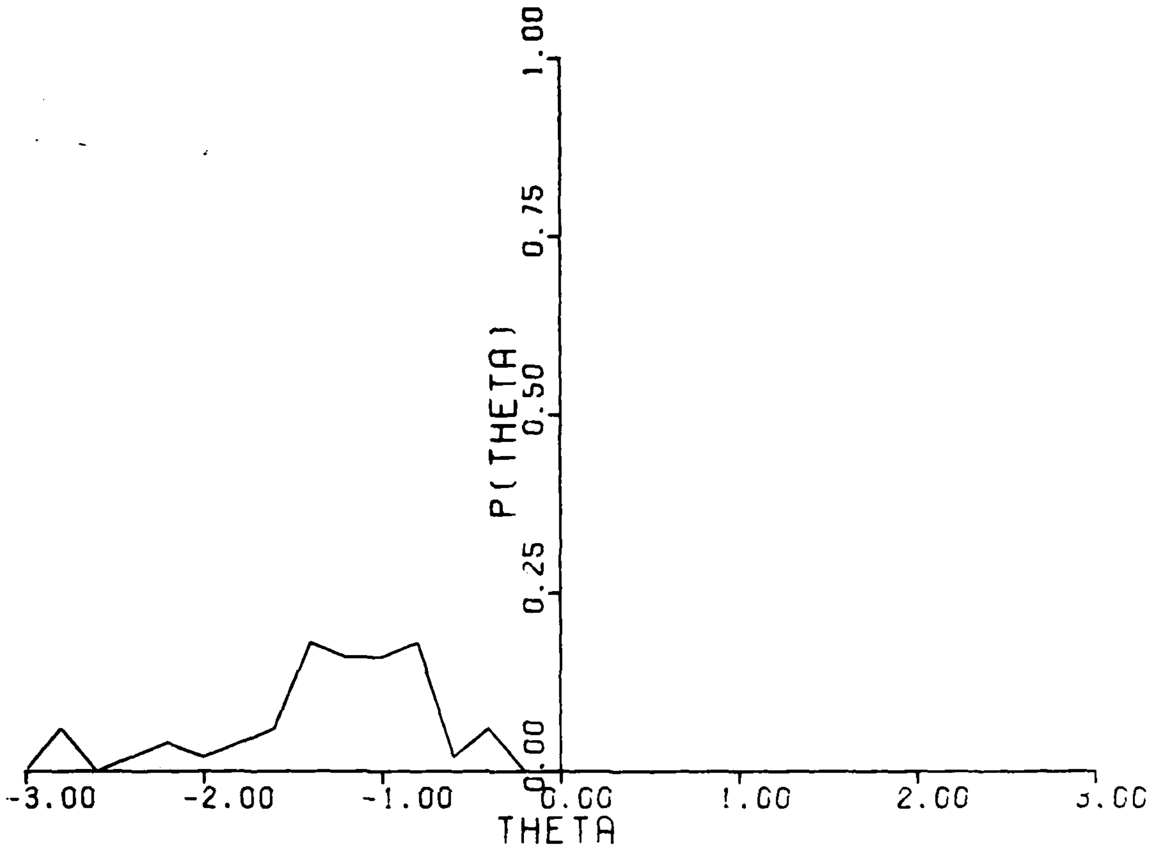
SIGNAL TO NOISE RATIO =40.00DB

AT PULSE 50

MEAN = 2.90

VARIANCE = 0.00

FIG G6 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 040.00DB
AND AT PULSE50



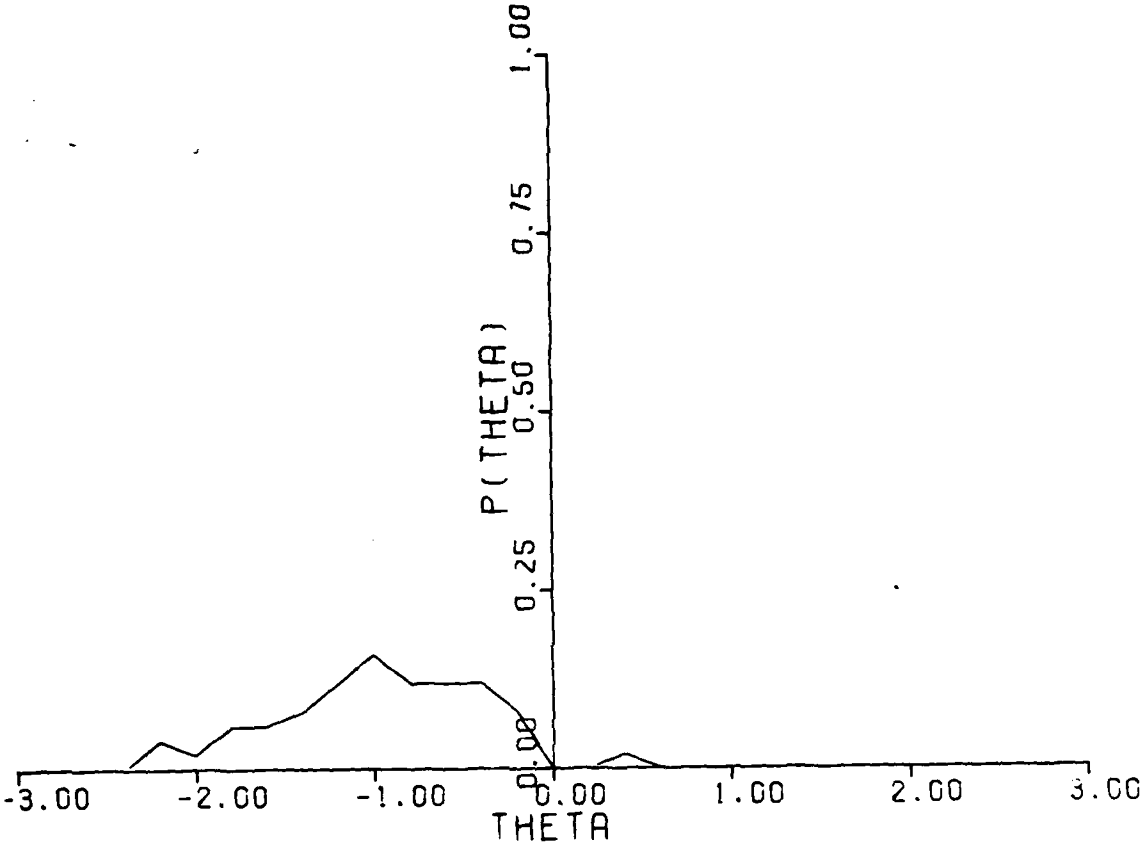
SIGNAL TO NOISE RATIO =13.98DB

AT PULSE 1

MEAN = -1.20

VARIANCE = .34

FIG G7 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 013.98DB
AND AT PULSE 1



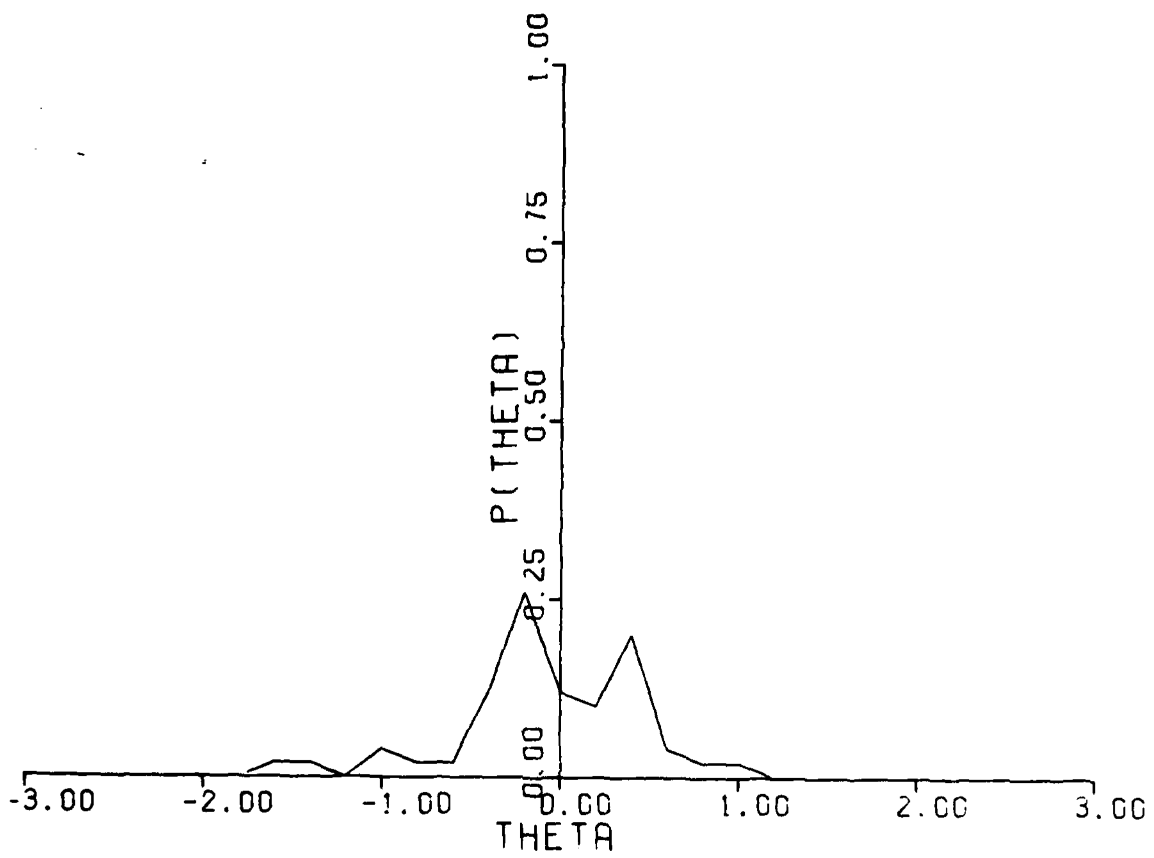
SIGNAL TO NOISE RATIO =13.98DB

AT PULSE 10

MEAN = -.87

VARIANCE = .31

FIG G8 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 013.98DB
AND AT PULSE10



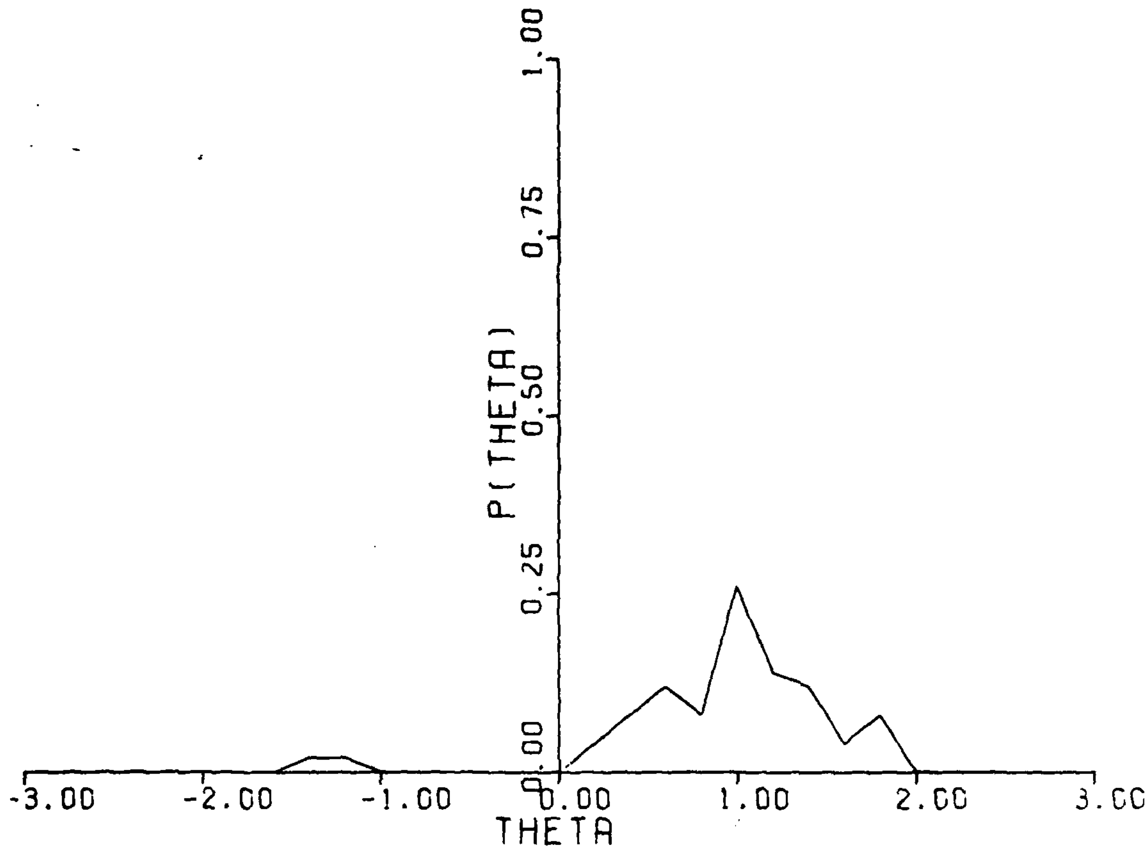
SIGNAL TO NOISE RATIO =13.98DB

AT PULSE 20

MEAN = .03

VARIANCE = .26

FIG G9 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 013.98DB
AND AT PULSE20



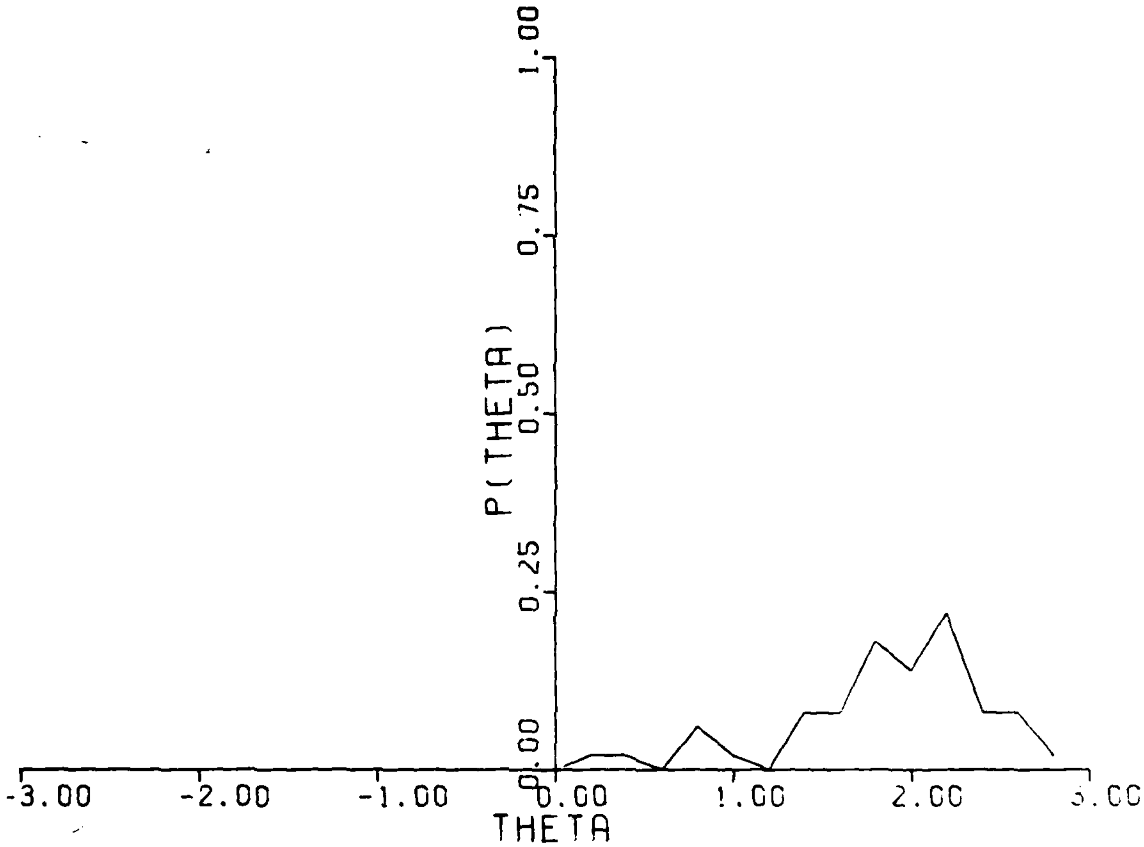
SIGNAL TO NOISE RATIO =13.98DB

AT PULSE 30

MEAN = 1.03

VARIANCE = .37

FIG G10 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 013.98DB
AND AT PULSE30



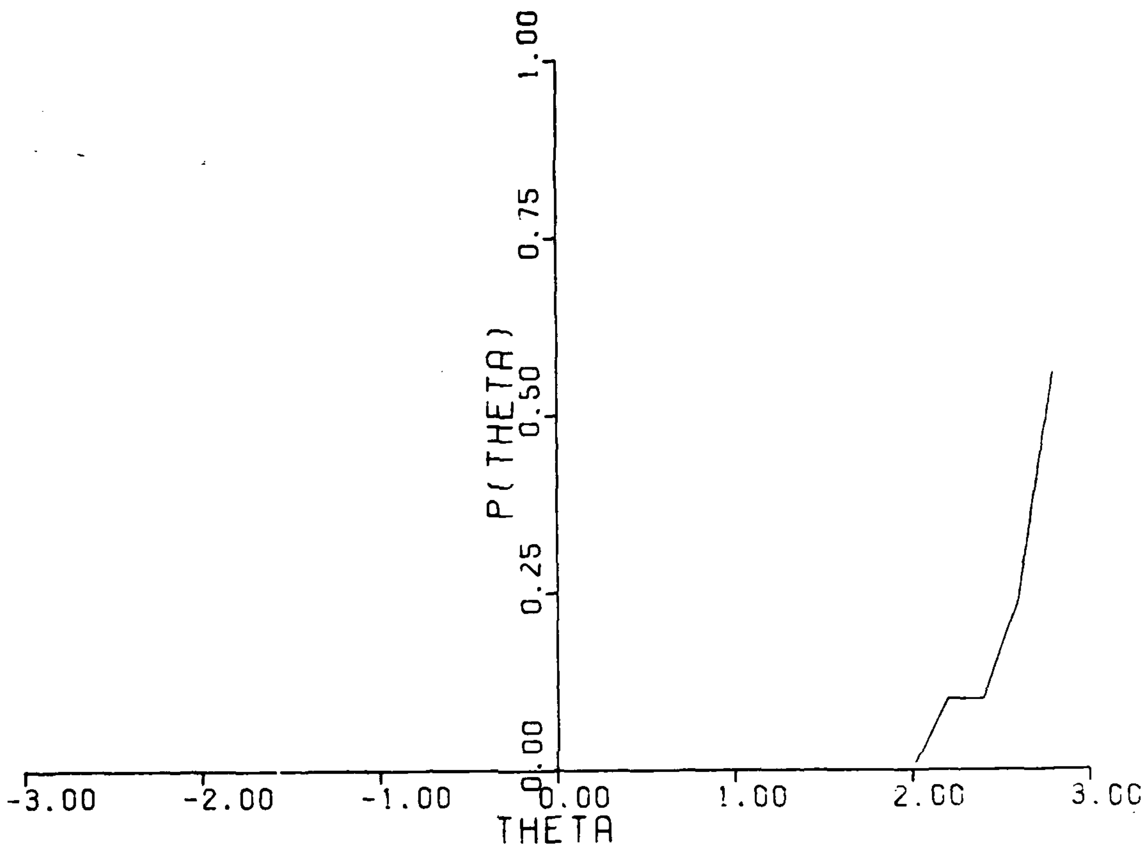
SIGNAL TO NOISE RATIO =13.38DB

AT PULSE 40

MEAN = 1.96

VARIANCE = .32

FIG G11 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 013.98DB
AND AT PULSE48



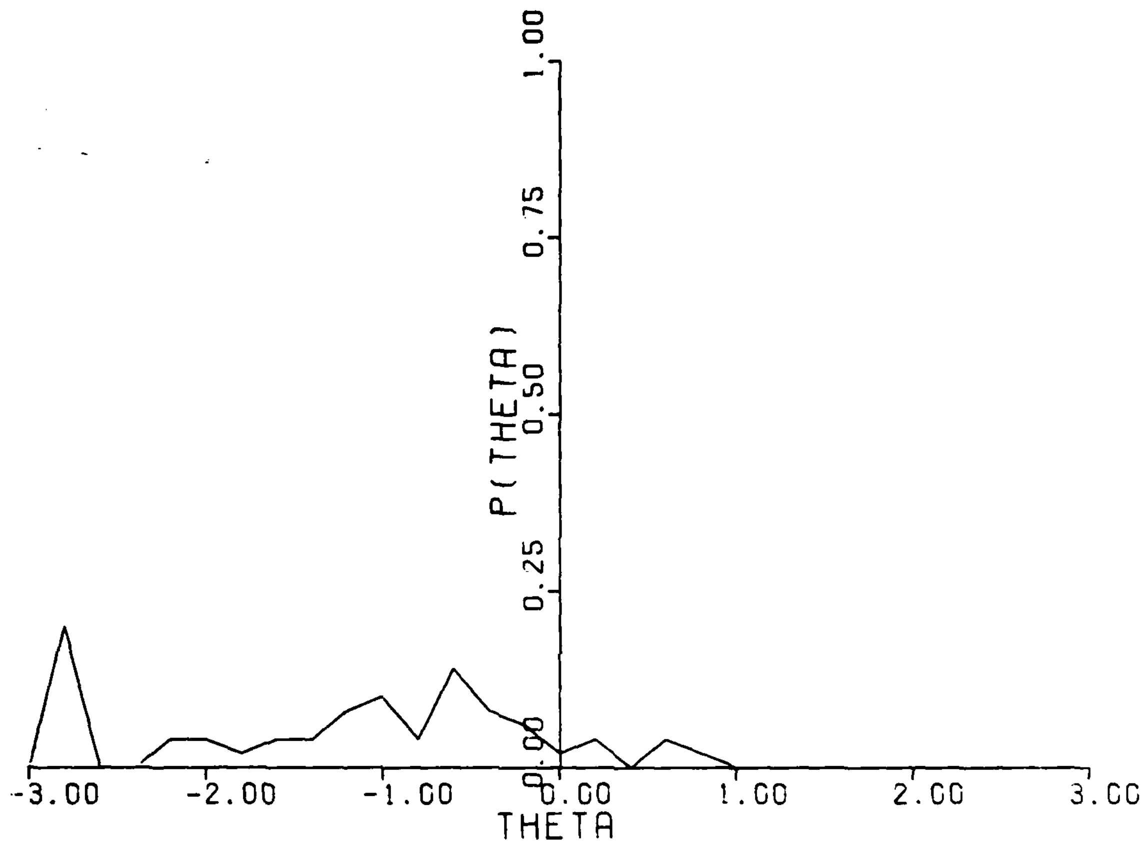
SIGNAL TO NOISE RATIO =13.98DB

AT PULSE 50

MEAN = 2.75

VARIANCE = .04

FIGG12 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO 013.98DB
AND AT PULSE50



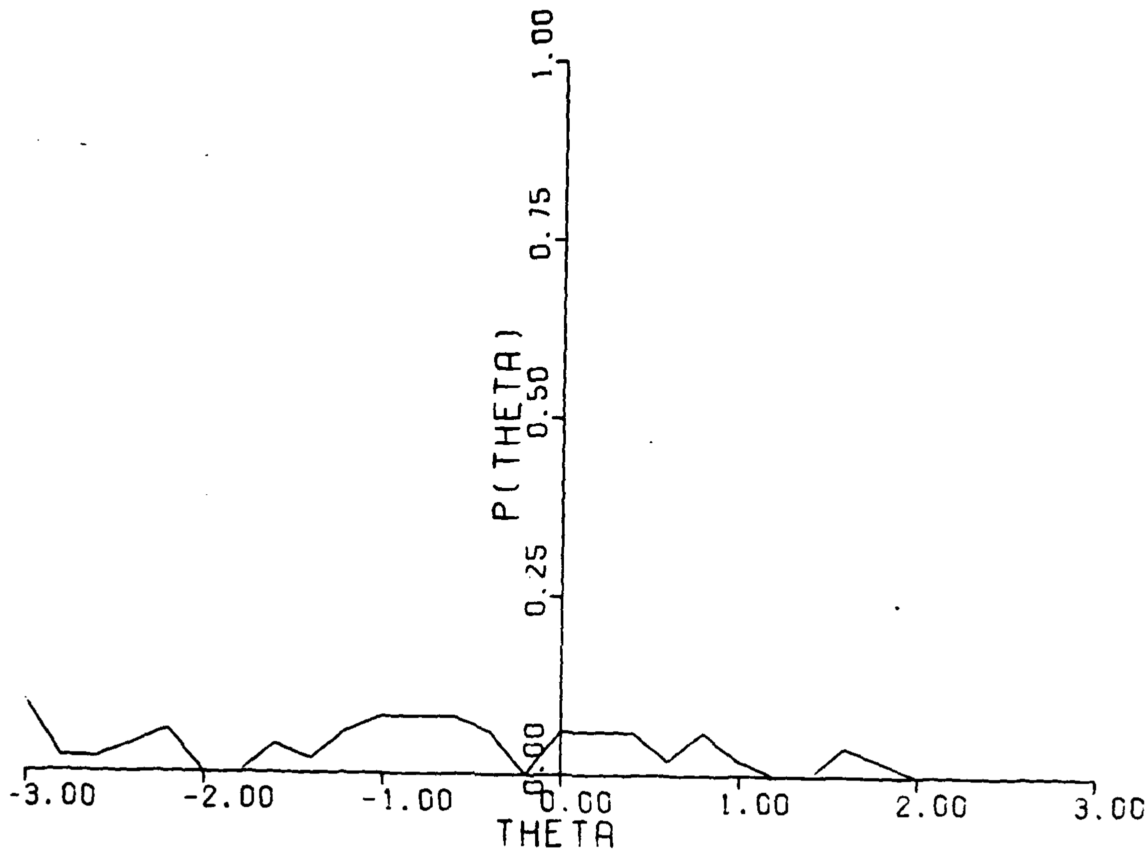
SIGNAL TO NOISE RATIO = 6.0208

AT PULSE 1

MEAN = -1.09

VARIANCE = 1.08

FIG G13 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO O 5.6208
AND AT PULSE 1



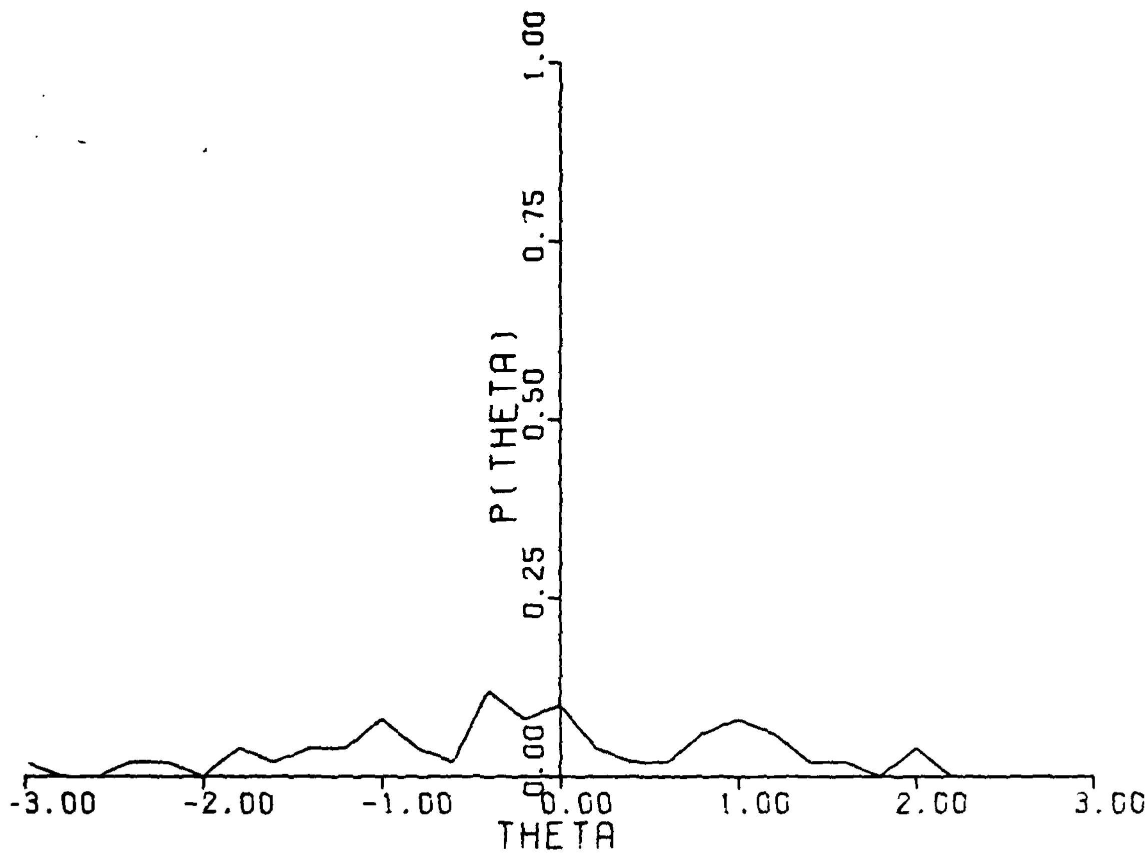
SIGNAL TO NOISE RATIO = 6.02DB

AT PULSE 10

MEAN = -.70

VARIANCE = 1.72

FIG G14 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO D 6.02DB
AND AT PULSE10



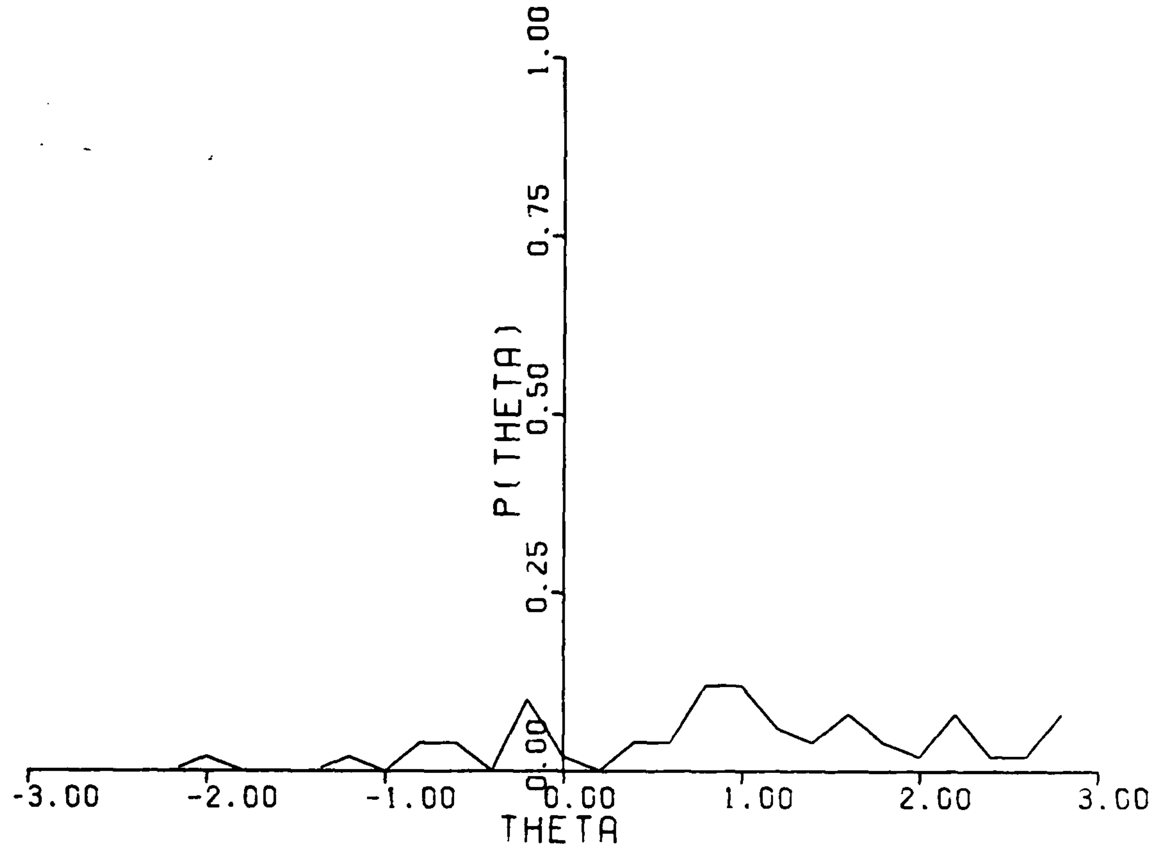
SIGNAL TO NOISE RATIO = 6.12DB

AT PULSE 20

MEAN = -.38

VARIANCE = 1.28

FIG G15 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO O 6.02DB
AND AT PULSE20



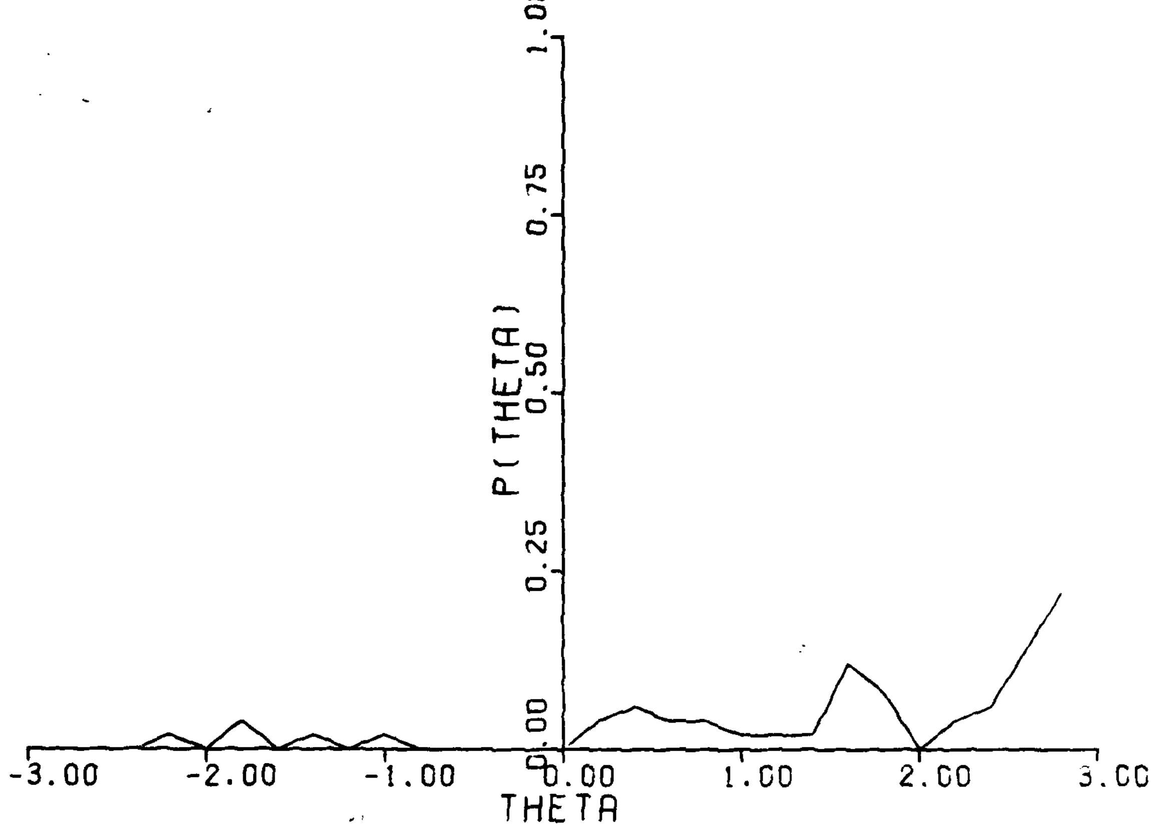
SIGNAL TO NOISE RATIO = 6.02DB

AT PULSE 30

MEAN = 1.08

VARIANCE = 1.25

FIG G16 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO O 5.02DB
AND AT PULSE30



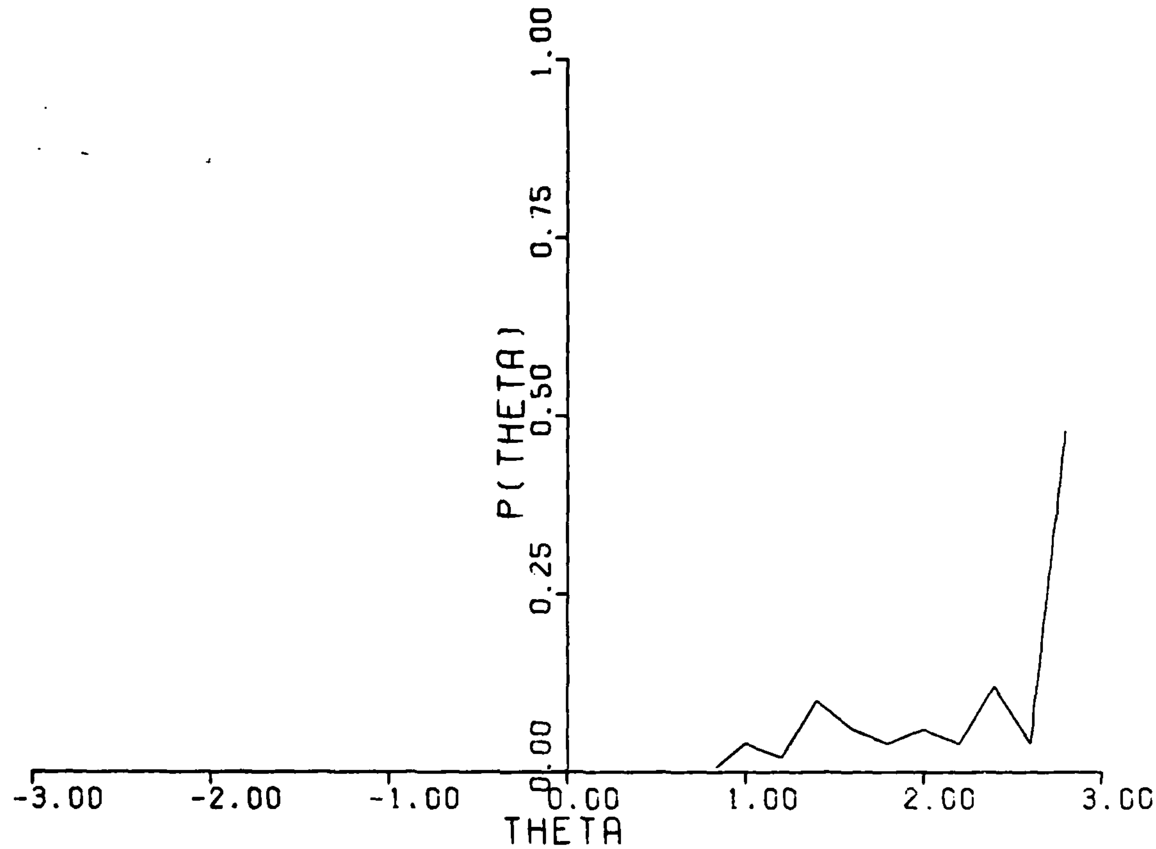
SIGNAL TO NOISE RATIO = 6.'20B

AT PULSE 4C

MEAN = 1.64

VARIANCE = 1.82

FIG G17 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO O 6.020B
AND AT PULSE40



SIGNAL TO NOISE RATIO = 6.02DB

AT PULSE 50

MEAN = 2.42

VARIANCE = .34

FIG G18 PLOT OF THE PROBALITY DENSITY FUNCTION
OF THETA ESTIMATE VERS THETA
FOR SIGNAL TO NOISE RATIO O 6.02DB
AND AT PULSE50

VITA

Peter M. Cronk was born on 20 September 1949 in Gympie, Queensland, Australia. He graduated from high school in Cairns, Queensland in 1966, at which time he enlisted in the Royal Australian Air Force. In 1972 he was awarded a cadetship at Diploma Cadet Squadron where he attended the Royal Melbourne Institue of Technology, (RMIT). He received the Diploma of Communications Engineering from RMIT in December 1976, and graduated from Officer Training School in May 1977. After commissioning, he served as an instructor at School of Radio, RAAF Base, Laverton until entering the School of Engineering of the Air Force Institue of Technology in June 1980.

Permanent Address: 47 Dover St.
St. Albans,
Victoria,
3021
Australia

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~~FREDERIC C. LYNCH~~
Director ~~of Public~~ Affairs

Lynn Wolaver
LYNN E. WOLAVER
Dean for Research and Professional Development

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Dean for Research and Professional Development
Air Force Institute of Technology (ATC)
Wright-Patterson AFB, OH 45433

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An Amplitude Comparison Monopulse radar is modeled using additive channel and system noise to the received signals. The amplitude of the incoming signal and the angle off boresight are estimated under the Maximum Likelihood Criteria. An ensemble of estimates of the angle off boresight are used to derive probability density functions for the estimate angle off boresight. From these probability density functions, a criteria for predicting break lock is derived.

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