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NUMERICAL RESULTS OF THE THEORY OF DIFFRACTION OF RADIO WAVES A--ETC(U)
MAY 82 P A AZRILYANT, M G BELKINA
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NUMERICAL RESULTS OF THE THEORY OF DIFFRACTION OF RADIO WAVES AROUND THE SURFACE OF THE EARTH

(Selected Sections)

by

P.A. Azrilyant, M.G. Belkina

FOREIGN TECHNOLOGY DIVISION

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Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
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М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*<u>ye</u> initially, after vowels, and after ъ, ь; <u>e</u> elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

GRAPHICS DISCLAIMER

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NUMERICAL RESULTS OF THE THEORY OF DIFFRACTION OF RADIO WAVES AROUND THE SURFACE OF THE EARTH

P. A. Azrilyant, M. G. Belkina

The monograph contains a collection, in the form of tables and graphs, of the results of the calculation of the attenuation factor of the electromagnetic field of radio waves with horizontal and vertical polarizations in the case of their normal propagation around the Earth.

The main calculations of the attenuation factor are made in sequence and encompass the areas of the shadow and semishadow.

The joining of the diffraction curves with curves based on reflection formulas is investigated.

Additional formulas, tables and graphs are given which make it possible with the help of simple operations to make a complete calculation of the normal propagation of radio waves (beginning from the illuminated area and ending with the area of deep shadow). An example is given of the application of the numerical results of the work to the calculation of electric field intensity curves for certain wavelengths.

The results obtained are applicable in the case of horizontal polarization for practically any waves and any electric properties of the terrestrial surface, and in the case of vertical they are calculated for the extreme cases of very short and very long waves, when the terrestrial surface can be considered approximately as ideally reflecting.

The book is intended for radio engineers engaged in the calculation of radio-wave propagation and of radar devices.

1. Basic Formulas

The present work deals with the calculation of the attenuation factor, characterizing the diffraction of radio waves around the terrestrial surface.

Assume r, ϑ, φ - spherical coordinates, selected so that the emitter is positioned with $\vartheta=0$. Then in an absolute system of units the electromagnetic fields of elementary dipoles, emitting radio waves above the surface of the uniform spherical Earth, according to the theory of diffraction have the following form (dependence on time is understood in the form $e^{-i\omega t} = e^{-ickt}$).

For a vertical electric dipole

$$\left.\begin{aligned} E_r = -H_\varphi &= \\ = \frac{1}{c} ikIl \frac{e^{ika\theta}}{a\sqrt{\theta\sin\theta}} & V(x, y_1, y_2, q) \\ E_\theta = -\frac{1}{c}\frac{kIl}{M}\frac{e^{ika\theta}}{a\sqrt{\theta\sin\theta}} & \frac{\partial V(x, y_1, y_2, q)}{\partial y_1} \\ E_\varphi = H_r = H_\theta &= 0 \end{aligned}\right\}. \qquad (1.1)$$

For a vertical magnetic dipole (horizontal frame)

$$\left.\begin{aligned} H_r = E_\varphi &= \\ = \frac{1}{c} k^2 IS \frac{e^{ika\theta}}{a\sqrt{\theta\sin\theta}} & V(x, y_1, y_2, q') \\ H_\theta = \frac{1}{c}\frac{ik^2 IS}{M}\frac{e^{ika\theta}}{a\sqrt{\theta\sin\theta}} & \frac{\partial V(x, y_1, y_2, q')}{\partial y_1} \\ E_r = E_\theta = H_\varphi &= 0 \end{aligned}\right\}. \qquad (1.2)$$

Here I - current in the elementary dipole or loop, l - length of dipole, S - area of loop. Consequently the electric moment of the dipole is equal to $p = \frac{i}{ck} Il$, and the magnetic moment of the loop $m = \frac{1}{c} IS$. a denotes the geometric radius of the Earth.

For the horizontal electric or magnetic dipole the formulas for the fields have a somewhat more complex form.

The horizontal electric dipole

$$
\left.\begin{aligned}
E_\varphi = H_r = \\
= -\frac{1}{c} ikIl \frac{e^{ika\vartheta}}{a\sqrt{\vartheta \sin\vartheta}} V(x, y_1, y_2, q') \sin\varphi \\
E_r = -H_\varphi = \\
= \frac{1}{c} \frac{kIl}{M} \frac{e^{ika\vartheta}}{a\sqrt{\vartheta \sin\vartheta}} \frac{\partial V(x, y_1, y_2, q)}{\partial y_2} \cos\varphi \\
H_\vartheta = \\
= \frac{1}{c} \frac{kIl}{M} \frac{e^{ika\vartheta}}{a\sqrt{\vartheta \sin\vartheta}} \frac{\partial V(x, y_1, y_2, q')}{\partial y_1} \sin\varphi \\
E_\vartheta = 0
\end{aligned}\right\}. \tag{1.3}
$$

The horizontal magnetic dipole (vertical frame)

$$
\left.\begin{aligned}
H_\varphi = -E_r = \\
= -\frac{1}{c} k^2 IS \frac{e^{ika\vartheta}}{a\sqrt{\vartheta \sin\vartheta}} V(x, y_1, y_2, q) \sin\varphi \\
H_r = E_\varphi = \\
= -\frac{1}{c} \frac{ik^2 IS}{M} \frac{e^{ika\vartheta}}{a\sqrt{\vartheta \sin\vartheta}} \frac{\partial V(x, y_1, y_2, q')}{\partial y_2} \cos\varphi \\
E_\vartheta = \frac{1}{c} \frac{ik^2 IS}{M} \frac{e^{ika\vartheta}}{a\sqrt{\vartheta \sin\vartheta}} \frac{\partial V(x, y_1, y_2, q)}{\partial y_1} \sin\varphi \\
H_\vartheta = 0
\end{aligned}\right\}. \tag{1.4}
$$

In these formulas it is assumed that the moment of the horizontal dipole is oriented in the direction $\varphi = 0$.

The function V entering into these formulas depends on three dimensionless coordinates x, y_1, y_2 and the dimensionless complex parameter q or q'. These variables are connected with the heights of the points of radiation and reception h_1 and h_2, the distance between them on the Earth's surface s, wavelength λ, conductivity σ, and dielectric constant of the soil ϵ by the following correlations:

$$
\left.\begin{aligned}
x = M\frac{s}{a^*} = \frac{s}{s_0(\lambda)} \\
y = \frac{kh}{M} = \frac{h}{h_0(\lambda)}
\end{aligned}\right\}, \tag{1.5}
$$

$$
q = iM \frac{\sqrt{\epsilon - 1 + i\frac{4\pi\sigma}{\omega}}}{\epsilon + i\frac{4\pi\sigma}{\omega}}, \tag{1.6}
$$

$$q' = q\left(\varepsilon + i\frac{4\pi\sigma}{\omega}\right) = iM\sqrt{\varepsilon - 1 + i\frac{4\pi\sigma}{\omega}}, \qquad (1.7)$$

where

$$M = \left(\frac{ka^*}{2}\right)^{\frac{1}{3}} = \left(\frac{\pi a^*}{\lambda}\right)^{\frac{1}{3}}.$$

is the large parameter of the problem.

Through a^* the effective radius of the Earth is designated, taking into account the change in the refractive index of the terrestrial atmosphere in that case when it depends on height linearly (see [3]). In practice the values of a^* can be most diverse, from $a^* = a$ to $a^* = \infty$.

In the case of a uniform atmosphere the effective radius of the Earth a^* is equal to its geometric radius a.

The variables $S_0(\lambda)$ and $h_0(\lambda)$ are the standard distance and height, depending on wavelength λ and on effective radius a^*, i.e., on atmospheric conditions.

The function $V(x, y_1, y_2, q)$ is called the attenuation factor.

For it V. A. Fok [1] gave the following integral representation

$$V(x, y_1, y_2, q) = e^{-i\frac{\pi}{4}}\sqrt{\frac{x}{\pi}}\int_\Gamma e^{ixt} F(t, y_1, y_2, q)\, dt. \qquad (1.8)$$

Here Γ - contour in the plane of the complex variable from $e^{i\frac{2\pi}{3}}$ to 0 and from 0 to ∞, function $F(t, y_1, y_2, q)$ for $y_2 > y_1$ is determined by the formula

$$F(t, y_1, y_2, q) = w(t - y_2)\Phi(t, y_1, q), \qquad (1.9)$$

where

$$\Phi(t, y_1, q) = v(t - y_1) - \frac{v'(t) - qv(t)}{w'(t) - qw(t)}\, w(t - y_1) \qquad (1.10)$$

or

$$\Phi(t, y_1, q) = \frac{i}{2}\left\{w_2(t - y_1) - \frac{w_2'(t) - qw_2(t)}{w'(t) - qw(t)}\, w(t - y_1)\right\}, \qquad (1.11)$$

and $\omega(t)$ is the complex function of Airy, determined by V. A. Fok [2] using the formula

$$w(t)=\frac{1}{\sqrt{\pi}}\int_{\Gamma} e^{tz-\frac{1}{3}z^3}\,dz, \qquad (1.12)$$

Γ - the same contour as in formula (1.8).

This function is the solution of the differential equation

$$w''(t)=tw(t), \qquad (1.13)$$

and u(t) and $v(t)$ are those solutions of this equation, which in the case of real values of t are real and imaginary parts $\omega(t)$ and, finally,

$$w_2(t)=u(t)-iv(t). \qquad (1.14)$$

Let us note that replacement of the geometric coordinates and physical parameters by those mentioned reduces the number of variables on which the attenuation factor depends. Here the obviousness of the representation of the function is in no way disrupted, since the distances and heights are measured as if in a new scale, depending on wavelength λ and atmospheric conditions (taking into account the effective radius a^*). Transition to coordinates reduced from geometric is reduced to the division of the latter by the horizontal scale $S_0(\lambda)$ and the vertical scale $h_0(\lambda)$. The scales $S_0(\lambda)$ and $h_0(\lambda)$ for different values of effective radius a^* are given in Figure 1.

According to the assumption made in the conclusion, formulas (1.1)-(1.4) are suitable in the area of the shadow (without any limitations for values of the angle ϑ) and in the area of the semishadow, and in the illuminated area they give correct results only for a sliding incidence of a ray.

For nonelementary radiators the fields in the indicated areas are expressed with the help of formulas which are similar to formulas (1.1)-(1.4). It is possible to establish the form of these formulas on the basis of the principle that two sources, giving in free space the same field of radiation for the given azimuth φ in directions, making up small angles with the horizon, create in this azimuth φ in all the listed areas the same diffraction fields. Differences can be manifested only in that part of the illuminated area where rays fall which are reflected from the Earth at angles which differ noticeably from $\frac{\pi}{2}$, but in this case formulas (1.1)-(1.4) in general are not applicable, and it is necessary to use formulas of the theory of propagation of radio waves over a flat Earth.

As an example of the action of the indicated principle one can cite the vertical magnetic and horizontal electric dipoles. If the dipole moments are taken such that

$$p = -m,$$

then in the azimuthal direction $\varphi = \frac{\pi}{2}$ near the horizontal plane they will give, as it is easy to comprehend,the same field of radiation in free space. Therefore their diffraction fields on the horizon and beyond it , as this is easy to check from formulas (1.2) and (1.3), will coincide.

In the derivation of formulas (1.1)-(1.4) it was also assumed that the refractive index of the soil $\eta = \sqrt{\varepsilon + i\frac{4\pi\sigma}{\omega}}$ is a large complex number.

Now let us consider the case of incidence of a flat wave on a sphere with a large complex refractive index.

Relative to problems of the propagation of radio waves this case can be interpreted as a problem of diffraction around the surface of the Earth of radio waves which are arriving from interplanetary space (nonuniformity of the atmosphere in the problem of a plane wave we disregard).

Assume that the field of the incident wave is given by the formula

$$E_x^0 = -H_y^0 = e^{-ikr\cos\vartheta} \qquad (1.15)$$

(r, ϑ, φ - spherical coordinates, and the direction of propagation of the incident wave is $\vartheta = \pi$). Then the diffraction field near the surface of a sphere, at small distances from it in comparison with the radius of the sphere, is given by the formulas

$$
\left.
\begin{aligned}
E_r &= -H_\varphi = \frac{e^{ika\left(\vartheta-\frac{\pi}{2}\right)}}{\sqrt{\sin\vartheta}} V_1(z,y,q)\cos\varphi \\
E_\varphi &= H_r = \frac{e^{ika\left(\vartheta-\frac{\pi}{2}\right)}}{\sqrt{\sin\vartheta}} V_1(z,y,q')\sin\varphi \\
E_\vartheta &= -\frac{i}{M}\frac{e^{ika\left(\vartheta-\frac{\pi}{2}\right)}}{\sqrt{\sin\vartheta}}\frac{\partial V_1(z,y,q)}{\partial y}\sin\varphi \\
H_\vartheta &= \frac{i}{M}\frac{e^{ika\left(\vartheta-\frac{\pi}{2}\right)}}{\sqrt{\sin\vartheta}}\frac{\partial V_1(z,y,q')}{\partial y}\cos\varphi
\end{aligned}
\right\} \qquad (1.16)
$$

The function $V_1(z, y, q)$, appearing in these formulas, which we will call the attenuation factor for the field of an incident plane wave, was introduced by V. A. Fok and defined by him in the form of the integral (see [1])

$$
V_1(z,y,q) = \frac{1}{\sqrt{\pi}}\int_\Gamma e^{izt}\Phi(t,y,q)\,dt. \qquad (1.17)
$$

Variables z and y in this problem are determined by the formulas

$$
z = \left(\frac{ka}{2}\right)^{\frac{1}{3}}\left(\vartheta-\frac{\pi}{2}\right) = \frac{s}{s_0(\lambda)}, \qquad (1.18)
$$

$$
y = \frac{k(r-a)}{\left(\frac{ka}{2}\right)^{\frac{1}{3}}} = \frac{h}{h_0(\lambda)}. \qquad (1.19)
$$

Thus z is proportional to the angular distance or the distance on the surface of the Earth from the point of observation to the boundary of the geometric shadow $\vartheta = \frac{\pi}{2}$ on the sphere, and in the area of the shadow $z > 0$, in the illuminated area $z < 0$, and y is the reduced height of the point of observation above the surface of the sphere. Let us note that since the nonuniformity of the atmosphere in this problem is not taken into account, then (1.18) and (1.19) and the expressions for q and q' (1.6), (1.7) include the geometric, and not the effective radius.

Formulas (1.16), just as formulas (1.1)-(1.4), are applicable in the area of the shadow, semishadow, and in that part of the illuminated area where the glancing angle of the incident wave is small. All of these formulas are given in absolute units.

In order to switch from absolute units to practical it follows to express current in amperes, length in meters, and then the electric field is obtained in V/m, if 30 ohms is substituted for 1/s [1/c]. In order to obtain the magnetic field in A/m it is necessary to replace 1/c by $1/4\pi$.

In the present work results are given from calculations of the attenuation factors $V(x, y_1, y_2, q)$ and $V_1(z, y, q)$, entering into the formulas for the components of the fields of the vertical and horizontal (electric and magnetic) dipoles and the field of a plane incident wave. As concerns the derivatives of these functions, entering into the same formulas, then they do not have great importance, since the field components expressed through them in the case of finite values of the variables x or z in order of magnitude are M times ($M = \left(\frac{ka}{2}\right)^{1/2}$ - large parameter) less than those components which are expressed through these functions themselves. Therefore the derivative attenuation factors $V(x, y_1, y_2, q)$ and $V_1(z, y, q)$ we did not calculate with the exception of those which were necessary for determination of the functions themselves for small values of heights adduced.

The subsequent paragraphs are devoted to the investigation of different formulas and conditions of their applicability. Those interested only in numerical results can switch directly to §6, where a description is given of the tables and graphs, and schemes are added which indicate which tables, graphs and formulas should be used in different areas.

2. Series of Deductions. Dependence on Soil Properties

The integral representations (1.8) and (1.17) of the attenuation factors V and V_1, possessing a sufficient degree of generality, usually are too complex for direct calculations.

However, these integrals can be presented in the form of series of deductions in points t_s, being the roots of the equation

$$w'(t) - qw(t) = 0. \qquad (2.1)$$

The roots t_s, corresponding to the given value of q, form a sequence, increasing in absolute value and situated in the first quadrant near the ray arc $t = \frac{\pi}{3}$, and for q=0 and q=∞ - on the ray itself.

The series of deductions have the form

$$V(x, y_1, y_2, q) = = e^{i\frac{\pi}{4}} 2\sqrt{\pi x} \sum_{s=1}^{\infty} \frac{e^{ixt_s}}{t_s - q^2} \frac{w(t_s - y_1)}{w(t_s)} \frac{w(t_s - y_2)}{w(t_s)} \tag{2.2}$$

$$V_1(z, y, q) = i2\sqrt{\pi} \sum_{s=1}^{\infty} \frac{e^{ixt_s}}{(t_s - q^2) w(t_s)} \frac{w(t_s - y)}{w(t_s)} \tag{2.3}$$

The series (2.2) is convergent at any possible values of x, y_1, y_2 and can be considered on a level with integral (1.8) by determination of the function $V(x, y_1, y_2, q)$, although the area of practical convergence of this series is limited; with low values of x and large values of y it, without taking the remainder term into account, becomes unsuitable for calculations (see §4).

Series (2.3) converges only when $z > 0$ and makes no sense when $z < 0$, therefore the determination of function $V_1(z, y, q)$ is possible only with the help of the contour integral (1.17). However, in the area of shadow and semishadow these series converge well.

Let us consider the extreme cases of zero and infinite values of q. The roots corresponding to q=∞ we designate t_s^0, so that

$$w(t_s^0) = 0. \tag{2.4}$$

and the roots for q=0 through t_s', so that

$$w'(t_s') = 0. \tag{2.5}$$

Then, using equation (2.1) and converting in series (2.2) and (2.3) to the limit $q=\infty$, we obtain

$$V(x, y_1, y_2, \infty) = -e^{i\frac{\pi}{4}} 2\sqrt{\pi x} \sum_{s=1}^{\infty} e^{ixt_s^0} g_s(y_1) g_s(y_2) \tag{2.6}$$

$$V_1(x, y, \infty) = i2\sqrt{\pi x} \sum_{s=1}^{\infty} \frac{e^{ixt_s^0}}{w(t_s^0)} g_s(y), \tag{2.7}$$

where it is assumed that

$$g_s(y) = -\frac{w(t_s^0 - y)}{w'(t_s^0)}. \tag{2.8}$$

For $q=0$ from (2.2) and (2.3) the immediate result is

$$V(x, y_1 y_2 0) = e^{i\frac{\pi}{4}} 2\sqrt{\pi x} \sum_{s=1}^{\infty} \frac{e^{ixt_s'}}{t_s'} f_s(y_1) f_s(y_2) \tag{2.9}$$

$$V_1(x, y, 0) = i2\sqrt{\pi x} \sum_{s=1}^{\infty} \frac{e^{ixt_s'}}{t_s' w(t_s')} f_s(y), \tag{2.10}$$

where

$$f_s(y) = \frac{w(t_s' - y)}{w(t_s')}. \tag{2.11}$$

Now we will explain under what physical conditions the extreme cases $q=0$ and $q=\infty$ are realized.

The complex parameters q and q', on which the attenuation factors V and V_1 depend, are connected with wavelength λ and the soil parameters σ and ε .

According to (1.7) the parameter q' is the product of two large co-factors. Therefore it is always possible to consider

$$q' = \infty. \tag{2.12}$$

As concerns the parameter q, then it can take the most diverse complex values, which is clear, for example, from Figure 1, where the dependence of absolute magnitude of q on wavelength λ is shown

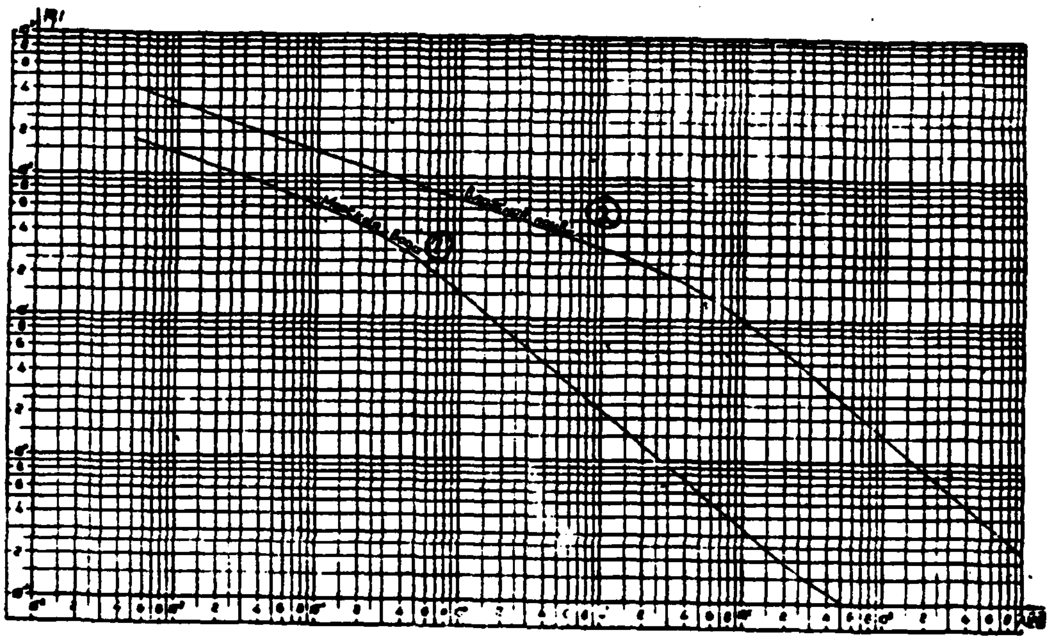

Figure 1. Dependence of $|q|$ on wavelength λ for sea water and wet soil.
Key: (1) Sea water; (2) Wet soil.

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for sea water and wet soil.

In [6] an investigation was made of the dependence of attenuation factors V and V_1 on the complex-valued parameter on the surface of the Earth. In this work the complex-valued parameter q was expressed through the material parameters α and lgn in the following manner

$$q = \frac{i n^{1/4}}{\sqrt{i + \alpha n}}. \quad (2.13)$$

The calculations were made for values of α=0, 0.01, 0.02, 0.03 and values of lgn, changing from - 1 to 3, i.e., for absolute values of q, lying approximately within the limits from $|q| \sim 0.1$ to $|q| \sim 100$ (Figure 1). Here it turned out that the dependence of V and V_1 on the parameter q is very significant when $|q| \sim 1$, where the moduli of the functions in the case of a fixed χ or χ and with an increase of lgn decrease sharply, and the phases have a maximum (see [6], drawings 4-19).

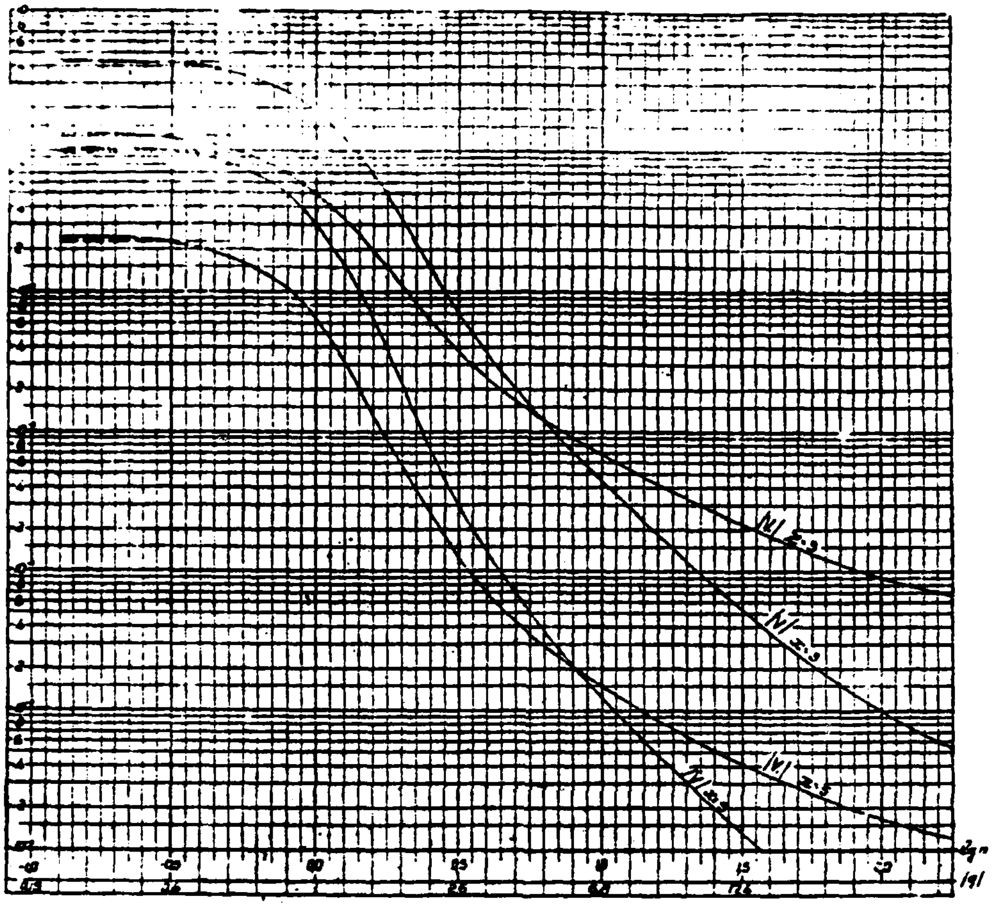

Figure 2. Dependence of attenuation factors on the complex-valued parameter q in the case of fixed values of $\mathcal{X}$ or $\mathcal{Z}$ and null values of heights.

However, for $|q| \gg 1$ it is possible to use the values of functions V and V_1 for $q = \infty$, and when $|q| \ll 1$ their values for q=0.

Figure 2 illustrates this circumstance for heights of source and point of observation equal to zero. Plotted on it in a logarithmic scale are the moduli of functions V and V_1 in the case of fixed values of $\mathcal{X}$ or $\mathcal{Z}$ and the values of these functions for q=0. (On the axis of abscissas here the values of lgn and their corresponding values of $|q|$ are plotted.)

In Figure 3 the moduli of the derivatives

$$\frac{\partial^2 V(x,0,0,q)}{\partial y_1 \partial y_2} = q^2 V(x,0,0,q)$$

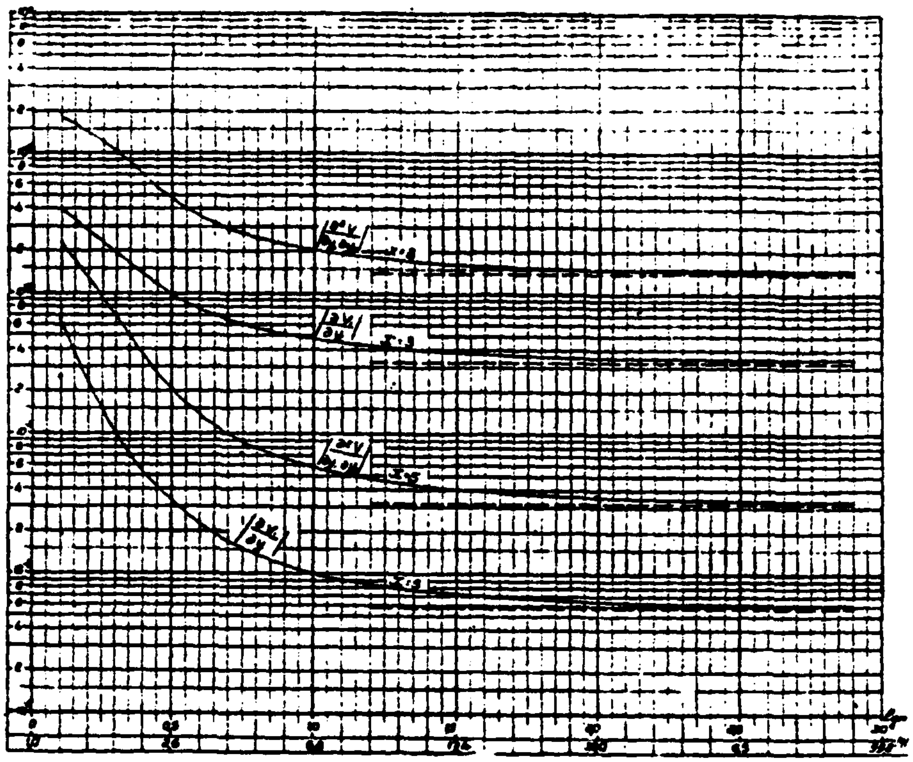

Figure 3. Derivatives of attenuation factors in dependence on the complex-valued parameter q with fixed values of χ or z and zero values of heights.

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and

$$\frac{\partial V_1(z,0,q)}{\partial y} = -q\, V_1(z,0,q)$$

are compared for large values of $|q|$ (lgn) with their values when $q=\infty$. These derivatives on the strength of the conversion of the functions themselves when $q \to \infty$ to zero determine them in the case of small values of y and large $|q|$.

Thus we see that the consideration of the extreme values of $q=0$ and $q=\infty$ also makes it possible to encompass cases when $|q| < 0.4$ and $|q| > 40$. Furthermore, on the strength of (2.12) the case $q=\infty$ makes it possible to obtain the main components of the fields of the horizontal electric dipole.
Let us note also that the case of an ideally conducting spherical surface corresponds namely to the extreme values of q=0 (for vertical

polarization) and $q=\infty$ (for horizontal).

For the calculation of attenuation factors in the case of arbitrary values of the complex-valued parameter q in the area of the semishadow, where at least five members of the series (2.2) and (2.3) have to be taken into account, a colossal volume of operations is required, which we were not able to carry out.

On the strength of the calculation difficulties indicated the data available in the literature are still very far from an exhaustive numerical investigation of the diffraction field for any soils.

Thus in [7] results are given (in the form of graphs) for different q and heights which are not equal to zero, but for the area of deep shadow, where it is possible to be limited to the first term of the series of deductions and only for attenuation factor V. Similar calculations were made much earlier by B. A. Vvedenskiy and their results were drawn up in the form of a nomogram [8].

In the book by Bremmer [9] results are given in the area of semishadow for two specific soils (two values of ε and σ), and either for heights equal to zero (on various λ) or for several heights not equal to zero, but only for two fixed values of λ. Here also only function V is given.

In work [6] results are given from calculations on two members of a series of deductions for heights equal to zero, but for a whole number of soils and for wavelengths λ which are changing in wide limits. Furthermore, there along with the attenuation factor V the attenuation factor for the field of a plane wave V_1 is also given.

Below results are given from the calculation of the attenuation factor for arbitrary values of adduced heights in the area of the shadow and semishadow, and the parameter q takes values of q=0 and $q=\infty$. Thus we encompass, first of all, the propagation of radio waves having a horizontal polarization for any wavelengths and soils; secondly, propagation of radio waves of vertical polarization for wavelengths and readily conducting soils (q=0), and also of short waves and poorly conducting soils ($q=\infty$).

The main calculations were made on five members of the series (2.6), (2.7), (2.9) and (2.10). In individual cases up to 10-15 members of these series were used, and for construction of the connection graph up to 20 members were taken. For large and small values of adduced heights and distances different limiting formulas were used (§§3 and 4). In the area of the shadow the tables and graphs are reduced to those values of x, y_1, y_2 (or z and y), for which in the series (2.6), (2.7), (2.9) and (2.10) it is permissible to take only the first term alone.

For calculations based on monomial formulas we present the tables and graphs of the factors entering into them, so that calculations using these formulas no longer present a difficulty.

3. Attenuation Factors in the Case of Large Values of y

We will consider the formulas for the attenuation factor $V(x, y_1, y_2, q)$ in the case of large values of one or both cited heights y_1 and y_2.

For example, under the condition $y_2 \gg 1$ the expression for $V(x, y_1, y_2, q)$ can be simplified considerably, reducing it to the function only of two variables and the parameter in the following manner. In the case of large x and y_2 and under the condition that the variable $z = x - \sqrt{y}$ is not a large negative number, the main sector of integration in (1.8) corresponds to values of t of an order of a unit. And since y_2 is great, then to the factor $w(t - y_2)$ entering into the function $F(t, y_1, y_2, q)$ it is possible to apply the asymptotic expression (see (8.03) in work [5])

$$w(t-y) = e^{i\frac{\pi}{4}} (y-t)^{-\frac{1}{4}} e^{i\frac{2}{3}(y-t)^{3/2}} \quad (3.1)$$

and approximately with an accuracy to terms of an order of $\frac{1}{y^{3/2}}$

$$w(t-y) = e^{i\frac{\pi}{4}} y^{-\frac{1}{4}} \left(1+\frac{t}{4y}\right) e^{i\frac{2}{3}y^{3/2} - i\sqrt{y}t + i\frac{t^2}{4\sqrt{y}}}, \quad (3.2)$$

or with an accuracy to $\frac{1}{\sqrt{y}}$

$$w(t-y) = e^{i\frac{\pi}{4}} y^{-\frac{1}{4}} e^{i\frac{2}{3}y^{3/2} - i\sqrt{y}t}. \quad (3.2a)$$

Substituting (3.2a) into (1.8), we obtain

$$V(x, y_1, y_2, q) = e^{i\frac{2}{3}y_2^{3/2}} \sqrt[4]{\frac{x^2}{y_2}} V_1(z, y_1, q), \tag{3.3}$$

where it is assumed that

$$z = x - \sqrt{y_2}. \tag{3.4}$$

Thus the function V_1, being the attenuation factor for the field of a plane wave, simultaneously makes it possible to calculate the attenuation factor of a spherical wave $V(x, y_1, y_2, q)$ for large values of y_2, if only $z = x - \sqrt{y_2}$ is not a large negative number.

Let us note that function V_1 in formula (3.3), in contrast to the case when V_1 enters into the formula for the field of a plane incident wave, can also be used for a nonuniform atmosphere with a linear course of the refractive index, if in x, y_1, y_2 and q instead of geometric the effective radius of the Earth a^* is introduced.

It is more accurate to replace the evaluation $y_2 \gg 1$ of applicability of formula (3.3) by the condition

$$\frac{1}{\sqrt{y_2}} \ll 1. \tag{3.5}$$

However, in the area of deep shadow, there where the first term of series (2.2) is main, it is possible to obtain a correlation between V and V_1 with a weaker limitation on y_2. In this area it is possible to use the concept (3.2) to series (2.2), preserving in it terms of an order of $\frac{1}{\sqrt{y}}$ and $\frac{1}{y}$.

Designating

$$M_1(y, q) = y^{-\frac{1}{4}} \left(1 + \frac{t_1}{4y}\right) e^{i\frac{t_1^2}{4\sqrt{y}}} \approx$$

$$\approx y^{-\frac{1}{4}} e^{i\frac{t_1^2}{4\sqrt{y}} + \frac{t_1}{4y}}, \tag{3.6}$$

we obtain

$$V(x, y_1, y_2, q) =$$

$$= \sqrt{x}\, e^{i\frac{2}{3}y_2^{3/2}} M_1(y_2, q) V_1(z, y_1, q). \tag{3.7}$$

In this formula it is required only that

$$\frac{1}{y_2^{1/6}} \ll 1, \qquad (3.8)$$

but then it is applicable only in the area of deep shadow (let us say when $z - \sqrt{y_1} > 1$).

Formula (3.3) is also suitable in the área of semishadow, if only z is not a large negative number (virtually when $z > -2$), but imposes a stronger limitation on y_2.

If in function $V(x, y_1, y_2, q)$ both y_1 and y_2 are great, then it is expressed approximately through function V_{11}, determined by the formula

$$V_{11}(\zeta, \mu, q) = V^{(0)}(\zeta, \mu) + V^{(1)}(\zeta, q) + V^{(2)}(\zeta, q), \qquad (3.9)$$

where

$$V^{(1)}(\zeta, q) = -e^{-i\frac{\pi}{12}} \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-\zeta t e^{i\frac{\pi}{6}}} \frac{v'(t) + e^{-i\frac{\pi}{3}} q v(t)}{w_2'(t) + e^{-i\frac{\pi}{3}} q w_2(t)} dt, \qquad (3.10)$$

$$V^{(2)}(\zeta, q) = -e^{+i\frac{\pi}{4}} \frac{1}{\sqrt{\pi}} \int_0^\infty e^{\zeta t} \frac{v'(t) - q v(t)}{w'(t) - q w(t)} dt, \qquad (3.11)$$

$$V^{(0)}(\zeta, \mu) = \begin{cases} e^{-i\frac{\pi}{4}} \mu e^{-i\tau^2} \dfrac{1}{\sqrt{\pi}} \displaystyle\int_\tau^\infty e^{is^2} ds & (\zeta > 0), \\ -e^{-i\frac{\pi}{4}} \mu e^{-i\tau^2} \dfrac{1}{\sqrt{\pi}} \displaystyle\int_{-\infty}^\tau e^{is^2} ds & (\zeta < 0) \end{cases} \qquad (3.12)$$

and

$$\tau = \mu\zeta, \qquad (3.13)$$

$$\mu = \sqrt{\frac{\sqrt{y_1}\sqrt{y_2}}{\sqrt{y_1} + \sqrt{y_2}}}. \qquad (3.14)$$

Thus function V_{11}, being a function of three variables, is expressed in the form of the sum of three functions, each of which depends only on two variables. It is noteworthy that the term $V^{(0)}$ does not depend on the electric properties of the soil and is expressed through the Fresnel integral, which is used widely in the mathematical theory of diffraction. Below we will examine the physical essence of this circumstance.

The attenuation factor $V(x, y_1, y_2, q)$ is expressed through the function V_{11} in the following manner: when $\zeta > 0$

$$V(x, y_1, y_2, q) = \sqrt{x}\, e^{i\frac{2}{3}(y_1^{3/2} + y_2^{3/2})} (y_1 y_2)^{-\frac{1}{4}} V_{11}(\zeta, \mu, q), \tag{3.15}$$

and when $\zeta < 0$

$$V(x, y_1, y_2, q) = e^{i\left[-\frac{x^3}{12} + \frac{1}{2}x(y_1 + y_2) + \frac{(y_1 - y_2)^2}{4x}\right]} + \sqrt{x}\, e^{i\frac{2}{3}(y_1^{3/2} + y_2^{3/2})} (y_1 y_2)^{-\frac{1}{4}} V_{11}(\zeta, \mu, q), \tag{3.16}$$

where the variable ζ is defined by the correlation

$$\zeta = x - \sqrt{y_1} - \sqrt{y_2}, \tag{3.17}$$

and the positive values of ζ correspond to the shaded, and the negative - to the illuminated areas.

Analogously with large values of reduced height y the function V_1 is expressed through the function V_{11}, and when $\zeta > 0$, in the shadow we have

$$V_1(x, y, q) = e^{i\frac{2}{3}y^{3/2}} y^{-\frac{1}{4}} V_{11}(\zeta, \mu, q), \tag{3.18}$$

and when $\zeta < 0$, in the illuminated area

$$V_1(x, y, q) = e^{-i\frac{x^3}{3} + ixy} + e^{i\frac{2}{3}y^{3/2}} y^{-\frac{1}{4}} V_{11}(\zeta, \mu, q). \tag{3.19}$$

In these formulas by ζ and μ should be understood the magnitudes

$$\zeta = z - \sqrt{y},$$ (3.20)

$$\mu = y^{\frac{1}{4}}.$$ (3.21)

Parameter μ under the conditions

$$y_1 \gg 1,\ y_2 \gg 1 \text{ or } y \gg 1$$ (3.22)

also should be large, although for the practical realization of this values y_1 and y_2 should be very great. Thus, for example, if $y_1 = y_2 = 20$, then $\mu = 1.5$, and if $y_1 = y_2 = 10^1$, then $\mu = 7.2$.

If the following condition is fulfilled

$$|\tau| = |\zeta\mu| \gg 1,$$ (3.23)

then we have the asymptotic formula

$$\int_{-\infty}^{\tau} e^{is^2} ds = \frac{e^{i\tau^2}}{2i\tau} \quad (\tau < 0)$$ (3.24)

and then

$$V^{(0)} = -e^{-i\frac{3\pi}{4}} \frac{1}{2\sqrt{\pi}} \frac{1}{\zeta}.$$ (3.25)

In this case the dependence of function V_{11} on μ vanishes and it becomes a function only of the variables ζ and q:

$$V_{11}(\zeta, q) = -e^{-i\frac{3\pi}{4}} \frac{1}{2\sqrt{\pi}\zeta} + V^{(1)}(\zeta, q) + V^{(2)}(\zeta, q).$$ (3.26)

Formally $V_{11}(\zeta, q)$ is the value $V_{11}(\zeta, \mu q)$ when $\mu = \infty$

$$V_{11}(\zeta, q) = V_{11}(\zeta, \infty, q)$$ (3.27)

If $\zeta > 0$ (area of shadow), then the function $V_{11}(\zeta, q)$ along with formula (3.27) can be calculated based on the series of deductions

$$V_{11}(\zeta, q) = e^{i\frac{3\pi}{4}} 2\sqrt{\pi} \sum_{s=1}^{\infty} \frac{e^{\zeta t_s}}{t_s - q^2} \frac{1}{w^2(t_s)},$$ (3.28)

which is obtained formally from formula (2.3), if in it instead of $w(t_s - y)$ its asymptotic expression (3.2a) is taken. Actually for the applicability of formula (3.28) it is required that the condition (3.23) take place.

Formulas (3.9)-(3.21), giving the solution to the problem of finding the diffraction field for large values of reduced heights, were obtained by V. A. Fok [10]. The conclusion, for example, of formulas (3.18) and (3.19) are based on the fact that in the integral for V_1 taken on the broken path Γ on the sector from $t = e^{i\frac{2\pi}{3}}\infty$ to t=0, for the function $\Phi(t, y, q)$ the expression (1.10) is taken, and on the sector from t=0 to $t=\infty$ - the expression (1.11). The first terms of each of the integrals are joined into one expression, which is converted to the form

$$\frac{i}{2\pi}\int e^{-\frac{\xi^3}{3} - i\xi y} \frac{d\xi}{iz + \xi}, \tag{3.29}$$

where the integral is taken based on the infinite contour from $t = e^{i\frac{4\pi}{3}}\infty$ to $t = \infty$, enveloping the point $\xi = -iz$ from above. In the case of large values of y this integral, calculated by the method of stationary phase, gives that term in the expression for V_1 which is connected with the function $V^{(0)}$. In the illuminated area this integral, furthermore, gives the incident wave which is written out separately in formula (2.19) (for V_1 this is a plane wave).

The term connected with the function $V^{(1)}$ is obtained in the case of large values of y (when instead of $w(t - y)$ it is possible to take expression (3.2a)) from the integral from the second term of function Φ on the left half of the circuit Γ, and the term connected with $V^{(2)}$ - from the integral from the second term of the function Φ on the right half of the contour Γ.

In a similar manner formulas (3.15) and (3.16) are obtained for the function V.

Now we will analyze the physical sense of formulas (3.15), (3.16), (3.18) and (3.19). We will begin with the two latter ones.

With fulfillment of condition (3.23) the term $V^{(0)}$ in expression (3.9) for V_{11} is main in the case of finite values of $\mu\zeta$, since in order of magnitude it is μ times greater than $V^{(1)}$ and $V^{(2)}$. Since μ increases very slowly with an increase of y_1 and y_2, then $V^{(0)}$ can be considered main in comparison with $V^{(1)}+V^{(2)}$ only in the extreme case. However, $V^{(1)}+V^{(2)}$ change very slowly and therefore their sum can be considered as a background, on which the diffraction picture given by $V^{(0)}$ is superimposed. Setting this background off to the side, we can write the following expressions for V_1:

when $\zeta > 0$

$$V_1(z, y, q) = e^{i\left(\frac{2}{3} y^{3/2} - \tau^3\right)} \frac{e^{-i\frac{\pi}{4}}}{\sqrt{\pi}} \int_{\tau}^{\infty} e^{is^2} ds, \qquad (3.30)$$

and when $\zeta < 0$

$$V_1(z, y, q) = e^{-\frac{i\zeta^3}{3} + i\zeta y} - e^{i\left(\frac{2}{3} y^{3/2} - \tau^3\right)} \frac{e^{-i\frac{\pi}{4}}}{\sqrt{\pi}} \int_{-\infty}^{\tau} e^{is^2} ds, \qquad (3.31)$$

where

$$\tau = \sqrt[4]{y}\,\zeta. \qquad (3.32)$$

In the area where these formulas are applicable, the field in the case of diffraction of a plane wave on the spherical terrestrial Earth's surface in the point of observation P (Figure 4) is obtained with an accuracy to almost a constant term the same as in the case of diffraction on a screen passing through the center of the terrestrial sphere, and the edge of this screen is the line of intersection of the plane of the horizon for point P in the geometric boundary of the shadow of the incident plane wave (Figure 4).

Actually we will designate through ψ the angle of diffraction, i.e., the angle between the direction of propagation of the incident plane wave and the direction of the radius-vector r=TP, drawn from the edge of the screen perpendicular to it in the point of observation P; in this case we consider $\psi > 0$, if P lies in the shaded area (as in Figure 4), and in the opposite case $\psi < 0$. Then based on the theory of diffraction on a half-plane the fields are expressed through the function

$$U = e^{ikr\cos\psi}\frac{e^{-i\frac{\pi}{4}}}{\sqrt{\pi}}\int_{\tau}^{\infty} e^{is^2}ds \text{ shadow (тень)}, \tag{3.33}$$

$$U = e^{ikr\cos\psi} - e^{ikr\cos\psi}\frac{e^{-i\frac{\pi}{4}}}{\sqrt{\pi}}\int_{-\infty}^{\tau} e^{is^2}ds \text{ light (свет)}, \tag{3.34}$$

where

$$\tau = \sqrt{2kr}\sin\frac{\psi}{2}. \tag{3.35}$$

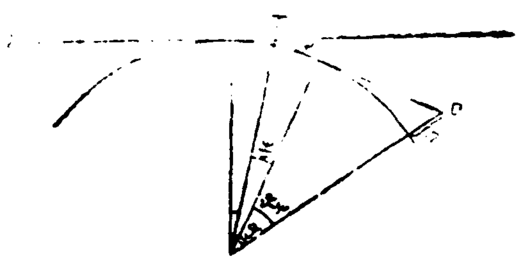

Figure 4. Equivalent screen for a plane wave.

Simple geometric considerations on the basis of Figure 4 lead to the fact that for small angles of diffraction ψ, large values of y and finite ζ, we have the correlations

$$\left.\begin{aligned} \zeta &= M\psi \\ kr &= 2M^2\sqrt{y} \\ \tau &= \sqrt[4]{y}\,\zeta \end{aligned}\right\}. \tag{3.36}$$

Thus the Fresnel integrals, through which the attenuation factor in formulas (3.30), (3.31) is expressed, have the same arguments as the diffraction field of the screen in Figure 4.

The factor $e^{-i\tau_2}$ in front of the integrals in (3.30) and (3.31) is equal, as is readily checked from (3.36), to the factor $e^{-ikr\frac{\psi^2}{2}}$, obtained in the case of using the approximate formula

$$\cos\psi = 1 - \frac{\psi^2}{2}.$$

The term $e^{\frac{iz^3}{3} + izy}$, giving the incident plane wave in formula (3.31), corresponds to the term $e^{ikr\cos\psi}$ in formula (3.34). The remaining differences in formulas (3.30), (3:31) and (3.33), (3.34) are connected simply with a different reading of phases in functions V_1 and U.

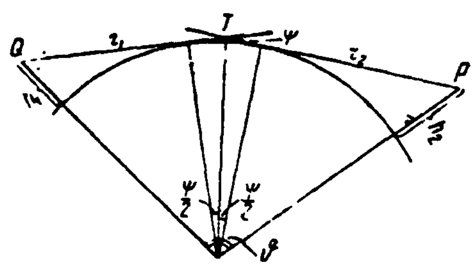

Figure 5. Equivalent screen for a spherical wave.

The same physical essence is held by the term $V^{(0)}$ in the expression for the attenuation factor V [formulas (3.15), (3.16)]. In this case the distances $TQ=r_1$ and $TP=r_2$ (Figure 5) are connected with the cited heights of the corresponding points by the correlations

$$kr_1 = 2M^2\sqrt{y_1}, \quad kr_2 = 2M^2\sqrt{y_2}. \qquad (3.37)$$

The screen itself is defined as a half-plane, passing through the center of the Earth and having as its own edge the straight line of intersection of the horizontal planes for the corresponding points P and Q. The angle of diffraction ψ is defined as the angle between these planes and is considered positive, if P is not found in the zone of direct visibility of point Q. Correlations (3.37) are valid under the assumption that angle ψ is small.

As is known from the classic theory of diffraction the field from a nontransparent screen of such a form is expressed (at small angles of diffraction, which only interest us) through the Fresnel integral from the argument

$$\tau = \sqrt{\frac{2kr_1 r_2}{r_1 + r_2}}\,\frac{\psi}{2}. \qquad (3.38)$$

Since as before the angle of diffraction ψ is connected with the variable ζ by the correlation

$$\zeta = M\psi, \qquad (3.39)$$

then formula (3.38) gives

$$\tau = \mu\zeta, \quad \mu = \sqrt{\frac{\sqrt{y_1}\,\sqrt{y_2}}{\sqrt{y_1} + \sqrt{y_2}}}, \qquad (3.40)$$

which coincides with (3.13) and (3.14).

Thus the term $V^{(0)}$ in the expression for the function $V_{11}(\zeta, \mu, q)$ corresponds to the standard Fresnel diffraction, which according to the Huygens principle is obtained from the nontransparent Earth the same as from a nontransparent screen, positioned so that it produces (for the given positioning of the corresponding points) the same geometric boundary of shadow as the Earth.

The diffraction field is obtained according to the Huygens principle as a result of the effect of that part of the incident plane or spherical wave which for the given point of observation P is not obstructed by a nontransparent obstacle.

In previous works on the theory of diffraction propagation of radio waves around the surface of the Earth repeated attempts were made to apply the Huygens principle and to establish a connection between diffraction around the Earth and diffraction on the edge of a screen. However, those elementary considerations, which were expressed in this connection, did not make it possible to determine the boundaries of applicability both of the Huygens principle and of those calculation formulas of the type (3.33), (3.34) which are obtained with its help and coincide with formulas of diffraction on a wedge or half-plane.

In particular these formulas were applied to the case of a deep shadow (large positive ζ), where they, according to formula (3.25), give an attenuation of the field inversely proportional to ζ , while according to strict theory (formula 3.28) the field fades with an increase of ζ exponentially. Therefore it is not surprising that the Huygens principle gave a field which is many times stronger than that actually observed. Even in the comparatively recently published book by Bremmer [9] in the interpretation of this interesting and important question total helplessness is revealed. Only in the works by V. A. Fok, stemming from the boundary integral as the main expression for the attenuation factor V (and not from a series of deductions as this is usually accepted), this question was cleared up, which then made it possible to effectively calculate the attenuation factor for high values of cited heights y [10].

From a comparison of formulas (3.26) and (3.28) it is evident that only in the deep shadow the term $V^{(0)}$ is not main; $V^{(1)}+V^{(2)}$ has the same order of magnitude, and the sum of all three terms has the form (3.28), i.e., with $\zeta \to \infty$ it decreases considerably more rapidly.

In general, as we already mentioned, the term $V^{(0)}$ in the expression for V_{11} is main only for high values of y . Even when y =1000 $\mu = \sqrt[4]{y}$ =5.6, so that when ζ =0 the term $V^{(0)}$ is only 10 times greater in absolute magnitude than the summ $V^{(1)}+V^{(2)}$. Therefore it can almost never be disregarded. The term $V^{(0)}$ can be considered main only because at low values of ζ it gives a rapid course of the functions V_{11}, V_1 and V: in the case of positive values of ζ thanks to this term the functions V_1 and V increase rapidly from low to finite values, and with negative values of ζ - they oscillate rapidly.

The terms $V^{(1)}$ and $V^{(2)}$, depending on the electric properties of the soil, determine the influence of the material and form of the "nontransparent obstacle" - the Earth, on the diffraction field - the circumstances, which the classic theory of diffraction (Fresnel-Kirchhoff) does not take into account. In addition to the parameter q these terms depend only on ζ and therefore at low values of ζ can be considered almost constant.

Let us note that the function $V_{11}(\zeta, q)$ [see (3.26)] emerges in the solution of problems of diffraction not only on a sphere, but also on bodies of another form. Thus these functions were used in the work of A. S. Goryainov [11], who considered diffraction on an infinite round cylinder. In this work the following designations are used:

$$\left.\begin{aligned} \hat{f}(\zeta) &= -V_{11}(\zeta, \infty) \\ \hat{q}(\zeta) &= -V_{11}(\zeta, 0) \\ \tilde{f}(\zeta) &= -[V^{(1)}(\zeta, \infty) + V^{(2)}(\zeta, \infty)] \\ \tilde{q}(\zeta) &= -[V^{(1)}(\zeta, 0) + V^{(2)}(\zeta, 0)] \end{aligned}\right\} \tag{3.41}$$

4. Attenuation Factors at Low Values of y

In the previous section we considered formulas for the calculation of attenuation factors in the case of large values of y. The case of low values of y should also be considered in particular and the formulas necessary for calculations given. The difficulty here is that at low y the area of half-shadow has corresponding low values of x and z, at which the series of deductions converge very slowly.

Therefore here it is necessary either to calculate the attenuation factors directly based on their integral presentations, using for simplification of the integrands the fact that $y \ll 1$, or to calculate in series of deductions the remainder terms, which in the area of half-shadow at small y no longer can be disregarded.

Let us note first of all that if function $f(y)$ satisfies the equation

$$f''(y) = (t - y) f(y) \tag{4.1}$$

with the initial conditions

$$f(0) = 1, \; f'(0) = -q, \tag{4.2}$$

then

$$f(y) = \operatorname{ch}(y\sqrt{t}) - \frac{q}{\sqrt{t}}\operatorname{sh}(y\sqrt{t}), \tag{4.3}$$

if only

$$\frac{y^2}{2|\sqrt{t}|} \ll 1 \tag{4.4}$$

(see [5], pages 45-46).

It is easy to see that the height factor $\frac{w(t_s - y)}{w(t_s)}$, entering into the series of deductions (2.2) and (2.3), satisfies equation (4.1) with the initial conditions (4.2).

Therefore

$$\frac{w(t_s - y)}{w(t_s)} = \operatorname{ch}(y\sqrt{t_s}) - \frac{q}{\sqrt{t_s}}\operatorname{sh}(y\sqrt{t_s}), \tag{4.5}$$

under the condition

$$\frac{y^2}{2|\sqrt{t_s}|} \ll 1. \tag{4.6}$$

If y_1 and y_2 satisfy the condition (4.6), then, substituting (4.5) for y_1 and y_2 in (2.2), we obtain

$$V(x, y_1, y_2, \infty) = -e^{i\frac{\pi}{4}} 2\sqrt{\pi x} \times$$
$$\times \sum_{s=1}^{\infty} \frac{e^{ixt_s^0}}{t_s^0} \operatorname{sh}(y_1\sqrt{t_s^0}) \operatorname{sh}(y_2\sqrt{t_s^0}) \tag{4.7}$$

$$V(x_1, y_1, y_2, 0) = e^{i\frac{\pi}{4}} 2\sqrt{\pi x} \times$$
$$\times \sum_{s=1}^{\infty} \frac{e^{ixt'_s}}{t'_s} \operatorname{ch}(y_1\sqrt{t'_s}) \operatorname{ch}(y_2\sqrt{t'_s}). \tag{4.8}$$

Introducing the designations

$$\phi(x, \eta) = e^{-i\frac{3\pi}{4}} \frac{\sqrt{\pi x}}{2} \sum_{s=1}^{\infty} \frac{e^{ixt_s^0 + \eta\sqrt{t_s^0}}}{t_s^0}, \tag{4.9}$$

$$\varphi(x, \eta) = e^{i\frac{\pi}{4}} \frac{\sqrt{\pi x}}{2} \sum_{s=1}^{\infty} \frac{e^{ixt'_s + \eta\sqrt{t'_s}}}{t'_s}. \tag{4.10}$$

we obtain

$$V(x,y_1,y_2,\infty) =$$
$$= \psi(x,y_1+y_2) - \psi(x,y_1-y_2) -$$
$$-\psi(x,-y_1+y_2) + \psi(x,-y_1-y_2), \quad (4.11)$$

$$V(x,y_1,y_2 0) =$$
$$= \varphi(x,y_1+y_2) + \varphi(x,y_1-y_2) +$$
$$+ \varphi(x,-y_1+y_2) + \varphi(x,-y_1-y_2). \quad (4.12)$$

For the calculation of $\psi(x,\eta)$ and $\varphi(x,\eta)$ the series (4.9) and (4.10) are divided into two parts: from s=1 to s=N-1 and from s=N to s=∞ (we selected N=6), and the first sum is calculated directly, and the second - remainder of the series - is replaced approximately by the integral, since when s $\geqslant$ 6 it is already possible to consider that

$$t_s^0 = \left[\frac{3}{2}\left(s-\frac{1}{4}\right)\pi\right]^{\frac{2}{3}} e^{i\frac{\pi}{3}}$$

$$t_s' = \left[\frac{3}{2}\left(s-\frac{3}{4}\right)\pi\right]^{\frac{2}{3}} e^{i\frac{\pi}{3}}.$$

The final formulas have the form (compare [4], p. 46-47)

$$\psi(x,\eta) = e^{-i\frac{3\pi}{4}} \frac{\sqrt{\pi x}}{2} \sum_{s=1}^{N-1} \frac{e^{ixt_s^0 + \eta\sqrt{t_s^0}}}{t_s^0} -$$
$$- e^{i\frac{\eta^2}{4x}} \frac{1}{\sqrt{\pi}} \int_a^{\infty} e^{-p^2}\, dp, \quad (4.13)$$

$$\varphi(x,\eta) = e^{i\frac{\pi}{4}} \frac{\sqrt{\pi x}}{2} \sum_{s=1}^{N-1} \frac{e^{ixt_s' + \eta\sqrt{t_s'}}}{t_s'} +$$
$$+ e^{i\frac{\eta^2}{4x}} \frac{1}{\sqrt{\pi}} \int_{a'}^{\infty} e^{-p^2}\, dp, \quad (4.14)$$

$$\alpha^{0} = \sqrt{x}\left[\frac{3}{2}\left(N-\frac{3}{4}\right)\pi\right]^{\frac{1}{3}} e^{-i\frac{\pi}{12}} - \frac{\eta}{2\sqrt{x}} e^{i\frac{\pi}{4}}, \qquad (4.15)$$

$$\alpha' = \sqrt{x}\left[\frac{3}{2}\left(N-\frac{5}{4}\right)\pi\right]^{\frac{1}{3}} e^{-i\frac{\pi}{12}} - \frac{\eta}{2\sqrt{x}} e^{i\frac{\pi}{4}}. \qquad (4.16)$$

If at low values of y_1 and y_2 the value of x is not small ($x \gtrsim 1$), then, since the factor when $\sqrt{x}$ in (4.15) and (4.16) is a number of order 3, the integrals in (4.13) and (4.14) are a small correction to the first N-1 members of the series and they can be disregarded. In the opposite case these integrals have the same order as the sums.

From such considerations, as was pointed out by V. A. Fok, for small values of y it is possible to obtain the formula

$$V_1(z,y,q) = \frac{1}{\sqrt{\pi}}\int_{\Gamma} e^{izt}\frac{\mathrm{ch}(y\sqrt{t}) - \frac{q}{\sqrt{t}}\mathrm{sh}(y\sqrt{t})}{w'(t) - qw(t)}dt, \qquad (4.17)$$

or, dividing the contour Γ into parts, as in §3,

$$V_1(z,y,q) = \frac{1}{\sqrt{\pi}}\int_0^{\infty} e^{izt}\frac{\mathrm{ch}(y\sqrt{t}) - \frac{q}{\sqrt{t}}\mathrm{sh}(y\sqrt{t})}{w'(t) - qw(t)}dt + \frac{1}{\sqrt{\pi}}\int_0^{\infty} e^{-zte^{i\frac{\pi}{6}}}\frac{\mathrm{ch}(Y) - e^{-i\frac{\pi}{3}}\frac{q}{\sqrt{t}}\mathrm{sh}(Y)}{w_2'(t) + e^{-i\frac{\pi}{3}}qw_2(t)}dt, \qquad (4.18)$$

where

$$Y = y\sqrt{t}\, e^{i\frac{\pi}{3}}.$$

Using the last formula calculations were made for graphs of connection in Figures 6-9. However, this formula gives approximately the same result as the much simpler expressions (4.23), (4.24), which are given below.

We will also use the following formulas

A. $y_1 \ll 1$

$$V(x, y_1, y_2, \infty) = y_1 \frac{\partial V(x, 0, y_2, \infty)}{\partial y_1}, \quad (4.19)$$

$$V(x, y_1, y_2, 0) = V(x, 0, y_2, 0). \quad (4.20)$$

B. $y_1 \ll 1,\ y_2 \ll 1$

$$V(x, y_1, y_2, \infty) = y_1 y_2 \frac{\partial^2 V(x, 0, 0, \infty)}{\partial y_1 \partial y_2} \quad (4.21)$$

$$V(x, y_1, y_2, 0) = V(x, 0, 0, 0) \quad (4.22)$$

C. $y \ll 1$

$$V_1(z, y, \infty) = y \frac{\partial V_1(z, 0, \infty)}{\partial y} = y f(z) \quad (4.23)$$

$$V_1(z, y, 0) = V_1(z, 0, 0) = g(z). \quad (4.24)$$

where

$$\frac{\partial V(x, 0, y_2, \infty)}{\partial y_1} =$$

$$= e^{-i\frac{3\pi}{4}} 2\sqrt{\pi x} \sum_{s=1}^{\infty} e^{ixt_s^0} g_s(y_2), \quad (4.25)$$

$$\frac{\partial^2 V(x, 0, 0, \infty)}{\partial y_1 \partial y_2} = e^{-i\frac{3\pi}{4}} 2\sqrt{\pi x} \sum_{s=1}^{\infty} e^{ixt_s'} \quad (4.26)$$

$$f(z) = \frac{\partial V_1(z, 0, \infty)}{\partial y} =$$

$$= i 2\sqrt{\pi} \sum_{s=1}^{\infty} \frac{e^{izt_s^0}}{w'(t_s^0)} = \frac{1}{\sqrt{\pi}} \int_\Gamma \frac{e^{izt}}{w(t)} dt \quad (4.27)$$

$$g(z) = V_1(z, 0, 0) =$$

$$= i 2\sqrt{\pi} \sum_{s=1}^{\infty} \frac{e^{izt_s'}}{t_s' w(t_s')} = \frac{1}{\sqrt{\pi}} \int_\Gamma \frac{e^{izt}}{w'(t)} dt. \quad (4.28)$$

(Functions $f(z)$ and $g(z)$ play an important role in a number of other problems and have been tabulated in detail.

Let us note that formulas (4.21) and (4.22) can be obtained also from formulas (4.7) and (4.8), if in all essential terms of series (4.7) and (4.8) it is permissible to replace the hyperbolic functions by the first terms of their expansions into a power series. An analysis of this requirement gives the following condition of applicability of formulas (4.21) and (4.22)

$$\frac{y}{\sqrt{x}} \ll 1. \qquad (4.29)$$

In a similar way formulas (4.23) and (4.24) can be derived from (4.18), and the condition of their applicability, obtained from the requirement of the possibility of replacement on the main sector of integration of the hyperbolic functions by the first terms of their expansion, has the form

$$\left.\begin{array}{l} y|z| \ll 1 \ \text{for} \ -z \gg 1 \\ y \ll 1 \ \text{for} \ |z| < 1 \ \text{и} \ z > 0 \end{array}\right\}. \qquad (4.30)$$

5. Reflection Formulas. Joining with Diffraction Formulas

As is known, for the calculation of field intensity in the illuminated area at relatively small distances from the source reflection formulas are used. These are based on the concept that the reflection of beams from the flat surface of the Earth according to the laws of geometric optics is such that an electromagnetic field in a given point of the illuminated area emerges as a result of the interference of the "direct" and "reflected" radio beam. As theory shows, for the ideally reflecting flat Earth this reflecting treatment gives completely accurate results (with any polarization of the propagating radio waves), at the same time that for the flat Earth, possessing a finite conductivity, the reflecting formulas are only approximately correct, and in the case of glancing incidence of the beams for waves, of vertical polarization for example, should be replaced by Weil-Van der Pol formulas.

The application of the stationary phase method to contour integrals (1.8) and (1.17) for $V(x, y_1, y_2, q)$ and $V_1(z, y, q)$ makes it possible to obtain in the illuminated area for these functions the following formulas (see [1] and [5]):

$$V(x, y_1, y_2, q) = e^{i\omega} - \frac{q - ip}{q + ip}\sqrt{A}\, e^{i\omega^*}, \tag{5.1}$$

$$V_1(z, y, q) = e^{i\varphi} - \frac{q - ip}{q + ip}\sqrt{\frac{p}{\sigma}}\, e^{i\varphi^*}. \tag{5.2}$$

These are reflection formulas for the attenuation factors V and V_1, taking into account the curvature of the Earth and generalizing the known reflection formulas for a flat Earth. The first term in formulas (5.1) and (5.2) gives the incident wave (spherical for V and plane for V_1), the second term - reflected wave: ω^* or φ^* is its phase, $\frac{q - i\rho}{q + i\rho}$ - coefficient of Fresnel reflection [for glancing incidence, since only for this case are formulas (1.1)-(1.4) and (1.16) valid], $\sqrt{A}$ and $\sqrt{\frac{\rho}{\sigma}}$ - factors, taking into account the additional convergence of rays as a result of their reflection from the spherical surface of the Earth.

The parameter p, designated in like manner in (5.1) and (5.2), and having the same geometric sense in both cases (which we will speak of below), is determined for V from the equation

$$\sqrt{y_1 + p^2} + \sqrt{y_2 + p^2} = 2p + x, \tag{5.3}$$

and for V_1 according to the formula

$$p = \frac{1}{3}\left(\sqrt{z^2 + 3y} - 2z\right). \tag{5.4}$$

The solution of equation (5.3) can be presented in the form

$$p = \frac{y_1}{x + \tilde{z}} - \frac{x + \tilde{z}}{4} = \frac{y_2}{x - \tilde{z}} - \frac{x - \tilde{z}}{4}, \tag{5.5}$$

where

$$\tilde{z} = 2\rho \sin\frac{\alpha}{3}, \tag{5.6}$$

$$\rho^2 = \frac{1}{3}(x^2 + 2y_1 + 2y_2) \quad (\rho > 0), \tag{5.7}$$

$$\sin\alpha = \frac{x(y_1 - y_2)}{\rho^3} \left(-\frac{\pi}{2} < \alpha < \frac{\pi}{2}\right). \tag{5.8}$$

Further

$$\omega = \frac{(y_1 - y_2)^2}{4x} + \frac{1}{2}x(y_1 + y_2) - \frac{1}{12}x^3, \tag{5.9}$$

$$\omega^* = -3p^2 x + 2p(y_1 + y_2 - x^2) + $$
$$+ x(y_1 + y_2) - \frac{1}{3}x^3, \tag{5.10}$$

$$A = \frac{px}{3px + x^2 - y_1 - y_2}. \tag{5.11}$$

For V_1 we have the correlations

$$\varphi = zy - \frac{z^3}{3}, \tag{5.12}$$

$$\varphi^* = \frac{1}{27}\left(4\sigma^3 - 3\sigma^2 z - 2z^3\right), \tag{5.13}$$

$$\sigma = \sqrt{z^2 + 3y}. \tag{5.14}$$

The geometric sense of the parameter p lies in the fact that in characterizes the area in which the point x, y_1, y_2 or z, y is found.

If γ is the angle of incidence of the beam, then

$$p = M\cos\gamma, \tag{5.15}$$

from which it is immediately clear that p=0 if the point lies on the geometric boundary of the shadow, p > 0 if the point lies in the area of light, and, finally, p < 0 if it lies in the shadow. Let us stress that this important factor does not depend on the properties of the soil.

For V_{11} it is also possible to obtain a "reflection" formula, i.e., a formula which is applicable in the case of large negative ζ. For this in the second member of formula (5.2) (V_{11} in the area of light by definition does not contain an incident wave) it is necessary to introduce in place of x the variable $\zeta = x - \sqrt{y}$ and to consider that x and y are great, and ζ is finite (and negative).

Then the result is that V_{11} does not depend on μ and has the form

$$V_{11} = -\frac{q - ip}{q + ip} \frac{1}{2} \sqrt{-\zeta}\, e^{-\frac{i}{12}\zeta^3}, \tag{5.16}$$

where

$$p = -\frac{1}{2}\zeta. \tag{5.17}$$

Let us note that the reflection formulas for V, V_1 and V_{11} convert one another in the case of large values of y in the same manner as the diffraction formulas for these functions.

Reflection formulas of the type (5.1) frequenly are used for calculations, considering in them $\sqrt{A}=1$, i.e., disregarding the additional divergence of beams as a result of their reflection from the Earth.

Figure 6 gives an example of curves, calculated using formula (5.1) (solid curves) and in the case of $\sqrt{A}=1$ (broken curves). As it should be expected, the latter oscillate more sharply and, in contrast to the first, reach in the minimum of zero the same position of the maximum and minimums for both curves.

As theoretical investigations show, the reflection formulas are applicable in the illuminated area under the condition that the parameter p is positive and great.

For refinement of how great the parameter p should be in order that the reflections would give a specific accuracy, we made a series of calculations on the joining of reflection formulas with diffraction.

Figures 2-5 give the results of investigations of functions $V(x, y_1, y_2, \infty)$ and $V(x, y_1, y_2, 0)$ by diffraction series (2.6) and (2.9) and by refelction formula (5.1) for $y_1 = y_2 = 0.4$, 0.8, 2.4, 4.8.

The oscillating parts of the curves are constructed according to the reflection formula. The values of parameter p are plotted on them. The figures in brackets show on what number of terms of series (2.6) or (2.9) the given points are calculated. The little circles denote the points which are calculated based on more than five members of the series.

In analyzing these drawings we see that already with p=0.7 for q=∞, and for q=0 even when p=0.5 there is an approximate joining of curves constructed by reflection and diffraction formulas.

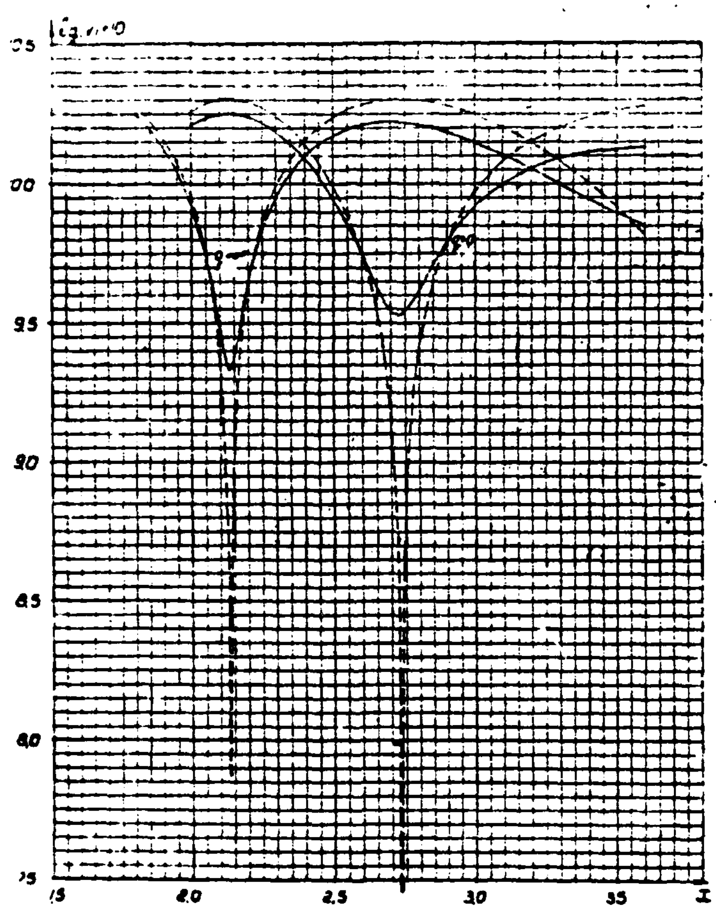

Figure 6. Influence of additional divergence on curves constructed using the reflection formula.

This result shows that reflection formulas actually have a wider area of applicability than this would be expected on the basis of general analytical evaluations.

In this case for q=∞ the joining takes place after the passing of the last maximum of absolute value of function V, i.e., in the area of its monotony, where it is possible yet to be limited to five members of the series of deductions (2.6) (for not too small χ).

For q=0 (also for not too small χ) the joining takes place on the last maximum of function V when $p \approx 0.5-0.7$ and also in the area of suitability of the five-term formula. However, the circumstance that the joining takes place on the maximum hampers the graphic representation of the results of calculations based on a five-member series (see §6 below). The joining of reflection and diffraction formulas for the attenuation factor V_1 is given in drawings 6-9.

Here the series of deductions (2.7) and (2.10) do not make it possible to obtain the joining with reflection formulas (even if 25 terms are taken in them) and we had to use other methods. Thus for y =4.8 formulas (3.18)-(3.19) already turned out to be suitable for large values of y. For y=0.4 and y=0.01 we used formulas (4.23) and (4.24), and for y=0.4 also formula (4.18).

In draw. 6-9 the points obtained with formulas (3.18) and (3.19) are noted by little circles, points calculated with formulas (4.23) and (4.24) are noted by triangles, and, finally, the results of calculations by (4.18) - by small squares.

The broken curve in drawing 8 represents

$$V_1(z, 0, 0) = g(z),$$

- the extreme curve, to which $V_1(z, y, 0)$ strives when $y \to 0$ [formula (4.24)], although the striving to the limit is nonuniform and depends on the value of z. Thus in the area of shadow and semishadow already for y=0.4 the function $V_1(z, y, 0)$ is very close to $g(z)$, in the area of light $V_1(z, y, 0)$ for y=0.4 has a completely different course than for $g(z)$. However, the smaller the value of y, the further into the area of light is formula (4.24) suitable. Thus, when y=0.01 for $z < -1.2$ the points, calculated by the reflection formula (noted by crosses), actually coincide with $g(z)$. The same is true when $q=\infty$ [formula (4.23)].

These circumstances agree with the evaluation of (4.30) given in §4 for the applicability of formulas (4.23) and (4.24). As was already stated above, formula (4.18) gives the same results.

The joining of the reflection and diffraction formulas for $V_1(z, y, q)$ takes place at those same values of p as for $V(\chi, y_1, y_2, q)$, namely when p=0.7 for $q=\infty$ and when $p \sim 0.5-0.7$ for q=0. In both cases the joining takes place on the maximum of the absolute magnitude of the function or even further to the left. The joining of V_{11} with formula (5.16) is shown in drawings 36-39. It takes place when $p \sim 0.7-1.0$.

Let us note that the joining of reflection and diffraction formulas for $V_1(x, y, q)$ takes place at positive or small negative values of x, i.e., in the area where formula (3.3) of connection between V and V_1 is applicable. This makes it possible for us to draw the conclusion that the joining of diffraction and reflection formulas for $V(x, y_1, y_2, q)$ for the case when $y_2 \gg 1$ and y_1 is finite or small takes place also when $p \sim 0.7$. The joining of V_{11} with formula (5.16) shown in drawings 36-39 makes it possible for us to make such a conclusion for $y_1 \gg 1$, $y_2 \gg 1$.

For determination of the area of applicability of reflection formulas, i.e., the area where $p \geqslant 0.7$, we present special graphs (drawings 10 and 23 - see §6).

6. Description of Graphs and Tables

As was already stated above, we will give the tables and graphs of attenuation factors V and V_1 for the values $q = \infty$ and $q=0$.

The calculations of the function V for these values of q were made for several fixed ratios of cited heights.

$$m = \frac{y_1}{y_2} = 1;\ 0{,}75;\ 0{,}5;\ 0{,}25;$$

and values of x from 0.5 to 6 ($q=\infty$) or to 7 ($q=0$) and y_1 from 0.1 to 4.8, and calculations were made for all pairs (x, y_1), for which by using a five-member series it was possible to obtain results if only with graphic accuracy.

Values of $y > 4.8$ were not taken in these calculations, since for them it is possible to obtain the attenuation factor V with the help of function V_1 using formulas (3.3) and (3.7) or (3.15)-(3.16).

For values of $x > 6$ the functions V and V_1 did not have to be calculated, since for such x and $y_1, y_2 < 4.8$ already the use of only the first member of the series of deductions is permissible (see below).

Results of the calculation of function V using the five-member series are given in tables 1-16.

The function of attenuation V_1 was calculated also for y =0.1,. .., 4.8 and values z from 0.5 to 6 ($q=\infty$) or to 7 ($q=0$) (tables 17-20).

Tables 21-24 give the first five height factors and their derivatives for y =0.1, 0.2,...., 5.0.

Furthermore tables 25-33 give auxiliary functions for calculations using formulas (4.11), (4.12), (4.19)-(4.24).

In the tables given the error can reach one and a half units of the last sign written out.

Small type is used for the figures in which the error can reach three units.

In practical calculations it is most convenient to use the graphs which are appended to this work.

Drawing 1 gives the standard distance $s_0(\lambda)$ and the standard height $h_0(\lambda)$ for obtaining the reduced distances and heights by using formulas (1.5), (1.18), (1.19). On the axis of abscissas in a logarithmic scale is plotted the length of the wave λ in meters. On the axis of ordinates, also in a logarithmic scale, are plotted the variables $s_0(\lambda)$ and $h_0(\lambda)$ in meters. Curves are given for different values of effective radius a^* from $a^*=a$ to $a^*=4a$.

The basis for the graphs of the attenuation factor V for $q=\infty$ (drawings 11-14) and $q=0$ (drawings 15-18) are tables 1-16, in which this factor is given for fixed values of the ratio $m=\frac{y_1}{y_2}$. Furthermore for these graphs all the auxiliary formulas for large and small values of y were used and a series of precalculations was made for values of m which are not found in tables (1-16).

In the most simplest manner the function V depends on x in such a way that based on this variable the linear interpolation is permissible. Therefore all the graphs were constructed for fixed values of x, taken through 0.5.

On the axis of abscissas of all these graphs is plotted the variable

$$m=\frac{y_1}{y_2}<1.$$

On the axis of ordinates in drawings 11, 12, 15 and 16 the lg |V| + 10 or arc V are plotted. In each sketch a family of curves is given for fixed values of y_1. For each of the values of $q=\infty$ and q=0 a drawing is given which consists of two bands, on which such graphs are grouped for fixed values of x which are arranged in increasing order.

For convenience of interpolation the graph for x=3.5, with which the first band always ends, is repeated on the second. The broken lines which intersect the families of curves represent the line of the horizon p=0 (or, which is the same, $x-\sqrt{y_1}-\sqrt{y_2}=0$).

In drawings 13, 14, 17, 18 on the axis of ordinates y_1(less than y) is plotted, and on the diagram the lines of equal values of the function are drawn. The broken curve gives the line of the horizon, and the dot-and-dash curve represents the line p=0.7, above which the reflection formula is applicable.

As was noted in the previous section, for $q=\infty$ the closing of the reflection and diffraction formulas takes place to the right of the maximum, closest to the area of the shadow, of the modulus of the attenuation factor in the area of monotonicity of the function (see Fig. 2), whereas for q=0 this closing takes place on the maximum itself or even to the left. This explains why for q=0 the graphs do not reach the dot-and-dash line p=0.7 everywhere, i.e., the reflection formulas, because the graphic representation of a family of non-monotone functions is very difficult.

Drawings 19-22 give the attenuation factor V for small values of x, y_1 and y_2. For each of the values x=0.007, 0.01, 0.03, 0.05, 0.1 and 0.25 a separate graph is given, and all these graphs are assembled on one sheet.

On the axis of abscissas $y_2 \leqslant 0.1$ is plotted, and on the axis of ordinates - the function |V| or arc V and for each value of $y_1 < 0.1$ a separate curve is given. For intermediate values of y_1 the function can be obtained either by interpolation on the graph or by formulas (4.11), (4.12) with the help of tables 25, 26.

For the attenuation factor $V_1(z,y,q)$ we give graphs of three types (for each of two values of q): dependence of lg $|V_1|$ and arc V_1 on z with fixed values of y (drawings 24, 25, 30, 31), the dependence of these functions on y with fixed values of z (drawings 26, 27, 32, 33), and, finally, their level lines (drawings 28, 29, 34, 35). On graphs of the last type the variable z is plotted on the axis of abscissas, and on the axis of ordinates - the variable y. The broken line (on all the graphs) denotes the line of the horizon, the dot-and-dash - the line p=0.7 - the boundary of applicability of reflection formulas.

For V_1, as was already pointed out above, the series of deductions converge more poorly, and it is not possible to achieve closing/joining with negative formulas on a mass scale.

However, for large or small values of y it is always possible to reach the areas $p \geqslant 0.7$ with the help of formulas (3.18)-(3.19) and (4.23), (4.24). The enumerated graphs of the attenuation factors V and V_1 we preface with diagrams (drawings 10 and 23) which give the areas of applicability of the reflection formulas, i.e., the areas where $p \geqslant 0.7$, and these diagrams are given in reduced and geometric coordinates.

In drawing 10 the graph of applicability of reflection formula (5.1) for function V is given.

On the axis of abscissas of this graph the ratio $m = \frac{y_1}{y_2}$ is plotted, and on the axis of ordinates - y_1. For each value of x (through 0.5) curves are given on which the condition p=0.7 takes place. Each curve ddvides the area of all (y_1, y_2) into two, in one of which (upper) at a given value of x the reflection formula is applicable.

Let us note that y_1 increases upwards, and y_2 increases with movement to the left, when $m = \frac{y_1}{y_2}$ decreases. Thus the upper area corresponds to greater heights, and the lower - to lesser.

Since the distance and height are connected with the reduced coordinates by the correlations

$$s = s_0(\lambda) x$$
$$h = h_0(\lambda) y,$$

then, in order to switch from x and y to s and h, it is sufficient to change the scale on the axis of ordinates and to replace the values of x on the curves in accordance with the indicated formulas. On our graph scales of geometric heights (in meters) and distances (in kilometers) are given for several wavelengths λ and effective radius of the Earth a^*=10,000 km.

Drawing 23 gives the areas of applicability of reflection formula (5.2) for function V_1. Here parameter p depends only on two variables z and y. On the axis of abscissas the variable z is plotted, and the distances (in kilometers), corresponding to several wavelengths λ, of the observation point from the boundary of the geometric shadow on the terrestrial surface with a plus sign in the shadow and minus in the illuminated area.

On the axis of ordinates the reduced height y is plotted, and the heights in meters for those same wavelengths λ. The reflection formula is applicable above the curve.

In drawings 36-39 the function $V_{11}(\zeta, \mu, q)$ is given for $q=\infty$ and q=0. On all the drawings the variable ζ is plotted on the axis of abscissas, and a separate curve corresponds to each value of μ. The heavier line on the drawing denotes the curves which correspond to $\mu=\infty$. On the sector $\zeta > 0$ for $q=\infty$ the curves are intercepted in those places where, as investigations show, the function V_{11} is no longer applicable for the calculation of V and V_1 using formulas (3.15)-(3.16) and (3.18)-(3.19).

Furthermore, in drawings 37a and 39a the actual and imaginary parts of function $V^{(1)}+V^{(2)}$ are given for $q=\infty$ and q=0 correspondingly. Since the calculation of $V^{(0)}$ using formula (3.12) is reduced to the use of the known tables, then drawings 37a and 39a make it possible easily to obtain the function V_{11} for $q=\infty$ and q=0 with any values of the parameter μ.

For the functions $\frac{\partial V(x, 0, y_2, \infty)}{\partial y_1}$ and $V(x, 0, y_2, 0)$, making it possible to find V for $y_1 \ll 1$ using formulas (4.19) and (4.20) and depending, just as $V_1(z,y,q)$ on two variables x and y_2, the same graphs are given as for V_1: dependence of the logarithms of the moduli and arguments of functions on x with fixed values of y_2 (drawings 40, 41, 46, 47); their dependence on y_2 with fixed values of x (drawings 42, 43, 48, 49; and the level lines (drawings 44, 45, 50, 51). On the latter on the axis of abscissas the reduced distance x is plotted, and on the axis of ordinates the reduced height y_2.

On the following graphs we give the functions from one variable (with the same fixed values of $q=\infty$ and $q=0$). In drawing 52 the logarithms of the moduli and phases of functions $\frac{\partial^2 V(x,0,0,\infty)}{\partial y_1 \partial y_2}$ and $V(x,0,0,0)$ are given simultaneously, and in drawing 53 the functions $g(z)$ and $f(z)$, making it possible to determine V for $y_1, y_2 \ll 1$ using formulas (4.21) and (4.22) and V_1 for $y \ll 1$ using formulas (4.23) and (4.24).

The graphs in drawings 54 and 55 are intended for determination of V and V_1 using monomial formulas, i.e., using the first terms of series (2.6), (2.9), (2.7), (2.10) for large values of x and z.

Drawing 54 gives the factors, depending on the reduced distance x and z,

$$\left.\begin{aligned} a(x,\infty) &= -e^{i\frac{\pi}{4}} 2\sqrt{\pi x}\, e^{ixt_1^{\circ}} \\ a(x,0) &= e^{i\frac{\pi}{4}} 2\sqrt{\pi x}\, \frac{e^{ixt_1'}}{t_1'} \end{aligned}\right\}, \quad (6.1)$$

$$\left.\begin{aligned} a_1(z,\infty) &= i2\sqrt{\pi}\, \frac{e^{izt_1^{\circ}}}{w'(t_1^{\circ})} \\ a_1(z,0) &= i2\sqrt{\pi}\, \frac{e^{izt_1'}}{t_1' w(t_1')} \end{aligned}\right\}, \quad (6.2)$$

and in drawing 55 - height factors $g_1(y)$ and $f_1(y)$.

Functions V and V_1 are calculated using the formulas

$$\left.\begin{aligned} V(x,y_1,y_2,\infty) &= a(x,\infty) g_1(y_1) g_1(y_2) \\ V(x,y_1,y_2,0) &= a(x,0) f_1(y_1) f_1(y_2) \end{aligned}\right\}, \quad (6.3)$$

$$\left.\begin{aligned} V_1(z,y,\infty) &= a_1(z,\infty) g_1(y) \\ V_1(z,y,0) &= a_1(z,0) f_1(y) \end{aligned}\right\}. \quad (6.4)$$

Drawing 56 gives the factor $M_1(y,q)$, connecting V and V_1 in the area of deep shadow, and also $y^{-\frac{1}{4}}$ for connection of these functions between one another in the area of semishadow and with function V_{11}, and in drawing 57 the phase $\frac{2}{3} y^{3/2}$, necessary for these formulas of connection (3.3), (3.7), (3.15)-(3.16), (3.18)-(3.19).

Since this phase increases exceedingly rapidly, then we will give only its supplement to the whole number of periods.

In order to facilitate the application of our results to practical calculations, we present here two schemes in which it is indicated which drawings and formulas should be used with the different values of arguments x, y_1, y_2 or z, y.

Calculation of V(x, y_1, y_2, q)

(1) Область значений x (z или ζ)	(2) Область значений y_1	(3) Область значений y_2	(4) Таблицы	(5) Рисунки	(6) Формулы	(7) Примечание
$1 < x < 6$ (7)	$0,1 \leq y_1 \leq 4,8$	$0,1 \leq y_2 \leq 4,8$	1—16	11—18		
$0 < x < 0,25$	$0 \leq y_1 \leq 0,1$	$0 \leq y_2 \leq 0,1$	25, 26	19—22	(4.11), (4.12)	(8) для $y < 0,1$
$x > 6$	$0 \leq y_1 \leq 5$	$0 \leq y_2 \leq 5$		54, 55	(6.3)	(9) $g_1(v) \approx y$, $f_1(v) \approx 1$
$z = x - \sqrt{y_2} \geq 0,5$ (1)	$y_1 > 4,8$	$0,1 \leq y_2 \leq 4,8$	17—20	24—35, 56, 57	(3.3), (3.7)	(3.7) применима в глубокой тени и при $y > 4,8$ (10) (3,3) применима при $y > 15(10)$, $z > -2$
$\zeta = x - \sqrt{y_1} - \sqrt{y_2} > -2$	$y_1 > 4,8$	$y_2 > 4,8$		36—39, 56, 57	(3.15), (3.16)	
$0,5(1) \leq x \leq 6$ (7)	$0 < y_1 \leq 0,1$	$0,1 \leq y_2 \leq 4,8$	27—30	40—51	(4.19), (4.20)	
$0,3 < x \leq 6$ (7)	$0 \leq y_1 \leq 0,1$	$0 \leq y_2 < 0,1$	31, 32	52	(4.21), (4.22)	(11)
$0,5(1) \leq x \leq 6$ (7)	$0 \leq y_1 \leq 0,1$	$y_2 > 4,8$	17—20, 33	24—35, 56, 57 и 53	(3.3), (3.7) и (4.23), (4.24)	Сначала применить вторую пару формул, а затем первую

Key: (1) Area of values x (z or ζ); (2) Area of values y_1; (3) Area of values y_2; (4) Tables; (5) Drawings; (6) Formulas; (7) Notes; (8) for; (9) (3.7) applicable in deep shadow and when $y > 4.8$; (10) (3.3) applicable when $y > 15(10)$, $z > -2$; (11) Initially the second pair of formulas is used, then the first.

Calculation of V_1(z, y, q)

(1) Область значений z (или ζ)	(2) Область значений y	(3) Таблицы	(4) Рисунки	(5) Вспомогательные формулы
$0,5(1) < z < 4,8$	$0,1 < y < 4,8$	17—20	24—35	
$\zeta = z - \sqrt{y} > -2$	$y > 4,8$		36—39, 56, 57	(3.18), (3.19)
$-5 < z < 5$	$0 < y < 0,1$	33	53	(4.23), (4.24)

Key: (1) Area of values z (or ζ); (2) Area of values y; (3) Tables; (4) Drawings; (5) Auxiliary formulas.

7. Examples of the Calculation of Field Intensity

We will give several examples of calculations of field intensity based on the tables and graphs of our work.

The horizontal component E_φ of field intensity, created by a horizontal electric dipole in the azimuth plane $\varphi = \frac{\pi}{2}$, perpendicular to the moment of the dipole itself, has the form [compare formula (1.3)]

$$E_\varphi = 3.10^5 \frac{|V(x, y_1, y_2, \infty)|}{s} \mu V/m, \qquad (7.1)$$

where $V(x, y_1, y_2, \infty)$ is the attenuation factor, and s - distance in kilometers. (The factor $\sqrt{\frac{\sin\theta}{\theta}}$ in practice can be disregarded within the limits of 1,000 km.)

Calculation formula (7.1) gives an amplitude value of horizontal component of the electric field with that moment of the dipole when in free space it radiates a power of one kilowatt. If the power of radiation (as this is often used) in free space is equal to $\frac{1}{2}$ kW and it is desired to know the effective (mean-square) value of the field, then the intensity obtained by formula (7.1) has to be cut in half.

Drawings 58-60 and 61, 62 give the graphs of the electric field according to formula (7.1) for different heights of the corresponding points when λ =10 cm and λ =7 m. In both cases the effective radius is taken equal to a^*=10,000 km. On the left scale μV/m is given, and on the right - the same values in decibels. For construction of these curves it is necessary to use graphs of all types: basic, containing the attenuation factor V (drawing 11 or 13), graphs of V for small x, y_1 and y_2 (drawing 19), the functions

$$\frac{\partial V(x, 0, y_2, \infty)}{\partial y_1} \text{ and } \frac{\partial^2 V(x, 0, 0, \infty)}{\partial y_1 \partial y_2}$$

(drawings 40-44 and 52) for the case when one or both y are small, and x is not very small, finally the functions $V_1(z, y, \infty)$ and $V_{11}(\zeta, \mu, \infty)$, where $z = x - \sqrt{y_2}$, and $\zeta = x - \sqrt{y_1} - \sqrt{y_2}$ (drawings 24-28 and 36) when one or both y are great. When function V_1 is used it is necessary to observe care in the selection of the transient factor from V_1 to V, keeping in mind that M_1 is applicable only in the area of deep shadow, and $y^{-1/4}$ is applicable in the areas of semishadow and light, however, with not too large negative $z = x - \sqrt{y_2}$. There where function V_1 is not applicable the reflection formula already functions.

Using the corresponding graphs it is possible in a completely analogous manner and in the same areas to obtain curves for the phase of the field components.

Drawing 63 gives comparative curves of field intensity for values of effective radius $a^* = a$=6,370 km and $a^* = 4a$=25,500 km, with wavelengths of λ=3 cm, 10 m and 100 m.

The examples given show that on the basis of the numerical and graphic material collected in this work it is possible for any specific case to calculate the intensity and phase of the field, given by the theory of diffraction propagation of radio waves in a homogeneous atmosphere above a spherical Earth, when the latter can be considered an ideal reflector.

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