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THE INFLUENCE OF COMPRESSIBILITY ON THE EQUATIONS OF THE RAPID DISTORTION THEORY OF TURBULENCE

by

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SUMMARY

A mathematical theory of the effect of rapid distortion on turbulence level, originally developed by Batchelor[1] for incompressible flow, has been extended to account for compressibility. Applied to a wind-tunnel contraction cone, it shows that the effectiveness of the contraction in reducing the turbulence level and non-uniformities in the mean flow is enhanced as the outlet Mach number is increased.

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1 INTRODUCTION

It is well known that a sudden change in the cross-sectional area of a stream of fluid has a substantial effect on the turbulent kinetic energy as well as on the distribution of mean velocity. This fact is used in the design of wind tunnels, in that the working section is generally preceded by a contraction cone in order to obtain a near-uniform flow with a low level of turbulence. The turbulence intensity is not in fact reduced. Rather, it increases through the contraction, but the acceleration of the mean flow is correspondingly greater and the relative turbulence is therefore decreased.

A mathematical theory of such rapid distortions has been developed. Taylor[2] showed how the problem may be linearised and looked at the effect of a contraction on a sinusoidal disturbance, taken to represent one component of the turbulent motion. Batchelor[1] and Batchelor and Proudman[3] then examined the whole spectrum of these disturbances, effectively summing over all contributions, and looked at arbitrary as well as symmetric contractions.

However, the effect of compressibility on the reduction of the level of turbulence appears not to have been looked at before and is considered in this Report. It emerges that its effect is beneficial. As the Mach number of the flow in the working section increases, the contraction ratio needed to achieve a given reduction in the relative turbulence, decreases. It will be shown that the major effect of compressibility is on the acceleration of the mean flow. In contrast, there is only a slight increase in the turbulent kinetic energy.

2 EXTENSION OF THE INCOMPRESSIBLE THEORY

For an incompressible flow, Batchelor[1] calculates the effect of rapid distortion on turbulent kinetic energy by linearising the equations of motion and relating the vorticities before and after the contraction. The same procedure may be repeated for flows in which compressibility must be accounted for.

Consider then a turbulent, compressible fluid, in steady, uniform mean motion, suddenly deformed. Define ρ and $\underline{u}$ to be the mean density and velocity, $\underline{x}$ to be the position of a general element of fluid and ρ' and $\underline{u}'$ to be the turbulent fluctuations in density and velocity. The implication of the assumption that $|\underline{u}'|/|\underline{u}|$ is small, which allows linearisation of the problem is broadly this: the distortion is so rapid that it alone affects the physical attributes of a particle of fluid, such as velocity, density and its relative displacement from any other particle.

More rigorously, it is assumed that the decay of turbulent energy occurs on a much greater time scale than that of the distortion itself. Thus, the convective terms in the turbulent kinetic energy equation will be dominant. Now, the rate of decay of turbulent energy is proportional to $|\underline{u}'|^3/\ell$, (Batchelor[1]), where ℓ is the length scale of the turbulent fluctuations and the time scale for decay is therefore $\ell/|\underline{u}'|$, while the time scale of the distortion is of order $D/|\underline{u}|$, where D is the length of the distortion. The condition under which rapid distortion theory may be applied is therefore

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$$\frac{|\underline{u}'|}{|\underline{u}|} \ll \frac{\ell}{D} \quad . \qquad (2\text{-}1)$$

In general, ℓ is small, so that the distortion must indeed be rapid, if the linearisation shown below is to be valid.

When (2-1) holds, there is insufficient time for viscous dissipation to affect the turbulent velocity and Cauchy's equation can be used to link the vorticity fluctuations before and after distortion. This equation, given by Lamb[4], is

$$\underline{\omega}''(\underline{x}) = \sigma S_{ij}\underline{\omega}'(\underline{a}) \qquad (2\text{-}2)$$

with

$$\sigma = \frac{\rho_{\underline{x}}}{\rho_{\underline{a}}} \qquad (2\text{-}3)$$

and

$$S_{ij} = \frac{\partial x_i}{\partial a_j} \qquad (2\text{-}4)$$

where the mean density of the fluid is $\rho_{\underline{x}}$ after distortion and $\rho_{\underline{a}}$ before distortion, while $\underline{x}$ and $\underline{\omega}''$ are the position and vorticity of an element of fluid that was originally at $\underline{a}$, with vorticity $\underline{\omega}'$. This analysis does not consider the way in which vorticity changes during the contraction and assumes uniform mean flow before and after distortion.

At the same time, because the Mach number of the turbulent fluctuations is small, the turbulent part of the flow is effectively incompressible and conservation of mass implies that

$$\rho_{\underline{a}}\underline{\nabla} \cdot \underline{u}' = 0 = \rho_{\underline{x}}\underline{\nabla} \cdot \underline{u}'' \quad . \qquad (2\text{-}5)$$

The derivation of this equation is shown in Appendix A.

Also, to first order in $|\underline{u}'|$, the distortion tensor, S_{ij} is related only to the mean velocity and is therefore uniform. If the principle axes of strain are used S_{ij} becomes a diagonal matrix. Analysis of the problem is therefore simplified and the continuity equation for the mean flow becomes, in Lagrangian form

$$e_1 e_2 e_3 = \frac{1}{\sigma} \qquad (2\text{-}6)$$

where $e_i = S_{ii}$.

Of course, the arguments presented above give a highly idealised description of the type of distortion that would be of interest in practice, since other effects will generally be present. For instance, if the distortion is caused by a wind-tunnel contraction cone, there will be enough time during the contraction for energy exchange between components of turbulence. The decay of turbulence must also be expected to be significant, while boundary layers forming on the tunnel walls affect the mean velocity distribution and so the distortion tensor will not be strictly uniform. The work which follows should therefore be thought of as being strictly valid only in the limit as such secondary effects vanish. It is nevertheless useful, because it gives a good indication of the qualitative effect of the contraction on the level of turbulence.

3 ANALYSIS FOR A GENERAL DISTORTION

Equations (2-2) to (2-6) form a basis for the following analysis. For the sake of generality, arbitrary values of e_1, e_2 and e_3 are assumed at first. Specific cases of interest will be considered in section 4.

In order to represent the properties of turbulence, which is a random phenomenon, Batchelor's approach is followed and Fourier resolutions of the velocity fluctuations $\underline{u}'$ and $\underline{u}''$ are made, thus:

$$\underline{u}'(\underline{a}) = \int e^{i\underline{\kappa}\cdot\underline{a}} d\underline{z}'(\underline{\kappa}) \ , \qquad \underline{u}''(\underline{x}) = \int e^{i\underline{\chi}\cdot\underline{x}} d\underline{z}''(\chi) \ , \tag{3-1}$$

where integration is carried out over all wave number space. The functions $d\underline{z}$ (indices ' and " will be dropped unless it is necessary to distinguish between quantities before and after distortion) are statistically random and provide a measure of the strength of the turbulent fluctuation at a particular wave number. Because $\underline{z}(\underline{\kappa})$ is not in general bounded, the integral is of the Fourier-Stieltjes type. Further discussion of the properties of $\underline{z}$ can be found in Batchelor[1].

The wave number vectors $\underline{\kappa}$ and $\underline{\chi}$ are related via the distortion tensor S_{ij} so that

$$\chi_i = \frac{\kappa_i}{e_i} \qquad i = 1,2,3, \tag{3-2}$$

while, from (2-6) the differential volumes $d\kappa_1$, $d\kappa_2$, $d\kappa_3$ and $d\chi_1$, $d\chi_2$, $d\chi_3$ are related by the equation

$$\sigma d\kappa_1 d\kappa_2 d\kappa_3 = d\chi_1 d\chi_2 d\chi_3 \ . \tag{3-3}$$

A relationship between $d\underline{z}'$ and $d\underline{z}''$ may also be found on substitution of (3-1) into the continuity equation (2-5) which yields

$$\underline{\kappa} \cdot d\underline{z}' = 0 = \underline{\chi} \cdot d\underline{z}'' \ . \tag{3-4}$$

The energy spectrum tensor $\phi_{ij}(\underline{\kappa})$ is defined by

$$\overline{u_i(\underline{x})u_j(\underline{x} + \underline{r})} = \int_{\kappa} \phi_{ij}(\underline{\kappa}) e^{i\underline{\kappa}\cdot\underline{r}} d\kappa_1 d\kappa_2 d\kappa_3 \tag{3-5}$$

where $\int_{\kappa} d\kappa_1 d\kappa_2 d\kappa_3$ is taken over all wave space. Batchelor[1] shows that this definition is equivalent to

$$\phi_{ij}(\underline{\kappa}) = \lim_{d\kappa_1 d\kappa_2 d\kappa_3 \to 0} \left(\frac{dz_i^* dz_j}{d\kappa_1 d\kappa_2 d\kappa_3} \right) \tag{3-6}$$

where dz_i^* is the complex conjugate of dz_i .

Now equation (2-5) implies that, with the summation convention used for repeated indices,

$$\frac{\partial u_j(\underline{x} + \underline{r})}{\partial r_j} = 0 \tag{3-7}$$

and, with (3-5) this gives that

$$\kappa_j \phi_{ij} = 0 \tag{3-8}$$

or, on use of the equivalent expression to (3-5) for $\overline{u_i(\underline{x} - \underline{r}) u_j(\underline{x})}$,

$$\kappa_i \phi_{ij} = 0 \ . \tag{3-9}$$

As a consequence of the definition, it can be shown (see Appendix B) that the energy spectrum tensor ϕ_{ij} is completely defined by its diagonal elements. Thus, to describe ϕ''_{ij} , only $\{\phi''_{ii}\}_{i=1}^{3}$ need be given and those only in terms of $\{\phi'_{jj}\}_{j=1}^{3}$, if it is assumed that these are already known. To find such a relation, take the curl of equation (2-2), which gives

$$-\nabla^2 u''_r(\underline{x}) = \sigma \varepsilon_{rsi} \varepsilon_{ipq} \frac{e_i}{e_s} \frac{\partial^2 u'_q(\underline{a})}{\partial a_p \partial a_s} \tag{3-10}$$

and, with (3-1), implies that

$$\chi^2 dz''_r(\underline{\chi}) = - \sigma \varepsilon_{rsi} \varepsilon_{ipq} \frac{e_i}{e_s} \kappa_p \kappa_s dz'_q(\underline{\kappa}) \tag{3-11}$$

where ε_{ijk} is the alternating tensor.

Expanded, this gives three equations of the form

$$\chi^2 dz''_1(\underline{\chi}) = - \sigma \left\{ dz'_1(\underline{\kappa}) \left(- \frac{e_2}{e_3} \kappa_3^2 - \frac{e_3}{e_2} \kappa_2^2 \right) + \frac{e_3}{e_2} \kappa_1 \kappa_2 dz'_2 + \frac{e_2}{e_3} \kappa_1 \kappa_3 dz'_3 \right\} \tag{3-12}$$

or, using equations (2-6) and (3-2)

$$dz''_1(\underline{\chi}) = \frac{dz'_1(\underline{\kappa})}{e_1} - \frac{\kappa_1}{\chi^2 e_1} \sum_{i=1}^{3} \frac{\kappa_i dz'_i(\underline{\kappa})}{e_i^2} \ . \tag{3-13}$$

It is worth noting here that the explicit appearance of the density ratio σ has been eliminated by use of the continuity equation (2-6). The equation is then identical to that obtained by Batchelor[1] for incompressible flow. However, the dependence upon σ is implicit in the values of e_i.

It follows from equations (3-3), (3-6) and (3-17) that the energy spectrum tensor is given by

$$\phi''_{re}(\underline{\chi}) = \frac{\sigma}{\chi^4} \varepsilon_{rsj}\varepsilon_{jpq} \frac{e_j}{e_s} \varepsilon_{\ell mv}\varepsilon_{vab} \frac{e_v}{e_m} \kappa_p\kappa_s\kappa_a\kappa_m \phi'_{qb}(\underline{\kappa}) \tag{3-14}$$

which, after some manipulation, and use of the relations given in Appendix B, yields three equations, for the diagonal elements of ϕ_{ij}, of the form

$$\begin{aligned}\phi''_{11}(\underline{\chi}) = \frac{1}{\sigma}\Bigg\{&\frac{\phi'_{11}}{e_1^2} + \frac{\kappa_1^2\phi'_{11}}{e_1^2\chi^4}\left[\left(\frac{\kappa_2^2}{e_2^2} + \frac{\kappa_3^2}{e_3^2}\right)\left(\frac{1}{e_2^2} + \frac{1}{e_3^2} - \frac{2}{e_1^2}\right) + \kappa_1^2\left(\sigma^2 e_1^2 - \frac{1}{e_1^4}\right)\right] \\ &+ \sigma^2\kappa_2^2\frac{\left(e_3^2 - e_2^2\right)}{e_2^2\chi^4}\phi'_{22}\left[\kappa_1^2 + \kappa_2^2 + \kappa_3^2\frac{e_2^2}{e_3^2}\right] + \sigma^2\kappa_3^2\frac{\left(e_2^2 - e_3^2\right)}{e_3^2\chi^4}\phi'_{33}\left[\kappa_1^2 + \frac{e_3^2}{e_2^2}\kappa_2^2 + \kappa_3^2\right]\Bigg\}.\end{aligned} \qquad \text{......(3-15)}$$

It is now possible to look at the ratios of the longitudinal and lateral components of the turbulence intensity, given by μ and ν,

where

$$\frac{\overline{u}_1''^2}{\overline{u}_1'^2} = \mu = \frac{\int_\chi \phi''_{11} d\chi_1 d\chi_2 d\chi_3}{\int_\kappa \phi'_{11} d\kappa_1 d\kappa_2 d\kappa_3} \tag{3-16}$$

$$\frac{\overline{u}_2''^2 + \overline{u}_3''^2}{\overline{u}_2'^2 + \overline{u}_3'^2} = \nu = \frac{\int_\chi (\phi''_{22} + \phi''_{33}) d\chi_1 d\chi_2 d\chi_3}{\int_\kappa (\phi'_{22} + \phi'_{33}) d\kappa_1 d\kappa_2 d\kappa_3} \tag{3-17}$$

where x_1 is chosen as the longitudinal coordinate.

4 APPLICATION TO A WIND-TUNNEL CONTRACTION CONE

Consider the problem of a symmetric wind-tunnel contraction of ratio C, as in Fig 1, and say a cylindrical element of fluid has radius r and length L. The ratio of the cross-sectional area of the cylinder to that of the tunnel will not change through the contraction cone, so that the radius of the element after distortion will be $r/\sqrt{C}$ and, since mass is conserved, its length will become LC/σ. From this argument, it follows that the values of e_i are given by

$$e_1 = \frac{C}{\sigma} , \quad e_2 = e_3 = \frac{1}{\sqrt{C}} . \tag{4-1}$$

A simple way to model the turbulence upstream of the contraction is to assume that it is isotropic and write, following Batchelor[1],

$$\phi'_{11} = \frac{E(\kappa)\left(\kappa_2^2 + \kappa_3^2\right)}{4\pi\kappa^4} , \quad \phi'_{22} = \frac{E(\kappa)\left(\kappa_1^2 + \kappa_3^2\right)}{4\pi\kappa^4} , \quad \phi'_{33} = \frac{E(\kappa)\left(\kappa_1^2 + \kappa_2^2\right)}{4\pi\kappa^4} . \tag{4-2}$$

Substitution of these values and the values of e_i into equation (3-15) and the similar expressions for ϕ''_{22} and ϕ''_{33} leads, after further manipulation to the equations

$$\phi''_{11} = \frac{\sigma E(\kappa)\left(\kappa_2^2 + \kappa_3^2\right)}{4\pi\chi^4} , \tag{4-3}$$

$$\phi''_{22} + \phi''_{33} = \frac{1}{\sigma}\,\frac{E(\kappa)}{4\pi\kappa^2\chi^4}\left[C\chi^4 + \frac{\kappa_1^2\kappa^2\sigma^4}{C^3}\right] , \tag{4-4}$$

which, in equations (3-16) and (3-17), yield

$$\mu = \frac{\sigma^2 \int_0^\pi \left(\frac{\sin^3\theta}{f(\theta)}\right) d\theta}{\int_0^\pi \sin^3\theta \, d\theta} \tag{4-5}$$

$$\nu = \frac{\int_0^\pi C \sin\theta\left(1 + \frac{\sigma^4\cos^2\theta}{C^4 f(\theta)}\right) d\theta}{\int_0^\pi \sin\theta(1 + \cos^2\theta)\, d\theta} \tag{4-6}$$

where $\left(\kappa_2^2 + \kappa_3^2\right)^{\frac{1}{2}} = \kappa \sin\theta$, $\kappa_1 = \kappa\cos\theta$, $\int_\kappa d\kappa_1 d\kappa_2 d\kappa_3 = \int_0^\pi \int_0^\infty 2\pi\kappa^2 \sin\theta \, d\kappa d\theta$,

and $f(\theta) = \left(\frac{\sigma^2}{C^2}\cos^2\theta + \sin^2\theta\right)^2$.

If a new parameter, α , is defined by

$$\alpha^2 = 1 - \frac{\sigma^2}{C^3} \tag{4-7}$$

equations (4-5) and (4-6) reduce to

$$\mu = \frac{3\sigma^2}{4C^2}\left[\frac{1+\alpha^2}{2\alpha^3}\log_e\left(\frac{1+\alpha}{1-\alpha}\right) - \frac{1}{\alpha^2}\right] \tag{4-8}$$

$$\nu = \frac{3C}{4}\left[1 + \frac{\sigma^2}{C^3}\left(\frac{1}{2\alpha^2} - \frac{1-\alpha^2}{4\alpha^3}\log_e\left(\frac{1+\alpha}{1-\alpha}\right)\right)\right] , \tag{4-9}$$

the total turbulence intensity is changed, through the contraction, by a factor $\overline{u''^2_{ii}}/\overline{u'^2_{ii}}$, which, because the turbulence is initially isotropic, is given by

$$\tfrac{1}{3}(\mu + 2\nu) = \frac{C}{2} + \frac{\sigma^2}{4C^2\alpha}\log_e\left(\frac{1+\alpha}{1-\alpha}\right) . \tag{4-10}$$

The effect of compressibility through the distortion can now be seen. When $\sigma = 1$ and the flow is incompressible, the expressions (4-8) to (4-10) reduce, as expected, to those given by Batchelor[1]. Explicitly, the dependence of $\mu + 2\nu$ on σ (< 1 if $M \neq 0$) is quadratic, but there is also an implicit dependence on σ via α . Fig 2 shows σ, μ and ν plotted against a range of values of the contraction ratio, taking the outlet Mach number to be unity. Clearly, the density ratio drops quickly and, for $C > 3$, is close to the ratio of outlet density to stagnation density, which is 0.634. The value of α , which is zero at C = 1, then tends to unity as the term σ^2/C^3 becomes small. Correspondingly, μ becomes small, showing the strong effect of the contraction cone on the longitudinal turbulence intensity, while ν approximates to $3C/4$ which shows that the lateral intensity increases through the contraction. In equation (4-10) it can be seen that for $C \geqslant 3$ the lateral dominates the longitudinal contribution and that the total turbulence intensity increases by a factor which is approximately $C/2$, independent of the effect of compressibility.

However, the factor of real interest is that which applies to the reduction of percentage turbulence, *ie* the ratio of the square root of the turbulence intensity, a measure of the turbulent velocity fluctuations, to the mean velocity. Because mass flux is conserved through the contraction cone, the ratio of the mean velocities before and after distortion is C/σ . The percentage level of turbulence is therefore reduced by a factor F

where

$$F = \frac{\sigma}{C}\sqrt{\frac{C}{2} + \frac{\sigma^2}{4C^2\alpha}\log_e\left(\frac{1+\alpha}{1-\alpha}\right)} \tag{4-11}$$

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and compressibility again has a significant effect. For large values of C, a good approximation to F is

$$F = \sigma\sqrt{\frac{1}{2C}} \qquad (4\text{-}12)$$

that is, F is σ times the reduction obtained for incompressible flow. This shows that the contraction ratio, needed to obtain a particular reduction in percentage turbulence, decreases as the outlet Mach number increases. Fig 3 compares values of C for various percentage turbulence reductions at $M = 0$, $M = 0.5$ and $M = 1$. Further results are shown in Winter and Boyd[5].

Since the purpose of a wind-tunnel contraction is also to reduce non-uniformities in the mean flow, it is valuable to look at the effect of compressibility in this context. The wave number vector is now of the form $(0, \kappa_2, \kappa_3)$ and, if the contraction is still symmetric, equation (3-13) gives, with equation (3-4),

$$\left.\begin{aligned} dz''_1 &= \frac{\sigma}{C}\, dz'_1 \\ dz''_i &= C^{\frac{1}{2}} dz'_i \;; \end{aligned}\right\} \quad i = 2,3 \qquad (4\text{-}13)$$

which, with equations (3-6), (3-16) and (3-17) show that the percentage level of non-uniformity in the streamwise direction is reduced by a factor of σ^2/C^2 while the angle of flow to the tunnel centre line, caused by the non-uniformities, is reduced by a factor of $\sigma/C^{\frac{1}{2}}$. In both cases, compressibility improves the effect of the contraction.

5 CONCLUSIONS

(1) The theory of rapid distortion, derived by Batchelor[1], has been extended to account for the effect of compressibility.

(2) The theory has been applied to the flow through a contraction and it emerges that compressibility has only a small effect on the change of turbulence intensity, but has a significant effect on the reduction of percentage total turbulence.

(3) The effect of compressibility is favourable. The contraction ratio needed to achieve a given reduction in the level of turbulence or in the level of non-uniformity, decreases as the outlet Mach number increases.

Appendix A

DERIVATION OF EQUATION (2-5)

It is assumed that the flow before or after distortion has uniform mean density ρ and uniform mean velocity U in the longitudinal direction x_1. It is further assumed that the turbulent fluctuations in this region are, to first order in $O(|\underline{u}'|/U)$, simply convected by the fluid, so that

$$\frac{d\underline{u}'}{dt} = 0 = \frac{d\rho'}{dt} \quad . \tag{A-1}$$

In fact, this assumes that there is no mechanism present, to first order, by which turbulence is created or destroyed and implies that the fluctuation in pressure, p', is $O(|\underline{u}'|^2/U^2)$.

Now, the principle of continuity of mass gives

$$\frac{\partial}{\partial t}(\rho + \rho') + \frac{\partial}{\partial x_1}(\rho + \rho')(U + u_1') + \frac{\partial}{\partial x_2}(\rho + \rho')u_2' + \frac{\partial}{\partial x_s}(\rho + \rho')u_3' = 0 \quad , \tag{A-2}$$

or, to first order in $|\underline{u}'|/U$, since ρ and U are constant,

$$\frac{\partial \rho'}{\partial t} + U\frac{\partial \rho'}{\partial x_1} + \rho\left(\frac{\partial u_1'}{\partial x_1} + \frac{\partial u_2'}{\partial x_2} + \frac{\partial u_3'}{\partial x_3}\right) = 0 \quad . \tag{A-3}$$

Combination of this equation with equation (A-1) gives the required result, equation (2-5) or

$$\rho\left(\frac{\partial u_1'}{\partial x_1} + \frac{\partial u_2'}{\partial x_2} + \frac{\partial u_3'}{\partial x_3}\right) = 0 \tag{A-4}$$

which states that the turbulent velocity fluctuations are incompressible to first order.

Appendix B

THE FORM OF THE ENERGY SPECTRUM TENSOR

The energy spectrum tensor ϕ_{ij}, as defined in equation (3-6), is clearly hermitian, so its diagonal elements are all real, while its off-diagonal elements are related by three expressions like

$$\phi_{ij} = \phi^*_{ji} \qquad j \neq i . \tag{B-1}$$

There are, therefore, only nine independent, non-zero quantities in the set $\{re\phi_{ij}, im\phi_{ij}\}^3_{i,j=1}$. Equations (3-8) and (3-9) together with relations, which follow from equation (3-6), of the form

$$\phi_{ii}\phi_{jj} = \phi_{ij}\phi_{ji} \; ; \qquad i \neq j \text{ , both fixed} \tag{B-2}$$

apparently give nine equations relating the members of this set. However, on closer inspection, it emerges that only six of these nine are independent and these are: from equations (3-8) and (3-9), with (B-1)

$$\left.\begin{aligned} \kappa_1 re\phi_{11} + \kappa_2 re\phi_{12} + \kappa_3 re\phi_{13} &= 0 \\ \kappa_1 re\phi_{12} + \kappa_2 re\phi_{22} + \kappa_3 re\phi_{23} &= 0 \\ \kappa_1 re\phi_{13} + \kappa_2 re\phi_{23} + \kappa_3 re\phi_{33} &= 0 \\ \kappa_2 im\phi_{12} + \kappa_3 im\phi_{13} &= 0 \\ -\kappa_1 im\phi_{12} + \kappa_3 im\phi_{23} &= 0 \end{aligned}\right\} \tag{B-3}$$

and from equation (B-2) with (B-1)

$$\phi_{11}\phi_{22} = (re\phi_{12})^2 + (im\phi_{12})^2 \quad . \tag{B-4}$$

Three further equations only are needed now to define ϕ_{ij} completely and are provided by equations giving the diagonal elements, ϕ_{11}, ϕ_{22} and ϕ_{33}. Thus, the energy spectrum tensor ϕ_{ij} is completely determined by its diagonal elements.

LIST OF SYMBOLS

$\underline{a}$	position before distortion = (a_1, a_2, a_3)
C	contraction ratio
D	length scale of distortion
$E(\kappa)$	energy function (equation (4-2))
F	factor of reduction of percentage turbulence through distortion
i	$\sqrt{-1}$
ℓ	turbulent length scale
L	length of cylindrical element of fluid before distortion
M	outlet Mach number
p	pressure
r	radius of cylindrical element of fluid before distortion
$\underline{r}$	position vector = (r_1, r_2, r_3)
S_{ij}	distortion tensor
U	mean velocity vector = (U_1, U_2, U_3)
$\underline{u}'$, $\underline{u}''$	turbulent velocity fluctuations
$\underline{x}$	position after distortion = (x_1, x_2, x_3)
$d\underline{z}$	measure of the strength of turbulent velocity fluctuations = (dz_1, dz_2, dz_3)
$d\underline{z}^*$	complex conjugate of $d\underline{z}$
α	defined in equation (4-7) = $\sqrt{1 - (\sigma^2/C^3)}$
ε_{ijk}	the alternating tensor = 0 $i = j$, $j = k$ or $k = i$ = 1 ijk are cyclic = −1 ijk are anti-cyclic
$\underline{\kappa}$	wave number vector before distortion = $(\kappa_1, \kappa_2, \kappa_3)$
μ	ratio of longitudinal components of turbulence intensities before and after distortion
ν	ratio of lateral components of turbulence intensities before and after distortion
ρ	mean density
ρ'	turbulent fluctuation of density
σ	ratio of mean densities before and after contraction
ϕ_{ij}	energy spectrum tensor
χ	wave number vector after distortion = (χ_1, χ_2, χ_3)
$\underline{\omega}$	vorticity

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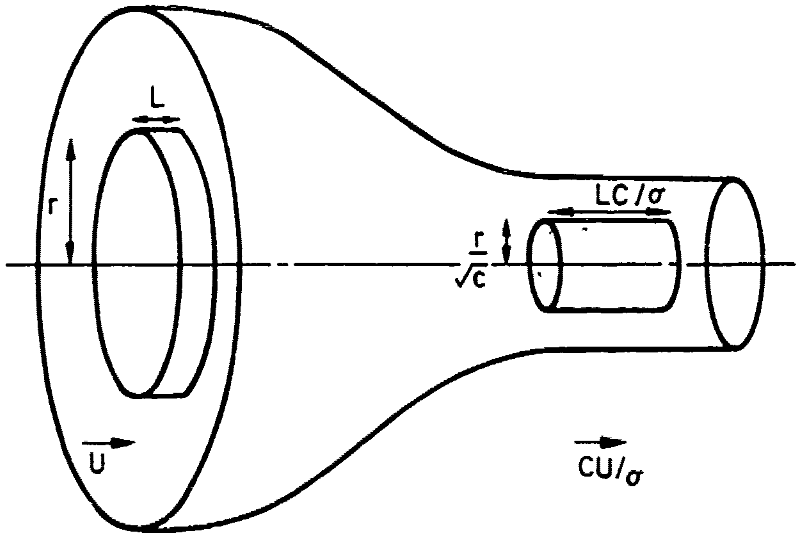

Fig 1 Distortion of fluid through contraction

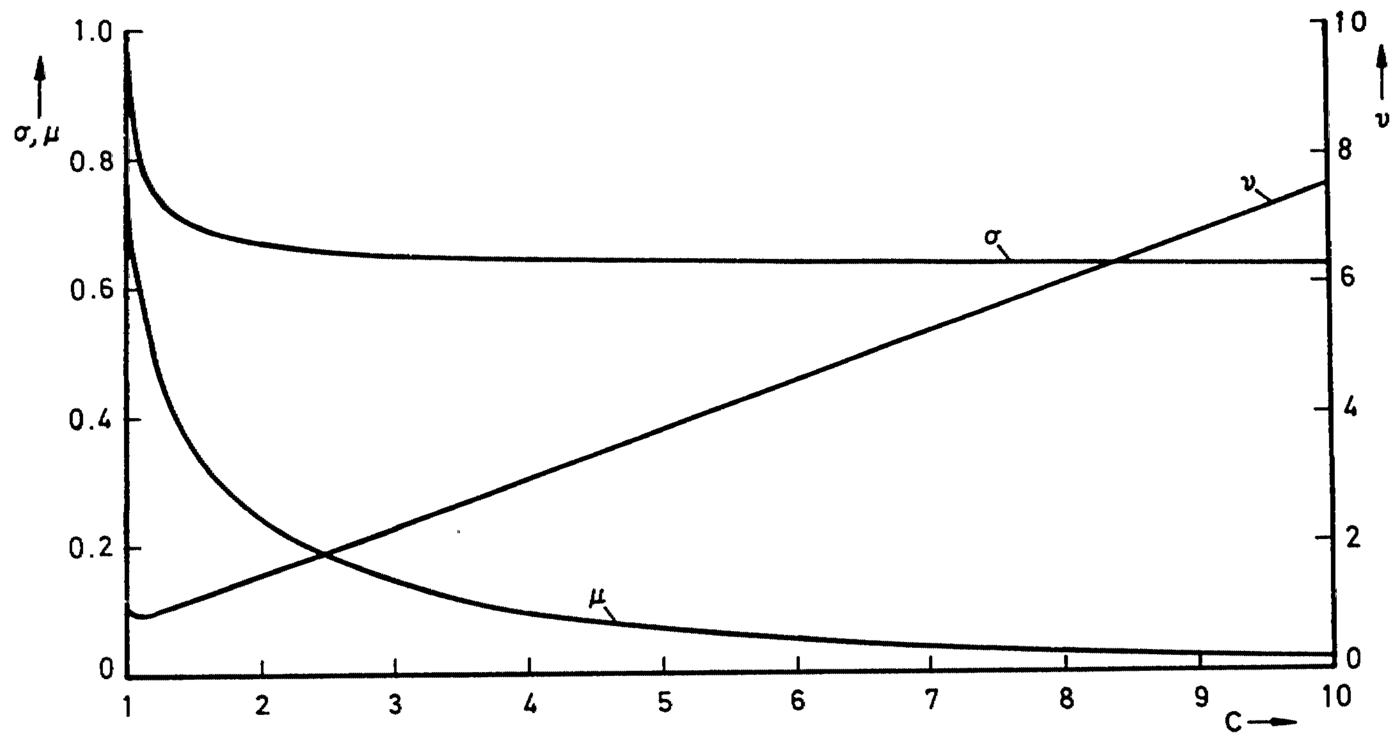

Fig 2 Dependence of density and of longitudinal and lateral turbulence ratios on contraction ratio for M = 1

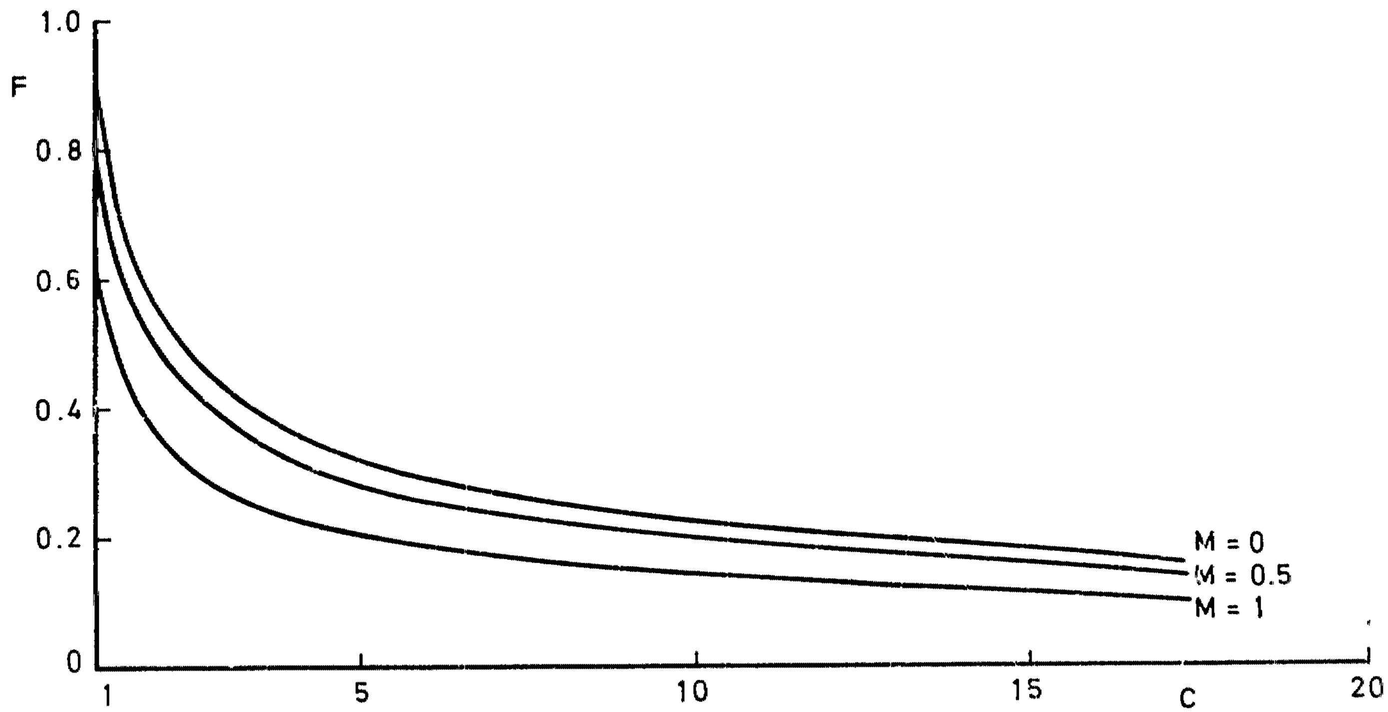

Fig 3 Effect of Mach number on turbulence reduction through contraction, ratio C

REPORT DOCUMENTATION PAGE

Overall security classification of this page

UNLIMITED

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17. Abstract

A mathematical theory of the effect of rapid distortion on turbulence level, originally developed by Batchelor for incompressible flow, has been extended to account for compressibility. Applied to a wind-tunnel contraction cone, it shows that the effectiveness of the contraction in reducing the turbulence level and non-uniformities in the mean flow is enhanced as the outlet Mach number is increased.

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