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MX SURVIVABILITY:

PASSIVE AND ACTIVE DEFENSE

THESIS

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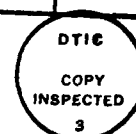S   D
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THESIS

Presented to the Faculty of the School of Engineering

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by

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Abstract

This thesis investigates MX survivability when a layered
or terminal defense system is deployed with various numbers
of multiple protective shelters (MPS). The layered defense
system defends the MX with an exoatmospheric layer which is
augmented by an endoatmospheric layer. The exoatmospheric
layer protects the MX with longwave infrared (LWIR) guided
interceptors which must directly impact an incoming RV at
approximately 300,000 feet altitude to destroy it. The endo-
atmospheric layer consists of a terminal BMD system known as
Low Altitude Defense (LoAD) which defends the MX with three
hypersonic, nuclear armed interceptor missiles. The terminal
defense system consists of either one or two LoAD systems.
This research effort determines the most cost effective defense
system, and draws conclusions on these systems based upon
quantitative and qualitative (ex., political) considerations.

MX SURVIVABILITY: PASSIVE AND ACTIVE DEFENSE

I. Background

General Topic

According to a statement by former Secretary of Defense
Brown in October 1980, America's land-based ballistic missile
force may now be vulnerable to Soviet ICBM attack (Ref 23:16).
Two causes cited are the rapidly increasing numbers of Soviet
reentry vehicles (RVs) and technological advances allowing
RV accuracy to increase to unpredicted levels in this decade.

In 1976, the U.S. initiated full scale development of a
more secure basing mode consisting of MX missiles in multiple
protective shelters (MPS). The strategy is to base 200 mis-
siles in 4600 horizontal shelters, with one missile located
in each group of 23 shelters (see Figure 1). After hardening
to a certain level, this obscuration of a missile's precise
location appears to be the only viable passive technique left
to increase U.S. ICBM survivability (Ref 20:15). The probabil-
ity of a single enemy RV destroying one MX missile cannot
exceed one in 23 as long as the location of the missile within
the cluster of 23 shelters remains unknown (i.e., preservation
of location uncertainty is maintained). Although President
Reagan has proposed that the MX be initially based in existing
silos, this proposal has yet to be ratified by the Congress.
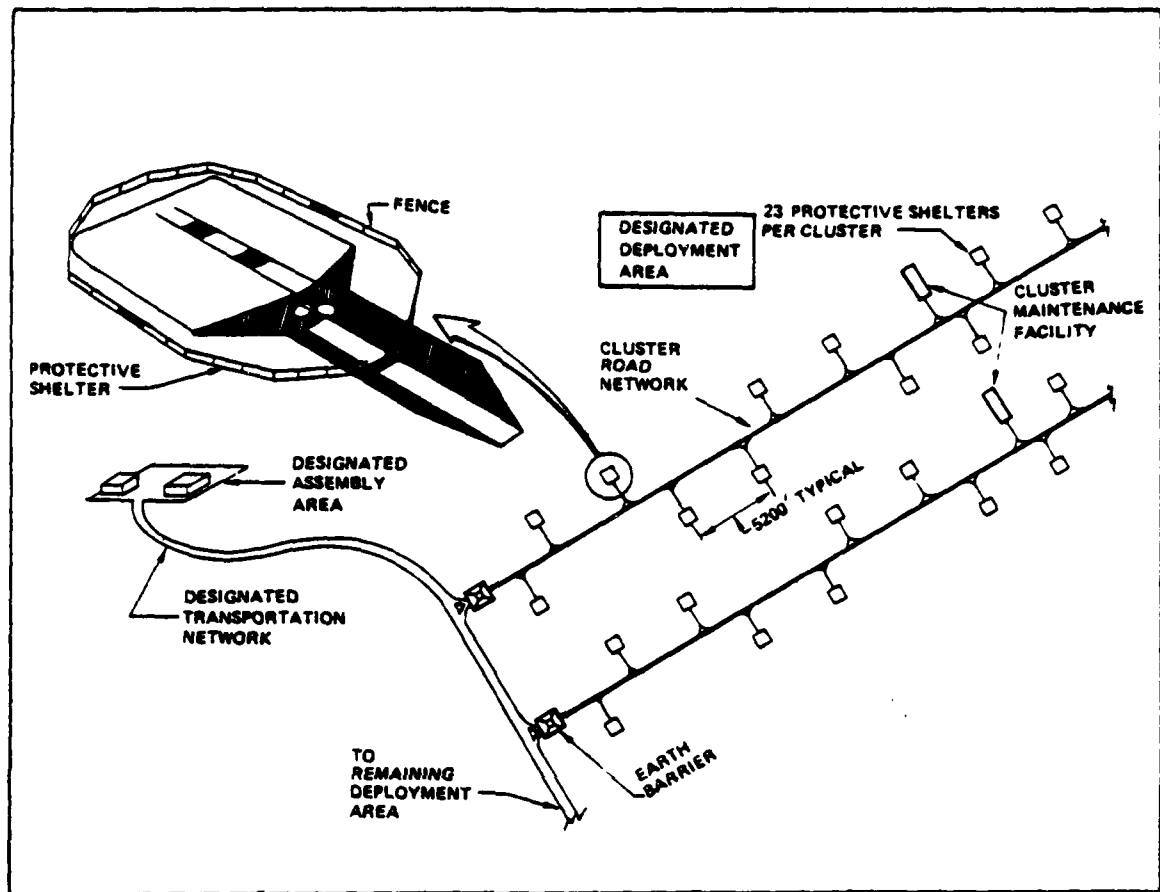
Figure 1. MX MPS Basing Plan (Ref 15)

Furthermore, the final MX basing plan has not been determined,
and MPS is certainly a viable option which needs to be studied.

As advances in Soviet technology improve the effective-
ness of each RV, the probability of one RV successfully destroy-
ing one MX shelter (an easily locatable target) will approach
one. At this point, 4600 RVs could destroy all 4600 shelters
and the 200 MX missiles. Due to this threat, the Army (mili-
tary department responsible for ICBM defense) is currently
studying various types of ballistic missile defense (BMD)

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systems. This study will investigate MX survivability as a function of several possible defense systems.

General Situation

Since the strategic capability of the two major military powers has drawn equal in the past decade, the U.S. strategy for nuclear deterrence has been one of approximate equality of strategic offensive capability (i.e., essential equivalence) (Ref 1:2). Defensive forces have not been a part of U.S. Strategic Policy since the signing of the Anti-Ballistic Missile (ABM) Treaty with the Soviet Union in 1972 and the abandonment of the U.S. operational Safeguard ABM site in North Dakota. The U.S. considered defensive weapon systems that might threaten Soviet deterrent capabilities as destabilizing (Ref 1:2) and adopted the idea in the 1970s that modernization of offensive forces and BMD development/deployment were mutually exclusive policies (Ref 20:12). It was felt that U.S. strategic deterrent goals could be met with offensive weapons, exclusively.

In recognition of the growing strategic imbalance and vulnerability of our ICBMs, a change in the U.S. nuclear targeting policy was formalized in 1980 (Ref 8:65-66). It is now a "countervailing" strategy that allows attack of military targets (counterforce targets), industrial targets/ population centers (countervalue targets), or a combination of these two.

In order to retain the countervailing strategy (with its implied assured destruction and damage limiting roles) and still promote arms reduction, the U.S. strategic force structure must be robust enough to withstand technological surprise, be somewhat insensitive to arms-control "cheating," respond economically to threat growth, and finally, promote crisis stability. It does not seem possible to achieve these goals with strictly an offensive force structure (Ref 8:3 and 20:15).

It has been suggested by several reliable sources and verified by initial studies that a properly configured strategic force of offensive missiles and a ballistic missile defense would permit continued deterrence, allow meeting the goals stated above, and still have significant economic advantages over an all-offensive force. Additionally, this force could achieve the above regardless of Soviet behavior/ response (Ref 8:3 and 20:11).

For example, if the U.S. deployed two ABMs per MX cluster (i.e., one MX missile and 23 shelters), Soviet strategy must target at least three RVs at each shelter (two to defeat the BMD and one to destroy the MX) to insure destruction of the one missile in an MX cluster -- a total of 69 RVs. Therefore, Soviet destruction of the planned U.S. MX configuration of 200 MX clusters would require a total of 13,800 RVs. With this type of leverage, it is possible to envision a reduction in the total number of shelters needed for a preset level of ICBM survivability.

Due to the impact of MX on ecology, the economy, and the defense posture, additional study of MX survivability with and without various defense elements is beneficial if for no other reason than to clearly elucidate all possible alternatives within technological reach. An extensive search of unclassified literature (for instance, Refs 14, 17, 27) gives no indication that the layered BMD problem described has been researched in total using simulation (see Ref 22 for a simulation of the low altitude component only). All discovered references use mathematical models and very narrowly defined Ballistic Missile Defense systems. The problem, as defined in this paper, lends itself very well to the use of simulation because of the relatively undefined nature of this postulated defense system. A general analysis of an undefined BMD system, using simulation, will be a helpful first step toward investigating the capabilities of a particular system if one is defined in more detail later.

Problem Statement

This thesis addresses MX survivability with and without employment of a layered BMD system, as well as the cost effectiveness of survivability alternatives. A layered BMD system consists of two elements, (1) an exoatmospheric component, and (2) an endoatmospheric component. The endoatmospheric component will be the Low Altitude Defense (LoAD) system as defined by James Moore (Ref 22).

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The effect of the layered defense on MX survivability in many varied situations will be examined and analyzed, as will the effect of different numbers of MX shelters per complex. This study should quantify the cost effectiveness and survivability of defense alternatives as the MX is defended by different configurations against a postulated Soviet attack. The overall results should delineate the effectiveness and relative costs of (1) various active defenses and (2) various numbers of MX shelters when confronted by a Soviet threat.

Overall Objective

The objective of this study is to develop a methodology, using simulation, which quantifies MX survivability for various combinations of passive and active defense.

Specific Goals

Four specific goals have been established for this study:

1. Construct a computer model which will compute MX survivability under certain MPS and defense configurations, and a predetermined Soviet threat.

2. Explore the tradeoff relationship between the number of MX shelters per complex and a layered defense system.

3. Explore the tradeoff relationship between the number of MX shelters per complex and terminal defense systems alone.

4. Analyze the results and comment on the cost effectiveness of possible solutions to the U.S. ICBM vulnerability dilemma, while taking into account qualitative factors such as political implications.

Scope

When constructing the models, it will be essential to calculate the probability of kill (PK) of an attacking Soviet RV and the PK of the U.S. interceptors when launched at an incoming RV. To accomplish this, four data sets (RV, MPS, terminal defense, and layered defense) which contain the information required to perform the necessary calculations must be developed. Once values are selected for the required parameters, the effectiveness of various combinations of passive (MPS) and active (terminal or layered) defense techniques will be examined.

The research will begin with a description of the Soviet threat and the U.S. defense techniques utilized in this analysis. A description of the Soviet threat will include such factors as the number of incoming RVs, their yield, and CEP; while a description of the U.S. defenses will include such systems as the MPS, the terminal defense, the exoatmospheric defense, and the layered defense.

The following alternatives will be examined:

1. Given one terminal BMD system per MX complex and various numbers of MPS per MX complex, determine the number of exoatmospheric interceptors per complex necessary to obtain a predetermined level of survivability.

2. Given one terminal BMD system per complex and no exoatmospheric defense layer, determine the number of MPS per complex necessary to obtain a predetermined level of survivability.

3. Given two terminal BMD systems per complex and no exoatmospheric defense layer, determine the number of MPS per complex necessary to obtain a predetermined level of survivability.

During the analysis phase, the most significant variables of each data set can be identified while the effectiveness of the passive and active defense systems is determined.

Limitations

In order to facilitate this study, several assumptions have been made. Some of these are based on actual facts, while others are made to assist in the modeling process.

The number of interceptors per terminal defense unit is assumed to be three, to be consistent with the recent study of James Moore (Ref 22). It is also assumed that the U.S. will not launch offensive missiles during a Soviet attack and will ride out the first strike.

It is assumed that the Soviets will attack the MX complex uniformly (i.e., an equal number of RVs will be targeted at each complex and these RVs will be evenly distributed among the shelters in each complex).

Methodology

Modeling Phase. The system science paradigm was used to develop the models, which means that the models were developed through an iterative process of conceptualization, analysis and measurement, and computerization. This iterative process was an aid in obtaining all the desired parameters in the models (Ref 31).

The models are programmed in the simulation language Q-GERT developed by A. Alan B. Pritsker. Q-GERT was chosen because it is based upon queuing theory, and the entire system being modeled is envisioned as a queuing network with the attacking RVs and defense systems representing the customers and servers, respectively. Q-GERT also allows Fortran to be directly inserted and it readily utilizes probabilities (Ref 26).

Analysis Phase. The model output provides data to develop an equation (using regression analysis) in order to determine the following:

1. The number of shelters and the number of exo-atmospheric interceptors which provide a prescribed level of survivability with one terminal defense unit (TDU).

2. The number of shelters which provide a prescribed level of survivability with one TDU.

3. The number of shelters which provide a prescribed level of survivability with two TDUs.

With these data and equations in hand, each alternative was quantitatively (i.e., in dollars) examined, and conclusions were drawn based upon both quantitative and qualitative (ex., political and environmental) considerations. An outgrowth of this analysis will be the significant variables of this study and recommendations for further study in the area of Ballistic Missile Defense (BMD).

II. The Model

The system being modeled can be divided into four
separate subsystems: attack, target, exoatmospheric defense,
and endoatmospheric defense (Figure 2). The variables and
parameters of each subsystem, the values assigned to the
variables, and sensitivity analysis on these values will be
discussed in this chapter; as well as the probability of
kill methodology, the simulation models, and the verification
and validation of the models.

Attack Subsystem

The attack subsystem is composed of Soviet ICBMs which
transport the attacking RVs designed to destroy a hardened
MX shelter. By the mid-1980s, the Soviet Union is expected
to have approximately 6000 warheads in the one-megaton yield
range (Refs 28:23 and 26). It is believed that these war-
heads could be delivered in various configurations by the
SS-17, SS-18, SS-19, and older generation Soviet ICBMs. The
CEP of these warheads ranges from 0.1 nautical miles (NM)
to 0.25 NM (Refs 16:54; 28:23; and 26). By decreasing the
yield of each warhead, thus allowing more warheads per
missile, the number of attacking RVs could be increased with-
out increasing the number of ICBMs. This process of fraction-
ization causes fratracide to become a problem. Fratracide is
the destruction of an RV by another RV's detonation (Ref 9:34).

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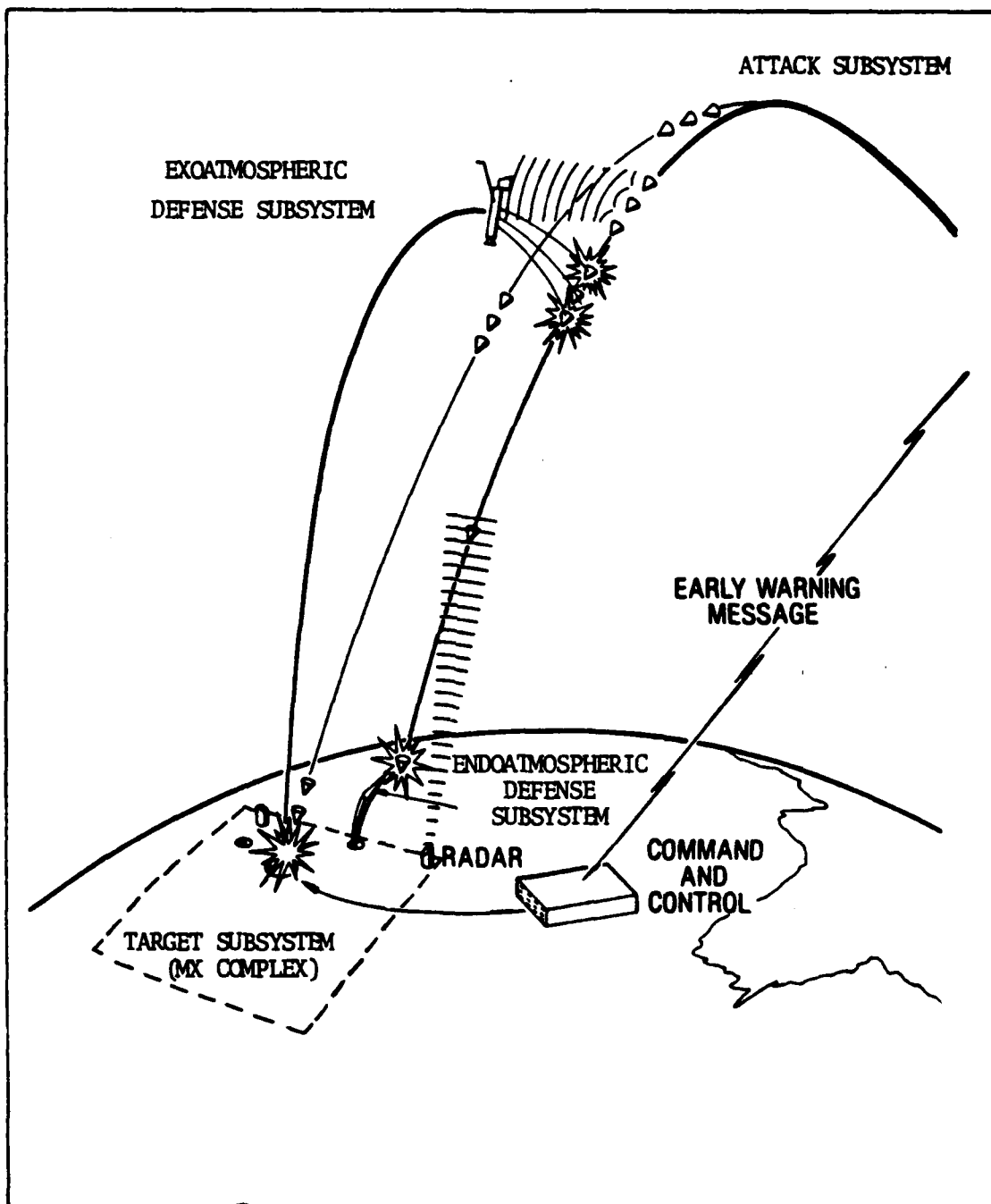
Figure 2. Model Subsystems (Ref 1)

There are basically two kinds of fratracide, local and area. Local and area fratracide deal respectively with the detonation of an earlier warhead assigned to the same target and the detonation of an earlier warhead assigned to a different, though nearby, target. (Ref 2:55). In a fratracide model developed by Steinbruner and Garwin, the second warhead's survivability level varies and the third and fourth warheads usually will not survive (Ref 35). In their model, the authors assume that individual warheads arriving at the same target must be separated by at least six minutes or they have a high probability of being destroyed by the first detonation. This thesis uses RV spacing of from several seconds to slightly over two minutes. "Normally it is assumed that at most two, and perhaps only one, warhead can be used per target without being overcome by fratracide" (Ref 2:58).

Target Subsystem

The target subsystem consists of 200 MX missiles located in from 1600 to 4600 hardened horizontal MX shelters, and any terminal defense units (TDUs) and its components (Figure 3). The one MX missile hidden in an MX complex will represent from 8 to 23 aimpoints, depending on the number of shelters in the complex. The destruction of more than one shelter by a single attacking RV is prevented by spacing the shelters approximately 5200 feet apart (Ref 15). The hardness of each horizontal shelter is dependent upon the shelter design selected. With

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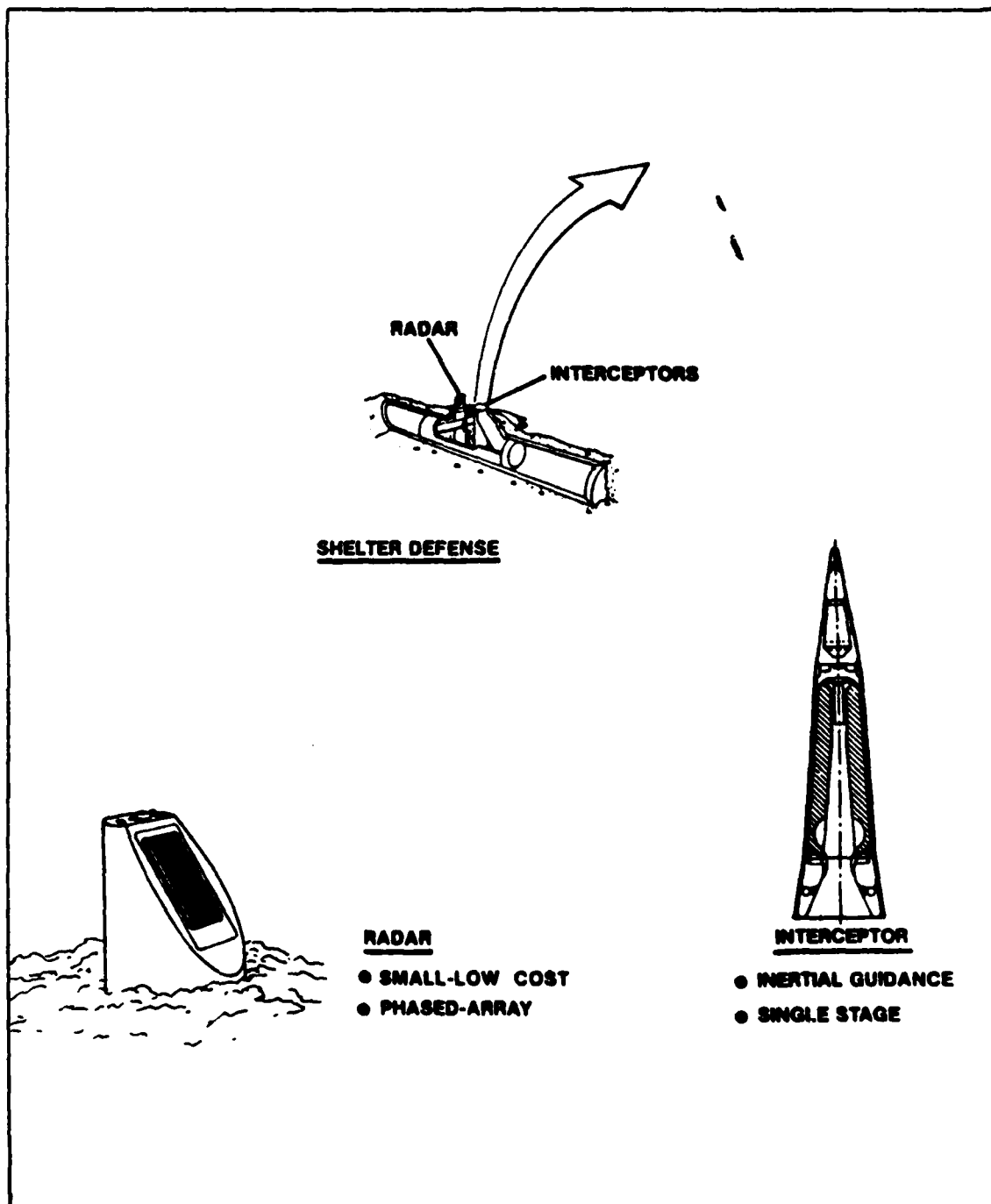
Figure 3. Terminal Defense Unit Components (Ref 1)

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four roof portals (to allow Soviet verification), the expected
hardness of an MX or TDU shelter to overpressure would be
approximately 600 pounds per square inch (psi) (Ref 19). A
decrease in the number and/or size of the roof portals, however,
could increase the expected hardness to over 1000 psi (Ref 16:58).

The MX and TDU shelters are also vulnerable to the crater-
ing caused by an attacking Soviet RV. Appendix A provides a
diagram of a horizontal shelter and the methodology for calcu-
lating the vulnerability to cratering for both a MX and TDU
shelter.

If a terminal defense system exists, its radar network
becomes part of the target subsystem. The TDU will not be
able to defend the MX if the radar network is destroyed, but
this consequence is overcome since it is felt that the TDU's
radar network can be made survivable (Ref 30).

Exoatmospheric Defense Subsystem

This defensive subsystem is currently envisioned as
having two phases: (1) data gathering/attack evaluation, and
(2) intercept. The attack data will come from a ballistic
information probe launched immediately when alerted to a
possible Soviet attack by other national resources. At an
altitude of approximately 300,000 feet the probe will scan the
attack azimuth with long wave infrared (LWIR) sensors and,
using on-board data processing, evaluate the attack, sending
the information back to command authorities. At this altitude,
the information probe can view the approaching vehicles

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against a cold space background - a requirement for accurate, timely IR detection. This phase is beyond the problem being studied and is not modeled or discussed hereafter.

The intercept phase occurs when a ballistic intercept probe carrying multiple high-speed interceptors is launched. A LWIR sensor on the intercept probe detects individual RVs and assigns each interceptor missile it is carrying to an RV. Each interceptor then uses LWIR terminal guidance and will directly impact the incoming RV to destroy it. This kill mechanism is very similar to that planned for the current F-15 launched antisatellite (ASAT) program.

"Impact-point prediction . . . is not expected to be able to resolve impacts among closely spaced shelters of any of the MPS emplacement schemes under consideration" (Ref 1:7). Therefore, the exoatmospheric BMD system cannot preferentially defend the MX or terminal defense unit, and must intercept each attacking RV that comes within the defense systems' field of view. The assumption that all interceptors can be fired while the entire attack cloud is scanned, coupled with the lack of preferential defense capability, results in the exo-atmospheric defense system firing its N interceptors at the first N attacking RVs on a one-to-one basis. Thus, the defense system is defending itself by not allowing any RV to come close enough to destroy it.

The circular error probable (CEP) of the exoatmospheric interceptors is assumed to be normally distributed in both

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the horizontal and vertical dimensions of the attacking RV plane. Since it is assumed that the exoatmospheric interceptor will pass through the plane perpendicular to the RV trajectory, the third dimension of depth is ignored and circular error probable rather than spherical error probable calculations are made.

Endoatmospheric Defense Subsystem

The endoatmospheric defense subsystem includes the terminal defense unit's (TDU's) radar network, three hypersonic nuclear armed interceptors, and a control unit. The control unit, the interceptors, and part of the radar network will be located in a hardened, horizontal shelter (Ref 12).

The radar network of this subsystem, operating without the exoatmospheric defense layer, might have three stages. The first stage would be an early-warning system which would detect attacking RVs targeted somewhere within the 200 MX missile field. This stage would not be needed if the endo-atmospheric defense system were operating as part of a layered system. The second stage might be an MX complex radar warning system. This stage would detect attacking RVs targeted on a particular MX complex. If attacking RVs are descending on the complex, the last stage of the radar network would begin to function. This radar would track incoming RVs and determine their precise target among the shelters of one MX complex (Ref 22:12).

The strategy selected for the terminal defense unit (TDU) allows a TDU to launch the first two interceptors at RVs aimed at either the MX or TDU shelters. The remaining interceptor will be used for MX defense only. Therefore, if an RV is attacking a TDU shelter and only one interceptor remains, the interceptor will not be launched, and the TDU will be subjected to an RV detonation which may or may not destroy the TDU (Ref 22:38). This strategy was selected because it was determined most effective in James Moore's recent study (Ref 22).

The control unit of the TDU determines whether or not an interceptor should be launched based upon the above strategy. The process of launching an interceptor requires the TDU to leave the shelter, acquire an attacking RV with its radar, launch an interceptor, and return to the shelter (Ref 22:13). All endoatmospheric interceptors are assumed to detonate at approximately 20,000 feet and attempt to destroy an attacking RV with neutron radiation. Circular error probable (CEP), rather than spherical error probable (SEP), is used since it is assumed that the interceptor passes through the RV plane as in the exoatmospheric defense layer.

System Variables

The subsystems previously described contain the following set of variables which have a direct or indirect effect on MX survivability.

The set of attack subsystem variables which can affect MX survivability are the number of RVs attacking an MX complex, the Soviet targeting strategy, and the yield and CEP of the attacking RVs. Since the RVs may be subjected to a shower of neutrons, their ability to survive the neutron fluence of the interceptor warhead is a major factor in determining MX survivability. The height of burst affects an attacking RV's overpressure and cratering capability which also have a direct impact on MX survivability. RV reliability is also an important factor.

There are various target subsystem variables which can affect MX survivability. Shelter hardness, which depends on shelter design and soil type, is a variable considered to have a definite effect on MX survivability. The number of shelters per complex, shelter spacing, and target altitude are other target subsystem variables which must be considered when determining MX survivability.

Exoatmospheric defense variables are the weapon radius (WR) and CEP of the interceptors, the number of interceptors, and the reliability of the infrared guidance system.

The set of endoatmospheric defense variables which can impact the survivability of the MX include the number of TDUs per complex, the number of interceptors per TDU, and the launch strategy of the TDU. Other important variables include the yield, CEP, and reliability of the interceptors; and the reliability of the TDU's radar network.

The interaction of all of the above variables determines the survivability of U.S. MX missiles. The residual effects of a nuclear detonation (ex., fallout) and the treaty banning above ground nuclear testing preclude using actual nuclear weapons to study MX survivability. Therefore, this study will use models which employ all variables considered important in determining the outcome of a Soviet attack on U.S. MX complexes.

Structural Model

The variables previously mentioned were selected because they were deemed to have a significant effect on MX survivability. Estimates of MX survivability are critical to attaining the objectives of this thesis. Some of these variables are given preassigned values, while others will be parameters in the models and will be varied on each run of the model.

The variables chosen for modeling the attack subsystem are the number of Soviet RVs attacking an MX complex; the targeting strategy; the yield, CEP, and height of burst of the RV; RV reliability; and the sure-safe and sure-kill neutron fluence of the attacking RV. Of these variables, only the number of attacking RVs is considered a parameter.

The target subsystem variables chosen for the modeling phase of this study are the number of shelters per complex, the sure-safe and sure-kill overpressure levels of a shelter, the type of soil in which the shelters are constructed, and the altitude of the area in which the shelters are located.

The number of shelters per complex is the only model parameter.

The variables included in modeling the exoatmospheric defense subsystem are the number of interceptors; the strategy, weapon radius, and CEP of the interceptors; and the reliability of the infrared (IR) guidance system. The number of exoatmospheric interceptors is the only exoatmospheric defense model parameter.

The variables chosen for modeling the endoatmospheric defense subsystem are the number of TDUs per complex; the number of interceptors per TDU; interceptor strategy, yield, CEP, and reliability; and the radar network reliability. The number of TDUs per complex is the only parameter selected for the modeling of the endoatmospheric defense system.

The variables which are not considered parameters and are given preassigned values establish limitations on the results of this thesis. The assumed targeting strategy is to randomly target each shelter until all shelters are targeted, and then randomly assign excess RVs to those same shelters. If there are twice as many RVs as shelters, then the Soviets will randomly target each shelter on a two-to-one basis. If there are fewer RVs than shelters, the RVs are just randomly targeted against the shelters. The two main ways to destroy a ground emplaced shelter are with overpressure, or cratering and ground shock. The optimum height of burst (HOB) for overpressure depends on weapon yield and could be several thousand feet above ground level. The optimum HOB

for cratering depends on weapon yield and soil type and could be several hundred feet below ground level. In the models, the attacking RVs detonate when they contact the earth's surface (i.e., height of burst equal to zero). While this height of burst is not optimum for either overpressure or cratering, it allows for sizeable contributions to shelter destruction from each effect and facilitates evaluation of both kill mechanisms. It also allows a two-dimensional (CEP) analysis instead of a three-dimensional (SEP) analysis of the RV-shelter interaction. The yield of the RVs will be one megaton since the Soviets will have approximately 6000 warheads in this range by 1985 (Ref 28:23 and 26). The CEP of the Soviet SS-18 is within the range of 0.12 to 0.25 NM (Ref 16:54), and for the purpose of this study is assumed to be 0.2 NM. Reasonable estimates for the sure-kill and sure-safe neutron fluence of the RVs have been established at 10^{17} and 10^{13} neutrons per square centimeter (N/CM^2), respectively (Ref 7). The reliability of the incoming RVs has been assumed to be one.

Approximate levels for the sure-kill and sure-safe overpressure of the MX and TDU shelters have been set at 1250 and 750 pounds per square inch (psi), respectively (Ref 22:36). The soil type in which the MX and TDU shelters are built is assumed to be dry soil and/or dry soft rock, representative of the West or Southwest U.S. where the shelters might be constructed. The shelters are assumed to be spaced sufficiently far apart to prevent one Soviet RV from destroying two

shelters. Each of the shelters is assumed to be at an altitude of 4000 feet (MSL), which is a representative altitude for the valleys of Utah and Nevada.

The diameter of the exoatmospheric interceptor is equal to that of the current antisatellite (ASAT) design (i.e., one foot) (Ref 10:244). This one foot diameter, coupled with a one foot wide mesh which will unfold prior to impact, results in a one and one-half foot weapons radius (Figure 4). Since it is assumed that the attacking RVs are cone shaped with a radius of approximately one and one-half feet, the effective weapons radius of the exoatmospheric interceptor is set at three feet (Figure 4). Since an unclassified value for long wave IR guidance accuracy was unavailable, the CEP of the exoatmospheric interceptor was chosen to be two feet based upon the resolution accuracy of a Department of Defense (DoD) infrared telescope currently in development (Ref 29:24), and the general capabilities of current air-to-air missiles. The strategy of the exoatmospheric defense system is to launch all of its N interceptors at the first N RVs attacking the U.S. MX complex on a one-to-one basis. The reliability of the interceptors and the IR guidance system have been assumed to be one.

The number of endoatmospheric interceptors per TDU has been taken as three based on current DoD plans (Ref 37). The yield of the nuclear-armed endoatmospheric interceptors is set at five kilotons (KTs), since it is assumed that the U.S. prefers to detonate as small a nuclear warhead as possible
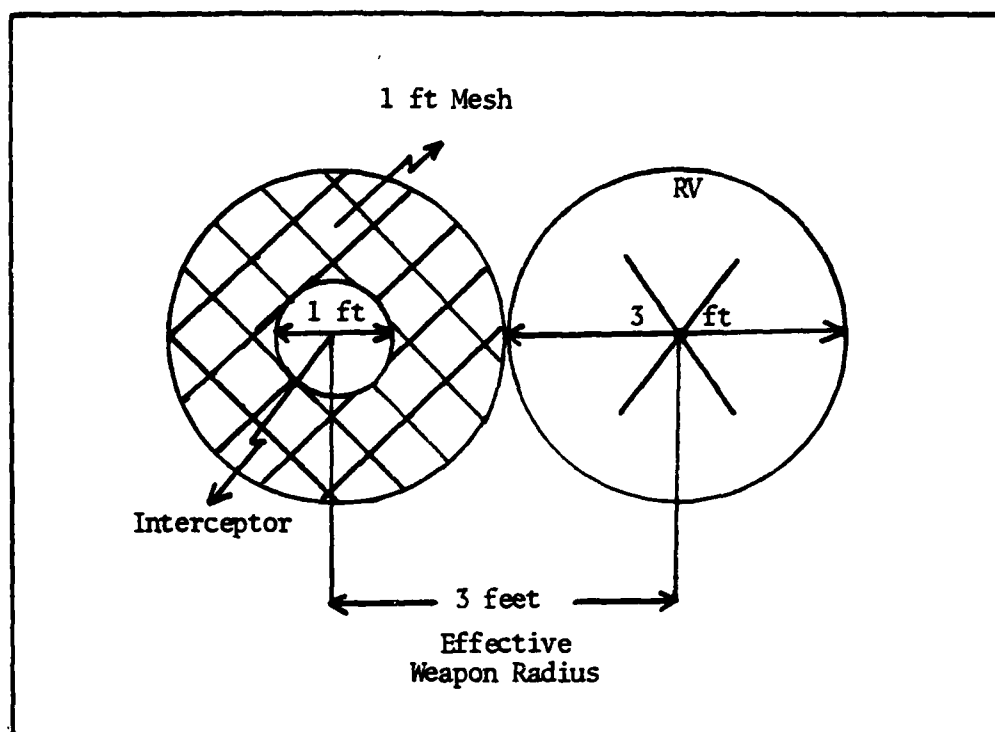
Figure 4. Exoatmospheric Interceptor and RV

within its atmosphere. A circular error probable (CEP)
of 600 feet is selected as being representative of U.S.
technological capabilities (Ref 22:21). This CEP is much
larger than the exoatmospheric interceptor's because the
endoatmospheric interceptor does not use terminal guidance
and must rely on more conventional guidance techniques.
Long wave IR terminal guidance is currently not feasible
in the endoatmospheric intercept phase due to the background
interference generated by the atmosphere at these wavelengths
(approximately 10^{-5} meters). As mentioned previously, the
strategy of the TDU allows launch of the first two interceptors
at RVs aimed at either the MX or TDU shelters, and the remain-

ing interceptor to be used for MX defense only (Ref 22:38). The interceptors and the radar network are assumed to be 100% reliable.

The ballistic missile defense models allow the number of shelters per complex, the number of attacking RVs, the number of exoatmospheric interceptors, and the number of TDUs per complex to be selected in order to determine a level of MX survivability.

Sensitivity of Model Variables

A few of the model variables which are given preassigned values are very critical in determining the effectiveness of the exoatmospheric interceptors, the endoatmospheric interceptors, and the Soviet RVs; all of which have an impact on MX survivability. Precise values of these variables are classified or unknown, which necessitates sensitivity analysis.

The critical variables which determine exoatmospheric interceptor probability of kill (PK) are the interceptor CEP and the effective weapons radius (interceptor radius plus RV radius) of the interceptor. The PKs of individual interceptors for various CEP and effective weapons radius (WR) combinations were calculated as shown in Appendix E and are presented in Table I. Since the radius of Soviet RVs is beyond U.S. control, the changes in effective weapon radius are essentially changes in interceptor weapon radius. An effective weapon radius larger than 3.5 feet was not examined

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TABLE I						
Exoatmospheric Interceptor PKs						
Effective WR (feet)	CEP (feet)					
	0.5	1.0	1.5	2.0	2.5	3.0
2.0	1.0	.94	.71	.50	.36	.27
2.5	1.0	.99	.85	.66	.50	.38
3.0	1.0	1.0	.94	.79	.63	.50
3.5	1.0	1.0	.98	.88	.74	.61

because it was felt that interceptors of this size/mass
would not be able to maneuver fast enough at the high
speeds experienced during an RV engagement. In general,
a one-half foot decrease in CEP provides a greater increase
in interceptor PK than a one-half foot increase in inter-
ceptor weapon radius. Therefore, interceptor CEP is considered
more critical than weapon radius.

Interceptor CEP and yield are the interceptor variables
which are critical in determining the PK of endoatmospheric
interceptors. Various combinations of interceptor CEP and
yield, and their corresponding PKs are shown in Table II. The
PKs were calculated using the neutron kill subroutines of the
simulation program (Appendix G). This table shows that
interceptor PK increases at a decreasing rate when yield is
increased and CEP is held constant, and interceptor PK

TABLE II

Endoatmospheric Interceptor PKs
(10^{13} N/CM^2 Sure-Safe, 10^{17} N/CM^2 Sure-Kill)

Yield (KT)	CEP (feet)			
	200	400	600	800
2.5	.75	.60	.49	.42
5.0	.83	.70	.60	.52
10.0	.89	.79	.70	.62
15.0	.92	.84	.76	.68
20.0	.94	.86	.79	.71

increases at a fairly constant rate when CEP is decreased
and yield is held constant. Therefore, as in the exoatmos-
pheric interceptor case, CEP is considered the most critical
interceptor variable in determining interceptor PK. Increas-
ing the yield of an interceptor may also be unacceptable
because larger nuclear warheads detonated in the atmosphere
cause increased nuclear fallout and peripheral blast damage.

Other important variables which impact the endoatmos-
pheric interceptors' PK are the sure-safe and sure-kill
neutron fluence levels of the attacking RVs. Various combin-
ations of sure-safe and sure-kill fluence levels are presented
in Table III for the interceptor yield and CEP used in this
thesis. The PKs in Table III were calculated using the neutron
kill subroutines of the simulation program (Appendix G). Since

the models in this study use sure-safe and sure-kill neutron intensity levels of 10^{13} and 10^{17} N/CM^2, respectively, this table shows that varying either the sure-safe or sure-kill fluence has a significant effect on interceptor PK.

TABLE III
Endoatmospheric Interceptor PKs
(CEP = 600 feet, Yield = 5 kilotons)

Sure-Kill Fluence (N/CM^2)	Sure-Safe Fluence (N/CM^2)			
	10^{11}	10^{12}	10^{13}	10^{14}
10^{15}	.985	.984	.983	.973
10^{16}	.915	.882	.813	.634
10^{17}	.800	.726	.599	.390
10^{18}	.680	.583	.447	.277

The critical variables that determine an attacking RV's PK are the yield and CEP of the RV, and the sure-safe and sure-kill overpressure levels of the MX or TDU shelter being attacked. The PKs of a single RV are shown in Tables IV, V, and VI for various combinations of RV CEP and yield, and MX shelter sure-safe and sure-kill overpressure levels. These PKs were calculated using the overpressure kill routines in the simulation program (Appendix G). In general, these tables show that, for yields greater than about 1000 kilotons, an

TABLE IV			
PK Against MX Shelter			
(500 psi Sure-Safe, 1000 psi Sure-Kill)			

Yield (KT)	CEP (NM)			
	0.10	0.14	0.20	0.25
250	.965	.599	.549	.190
500	1.0	.751	.572	.543
750	1.0	.986	.586	.557
1000	1.0	1.0	.601	.566
1250	1.0	1.0	.648	.574
1500	1.0	1.0	.763	.581

TABLE V			
PK Against MX Shelter			
(750 psi Sure-Safe, 1250 psi Sure-Kill			

Yield (KT)	CEP (NM)			
	0.10	0.14	0.20	0.25
250	.739	.599	.442	.160
500	1.0	.634	.572	.455
750	1.0	.805	.586	.557
1000	1.0	.992	.599	.566
1250	1.0	1.0	.609	.574
1500	1.0	1.0	.619	.581

TABLE VI			
PK Against MX Shelter			
(900 psi Sure-Safe, 1400 psi Sure-Kill)			

Yield (KT)	CEP (NM)			
	0.10	0.14	0.20	0.25
250	.677	.599	.271	.161
500	1.0	.635	.572	.282
750	1.0	.678	.586	.548
1000	1.0	.940	.599	.566
1250	1.0	.999	.609	.574
1500	1.0	1.0	.619	.581

increase in yield does not have a significant effect on RV PK; and the sure-safe and sure-kill overpressure levels are not critical in determining RV probability of kill. The most significant effect on RV PK occurs when the CEP of the reentry vehicles change from 0.2 NM to 0.14 NM, and vice versa. Considering that the models use sure-safe and sure-kill overpressure levels of 750 psi and 1250 psi, respectively, and a CEP and yield of 0.2 NM and 1000 kilotons, respectively, the attacking RVs CEP is considered the most critical variable in determining the probability of killing the MX.

Probabilities of Kill

In order to perform the probability of kill (PK) calculations for the exoatmospheric interceptors, the TDU interceptors, and the Soviet RVs, various assumptions must be made. It is assumed that the exact MX and TDU locations within the MX complex are unknown and thus the attacking RVs are randomly targeted among the entire complex of shelters. This and the fact that the exoatmospheric defense system cannot preferentially defend the MX complex make exoatmospheric intercept strategy immaterial to the model results. Another assumption is that the exoatmospheric defense system has the time and capability to scan the entire RV attack cloud while all the interceptors are being launched. The exoatmospheric interceptors have a direct impact kill mechanism with an effective "cookie-cutter" weapon radius (WR) of three feet.

The products of the detonation of nuclear-armed endo-atmospheric interceptors in the atmosphere are overpressure, dynamic pressure, thermal radiation, gamma rays, and neutrons. A reentry vehicle (RV) is designed to withstand the high temperatures which occur when it reenters the earth's atmosphere, hence it is probably capable of withstanding the effects of thermal radiation. The effects of overpressure and dynamic pressure also have little impact on attacking RVs because of their aerodynamic "low drag" design (Ref 6). Gamma rays are not an effective kill mechanism because the prompt (source)

gammas are such a small percentage of the total energy of
the explosion that the intensity levels are not sufficiently
high to cause the heating required to crack or disrupt the
fissile material in the RV. Secondary gamma fluence is also
too low because secondary gammas are produced by neutron-air
reactions over the entire volume of air populated by the
neutrons and not from a point source (the weapon) (Ref 4).
The neutrons created by the detonation of an interceptor,
however, can destroy an RV by heating the fissile material
if the neutron fluence is sufficiently high (Ref 11:1137).
Therefore, the endoatmospheric interceptors' kill mechanism
is the neutrons created by their detonation. Appendix D
presents the method for calculating endoatmospheric proba-
bility of kill (PK) based upon neutron fluence.

The effects of a surface burst nuclear explosion are
dynamic pressure, overpressure, neutrons, gamma rays, thermal
radiation, ground motion, and cratering. The thick, steel
and reinforced concrete walls of a shelter provide shielding
against gamma rays, neutrons, and thermal radiation (Ref 18).
The MX shelters will also be built on a suspension system to
prevent damage from ground motion (Ref 33). If a shelter is
built flush with the ground, the destructive sideloadings of
dynamic pressure can also be avoided (Ref 22:17). Although
a shelter can be designed to limit damage by overpressure,
a sufficiently high level of overpressure will destroy the
shelter. A shelter can also be destroyed if the shelter is

within the crater radius caused by the nuclear explosion,
or rendered inoperable if the ejecta caused by the crater
is thick enough to cover the door of a horizontal shelter.
Hence, a horizontal shelter can be destroyed or rendered
inoperable by the effects of overpressure and cratering.
Appendices A and C show the model procedures for computing
the Soviet RV probability of kill due to cratering and over-
pressure, respectively.

The Simulation

The models in this thesis simulate the defense of an MX
complex being attacked by Soviet RVs using the simulation
language Q-GERT. The computer programs and Q-GERT networks
are shown in Appendix G. The BUS deployment and defense
service times were included to provide realistic timing
control of the model flow, but they do not have a significant
effect on the model results. It is assumed that the defense
systems cannot be saturated.

The simulation first generates the required number of
attacking RVs which are dispensed from the RV BUS normally
distributed in time with a mean of five seconds and a
standard deviation of two seconds. If the number of RVs is
exactly twice the number of shelters, two RVs are randomly
assigned to each shelter. The number of RVs will never be
greater than twice the number of shelters due to the fratri-
cide limit previously mentioned. If the number of RVs is
larger than the number of shelters but smaller than twice

the number of shelters, the simulation insures that a minimum
of one RV is targeted at each shelter if the Soviets possess
enough RVs to target each shelter at least once. Excess RVs
are also randomly targeted on the shelters. After assigning
all the attacking RVs to shelters, all the RVs are given an
eleven minute flight time delay until they reach the exoatmos-
pheric defense, if one exists.

Layered Defense (Exoatmospheric Layer). The first layer,
the exoatmospheric defense, attempts to defend the MX complex
by launching N interceptors on a one-to-one basis at the first
N RVs encountered. Although the RVs are intercepted first-come-
first-serve, the exoatmospheric defense does not know the
intended target since the RVs were randomly assigned to t_e
targets. All RVs which do not encounter an interceptor or are
missed by an interceptor penetrate the exoatmospheric defense
layer and proceed to the endoatmospheric defense layer which
consists of one terminal defense unit (TDU).

Endoatmospheric Layer (One TDU). This layer attempts to
defend the MX and TDU shelters based upon the designated
interceptor strategy. If the TDU is destroyed, the MX cannot
be defended by this layer of defense. The attacking RVs are
defended against one at a time with a service time of five
seconds. The simulation computes the number of RVs which
impact the MX shelter and calculates the probability of the
Soviet attack killing the MX using the following equation:

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$$PK = 1 - (1 - PK_1)^n$$

where PK = probability of kill for the entire attack

                PK_1 = probability of kill for one RV

                n = number of RVs impacting the MX shelter.

The simulation then compares this PK to a random number to determine if the MX is destroyed.

Endoatmospheric Defense (Two TDUs). This simulation attempts to defend the MX and TDU shelters according to the following strategy. The strategy designates one of the TDUs as the primary and the remaining TDU as the backup. The primary TDU defends the MX complex until it is destroyed or has launched two of its three interceptors. The primary TDU will launch its first two interceptors at RVs aimed at either the MX or a TDU shelter, but will save its remaining interceptor for defending the MX. If the primary TDU is destroyed or has launched two of its three interceptors, the backup TDU assumes the defense role if it has not been destroyed. If both TDUs are destroyed, the MX cannot be defended by this layer of defense. If the backup TDU is destroyed or has launched all three of its interceptors, the primary TDU reassumes the defense role if it has not been destroyed. Both TDUs will not defend a TDU shelter which has been destroyed. Either TDU defends against attacking RVs one at a time with a service time of five seconds. The simulation

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computes the number of RVs which impact the MX shelter and calculates the probability of the Soviet attack killing the MX using the same equation as the one TDU model. The simulation then compares the probability of kill of the equation to a random number to determine whether or not the MX is destroyed.

Verification

Although the exoatmospheric and endoatmospheric defense models are combined to form a layered defense model, they are verified separately since they are considered independent of one another. Therefore, if both the exoatmospheric and endoatmospheric models are verified, the layered defense model will also be verified.

All defensive models were verified by simulating a 24 RV Soviet attack on 15 MX shelters and comparing the model results with results derived analytically. The output of one model run is MX destroyed or not destroyed. This type of output is a Bernoulli trial and the results of multiple Bernoulli trials can be characterized by the binomial distribution (Ref 32:191). A binomial distribution can be approximated by a normal distribution if the number of runs or trials (n) is sufficiently large, and the probability of the MX being destroyed, p, is close to one-half. In general, this approximation is good if $np > 3$ when $p \leq 0.5$ or $n(1-p) > 3$ when $p > 0.5$ (Ref 39). This is true of the models' outputs when $n = 1200$. The output of 1200 simulation

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runs is $\hat{p}$, the probability that the MX is destroyed. Hence, since we are using $\hat{p}$ as an estimate of the true probability of kill, p , we can be (1-α)100% confident that the error of the estimate will be less than a specified amount e (in decimal) when the sample size is:

$$N = \frac{Z_{\alpha/2}^2 \; \hat{p} \; (1-\hat{p})}{e^2}$$

where $Z_{\alpha/2}$ is the two-tailed standardized normal statistic (Ref 38:212).

In order to test the null hypothesis that the PK of the MX calculated by the model equals the analytical PK, a hypothesis test of the proportions using a normal distribution is used since the sample size (n = 1200) is sufficiently large. The hypotheses are:

$$H_0: \quad p_o = \hat{p}$$

$$H_1: \quad p_o \neq \hat{p}$$

The test is:

$$\hat{p} - Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{N}} \; < \; p \; < \; \hat{p} + Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{N}}$$

and if p_o falls within this acceptance region, then it is assumed that $p_o = \hat{p}$; otherwise, H_0 is rejected and it is concluded that $p_o \neq \hat{p}$ where

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p_0 = PK determined analytically

$\hat{p}$ = PK determined by the model

$Z_{\alpha/2}$ = two-tailed standardized normal statistic

n = number of simulation runs (Ref 38:209-213).

Exoatmospheric Defense Model. This model was verified by simulating 12 exoatmospheric interceptors against a 24 RV attack. The yield and CEP of the Soviet RVs were assumed to be sufficient to provide each RV with a 100% PK if not destroyed by an interceptor. The analytical calculation of an RV's probability of killing the MX, p_0, was performed as outlined in Appendix E, and results in a PK of 74.8%. The 1200 model simulations resulted in a PK of 74.9%, which is within 2.5% of the true mean. The 95% confidence interval for these data is 72.4% < p < 77.4%. Since the analytical PK falls within this confidence interval, the null hypothesis cannot be rejected and the exoatmospheric defense model functions properly.

Endoatmospheric Defense Model (One TDU). The verification of this model was accomplished using 1200 simulations of the Soviet attack on the MX complex. The yield and CEP of the Soviet RVs are set at one-megaton and 0.2 nautical miles, respectively. The analytical probability of an RV killing the MX, p_0, was calculated as shown in Appendix F, and results in a PK of 44.4%. The 1200 model simulations resulted

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in a PK of 42.2%, which is within 2.8% of the true mean.
The 95% confidence interval for these data is 39.8% < p < 45.4%.
Hence, since the analytical PK falls within this confidence
interval, the null hypothesis cannot be rejected and the endo-
atmospheric defense model with one TDU functions properly.

Layered Defense Model. As stated previously, the exo-
atmospheric and endoatmospheric (one TDU) models are considered
independent of one another. Therefore, since both of these
models have been verified, the layered defense model has been
verified and functions properly.

Endoatmospheric Defense Model (Two TDUs). The only
difference between this model and the one TDU model are the
situations in which the primary TDU transfers the responsi-
bility of defense to the backup TDU, and vice versa. Conse-
quently, only the situations which cause this transfer of
responsibility need to be verified. The verification of
these transfers was accomplished by tracing 25 simulations
of the Q-GERT model. Since all of these simulations per-
formed the transfer of defense responsibility successfully
and represent all possible situations that might occur, the
endoatmospheric model with two TDUs functions properly.

Validation

The exoatmospheric and endoatmospheric defense systems
that these models portray will not be feasible until the
mid and late 1980s, respectively. It is impossible to compare

the behavior of each model with the behavior of an actual

system which has not been built. An attempt has been made

to include those variables of the actual systems which are

anticipated to have a significant effect on MX survivability.

Reasonable assumptions were used to model those areas where

unclassified data were not available. These assumptions

are based upon an extensive search of unclassified litera-

ture and discussions with qualified individuals of the Aero-

nautical Systems and Armament Divisions of Air Force Systems

Command, and the Air Force Institute of Technology. All of

the models in this study follow directly from the systems

and data of current unclassified sources, and within the

stated limitations, the models are valid.

III. The Analysis

Experimental Design

The models developed in this study are designed to
furnish estimates of MX survivability when an MX complex is
defended by a layered defense, one terminal defense unit
(TDU), or two TDUs. With these models, the effects of exo-
atmospheric interceptors, protective shelters, and TDUs on
MX survivability can be examined.

Layered Defense Model. The experimental design of this
model consists of two factors (number of shelters and number
of interceptors) and one response (MX survivability) as shown
in Figure 5.
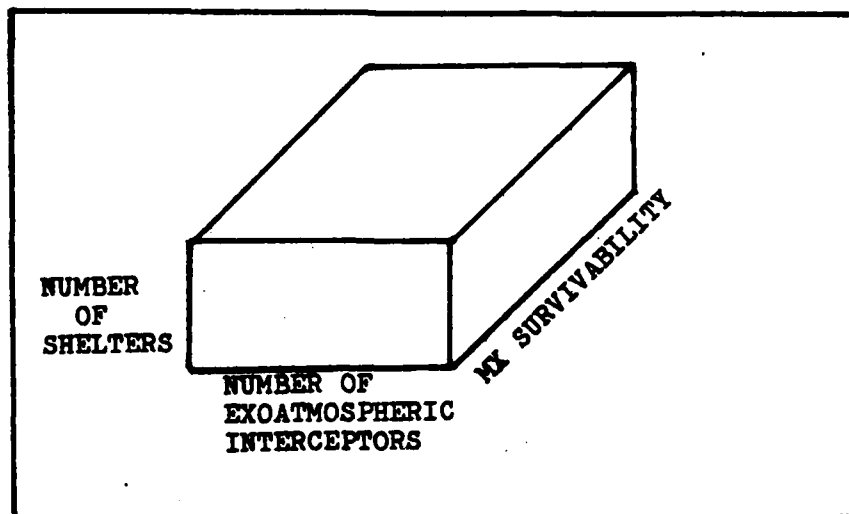
Figure 5. Layered Experimental Design

The model inputs are the number of attacking RVs, the number of shelters, and the number of exoatmospheric interceptors. After the required number of simulations, the model outputs MX survivability. The number of attacking RVs was set at either 16 (low density attack) or 24 (high density attack). The number of RVs was not considered a design factor in any of the experimental designs because each attack level was analyzed separately. The number of shelters was set at a discrete level from 8 to 23, and the number of interceptors was set at any integer value from one to the number of attacking RVs. The experimental model was run with 20 different combinations of shelters and interceptors, and the MX survivability of these combinations was recorded. Regression analysis was then used to develop an equation which fits these 20 data points, using the number of shelters and interceptors as the independent variables, and MX survivability as the dependent variable. The general form of the regression equation is:

$$Y = b_o + b_1X_1 + b_2X_2 + \ldots + b_nX_n \quad,$$

where

b_o through b_n = regression coefficients

X_1 through X_n = a form of the independent variables.

The equation which characterizes these data was used to
determine the number of exoatmospheric interceptors needed
to obtain a particular MX survivability, given a certain
number of shelters and attacking RVs. A form of this equa-
tion was selected based upon its large adjusted coefficient
of determination $(\bar{R}^2)$, since the main concern is total
predictive power and not the marginal predictive power of
each independent variable. A regression equation with an
adjusted R^2 greater than approximately 0.64 is considered
to have significant predictive power (Ref 39).

Endoatmospheric Defense Model (One TDU). This model's
experimental design consists of only one factor (number of
shelters) and one response (see Figure 6), since the number
of TDUs is fixed at one. The inputs to the model are the
number of attacking RVs and the number of shelters per com-
plex, and the model output is MX survivability. The number
of RVs is set at either 16 (low density attack) or 24 (high
density attack), and the number of shelters will be at a
discrete level from 8 to 23. The model was run for these
different shelter values, and the MX survivability for these
values was documented. Regression analysis was again used
to develop an equation which fits these data points, with
the number of shelters being the independent variable and MX
survivability being the dependent variable. In both the
one and two TDU cases, where all possible model outcomes are
determined by simulation, regression analysis provides a more
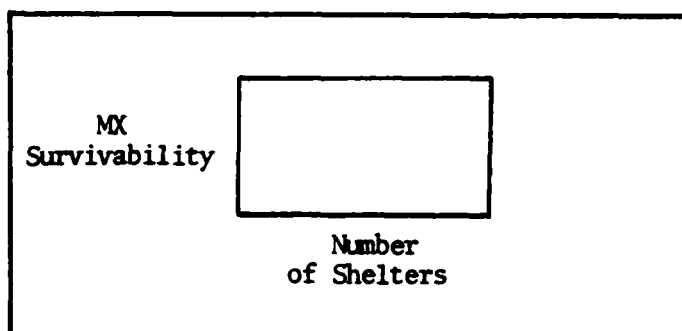
Figure 6.   One TDU Experimental Model

representative output by smoothing the stochastic processes
of the model.   The general form of the regression equation
is:

$$Y = b_0 + b_1X_1 + b_2X_2 + \ldots + b_nX_n \ .$$

A form of this equation was then used to determine the
number of shelters, which, when combined with the one TDU,
provides a predetermined level of MX survivability.

Endoatmospheric Defense Model (Two TDUs).   The experi-
mental design of the two TDU model consists of one factor
(number of shelters) and one response (MX survivability) as
depicted in Figure 7.   The model inputs are the number of
attacking RVs, which is set at 16 (low density attack), or
24 (high density attack), and the number of shelters per
complex, which is set at a discrete value from 8 to 23.
The model output is MX survivability.   As in the case of
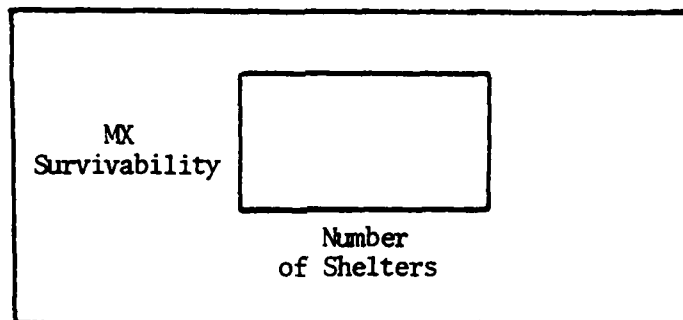
Figure 7. Two TDU Experimental Design

one TDU, the model was executed for these shelter values,
and the respective MX survivability was recorded. Regression
analysis was applied to these data points to develop an
equation with a large $\bar{R}^2$; with the number of shelters
being the independent variable and MX survivability being
the dependent variable. Once again, the general form of
the regression equation is:

$$Y = b_o + b_1 X_1 + b_2 X_2 + \ldots + b_n X_n .$$

A form of this equation was employed to determine the number
of shelters, which, when deployed with two TDUs, yields a
predetermined level of MX survivability against an enemy
attack.

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Number of Sample Runs

The effectiveness of all the models is dependent upon the number of runs required to achieve a particular level of confidence in the results. The results of the models can be characterized by the binomial distribution because the output of one model run is a Bernoulli trial. For reasonably large sample sizes, the binomial distribution can be approximated by the normal distribution. It can be shown that, in a worse case situation, the equation in Chapter II (verification) reduces to:

$$N \leq Z^2_{\alpha/2}/(4e^2)$$

where N is the maximum number of model runs and is an upper bound for any degree of confidence, e is the desired interval (in decimal) about the true mean, and $Z_{\alpha/2}$ is the standardized normal statistic for the probability sought (Ref 32:191-192). Since it is desired that the model results differ from the true mean by less than approximately 3% with a 95% confidence interval $(Z_{\alpha/2} = 1.96)$, the maximum number of model runs required is 1067 or approximately 1200.

Cost Effectiveness

The following cost data (Ref 1:20) will be applied to the parameters of the layered and endoatmospheric defense systems which, according to the regression equations selected,

provide a predetermined level of MX survivability.

Applying this cost data results in a total cost figure for each defense configuration. A comparison of these costs will provide the most cost-effective solution for defending the MX, since all of the alternatives considered provide an equivalent level of survivability.

	TABLE VII	
	Costs of Offensive and Defensive Systems	
Parameter	Fixed Cost ($M)	Variable Cost ($M)
MX Missile	8000	$60(M)^{.78}$
Shelter	5000	$3S$
Exoatmospheric Interceptor	7000	$60(X)^{.78}$
Endoatmospheric Interceptor	5000	$16(N)^{.78}$

where

    M = number of MX missiles

    S = number of shelters

    X = number of exoatmospheric interceptors

    N = number of endoatmospheric interceptors (Ref 1:20).

Model Runs

The layered defense model was run 1200 times for twenty different combinations of shelters and exoatmospheric interceptors being attacked by 16 and 24 Soviet RVs. For both the one and two TDU endoatmospheric models, the model was executed 1200 times for the twelve shelter configurations (12 through 23) under attack by 24 Soviet RVs, and 1200 times for the 16 shelter configurations (8 through 23) under attack by 16 Soviet RVs. The lower shelter value is restricted due to the fratracide limit of a maximum of two RVs being targeted against each shelter.

Results

The output of a model simulation is MX survivability (MXS), and the results of the model runs mentioned above are presented in Tables VIII through XIII.

TABLE VIII

MX Survivability Against a 24 RV Attack

(Layered Defense)

Number of Shelters	Number of Exoatmospheric Interceptors	MX Survivability (%)
12	1	44.3
12	4	48.9
12	5	51.8
12	6	56.1
12	8	61.5
13	1	57.9
13	2	62.7
13	15	82.8
14	1	61.4
14	10	73.7
15	11	75.3
16	20	83.6
17	4	68.0
17	16	80.8
17	23	87.7
18	20	84.5
20	20	84.5
23	20	85.4
23	23	94.0
23	24	99.1

TABLE IX
MX Survivability Against a 24 RV Attack
(One TDU Defense)

Number of Shelters	MX Survivability (%)
12	40.9
13	57.4
14	58.8
15	60.1
16	60.3
17	60.0
18	63.0
19	62.6
20	62.1
21	63.0
22	64.2
23	71.9

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TABLE X	
MX Survivability Against a 24 RV Attack	
(Two TDU Defense)	
Number of Shelters	MX Survivability (%)
12	50.3
13	60.0
14	65.3
15	65.0
16	68.0
17	65.0
18	66.7
19	71.3
20	69.2
21	68.8
22	69.7
23	73.3

TABLE XI
MX Survivability Against a 16 RV Attack
(Layered Defense)

Number of Shelters	Number of Exoatmospheric Interceptors	MX Survivability (%)
8	1	45.4
8	2	49.8
8	5	59.7
9	1	62.5
10	2	63.5
10	10	80.5
12	5	70.4
12	16	92.5
14	1	63.4
15	12	85.8
16	8	83.5
17	4	87.3
17	12	93.7
18	10	93.2
20	5	92.0
20	14	96.8
20	16	98.0
23	2	89.4
23	8	94.2
23	16	97.4

TABLE XII
MX Survivability Against a 16 RV Attack
(One TDU Defense)

Number of Shelters	MX Survivability (%)
8	40.4
9	60.7
10	58.1
11	60.9
12	62.2
13	62.0
14	64.5
15	69.3
16	68.9
17	69.9
18	85.1
19	84.0
20	84.7
21	85.6
22	86.6
23	86.7

TABLE XIII	
MX Survivability Against a 16 RV Attack	
(Two TDU Defense)	
Number of Shelters	MX Survivability (%)
8	40.4
9	62.6
10	63.7
11	67.0
12	67.4
13	70.4
14	68.4
15	74.0
16	76.5
17	80.9
18	84.9
19	86.3
20	85.6
21	87.1
22	87.6
23	87.7

Analysis

The experimental models discussed previously were used
to develop the equations which characterize the results for
each of the defense systems. Various equations (semilog,
reciprocal, polynomial, and linear) which might characterize
the true relationship of the data were evaluated. Regression
analysis was used because the equations obtained from this
process provide the following advantages:

1. Conserves computer resources (i.e., limits the
    number of model runs required).

2. Smooths the stochastic processes of the models.

3. Easily applicable to different survivability
    levels without further computer simulations.

The layered defense equation can be used to develop the
combinations of the number of exoatmospheric interceptors and
shelters which provide a prescribed level of survivability,
while both the one and two TDU equations can be used directly
to obtain the number of shelters which provide the prescribed
survivability. For assured destruction, the U.S. retaliatory
strike force has been arbitrarily chosen to consist of approx-
imately 1200 warheads that can be delivered against Soviet
targets of value, or 120 MX-equivalent payloads (Ref 1:10).
This equates to a prescribed MX survivability of 60%. When
using the equations to determine the number of interceptors
and/or shelters which provide 60% MX survivability, the
result may consist of a fraction of an interceptor or shelter.
Since 60% is the chosen requirement for U.S. MX survivability,

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the general rule for rounding off in this study is to round
up to the next highest number of interceptors or shelters.
Fractions of interceptors or shelters less than one-tenth
are assumed to be negligible and therefore will be rounded
down to the nearest integer value.

High Density Attack (24 RVs).

Layered Defense Model. A regression analysis of
the 20 data points collected for this model was performed
using the Statistical Package for the Social Sciences (SPSS).
Of the equations evaluated, the following model best charac-
terizes the true relationship of the data:

$$MX\ Survivability = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3, \quad \overline{R}^2 = 0.905$$

where F-Ratio

			F-Ratio
b_0	=	1.034	44.57
b_1	=	.02173	13.46
b_2	=	-6.518	10.90
b_3	=	-.000632	3.14

X_1 = number of exoatmospheric interceptors

X_2 = 1/number of shelters

X_3 = X_1* (number of shelters)

This equation was selected because it has the best predictive
power (i.e., largest significant adjusted R^2) and all of the

parameters estimated by this model (b_0, b_1, b_2, and b_3) were significantly different than zero (at 90% confidence) as indicated by their partial F-ratios.

Using the equation selected, Table XIV shows the combinations of exoatmospheric interceptors and shelters which provide at least 60% MX survivability. All of the combinations which do not require any exoatmospheric interceptors to achieve 60% MX survivability are actually an endoatmospheric defense with one TDU and not a layered defense.

TABEL XIV	
60% MX Survivability	
(Layered Defense - 24 RV Attack)	
Number of Shelters	Number of Exoatmospheric Interceptors
12	8
13	5
14	3
15	0

Therefore, for the layered defense model, the cost data will only be applied to the three combinations which require exoatmospheric interceptors. Table XV presents the total cost to provide all 200 MX complexes with any of the three layered defenses.

TABLE XV
Total Cost of 200 MX Complexes (Layered Defense - 24 RV Attack)

Number of Shelters	Number of Exoatmospheric Interceptors	Total Cost ($ Millions)
12	8	57,230
13	5	52,017
14	3	43,303

This table shows that a defense consisting of 14 shelters, three exoatmospheric interceptors, one TDU, and one MX in each of the 200 MX complexes is the most cost-effective layered defense option against a 24 RV attack.

Endoatmospheric Defense Model (One TDU). An equation which fits the twelve data points was obtained using SPSS, and the following equation was chosen as the best fit for the data:

$$\text{MX Survivability} = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 \text{ , } \overline{R}^2 = 0.823$$

where   F-Ratio

b_0 = 11.0566   26.6

b_1 = -67.9109   26.4

b_2 = -.44839   22.1

b_3 = .00024   20.7

X_1 = 1/number of shelters

X_2 = number of shelters

X_3 = number of shelters cubed

Of all the models examined, the equation selected has the best predictive power (i.e., largest significant $\bar{R}^2$) and the partial F-ratios of this model indicate that all of the model parameters estimated (i.e., b_0, b_1, b_2, and b_3) were significantly different from zero.

Substituting 60% MX survivability into this equation yields 14.39 shelters. Therefore, 15 shelters are needed to obtain 60% MX survivability with one terminal defense unit. The total cost for developing all 200 MX complexes with one TDU per complex and 15 shelters per complex is presented in Table XVI.

TABLE XVI		
Total Cost of 200 MX Complexes (One TDU Defense - 24 RV Attack)		
Number of Shelters	Number of Endoatmospheric Interceptors	Total Cost ($ Millions)
15	3	33,090.77

<u>Endoatmospheric Defense Model (Two TDUs)</u>. Applying
regression analysis to the twelve data points using SPSS
resulted in the following equation having the best adjusted
coefficient of determination ($\overline{R}^2$) and model parameters (i.e.,
b_0, b_1, b_2, and b_3) significantly different from zero:

$$\text{MX Survivability} = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 \ , \ \overline{R}^2 = 0.897$$

where			F-Ratio
b_0	=	6.5358	12.9
b_1	=	-40.0277	12.68
b_2	=	-.24153	8.86
b_3	=	.0001225	7.45
X_1	=	1/number of shelters	
X_2	=	number of shelters	
X_3	=	number of shelters cubed.	

Other models examined were semilog, linear, and log models.

For 60% MX survivability, the equation selected yields
13.26 shelters. Hence, 14 shelters are required to obtain
60% MX survivability with two terminal defense units.
Table XVII shows the total cost for defending all 2C0 MX
complexes with two TDUs per complex and 14 shelters per
complex.

TABLE XVII		
Total Cost of 200 MX Complexes		
(Two TDU Defense - 24 RV Attack)		
Number of Shelters	Number of Endoatmospheric Interceptors	Total Cost ($ Millions)
14	6	34,176.07

Low Density Attack (16 RVs).

Layered Defense Model. A regression analysis of the 20 data points collected for this model was performed using SPSS. Of the equations evaluated, the following model best characterizes the true relationship of the data:

$$\text{MX Survivability} = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 \quad , \quad R^2 = 0.937$$

where   F-Ratio

     b_0 = .24729   31.5

     b_1 = .02974   92.78

     b_2 = .03971   40.03

     b_3 = -.0015589   17.57

     X_1 = number of shelters

     X_2 = number of exoatmospheric interceptors

     X_3 = $X_1 * X_2$

This equation was selected because it had the best predictive power and all of the parameters estimated by this model (i.e., b_0, b_1, b_2, and b_3) were significantly different than zero (at 90%) as indicated by their partial F-ratios.

Using the equation selected, Table XVIII shows the combinations of exoatmospheric interceptors and shelters which provide 60% MX survivability. All of the combinations which do not require any exoatmospheric interceptors to achieve 60% MX survivability are again actually an endo-atmospheric defense with one TDU and not a layered defense.

TABLE XVIII	
60% MX Survivability	
(Layered Defense - 16 RV Attack)	
Number of Shelters	Number of Exoatmospheric Interceptors
8	5
9	4
10	3
11	2
12	0

Therefore, for the layered defense model, the cost data will only be applied to the four combinations which require exo-atmospheric interceptors. Table XIX presents the total cost to provide all 200 MX complexes with any of the layered defenses.

TABLE XIX

Total Cost of 200 MX Complexes
(Layered Defense - 16 RV Attack)

Number of Shelters	Number of Exoatmospheric Interceptors	Total Cost ($ Millions)
8	5	49,017
9	4	47,520
10	3	45,903
11	2	44,114

This table shows that a defense consisting of 11 shelters, two exoatmospheric interceptors, one TDU, and one MX in each of the 200 MX complexes is the most cost-effective layered defense option against a 16 RV attack.

Endoatmospheric Defense Model (One TDU). An equation which fits the 16 data points was obtained using SPSS, and the following equation was chosen as the best fit for the data:

$$\text{MX Survivability} = b_o + b_1 X_1 \,, \quad R^2 = 0.881$$

where   F-Ratio

b_o = .2884   49.26

b_1 = .02695   112.46

X_1 = number of shelters

Of all the models examined, the equation selected has the best predictive power and the partial F-ratios of this model indicate that all of the model parameters estimated (i.e., b_0 and b_1) are significantly different from zero.

Substituting 60% MX survivability into this equation yields 11.56 shelters. Therefore, 12 shelters are needed to obtain 60% MX survivability with one terminal defense unit. The total cost for defending all 200 MX complexes with one TDU per complex and 12 shelters per complex is presented in Table XX.

TABLE XX
Total Cost of 200 MX Complexes
(One TDU Defense - 16 RV Attack)

Number of Shelters	Number of Endoatmospheric Interceptors	Total Cost ($ Millions)
12	3	31,290.77

Endoatmospheric Defense Model (Two TDUs). Applying regression analysis to the 16 data points using SPSS resulted in the following equation having the best adjusted coefficient of determination ($\bar{R}^2$) and model parameters (i.e., b_0 and b_1) significantly different from zero at the 90% level:

$$\text{MX Survivability} = b_0 + b_1 X_1 \,, \quad \overline{R}^2 = 0.948$$

where   F-Ratio

b_0 = -.151528   7.613

b_1 = .334915   273.42

X_1 = ℓn (shelters).

For 60% MX survivability, the equation selected yields
9.43 shelters. Hence, 10 shelters are required to obtain
60% MX survivability with two terminal defense units.
Table XXI shows the total cost for defending all 200 MX
complexes with two TDUs per complex and 10 shelters per
complex.

TABLE XXI		
Total Cost of 200 MX Complexes (Two TDU Defense - 16 RV Attack)		
Number of Shelters	Number of Endoatmospheric Interceptors	Total Cost ($ Millions)
10	6	31,776.07

Summary

High Density Attack (24 RVs). Of the three defense systems investigated, the one TDU defense system is the most cost-effective. The terminal BMD system with two TDUs is almost as cost-effective as one TDU, but both terminal BMD strategies are much more cost effective than the layered defense strategy.

Low Density Attack (16 RVs). Of the three defense systems investigated, the one TDU defense system is again the most cost-effective and, as in the high density attack, both terminal BMD strategies are much more cost-effective than the layered defense strategy.

IV. Conclusions

Based on the results obtained from this study, the development cost of the exoatmospheric component causes the layered defense system to be non-competitive with the terminal defense systems when cost is the only criteria. It is possible that a substantial portion of the development cost could be overcome if the exoatmospheric defense component could preferentially defend the MX complex (i.e., intercept only those RVs aimed at itself or the actual MX). If preferential defense by the exoatmospheric defense layer can be achieved, the endoatmospheric portion of the layered defense can be eliminated because of the relatively high PK of an exoatmospheric interceptor with a three foot effective weapons radius and one and one-half foot CEP. Hence, the layered defense system would become solely an exoatmospheric defense system.

The two TDU effectiveness (i.e., MX survivability) was only marginally better than the one TDU effectiveness, primarily because there are three shelters in each complex to defend rather than two as in the one TDU case. Additional factors to consider here are the possibility of detonating twice as many nuclear weapons (six vice three) at a low altitude over friendly territory with two TDUs, and the increased amount of high-neutron yielding fissile material that must be withdrawn from offensive warhead production.

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Due to the nuclear kill mechanism of the LoADs system, it is only effective in defending hard targets and cannot be used to defend soft area targets. The exoatmospheric interceptors help in this regard and can reduce the number of nuclear atmospheric detonations required. A superior system would preferentially intercept all RVs at high altitudes without nuclear kill, thus being able to defend soft area targets as well as hard point targets. The results do clearly indicate that significant flexibility exists for tradeoff between the number of protective shelters and various defense configurations for a preset level of MX survivability.

While the actual number of shelters or interceptors generated by the regression models in this study provide an indication of the magnitudes involved, these regression model outputs vary from the simulation model outputs and, as with all model outputs, should be employed with caution. Therefore, absolute recommendations should not be made, but the relative comparisons of the layered, one TDU, and two TDU defense systems in this study can be drawn.

Several very sensitive variables were discovered in the course of this study and the values used for these variables will determine the outcome of the models. These variables were sure-safe and sure-kill neutron intensity levels of the RVs, CEPs for the RVs and all of the interceptors, and the effective weapon radius of the exoatmospheric interceptors. Shelter hardness was not a critical variable as long as the

sure-safe and sure-kill intensities remained 500 psi apart. Values closer together would make this variable more sensitive.

While the layered defense system is not justified on cost considerations alone, the research required in building the system could lead to tremendous improvements such as preferential defense by the exoatmospheric defense layer, direct impact/conventional warhead kill within the atmosphere, and an absolute degree of confidence in the system's ability because it could be tested in actual RV intercept situations where the nuclear intercept cannot.

V. Recommendations for Further Study

Soviet Union parity with the U.S in the nuclear arms race has stimulated considerable interest in BMD as an addition to the current MPS basing mode; and hence efforts in BMD, beyond this study, could prove very helpful. A scenario using classified values for the parameters and variables of these models is a necessity for an accurate representation of reality.

Various areas of the analysis and models in this study which could be expanded or enhanced are stated below:

1. The RV-shelter interaction of the model could be enhanced to include ground shock as another possible RV-kill mechanism.

2. The survivability and attack levels chosen in the analysis phase could be expanded to investigate several attack and defense scenarios.

3. The cost data used in the analysis phase could be enhanced by obtaining more detailed cost estimates.

4. The computer model could be enhanced to more efficiently assign RVs to targets for all attack and defense configurations. This would eliminate the model modifications in Appendix G.

5. The computer model could be expanded to include other possible interceptor strategies.

Finally, as both the exoatmospheric and endoatmospheric defense subsystems are researched, developed, and refined, the requirements for future efforts in the area of BMD will expand.

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APPENDICES

Appendix A

Probability of Kill Due to Cratering

MX Shelter

A surface burst nuclear explosion creates a crater.
The crater ejecta consists of soil or rock debris that are
thrown beyond the boundaries of the apparent crater (Ref 18:
255). It is assumed that the shelter door shown in Figure 8
will be totally covered at 2/3 the radius of the ejecta (R_e)
since ejecta thickness is approximately 16.86 feet at 0.84 R_e.
In addition, any object within the apparent crater radius
(R_a) will be destroyed.

To calculate both R_e and R_a, the following equations
are used:

$$R_a = R_a^R (Y^{.3})$$

$$R_e = 2.15 R_a$$

where Y is the yield of the weapon in kilotons and R_a^R
is the apparent crater radius created by a one kiloton weapon.
For a surface burst in dry soil or dry hard rock, R_a^R is
61 feet (Ref 18:254-256). The probability of kill (PK) for
cratering is defined by the circular normal function and can
be found using the following equation.

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$$PK = \int_{-(2/3R_e+85.5)}^{+(R_a+85.5)} \frac{1}{\sqrt{2\pi}\,\sigma_y} e^{-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2} dy \int_{-(R_a+66.5)}^{+(R_a+66.5)} \frac{1}{\sqrt{2\pi}\,\sigma_x} e^{-\frac{1}{2}\left(\frac{x}{\sigma_x}\right)^2} dx$$

where

$$\sigma_x = \sigma_y = CEP \sqrt{2\ln 2} = \sigma$$

Letting $z_1 = \frac{x}{\sigma} \rightarrow dx = \sigma dz$, and $z_2 = \frac{y}{\sigma} \rightarrow dy = \sigma dz$ and substituting into the above equation yields

$$PK = \int_{\frac{-(2/3R_e+85.5)}{\sigma}}^{\frac{+(R_a+85.5)}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z_1^2} dz_1 \int_{\frac{-(R_a+66.5)}{\sigma}}^{\frac{+(R_a+66.5)}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z_2^2} dz_2$$

The MX integration limits in the y direction were chosen assuming the apparent crater radius (R_a) had to at least touch the back of the shelter for any chance of destruction from cratering (+y direction) and the ejecta at $2/3\ R_e$ has to cover the door (-y direction). For the x-direction, it was assumed that the R_a must at least touch the sides of the shelter.

Using a circular error probable (CEP) of 0.2 NM (1215.2 feet), a yield of 1000 kilotons, and the dimensions of an MX shelter (Figure 8), the following results are obtained:
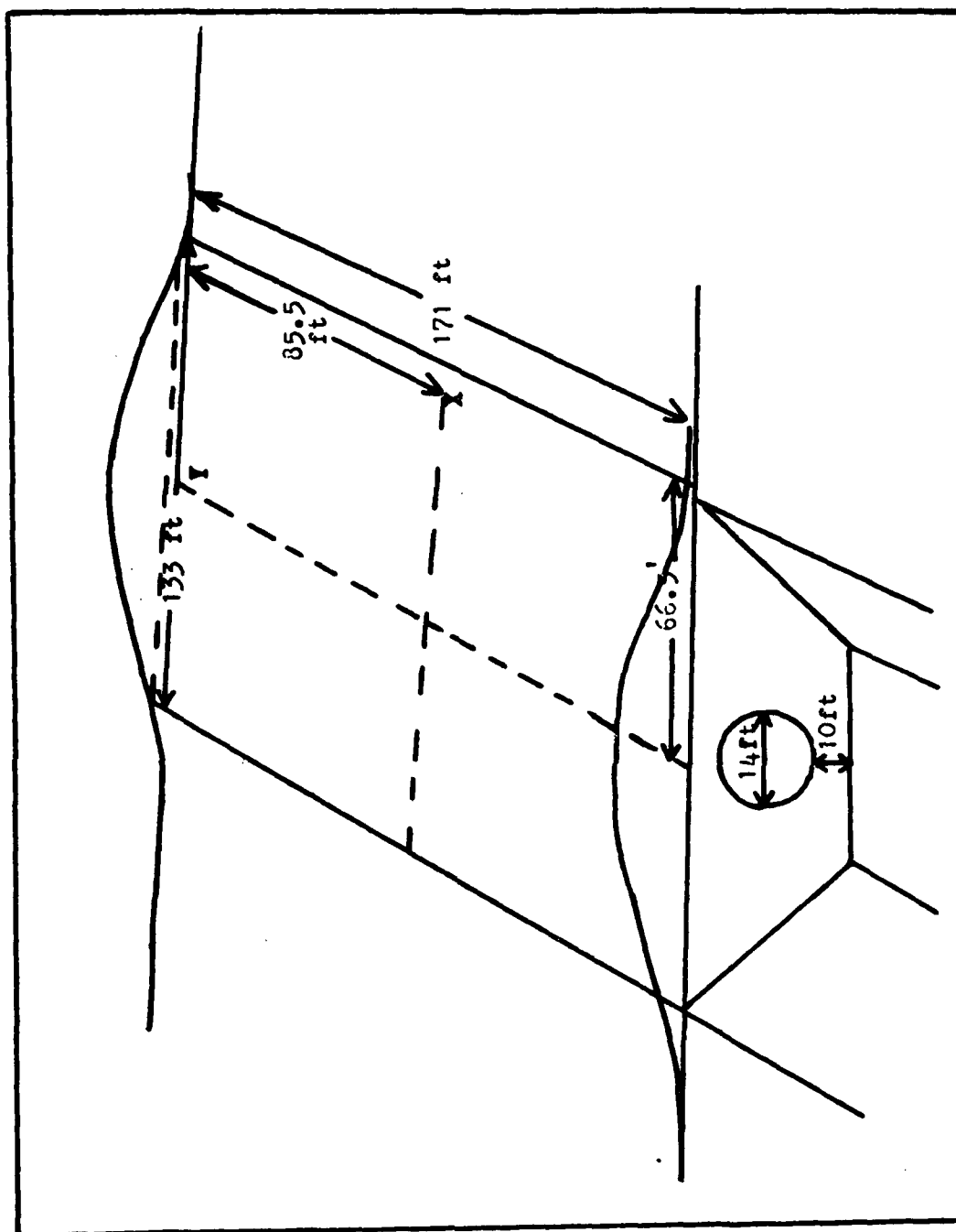
Figure 8. MX Shelter (Ref 24)

$$R_a = (61 \text{ feet})(1000^{.3}) = 484.54 \text{ feet}$$

$$R_e = (494.54 \text{ feet})(2.15) = 1041.76 \text{ feet}$$

$$2/3R_e = 694.51 \text{ feet}$$

$$\sigma_x = \sigma_y = \frac{1215.2 \text{ feet}}{\sqrt{2} \ln 2} = 1032.1 \text{ feet.}$$

Therefore,

$$PK = \int_{\frac{-(694.51+85.5)}{1032.1}}^{\frac{+(484.54+85.5)}{1032.1}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z_1^2} dz_1 \int_{\frac{-(484.54+66.5)}{1032.1}}^{\frac{+(484.54+66.5)}{1032.1}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z_2^2} dz_2$$

$$PK = \int_{-.756}^{+.5523} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z_1^2} dz_1 \int_{-.534}^{+.534} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z_2^2} dz_2$$

Using a standard normal table (Ref 37),

$$PK = (.7096 - .2248) \times (.7033 - .2967)$$

$$PK = (.4848) \times (.4066) = .1971.$$

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Terminal Defense Unit (TDU) Shelter

A TDU shelter is susceptible to the same cratering effects as the MX shelter, except that the limits of integration for cratering a TDU shelter will be different because the TDU is designed to punch out of the top of an MX shelter (Figure 9) (Ref 37). Covering any portion of
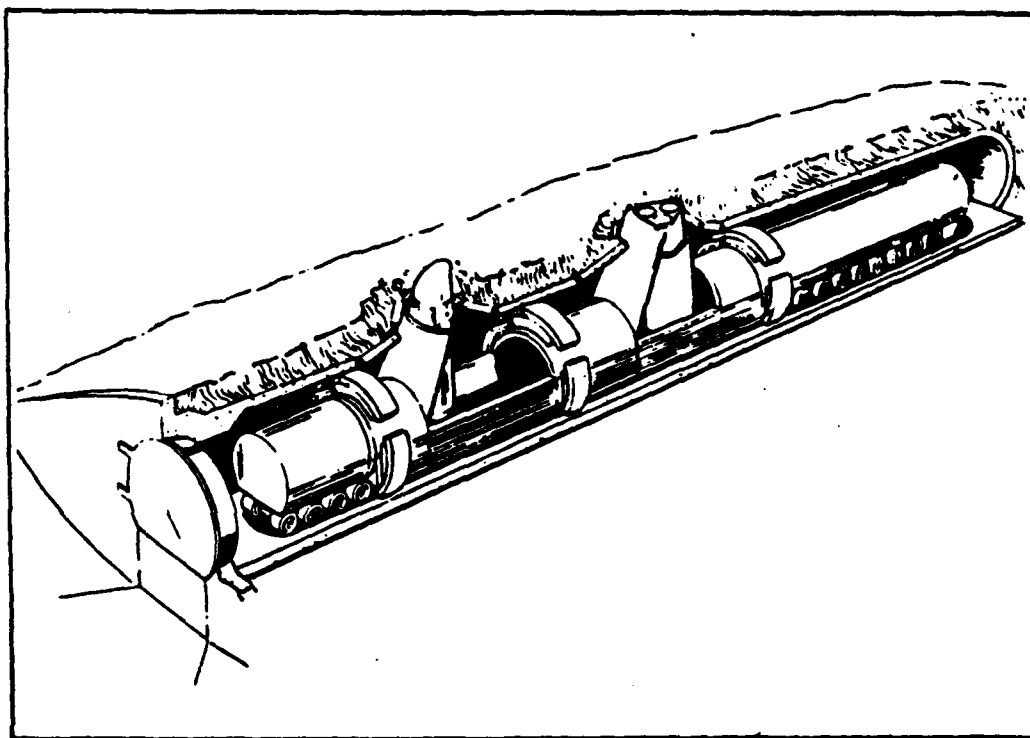
Figure 9. TDU Shelter (Ref 37)

the shelter with *ejecta is, therefore, assumed to be inef-fective in prohibiting TDU operation. The probability of

kill (PK) of cratering is defined by the circular normal
function and can be found using the following equation:

$$PK = \int_{\frac{-(R_a+85.5)}{\sigma}}^{\frac{+(R_a+85.5)}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z_1^2} dz_1 \int_{\frac{-(R_a+66.5)}{\sigma}}^{\frac{+(R_a+66.5)}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z_2^2} dz_2$$

The TDU integration limits in both directions were chosen
assuming R_a must touch the perimeter of the shelter for
any chance of destruction from cratering.

Using the same CEP, yield, R_a, and R_e as the MX
calculations, the probability of killing (PK) a TDU can be
calculated as follows:

$$PK = \int_{\frac{-(484.54+85.5)}{1032.1}}^{\frac{+(484.54+85.5)}{1032.1}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z_1^2} dz_1 \int_{\frac{-(484.54+66.5)}{1032.1}}^{\frac{+(484.54+66.5)}{1032.1}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z_2^2} dz_2$$

$$PK = \int_{-.5523}^{+.5523} \frac{1}{\sqrt{2\pi}} \ e^{-\frac{1}{2}z_1^2} \ dz_1 \ \int_{-.534}^{+.534} \frac{1}{\sqrt{2\pi}} \ e^{-\frac{1}{2}z_2^2} \ dz_2$$

Using a standard normal table (Ref 37),

$$PK = (.7096 - .2904) \ X \ (.7033 - .2967)$$

$$PK = (.4192) \ X \ (.4066) = .1704$$

Appendix B

Ten Cell Model

A methodology used to calculate the probability of
kill (PK) for a particular weapon against a specific target
is based on the ten cell model. This model places ten cells
of equal probability of hit centered around the designated
ground zero (DGZ), which is the aim point of the weapon.
Thus, the weapon has a 10% chance of impacting each cell.
The inputs required for this model are the sure-safe and
sure-kill weapon effect intensities of the target, the CEP
of the weapon, and the distance of the target of interest
from the DGZ. Figure 10 depicts this cell structure.
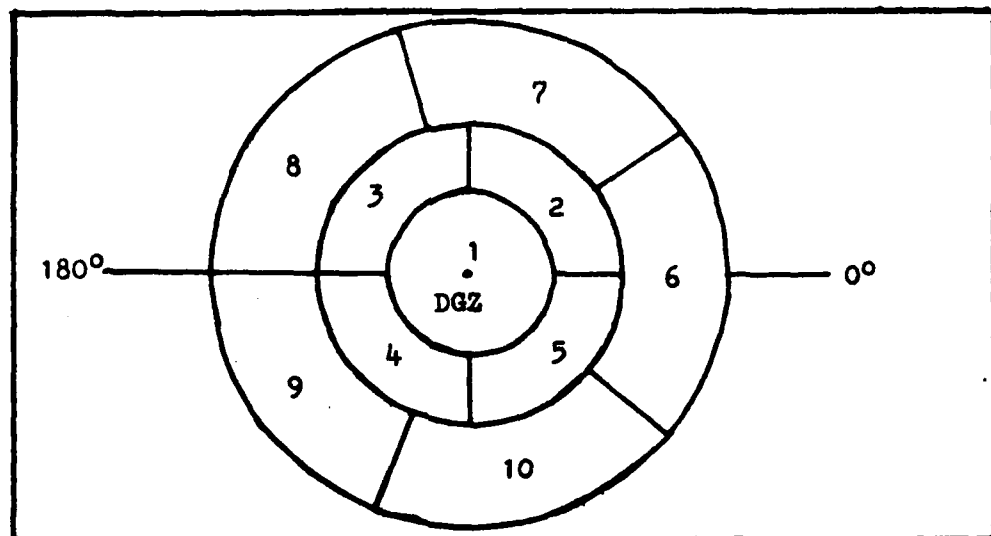
Figure 10. Ten Cells of Equal Probability of Hit

Note that the distance from DGZ is infinity for the outer-
most circle of the model. The case where a target is a
given distance from the DGZ and a weapon impacts at some
third point is the most general case and is depicted in
Figure 11. The PK of a target at a distance X from the
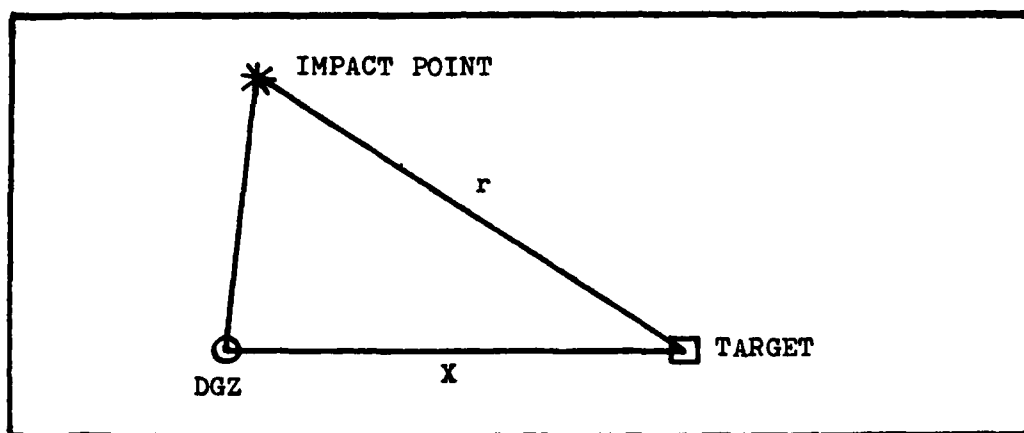
Figure 11.   Weapon Impact

DGZ is a function of the probability of hit (PH) at a
point described by ρ , ϕ , and the probability of damage
(PD) of the target at a distance r from the point of
impact. Thus, the following equation describes the situation:

PK(X) =   (Prob of Hitting Cell i)

X (Prob of Damage Given a Hit on Cell i)

or

$$PK(X) = \int_0^{2\pi} \int_0^{\infty} PH(\rho,\phi) \, PD(r) r dr d\phi$$

Placing the ten cell model over Figure 11 and treating each cell as a discrete impact point produces the following equation:

$$PK(X) \sum_{i=1}^{N_T} PH(\rho_i,\phi_i) \, \Delta A_i \, PD(r_i) \tag{1}$$

If we know a probability damage function based on intensity $(PD(I))$, this can be substituted directly for the probability damage function based on range $(PD(r))$. The variables are defined as follows:

r_i = distance from target to the center of cell i;

I = weapon effect intensity at the center of cell i;

ρ_i = distance from the DGZ to the outer edge of cell i;

ϕ_i = angle of cell i in relation to the DGZ;

ΔA_i = area of cell i;

N_T = number of cells in the model;

$<\rho_i>$ = distance from the DGZ to the probabilistic center of cell i;

$<\phi_i>$ = angle at which the probabilistic center of cell i is located;

n_i = number of cells in ring i;

N_i = number of cells in ring i plus all cells inside of ring i.

Figure 12 illustrates the geometry of these variables.

Figure 12. Ten Cell Model on a Target

For the ten cell model, the following values can be assigned:

N_T = 10 N_1 = 1 N_2 = 5 N_3 = 10

n_1 = 1 n_2 = 4 n_3 = 5 ρ_3 = ∞

$PH(\rho_i, \phi_i) \Delta A_i = 1/N_T$ for each i since the model is constructed so that an attacking weapon has an equal probability of hitting within each cell. Thus, Eq (1) can be written:

$$PK(X) = \frac{1}{N_T} \sum_{i=1}^{N_T} PD(I) \quad . \quad \text{From the Law of Cosines,}$$

$$r_i^2 = {<\rho_i>}^2 + x^2 - 2 <\rho_i> x \cos <\phi_i> \quad .$$

To solve for $<\rho_i>$, ρ_i must be found. Since each cell of the model represents an equal probability of hit and since the probability of hit is distributed circular normal, the following equality can be used:

$$P(\text{hit from DGZ to Ring i}) = \frac{N_i}{N_T}$$

$$= \int_0^{2\pi} \int_0^{\rho_i} \frac{e^{-\frac{1}{2}(\rho/\sigma)^2}}{2\pi\sigma^2} \rho d\rho d\theta \quad .$$

Integrating over θ and simplifying gives:

$$\frac{N_i}{N_T} = \frac{1}{\sigma^2} \int_0^{\rho_i} e^{-\frac{1}{2}(\rho/\sigma)^2} \rho d\rho$$

Letting $Z = \rho/\sigma$ (implies $dZ = d\rho/\sigma$) and solving for ρ_i gives:

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$$\frac{N_i}{N_T} = -\int_0^{\rho_i/\sigma} e^{-\frac{1}{2}z^2}(-Z)dZ = -[e^{-\frac{1}{2}(\rho_i/\sigma)^2} - e^0]$$

$$= 1 - e^{-\frac{1}{2}(\rho_i/\sigma)^2}$$

Simplifying results in the following expression for ρ_i :

$$e^{-\frac{1}{2}(\rho_i/\sigma)^2} = 1 - \frac{N_i}{N_T} \quad \text{or} \quad (\rho_i/\sigma)^2 = -2\ln(1 - \frac{N_i}{N_T}) . \quad (2)$$

To find an expression for σ^2 , we note that the CEP can be defined as:

$$\frac{1}{2} = \int_0^{CEP} e^{-\frac{1}{2}(r/\sigma)^2} \frac{r}{\sigma^2} dr = 1 - e^{-\frac{1}{2}(CEP/\sigma)^2}$$

Simplifying:

$$(CEP/\sigma)^2 = -2(\ln 1 - \ln 2) \quad \text{or} \quad (CEP/\sigma)^2 = 2\ln 2 ,$$

which implies

$$\sigma^2 = \frac{CEP^2}{2\ln 2} \qquad (3)$$

Substituting this last result into Eq (2) and simplifying yields:

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$$\frac{\rho_i}{CEP} = \left(\frac{-\ell n(1 - N_i/N_T)}{\ell n2}\right)^{\frac{1}{2}} \tag{4}$$

Now $\langle\rho_i\rangle$ must be found in terms of known quantities. We are finding the center of a two-dimensional Gaussian curve so:

$$\langle\rho_i\rangle = \frac{\displaystyle\int_{\rho_{i-1}}^{\rho_i} \rho \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2}(\rho/\sigma)^2} 2\pi\rho d\rho}{\displaystyle\int_{\rho_{i-1}}^{\rho_i} \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2}(\rho/\sigma)^2} 2\pi\rho d\rho}$$

the denominator is just the fraction of cells between rings $i-1$ and i : n_i/N_T . Therefore,

$$\langle\rho_i\rangle = \frac{N_T}{n_i} \int_{\rho_{i-1}}^{\rho_i} \frac{\rho}{\sigma^2} e^{-\frac{1}{2}(\rho/\sigma)^2} \rho d\rho \quad .$$

Letting $Z = \rho/\sigma$ (therefore $dZ = d\rho/\sigma$) and when $\rho = \rho_i$, $Z = \rho_i/\sigma$

$$\langle\rho_i\rangle = \frac{N_T}{n_i} \sigma \int_{\frac{\rho_{i-1}}{\sigma}}^{\rho_i/\sigma} z e^{-\frac{1}{2}z^2} dz \quad .$$

Integrating by parts with $U = Z$ and $dV = e^{-\frac{1}{2}z^2} z dz$ (implies $dU = dZ$ and $V = -e^{-\frac{1}{2}Z^2}$) gives:

$$\langle\rho_i\rangle = \frac{N_T}{n_i} \sigma\left\{\left[-Ze^{-\frac{1}{2}Z^2}\right]_{\frac{\rho_{i-1}}{\sigma}}^{\rho_i/\sigma} - \int_{\frac{\rho_{i-1}}{\sigma}}^{\rho_i/\sigma} - e^{-\frac{1}{2}Z^2} dZ\right\} \tag{5}$$

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The integral in Eq (5) can be written as the difference between cumulative normal functions:

$$\int_{\frac{\rho_{i-1}}{\sigma}}^{\rho_i/\sigma} -e^{-\frac{1}{2}Z^2}\,dZ = -\sqrt{2\pi}\left[\int_0^{\rho_i/\sigma} \frac{e^{-\frac{1}{2}Z^2}}{\sqrt{2\pi}}\,dZ\right.$$

$$\left. -\int_0^{\rho_{i-1}/\sigma} \frac{e^{-\frac{1}{2}Z^2}}{\sqrt{2\pi}}\,dZ\right]$$

Therefore, Eq (5) can be rewritten as

$$<\rho_i> = \frac{N_T}{n_i}\,\sigma\{\frac{\rho_{i-1}}{\sigma}\,e^{-\frac{1}{2}(\rho_{i-1}/\sigma)^2} - \frac{\rho_i}{\sigma}\,e^{-\frac{1}{2}(\rho_i/\sigma)^2}$$

$$+ \sqrt{2\pi}\,\left[F(\frac{\rho_i}{\sigma}) - F(\frac{\rho_{i-1}}{\sigma})\right]\}$$

where F represents the cumulative normal function. From Eq (3),

$$\sigma = \frac{CEP}{\sqrt{2\ell n2}}$$

and substituting into the last equation, yields

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$$\langle \rho_i \rangle = \frac{N_T}{n_i} \frac{CEP}{\sqrt{2\ln 2}} \{ \frac{\sqrt{2\ln 2}}{CEP} \rho_{i-1} e^{-\ln 2 (\rho_{i-1}/CEP)^2}$$

$$- \frac{\sqrt{2\ln 2}}{CEP} \rho_i e^{-\ln 2 (\rho_i/CEP)^2}$$

$$+ \sqrt{2\pi} [F(\frac{\sqrt{2\ln 2} \ \rho_i}{CEP}) - F(\frac{\sqrt{2\ln 2}}{CEP} \rho_{i-1})] \}$$

Simplifying:

$$\frac{\langle \rho_i \rangle}{CEP} = \frac{N_T}{n_i} \{ \frac{\rho_{i-1}}{CEP} e^{-\ln 2 (\rho_{i-1}/CEP)^2}$$

$$- \frac{\rho_i}{CEP} e^{-\ln 2 (\rho_i/CEP)^2}$$

$$+ \sqrt{\pi/\ln 2} [F(\frac{\sqrt{2\ln 2}}{CEP} \rho_i) - F(\frac{\sqrt{2\ln 2}}{CEP} \rho_{i-1})] \} \tag{6}$$

Because the two-dimensional Gaussian distribution is symmetric with respect to the angle ϕ , the angle to the cell center $\langle \phi_i \rangle$ is found by simply dividing the number of cells in ring i into 360°. This will provide an equal area in each cell of a particular ring.

Equation (4) can now be used for finding $\frac{\rho_i}{CEP}$ and Eq (6) will find $\frac{\langle \rho_i \rangle}{CEP}$. The following values for the variables of the ten cell model can be found (Table XXII).

			TABLE XXII			
			Ten Cell Model Values			
Ring #	Cell	Cells in Ring i	Cells Inside N_i	ρ_i/CEP	$<\rho_i>/CEP$	$<\phi_i>$
1	1	1	1	0.39	0	N/A
2	2	4	5	1.00	0.7119	45°
2	3	4	5	1.00	0.7119	135°
2	4	4	5	1.00	0.7119	225°
2	5	4	5	1.00	0.7119	315°
3	6	5	10	∞	1.507	0°
3	7	5	10	∞	1.507	72°
3	8	5	10	∞	1.507	144°
3	9	5	10	∞	1.507	216°
3	10	5	10	∞	1.507	288°

It was previously shown that:

$$PK(X) \;=\; \frac{1}{N_T} \sum_{i=1}^{N_T} PD(I) \;.$$

To find PK(X) , PD(I) must be determined. One well accepted
approach is to define PD(I) as a cumulative log-normal
function:

$$PD(I) = \int_0^I \frac{1}{\sqrt{2\pi}\beta I'} \, e^{-\frac{1}{2}\left(\frac{\ln I' - \ln I_{.5}}{\beta}\right)^2} dI'$$

Here β is the slope of the intensity versus PD curve (plotted as a straight line on log-log paper) and $\ln I_{.5}$ is the intensity at $PD(I) = 0.5$, now called α. Letting $Z = \left(\frac{\ln I' - \alpha}{\beta}\right)$ (therefore $dZ = \frac{1}{\beta I} dI$) we get:

$$PD(I) = \int_{-\infty}^{Z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}Z^2} dZ$$

or $PD(I)$ equals the standardized normal probability density function evaluated from $-\infty$ to Z.

By defining sure-kill (I_{sk}) and sure-safe (I_{ss}) probabilities at 98% kill and 2% kill, respectively, the area under the normal curve is 0.98 and 0.02, respectively, and the upper limits of integration are just equal to the Z value in a normal table corresponding to these areas. Therefore:

$$\frac{\ln I_{sk} - \alpha}{\beta} = +2.054 \quad \text{and} \quad \frac{\ln I_{ss} - \alpha}{\beta} = -2.054 \ .$$

Solving these equations simultaneously, we get

$$\alpha = \frac{1}{2} \ln\{(I_{sk})(I_{ss})\} \quad \text{and} \quad \beta = \frac{1}{2(2.054)} \ln\left(\frac{I_{sk}}{I_{ss}}\right) \ ,$$

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where I_{ss} is the sure-safe intensity and I_{sk} is the sure-kill intensity.

Now the values of PK(X) can be computed as functions of intensity. The required inputs are the yield and CEP of the attacking weapons, the sure-safe and sure-kill intensities (for the weapon effects of interest), and the distance of the target from the DGZ. Given these inputs, the ten cell model can determine the probability of kill of a weapon against a target. In this study, the weapon effects used are neutrons for determining interceptor PK, and overpressure and cratering for determining RV PK.

Appendix C

Probability of Kill Routine for Overpressure

To calculate the probability of kill (PK) of the shelters
due to overpressure, a procedure based on the ten cell model
is used. The ten cell model (Appendix B) requires the sure-
safe and sure-kill overpressures of the shelter. The sure-
safe overpressure is the overpressure at which survival is
expected 98 percent of the time, and the sure-kill overpressure
is the overpressure at which destruction is expected 98 per-
cent of the time (Ref 5).

The overpressure caused by a surface burst which creates
a crater is similar to that of a "free-air" burst with one
and one-half times the yield (Ref 21). The graph of peak
overpressure (psi) versus distance from the burst in feet or
meters for a "free-air" burst of a one kiloton weapon at sea
level is shown in Figure 13 (Ref 3). An equation approximating
this graph is as follows:

$$PSI = e^{[.19(\ell nSR)^2 - 1.5(\ell nSR) - .1]}$$

where

SR = scaled distance from burst in kilometers

$$SR = (\text{Actual Distance})\ \frac{\left(\frac{\text{Atmospheric Pressure at Burst Altitude}}{\text{Atmospheric Pressure at Sea Level}}\right)^{1/3}}{(1.5\ \text{Yield})^{1/3}}$$

(Ref 25)
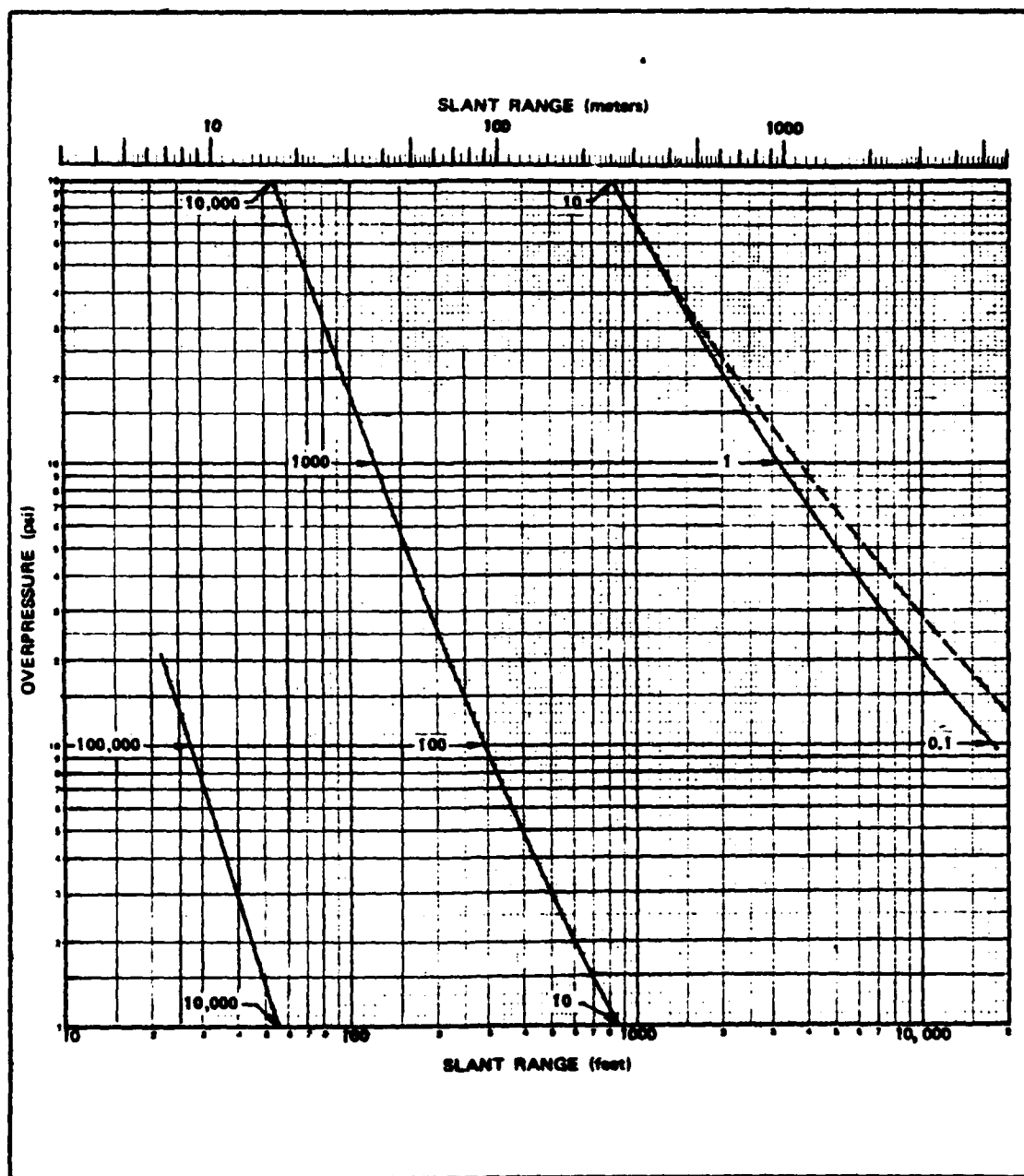
Figure 13. Peak Overpressure from a 1-Kiloton
Free-Air Burst at Sea Level (Ref 3)

Table XXIII checks the difference between the values read
from the graph and the values provided by the equation. The
equation provides an approximation, which on the average is
within 5.1 percent of the graph.

TABLE XXIII			
Graph versus Equation			
Slant Range (feet)	PSI (graph)	PSI (equation)	Difference
30	71,000	68,151	2849
150	550	565	15
400	46	42	4
1000	7	7	0
5000	.5	.55	.05

The equation given above provides the scaled overpressure for
a one kiloton weapon at sea level. The actual overpressure
(PSI_A) is found using the following equation:

$$PSI_A = PSI \left(\frac{\text{Atmospheric Pressure at Burst Altitude}}{\text{Atmospheric Pressure at Sea Level}} \right)$$

Thus, the overpressure of a surface burst can be found using
the following equation:

$$PSI_A = e^{[.19(\ell nSR)^2 - 1.5(\ell nSR) - .1]} \frac{\text{Atmospheric Pressure at Burst Altitude}}{\text{Atmospheric Pressure at Sea Level}}$$

where

SR = scaled distance from burst in kilometers.

Appendix D

Probability of Kill Routine for Neutron Fluence

To calculate the probability of kill (PK) of the RVs due
to neutron fluence, the ten cell model is used (Appendix B).
The sure-safe and sure-kill neutron fluence levels of an RV
are assumed to be 10^{13} and 10^{17} neutrons per square centimeter
(N/CM^2), respectively (Ref 22). Although the actual fluence
levels depend on the RV design, these unclassified values
chosen appear to be reasonable (Ref 7).

To calculate the actual neutron fluence at various ranges,
the number of neutrons produced per kiloton yield of the
terminal interceptor must be known. Since a thermonuclear
device produces more neutrons per kiloton than a pure fission
device, the terminal interceptor warhead is assumed to be a
50-50 fission-fusion device. The neutrons per kiloton yield
for a 50-50 thermonuclear device is approximately 3.16×10^{23}
neutrons per kiloton (Ref 4).

The $4\pi R^2$ neutron fluence versus mass integral (MI)
where R is the distance from burst point to target is shown
in Figure 14. The mass integral (MI) is the amount of air
that a neutron must traverse in traveling from the burst point
to the target. An equation which fits this graph is given
by:

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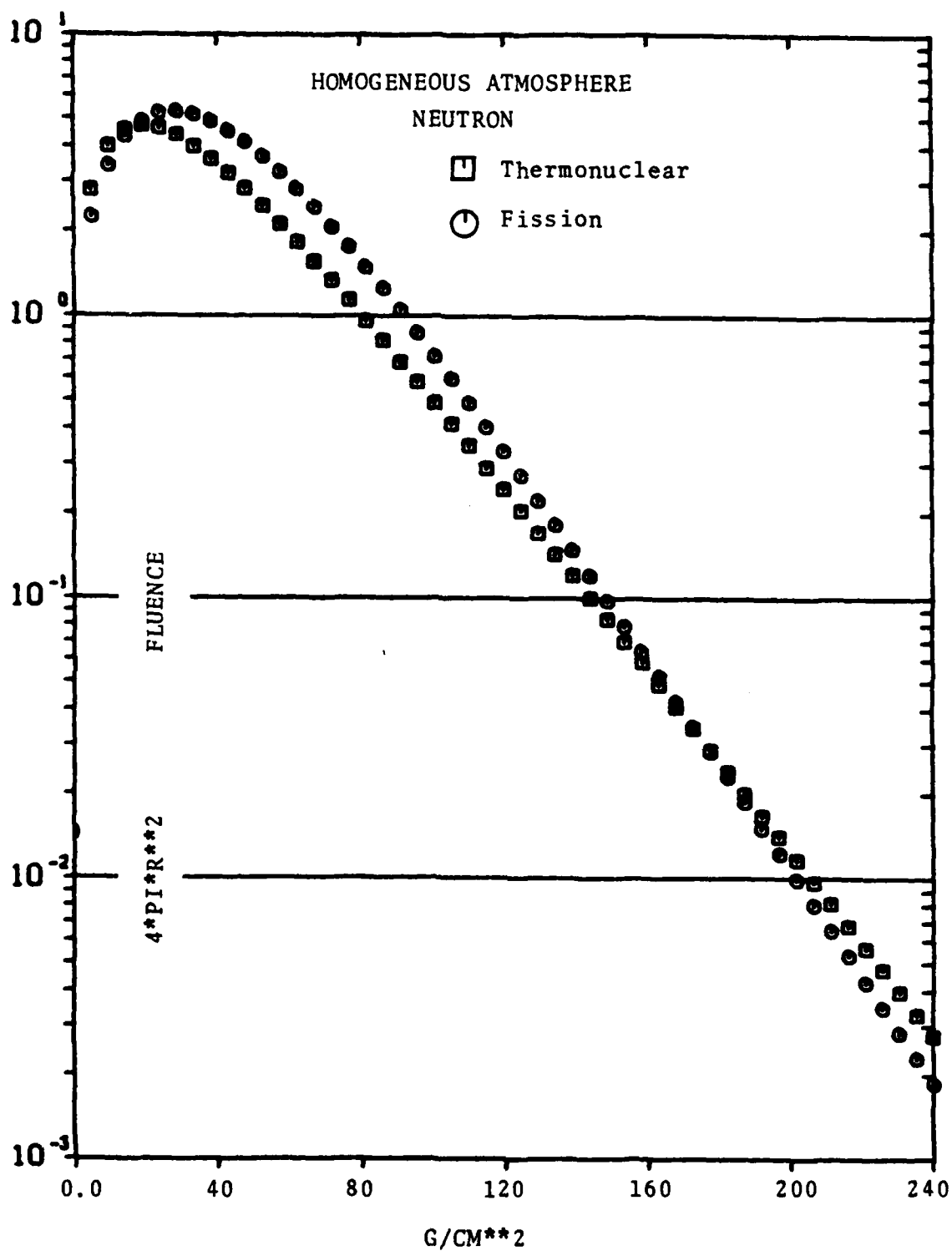
Figure 14. 4πR² Neutron Fluence for Fission and Thermonuclear Sources (Ref 13:55)

$$4\pi R^2 \text{ Fluence } = \exp[-6.775 + .5269\text{x}10^{-2}(MI) - .54364\text{x}10^{-5}(MI)^2$$

$$- .21468\text{x}10^{-3}(MI)^{3/2} - 3.8214(MI)^{1/2}$$

$$+ 10.875(MI)^{1/3} - 1.3975(\ln(MI))] \quad \text{(Ref 13:51-55)}.$$

It is assumed that the actual terminal intercept will occur at approximately 20,000 feet altitude. Since the terminal interceptor's probability of kill is determined when it passes through the RV altitude, a homogeneous atmosphere with a density (ρ) of 0.65312 kilograms per meter cubed (KG/M^3) was assumed. This value is used to calculate the MI in a homogeneous atmosphere as follows:

$$MI = \rho R$$

where R is the distance from the target to the burst point.

Appendix E

Analytical Attack Probability of Kill
for Exoatmospheric Defense Model

$$1 \quad - \quad A$$

$$A = \frac{B*D}{C}$$
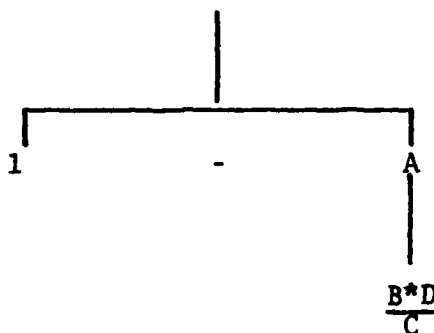
where

A = Upper layer of defense probability of kill

B = Number of exoatmospheric interceptors

C = Number of attacking RVs

D = Exoatmospheric interceptor probability
of kill

D = $1 - \exp(-.694(WR/CE)^2)$

Analytical Attack Probability of Kill
for Endoatmospheric Defense Model (One TDU)

RV Attack Probability of Killing the MX

```
                        1  -  (A*B)

                 1  -  C           1  -  D

          (1-E)  *  F  *  G      (1-H)  *  I  *  G

           J  *  K  *  L           M  *  N  *  L
                     |                      1 - P
                     1
                                       Q * R * S
          1  -  T             1  -  T1            1/6

     1/6(U1) + (W)2/6        3/6(U1)  +  (W)2/6

   1 - (U*V)  (1-X) * Y * Z   1 - (U*V)   (1-X) * Y * Z

  1 - W  1 - AA DD * EE * L  1 - W  1 - AA DD * EE * L
                  1    1                     1    1

        (1-BB)*CC  *  Z            (1-BB)*CC * Z

         FF  *  GG  *  L            FF  *  GG  *  L

     1 - JJ    1  -  HH         1 - JJ     1  -  HH

   (1-L)*Y * Z   Q * R * II   (1-L)*Y * Z    Q * R * II
                        2/3                        2/3
```
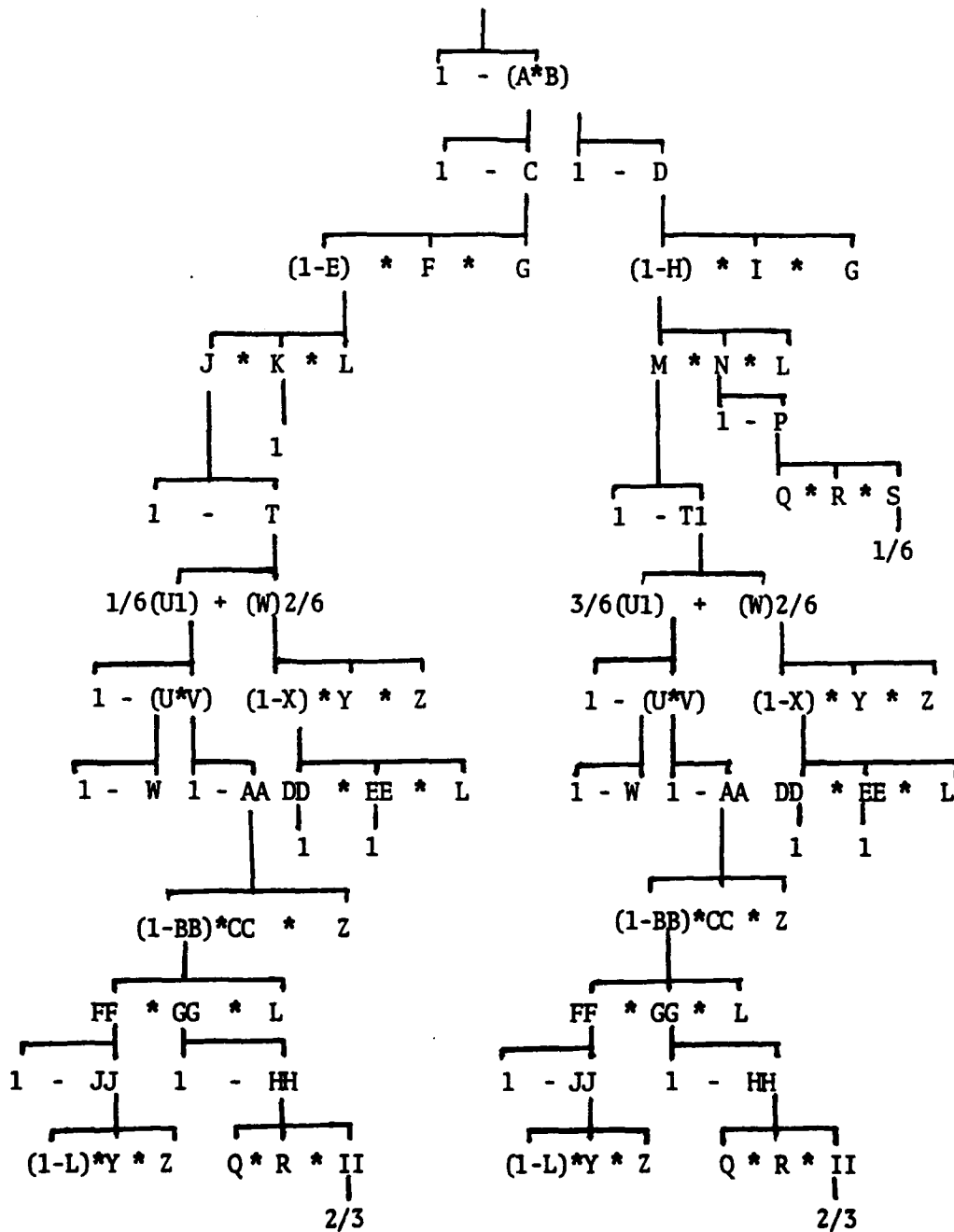
where

A = Probability that the MX survives the 1st RV
targeted at it

B = Probability that the MX survives the 2nd RV
targeted at it

C = Probability that the MX is destroyed by the 1st
RV targeted at it

D = Probability that the MX is destroyed by the 2nd
RV targeted at it

E = Probability of destroying the 1st RV targeted
at the MX

F = Probability of the 1st RV targeted at the MX
actually being targeted at the MX (# of RVs/#
of shelters or 1, whichever is smaller)

G = Attacking RV probability of destroying an MX
shelter (0.5986 for a 1000 KT RV with a CEP
of 0.2 NM)

H = Probability of destroying the 2nd RV targeted
at the MX

I = Probability of the 2nd RV targeted at the MX
actually being targeted at the MX ((# of RVs-
# of shelters)/# of shelters or $\emptyset$, whichever
is larger)

J = Probability of the terminal defense unit (TDU)
being active for the 1st RV targeted at the MX

K = Probability that the TDU has interceptors available
for the 1st RV targeted at the MX

L = Defensive interceptor probability of kill
(0.599 for a 5 KT interceptor with a CEP of
600 feet)

M = Probability of the TDU being active for the
2nd RV targeted at the MX

N = Probability that the TDU has interceptors
available for the 2nd RV targeted at the MX

P = Probability that the TDU does not have inter-
ceptors available for the 2nd RV targeted at
the MX

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Q = Probability of 2 RVs being targeted at the TDU
((# of RVs - # of shelters)/# of shelters or
0, whichever is larger)

R = Probability of 2 RVs being targeted at the MX
shelter ((# of RVs - # of shelters)/# of shelters
or 0, whichever is larger)

S = Probability of having no opportunity to intercept
the 2nd RV targeted at the MX

T = Probability that the TDU is dead before the
1st RV targeted at the MX can be intercepted

T1 = Probability that the TDU is dead before the
2nd RV targeted at the MX can be intercepted

U1 = Probability that the TDU is destoryed by both
RVs targeted at it

U = Probability that the TDU survives the 1st RV
targeted at it

V = Probability that the TDU survives the 2nd
RV targeted at it

W = Probability that the TDU is destroyed by the
1st RV targeted at it

X = Probability of destroying the 1st RV targeted
at the TDU

Y = Probability of the 1st RV targeted at the TDU
actually being targeted at the TDU (# of RVs/
# of shelters or 1, whichever is smaller)

Z = Attacking RV probability of destroying a TDU
shelter (0.5852 for a 1000 KT RV with a CEP
of 0.2 NM)

AA = Probability that the TDU is destoryed by the
2nd RV targeted at it

BB = Probability of destroying the 2nd RV targeted
at the TDU

CC = Probability of the 2nd RV targeted at the TDU
actually being targeted at the TDU ((# of
RVs - # of shelters)/# of shelters or 0, which-
ever is larger)

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DD = Probability of the TDU being active for the 1st RV targeted at the TDU

EE = Probability that the TDU has interceptors available for the 1st RV targeted at the TDU

FF = Probability of the TDU being active for the 2nd RV targeted at the TDU

GG = Probability that the TDU has interceptors available for the 2nd RV targeted at the TDU

HH = Probability that the TDU does not have interceptors available for the 2nd RV targeted at the TDU

II = Probability of having no opportunity to intercept the 2nd RV targeted at the TDU

JJ = Probability that the TDU is destoryed by the 1st RV targeted at the TDU

Appendix G

Computer Model Listings

and

Q-GERT Networks

Q-GERT Listing (Layered Defense Model)

```
GEN,SHERET,LAYER1,11,6,1981,5,0,0,20.,600,S,0.,4*
SOU,1,0,1,A,M*
VAS,1,1,IN,1,2,CO,10,4,CO,12* #COUNTER,SHEL-2,SHELTERS              #
ACT,1,1,NO,1,1/GENERATE,(9)A1.LT.16* GENERATE RV ARIV              #
PAR,1,,.083,0.,,,.033,9* RVS ARRIVE NORMALLY. MEAN=5 SEC,SD=2 SEC
ACT,1,3,CO,0.,,2/RVARIV,(9)A1.LE.16* RVS ARRIVE TO SYSTEM          #
REG,3,1,1,F* DETERMINE IF RVS ARE TWICE # OF SHELTERS
ACT,3,41,CO,1.,,3/TWOPER,(9)A4.EQ.8* RVS TWICE # OF SHELTERS       #
ACT,3,30,CO,0.,,4/LTTWOPER,(9)A4.GT.8* RVS LESS THAN TWICE         #
REG,41,1,1,P* TWICE # OF RVS ASSIGNED TO SHELTERS
ACT,41,42,CO,0.,,5/MPS,(8).8333* ASSIGN RVS TO MPS                 #
ACT,41,43,CO,0.,,5/DU,(8).08335* ASSIGN RVS TO DU                  #
ACT,41,44,CO,0.,,5/MX,(8).08335* ASSIGN RVS TO MX                  #
QUE,42/MPSQUE,0,0,D,F,41* MPS QUEUE                                #
QUE,43/DUQUE,0,0,D,F,41* DU QUEUE
QUE,44/MXQUE,0,0,D,F,41* MX QUEUE
VAS,42,3,CO,1* ASSIGN MPS ID #
VAS,43,3,CO,2* ASSIGN DU ID #
VAS,44,3,CO,3* ASSIGN MX ID #
ACT,42,19,CO,10.,,7/MPSDELAY,12* SEND MPS RVS TO MAIN RV QUEUE(RV-4
ACT,43,19,CO,10.,,8/DUDELAY,2* SEND DU RVS TO MAIN RV QUEUE
ACT,44,19,CO,10.,,9/MXDELAY,2* SEND MX RVS TO MAIN RV QUEUE
QUE,30,0,,D,F* LESS THAN TWICE THE # OF RVS QUEUE
ACT,30,31,CO,1,22/CAPCK,24* CHECK MPS/DU/MX QUE STATUS
REG,31,1,1,P* LESS THAN TWICE THE # OF RVS ASSIGNED TO SHELTERS
ACT,31,32,CO,0.,,6/MPS,(8).8333* ASSIGN RVS TO MPS                 #
ACT,31,33,CO,0.,,6/DU,(8).08335* ASSIGN RVS TO DU                  #
ACT,31,34,CO,0.,,6/MX,(8).08335* ASSIGN RVS TO MX                  #
QUE,32/MPSQUE,0,0,D,F,31* MPS QUEUE                                #
QUE,33/DUQUE,0,0,D,F,31* DU QUEUE
QUE,34/MXQUE,0,0,D,F,31* MX QUEUE
ACT,32,19,CO,10.,,10/MPSDELAY,14* SEND MPS RVS TO MAIN RV QUEUE
ACT,33,19,CO,10.,,11/DUDELAY,2* SEND DU RVS TO MAIN RV QUEUE
ACT,34,19,CO,10.,,12/MXDELAY,2* SEND MX RVS TO MAIN RV QUEUE
VAS,32,3,CO,1,2,US,2* ASSIGN MPS ID #
VAS,33,3,CO,2,2,US,2* ASSIGN DU ID #
VAS,34,3,CO,3,2,US,2* ASSIGN MX ID #
QUE,19/RVQUE,0,,D,F* MAIN RV QUEUE
VAS,19,4,CO,1* ASSIGN ALIVE (1) CODE
ACT,19,20,CO,0.,,13/RVDELAY* RV DELAY
REG,20,1,1,F* DETERMINE RVS TO SEND TO PROBE
ACT,20,21,CO,0.,,14/TOPROBE,(9)A1.LE.5* SEND RVS TO PROBE QUE      #
ACT,20,51,CO,,5,15/TOLOADS,(9)A1.GT.5* EXCESS RVS FOR STAT COL     #
QUE,51/RVACTIVE,0,,D,F*
QUE,21/PROBEQUE,0,,D,F,,10.,,5* PROBE QUEUE
ACT,21,23,US,3,17/INTERCEPTOR,5* INTERCEPTOR/RV INTERACTION        #
PAR,2,0.,,,,1.6988,9* X&Y DISTRIBUTION
STA,23/P1STATS,1,1,D,I,10.,,.5* PROBE 1 STATS
ACT,23,24,CO,0,18/DELAY*
```

```
QUE,24/CKSTATUS,0,,D,F*
ACT,24,50,US,1,19/CKALIVE,5*        SEND ACTIVE RVS TO NODE 51        *
STA,50/DEADSTAT,1,1,D,I,10.,,,5* COLLECTS STATS ON KILLED RVS
ACT,51,53,CO,3.,,20/LDDELAY,24*   DELAY UNTIL LOADS INTERACTION
QUE,53/LDSQUE,0,,D,F*  LOADS QUEUE
ACT,53,54,US,4,21/LDSSERV,24*  LOADS INTERACTION
REG,54,1,1,D*
ACT,54,55,US,5,23/MXSTATUS,24*  CK STATUS OF RVS & MX
STA,55/LDSTATS,1,1,D,I,13.,,,5*  LOADS STATISTICS
FIN*
```

Q-GERT Listing (Endoatmospheric Defense Model - One TDU)

```
GEN,SHERET,LOADS,11,16,1981,1,0,0,20.,600,S,0.,,4*
SOU,1,0,1,A,M*
VAS,1,1,IN,1,2,CO,10,4,CO,12*       *COUNTER,SHEL-2,SHELTERS           *
ACT,1,1,NO,1,1/GENERATE,(9)A1.LT.16* GENERATE RV ARIV                 *
PAR,1,,.083,0.,,,,.033,6*  RVS ARRIVE NORMALLY. MEAN=5 SEC,SD=2 SEC
ACT,1,3,CO,0.,,2/RVARIV,(9)A1.LE.16* RVS ARRIVE TO SYSTEM             *
REG,3,1,1,F*  DETERMINE IF RVS ARE TWICE * OF SHELERS
ACT,3,41,CO,1.,,3/TWOPER,(9)A4.EQ.8* RVS TWICE * OF SHELTERS          *
ACT,3,30,CO,0.,,4/LTTWOPER,(9)A4.GT.8* RVS LESS THAN TWICE           *
REG,41,1,1,P*  TWICE * OF RVS ASSIGNED TO SHELERS
ACT,41,42,CO,0.,,5/MPS,(8).8333* ASSIGN RVS TO MPS                    *
ACT,41,43,CO,0.,,5/DU,(8).08335* ASSIGN RVS TO DU                     *
ACT,41,44,CO,0.,,5/MX,(8).08335* ASSIGN RVS TO MX                     *
QUE,42/MPSQUE,0,0,D,F,41*  MPS QUEUE                                  *
QUE,43/DUQUE,0,0,D,F,41*  DU QUEUE
QUE,44/MXQUE,0,0,D,F,41*  MX QUEUE
VAS,42,3,CO,1*  ASSIGN MPS ID *
VAS,43,3,CO,2*  ASSIGN DU ID *
VAS,44,3,CO,3*  ASSIGN MX ID *
ACT,42,19,CO,10.,,7/MPSDELAY,12* SEND MPS RVS TO MAIN RV QUEUE(RV-4)
ACT,43,19,CO,10.,,8/DUDELAY,2* SEND DU RVS TO MAIN RV QUEUE
ACT,44,19,CO,10.,,9/MXDELAY,2* SEND MX RVS TO MAIN RV QUEUE
QUE,30,0,,D,F* LESS THAN TWICE THE * OF RVS QUEUE
ACT,30,31,CO,1.,,22/CAPCK,24*  CHECK MPS/DU/MX QUE STATUS
REG,31,1,1,P* LESS THAN TWICE THE * OF RVS ASSIGNED TO SHELTERS
ACT,31,32,CO,0.,,6/MPS,(8).8333*  ASSIGN RVS TO MPS                   *
ACT,31,33,CO,0.,,6/DU,(8).08335*  ASSIGN RVS TO DU                    *
ACT,31,34,CO,0.,,6/MX,(8).08335*  ASSIGN RVS TO MX                    *
QUE,32/MPSQUE,0,0,D,F,31*  MPS QUEUE                                  *
QUE,33/DUQUE,0,0,D,F,31*  DU QUEUE
QUE,34/MXQUE,0,0,D,F,31*  MX QUEUE
ACT,32,19,CO,10.,,10/MPSDELAY,14*  SEND MPS RVS TO MAIN RV QUEUE
ACT,33,19,CO,10.,,11/DUDELAY,2*  SEND DU RVS TO MAIN RV QUEUE
ACT,34,19,CO,10.,,12/MXDELAY,2*  SEND MX RVS TO AN RV QUEUE
VAS,32,3,CO,1,2,US,2*  ASSIGN MPS ID *
VAS,33,3,CO,2,2,US,2*  ASSIGN DU ID *
```

```
VAS,34,3,CO,3,2,US,2* ASSIGN MX ID #
QUE,19/RVQUE,0,,D,F*  MAIN RV QUEUE
VAS,19,4,CO,1*  ASSIGN ALIVE (1) CODE
ACT,19,53,CO,3,,20/TOLOADS,24*  SEND RVS TO LOADS
QUE,53/LDSQUE,0,,D,F*  LOADS QUEUE
ACT,53,54,US,4,21/LDSSERV,24*  LOADS INTERACTION
REG,54,1,1,D*
ACT,54,55,US,5,23/MXSTATUS,24*  CK STATUS OF RVS & MX
STA,55/LDSTATS,1,1,D,I,13,,.5*  LOADS STATISTICS
FIN*
```

Q-GERT Listing (Endoatmospheric Defense Model - Two TDUs)

```
GEN,SHERET,LDS2DU,11,20,1981,2,0,0,35,,1200,S,0.,,5*
SOU,1,0,1,A,M*
VAS,1,1,IN,1,2,CO,9,4,CO,12* #COUNTER,SHEL-3,SHELTERS             #
ACT,1,1,NO,1,1/GENERATE,(9)A1.LT.16* GENERATE RV ARIV           #
PAR,1,.083,0.,,,.033,3*  RVS ARRIVE NORMALLY. MEAN=5 SEC,SD=2 SEC
ACT,1,3,CO,0,,2/RVARIV,(9)A1.LE.16* RVS ARRIVE TO SYSTEM        #
REG,3,1,1,F*  DETERMINE IF RVS ARE TWICE # OF SHELERS
ACT,3,41,CO,1,,3/TWOPER,(9)A4.EQ.8* RVS TWICE # OF SHELTERS      #
ACT,3,30,CO,0,,4/LTTWOPER,(9)A4.GT.8* RVS LESS THAN TWICE       #
REG,41,1,1,P*  TWICE # OF RVS ASSIGNED TO SHELERS
ACT,41,42,CO,0,,5/MPS,(8).75* ASSIGN RVS TO MPS         (12 SHEL)
ACT,41,43,CO,0,,5/DU,(8).0833333334* ASSIGN RVS TO DU 1       (12 SHEL)
ACT,41,44,CO,0,,5/MX,(8).0833333333* ASSIGN RVS TO MX        (12 SHEL)
ACT,41,45,CO,0,,5/DU2,(8).0833333333* ASSIGN RVS TO DU2       (12 SHEL)
QUE,42/MPSQUE,0,0,D,F,41*  MPS QUEUE                            #
QUE,43/DUQUE,0,0,D,F,41*  DU 1 QUEUE
QUE,44/MXQUE,0,0,D,F,41*  MX QUEUE
QUE,45/DU2QUE,0,0,D,F,41* DU2 QUEUE
VAS,42,3,CO,1*  ASSIGN MPS ID #
VAS,43,3,CO,2*  ASSIGN DU 1 ID #
VAS,44,3,CO,3*  ASSIGN MX ID #
VAS,45,3,CO,4* ASSIGN DU2 ID #
ACT,42,19,CO,10,,7/MPSDELAY,10* SEND MPS RVS TO MAIN RV QUEUE(RV-6)
ACT,43,19,CO,10,,8/DUDELAY,2* SEND DU RVS TO MAIN RV QUEUE
ACT,44,19,CO,10,,9/MXDELAY,2* SEND MX RVS TO MAIN RV QUEUE
ACT,45,19,CO,10. 99/DUDELAY,2*
QUE,30,0,,D,F* LESS THAN TWICE THE # OF RVS QUEUE
ACT,30,31,CO,1,,22/CAPCK,24*  CHECK MPS/DU/MX QUE STATUS
REG,31,1,1,P* LESS THAN TWICE THE # OF RVS ASSIGNED TO SHELTERS
ACT,31,32,CO,0,,6/MPS,(8).75*  ASSIGN RVS TO MPS    (12 SHEL)
ACT,31,33,CO,0,,6/DU,(8).0833333334*  ASSIGN RVS TO DU 1    (12 SHEL)
ACT,31,34,CO,0,,6/MX,(8).0833333333*  ASSIGN RVS TO MX    (12 SHEL)
ACT,31,35,CO,0,,6/DU,(8).0833333333* ASSIGN RVS TO DU2  (12 SHEL)
QUE,32/MPSQUE,0,0,D,F,31*  MPSQUE                               #
QUE,33/DUQUE,0,0,D,F,31*  DU QUEUE
QUE,34/MXQUE,0,0,D,F,31*  MX QUEUE
```

```
QUE,35/DU2QUE,0,0,D,F,31* DU2 QUEUE
ACT,32,19,CO,10,,10/MPSDELAY,13*   SEND MPS RVS TO MAIN RV QUEUE(RV-3)
ACT,33,19,CO,10,,11/DUDELAY,2*   SEND DU RVS TO MAIN RV QUEUE
ACT,34,19,CO,10,,12/MXDELAY,2*   SEND MX RVS TO MAIN RV QUEUE
ACT,35,19,CO,10,,98/DUDELAY,2* SEND DU2 RVS TO MAIN RV QUEUE
VAS,32,3,CO,1,2,US,2*   ASSIGN MPS ID #
VAS,33,3,CO,2,2,US,2*   ASSIGN DU 1 ID #
VAS,34,3,CO,3,2,US,2* ASSIGN MX ID #
VAS,35,3,CO,4,2,US,2* ASSIGN DU2 ID #
QUE,19/RVQUE,0,,D,F*   MAIN RV QUEUE

VAS,19,4,CO,1*   ASSIGN ALIVE (1) CODE
ACT,19,53,CO,1,,20/TOLOADS,24*   SEND RVS TO LOADS
REG,53,1,1,D*
ACT,53,54,CO,0,,31/LDSSERV,24*   LOADS INTERACTION
REG,54,1,1,D*
VAS,54,5,US,4*
ACT,54,57,US,5,32/MXSTATUS,24*   CK STATUS OF RVS & MX
STA,57/LDSTATS,1,1,D,I,13,,,5*   LOADS STATISTICS
REG,55,1,1,D*
ACT,55,56,CO,0,,33/DU2SERV,24* RV/DU2 INTERACTION
REG,56,1,1,D*
VAS,56,5,US,6*
ACT,56,58,US,5,34/MXSTATUS,24* DETERMINE MX STATUS AFTER ATTACK
STA,58/DU2STAT,1,1,D,I,13,,,5* DU2 STATS
FIN*
```

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```
        SUBROUTINE US(ISN,DTIM)
        COMMON/USER/MISS,X,Y,ZZ,DU,DUPK,MXPK,JI,L,PK
        COMMON/QVAR/NDE,NFTBU(100),NREL(100),NRELP(100),NREL2(100),
       1NRUN,NRUNS,NTC(100),PARAM(100,4),TBEG,TNOW
C**DECLARE,VARIABLES **
        INTEGER DU,JI,L,I,JK
        REAL MISS,ZZ,X,Y,NO,DUPK,MXPK,PK,RA,RE,RCPK,NALPHA,NBETA
        REAL PIR2F,MI,PI,PERKT,SR,ATT(4),DENS,F,C(7),Z,CPK
        REAL SOP,OP,CEP,YIELD,OPALPHA,OPBETA,T,YLD,TR,MXPS
        DATA JK,CPK/0,0./
        Z=0.
        GO TO (1,2,3,4,5),ISN
  1     DTIM=1.
        ATT(1)=GATRB(1)
        ATT(2)=GATRB(2)
        ATT(3)=GATRB(3)
        ATT(4)=GATRB(4)
        IF(GATRB(4).GT..5) THEN
          CALL STAGO(19,51,0.,1,ATT)
        ENDIF
        RETURN
C** ENSURES CORRECT RV TARGETING. # SHELTERS > 1/2 # RVS.
  2     DTIM=0.
        TR=REMST(22)
        IF((NTC(32).GE.13).AND.(NTC(34).EQ.2)) THEN
          CALL STAGO(22,33,TR,0,ATT)
        ELSEIF((NTC(32).GE.13).AND.(NTC(33).EQ.2))THEN
          CALL STAGO(22,34,TR,0,ATT)
        ELSEIF((NTC(32).GE.14).AND.(NTC(34).EQ.1)) THEN
          CALL STAGO(22,33,TR,0,ATT)
        ELSEIF((NTC(32).GE.14).AND.(NTC(33).EQ.1)) THEN
          CALL STAGO(22,34,TR,0,ATT)
        ELSEIF(NTC(32).EQ.14) THEN
          RN=DRAND(9)
          IF(RN.LE..5) THEN
            CALL STAGO(22,33,TR,0,ATT)
          ELSE
            CALL STAGO(22,34,TR,0,ATT)
          ENDIF
        ENDIF
        RETURN
C**     SEND RV TO INTERCEPTORS   **
C** FIRE INTERCEPTOR AT RV & DETERMINE HIT/MISS **
  3     X=NO(2)
        Y=NO(2)
        ZZ=(X*X+Y*Y)**.5
        MISS=ZZ-1.5
        MISS=ABS(MISS)
```

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```fortran
C** IS RV WITHIN INTERCEPTOR WEAPON RADIUS? (KILL) **
      IF(MISS.LE.1.5) THEN
        CALL PATRB(0,4)
      ENDIF
C** 30 SEC FROM DETECTION TILL INTECEPTOR/RV ENCOUNTER **
      DTIM=.5
      RETURN
C** SEND RVS NOT KILLED TO LOWER LAYER (LOADS) **
C** IS RV TARGETED AT MPS? **
   4  IF (GATRB(3).EQ.1.) THEN
        DTIM=0.
        RETURN
      ENDIF
C** CHECK STATUS OF DU **
      IF(DU.GT..5) THEN
        DTIM=.08333
        GO TO 100
      ENDIF
        IF (JI.GE.2) THEN
C** IS THIRD RV TARGETED AT DU? **
        IF (GATRB(3).EQ.2.) THEN
          DTIM=.08333
          GO TO 100
        ENDIF
      ENDIF
C** ALL INTERCEPTORS LAUNCHED **
      DTIM=.08333
      IF(JI.GE.3) THEN
      GO TO 100
      ENDIF
       JI=JI+1
C** INTERCEPTOR NEUTRON PK **
C**GLOSSARY**
C*    MI=MASS INTEGRAL (G/CM2)
C*    PIR2F=4*PI*R* 2 FLUENCE
C*    PERKT=NEUTRONS PER KILOTON
C*    INTALT=ALTITUDE OF INTERCEPT (KM)
C*    SR=DISTANCE BETWEN BURST AND TARGET (KM)
C*    DENS=AIR DENSITY AT INTERCEPT ALTITUDE (KG/M3)
C*    YIELD=THERMONUCLEAR YIELD (KT)
C*    NALPHA=NEUTRON PK FUNCTION ALPHA
C*    NBETA=NEUTRON PK FUNCTION BETA
C*    CEP=INTERCEPTOR CIRCULAR ERROR PROBABLE
C*    F=NEUTRON FLUENCE (NEUTRONS/CM2)
C*
C** INPUT DATA **
      CEP=600
      PI=3.1415927
      PERKT=3.16E23
      DENS=.65312
```

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```
            YIELD=5.
            DATA C/-.6775E1,.5269E-2,-.54364E-5,-.21468E-3,-.38214E1,
           %.10875E2,-.13975E1/
            NALPHA=34.539
            NBETA=2.242
C** CALCULATE SLANT RANGE USING 10 CELL MODEL **
            DO 200 I=1,10
            CALL TENCELL(SR,CEP,I)
C** CALCULATE MASS INTEGAL **
            IF(SR.LT..1) THEN
              SR=.001
            ENDIF
C** CHANGE SR TO KILOMETERS
            SR=SR*.0003048
            MI=(DENS)*SR
            MI=MI*100.
C** COMPUTE HOMOGENEOUS AIR 4PIR2 FLUENCE **
            PIR2F=EXP(C(1)+C(2)*MI+C(3)*MI**2.+C(4)*MI**1.5+C(5)*MI**.5+
           %C(6)*MI**(1./3.)+C(7)*(ALOG(MI)))
C** COMPUTE FLUENCE **
            F=((PIR2F)/(4.*PI*(SR**2.))*(PERKT)*(YIELD)*(1E-10))
C** Z=NORMAL TABLE ENTRY VALUE FOR NEUTRON FLUENCE **
            Z=(ALOG(F)-NALPHA)/NBETA
            CALL PROB(Z,CPK)
  200     CONTINUE
            RN=DRAND(9)
              IF(RN.LE.CPK) THEN
                CALL PATRB(0,4)
                CPK=0.
              RETURN
            ENDIF
            CPK=0.
C
C** RV KILL MECHANISM **
  100     DTIM=DTIM+.08333
            IF(GATRB(3).EQ.3.) THEN
            L=L+1
            ENDIF
C
C** RV OVERPRESSURE PK **
C**GLOSSARY**
C*      OPALPHA=OVERPRESSURE PK FUNCTION ALPHA
C*      OPBETA=OVERPRESSURE PK FUNCTION BETA
C*      SOP=SCALED OVERPRESSURE
C*      OP=OVERPRESSURE
C
C** INPUT DATA **
            CEP=1215.2
            YLD=1000.
            OPALPHA=6.8755
            OPBETA=.12435
```

```fortran
C** CALCULATE SLANT RANGE USING TEN CELL MODEL **
      DO 300 I=1,10
      CALL TENCELL(SR,CEP,I)
C** SCALE SR FOR PRESSURE AND YIELD **
      SR=SR*((12.69/14.7)**(1./3.))/((1.5*YLD)**(1./3.))
C** CHECK FOR CELL 1 **
      IF (SR.LT..1) THEN
      SOP=1E5
      ELSE
C** CHANGE SR TO KILOMETERS **
      SR=SR*.0003048
      SOP=EXP(.19*(ALOG(SR))**2-1.5*ALOG(SR)-.1)
      ENDIF
C** SCALE OVERPRESSURE FOR ALTITUDE **
      OP=SOP*(12.69/14.7)
C** Z=NORMAL TABLE ENTRY VALUE FOR RV OVERPRESSURE **
      Z=(ALOG(OP)-OPALPHA)/OPBETA
      CALL PROB(Z,CPK)
  300 CONTINUE
      ROPK=CPK
      CPK=0.
C
C** RV CRATERING PK **
C**GLOSSARY**
C*    RA=APPARENT CRATER RADIUS
C*    RCPK=RV CRATER PROB OF KILL
      RA=484.54
      RE=1041.76
      RE=RE*(2./3.)
      IF(GATRB(3).EQ.2) THEN
      RCPK=.1704
      ELSEIF(GATRB(3).EQ.3) THEN
      RCPK=.1971
      ENDIF
      PK=ROPK+(1-ROPK)*(RCPK)
C** COMPUTE CORRECT PK FOR DU SHELTER **
      IF(GATRB(3).EQ.2) THEN
      DUPK=PK
      RN=DRAND(9)
C** IS DU DESTROYED? **
      IF(RN.LE.DUPK) THEN
       DU=1
      RETURN
      ENDIF
      RETURN
      ENDIF
C** COMPUTE CORRECT PK FOR MX SHELTER **
      IF(GATRB(3).EQ.3) THEN
      IF(L.EQ.2) THEN
      MXPK=1-(1-MXPK)**2
      ELSEIF(L.EQ.1) THEN
```

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```
          MXPK=PK
          ENDIF
          RETURN
          ENDIF
C** IS MX DESTROYED?   **
  5       IF(NTC(54).EQ.NTC(51)) THEN
          JI=0
          L=0
          RN=DRAND(9)
          IF(RN.LE.MXPK) THEN
            PRINT*,'  MX DESTROYED'
           JK=JK+1
           IF(NRUN.GE.NRUNS) THEN
            MXPS=1.-(FLOAT(JK)/FLOAT(NRUNS))
            PRINT*,'>>> MX SURVIVABILITY= ',MXPS
           ENDIF
           ELSE
            IF(NRUN.GE.NRUNS) THEN
            MXPS=1.-(FLOAT(JK)/FLOAT(NRUNS))
             PRINT*,'>>> MX SURVIVABILITY= ',MXPS
            ENDIF
          ENDIF
          ENDIF
          RETURN
          END
          SUBROUTINE UI
          COMMON/USER/MISS,X,Y,ZZ,DU,DUPK,MXPK,JI,L,PK
          INTEGER DU,JI,L
          REAL MISS,X,Y,ZZ,DUPK,MXPK,PK,CPK,Z
          MISS=0.
          X=Y=ZZ=0.
          DU=0
          DUPK=0.
          MXPK=0.
          JI=L=0
          PK=0.
          CPK=0.
          Z=0.
          RETURN
          END
          SUBROUTINE TENCELL (SR,CEP,I)
C
          COMMON/USER/MISS,X,Y,ZZ,DU,DUPK,MXPK,JI,L,PK
C
C**GLOSSARY**
C*      THETA(I)=REFERENCE ANGLE TO RADII TO CELL CENTERS MEASURED
C*      FROM MAJOR AXIS COUNTER CLOCKWISE
C*      RHO(I)=CELL RADII (RHO/CEP)
C
C**DECLARE VARIABLES
```

```
      INTEGER I
      REAL THETA(10),RHO(10),CEP,SR
C** DATA INPUT **
      DATA THETA/0,45,135,225,315,0,72,144,216,288/
      DATA RHO/0,.7109,.7109,.7109,.7109,1.509,1.509,1.509,1.509,
     &1.509/
C** CALCULATE INTERCEPTOR SLANT RANGE **
      SR=RHO(I)*CEP
      RETURN
      END
C
      SUBROUTINE PROB(Z,CPK)
      COMMON/USER/MISS,X,Y,ZZ,DU,DUPK,MXPK,JI,L,PK
C
C**GLOSSARY**
C*    PK=PROBABILITY OF KILL
C*    CPK=CUM PROBABILITY OF KILL
C** DECLARE VARIABLES **
      REAL PK,CPK,Z
      IF(Z.LT.0.) THEN
      Z=ABS(Z)
C** CURVE FIT FOR STANDARD NORMAL CURVE **
      PK=.5*(1.+.19685*Z+.115194*Z*Z+.000344*Z*Z*Z+.019527*Z**4.)**(-4.)
      ELSE
      PK=1-.5*(1.+.196854*Z+.115194*Z*Z+.000344*Z**3.+.019527*Z**4.)
     &**(-4.)
      ENDIF
      PK=PK/10.
      CPK=CPK+PK
      RETURN
      END
```

USER Subroutines (Endoatmospheric Defense Model - One TDU)

```
      SUBROUTINE US(ISN,DTIM)
      COMMON/USER/MISS,X,Y,ZZ,DU,DUPK,MXPK,JI,L,PK
      COMMON/QVAR/NDE,NFTBU(100),NREL(100),NRELP(100),NREL2(100),
     &NRUN,NRUNS,NTC(100),PARAM(100,4),TBEG,TNOW
      INTEGER DU,JI,L,I,KK
      REAL MISS,ZZ,X,Y,NO,DUPK,MXPK,PK,RA,RE,RCPK,NALPHA,NBETA
      REAL PIR2F,MI,PI,PERKT,SR,ATT(4),DENS,F,C(7),Z,CPK
      REAL SOP,OP,CEP,YIELD,OPALPHA,OPBETA,YLD,TR,MXPS
      DATA CPK,KK/0.,0/
      GO TO (1,2,3,4,5),ISN
1     DTIM=1.
      RETURN
C** ENSURES CORRECT RV TARGETING. # SHELTERS > 1/2 # RVS.
```

```
2       DTIM=.001
        TR=REMST(22)
        IF((NTC(32).GE.13).AND.(NTC(34).EQ.2)) THEN
          CALL STAGO(22,33,TR,0,ATT)
        ELSEIF((NTC(32).GE.13).AND.(NTC(33).EQ.2))THEN
          CALL STAGO(22,34,TR,0,ATT)
        ELSEIF((NTC(32).GE.14).AND.(NTC(34).EQ.1)) THEN
          CALL STAGO(22,33,TR,0,ATT)
        ELSEIF((NTC(32).GE.14).AND.(NTC(33).EQ.1)) THEN
          CALL STAGO(22,34,TR,0,ATT)
        ELSEIF(NTC(32).EQ.14) THEN
          RN=DRAND(6)
          IF(RN.LE..5) THEN
            CALL STAGO(22,33,TR,0,ATT)
          ELSE
            CALL STAGO(22,34,TR,0,ATT)
          ENDIF
        ENDIF
        RETURN
3       DTIM=1.
        RETURN
C**     SEND RV TO LOADS INTERCEPTORS  **
C** IS RV TARGETED AT MPS ? **
4       IF (GATRB(3).EQ.1.) THEN
        DTIM=0.
        RETURN
        ENDIF
C** CHECK STATUS OF DU **
        IF(DU.GT..5) THEN
        DTIM=.08333
        GO TO 100
        ENDIF
        IF (JI.GE.2) THEN
C** IS THIRD RV TARGETED AT DU? **
        IF (GATRB(3).EQ.2.) THEN
          DTIM=.08333
          GO TO 100
        ENDIF
        ENDIF
C** ALL INTERCEPTORS LAUNCHED **
        DTIM=.08333
        IF(JI.GE.3) THEN
        GO TO 100
        ENDIF
        JI=JI+1
C** INTERCEPTOR NEUTRON PK **
C**GLOSSARY**
C*      MI=MASS INTEGRAL (G/CM2)
C*      PIR2F=4*PI*R* 2 FLUENCE
C*      PERKT=NEUTRONS PER KILOTON
```

```
C*      INTALT=ALTITUDE OF INTERCEPT (KM)
C*      SR=DISTANCE BETWEN BURST AND TARGET (KM)
C*      DENS=AIR DENSITY AT INTERCEPT ALTITUDE (KG/M3)
C*      YIELD=THERMONUCLEAR YIELD (KT)
C*      NALPHA=NEUTRON PK FUNCTION ALPHA
C*      NBETA=NEUTRON PK FUNCTION BETA
C*      CEP=INTERCEPTOR CIRCULAR ERROR PROBABLE
C*      F=NEUTRON FLUENCE (NEUTRONS/CM2)
C*
C** INPUT DATA **
        CEP=600.
        PI=3.1415927
        PERKT=3.16E23
C**  4000 FT DENSITY INDICATIVE OF MX BASING AREA **
        DENS=.65312
        YIELD=5.
        DATA C/-.6775E1,.5269E-2,-.54364E-5,-.21468E-3,-.38214E1,
       %.10875E2,-.13975E1/
        NALPHA=34.539
        NBETA=2.242
C** CALCULATE SLANT RANGE USING 10 CELL MODEL **
        DO 200 I=1,10
        CALL TENCELL(SR,CEP,I)
C** CALCULATE MASS INTEGAL **
        IF(SR.LT..1) THEN
          SR=.001
        ENDIF
C** CHANGE SR TO KILOMETERS
        SR=SR*.0003048
        MI=(DENS)*SR
        MI=MI*100.
C** COMPUTE HOMOGENEOUS AIR 4PIR2 FLUENCE **
        PIR2F=EXP(C(1)+C(2)*MI+C(3)*MI**2.+C(4)*MI**1.5+C(5)*MI**.5+
       %C(6)*MI**(1./3.)+C(7)*(LOG(MI)))
C** COMPUTE FLUENCE **
        F=((PIR2F)/(4.*PI*(SR**2.))*(PERKT)*(YIELD)*(1E-10))
C** Z=NORMAL TABLE ENTRY VALUE FOR NEUTRON FLUENCE **
        Z=(LOG(F)-NALPHA)/NBETA
        CALL PROB(Z,CPK)
 200    CONTINUE
        RN=DRAND(6)
         IF(RN.LE.CPK) THEN
            CALL PATRB(0,4)
            CPK=0.
           RETURN
         ENDIF
         CPK=0.
C
C** RV KILL MECHANISM **
 100    DTIM=DTIM+.08333
```

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```
              IF(GATRB(3).EQ.3.) THEN
               L=L+1
              ENDIF
C
C** RV OVERPRESSURE PK **
C**GLOSSARY**
C*     OPALPHA=OVERPRESSURE PK FUNCTION ALPHA
C*     OPBETA=OVERPRESSURE PK FUNCTION BETA
C*     SOP=SCALED OVERPRESSURE
C*     OP=OVERPRESSURE
C
C** INPUT DATA **
      CEP=1215.2
      Y=1000.
      OPALPHA=6.8755
      OPBETA=.12435
C** CALCULATE SLANT RANGE USING TEN CELL MODEL **
      DO 300 I=1,10
      CALL TENCELL(SR,CEP,I)
C** SCALE SR FOR PRESSURE AND YIELD **
      SR=SR*((12.69/14.7)**(1./3.))/((1.5*Y)**(1./3.))
C** CHECK FOR CELL 1 **
      IF (SR.LT..1) THEN
      SOP=1E5
      ELSE
C** CHANGE SR TO KILOMETERS **
      SR=SR*.0003048
      SOP=EXP(.19*(LOG(SR))**2-1.5*LOG(SR)-.1)
      ENDIF
C** SCALE OVERPRESSURE FOR ALTITUDE **
      OP=SOP*(12.69/14.7)
C** Z=NORMAL TABLE ENTRY VALUE FOR RV OVERPRESSURE **
      Z=(LOG(OP)-OPALPHA)/OPBETA
      CALL PROB(Z,CPK)
  300 CONTINUE
      ROPK=CPK
      CPK=0.
C
C** RV CRATERING PK **
C**GLOSSARY**
C*     RA=APPARENT CRATER RADIUS
C*     RE=RADIUS OF EJECTA
C*     RCPK=RV CRATER PROB OF KILL
      RA=484.54
      RE=1041.76
      RE=RE*(2./3.)
      IF(GATRB(3).EQ.2) THEN
      RCPK=.1704
      ELSEIF(GATRB(3).EQ.3) THEN
      RCPK=.1971
      ENDIF
```

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```
                PK=ROPK+(1-ROPK)*(RCPK)
C** COMPUTE CORRECT PK FOR DU SHELTER **
          IF(GATRB(3).EQ.2) THEN
          DUPK=PK
          RN=DRAND(6)
C** IS DU DESTROYED? **
          IF(RN.LE.DUPK) THEN
           DU=1
           RETURN
          ENDIF
          RETURN
          ENDIF
C** COMPUTE CORRECT PK FOR MX SHELTER **
          IF(GATRB(3).EQ.3) THEN
          IF(L.EQ.2) THEN
          MXPK=1-(1-MXPK)**2
          ELSEIF(L.EQ.1) THEN
          MXPK=PK
          ENDIF
          RETURN
          ENDIF
C** IS MX DESTROYED?  **
    5     IF(NTC(54).EQ.NTC(19)) THEN
           JI=0
           L=0
           RN=DRAND(6)
           IF(RN.LE.MXPK) THEN
             PRINT*,'  MX DESTROYED'
             KK=KK+1
             IF(NRUN.GE.NRUNS) THEN
              MXPS=1.-(FLOAT(KK)/FLOAT(NRUNS))
              PRINT*,'>>>MX SURVIVABILITY= ',MXPS
             ENDIF
           ELSE
             IF(NRUN.GE.NRUNS) THEN
              MXPS=1.-(FLOAT(KK)/FLOAT(NRUNS))
              PRINT*,'>>>MX SURVIVABILITY= ',MXPS
             ENDIF
           ENDIF
          ENDIF
          RETURN
          END
          SUBROUTINE UI
          COMMON/USER/MISS,X,Y,ZZ,DU,DUPK,MXPK,JI,L,PK
          INTEGER DU,JI,L
          REAL MISS,X,Y,ZZ,DUPK,MXPK,PK,CPK,Z
          MISS=0.
          X=Y=ZZ=0.
          DU=0
          DUPK=0.
          MXPK=0.
```

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```
          JI=L=0
          PK=CPK=Z=0.
          RETURN
          END
          SUBROUTINE TENCELL (SR,CEP,I)
C
          COMMON/USER/MISS,X,Y,ZZ,DU,DUPK,MXPK,JI,L,PK
C
C**GLOSSARY**
C*      THETA(I)=REFERENCE ANGLE TO RADII TO CELL CENTERS MEASURED
C*      FROM MAJOR AXIS COUNTER CLOCKWISE.
C*      RHO(I)=CELL RADII (RHO/CEP)
C
C**DECLARE VARIABLES
          INTEGER I
          REAL THETA(10),RHO(10),CEP,SR
C** DATA INPUT **
          DATA THETA/0,45,135,225,315,0,72,144,216,288/
          DATA RHO/0,.7109,.7109,.7109,.7109,1.509,1.509,1.509,1.509,
         &1.509/
C** CALCULATE INTERCEPTOR SLANT RANGE **
          SR=RHO(I)*CEP
          RETURN
          END
C
          SUBROUTINE PROB(Z,CPK)
          COMMON/USER/MISS,X,Y,ZZ,DU,DUPK,MXPK,JI,L,PK
C
C**GLOSSARY**
C*      PK=PROBABILITY OF KILL
C*      CPK=CUM PROBABILITY OF KILL
C** DECLARE VARIABLES **
          REAL PK,CPK,Z
          IF(Z.LT.0.) THEN
          Z=ABS(Z)
C** CURVE FIT FOR STANDARD NORMAL CURVE **
          PK=.5*(1.+.19685*Z+.115194*Z*Z+.000344*Z*Z*Z+.019527*Z**4.)**(-4.)
          ELSE
          PK=1-.5*(1.+.196854*Z+.115194*Z*Z+.000344*Z**3.+.019527*Z**4.)
         &**(-4.)
          ENDIF
          PK=PK/10.
          CPK=CPK+PK
          RETURN
          END
```

USER Subroutines (Endoatmospheric Defense Model - Two TDUs)

```
      SUBROUTINE US(ISN,DTIM)
      COMMON/USER/MISS,X,Y,ZZ,DU1,DU2,DUPK,MXPK,JI,L,PK,JJ
      COMMON/QVAR/NDE,NFTBU(100),NREL(100),NRELP(100),NREL2(100),
     &NRUN,NRUNS,NTC(100),PARAM(100,4),TBEG,TNOW
      INTEGER JI,L,I,JJ,DU1,DU2,KK
      REAL MISS,ZZ,X,Y,NO,DUPK,MXPK,PK,RA,RE,RCPK,XX,MXPS
      REAL PIR2F,MI,PI,PERKT,SR,DENS,F,C(7),Z,CPK,RATT(5)
      REAL SOP,OP,CEP,YIELD,OPALPHA,OPBETA,NALPHA,NBETA
      DATA CPK,KK/0.,0/
      GO TO (1,2,3,4,5,6),ISN
1     DTIM=1.
      RETURN
C** ENSURES CORRECT RV TARGETING. # SHELTERS > 1/2 # RVS.
2     DTIM=0.0
      TR=REMST(22)
      IF((NTC(32).EQ.21).AND.(NTC(33).EQ.1).AND.(NTC(35).EQ.1))THEN
        CALL STAGO(22,34,TR,0,RATT)
      ELSEIF((NTC(32).EQ.21).AND.(NTC(33).EQ.1).AND.(NTC(34).EQ.1))THEN
        CALL STAGO(22,35,TR,0,RATT)
      ELSEIF((NTC(32).EQ.21).AND.(NTC(34).EQ.1).AND.(NTC(35).EQ.1))THEN
        CALL STAGO(22,33,TR,0,RATT)
      ELSEIF((NTC(32).EQ.20).AND.(NTC(33).EQ.2).AND.(NTC(35).EQ.1))THEN
        CALL STAGO(22,34,TR,0,RATT)
      ELSEIF((NTC(32).EQ.20).AND.(NTC(35).EQ.2).AND.(NTC(34).EQ.1))THEN
        CALL STAGO(22,33,TR,0,RATT)
      ELSEIF((NTC(32).EQ.20).AND.(NTC(34).EQ.2).AND.(NTC(35).EQ.1))THEN
        CALL STAGO(22,33,TR,0,RATT)
      ELSEIF((NTC(32).EQ.20).AND.(NTC(34).EQ.2).AND.(NTC(33).EQ.1))THEN
        CALL STAGO(22,35,TR,0,RATT)
      ELSEIF((NTC(32).EQ.20).AND.(NTC(35).EQ.2).AND.(NTC(33).EQ.1))THEN
        CALL STAGO(22,34,TR,0,RATT)
      ELSEIF((NTC(32).EQ.20).AND.(NTC(33).EQ.2).AND.(NTC(34).EQ.1))THEN
        CALL STAGO(22,35,TR,0,RATT)
      ELSEIF((NTC(32).EQ.19).AND.(NTC(33).EQ.2).AND.(NTC(34).EQ.2))THEN
        CALL STAGO(22,35,TR,0,RATT)
      ELSEIF((NTC(32).EQ.19).AND.(NTC(35).EQ.2).AND.(NTC(34).EQ.2))THEN
        CALL STAGO(22,33,TR,0,RATT)
      ELSEIF((NTC(32).EQ.19).AND.(NTC(33).EQ.2).AND.(NTC(35).EQ.2))THEN
        CALL STAGO(22,34,TR,0,RATT)
      ENDIF
      RETURN
3     DTIM=1.
      RETURN
C**   SEND RV TO LOADS INTERCEPTORS  **
C** IS RV TARGETED AT MPS? **
4     IF (GATRB(3).EQ.1.) THEN
      DTIM=0.
      RETURN
      ENDIF
```

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```
      DTIM=0.0
      IF((DU2.GT..5).AND.(GATRB(3).EQ.4)) GO TO 110
      IF(JI.GE.2) THEN
        IF((GATRB(3).EQ.2).OR.(GATRB(3).EQ.4)) THEN
          GO TO 110
        ENDIF
      ENDIF
C** CHECK STATUS OF DU1 **
      JI=JI+1
      IF((DU1.GT..5).AND.(DU2.GT..5)) GO TO 110
      IF ((DU1.GT..5).AND.(DU2.LT..5)) THEN
        CALL NODMOD(53,55)
        RETURN
      ENDIF
C** TWO INTERCEPTORS FIRED **
      IF((JI.GE.2).AND.(DU2.LT..5).AND.(JJ.LT.3)) THEN
C** IS THIRD RV TARGETED AT A DU? **
        CALL NODMOD(53,55)
      ENDIF
C** ALL INTERCEPTORS LAUNCHED **
      IF(JI.GE.4) THEN
        GO TO 110
      ENDIF
C** INTEREPTOR NEUTRON PK **
C**GLOSSARY**
C*    MI=MASS INTEGRAL (G/CM2)
C*    PIR2F=4*PI*R* 2 FLUENCE
C*    PERKT=NEUTRONS PER KILOTON
C*    INTALT=ALTITUDE OF INTERCEPT (KM)
C*    SR=DISTANCE BETWEN BURST AND TARGET (KM)
C*    DENS=AIR DENSITY AT INTERCEPT ALTITUDE (KG/M3)
C*    YIELD=THERMONUCLEAR YIELD (KT)
C*    NALPHA=NEUTRON PK FUNCTION ALPHA
C*    NBETA=NEUTRON PK FUNCTION BETA
C*    CEP=INTERCEPTOR CIRCULAR ERROR PROBABLE
C*    F=NEUTRON FLUENCE (NEUTRONS/CM2)
C*
C** INPUT DATA **
      CEP=600.
      PI=3.1415927
      PERKT=3.16E23
C**   4000 FT DENSITY INDICATIVE OF MX BASING AREA **
      DENS=.65312
      YIELD=5.
      DATA C/-.6775E1,.5269E-2,-.54364E-5,-.21468E-3,-.38214E1,
     &.10875E2,-.13975E1/
      NALPHA=34.539
      NBETA=2.242
C** CALCULATE SLANT RANGE USING 10 CELL MODEL **
      DO 200 I=1,10
```

```
      CALL TENCELL(SR,CEP,I)
C** CALCULATE MASS INTEGAL **
      IF(SR.LT..1) THEN
        SR=.001
      ENDIF
C** CHANGE SR TO KILOMETERS
      SR=SR*.0003048
      MI=(DENS)*SR
      MI=MI*100.
C** COMPUTE HOMOGENEOUS AIR 4PIR2 FLUENCE **
      PIR2F=EXP(C(1)+C(2)*MI+C(3)*MI**2.+C(4)*MI**1.5+C(5)*MI**.5+
     &C(6)*MI**(1./3.)+C(7)*(LOG(MI)))
C** COMPUTE FLUENCE **
      F=((PIR2F)/(4.*PI*(SR**2.))*(PERKT)*(YIELD)*(1E-10))
C** Z=NORMAL TABLE ENTRY VALUE FOR NEUTRON FLUENCE **
      Z=(LOG(F)-NALPHA)/NBETA
      CALL PROB(Z,CPK)
  200 CONTINUE
      RN=DRAND(6)
       IF(RN.LE.CPK) THEN
          CALL PATRB(0,4)
          CPK=0.
         RETURN
       ENDIF
       CPK=0.
C
C** RV KILL MECHANISM **
  110 IF(GATRB(3).EQ.3) THEN
      L=L+1
      ENDIF
C
C** RV OVERPRESSURE PK **
C**GLOSSARY**
C*    OPALPHA=OVERPRESSURE PK FUNCTION ALPHA
C*    OPBETA=OVERPRESSURE PK FUNCTION BETA
C*    SOP=SCALED OVERPRESSURE
C*    OP=OVERPRESSURE
C
C** INPUT DATA **
      CEP=1215.2
      Y=1000.
      OPALPHA=6.8755
      OPBETA=.12435
C** CALCULATE SLANT RANGE USING TEN CELL MODEL **
      DO 300 I=1,10
      CALL TENCELL(SR,CEP,I)
C** SCALE SR FOR PRESSURE AND YIELD **
      SR=SR*((12.69/14.7)**(1./3.))/((1.5*Y)**(1./3.))
C** CHECK FOR CELL 1 **
      IF (SR.LT..1) THEN
```

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```fortran
      SOP=1E5
      ELSE
C** CHANGE SR TO KILOMETERS **
      SR=SR*.0003048
      SOP=EXP(.19*(LOG(SR))**2-1.5*LOG(SR)-.1)
      ENDIF
C** SCALE OVERPRESSURE FOR ALTITUDE **
      OP=SOP*(12.69/14.7)
C** Z=NORMAL TABLE ENTRY VALUE FOR RV OVERPRESSURE **
      Z=(LOG(OP)-OPALPHA)/OPBETA
      CALL PROB(Z,CPK)
  300 CONTINUE
      ROPK=CPK
      CPK=0.
C
C** RV CRATERING PK **
C**GLOSSARY**
C*    RA=APPARENT CRATER RADIUS
C*    RE=RADIUS OF EJECTA
C*    RCPK=RV CRATER PROB OF KILL
      RA=484.54
      RE=1041.76
      RE=RE*(2./3.)
      IF((GATRB(3).EQ.2).OR.(GATRB(3).EQ.4))THEN
      RCPK=.1704
      ELSEIF(GATRB(3).EQ.3) THEN
      RCPK=.1971
      ENDIF
      PK=ROPK+(1-ROPK)*(RCPK)
C** COMPUTE CORRECT PK FOR DU SHELTER **
      IF(GATRB(3).EQ.2) THEN
      DUPK=PK
      RN=DRAND(6)
C** IS DU1 DESTROYED? **
      IF(RN.LE.DUPK) THEN
        DU1=1
        IF((DU1.GT..5).AND.(DU2.LT..5)) THEN
        CALL NODMOD(53,55)
        ENDIF
        RETURN
      ENDIF
      RETURN
      ENDIF
      IF(GATRB(3).EQ.4) THEN
        DUPK=PK
        RN=DRAND(6)
C** ID DU2 DESTROYED? **
        IF(RN.LE.DUPK) THEN
           DU2=1
           IF((DU2.GT..5).AND.(DU1.LT..5)) THEN
```

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```fortran
              CALL NODMOD(55,53)
            ENDIF
              RETURN
            ENDIF
            RETURN
          ENDIF
C** COMPUTE CORRECT PK FOR MX SHELTER **
      IF(GATRB(3).EQ.3) THEN
      IF(L.EQ.2) THEN
      MXPK=1-(1-MXPK)**2
      ELSEIF(L.EQ.1) THEN
      MXPK=PK
      ENDIF
      RETURN
      ENDIF
C** IS MX DESTROYED?  **
  5     XX=NTC(54) + NTC(56)
        DTIM=.167
        IF(NTC(19).EQ.XX)THEN
          JI=0
          JJ=0
          L=0
          RN=DRAND(6)
          IF(RN.LE.MXPK) THEN
            PRINT*,'  MX DESTROYED'
           KK=KK+1
           IF(NRUN.GE.NRUNS) THEN
            MXPS=1.-(FLOAT(KK)/FLOAT(NRUNS))
            PRINT*,'>>>MX SURVIVABILITY= ',MXPS
           ENDIF
           ELSE
            IF(NRUN.GE.NRUNS) THEN
             MXPS=1.-(FLOAT(KK)/FLOAT(NRUNS))
             PRINT*,'>>>MX SURVIVABILITY= ',MXPS
            ENDIF
          ENDIF
        ENDIF
        RETURN
C** IS RV TARGETED AT MPS? **
  6     IF(GATRB(3).EQ.1)THEN
          DTIM=0.
          RETURN
        ENDIF
        DTIM=0.0
        IF((DU1.GT..5).AND.(GATRB(3).EQ.2)) GO TO 100
        IF(DU2.GT..5) GO TO 100
C** HAVE TWO INTERCEPTORS BEEN FIRED? **
        IF(JJ.GE.2)THEN
C** IS THIRD RV TARGETED AT A DU? **
      IF((GATRB(3).EQ.2).OR.(GATRB(3).EQ.4))THEN
```

```fortran
          GO TO 100
        ENDIF
        ENDIF
        JJ=JJ+1
C** ALL INTERCEPTORS LAUNCHED **
        IF((JJ.GE.3).AND.(DU1.LE..5))THEN
          CALL NODMOD(55,53)
        ENDIF
        IF(JJ.GE.4) GO TO 100
C** NEUTRON KILL FUNCTION **
C**
        CEP=600.
        PI=3.1415927
        PERKT=3.16E23
        DENS=.65312
        YIELD=5.
        DATA C/-.6775E1,.5269E-2,-.54364E-5,-.21468E-3,-.38214E1,
      1.10875E2,-.13975E1/
        NALPHA=34.539
        NBETA=2.242
C** CALCULATE SLANT RANGE USING 10 CELL MODEL **
        DO 500 I=1,10
        CALL TENCELL(SR,CEP,I)
C** CALCULATE MASS INTEGRAL ***
        IF(SR.LT..1)THEN
          SR=.001
        ENDIF
C** CHG SR TO KM
        SR=SR*.0003048
        MI=DENS*SR
        MI=MI*100.
C** COMPUTE HOMOGENEOUS AIR 4PIR2 FLUENCE **
        PIR2F=EXP(C(1)+C(2)*MI+C(3)*MI**2.+C(4)*MI**1.5+C(5)*MI**.5+
      2C(6)*MI**(1./3.)+C(7)*(LOG(MI)))
C** COMPUTE FLUENCE **
        F=((PIR2F)/(4.*PI*(SR**2.))*(PERKT)*YIELD*1E-10)
C** Z=NORMAL TABLE ENTRY VALUE FOR NEUTRON FLUENCE **
        Z=(LOG(F)-NALPHA)/NBETA
        CALL PROB(Z,CPK)
  500   CONTINUE
        RN=DRAND(6)
        IF(RN.LE.CPK)THEN
          CALL PATRB(0,4)
          CPK=0.
        RETURN
        ENDIF
        CPK=0.
C** RV KILL MECHANISM **
  100   IF(GATRB(3).EQ.3) THEN
          L=L+1
        ENDIF
```

```
C** RV OVERPRESSURE PK **
      CEP=1215.2
      Y=1000.
      OPALPHA=6.8755
      OPBETA=.12435
C** CALCULATE SLANT RNG USING 10 CELL MODEL **
      DO 600 I=1,10
      CALL TENCELL(SR,CEP,I)
C** SCALE SR FOR PRESSURE AND YIELD **
      SR=SR*((12.69/14.7)**(1./3.))/((1.5*Y)**(1./3.))
C** CK FOR CELL ONE **
      IF(SR.LT..1)THEN
        SOP=1E5
      ELSE
C** CHG SR TO KILOMETERS **
      SR=SR*.0003048
      SOP=EXP(.19*(LOG(SR))**2.-1.5*LOG(SR)-.1)
      ENDIF
C** SCALE OVERPRESSURE FOR ALTITUDE **
      OP=SOP*(12.69/14.7)
      Z=(LOG(OP)-OPALPHA)/OPBETA
      CALL PROB(Z,CPK)
  600 CONTINUE
      ROPK=CPK
      CPK=0.
C
C** RV CRATERING PK **
      RA=484.54
      RE=1041.76
      RE=RE*(2./3.)
      IF((GATRB(3).EQ.2).OR.(GATRB(3).EQ.4)) THEN
        RCPK=.1704
      ELSEIF(GATRB(3).EQ.3)THEN
        RCPK=.1971
      ENDIF
      PK=ROPK+(1-ROPK)*RCPK
C** COMPUTE CORRECT PK FOR DU SHELTER **
      IF(GATRB(3).EQ.2)THEN
      DUPK=PK
      RN=DRAND(6)
C** IS DU1 DESTROYED? **
      IF(RN.LE.DUPK)THEN
        DU1=1
        RETURN
      ENDIF
      RETURN
      ENDIF
      IF(GATRB(3).EQ.4) THEN
        DUPK=PK
       RN=DRAND(6)
```

```fortran
C** IS DU2 DESTROYED? **
      IF(RN.LE.DUPK)THEN
         DU2=1
         IF((DU2.GT..5).AND.(DU1.LT..5)) THEN
          CALL NODMOD(55,53)
         ENDIF
          RETURN
      ENDIF
      RETURN
      ENDIF
C** COMPUTE CORRECT PK FOR MX SHELTER **
      IF(GATRB(3).EQ.3)THEN
        IF(L.EQ.2)THEN
           MXPK=1-(1-MXPK)**2
        ELSEIF(L.EQ.1)THEN
           MXPK=PK
        ENDIF
        RETURN
      ENDIF
      END
      SUBROUTINE UI
      COMMON/USER/MISS,X,Y,ZZ,DU1,DU2,DUPK,MXPK,JI,L,PK,JJ
      INTEGER DU,JI,L,JJ
      REAL MISS,X,Y,ZZ,DUPK,MXPK,PK,CPK,Z
      MISS=0.
      X=Y=ZZ=0.
      DU1=DU2=0
      DUPK=0.
      MXPK=0.
      JI=JJ=L=0
      PK=0.
      CPK=0.
      Z=0.
      RETURN
      END
      SUBROUTINE TENCELL (SR,CEP,I)
C
      COMMON/USER/MISS,X,Y,ZZ,DU1,DU2,DUPK,MXPK,JI,L,PK,JJ
C
C**GLOSSARY**
C*    THETA(I)=REFERENCE ANGLE TO RADII TO CELL CENTERS MEASURED
C*    FROM MAJOR AXIS COUNTER CLOCKWISE
C*    RHO(I)=CELL RADII (RHO/CEP)
C
C**DECLARE VARIABLES
      INTEGER I
      REAL THETA(10),RHO(10),CEP,SR
C** DATA INPUT **
      DATA THETA/0,45,135,225,315,0,72,144,216,288/
      DATA RHO/0,.7109,.7109,.7109,.7109,1.509,1.509,1.509,1.509,
     &1.509/
```

```
C** CALCULATE INTERCEPTOR SLANT RANGE **
      SR=RHO(I)*CEP
      RETURN
      END
C
      SUBROUTINE PROB(Z,CPK)
      COMMON/USER/MISS,X,Y,ZZ,DU1,DU2,DUPK,MXPK,JI,L,PK,JJ
C
C**GLOSSARY**
C*    PK=PROBABILITY OF KILL
C*    CPK=CUM PROBABILITY OF KILL
C** DECLARE VARIABLES **
      REAL PK,CPK,Z
      IF(Z.LT.0.) THEN
      Z=ABS(Z)
C** CURVE FIT FOR STANDARD NORMAL CURVE **
      PK=.5*(1.+.19685*Z+.115194*Z*Z+.000344*Z*Z*Z+.019527*Z**4.)**(-4.)
      ELSE
      PK=1-.5*(1.+.196854*Z+.115194*Z*Z+.000344*Z**3.+.019527*Z**4.)
     1**(-4.)
      ENDIF
      PK=PK/10.
      CPK=CPK+PK
      RETURN
      END
```
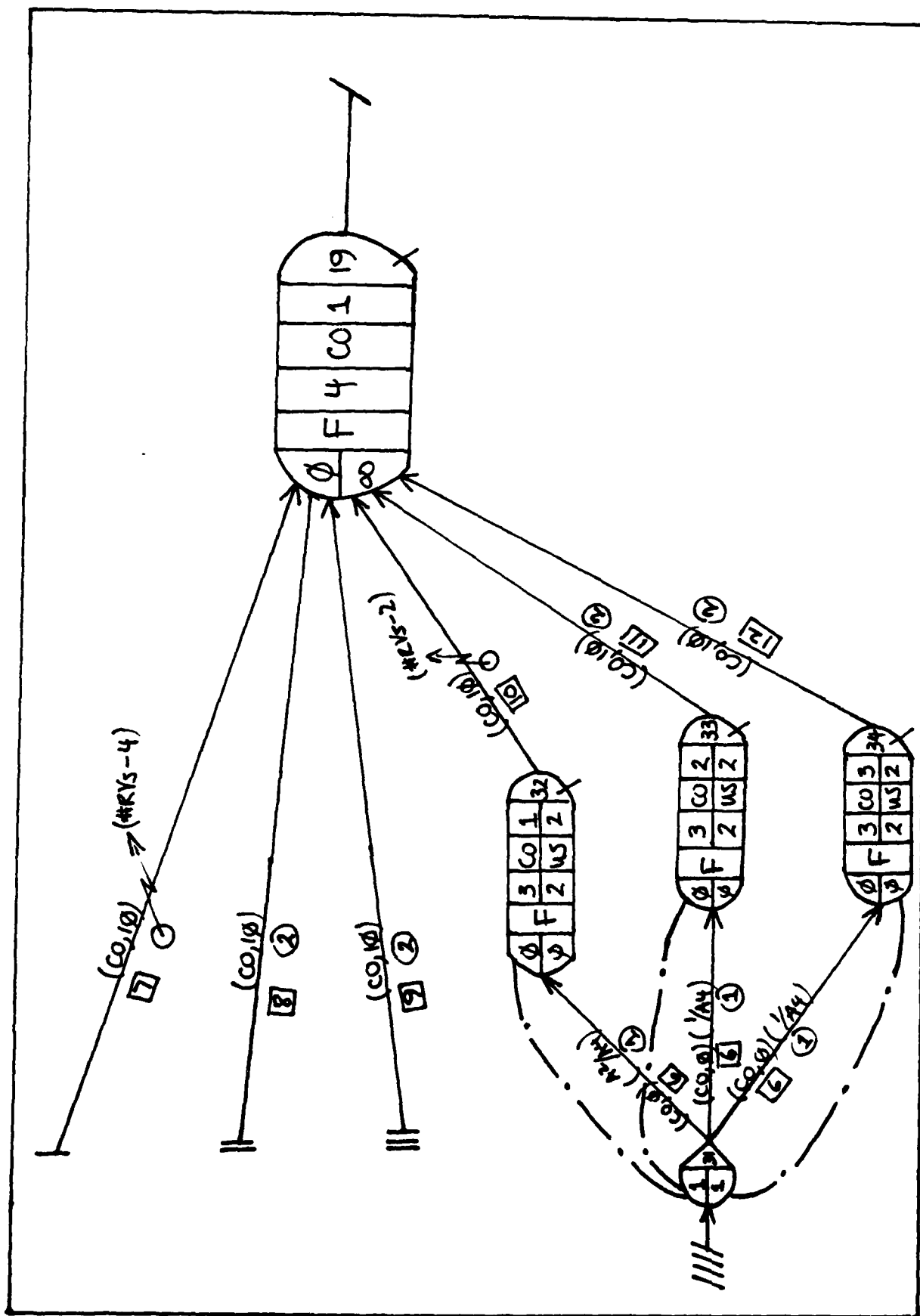
Figure 15-1. Q-GERT Network (Layered Defense)
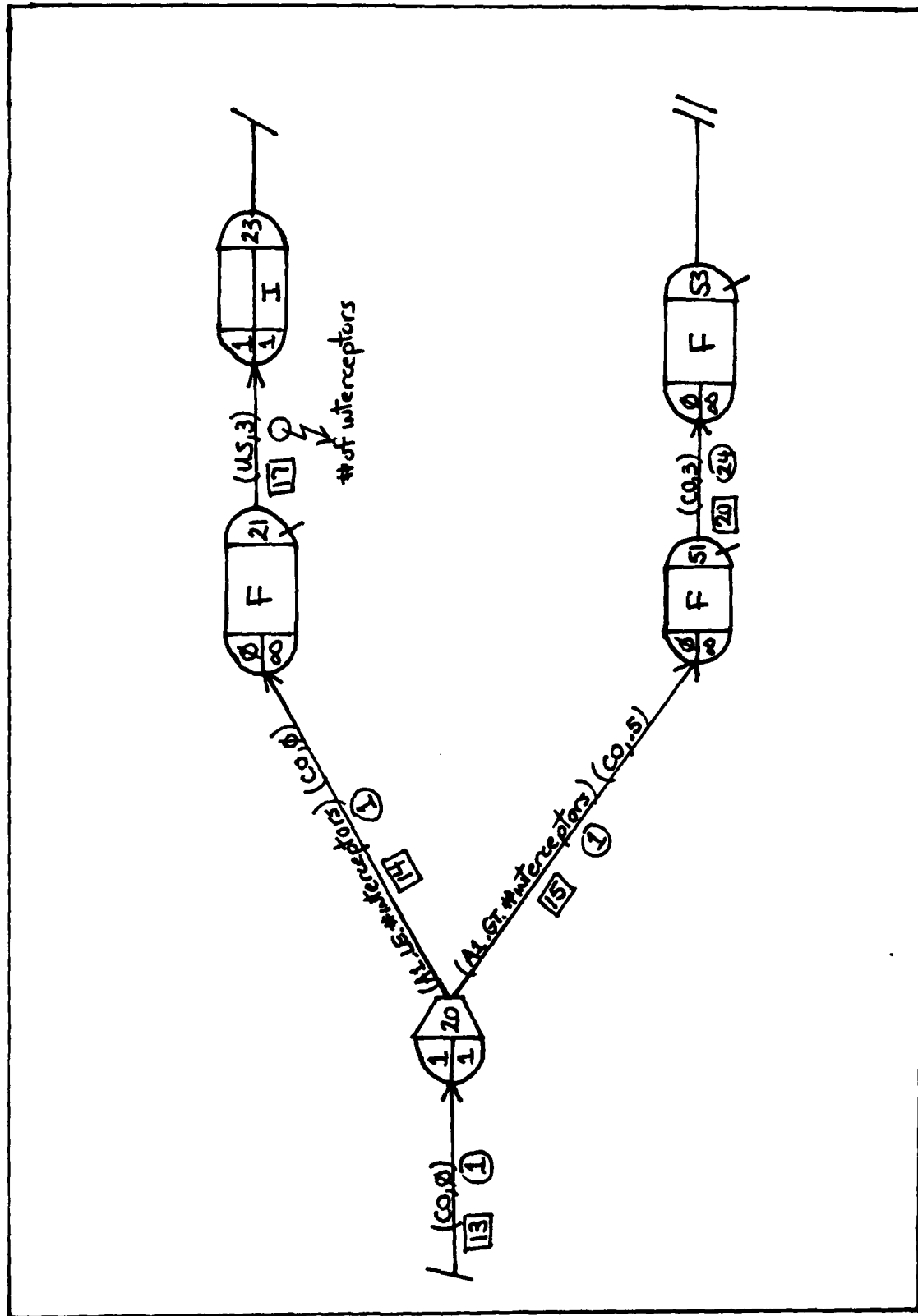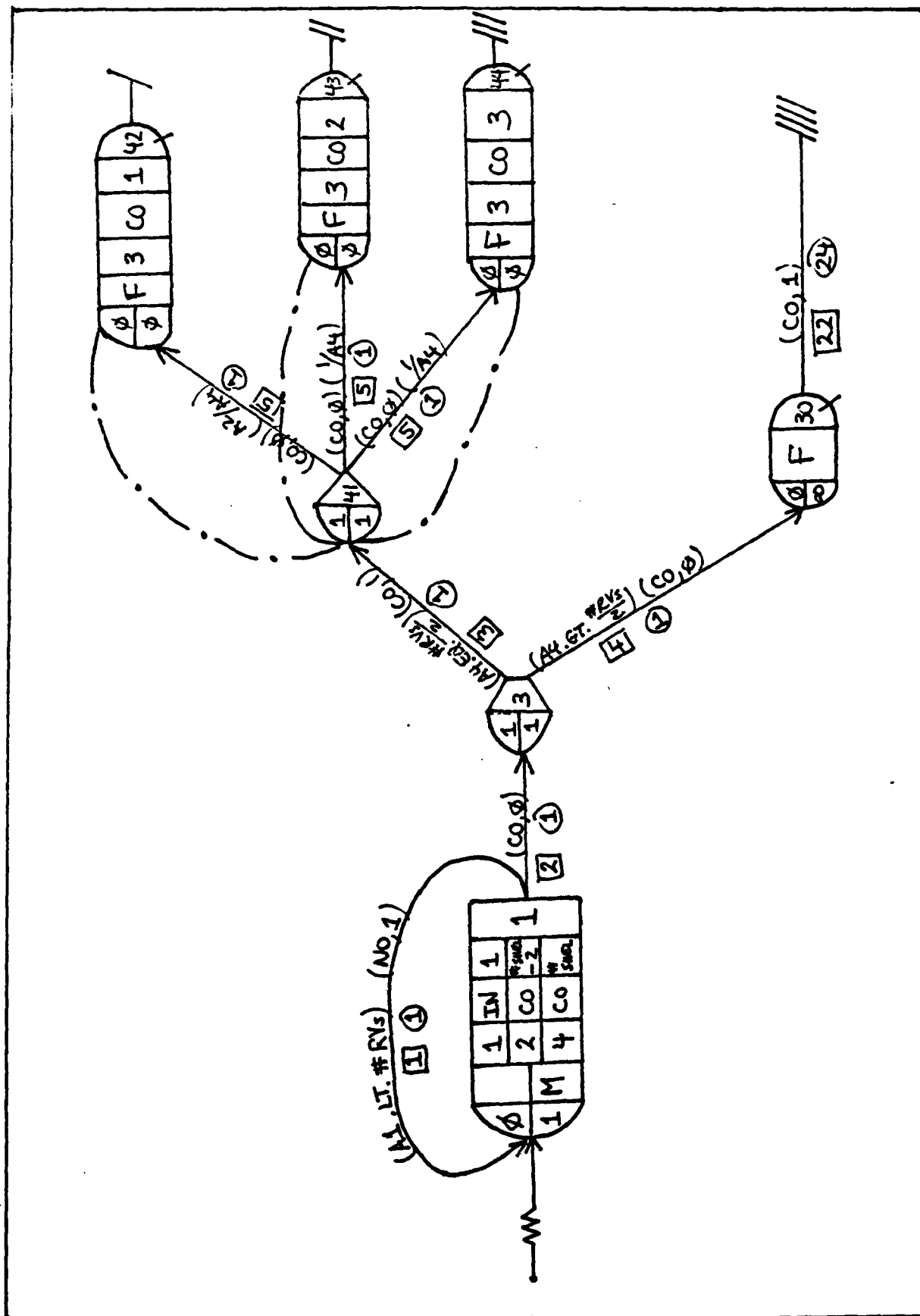
Figure 15-2. Q-GERT Network (Layered Defense)

Figure 15-3. Q-GERT Network (Layered Defense)

Figure 15-4. Q-GERT Network (Layered Defense)
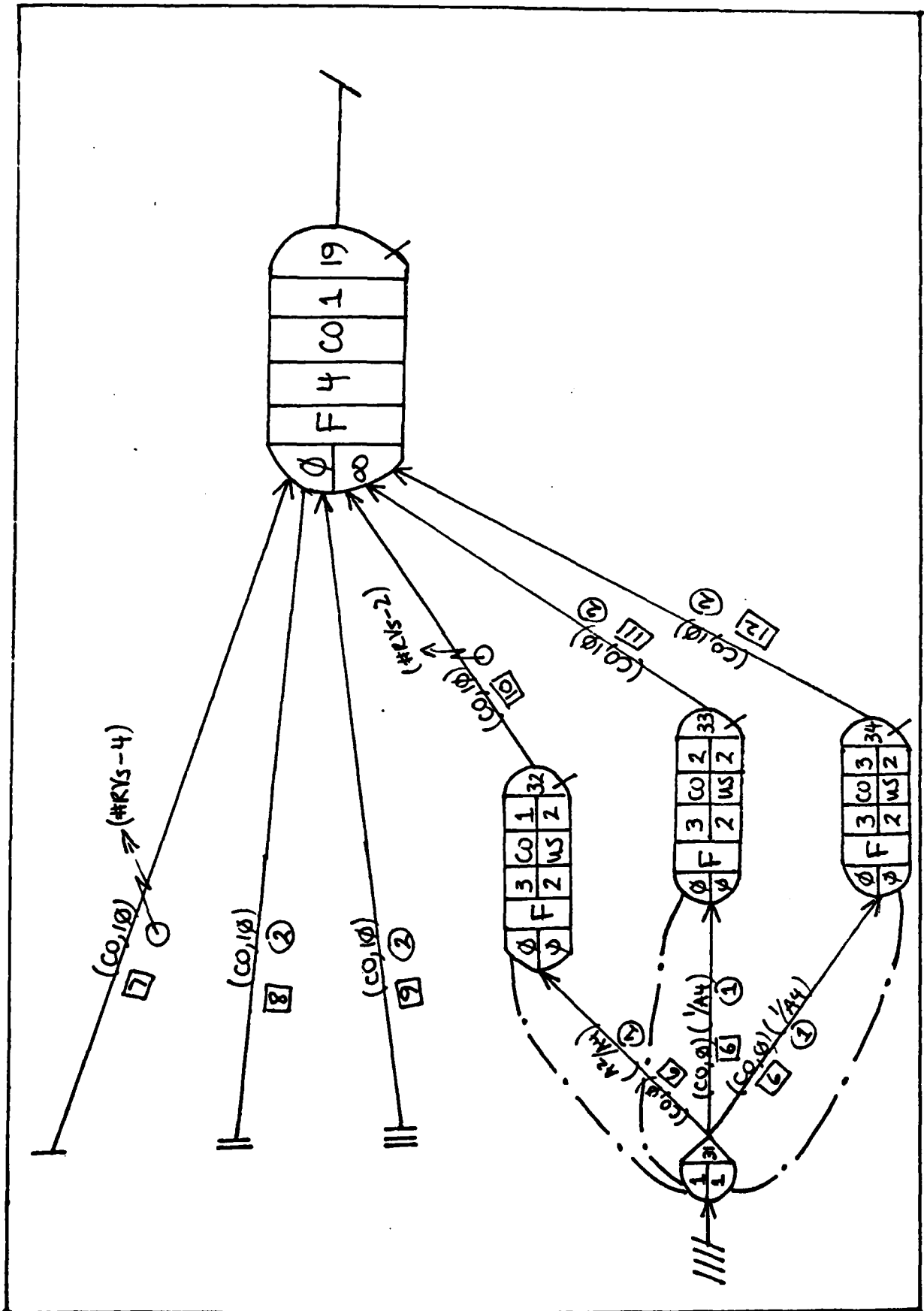
Figure 16-1. Q-GERT Network (One TDU Defense)

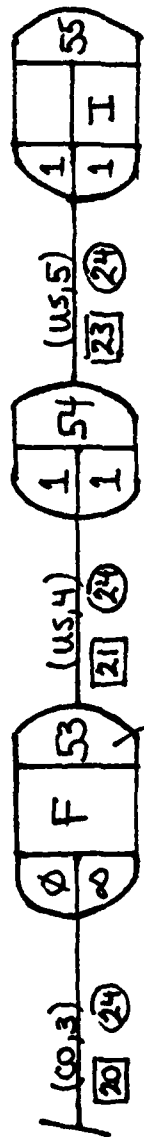Figure 16-2. Q-GERT Network (One TDU Defense)

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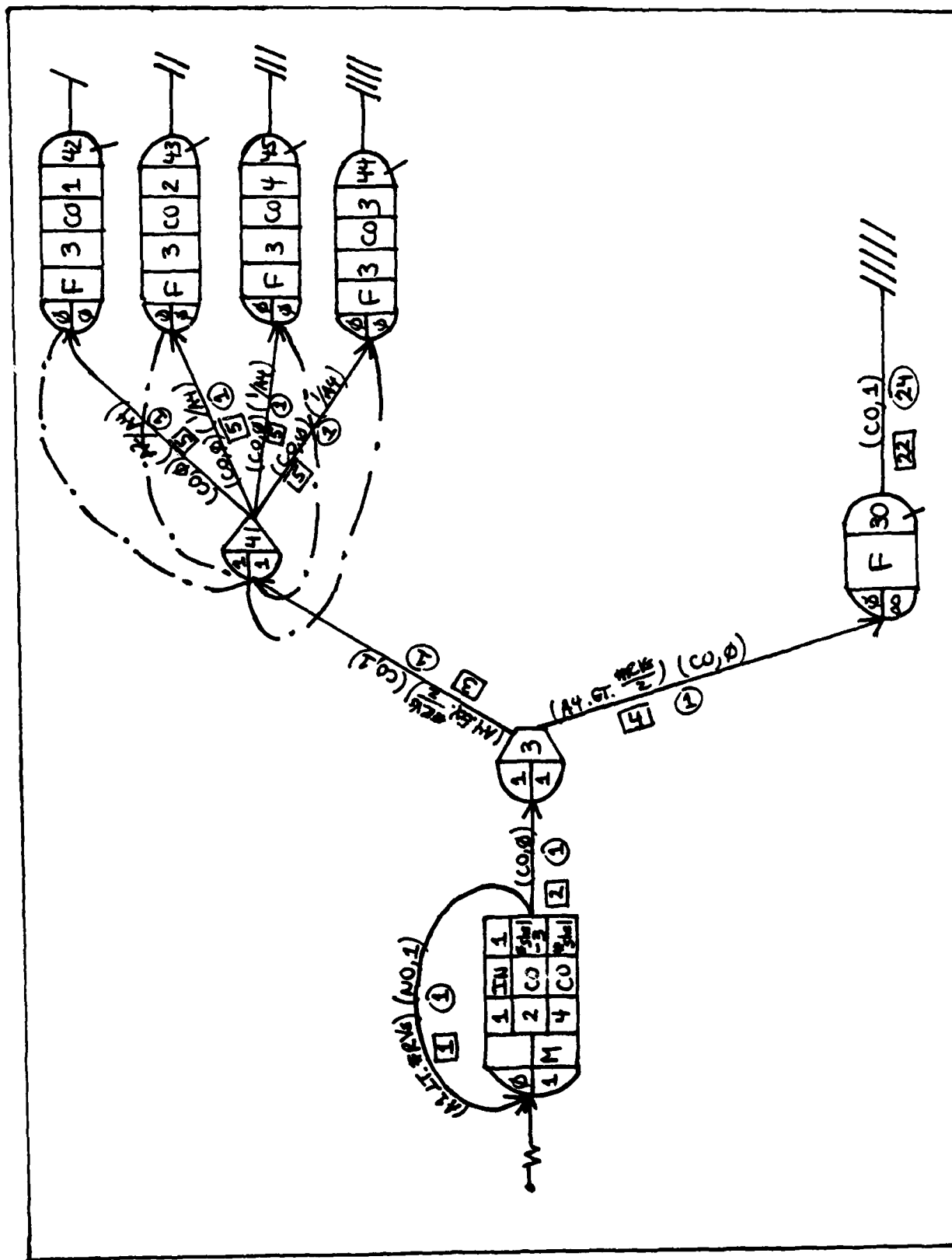
Figure 16-3. Q-GERT Network (One TDU Defense)

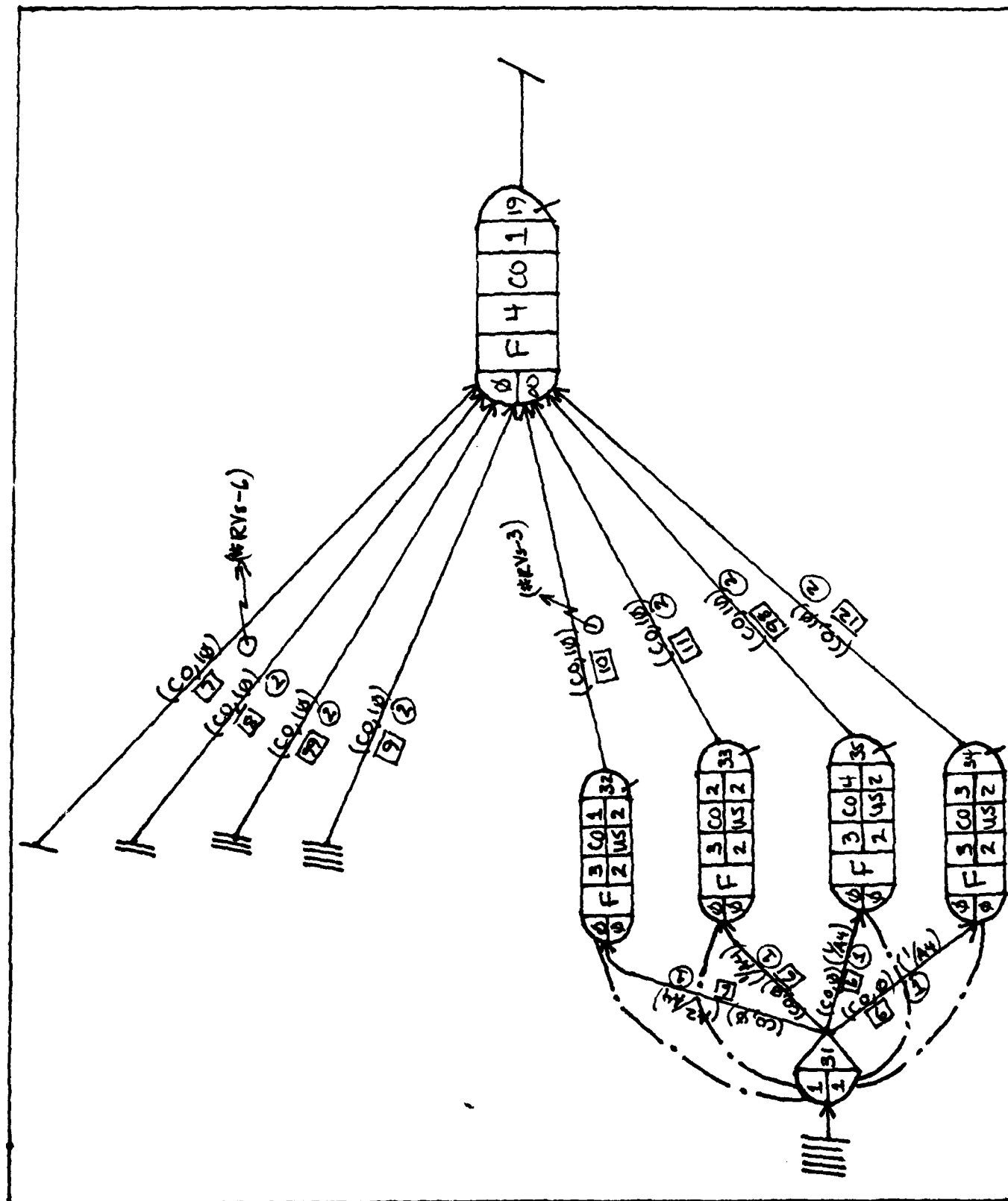Figure 17-1. Q-GERT Network (Two TDU Defense)
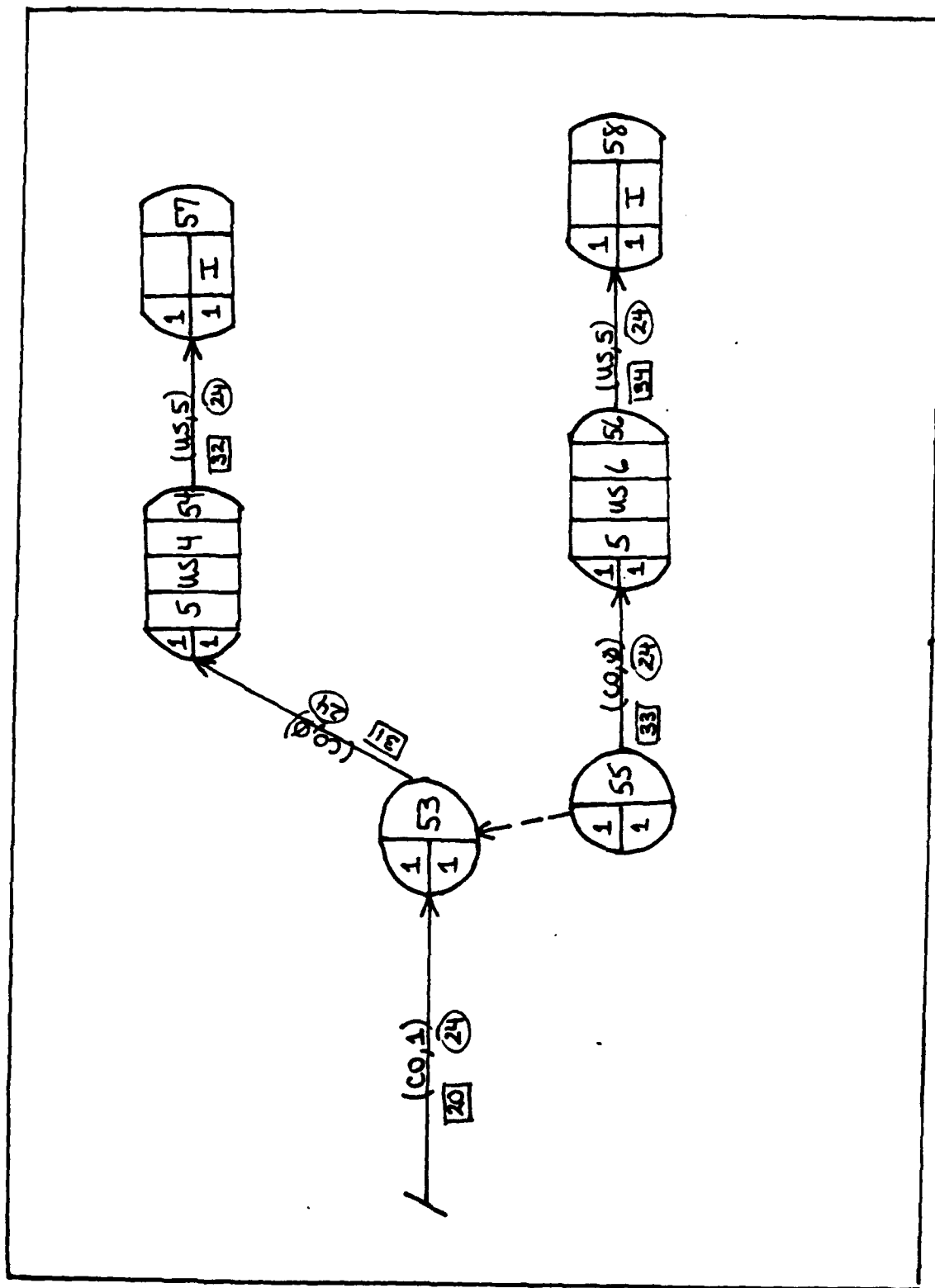
Figure 17-2. Q-GERT Network (Two TDU Defense)

Figure 17-3. Q-GERT Network (Two TDU Defense)

Model Modifications

Layered and One TDU Defense Models. If the number of shelters is greater than or equal to the number of attacking RVs, then the number of servers for activities 10, 11, and 12 must be changed to the number of shelters minus two, one, and one, respectively. When the number of attacking RVs is one more than the number of shelters, the number of servers should be modified as stated above and user subroutine two (US2) should be changed to the following:

```
2      DTIM=0.
       TR=REMST(22)
       IF(GATRB(4).EQ.23) THEN
           IF((NTC(32).EQ.21).AND.(NTC(33).EQ.1).AND.(NTC(34).EQ.1))THEN
               NREL2(32)=1
               NREL2(33)=1
               NREL2(34)=1
           ENDIF
       ENDIF
       RETURN
```

Two TDU Defense Models. If the number of shelters is greater than or equal to the number of attacking RVs, then the number of servers for activities 10, 11, 98, and 12 must be changed to the number of shelters minus three, one, one, and one, respectively. When the number of attacking RVs is one or two more than the number of shelters, the number of servers should be modified as stated above and user subroutine two (US2) should be changed to the following:

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```
2      DTIM=0.0
       TR=REMST(22)
       IF(GATRB(4).EQ.14)THEN
          IF((NTC(32).EQ.11).AND.(NTC(33).EQ.1).AND.(NTC(34).EQ.1).AND.
     &    (NTC(35).EQ.1))THEN
             NREL2(32)=1
             NREL2(33)=1
             NREL2(34)=1
             NREL2(35)=1
          ENDIF
       ELSEIF(GATRB(4).EQ.15)THEN
          IF((NTC(32).EQ.12).AND.(NTC(33).EQ.1).AND.(NTC(34).EQ.1).AND.
     &    (NTC(35).EQ.1))THEN
             NREL2(32)=1
             NREL2(33)=1
             NREL2(34)=1
             NREL2(35)=1
          ENDIF
       ENDIF
       RETURN
```

Q-GERT Network Documentation

Layered Defense Model.

(1) <u>RV Generation (Node 1)</u> - Generates the required
number of RVs using attribute 1 as a counter. These RVs
arrive normally distributed in time with a mean of five seconds
and a standard deviation of two seconds.

(2) <u>Attribute Assignment (Node 1)</u> - Attributes 2 and 4
are set equal to the number of shelters minus two and the
number of shelters, respectively. Both attributes are used
for probabilistic branching in the model.

(3) <u>Take First Branching (Node 3)</u> - If the number of
RVs is twice the number of shelters, the RVs branch to node
41 for random assignment to a shelter. If the number of RVs
is less than twice the number of shelters, the RVs branch

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to node 30 to perform an equivalent procedure. The number
of RVs will never be greater than twice the number of shelters
due to the fratracide limit previously mentioned.

(4) <u>RV Target Assignment (RVs Twice the Number of</u>
<u>Shelters</u> - At node 41, the RVs are probabilistically targeted
(using attributes 2 and 4) to either an empty MX shelter (node
42), the terminal defense unit (node 43; located in one of the
shelters), or the actual MX (node 44), and are assigned an
attribute 3 of 1, 2, or 3, respectively. The number of servers
and queue capacities are restricted and balkers are returned
to node 41 to insure that two RVs are targeted to each shelter.
From here, all RVs are sent to the interceptor queue (node 19)
with a flight time delay of ten minutes.

(5) <u>RV Target Assignment (RVs Less Than Twice the Number</u>
<u>of Shelters)</u> - At node 30, the RVs are sent to node 31 with a
time delay of one minute. At node 31, the RVs are probabilisti-
cally targeted (using attributes 2 and 4) to either an empty MX
shelter (node 32), a terminal defense unit (node 33), or the
actual MX (node 34), and assigned an attribute 3 of 1, 2, or 3,
respectively. When an RV is assigned to a target, it encounters
user subroutine two (US2) which insures that a minimum of one
RV is targeted at each shelter. The number of servers and
queue capacities are restricted and balkers are sent back to
node 31 which insures that all protective shelters are targeted.
After this procedure, all RVs are routed to the interceptor
queue (node 19) with a flight time delay equal to ten minutes.

(6) <u>Set RV Status</u> - At queue node 19, attribute 4 is
reset equal to one to indicate that all RVs are active.

(7) <u>Select RVs to be Intercepted by Exoatmospheric Layer</u> -
At node 20, if an RV's attribute 1 is less than or equal to
the number of interceptors available, the RV will be sent to
queue node 21 to await interaction with an exoatmospheric
interceptor. Although the RVs are intercepted first-come-
first-serve, the exoatmospheric defense does not know the RV's
intended target since the targets were randomly assigned in
steps 4 and 5 above. If an RV does not encounter an exoatmos-
pheric interceptor, the RV will penetrate this layer of defense
and await interception by the endoatmospheric layer which
consists of one terminal defense unit (TDU).

(8) <u>RV/Exoatmospheric Interceptor Engagement</u> - Each RV
in queue node 21 is serviced by an exoatmospheric interceptor
in accordance with user subroutine three (US3), which deter-
mines if the RV is destroyed using a CEP distribution coupled
with a "cookie-cutter" weapon radius as mentioned in the
structural model.

(9) <u>Determine RV Status</u> - After being serviced by the
exoatmospheric layer, each RV is examined by user subroutine
one (US1) which routes those RVs not destroyed to node 51 to
join those RVs which penetrated the exoatmospheric defense and
are awaiting interaction with the TDU. From node 51, all RVs
are sent to the TDU queue (node 53) with a flight time delay
of three minutes.

(10) RV/Endoatmospheric Interceptor Engagement - Each RV in queue node 53 is serviced by an endoatmospheric interceptor in accordance with user subroutine four (US4) which employs the strategy stated in the structural model. US4 determines if an RV has destroyed the TDU and the RV's probability of killing (PK) the MX. If the TDU is destroyed, the MX cannot be defended by this layer of defense.

(11) MX Status - After all of the RVs have been serviced by the defense, user subroutine five (US5) compares the probability of killing the MX to a random number to determine if the MX is destroyed. US5 computes the number of times the MX is destroyed in N simulation runs and prints out MX survivability.

Endoatmospheric Defense Model (One TDU).

(1) through (6) - Same as the layered defense model.

(7) Flight Time - All Rvs in queue node 19 are sent to queue node 53 with a flight time delay of three minutes.

(8) The remaining steps are the same as steps (10) through (11) of the layered defense model.

Endoatmospheric Defense Model (Two TDUs).

(1) Same as the layered defense model.

(2) Attribute Assignment (Node 1) - Attributes 2 and 4 are set equal to the number of shelters minus three and the number of shelters, respectively. Both attributes are used for probabilistic branching in the model.

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(3) Same as the layered defense model.

(4) RV Target Assignment (RVs Twice the Number of Shelters - At node 41, the RVs are probabilistically targeted (using attributes 2 and 4) to either an empty MX shelter (node 42), the primary TDU (node 43), the backup TDU (node 45), or the actual MX (node 44), and are assigned an attribute 3 of 1, 2, 4, or 3, respectively. The number of servers and queue capacities are restricted and balkers are returned to node 41 to insure that two RVs are targeted to each shelter. From here, all RVs are sent to the interceptor queue (node 19) with a flight time delay of ten minutes.

(5) RV Target Assignment (RVs Less Than Twice the Number of Shelters - At node 30, the RVs are sent to node 31 with a time delay of one minute. At node 31, the RVs are probabilistically targeted (using attributes 2 and 4) to either an empty MX shelter (node 32), the primary TDU (node 33), the backup TDU (node 35), or the actual MX (node 34), and are assigned an attribute 3 of 1, 2, 4, or 3, respectively. When an RV is assigned to a target, it encounters US2 which insures that a minimum of one RV is targeted at each shelter. The number of servers and queue capacities are restricted and balkers are sent back to node 31 which insures that all protective shelters are targeted. After this procedure, all RVs are routed to the interception queue (node 19) with a flight time delay equal to ten minutes.

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(6) Same as the layered defense model.

(7) <u>Flight Time</u> - All RVs in queue node 19 are sent to node 53 with a flight time delay of three minutes.

(8) <u>RV/Endoatmospheric Interceptor Engagement</u> - Each RV in node 53 is routed to node 54 with no time delay. Each RV in node 53 is serviced by the primary TDU according to US4 which utilizes the strategy outlined in the simulation section of this study. If the primary TDU is destroyed or has launched two of its three interceptors, node 53 is replaced by node 55, and the backup TDU assumes the defense role if it has not been destroyed. If both TDUs are destroyed, the MX cannot be defended by this defense structure. The backup TDU services each RV according to user subroutine six (US6). If the backup TDU is destroyed or has launched all three of its interceptors, node 55 is replaced by node 53 and the primary TDU reassumes the defense role if it has not been destroyed.

(9) Same as step (11) of the layered defense model.

Appendix H

List of Acronyms

ABM	-	Antiballistic Missile
ASAT	-	Antisatellite
BMD	-	Ballistic Missile Defense
CEP	-	Circular Error Probable
ICBM	-	Intercontinental Ballistic Missile
IR	-	Infrared
ISS	-	Intensity Sure-Safe
ISK	-	Intensity Sure-Kill
KT	-	Kilotons
LoAD	-	Low Altitude Defense
MPS	-	Multiple Protective Shelters
MX	-	Missile X
MXS	-	MX Survivability
N/CM^2	-	Neutrons per square centimeter
PK	-	Probability of Kill
PSI	-	Pounds per square inch
RV	-	Reentry Vehicle
TDU	-	Terminal Defense Unit
WR	-	Weapon Radius

Vita

Ellsworth Francis Rettammel was born on 4 October 1947 in Mauston, Wisconsin. He graduated from high school in New Lisbon, Wisconsin in 1965 and attended the University of Wisconsin - Whitewater, from which he received the degree of Bachelor of Science in Physics and Mathematics in 1969. He received a commission in the USAF through Officer's Training School in February 1970. He was married to Rose Mary in Houston, Texas before attending Undergraduate Navigator Training at Mather AFB. He received his navigator wings in December 1970 and remained at Mather AFB to specialize in Electronic Warfare Training. After attending B-52 combat crew training school, he served as a B-52 Electronic Warfare Officer (EWO) and instructor EWO in the 77th Bomb Squadron, Ellsworth AFB, South Dakota. Half of his three years at Ellsworth AFB were spent on temporary duty (TDY) to Guam where he accumulated over 900 combat hours in Southeast Asia. In 1975 he was reassigned to the 55th Strategic Reconnaissance Wing, where he served as an RC-135 EWO, Instructor EWO, and Wing Standardization Evaluator. In 1978 he transferred to the Headquarters Strategic Air Command (SAC) staff and spent two years as an action officer and branch chief HQ SAC/INC. He entered the School of Engineering, Air Force Institute of Technology, in August 1980.

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MX Survivability	Terminal Defense
Ballistic Missile Defense	Multiple Protective Shelters
Layered Defense	Cost Effectiveness

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This thesis investigates MX survivability when a layered or terminal defense system is deployed with various numbers of multiple protective shelters (MPS). The layered defense system defends the MX with an exoatmospheric layer which is augmented by an endoatmospheric layer. The exoatmospheric layer protects the MX with long-wave infrared (LWIR) guided interceptors which must directly impact an incoming RV at approximately 300,000 feet
(Continued on Reverse)

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BLOCK 20: ABSTRACT (Cont'd)

altitude to destroy it. The endoatmospheric layer consists of
a terminal BMD system known as Low Altitude Defense (LoAD) which
defends the MX with three hypersonic, nuclear armed interceptor
missiles. The terminal defense system consists of either one or
two LoAD systems. This research effort determines the most cost
effective defense system, and draws conclusions on these systems
based upon quantitative and qualitative (ex., political) con-
siderations.